ORNL-TM-4160

TRANSIENT COOLING OF A ONE-DIMENSIONAL THERMAL PLUME AND ITS APPLICATION FOR DETERMINING COLD SHOCK

D. A. Pilati

MASTER

OAK RIDGE NATIONAL LABORATORY
OPERATED BY UNION CARBIDE CORPORATION • FOR THE U.S. ATOMIC ENERGY COMMISSION

DISCLAIMER

ORNL-TM-4160

Contract No. W-7405-eng-26

Reactor Division

TRANSIENT COOLING OF A ONE-DIMENSIONAL THERMAL PLUME AND ITS APPLICATION FOR DETERMINING COLD SHOCK

D. A. Pilati

MAY 1973

NOTICE: This document contains information of a preliminary nature and was prepared primarily for internal use at the Oak Ridge National Laboratory. It is subject to revision or correction and therefore does not represent a final report.

OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

THIS PAGE

WAS INTENTIONALLY

LEFT BLANK

CONTENTS

THIS PAGE

WAS INTENTIONALLY

LEFT BLANK

FOREWORD

Oak Ridge National Laboratory is providing technical assistance to the U. S. Atomic Energy Commission Directorate of Licensing in its analyses of the environmental impact of power reactors as required by the National Environmental Policy Act. The assessment of the environmental impact of a nuclear station has required the development of new analytical techniques and the expansion of many existing ones.

This document is one of the spin-off benefits of this effort by the Commission and its National Laboratories to protect the environment. The work reported here, while originally completed in support of the analysis of a particular nuclear plant, has wide application in other areas.

Other documents in this series are listed below:

1. L. Dresner, Steady Temperature Distributions in the Far-Field Region Obtained by Solution of the Equation of Convective Diffusion in Two Dimensions, USAEC Report ORNL-TM-4119, Oak Ridge National Laboratory.

TRANSIENT COOLING OF A ONE-DIMENSIONAL THERMAL PLUME AND ITS APPLICATION FOR DETERMINING COLD SHOCK

D. A. Pilati

ABSTRACT

The abrupt termination of a thermal discharge may cause an impact to aquatic organisms that have been attracted to the discharge's heated water. To assess the cold shock impact, predictions of the transient temperature of the cooling thermal plume are required. An analytical solution is presented for the time-dependent temperature distribution of a one-dimensional thermal plume whose heat source is discontinued. The one-dimensional restriction limits the application of the model to well-mixed rivers and estuaries which might approximate this idealization. Examples of the model's applications are included for an arbitrary initial temperature profile. These examples illustrate how to predict the temperature effects for both mobile and immobile organisms. The sensitivity of the results to various values of the longitudinal dispersion coefficients has been investigated. The results illustrate the fact that the initial rate of temperature decrease is much greater than that predicted from surface cooling alone. Although this work is preliminary in nature, the method developed can be employed, in many cases, to obtain the temperature predictions required for a cold shock assessment.

Keywords: Thermal pollution, thermal transient, plume behavior, model, analytical model, computer program.

1. INTRODUCTION

The warming of our natural waters by heat rejected from steam-electric power plants has been of growing concern. The various problems attributed to this thermal discharge have been categorized as "thermal pollution." Much of the engineering research on thermal discharges has dealt with prediction of the temperature distribution in the receiving water body. The predicted temperatures are used in assessing the biological impact of a proposed steam-electric power plant.

Mathematical and physical models of thermal discharges into receiving water bodies (e.g., river, estuary, lake, etc.) have been developed. These models permit prediction of the steady-state temperature distribution in the vicinity of the discharge. (Mathematical models are termed "near-field" or "far-field" depending on the region for which they are valid.) The elevated temperature region within the water body defines the "thermal plume."

The maximum impact from temperature change in a water body may be greater due to the "cold shock" as a result of an abrupt termination of the thermal discharge rather than from elevated temperatures. If, for example, a steam-electric plant were rapidly shutdown during the winter, cold shock could occur. There is recent evidence[1] that fish can tolerate higher rates of temperature increase than temperature decrease. Fish kills from cold shock have already been reported.[2-6]

This paper presents work on predicting the temperature of a thermal plume when the heat source has been abruptly terminated. A one-dimensional model is employed which restricts the results to application in well-mixed rivers and estuaries where a one-dimensional idealization might be applicable. The basic heat transfer equation is presented for the assumption of constant depth and width for the water body. The resulting partial differential equation's solution is obtained for an arbitrary initial temperature distribution. In practice, the initial condition would be obtained from the steady-state, far-field model for the thermal discharge. A discussion is included on how the initial condition may be treated to retain a closed-form solution which facilitates computations.

Examples are given for an assumed initial temperature distribution. The application of the method to both mobile and immobile organisms is investigated for several values of the longitudinal dispersion coefficient.

2. HEAT TRANSFER EQUATION

This work describes the behavior of a thermal plume after abrupt termination of its discharge source. The heat transfer equation describing this phenomonon is presented below for the following assumptions:

1. The heat transfer and fluid flow is one-dimensional (in the x-direction).
2. The turbulent thermal diffusion and longitudinal thermal dispersion processes can be combined and expressed employing a constant longitudinal dispersion coefficient.
3. The only heat transfer from the water body is at the free surface.
4. Fluid properties are constant.
5. The depth of the water body is constant.

The resulting heat transfer equation is

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = E_L \frac{\partial^2 T}{\partial x^2} - \alpha_s T \, , \tag{1}$$

where T is the excess temperature (temperature above the ambient water temperature), u is the flow velocity in the x-direction, E_L is the dispersion coefficient, and α_s is the quantity $K_s/\rho C_p D$ (K_s, ρ, C_p and D are, respectively, the surface heat transfer coefficient, density, specific heat, and depth of the water). [Since α_s is somewhat temperature dependent, an average value should be used since it is assumed constant in Eq. (1).]

3. SOLUTION OF THE HEAT TRANSFER EQUATION

The heat transfer equation presented above can be simplified by a transformation. This is effected by restricting u to depend on t only and defining new variables X and ξ (X,t) where

$$x = X + \int_0^t u(t')\, dt' \tag{2}$$

$$T(x,t) = \xi\,(X,t)\, e^{-\alpha_s t} \tag{3}$$

The transformed equation becomes

$$\frac{\partial \xi}{\partial t} = E_L \frac{\partial^2 \xi}{\partial X^2} \quad . \tag{4}$$

The spatial transformation of Eq. (2) transforms the Eulerian coordinate x to a coordinate system which moves with the plume (Lagrangian X). Note that for a thermal discharge which is discontinued at t = 0; X = x and ξ (X,0) = T (x,0). Therefore, the initial condition for ξ is just the initial temperature distribution.

The equation for ξ (X,t) can now be solved assuming the water body is of infinite extent along x (and therefore, X). The solution is obtained by separating variables and employing Fourier transform techniques (see, for example, pages 540-543 of Kreyszig[7]). This results in the following integral relation:

$$\xi\,(X,t) = \frac{1}{2\sqrt{E_L t}} \int_{-\infty}^{\infty} \xi\,(X',0)\, \exp^{-(X-X')^2/4E_L t}\, dX' \quad . \tag{5}$$

This relation can be simplified by defining

$$\omega = \frac{X - X'}{2\sqrt{E_L t}} \tag{6}$$

from which,

$$\xi\,(X,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \xi\,(X + 2\,\omega\,\sqrt{E_L t}\,,\,0)\, \exp^{-\omega^2}\, d\omega \quad . \tag{7}$$

In general, the original temperature distribution is not a simple function. To obtain a closed-form solution the initial temperature distribution can be approximated by a series of n step functions as illustrated in Figure 1. Employing this approximation for the initial temperature distribution, Eq. (7) becomes

$$\xi\,(X,t) = \sum_{i=1}^{n} \frac{1}{\sqrt{\pi}} \int_{\frac{X_i - X}{2\sqrt{E_L t}}}^{\frac{X_{i+1} - X}{2\sqrt{E_L t}}} \xi_{o,i} \exp^{-\omega^2}\, d\omega\ . \qquad (8)$$

Incorporating the error function where,

$$\operatorname{erf}\,(y) \triangleq \frac{2}{\sqrt{\pi}} \int_o^y \exp^{-v^2}\, dv\ . \qquad (9)$$

and the fact that $\xi_{o,i} = T_{o,i}$, then Eq. (7) becomes

$$\xi\,(X,t) = 0.5 \sum_{i=1}^{n-1} T_{o,i} \left[\operatorname{erf}\left(\frac{X_{i+1} - X}{2\sqrt{E_L t}}\right) - \operatorname{erf}\left(\frac{X_i - X}{2\sqrt{E_L t}}\right)\right]. \qquad (10)$$

The temperature for any (x,t) can then be obtained from the transform relations, i.e.,

$$x = X + \int_o^t u(t')\, dt' \qquad (2)$$

and

$$T(x,t) = \xi\,(X,t)\, e^{-\alpha_s t} \qquad (3)$$

ORNL–DWG 73–1458

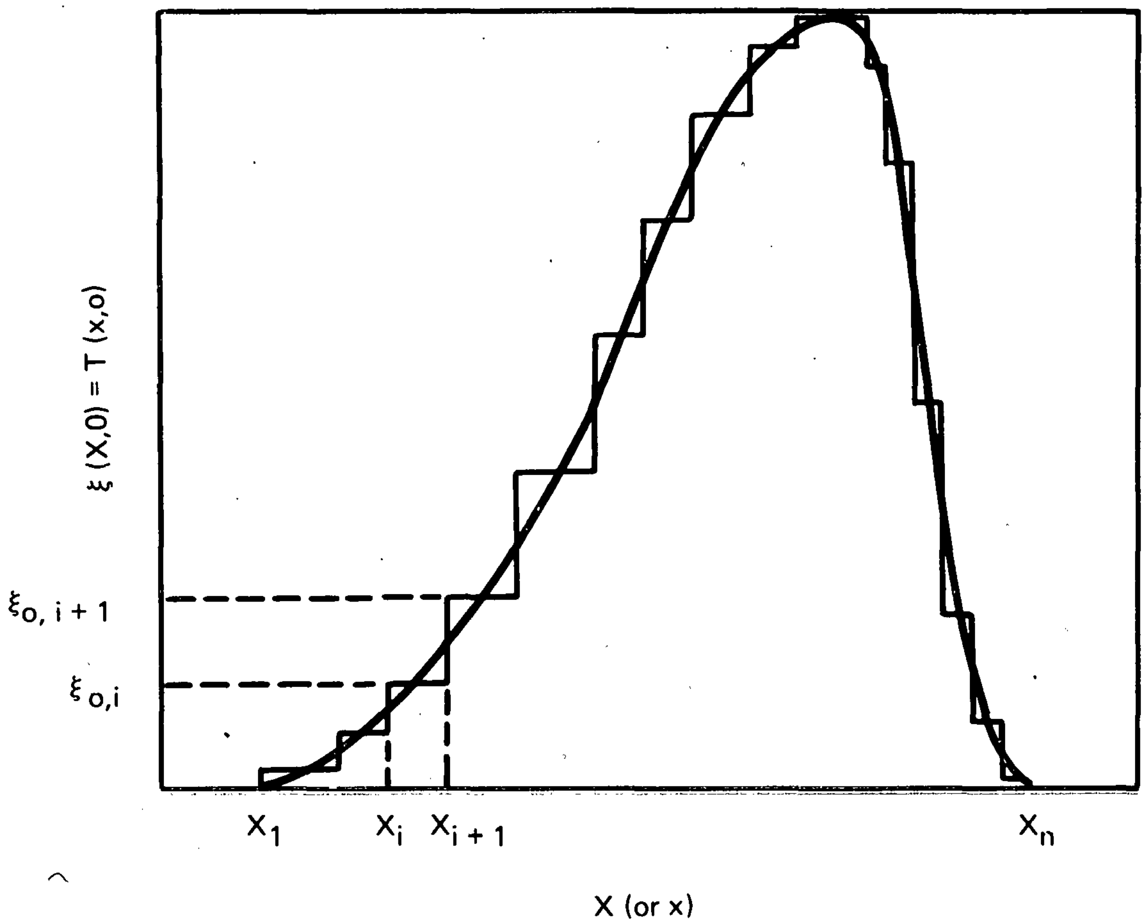

Fig. 1. Example of Approximating the Initial Temperature Distribution as a Series of n Steps.

4. APPLICATION OF THE MODEL

The results for an environmental impact assessment may not require the spatial dependence of the thermal plume. From a biological standpoint, the temperature-time history in the region of maximum temperature elevation may be all that is desired. Fish are very temperature sensitive and would therefore probably be able to follow the maximum temperature region. For this case, the transient-dependence of the heated water's temperature in the Lagrangian reference frame can be investigated. A case was run for an arbitrary initial temperature distribution and the assumed conditions shown in Fig. 2. The Lagrangian time-dependence of the plume temperature is also shown in Fig. 2 for 5, 40, and 160 hours after the hypothetical discharge termination.

The results depicted in Fig. 2 illustrate how the region of maximum temperature cools very rapidly just after the thermal discharge is discontinued. A better appreciation of this fact can be obtained by comparing the behavior of $(T/T_{i,m})_{max}$, where $T_{i,m}$ is the maximum initial temperature, with that predicted by surface cooling alone. If there is only surface cooling and no mixing, then

$$(T/T_{i,m})_{max} = \exp^{-\alpha_s t} \quad . \tag{11}$$

Figure 3 illustrates a comparison of the above relation with the one-dimensional model predictions for several dispersion coefficient values (in Fig. 2, $E_L = 10\ ft^2/sec$). The higher the dispersion coefficient, the more rapidly the maximum temperature decreases due to the increased longitudinal mixing.

Some organisms are not as mobile as fish (e.g., certain benthic organisms). Therefore, their impact can be assessed by assuming they do not move after the thermal discharge is terminated. Figure 4 illustrates the transient temperature behavior at the location of the maximum initial temperature for several values of dispersion coefficients. Again, the conditions in Fig. 2 have been employed. For this case,

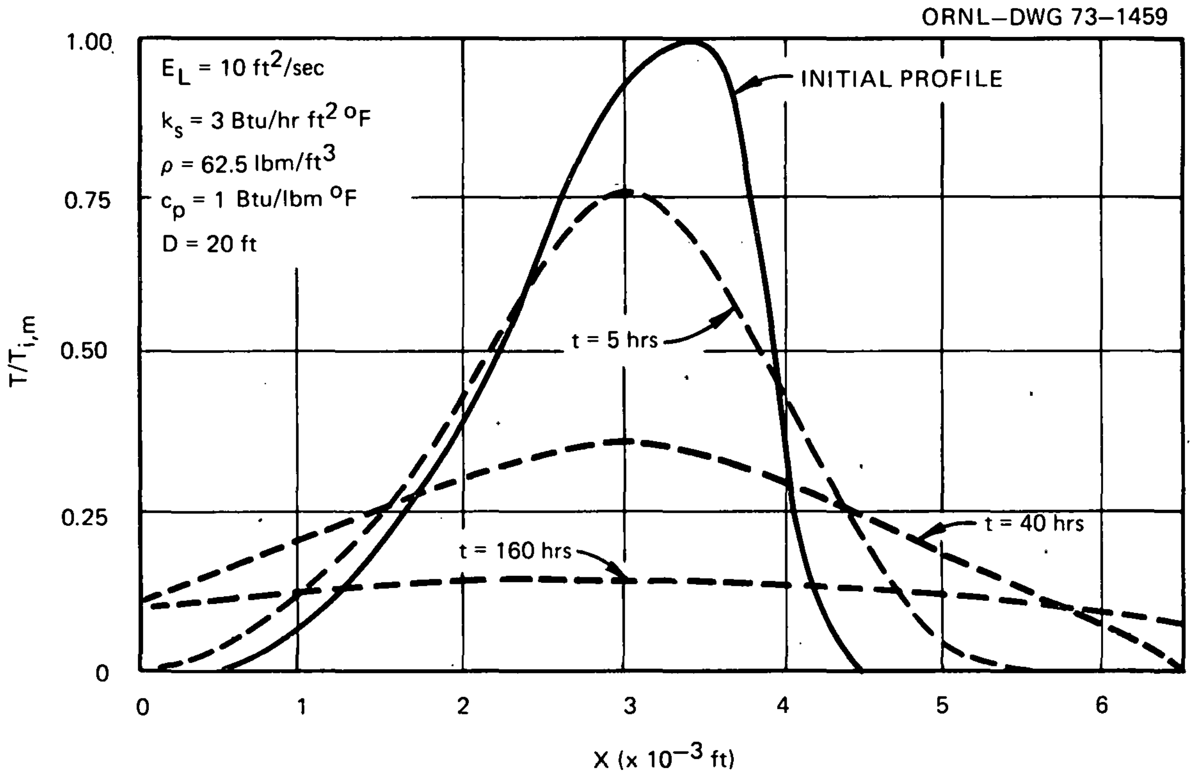

Fig. 2. Example of a Cooling Thermal Plume's Temperature at Several Times ($T_{i,m}$ is the Initial Maximum Excess Temperature).

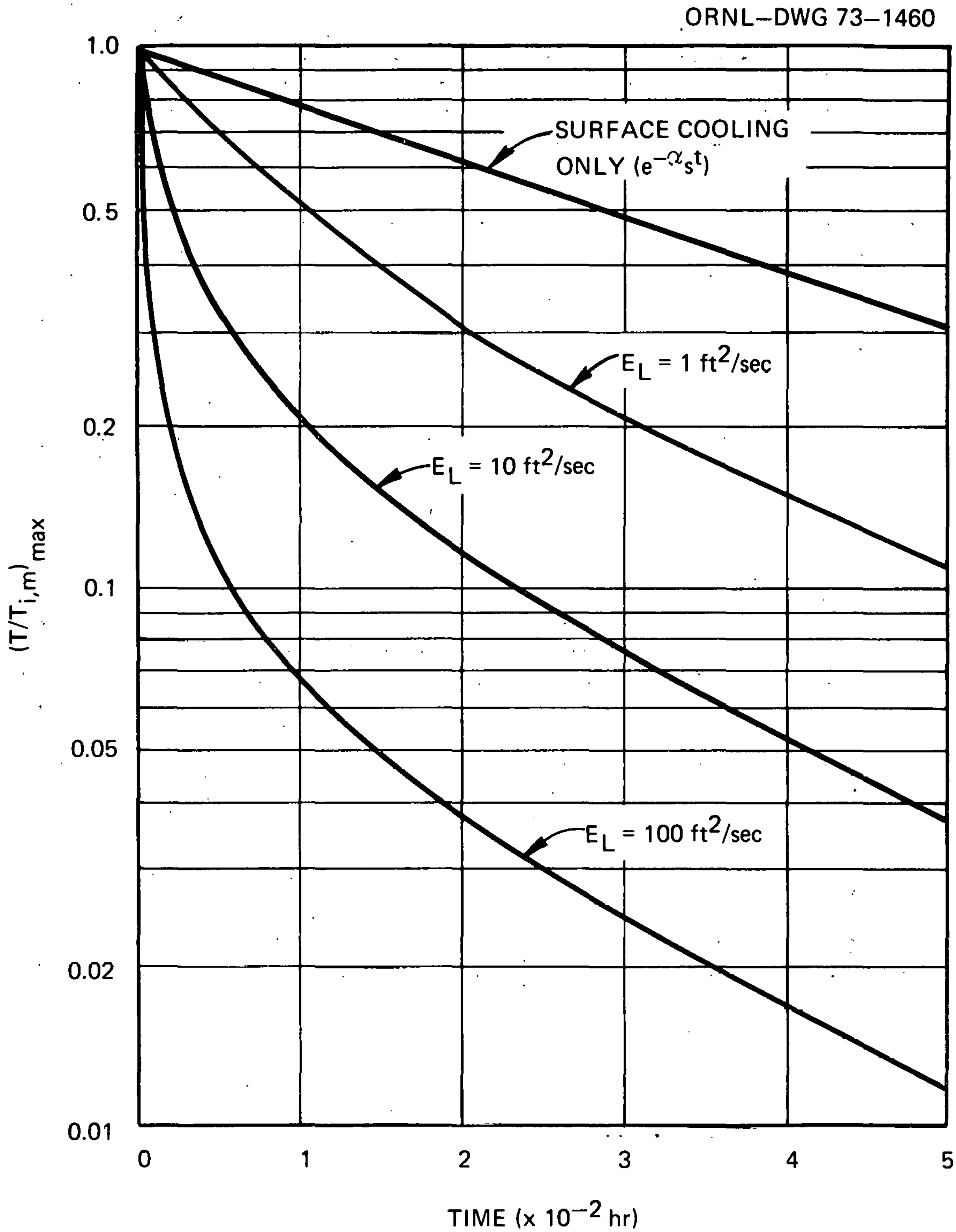

Fig. 3. The Time-Dependence of the Maximum Plume Temperature.

ORNL–DWG 73–2797

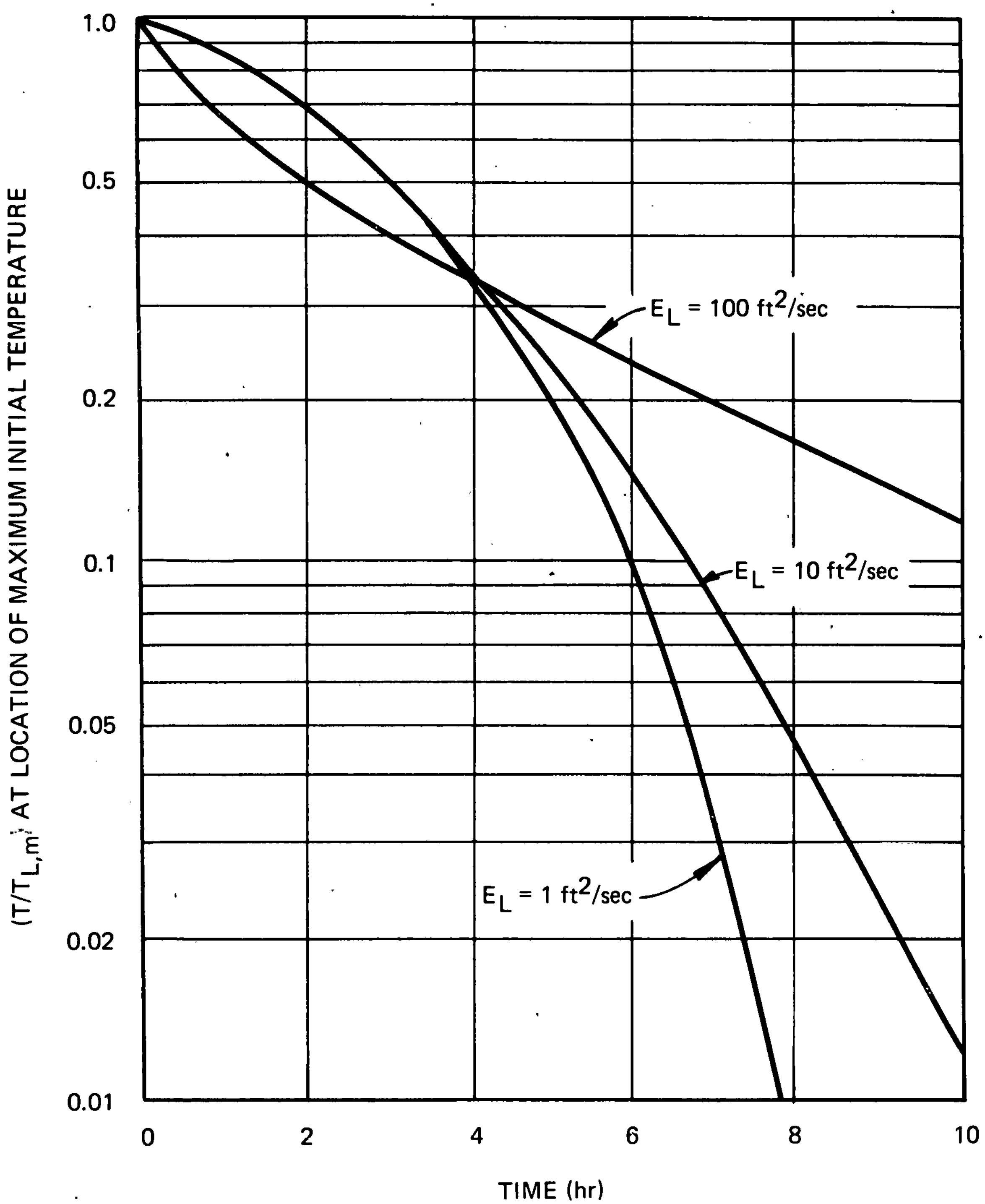

Fig. 4. The Time-Dependence of the Plume Temperature at the Location of Initial Maximum Temperature.

higher dispersion coefficients decrease the rate of temperature change. Although the plume tends downstream, increased dispersion inhibits the upstream cooling rate. Note that the time scale of Fig. 4 is not the same as Fig. 3. The figure 4 results are very dependent on the assumed flow velocity (.1 ft/sec).

In some instances the spatial dependence of the plume may also be of interest. This can be obtained for any time-dependent flow velocity. An idealized estuary flow velocity can be represented as

$$u = u_f + u_m \sin \frac{2\pi t}{t_p} \tag{12}$$

where u_f is the nontidal inflow velocity, u_m is the maximum tidal velocity, and t_p is the tidal period. An example employing the above time-dependent velocity is illustrated in Fig. 5. The same initial condition used in the previous examples was assumed. The thermal plume's temperature is the same as in Fig. 2 except that it has now been translated according to the flow velocity.

The FORTRAN IV computer listing employed for the above calculations is in the Appendix.

5. SUMMARY

A thermal discharge into our natural waters which is abruptly terminated can cause rapid temperature change which may result in the cold shock of aquatic organisms that have been attracted to the discharge's thermal plume. The cold shock phenomenon may be a greater environmental impact than the existence of the initial thermal plume.

A method has been developed to obtain the temperature of the thermal plume as it cools after the thermal discharge is discontinued. The one-dimensional idealization for a water body of constant width and depth results in an analytical solution which can easily be computed.

An arbitrary initial condition was assumed in order to display the model's application to predict fish cold shock. The longitudinal dispersion of heat causes the maximum plume temperature to decrease more rapidly

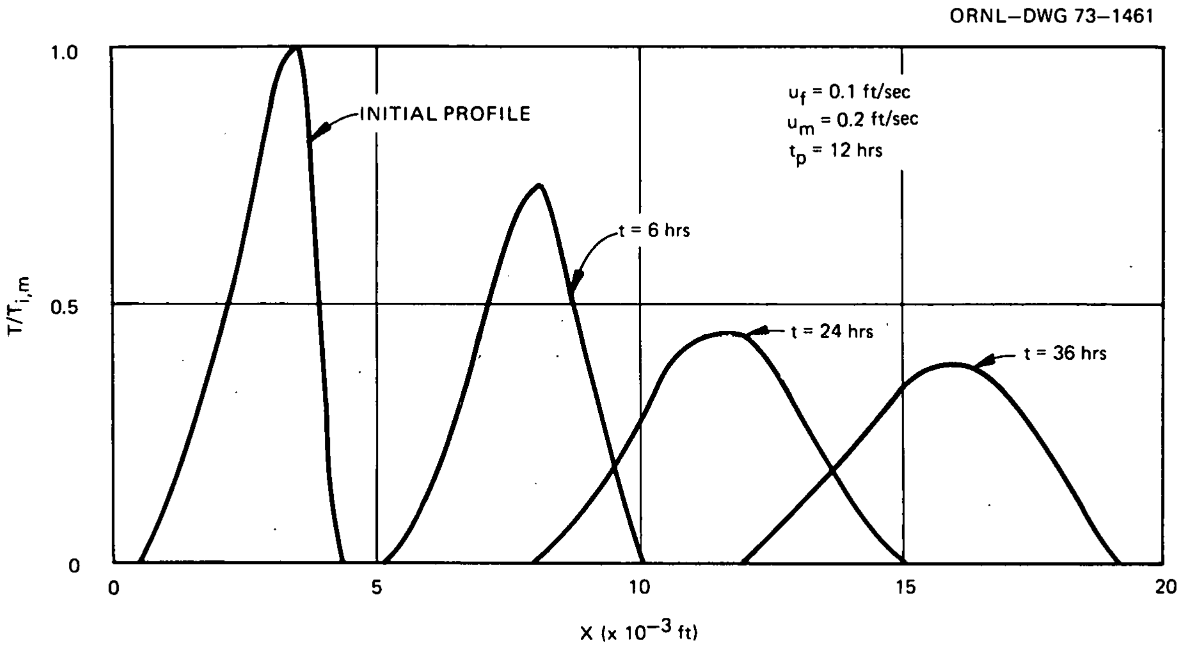

Fig. 5. Example of a Cooling Thermal Plume's Temperature and Position at Several Times for an Estuary.

than that predicted by surface cooling alone. The cooling rate of the maximum plume temperature was shown to be very sensitive to the dispersion coefficient.

The application of the model to poorly motil organisms was discussed. For these organisms, lower dispersion coefficients caused the greater cold shock.

It is felt that the model presented can provide adequate results for the prediction of possible cold shock from thermal discharges into water bodies that nearly satisfy the very restrictive idealizations. The ease of application of the presented model should increase its viability when compared to more complicated models which may be less restrictive.

REFERENCES

1. P. A. Krenkel, J. N. Speakman, Quantification of the Effects of Rate of Temperature Change on the Bluegill Sunfish (*Leponis Macrochirus*) in Radionuclides in Ecosystems, D. T. Nelson (ed.), U.S. AEC CONF-710501; (in press).

2. Thermal Fish Kill, Sport Fishing Institute Bull. No. 232: 7(1972).

3. Thermal Pollution Fish Kill, Sport Fishing Institute Bull. No. 223: 3-4(1971).

4. Pennsylvania Fish Commission Water Pollution Report No. 4170, Pennsylvania Fish Commission, Harrisburg, Pa. (1971).

5. J. G. Robinson, Fish Mortality Report, Lake Michigan, Port Sheldon Michigan Water Resources Commission, Lansing, Michigan (1970)(processed).

6. R. S. Norris, and J. R. Gammon, The Effect of the Heated Water Effluent on the Aquatic Biota of Little-Three-Mile Creek, Processed report, DePauw University, Greencastle, Ind.(1972).

7. E. Kreyszig, Advanced Engineering Mathematics, John Wiley and Sons, New York,(1965).

APPENDIX

The following pages contain the FORTRAN IV listing of the computer program employed for the calculations described in the text. To facilitate the use of the program, the following list has been prepared to relate the names of variables in the text to those used in the program.

In Text	In Program	Remarks
n	NXI	Number of input locations
K_s	KSURF	[Btu/(hr ft^2°F)]
C_p	CP	[Btu/(lbm°F)]
ρ	RHO	(lbm/ft^3)
D	D	(ft)
E_L	DISP	(ft^2/sec)
X_i	XI(I)	(ft)
$\xi_{o,i}$	TI(I)	(°F)
----	NXO	Number of desired output x-locations
----	NTO	Number of desired output times
U_f	UF	(ft/sec)
U_m	UM	(ft/sec)
t_p	XO(I)	Set of output x-locations (ft)
----	TIME(I)	Set of times at which output is printed (hr)
	TEMP	Output temperature (°F)

```
C   ONE DIMENSIONAL TRANSIENT RESPONSE FOR A HEATED PLUME WHOSE
C   SOURCE IS CUTOFF AT TIME=0.0.
      DIMENSION XI(50),TI(50),XO(50),TIME(50)
      DATA XI,TI,XO,TIME/200*0./
      REAL INT(50),KSURF
      DATA INT/50*0.0/
      READ (5,100) NXI,KSURF,CP,RHO,D,DISP
  100 FORMAT (I5,5F10.3)
      WRITE (6,131) NXI,KSURF,CP,RHO,D,DISP
  131 FORMAT ('1 NXI=',I3,3X,'KSURF=',F5.2,3X,'CP=',F5.2,3X,'RHO=',F5.2,
     13X,'DEPTH=',F6.1,3X,'DISP. COEF.=',F6.2)
      WRITE (6,132)
  132 FORMAT ('0  X(I)      TEMP(I)')
      DO 10 I=1,NXI
      READ (5,110) XI(I),TI(I)
  110 FORMAT (2F10.3)
      WRITE (6,133) XI(I),TI(I)
  133 FORMAT ('0',2F10.3)
   10 CONTINUE
      READ (5,120) NXO,NTO,UF,UM,TP
  120 FORMAT(2I5,3F10.3)
      WRITE (6,134) UF,UM,TP
  134 FORMAT ('0UF=',F10.3,' UM=',F10.3,' TP=',F10.3)
      DO 20 I=1,NXO
      READ (5,130) XO(I)
  130 FORMAT (F10.3)
   20 CONTINUE
      DO 30 I=1,NTO
      READ (5,130) TIME(I)
   30 CONTINUE
      PI=3.14159
      EX=KSURF/(RHO*CP*D)
      KK=NXI-1
      DO 1000 IT=1,NTO
      WRITE (6,140) TIME(IT)
  140 FORMAT ('1THE TIME AFTER SHUTDOWN IS',F7.1,'(HOURS).THE TEMPERATUR
     1E AT THE VARIOUS X LOCATIONS IS GIVEN BELOW.')
      WRITE (6,150)
  150 FORMAT ('0',3X,'X(FEET)          TEMPERATURE(F)')
      ARGEX=-EX*TIME(IT)
      SURFL=EXP(ARGEX)
      DEN=2.*(DISP*3600.*TIME(IT))**.5
      ARGCOS=2.*PI*TIME(IT)/TP
      XADJ=3600.*(UF*TIME(IT)+UM*TP*(1-COS(ARGCOS))/(2.*PI))
      DO 2000 IX=1,NXO
      CHI=XO(IX)-XADJ
      DO 3000 IN=1,NXI
      ARG=(XI(IN)-CHI)/DEN
      INT(IN)=ERF(ARG)
 3000 CONTINUE
      SI=0.0
      DO 4000 J=1,KK
      SI=SI+.5*TI(J)*(INT(J+1)-INT(J))
 4000 CONTINUE
      TEMP=SI*SURFL
      WRITE (6,160) XO(IX),TEMP
  160 FORMAT ('0',2X,E13.6,2X,E13.6)
 2000 CONTINUE
 1000 CONTINUE
      RETURN
      END
```

Internal Distribution

1. S. E. Beall
2. F. T. Binford
3. W. E. Browning
4. R. H. Bryan
5. R. H. Busey
6. V. R. Cain
7. W. B. Cottrell
8. C. C. Coutant
9. W. C. Cox
10. F. L. Culler
11. W. Davis, Jr.
12. L. Dresner
13. C. Easterly
14. A. Eraslan
15. A. P. Fraas
16. P. M. Garrett
17. C. P. Goodyear
18. J. A. Hafford
19. R. F. Hibbs
20. R. M. Hill
21. H. W. Hoffman
22. P. R. Kasten
23. G. W. Keilholtz
24. S. S. Kirslis
25. O. H. Klepper
26. M. I. Lundin
27. R. N. Lyon
28. H. C. McCurdy
29. R. E. McPherson
30. J. R. McWherter
31. A. J. Miller
32. M. Mitchell
33. D. J. Nelson
34. B. L. Nichols
35. A. M. Perry
36. E. Pickelsimer

37-41. D. A. Pilati

42. A. S. Quist
43. R. R. Rickard
44. R. C. Robertson
45. J. W. Roddy
46. M. W. Rosenthal

47-97. T. H. Row

98. R. M. Rush
99. P. J. Ryan
100. T. Shapiro
101. M. R. Sheldon
102. S. Siegel
103. M. Siman-Tov
104. O. Sisman
105. M. J. Skinner
106. W. T. Snyder
107. I. Spiewak
108. W. G. Stockdale
109. E. G. Struxness
110. D. A. Sundberg
111. E. H. Taylor
112. R. E. Thoma
113. J. Trabalka
114. D. B. Trauger
115. W. C. Tunnell
116. F. Vaslow
117. A. M. Weinberg
118. G. D. Whitman

119-127. R. P. Wichner

128. M. M. Yarosh
129. W. C. Yee

130-132. Central Research Library

133. Document Reference Section

134-136. Laboratory Records Department

137. Laboratory Records — RC
138. EISO Library (Bldg 2028)

External Distribution

139-156. Manager, Technical Information Center, AEC (For ACRS Members)

157. Research and Technical Support Division, AEC, ORO
158. Patent Office, AEC, ORO

159-160. Technical Information Center, AEC