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ELBOW: A FORTRAN PROGRAM FOR THE
CALCULATION OF STRESSES, STRESS INDICES,
AND FLEXIBILITY FACTORS FOR ELBOWS
AND CURVED PIPE

W. G. Dodge
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MASTER

OAK RIDGE NATIONAL LABORATORY

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FOREWORD

The work reported here was done at Oak Ridge National Laboratory (ORNL) as part of the ORNL Piping Program — Design Criteria for Piping, Pumps, and Valves — under the direction of W. L. Greenstreet, associate head, Solid Mechanics Department, and S. E. Moore, program coordinator. The program is funded by the U.S. Atomic Energy Commission (USAEC) under the Nuclear Safety Research and Development Program as the AEC-supported portion of an AEC-industry cooperative effort for the development of design criteria for nuclear power plant piping components, pumps, and valves. This joint effort is coordinated by the Pressure Vessel Research Committee (PVRC) of the Welding Research Council under the Subcommittee to Develop Stress Indices for Piping, Pumps, and Valves. J. L. Mershon, AEC Division of Reactor Development and Technology, is the USAEC cognizant engineer.

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ABSTRACT

A computer program is described for calculating the stresses, stress indices, and flexibility factors for in-plane and out-of-plane bending of elbows and curved pipe subjected to internal pressure. The results are obtained using a series solution as discussed by the authors in a separate report. Included is a listing of the computer program, a description of the input data, and an example of the output.

Keywords: curved pipe, pipe elbows, piping design, stress indices, flexibility factors, stress analysis, piping code, ASME Boiler and Pressure Vessel Section III, nuclear piping, PVRC.

INTRODUCTION

The determination of the stresses and flexibility for elbows and curved pipe is a significant part of piping design and analysis. When subjected to applied bending moments, the cross section of the elbow is distorted, producing greater flexibility and higher stresses than predicted by simple beam theory. The first rational explanation of this phenomenon was presented by von Karman¹ in 1911. Since that time many analytical and experimental investigations have been published.

We recently completed a study² to develop flexibility factors and stress indices for elbows and curved pipe for use with the simplified analysis procedures of current design codes. After an extensive review of currently available theoretical analyses, a modified version of the analytical method by Rodabaugh and George,³ which considered both in-plane and out-of-plane bending as well as the influence of internal pressure, was selected as the most appropriate. This analysis is a generalization of the work done by von Karman. The modifications to the analysis method

of Rodabaugh and George include a generalization of the "correction for transverse compression" recommended by Gross.⁴ The computer program ELBØW was written to implement the analysis procedure developed in the study.

The purpose of this report is to document the computer program ELBØW so that it may be generally available to the public. For a more complete discussion of the theoretical aspects see Ref. 2.

THE COMPUTER PROGRAM

The computer program ELBØW was written for the IBM 360 computer using the FORTRAN IV language; a listing is presented in the next section. The program requires less than 80K bytes of storage on the ORNL IBM 360/91 computer and executes at the rate of approximately 0.5 sec of CPU (central processing unit) time for each case. An example of the output from the program is given in Appendix A. The coordinate system used is shown in Fig. 1.

Program Input

Normally three input cards are required for each case, and multiple cases may be submitted. The first of these cards, the OPTION CARD, is used to specify which of the various execution options are selected. The options are requested by typing the underlined characters in the columns specified. If the GROUP option is used then all following cases will be treated with the same options and no additional option cards will be expected. The format for this card is:

OPTION CARD - (5(A8,2X))

- Col. (01-05) - GROUP - If the same options are desired for each case. Option cards are then not required for remaining cases.
- Col. (11-15) - TITLE - If the title card is to be included.
- Col. (21-27) - PUNCHED - If punched output of the stress index and flexibility factor is desired.

Fig. 1. Coordinate system and dimensions for analytical model of elbows.

Col. (31-38) - STRESSES - If printed output of the stresses is desired.

Col. (41-47) - PUNCHED - If punched output of the stresses is desired.

The second card, the TITLE CARD, is included only if the title option is specified. The alphanumeric data contained on this card is printed as a heading for the output. The format for this card is:

TITLE CARD - (20A4)

Col. (01-80) - The alphanumeric title.

The analytical results are in terms of a series solution which depends upon three dimensionless parameters:

$$\lambda = \frac{tR}{r^2 \sqrt{1 - \nu^2}},$$

$$\psi = \frac{pR}{Ert},$$

and

$$\gamma = R/r,$$

where

- t = thickness of the pipe wall (in.),
- R = bend radius (in.) of pipe centerline
- r = pipe radius (in.),
- p = internal pressure (psi),
- E = modulus of elasticity (psi),
- ν = Poisson's ratio.

The third card, called the DATA CARD, contains the numeric values of these parameters. The format for this card is:

DATA CARD - (E10.4, I5, 34X, I1)

Col. (01-10) - GAMMA - γ .

Col. (11-20) - LAMBDA - λ .

Col. (21-30) - PSI - ψ .

Col. (31-40) - NU - ν .

Col. (41-45) - DELPHI - The fixed point value of the increment in the angle ϕ (Fig. 1) between the points where stresses are to be calculated.

Col. (80) - ISTOP - Punch the number 1 if this is the last case of this job.

A brief description of the program input is included at the beginning of the program listing.

Program Operation

The computer program was written to evaluate as many terms in the series solution as needed to guarantee convergence. Previous authors had shown that the number of terms required for convergence increased with decreasing values of the elbow bend parameter λ , usually requiring between three and six terms for λ greater than about 0.1. The computer program was therefore written to compute up to 20 terms if necessary. Starting with a two-term series, the number of terms used in the series is increased until the flexibility factor has converged to within 1.0×10^{-8} . This should be sufficient to yield at least six-digit accuracy in the calculated stresses. For λ values on the order of 0.01, 15 or 16 terms were sufficient to achieve adequate convergence, whereas only 5 or 6 terms were required for $\lambda \approx 0.1$.

Truncating the series after N terms, a set of N linear equations for N + 1 unknown values of d_n is obtained (Eqs. A-8, Ref. 2):

$$0 = 3 + (5 + 6\lambda^2 + 24\psi)d_1 - \frac{5}{2} d_2 ,$$

$$0 = -\frac{5}{2} d_1 + (17 + 600\lambda^2 + 480\psi)d_2 - \frac{21}{2} d_3 ,$$

⋮
⋮
⋮

$$\begin{aligned}
0 = & -\frac{1}{2} (2n-3)(2n+1) d_{n-1} \\
& + \left[(4n^2 + 1) + (8n^3 - 2n)^2 \frac{\lambda^2}{6} + 8n^2 (4n^2 - 1) \psi \right] d_n \\
& - \frac{1}{2} (2n-1)(2n+3) d_{n+1} .
\end{aligned}$$

With $d_{N+1} = 0$, the values of $d_1, d_2, \dots, d_N$ are the solution to this set of equations.

After the series coefficients d_n have been determined, the flexibility factor is obtained using the equation (Eq. A-12, Ref. 2):

$$k_p = \frac{1}{1 + \frac{3}{2} d_1} .$$

The nondimensionalized stresses, normalized to the external moment over the section modulus, M/Z , where $Z = \pi r^2 t$, are calculated at intervals of DELPHI degrees around the circumference of the elbow. For in-plane bending the normalized longitudinal and circumferential stresses σ_ℓ^i and σ_c^i are given by the equations (Eqs. A-13 and E-7, Ref. 2):

$$\begin{aligned}
\sigma_\ell^i = & F \left[D \sin \varnothing + \frac{1}{2} \sum_{n=1}^N A_n \sin (2n+1)\varnothing \pm \frac{\nu\lambda}{2} \sum_{n=1}^N B_n \cos 2n\varnothing \right] , \\
\sigma_c^i = & F \left\{ -\frac{1}{\gamma} \left[D \cos \varnothing + \frac{1}{2} \sum_{n=1}^N \frac{A_n}{(2n+1)} \cos (2n+1)\varnothing \right] \cos \varnothing \right. \\
& \left. \pm \frac{\lambda}{2} \sum_{n=1}^N B_n \cos 2n\varnothing \right\} .
\end{aligned}$$

The normalized longitudinal and circumferential stresses σ_ℓ^o and σ_c^o due to an out-of-plane bending moment acting on the cross section of the elbow are given by (Eqs. A-15 and B-8, Ref. 2):

$$\sigma_\ell^o = F \left[D \cos \varnothing + \frac{1}{2} \sum_{n=1}^N A_n \cos (2n+1)\varnothing \pm \frac{\nu\lambda}{2} \sum_{n=1}^N -B_n \sin 2n\varnothing \right] ,$$

$$\sigma_c^o = F \left\{ \frac{1}{\gamma} \left[D \sin \varnothing + \frac{1}{2} \sum_{n=1}^N \frac{A_n}{(2n+1)} \sin (2n+1)\varnothing \right] \sin \varnothing \right. \\ \left. \pm \frac{\lambda}{2} \sum_{n=1}^N B_n \sin 2n\varnothing \right\},$$

where (Eq. A-17, Ref. 2)

$$F = \frac{k_p}{1 - \nu^2},$$

$$D = 1 + \frac{3}{2} d_1,$$

$$A_n = (1 - 2n)d_n + (2n + 3)d_{n+1},$$

$$B_n = (2n - 8n^3)d_n.$$

The normalized shear stress τ due to a torsional moment M_t acting on the cross section of the elbow is assumed to be constant as a function of $\varnothing$,

$$\tau = \frac{1}{2}.$$

The maximum normalized stress intensity $\bar{S}/(M/Z)$ for any combination of in-plane, out-of-plane, and torsional moments acting on the cross section of the elbow is determined and the corresponding stress index calculated following the procedure described in the section "Stress-Index Development" of Ref. 2. The flexibility factor and stress index are printed, and any additional output requested on the OPTION CARD is provided. A test on the variable ISTOP determines if input data cards for another case are to be read; otherwise, execution is stopped.

COMPUTER PROGRAM LISTING

```

C*****ELBOW 10
C   THIS PROGRAM CALCULATES THE STRESSES AND CORRESPONDING STRESS ELBOW 20
C   INDEX AND THE FLEXIBILITY FACTOR FOR A 'PERFECT' ELBOW ELBOW 30
C   USING THE THEORY GIVEN IN THE PAPER 'EFFECT OF INTERNAL ELBOW 40
C   PRESSURE ON FLEXIBILITY AND STRESS-INTENSIFICATION FACTORS ELBOW 50
C   OF CURVED PIPE AND WELDING ELBOWS' BY E.C. RODABAUGH AND ELBOW 60
C   H.H. GEORGE, TRANS. ASME 1957. THE CORRECTION FOR TRASVERSE ELBOW 70
C   COMPRESSION INTRODUCE BY N. GROSS IN THE PAPER 'EXPERIMENTS ELBOW 80
C   ON SHORT RADIUS PIPE-BENDS', INST. MECH. ENGR., 1952-3 IS ELBOW 90
C   ALSO INCLUDED. FOR A CONELETE DISCUSSION SEE ORNL-TM-3658. ELBO 100
C   ELBO 110
C*****ELBO 120
C   *** INPUT *** ELBO 130
C   ELBO 140
C   FIRST CARD - (OPTION CARD) - (A8,2X) ELBO 150
C   COL. (1-5) - GROUP - IF THE SAME OPTIONS ARE DESIRED FOR ELBO 160
C   EACH CASE (OPTION CARDS THEN ARE NOT ELBO 170
C   REQUIRED FOR REMAINING CASES). ELBO 180
C   COL. (11-15) - TITLE - IF TITLE CARD IS INCLUDED. ELBO 190
C   COL. (21-25) - PUNCHED - IF PUNCH OUTPUT OF STRESS INDEX AND ELBO 200
C   FLEXIBILITY FACTOR IS DESIRED. ELBO 210
C   COL. (31-38) - STRESSES - IF PRINT OUTPUT OF THE STRESSES ELBO 220
C   IS DESIRED. ELBO 230
C   COL. (41-45) - PUNCHED - IF PUNCH OUTPUT OF THE STRESSES IS ELBO 240
C   DESIRED. ELBO 250
C   ELBO 260
C   TITLE CARD - TO BE INCLUDED ONLY IF TITLE OPTION IS SPECIFIED. ELBO 270
C   (ONE PER CASE IF SPECIFIED) ELBO 280
C   COL. (1-80) - ALPHANUMERIC TITLE ELBO 290
C   ELBO 300
C   DATA CARD - (ONE PER CASE) - (4E10.4,I5,34X,I1) ELBO 310
C   COL. (1-10) - GAMMA - (BEND RADIUS)/(PIPE RADIUS) ELBO 320
C   COL. (11-20) - LAMEDA - (PIPE THICKNESS)*(GAMMA) DIVIDED BY ELBO 330
C   (PIPE RADIUS)*SQRT(1.-NU*NU) ELBO 340
C   COL. (21-30) - PSI - (PRESSURE)*(GAMMA)*(BEND RADIUS) DIVIDED ELBO 350
C   BY (MODULUS OF ELASTICITY)*(PIPE THICK) ELBO 360
C   COL. (31-40) - NU - POISSON'S RATIO ELBO 370
C   COL. (41-45) - DELPHI - ANGLE IN DEGRESS (INTEGER) BETWEEN ELBO 380
C   POINTS WHERE STRESSES ARE CALCULATED ELBO 390
C   COL. (80) - ISTOP - PUNCH THE NO. 1 IF THIS IS THE LAST ELBO 400
C   CASE OF THIS JOB ELBO 410
C*****ELBO 420
C   ELBO 430
C   INTEGER DELPHI,PHI,THETA,BETA,ETA ELBO 440
C   REAL LAMBDA,NU,MAXSI ELBO 450
C   DOUBLE PRECISION TEST,GROUP,TITLE,PUNCE,STRESS,DATE,SIO(2) ELBO 460
C   LOGICAL FLAG ELBO 470
C   DIMENSION TEST(5),ATITLE(20),SCII(361),SCIO(361),SLII(361), ELBO 480
C   1 SLIO(361),WI(361),SCOI(361),SCOO(361),SLOI(361), ELBO 481
C   2 SLOO(361),WO(361) ELBO 482
C   DATA GROUP,TITLE,STRESS,PUNCE/8HGROUP ,8HTITLE ,8HSTRESSES, ELBO 490
C   1 8HPUNCHED / ELBO 491
C   DATA ATITLE/20*4H /,SIO(1),SIO(2)/8H INSIDE,8H OUTSIDE/ ELBO 500
C   CALL IDAY(DATE) ELBO 510
C   ICALL = 1 ELBO 520
C   *****ELBO 530
C   READ INPUT ELBO 540
C   *****ELBO 550
C   10 READ 1000, (TEST(I),I=1,5) ELBO 560
C   1000 FORMAT(5(A8,2X)) ELBO 570

```

```

20 PRINT 1010,DATE ELBO 580
1010 FORMAT(1H1,100X,11HJOB RUN ON ,A8) ELBO 590
      IT1 = ICLOCK(0) ELBO 600
      IF(TEST(2).NE.TITLE) GO TO 30 ELBO 610
      READ 1020, (ATITLE(I),I=1,20) ELBO 620
1020 FORMAT(20A4) ELBO 630
      PRINT 1030, (ATITLE(I),I=1,20) ELBO 640
1030 FORMAT(1H0,27X,20A4/1H-,16X,24H*** BEND PARAMETERS *** ) ELBO 650
30 READ 1040, GAMMA,LAMBDA,PSI,NU,DELPHI,ISTOP ELBO 660
1040 FORMAT(4E10.4,I5,33X,I2) ELBO 670
      IF(NU.EQ.0.) NU = .28 ELBO 680
      IF(DELPHI.EQ.0) DELPHI = 3 ELBO 690
      PRINT 1050, GAMMA,LAMBDA,PSI,NU ELBO 700
1050 FORMAT(1H0,24X,7HGAMMA =,F6.3/1H0,23X,8HLAMBDA =,F8.5/ ELBO 710
      1 1H0,26X,5HPSI =,F8.5/1H0,15X,15HPOISSON RATIO =,F5.2) ELBO 711
      PRINT 1060 ELBO 720
1060 FORMAT(1H-,15X,25H*** OPTIONS REQUESTED *** ) ELBO 730
      IF(TEST(3).EQ.PUNCE) PRINT 1070 ELBO 740
1070 FORMAT(1H0,16X,44HPUNCH OUTPUT - FLEXIBILITY FACTOR AND STRESS, ELBO 750
      1 6H INDEX ) ELBO 751
      IF(TEST(4).EQ.STRESS) PRINT 1080 ELBO 760
1080 FORMAT(1H0,16X,23HPUNCH OUTPUT - STRESSES ) ELBO 770
      IF(TEST(5).EQ.PUNCE) PRINT 1090 ELBO 780
1090 FORMAT(1H0,16X,23HPUNCH OUTPUT - STRESSES ) ELBO 790
C ***** ELBO 800
C SOLVE FOR STRESSES AND FLEXIBILITY ELBO 810
C ***** ELBO 820
      FLAG = .TRUE. ELBO 830
      CALL SOLVE(GAMMA,LAMBDA,PSI,NU,DELPHI,FLAG,SLIO,SLII,SCIO,SCII, ELBO 840
      1 SLOO,SLOI,SCOO,SCOI,WI,WO,FLEX) ELBO 841
      IF(FLAG.EQ..FALSE.) GO TO 110 ELBO 850
C ***** ELBO 860
C CALCULATE STRESS INDEX ELBO 870
C ***** ELBO 880
      CALL SINDEX(SLIO,SLII,SCIO,SCII,SLOO,SLOI,SCOO,SCOI,DELPHI,MAXSI, ELBO 890
      1 THETA,BETA,ETA,IO,ICALL) ELBO 891
C ***** ELBO 900
C OUTPUT THE RESULTS ELBO 910
C ***** ELBO 920
      PRINT 1100, FLEX,MAXSI,SIO(IO),THETA ELBO 930
1100 FORMAT(1H-/61X,15H*** RESULTS *** /1H0,16X,18HFLEXIBILITY FACTOR, ELBO 940
      1 2H =,F7.3/1H0,17X,19HMAX. STRESS INDEX =,F7.3, ELBO 941
      2 16H OCCURING ON THE,A8,21H OF THE WALL AT PHI =,I4,5H DEG., ELBO 942
      3 9H DUE TO A) ELBO 943
      PRINT 1110, ETA,BETA ELBO 950
1110 FORMAT(1H0,44X,33HMOMENT VECTOR FORMING AN ANGLE OF,I3,6H DEG. , ELBO 950
      1 19HWITH THE TRANSVERSE/1H0,44X,16HPLANE AND WHOSE , ELBO 961
      2 38HPROJECTION ON THIS PLANE LIES AT PHI =,I3,2H ./1H0,15X, ELBO 962
      3 10H*****/16X,34HPHI IS THE ANGLE AROUND THE CROSS , ELBO 963
      4 53HSECTION (C.C. POSITIVE) SUCH THAT -90 DEG. IS TOWARD , ELBO 964
      5 10HTHE CENTER/26X,17HOF THE TUBE BEND. ) ELBO 965
      IF(TEST(4).NE.STRESS)GO TO 90 ELBO 970
C ***** ELBO 980
C OUTPUT STRESSES FOR IN-PLANE BENDING ELBO 990
C ***** ELB 1000
      PRINT 1120 ELB 1010
1120 FORMAT(1H1,51X,32H*** STRESSES AND DEFLECTIONS *** /57X, ELB 1020
      1 21H(NONDIMENSIONAL FORM)/1H0) ELB 1021
      PHI = 1 ELB 1030
      LINE = 12 ELB 1040

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      LOOP = 1
40  PRINT 1130
1130 FORMAT (1H+,59X,16HIN-PLANE BENDING )
      PRINT 1140
1140 FORMAT (1H0,30X,15HCIRCUMFERENTIAL,26X,12HLONGITUDINAL,19X,
1      11H WALL DEF. /19X,3HPhi,5X,6HINSIDE,11X,7HOUTSIDE,15X,
2      6HINSIDE,11X,7HOUTSIDE /1H )
      LINE = LINE + 5
50  NPH = PHI-1
      PRINT 1150, NPH,SCII (PHI), SCIO (PHI), SLII (PHI), SLIO (PHI), WI (PHI)
1150 FORMAT (19X,I3,4X,2 (2 (P7.3,10X),5X),P7.3)
      LINE = LINE + 1
      PHI = PHI + DELPHI
      IF (PHI.GT.360) GO TO 60
      IF (LINE.LT.59) GO TO 50
      IF (LOOP.EQ.1) PRINT 1160
1160 FORMAT (1H0,15X,10H***** /16X,24HTO OBTAIN TRUE STRESSES ,
1      50HMULTIPLY BY THE BENDING MOMENT AT THE SECTION AND ,
2      21HDIVIDE BY THE SECTION/25X,21HMODULUS OF THE CROSS ,
3      8HSECTION./16X,36HTO OBTAIN TRUE DEFLECTIONS MULTIPLY ,
4      55HBY THE PRODUCT OF THE BEND RADIUS, THE BENDING MOMENT,
5      7HAND THE/26X,37HFLEXIBILITY FACTOR AND DIVIDE BY THE ,
6      49HPRODUCT OF THE SECTION MODULUS AND THE MODULUS OF/26X,
7      11HELASTICITY./16X,34HPhi IS THE ANGLE AROUND THE CROSS ,
8      53HSECTION (C.C. POSITIVE) SUCH THAT -90 DEG. IS TOWARD ,
9      10HTHE CENTER/26X,17HOF THE TUBE BEND. )
      LOOP = 2
      LINE = 0
      PRINT 1170
1170 FORMAT (1H1)
      GO TO 40
C *****
C OUTPUT STRESSES FOR OUT-OF-PLANE BENDING
C *****
60  LINE = 9
      LOOP = 1
      PHI = 1
      PRINT 1170
70  PRINT 1180
1180 FORMAT (1H+,57X,20HOUT-OF-PLANE BENDING )
      PRINT 1140
      LTNR = LTNR + 5
80  NPH = PHI-1
      PRINT 1150, NPH, SCOI (PHI), SCOO (PHI), SLOI (PHI), SLOO (PHI), WO (PHI)
      LINE = LINE + 1
      PHI = PHI + DELPHI
      IF (PHI.GT.360) GO TO 90
      IF (LINE.LT.59) GO TO 80
      LINE = 0
      IF (LOOP.EQ.1) PRINT 1160
      LOOP = 2
      PRINT 1170
      GO TO 70
C *****
C OUTPUT PUNCHED INFORMATION
C *****
90  IF ((TEST(3).NE.PUNCE).AND.(TEST(5).NE.PUNCE)) GO TO 110
      PUNCH 1190, GAMMA,LAMBDA,PSI,NU,FLEX,MAXSI,DELPHI,THETA,BETA,ETA
1190 FORMAT (6F10.4,4I5)
      IF (TEST(5).NE.PUNCE) GO TO 110

```

DO 100 PHI=1,361,DELPHI	ELB 1540
NPH = PHI-1	ELB 1550
100 PUNCH 1200, NPH,SCII(PHI),SCIO(PHI),SLII(PHI),SLIO(PHI),WI(PHI),	ELB 1560
1 SCOI(PHI),SCOO(PHI),SLOI(PHI),SLOO(PHI),WO(PHI)	ELB 1561
1200 FORMAT(I4,2(4F8.4,F6.3))	ELB 1570
110 IT2 = ICLOCK(0)	ELB 1580
FT = FLOAT(IT2-IT1)/100.	ELB 1590
PRINT 1210, FT	ELB 1600
1210 FORMAT(1H0,16X,18HTHIS CASE REQUIRED,F5.1,8H SECONDS)	ELB 1610
IF (ISTOP.EQ.0) GO TO 120	ELB 1620
STOP	ELB 1630
120 IF (TEST(1).EQ.GROUP) GO TO 20	ELB 1640
GO TO 10	ELB 1650
END	ELB 1660

```

SUBROUTINE SOLVE (GAMMA,LAMBDA,PSI,NU,DELPHI,FLAG,SLIO,SLII,SCIO, SOLVE 10
1 SCII,SLOO,SLOI,SCOO,SCOI,WI,WO,FLEX) SOLVE 11
C ***** SOLVE 20
C THIS PROGRAM SOLVES FOR THE STRESSES AND DEFLECTION IN PROGRAM SOLVE 30
C ELBOW SOLVE 40
C ***** SOLVE 50
INTEGER DELPHI,PHI SOLVE 60
REAL LAMBDA,NU SOLVE 70
LOGICAL FLAG SOLVE 80
DIMENSION SCII(361),SCIO(361),SLII(361),SLIO(361),SCOI(361), SOLVE 90
1 SCOO(361),SLOI(361),SLOO(361),WI(361),WO(361),D(20), SOLVE 91
2 ARRAY(20,20) SOLVE 92
DOUBLE PRECISION ATEM(20,20),PR(20),DET SOLV 100
C1NU = 1./(1.-NU*NU) SOLV 110
C3NU = .5*C1NU SOLV 120
C ***** SOLV 130
C CONSTRUCT THE EQUATIONS WHICH DETERMINE THE COEFFICIENTS OF THE SOLV 140
C SERIES REPRESENTATION SOLV 150
C ***** SOLV 160
DO 10 I=1,20 SOLV 170
DO 10 J=1,20 SOLV 180
10 ARRAY(I,J) = 0. SOLV 190
D(1) = -6. SOLV 200
ARRAY(1,1) = 10.0+12.0*LAMBDA**2+48.0*PSI SOLV 210
DO 20 N=2,20 SOLV 220
ID = N-1 SOLV 230
ARRAY(ID,N) = -1.*FLOAT((2*ID+3)*(2*ID-1)) SOLV 240
ARRAY(N,ID) = ARRAY(ID,N) SOLV 250
FSQ4 = 4*N*N SOLV 260
FSQ4M1 = FSQ4-1. SOLV 270
DEN1 = 2.*(FSQ4+1.) SOLV 280
DEN2 = FSQ4*FSQ4M1**2 SOLV 290
DEN3 = 4.*FSQ4*FSQ4M1 SOLV 300
ARRAY(N,N) = DEN1+DEN2*LAMBDA**2/3.+DEN3*PSI SOLV 310
20 D(N) = 0. SOLV 320
C ***** SOLV 330
C SOLVE FOR THE COEFFICIENTS OF A 'IK' TERM SERIES REPRESENTATION SOLV 340
C ***** SOLV 350
DO 50 IK=2,20 SOLV 360
DO 30 IX=1,IK SOLV 370
PR(IX) = D(IX) SOLV 380
DO 30 IY=1,IK SOLV 390
30 ATEM(IX,IY) = ARRAY(IX,IY) SOLV 400
DET = 0. SOLV 410
CALL MATQD(ATEM,PR,IK,1,DET,20,20,IPLAG) SOLV 420
IF(IPLAG.EQ.0) GO TO 4C SOLV 430
FLAG = .FALSE. SOLV 440
PRINT 1000 SOLV 450
1000 FORMAT(1H0,16X,46H*** ERROR - COEFFICIENT MATRIX OF SIMULTANEOUS, SOLV 460
1 22H EQUATIONS IS SINGULAR ) SOLV 461
RETURN SOLV 470
40 NT = IK SOLV 480
C ***** SOLV 490
C CHECK TO SEE IF A 'IK' TERM SERIES IS ADEQUATE SOLV 500
C ***** SOLV 510
EPT = DABS(ARRAY(IX,IK)*PR(IX)**2) SOLV 520
50 IF(EPT.LT.1.E-8) GO TO 60 SOLV 530
PRINT 1010 SOLV 540
1010 FORMAT(1H0,16X,46H*** UNABLE TO OBTAIN SUFFICIENT ACCURACY WITH , SOLV 550
1 40H20 TERM EXPANSION - RESULTS QUESTIONABLE) SOLV 551

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      SLIB = NU*SCIB                                SOL 1160
      SLOB = NU*SCOB                                SOL 1170
      SCOT = CON3*(CON5+.5*CSLOM)                   SOL 1180
      SCIT = -CON3*(CON6+.5*CSLIM)                   SOL 1190
C *****SOL 1200
C CALCULATE THE LONGITUDINAL AND CIRCUMFERENTIAL STRESSES SOL 1210
C SOL 1220
C      SLIO - LONGITUDINAL STRESS ON OUTSIDE OF THE PIPE WALL DUE SOL 1230
C           TO IN-PLANE BENDING SOL 1240
C      SCOI - CIRCUMFERENTIAL STRESS ON INSIDE OF THE PIPE WALL SOL 1250
C           DUE TO OUT-OF-PLANE BENDING SOL 1260
C           ETC. SOL 1270
C *****SOL 1280
      SLIO(NPH) = SLIM + SLIB                        SOL 1290
      SLII(NPH) = SLIM - SLIB                        SOL 1300
      SCIO(NPH) = SCIT + SCIB                        SOL 1310
      SCII(NPH) = SCIO(NPH) - 2.*SCIB                SOL 1320
      SLOO(NPH) = SLOM + SLOB                        SOL 1330
      SLOI(NPH) = SLOM - SLOB                        SOL 1340
      SCOO(NPH) = SCOT + SCOB                        SOL 1350
80  SCOI(NPH) = SCOO(NPH) - 2.*SCOB                 SOL 1360
      RETURN                                         SOL 1370
      END                                           SOL 1380

```

```

SUBROUTINE SINDE (SLIO,SLII,SCIO,SCII,SLOO,SLOI,SCOO,SCOI,DELPHI, SINDE 10
1      MAXSI,THETA,BETA,ETA,IO,ICALL) SINDE 11
C ***** SINDE 20
C THIS PROGRAM CALCULATES THE STRESS INDEX FOR ELBOW SINDE 30
C ***** SINDE 40
C INTEGER DELPHI,PHI,THETA,BETA,ETA SINDE 50
C REAL MAXSI SINDE 60
C DIMENSION SCII(361),SCIO(361),SLII(361),SLIO(361),CBETA(181), SINDE 70
1      SCOI(361),SCOO(361),SLOI(361),SLOO(361),SBETA(181), SINDE 71
2      CETA(91),SETA(91) SINDE 72
C IF(ICALL.EQ.2)GO TO 30 SINDE 80
C ***** SINDE 90
C STORE TRIGOMETRIC VALUES. SIND 100
C ***** SIND 110
DO 10 ETA=1,91 SIND 120
ET = .017453*FLOAT(ETA-1) SIND 130
CETA(ETA) = COS(ET) SIND 140
10 SETA(ETA) = SIN(ET) SIND 150
DO 20 BETA=1,181 SIND 160
BT = .017453*FLOAT(BETA-91) SIND 170
CBETA(BETA) = COS(BT) SIND 180
20 SBETA(BETA) = SIN(BT) SIND 190
ICALL = 2 SIND 200
C ***** SIND 210
C PREFORM A COURSE SEARCH, STORING THE MAXIMUM. SIND 220
C ***** SIND 230
30 IO = 1 SIND 240
40 LOOP = 1 SIND 250
KBETA = 1 SIND 260
LBETA = 181 SIND 270
KPHI = 1 SIND 280
LPHI = 361 SIND 290
MAXSI = 0. SIND 300
N = 15./FLOAT(DELPHI) + .5 SIND 310
IPHI = N*DELPHI SIND 320
IBETA = 10 SIND 330
50 NPHI = KPHI SIND 340
60 PHI = MOD(NPHI+359,360) + 1 SIND 350
IF(IO.EQ.2)GO TO 70 SIND 360
A = SLII(PHI) - SCII(PHI) SIND 370
B = SLOI(PHI) - SCOI(PHI) SIND 380
C = SLII(PHI) + SCII(PHI) SIND 390
D = SLOI(PHI) + SCOI(PHI) SIND 400
GO TO 80 SIND 410
70 A = SLIO(PHI) - SCIO(PHI) SIND 420
B = SLOO(PHI) - SCOO(PHI) SIND 430
C = SLIO(PHI) + SCIO(PHI) SIND 440
D = SLOO(PHI) + SCOO(PHI) SIND 450
80 NBETA = KBETA SIND 460
C ***** SIND 470
C EVALUATE EQUATIONS 15 AT EACH BETA. SIND 480
C ***** SIND 490
90 BETA = MOD(NBETA+179,180) + 1 SIND 500
CON1 = A*CBETA(BETA) + B*SBETA(BETA) SIND 510
CON2 = C*CBETA(BETA) + D*SBETA(BETA) SIND 520
CON1 = CON1*CON1 SIND 530
ARG = 1. + CON1*(CON1 - CON2*CON2 - 2.) SIND 540
IF(ARG.LT.0.)GO TO 100 SIND 550
ETA = ABS(57.2964*ATAN(SQRT(ARG)/CON2)) + 1.5 SIND 560
CON5 = SQRT(CON1*CETA(ETA)*CETA(ETA) + SETA(ETA)*SETA(ETA)) SIND 570

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CON6 = CON2*CETA(ETA). SIND 580
S = .5*AMAX1(ABS(CON6+CON5),ABS(CON6-CON5)) SIND 590
IF(S.LT.MAXSI)GO TO 100 SIND 600
MAXSI = S SIND 610
IOS = IO SIND 620
THETA = PHI SIND 630
MBETA = BETA SIND 640
META = ETA SIND 650
100 NBETA = NBETA + IBETA SIND 660
IF(NBETA.LE.LBETA)GO TO 90 SIND 670
C ***** SIND 680
C EVALUATE EQUATION 14. SIND 690
C ***** SIND 700
S = SQRT(A*A + B*B) SIND 710
IF(S.LT.MAXSI)GO TO 110 SIND 720
MAXSI = S SIND 730
META = 1 SIND 740
MBETA = 57.296*ATAN(B/A) + 91.5 SIND 750
IOS = IO SIND 760
C ***** SIND 770
C EVALUATE EQUATION 18. SIND 780
C ***** SIND 790
110 SI = .5*(A+C) SIND 800
SO = .5*(B+D) SIND 810
S = SQRT(SI*SI + SO*SO) SIND 820
IF(S.LT.MAXSI)GO TO 120 SIND 830
MAXSI = S SIND 840
META = 1 SIND 850
MBETA = 57.296*ATAN(SO/SI) + 91.5 SIND 860
THETA = PHI SIND 870
IOS = IO SIND 880
120 SI = .5*(C-A) SIND 890
C ***** SIND 900
C EVALUATE EQUATION 19. SIND 910
C ***** SIND 920
SO = .5*(D-B) SIND 930
S = SQRT(SI*SI + SO*SO) SIND 940
IF(S.LT.MAXSI)GO TO 130 SIND 950
MAXSI = S SIND 960
META = 1 SIND 970
MBETA = 57.296*ATAN(SO/SI) + 91.5 SIND 980
THETA = PHI SIND 990
130 NPHI = NPHI + IPHI SIN 1000
IF(NPHI.LE.LPHI)GO TO 60 SIN 1010
IF(IO.EQ.2)GO TO 140 SIN 1020
IO = 2 SIN 1030
GO TO 50 SIN 1040
140 IF(LOOP.EQ.2)GO TO 150 SIN 1050
LOOP = 2 SIN 1060
C ***** SIN 1070
C PREFORM A FINE SEARCH IN THE NEIGHBORHOOD OF INITIAL MAXIMUM. SIN 1080
C ***** SIN 1090
N = .8*FLOAT(IPHI)/FLOAT(DELPHI) + .5 SIN 1100
IPHI = N*DELPHI SIN 1110
KPHI = THETA - IPHI SIN 1120
LPHI = THETA + IPHI SIN 1130
IPHI = DELPHI SIN 1140
KBETA = MBETA - 8 SIN 1150
LBETA = MBETA + 8 SIN 1160
IBETA = 2 SIN 1170

```

	IO = 1	SIN 1180
	GO TO 50	SIN 1190
C	*****	SIN 1200
C	RETURN THE RESULTS.	SIN 1210
C	*****	SIN 1220
150	IO = IOS	SIN 1230
	BETA = 91 - MBETA	SIN 1240
	ETA = META - 1	SIN 1250
	THETA = THETA - 1	SIN 1260
	RETURN	SIN 1270
	END	SIN 1280

```

SUBROUTINE MATQD (A,X,NR,NV,DET,NA,NX,IFLAG)
C *****
C THIS IS AS ORNL SIMULTANEOUS EQUATION SOLVER. P04013 OF 1167
C *****
DOUBLE PRECISION A,X,DET,PIVOT,Z,SUM
DIMENSION A(1),X(1)
IFLAG = 0
NR1=NR-1
DO 120 K=1,NR1
IR1=K+1
PIVOT=0.0
DO 20 I=K,NR
IK=(K-1)*NA+I
Z=DABS(A(IK))
IF (Z-PIVOT) 20,20,10
10 PIVOT=Z
IPR=I
20 CONTINUE
IF (PIVOT) 40,30,40
30 IFLAG = 1
RETURN
40 IF (IPR-K) 50,80,50
50 DO 60 J=K,NR
IPRJ=(J-1)*NA+IPR
Z=A(IPRJ)
KJ=(J-1)*NA+K
A(IPRJ)=A(KJ)
60 A(KJ)=Z
DO 70 J=1,NV
IPRJ=(J-1)*NX+IPR
Z=X(IPRJ)
KJ=(J-1)*NX+K
X(IPRJ)=X(KJ)
70 X(KJ)=Z
DET=-DET
80 KK=(K-1)*NA+K
DET=DET*A(KK)
DO 90 J=IR1,NR
KJ=(J-1)*NA+K
A(KJ)=A(KJ)/A(KK)
DO 90 I=IR1,NR
IJ=(J-1)*NA+I
IK=(K-1)*NA+I
90 A(IJ)=A(IJ)-A(IK)*A(KJ)
DO 120 J=1,NV
KJ=(J-1)*NX+K
IF (X(KJ)) 100,120,100
100 X(KJ)=X(KJ)/A(KK)
DO 110 I=IR1,NR
IJ=(J-1)*NX+I
IK=(K-1)*NA+I
110 X(IJ)=X(IJ)-A(IK)*X(KJ)
120 CONTINUE
NRNR=(NR-1)*NA+NR
IF (A(NRNR)) 130,30,130
130 DET=DET*A(NRNR)
DO 150 J=1,NV
NRJ=(J-1)*NX+NR
X(NRJ)=X(NRJ)/A(NRNR)
DO 150 K=1,NR1

```

```

MATQD 10
MATQD 20
MATQD 30
MATQD 40
MATQD 50
MATQD 60
MATQD 70
MATQD 80
MATQD 90
MATQ 100
MATQ 110
MATQ 120
MATQ 130
MATQ 140
MATQ 150
MATQ 160
MATQ 170
MATQ 180
MATQ 190
MATQ 200
MATQ 210
MATQ 220
MATQ 230
MATQ 240
MATQ 250
MATQ 260
MATQ 270
MATQ 280
MATQ 290
MATQ 300
MATQ 310
MATQ 320
MATQ 330
MATQ 340
MATQ 350
MATQ 360
MATQ 370
MATQ 380
MATQ 390
MATQ 400
MATQ 410
MATQ 420
MATQ 430
MATQ 440
MATQ 450
MATQ 460
MATQ 470
MATQ 480
MATQ 490
MATQ 500
MATQ 510
MATQ 520
MATQ 530
MATQ 540
MATQ 550
MATQ 560
MATQ 570
MATQ 580
MATQ 590
MATQ 600

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```

I=NR-K
SUM=0.0
DO 140 L=I,NR1
IL=L*NA+I
LJ=(J-1)*NX+(L+1)
140 SUM=SUM+A(IL)*X(LJ)
IJ=(J-1)*NX+I
150 X(IJ)=X(IJ)-SUM
RETURN
END
```

MATQ 610
MATQ 620
MATQ 630
MATQ 640
MATQ 650
MATQ 660
MATQ 670
MATQ 680
MATQ 690
MATQ 700

CONCLUSION

The computer program described herein is a very efficient and easily used tool for determining stresses, stress indices, and flexibility factors for elbows and curved pipe. Comparison with experimental results indicates that the results accurately represent the maximum stresses which occur at the center of the bend. Since end effects are not included in the analysis which was used, the calculated stresses and flexibility of the elbow may be larger than the actual values. A more complicated analysis will be required if end effects are important.

REFERENCES

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2. W. G. Dodge and S. E. Moore, Stress Indices and Flexibility Factors for Moment Loadings on Elbows and Curved Pipe, ORNL-TM-3658 (March 1972); also in Welding Research Council Bulletin 179, December 1972.
3. E. C. Rodabaugh and H. H. George, "Effect of Internal Pressure on Flexibility and Stress-Intensification Factors of Curved Pipe or Welding Elbows," Trans. ASME 79, 939-48 (1957).
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Appendix A

EXAMPLE OF COMPUTER OUTPUT

This Appendix contains an example of the output from the program ELBOW. The dimensionless parameters were chosen to correspond to the experimental data given in Ref. 3 for an elbow subjected to an in-plane bending moment and having 400 psi internal pressure. On the OPTION CARD the following options were selected: title, punched output of stress index and flexibility factors, and printed and punched output of the stresses.

The first page of output contains the printout of the program input and the options requested, as well as the calculated values of the stress index and flexibility factors. The circumferential location of the point of maximum stress intensity and the direction of the moment vector producing this stress are identified. The angle between the moment vector and the transverse plane is the angle η shown in Fig. A.1. The angle of its projection in the transverse plane is given in the figure as β .

The remaining pages of output contain the normalized stresses due to in-plane and out-of-plane bending respectively. These stresses are given in a nondimensional form, i.e., $\sigma/(M/Z)$, as indicated. This case required 0.4 sec of CPU time.

EXAMPLE OF COMPUTER PROGRAM OUTPUT

JOB RUN ON 11-22-72

RODABAUGH AND GEORGE'S 30 INCH ELBOW (USED AS AN EXAMPLE OF PROGRAM OUTPUT)

*** BEND PARAMETERS ***

GAMMA = 3.000
 LAMBDA = 0.11180
 PSI = 0.00370
 POISSON RATIO = 0.30

*** OPTIONS REQUESTED ***

PUNCH OUTPUT - FLEXIBILITY FACTOR AND STRESS INDEX
 PRINT OUTPUT - STRESSES
 PUNCH OUTPUT - STRESSES

*** RESULTS ***

FLEXIBILITY FACTOR = 11.754

MAX. STRESS INDEX = 6.651 OCCURRING ON THE INSIDE OF THE WALL AT PHI = 0 DEG. DUE TO A
 MOMENT VECTOR FORMING AN ANGLE OF 0 DEG. WITH THE TRANSVERSE
 PLANE AND WHOSE PROJECTION ON THIS PLANE LIES AT PHI = 0 .

PHI IS THE ANGLE AROUND THE CROSS SECTION (C.C. POSITIVE) SUCH THAT -90 DEG. IS TOWARD THE CENTER
 OF THE TUBE BEND.

*** STRESSES AND DEFLECTIONS ***
(NONDIMENSIONAL FORM)

IN-PLANE BENDING

PHI DEG.	CIRCUMFERENTIAL		LONGITUDINAL		WALL DEP.
	INSIDE	OUTSIDE	INSIDE	OUTSIDE	
0	-6.651	-5.332	-1.798	-1.798	1.488
5	-6.328	5.047	-0.604	2.809	1.451
10	-5.415	4.239	0.613	3.509	1.344
15	-4.061	3.045	1.696	3.828	1.176
20	-2.482	1.656	2.520	3.761	0.961
25	-0.904	0.276	3.013	3.367	0.718
30	0.476	-0.920	3.165	2.746	0.462
35	1.527	-1.816	3.022	2.020	0.210
40	2.192	-2.362	2.665	1.299	-0.026
45	2.490	-2.577	2.189	0.669	-0.238
50	2.487	-2.522	1.684	0.181	-0.422
55	2.278	-2.285	1.218	-0.151	-0.577
60	1.958	-1.953	0.832	-0.341	-0.703
65	1.608	-1.600	0.544	-0.419	-0.803
70	1.285	-1.278	0.347	-0.422	-0.880
75	1.022	-1.018	0.226	-0.386	-0.937
80	0.833	-0.832	0.160	-0.340	-0.976
85	0.722	-0.721	0.129	-0.304	-0.999
90	0.685	-0.685	0.120	-0.291	-1.007
95	0.722	-0.721	0.129	-0.304	-0.999
100	0.833	-0.832	0.160	-0.340	-0.976
105	1.022	-1.018	0.226	-0.386	-0.937
110	1.285	-1.278	0.347	-0.422	-0.880
115	1.608	-1.600	0.544	-0.419	-0.803
120	1.958	-1.953	0.832	-0.341	-0.703
125	2.278	-2.285	1.218	-0.151	-0.577
130	2.487	-2.522	1.684	0.181	-0.422
135	2.490	-2.577	2.189	0.669	-0.238
140	2.192	-2.362	2.665	1.299	-0.026
145	1.527	-1.816	3.022	2.020	0.210
150	0.476	-0.920	3.165	2.746	0.462
155	-0.904	0.276	3.013	3.367	0.718
160	-2.482	1.656	2.520	3.761	0.961
165	-4.061	3.045	1.696	3.828	1.176
170	-5.415	4.239	0.613	3.509	1.344
175	-6.328	5.047	-0.604	2.809	1.451
180	-6.651	5.332	-1.798	1.798	1.488
185	-6.328	5.047	-2.809	0.604	1.451
190	-5.415	4.239	-3.509	-0.613	1.344
195	-4.061	3.045	-3.828	-1.696	1.176
200	-2.482	1.657	-3.761	-2.520	0.961
205	-0.904	0.276	-3.367	-3.013	0.718

TO OBTAIN TRUE STRESSES MULTIPLY BY THE BENDING MOMENT AT THE SECTION AND DIVIDE BY THE SECTION MODULUS OF THE CROSS SECTION.

TO OBTAIN TRUE DEFLECTIONS MULTIPLY BY THE PRODUCT OF THE BEND RADIUS, THE BENDING MOMENT, AND THE FLEXIBILITY FACTOR AND DIVIDE BY THE PRODUCT OF THE SECTION MODULUS AND THE MODULUS OF ELASTICITY.

PHI IS THE ANGLE AROUND THE CROSS SECTION (C.C. POSITIVE) SUCH THAT -90 DEG. IS TOWARD THE CENTER OF THE TUBE BEND.

IN-PLANE BENDING

PHI	CIRCUMFERENTIAL		LONGITUDINAL		WALL DEP.
	INSIDE	OUTSIDE	INSIDE	OUTSIDE	
210	0.476	-0.920	-2.746	-3.165	0.462
215	1.527	-1.816	-2.020	-3.022	0.210
220	2.192	-2.362	-1.299	-2.665	-0.026
225	2.490	-2.577	-0.669	-2.189	-0.238
230	2.487	-2.522	-0.181	-1.684	-0.422
235	2.278	-2.285	0.151	-1.218	-0.577
240	1.958	-1.953	0.341	-0.832	-0.703
245	1.608	-1.600	0.419	-0.544	-0.803
250	1.285	-1.278	0.422	-0.347	-0.880
255	1.022	-1.018	0.386	-0.226	-0.937
260	0.833	-0.832	0.340	-0.160	-0.976
265	0.722	-0.721	0.304	-0.129	-0.999
270	0.685	-0.685	0.291	-0.120	-1.007
275	0.722	-0.721	0.304	-0.129	-0.999
280	0.833	-0.832	0.340	-0.160	-0.976
285	1.022	-1.018	0.386	-0.226	-0.937
290	1.285	-1.278	0.422	-0.347	-0.880
295	1.608	-1.600	0.419	-0.544	-0.803
300	1.958	-1.953	0.341	-0.832	-0.703
305	2.278	-2.285	0.151	-1.218	-0.577
310	2.487	-2.522	-0.181	-1.684	-0.422
315	2.490	-2.577	-0.669	-2.189	-0.238
320	2.192	-2.362	-1.299	-2.665	-0.026
325	1.527	-1.816	-2.020	-3.022	0.210
330	0.476	-0.920	-2.746	-3.165	0.462
335	-0.904	0.276	-3.367	-3.013	0.718
340	-2.482	1.656	-3.761	-2.520	0.961
345	-4.061	3.045	-3.828	-1.696	1.175
350	-5.415	4.239	-3.509	-0.613	1.344
355	-6.328	5.047	-2.809	0.604	1.451

OUT-OF-PLANE BENDING

PHI	CIRCUMFERENTIAL		LONGITUDINAL		WALL DEF.
	INSIDE	OUTSIDE	INSIDE	OUTSIDE	
0	0.0	0.0	4.204	4.204	0.0
5	1.853	-1.613	4.535	3.495	-0.308
10	3.459	-3.006	4.449	2.510	-0.596
15	4.623	-4.005	3.980	1.391	-0.846
20	5.235	-4.511	3.219	0.295	-1.044
25	5.286	-4.520	2.297	-0.645	-1.184
30	4.857	-4.105	1.351	-1.337	-1.263
35	4.092	-3.396	0.502	-1.745	-1.285
40	3.158	-2.545	-0.170	-1.881	-1.259
45	2.210	-1.691	-0.628	-1.798	-1.192
50	1.368	-0.941	-0.875	-1.567	-1.097
55	0.699	-0.357	-0.946	-1.263	-0.980
60	0.227	0.041	-0.891	-0.946	-0.850
65	-0.061	0.267	-0.759	-0.661	-0.713
70	-0.198	0.353	-0.595	-0.429	-0.572
75	-0.227	0.337	-0.427	-0.257	-0.429
80	-0.184	0.255	-0.271	-0.139	-0.286
85	-0.101	0.136	-0.130	-0.059	-0.143
90	-0.000	0.000	-0.000	-0.000	-0.000
95	0.101	-0.136	0.130	0.059	0.143
100	0.184	-0.255	0.271	0.139	0.286
105	0.227	-0.337	0.427	0.257	0.429
110	0.198	-0.353	0.595	0.429	0.572
115	0.061	-0.267	0.759	0.661	0.713
120	-0.227	-0.041	0.891	0.946	0.850
125	-0.699	0.357	0.946	1.263	0.980
130	-1.368	0.941	0.875	1.567	1.097
135	-2.210	1.691	0.628	1.798	1.192
140	-3.158	2.545	0.170	1.881	1.259
145	-4.092	3.396	-0.502	1.745	1.285
150	-4.857	4.104	-1.351	1.337	1.263
155	-5.286	4.520	-2.297	0.645	1.184
160	-5.235	4.511	-3.219	-0.295	1.044
165	-4.623	4.005	-3.980	-1.391	0.846
170	-3.460	3.006	-4.449	-2.510	0.596
175	-1.853	1.613	-4.535	-3.495	0.308
180	-0.000	0.000	-4.204	-4.204	0.000
185	1.853	-1.613	-3.495	-4.535	-0.308
190	3.459	-3.006	-2.510	-4.449	-0.596
195	4.623	-4.005	-1.391	-3.980	-0.846
200	5.235	-4.511	-0.295	-3.219	-1.044
205	5.286	-4.520	0.645	-2.297	-1.184
210	4.857	-4.105	1.337	-1.351	-1.263
215	4.092	-3.396	1.745	-0.502	-1.285
220	3.158	-2.545	1.881	0.170	-1.259

TO OBTAIN TRUE STRESSES MULTIPLY BY THE BENDING MOMENT AT THE SECTION AND DIVIDE BY THE SECTION MODULUS OF THE CROSS SECTION.

TO OBTAIN TRUE DEFLECTIONS MULTIPLY BY THE PRODUCT OF THE BEND RADIUS, THE BENDING MOMENT, AND THE FLEXIBILITY FACTOR AND DIVIDE BY THE PRODUCT OF THE SECTION MODULUS AND THE MODULUS OF ELASTICITY.

PHI IS THE ANGLE AROUND THE CROSS SECTION (C.C. POSITIVE) SUCH THAT -90 DEG. IS TOWARD THE CENTER OF THE TUBE BEND.

OUT-OF-PLANE BENDING

PHI	CIRCUMFERENTIAL		LONGITUDINAL		WALL DEP.
	INSIDE	OUTSIDE	INSIDE	OUTSIDE	
225	2.210	-1.691	1.798	0.628	-1.192
230	1.368	-0.941	1.568	0.875	-1.097
235	0.699	-0.357	1.263	0.946	-0.980
240	0.227	0.041	0.946	0.891	-0.850
245	-0.061	0.267	0.661	0.759	-0.713
250	-0.198	0.353	0.429	0.595	-0.572
255	-0.227	0.337	0.257	0.427	-0.429
260	-0.184	0.255	0.139	0.271	-0.286
265	-0.101	0.136	0.059	0.130	-0.143
270	-0.000	0.000	-0.000	-0.000	-0.000
275	0.101	-0.136	-0.059	-0.130	0.143
280	0.184	-0.255	-0.139	-0.271	0.286
285	0.227	-0.337	-0.257	-0.427	0.429
290	0.198	-0.353	-0.429	-0.595	0.572
295	0.061	-0.267	-0.661	-0.759	0.713
300	-0.227	-0.041	-0.946	-0.091	0.850
305	-0.699	0.357	-1.263	-0.946	0.980
310	-1.368	0.941	-1.567	-0.875	1.097
315	-2.210	1.691	-1.798	-0.628	1.192
320	-3.158	2.545	-1.881	-0.170	1.259
325	-4.092	3.396	-1.745	0.502	1.285
330	-4.857	4.104	-1.337	1.351	1.263
335	-5.286	4.520	-0.645	2.297	1.184
340	-5.235	4.511	0.295	3.219	1.044
345	-4.623	4.005	1.391	3.980	0.846
350	-3.460	3.006	2.510	4.449	0.596
355	-1.853	1.613	3.495	4.535	0.308

THIS CASE REQUIRED 0.4 SECONDS

IHC002I STOP 0

Fig. A.1. Moment vector loadings on an elbow or curved pipe.

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