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A FINITE ELEMENT COMPUTER CODE
(AXICRP)
FOR CREEP ANALYSIS

October 1969

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A FINITE ELEMENT COMPUTER CODE (AXICRP)
FOR CREEP ANALYSIS

By

W. H. Sutherland

Systems Engineering Department
Physics and Engineering Division

October 1969

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W. H. Sutherland

ABSTRACT

This paper describes and shows application of the finite element computer code AXICRP developed for the purpose of solving creep problems for plane stress, plane strain, and axisymmetric bodies of revolution. The code represents an extension of an elastic finite element code to include the nonlinear effects arising from creep by the use of an incremental procedure. This procedure permits the utilization of any creep law expressible in the general functional form

$$\Delta \epsilon_e^C = F(\sigma_e, \epsilon_e^C, T, t).$$

Principal application has been to obtain the creep solution for a Fast Flux Test Facility (FFTF) hexagonal flow duct. Other applications to a thick-walled cylinder and a rectangular beam demonstrate the variety of structural geometries which can be analyzed by the program.

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INTRODUCTION

Creep deformation is an important design consideration in any structure required to operate at high temperatures over a long period. Moreover, recent evidence ⁽¹⁻⁵⁾ indicates that in nuclear reactors, significant irradiation induced creep may occur at low temperatures.

The Liquid Metal Fast Breeder Reactor (LMFBR) program is concerned with many components which have potential creep deformation problems. A prime example is the Fast Flux Test Facility (FFTF).

Reactor core components for the FFTF (such as fuel pins and flow ducts) may incur significant creep deformation at temperatures well below the level that thermally activated creep deformation would be significant. Furthermore, creep deformations are particularly important in those fast reactor core components where safe and efficient functioning of the components depends upon their shape and position. Significant reactor downtime could result from designs which incorrectly assess the amount of in-reactor creep.

This report describes applications of the finite element computer code AXICRP: a code that was developed to solve creep problems for plane stress, plane strain, and axisymmetric bodies of revolution. AXICRP represents the extension of an elastic finite element code ⁽⁶⁾ to include the nonlinear effects arising from creep by the use of an incremental procedure. ⁽⁷⁾ The basic finite element method was selected for modification because of its ability to handle arbitrary geometries, mixed boundary conditions, and structural discontinuities all within the same computer code.

The AXICRP program has been used for the solution of the following creep problems

- 1) Axisymmetric body of revolution application
Plane strain section of a thick-walled cylinder with internal pressure loading.
- 2) Plane strain application
Plane strain section of a hexagonal flow duct with internal pressure loading.
- 3) Plane stress application
Simply-supported beam of rectangular cross-section with a concentrated load at the center.

SUMMARY AND CONCLUSIONS

This document contains a description of the finite element computer code AXICRP developed for the purpose of performing creep analyses for plane stress, plane strain, and axisymmetric bodies of revolution. Although developed and utilized for stress analysis of FFTF reactor core components, the code is generally applicable to structural problems. Examples of each type of problem have been presented and show that the program can predict the complete time history response for complicated structures assumed to creep under constant load. Furthermore, the program can utilize any creep law expressible in the general functional form

$$\Delta \epsilon_e^C = F(\sigma_e, \epsilon_e^C, T, t).$$

This general form permits consideration of strain hardening or age hardening laws, or any other empirical relationship experimentally derived.

The examples of the hexagonal flow duct, thick-walled cylinder, and rectangular beam demonstrate the wide variety of complex structural geometries which can be analyzed using the AXICRP program. However, the direct incremental treatment of time which has been used restricts the current program to

the consideration of creep under constant load. Additional refinement in the time treatment is necessary to economically handle transient and cyclic loading conditions. However, for design purposes, the current creep program AXICRP can be used to determine the worst creep conditions. Based upon these extreme conditions, critical design decisions involving creep considerations can be achieved.

METHOD OF ANALYSIS

The finite element method of solution used here for the development of a creep program represents a systematic procedure for the implementation of the theorem of Minimum Potential Energy. This theorem states: of all the displacement functions which satisfy the displacement boundary conditions and for which a compatible set of strains exist, the function which satisfies the equilibrium equations and the stress boundary conditions makes the potential energy an absolute minimum. The finite element procedure differs from the Rayleigh-Ritz procedure for solving stationary value problems only with respect to the choice of the displacement functions. The Rayleigh-Ritz procedure defines a smooth displacement function over the entire solid, whereas the finite element procedure uses many displacement functions with each function restricted to a small part of the solid called a finite element. Based on these prescribed displacement functions for the individual elements, the stiffness and load matrices for each element can be derived. The equilibrium equations for the total solid are obtained by matrix superposition of the individual element stiffness and load matrices, resulting in a system of linear algebraic equations. Thus, the finite element procedure has replaced the original set of differential equations of equilibrium by an equivalent set of algebraic equations. The solution of this set of algebraic equations represents the solution of the problem.

BASIC ELASTIC FINITE ELEMENT PROCEDURE

The potential energy (ϕ) for an elastic body is defined by

$$\phi = \frac{1}{2} \int_{vol} \sigma_{ij} \epsilon_{ij} dV - \int_{vol} w_i B_i dV - \int_{A_p} w_i P_i dA_p \quad (1)$$

where B_i = body forces

P_i = applied surface tractions

w_i = displacements within the element

A_p = area over which the applied surface tractions act.

Equation (1) written in matrix form becomes*

$$\phi = \frac{1}{2} \int_{vol} [\epsilon]^T [\sigma] dV - \int_{vol} \{w\}^T \{B\} dV - \int_{A_p} \{w\}^T \{P\} dA_p. \quad (2)$$

The finite element method of solution requires the definition of the potential energy for each of the elements (total number = M) making up the total solid and the potential energy of the entire solid is written as the sum of integrals over the volume and surface of each element.

$$\phi = \sum_{m=1}^M \left[\int_{vol} \frac{1}{2} [\epsilon]^T [\sigma] dV - \int_{vol} \{w\}^T \{B\} dV - \int_{A_p} \{w\}^T \{P\} dA_p \right]. \quad (3)$$

To generate the element stiffness and load matrices it is necessary to assume a displacement field within each element usually in the form of polynomials. The assumed displacement field must satisfy compatibility between elements of the system. Symbolically the displacement $\{w\}$ within the element can be defined as

$$\{w\} = [e] \{b\} \quad (4)$$

* The notation $[\]$ indicates a square matrix, $\{ \}$ indicates a column matrix.

where $\{b\}$ denotes the generalized displacements and $[e]$ is a geometry matrix associated with the order of polynomial that has been selected. Evaluating the displacements $\{w\}$ at the nodal points permits one to solve for the generalized displacements in terms of the nodal displacements $\{u\}$.

$$\{b\} = [h]\{u\}. \quad (5)$$

The strains $\{\epsilon\}$ within an element are obtained from Equation (4) by appropriate differentiation

$$\{\epsilon\} = [g]\{b\}. \quad (6)$$

The stresses at any point within an element for an elastic material are expressed by the following stress-strain relationship

$$\{\sigma\} = [c]\{\epsilon\} - \{\tau\} \quad (7)$$

where τ denotes the thermal effects.

Combining Equations (4), (5), (6), and (7) and substituting into Equation (3) gives the potential energy as a function of the nodal displacements.

$$\phi = \sum_{m=1}^M \left[\frac{1}{2} \int_{vol} \{u\}^T [h]^T [g]^T [c] [g] [h] \{u\} dV - \int_{vol} \{u\}^T [h]^T [g]^T \{\tau\} dV - \int_{vol} \{u\}^T [h]^T [e]^T \{B\} dV - \int_{A_p} \{u\}^T [h]^T [e]^T \{P\} dA_p \right]. \quad (8)$$

The potential energy is minimized by requiring that

$$\frac{\partial \phi}{\partial u_i} = 0 \quad \text{for } i = 1, 2, \dots, N \quad (9)$$

where N is the total number of nodal point displacements. Performing the operations indicated in Equation (9) yields the following relationship for the equilibrium of the finite element system:

$$\sum_{m=1}^M \int_{vol} [h]^T [g]^T [c] [g] [h] \{u\} dV = \sum_{m=1}^M \int_{vol} \left([h]^T [g]^T \{\tau\} + [h]^T [e]^T \{B\} \right) dV + \sum_{m=1}^M \int_{A_p} [h]^T [e]^T \{P\} dA_p \quad (10)$$

or written in symbolic form

$$[K] \{u\} = \{Q\} \quad (11)$$

$[K]$ represents the stiffness matrix for the complete finite element system given by the sum of the individual stiffness matrices:

$$[K] = \sum_{m=1}^M [k^m]. \quad (12)$$

The individual stiffness matrices are given by

$$[k^m] = \int_{vol} [h]^T [g]^T [c] [g] [h] dV. \quad (13)$$

The load vector $\{Q\}$ is defined by

$$\{Q\} = \sum_{m=1}^M \left(\{L^m\} + \{R^m\} \right) \quad (14)$$

where $\{L^m\}$ represents the body force vector given by

$$\{L^m\} = \int_{vol} \left([h]^T [g]^T \{\tau\} + [h]^T [e]^T \{B\} \right) dV \quad (15)$$

and $\{R^m\}$ represents the load vector due to surface forces given by

$$\{R^m\} = \int_{A_p} [h]^T [e]^T \{P\} dA_p. \quad (16)$$

Equation (11) represents the relationship between all nodal point forces and the corresponding nodal point displacements. After imposing the boundary conditions on Equation (11), the reduced stiffness matrix is inverted and the nodal point

displacements obtained. Knowing the nodal point displacements permits the evaluation of the strains using Equations (5) and (6). Equation (7) then permits the calculation of stresses corresponding to these strains. These results represent the complete solution to the elasticity problem.

EXTENSION TO INCLUDE CREEP BEHAVIOR

Inclusion of creep behavior forces the consideration of nonlinear stress-strain relationships. Substitution of the nonlinear stress-strain relationships into the governing field equations for the system results in a set of nonlinear equations. Two methods of analysis are currently used for the solution of nonlinear problems:

1. Direct iteration procedure: final solution is achieved by an iterative solution of the nonlinear equations.
2. Incremental procedure: final solution is achieved by solving a number of linearized problems distributed between the initial and final configuration.

The incremental procedure has been selected for the AXICRP program because it is completely general and provides a description of intermediate states. Furthermore, the incremental procedure is more easily adapted to the use of arbitrary forms of the creep law. The fundamental assumption in the incremental creep solution technique is that the time can be subdivided into sufficiently small time intervals such that the stress can be assumed to be constant within each time interval. If this assumption is valid, then the nonlinear creep problem can be solved as a series of linear problems for each time interval treating the incremental creep strains from the last time interval as initial strains for the current time interval. The basic procedure for deriving direct stiffness relationships has been shown by C. A. Felippa⁽⁸⁾ to be applicable for incremental quantities. Thus, for the incremental creep solution technique it becomes necessary to express all the relationships in incremental form.

First, assume the total change in strain during a time interval is the sum of the changes in the elastic $\{\Delta\epsilon^E\}$ and creep strains $\{\Delta\epsilon^C\}$

$$\{\Delta\epsilon\} = \{\Delta\epsilon^E\} + \{\Delta\epsilon^C\}. \quad (17)$$

The incremental form of Hooke's law for an elastic body relates the change in elastic strains to the incremental stresses.

$$\{\Delta\sigma\} = [E]\{\Delta\epsilon^E\} \quad (18)$$

Combining Equations (17) and (18) yields the relationship between incremental stress, incremental total strain, and incremental creep strain.

$$\{\Delta\sigma\} = [E](\{\Delta\epsilon\} - \{\Delta\epsilon^C\}) \quad (19)$$

Combining Equations (5) and (6) and writing in incremental form gives the incremental strain-displacement relations.

$$\{\Delta\epsilon\} = [g][h]\{\Delta u\} \quad (20)$$

The incremental relationships for the equilibrium Equations (11) can be written in the following manner where the load vector $\{Q\}$ has been replaced by the incremental creep forces $\{\Delta F^C\}$ resulting from the incremental creep strains occurring during the previous time interval

$$\begin{aligned} \{\Delta u\} &= [K]^{-1} \{\Delta F^C\} \\ \text{where } \{\Delta F^C\} &= [M] \{\Delta\epsilon^C\} \\ [M] &= \text{matrix consisting of geometry and material properties.} \end{aligned} \quad (21)$$

The specific form of the incremental creep law depends upon the material and environmental conditions, but its general functional form can be represented by the following equation

$$\begin{aligned} \text{where } \{\Delta\epsilon_e^C\} &= \{f(\sigma_e, \epsilon_e^C, t, T)\} \\ \{\epsilon_e^C\} &= \text{effective creep strain.} \end{aligned} \quad (22)$$

With the above incremental relationships defined, the creep solution proceeds as follows:

1. Solve the elastic problem for the initial time ($t = 0$) and determine the elastic stress distribution.
2. Assume for a small increment of time (Δt), that the stresses remain constant. From the assumed creep law (Equation 22), which will be an empirical relationship experimentally determined, calculate the incremental change in effective creep strain. The formulation in terms of effective stress and strain permits the use of uniaxial creep data.
3. From the effective incremental creep strains calculate a set of fictitious incremental creep forces $\{\Delta F^C\}$.
4. Substitute the fictitious incremental creep forces into Equation (21) -- equivalent to resolving the elastic problem with the original load vector replaced with this set of forces. This step assures equilibrium of the system after the creep associated with time (Δt) has occurred.
5. Substitute the incremental nodal displacements $\{\Delta u\}$ into Equation (20) to determine the total change in strain $\{\Delta \epsilon\}$.
6. The incremental stresses $\{\Delta \sigma\}$ may now be computed from Equation (19).
7. Add the incremental stresses $\{\Delta \sigma\}$ arising from the fictitious incremental creep forces to the initial elastic stresses to give the new stress distribution.
8. Repeat the procedure for the next time increment using the new stress distribution.

MULTIAXIAL STRESS-STRAIN RELATIONSHIPS

To permit the use of uniaxial creep data it is necessary to formulate the multiaxial stress-strain relationships in terms of effective stress (σ_e) and effective creep strain (ϵ_e^C).

The stress-strain relationships used are postulated in a form to satisfy the following conditions:

1. The stress-strain relationships for the multiaxial state of stress should degenerate to a uniaxial relationship for a uniaxial state of stress.
2. The volume of the body is assumed to remain constant during creep deformation.
3. A superimposed hydrostatic state of stress should not give rise to any change in the creep rate.
4. For an isotropic medium the principal directions of the strain rate and stress tensors should coincide.

The following stress-strain relationships satisfy these conditions:

$$\Delta \epsilon_{ij}^c = \frac{3}{2} \frac{\Delta \epsilon_e^c}{\sigma_e} S_{ij} \quad (23)$$

where the effective stress (σ_e), the incremental creep strain ($\Delta \epsilon_e^c$), and the deviatoric stress (S_{ij}) are defined by

$$\sigma_e = \left(\frac{3}{2} S_{ij} S_{ij} \right)^{\frac{1}{2}} \quad (24)$$

$$\Delta \epsilon_e^c = \left(\frac{2}{3} \Delta \epsilon_{ij}^c \Delta \epsilon_{ij}^c \right)^{\frac{1}{2}} \quad (25)$$

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \quad (26)$$

(Note: Summation is assumed for repeated indices.)

Note that the effective stress has been assumed proportional to the second invariant of the deviatoric stress tensor and condition (1) fixes the proportionality constant at $\sqrt{3/2}$. Equation (23) automatically satisfies condition (2) because S_{kk} is identically zero. Condition (3) is satisfied since S_{ij} and, therefore, σ_e are independent of a hydrostatic state of stress. Since S_{ij} and σ_{ij} are coaxial by definition, condition (4) is satisfied.

These stress-strain relationships permit the use of uniaxial creep data since the effective stress and incremental effective creep strain are equivalent to the uniaxial stress and creep strain. However, before an explicit form for the stress-strain relationships can be obtained it is necessary to decide on a specific functional form for $\Delta \epsilon_e^C$ and fit the resultant functional to experimental data. In general, the effective incremental creep strain is a function of the effective stress (σ_e), the total effective creep strain (ϵ_e^C), the temperature (T), the time (t), and the strain history of the material:

$$\Delta \epsilon_e^C = F(\sigma_e, \epsilon_e^C, T, t). \quad (27)$$

A completely satisfactory creep law for predicting creep strains under continuously changing stress conditions cannot be defined for all materials and all conditions. Two creep laws currently in use are the strain hardening and the age hardening laws. The strain hardening law assumes that the creep rate is a function only of the instantaneous values of the stress, temperature, and accumulated creep strain. The age hardening law assumes the creep rate depends only on the instantaneous values of stress, temperature, and a function of total time from the beginning of the creep process. However, the incremental finite element creep procedure is independent of the manner in which the creep strains are accumulated and, therefore, may be used with any creep law that can be expressed in the general functional form given by Equation (27).

DEVELOPMENT OF FINITE ELEMENT STIFFNESS RELATIONS

The basic assumptions used in the derivation of the finite element stiffness relations are:

1. The material is isotropic, homogeneous, and has identical creep properties in tension and compression.
2. Displacements and strains are assumed small so that small deformation theory is applicable.

3. Solution at time ($t = 0$) is entirely elastic and no time independent plastic strains are considered.
4. Program will accept any arbitrary creep law of the general functional form given by Equation (27).
5. System is assumed to creep at constant external load, and unloading is not considered.

AXISYMMETRIC ANALYSIS

The basic stiffness element used for the analysis of axisymmetric bodies under axisymmetric loads was a triangular ring element shown in Figure 1. Since the solution is not dependent upon θ it is only necessary to idealize geometrically a plane section in the r - z plane. Figure 2 shows the basic triangular element used where the z -axis represents the symmetry axis. The displacement field was assumed to be a linear polynomial in the (r,z) coordinates. Equation (4) in matrix form becomes

FIGURE 1. Basic Stiffness Element for Axisymmetric Analysis

FIGURE 2. *Triangular Axisymmetric Element*

$$\begin{Bmatrix} w_r(r, z) \\ w_z(r, z) \end{Bmatrix} = \begin{bmatrix} 1 & r & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & r & z \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix} \quad (28)$$

Evaluating the displacements at the nodal points permits one to solve for the generalized displacements $\{b\}$ in terms of the nodal displacements $\{u\}$

$$\{b\} = [h]\{u\} \quad (29)$$

where

$$[h] = \frac{1}{\beta} \begin{bmatrix} (r_j z_k - r_k z_j) & 0 & (r_k z_i - r_i z_k) & 0 & (r_i z_j - r_j z_i) & 0 \\ (z_j - z_k) & 0 & (z_k - z_i) & 0 & (z_i - z_j) & 0 \\ (r_k - r_j) & 0 & (r_i - r_k) & 0 & (r_j - r_i) & 0 \\ 0 & (r_j z_k - r_k z_j) & 0 & (r_k z_i - r_i z_k) & 0 & (r_i z_j - r_j z_i) \\ 0 & (z_j - z_k) & 0 & (z_k - z_i) & 0 & (z_i - z_j) \\ 0 & (r_k - r_j) & 0 & (r_i - r_k) & 0 & (r_j - r_i) \end{bmatrix}$$

and $\beta = r_i(z_j - z_k) + r_j(z_k - z_i) + r_k(z_i - z_j)$ (30)

Compatibility between elements is assured with this assumed displacement function since the displacements at the nodal points are identical and the displacements vary in a linear manner between nodal end points. The strain-displacement relations in cylindrical coordinates are given by

$$\epsilon_{rr} = \frac{\partial w_r}{\partial r}; \quad \epsilon_{\theta\theta} = \frac{w_r}{r}; \quad \epsilon_{zz} = \frac{\partial w_z}{\partial z}; \quad \gamma_{rz} = \frac{\partial w_r}{\partial z} + \frac{\partial w_z}{\partial r}. \quad (31)$$

Equation (6) for the strains becomes

$$\begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{\theta\theta} \\ \gamma_{rz} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1/r & 1 & z/r & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix} \quad (32)$$

For subsequent derivations it is important to have the strains constant within each element. To accomplish this for the circumferential strain ($\epsilon_{\theta\theta}$), the following mean values will be used for the r and z coordinates

$$r = \frac{1}{3} (r_i + r_j + r_k); \quad z = \frac{1}{3} (z_i + z_j + z_k).$$

To obtain the stress-strain relationships for the creep problem, the total strain ϵ_{ij} is assumed to be composed of an elastic strain (ϵ_{ij}^E), a creep strain (ϵ_{ij}^C) and a thermal strain ($\alpha\Delta T$).

$$\epsilon_{ij} = \epsilon_{ij}^E + \epsilon_{ij}^C + \delta_{ij} \alpha \Delta T$$

In matrix form, the elastic strain is given by

$$\{\epsilon^E\} = \{\epsilon\} - \{\epsilon^C\} - \{\alpha\Delta T\} \quad (33)$$

For an isotropic material the stress-strain relationship is given by

$$\sigma_{ij} = \frac{E}{(1 + \nu)} \epsilon_{ij}^E + \frac{E\nu}{(1 + \nu)(1 - 2\nu)} \delta_{ij} \epsilon_{kk}^E \quad (34)$$

Substituting Equation (33) into (34) and using the assumption that the creep strains are incompressible ($\epsilon_{kk}^C = 0$) yields the following stress-strain relationships:

$$\begin{aligned} \sigma_{ij} = & \frac{E}{(1 + \nu)} \epsilon_{ij} + \frac{E\nu}{(1 + \nu)(1 - 2\nu)} \delta_{ij} \epsilon_{kk} \\ & - \frac{E}{(1 + \nu)} \epsilon_{ij}^C - \frac{E}{(1 - 2\nu)} \delta_{ij} \alpha \Delta T \end{aligned} \quad (35)$$

In matrix form Equation (35) becomes

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \\ \sigma_{rz} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 \\ \nu & (1-\nu) & \nu & 0 \\ \nu & \nu & (1-\nu) & 0 \\ 0 & 0 & 0 & (1-2\nu) \end{bmatrix} \begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{\theta\theta} \\ \epsilon_{rz} \end{Bmatrix} - \frac{E}{(1+\nu)} \begin{Bmatrix} \epsilon_{rr}^c \\ \epsilon_{zz}^c \\ \epsilon_{\theta\theta}^c \\ \epsilon_{rz}^c \end{Bmatrix} - \frac{E\alpha\Delta T}{(1-2\nu)} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{Bmatrix} \quad (36)$$

or symbolically corresponding to Equation (7)

$$\{\sigma\} = [c] \{\epsilon\} - 2G \{\epsilon^c\} - \{\epsilon^T\} \quad (37)$$

where

$$G = \frac{E}{2(1+\nu)}$$

Combining Equation (28), (29), (30), (32), and (37) and substituting into Equation (3) gives the following expression for the potential energy,

$$\begin{aligned} \phi = \sum_{m=1}^M & \left[\int_{vol} \frac{1}{2} \{u\}^T [h]^T [g]^T [c] [g] [h] \{u\} dV \right. \\ & - 2G \int_{vol} \{u\}^T [h]^T [g]^T \{\epsilon^c\} dV \\ & - \int_{vol} \{u\}^T [h]^T [g]^T \{\epsilon^T\} dV - \int_{vol} \{u\}^T [h]^T [e]^T \{B\} dV \\ & \left. - \int_{A_p} \{u\}^T [h]^T [e]^T \{P\} dA_p \right] \end{aligned}$$

Minimizing the potential energy with respect to the nodal point displacements yields the following set of equilibrium equations including the effects of creep. Observe that creep effects play the same role as thermal effects.

$$\begin{aligned} \sum_{m=1}^M \int_{vol} [h]^T [g]^T [c] [g] [h] \{u\} dV &= \sum_{m=1}^M \left[2G \int_{vol} [h]^T [g]^T \{\epsilon^c\} dV \right. \\ &+ \int_{vol} [h]^T [g]^T \{\epsilon^T\} dV + \int_{vol} [h]^T [e]^T \{B\} dV + \int_{A_p} [h]^T [e]^T \{P\} dA_p \left. \right] \quad (39) \end{aligned}$$

or symbolically as in Equation (11)

$$[K]\{u\} = \{Q\}. \quad (40)$$

All of the quantities in Equation (40) have the same significance as in Equations (12) through (16) with the exception of the body force vector $\{L^m\}$ which has an additional term due to creep. (41)

$$\{L^m\} = \int_{vol} \left([h]^T [g]^T \{\epsilon^T\} + [h]^T [e]^T \{B\} + 2G [h]^T [g]^T \{\epsilon^C\} \right) dV$$

Imposing condition (5) and writing the equilibrium Equation (40) in incremental form it becomes apparent that the only nonvanishing term for the load vector $\{Q\}$ is associated with the incremental creep strains $\{\Delta\epsilon^C\}$

$$[K]\{\Delta u\} = \{\Delta F^C\} \quad (42)$$

where $\{\Delta F^C\}$ represents the fictitious incremental creep forces given by

$$\{\Delta F^C\} = 2G \int_{vol} [h]^T [g]^T \{\Delta\epsilon^C\} dV. \quad (43)$$

Since the incremental creep strains are constant within the element, it is possible to perform the integration independently of the strains and write Equation (42) in the form of Equation (21).

$$\{\Delta u\} = [K]^{-1} [M] \{\Delta\epsilon^C\} \quad (44)$$

where $[M] = 2G [h]^T \int_{vol} [g]^T dV$.

To complete the incremental creep solution procedure for an axisymmetric body, it is necessary to specify the specific functional form of the incremental creep law given by Equation (22). Based upon this value for the incremental effective creep strain $\Delta\epsilon_e^C$, it is possible to define the incremental creep strains $\{\Delta\epsilon^C\}$, using Equations (23), (24), and (26).

$$\begin{Bmatrix} \Delta \epsilon_{rr}^c \\ \Delta \epsilon_{zz}^c \\ \Delta \epsilon_{\theta\theta}^c \\ \Delta \epsilon_{rz}^c \end{Bmatrix} = \frac{\Delta \epsilon_e^c}{2\sigma_e} \begin{Bmatrix} 2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{zz} \\ 2\sigma_{zz} - \sigma_{rr} - \sigma_{\theta\theta} \\ 2\sigma_{\theta\theta} - \sigma_{rr} - \sigma_{zz} \\ 3\sigma_{rz} \end{Bmatrix} \quad (45)$$

$$\text{where } \sigma_e = \frac{1}{\sqrt{2}} \left[(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{\theta\theta} - \sigma_{zz})^2 + (\sigma_{rr} - \sigma_{zz})^2 + 6\sigma_{rz}^2 \right]^{1/2}.$$

Thus, the incremental form of Equation (32), and Equations (44) and (45) represent the incremental relationships needed to solve the creep problem for an axisymmetric body.

PLANE STRESS ANALYSIS

The basic stiffness element used for the plane stress analysis will be a triangular element of unit thickness oriented in the r - z plane as shown in Figure 3. The displacement field was assumed to be of the same form as for the axisymmetric problem. Thus, Equations (28), (29), and (30) are unchanged. The strain-displacement relations in rectangular coordinates are given by

$$\epsilon_{rr} = \frac{\partial w_r}{\partial r}; \quad \epsilon_{zz} = \frac{\partial w_z}{\partial z}; \quad \gamma_{rz} = \frac{\partial w_r}{\partial z} + \frac{\partial w_z}{\partial r} \quad (46)$$

Equation (6) for the strains becomes

$$\begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \gamma_{rz} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix} \quad (47)$$

FIGURE 3. Basic Stiffness Element for Plane Stress Analysis

For this case the stress-strain relationships (Equation 34) for an isotropic material must be modified to satisfy the plane stress conditions:

$$\sigma_{yy} = \sigma_{ry} = \sigma_{zy} = 0 \quad (48)$$

where y is the coordinate axis normal to the r - z plane. Thus, Equation (34) becomes in matrix form,

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{rz} \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu^2)}{(1+\nu)} \end{bmatrix} \begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{rz} \end{Bmatrix} \quad (49)$$

Substituting the appropriate form of Equation (33) into Equation (49) and imposing the assumption that the creep strains are incompressible ($\epsilon_{kk}^c = 0$) yields the following stress-strain relationship:

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{rz} \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu^2)}{(1+\nu)} \end{bmatrix} \begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{rz} \end{Bmatrix} - \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu^2)}{(1+\nu)} \end{bmatrix} \begin{Bmatrix} \epsilon_{rr}^c \\ \epsilon_{zz}^c \\ \epsilon_{rz}^c \end{Bmatrix} - \frac{E\alpha\Delta T}{(1-\nu)} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \quad (50)$$

Symbolically

$$\{\sigma\} = [A]\{\epsilon\} - [D]\{\epsilon^c\} - \{\epsilon^T\} \quad (51)$$

Combining Equations (28), (29), (30), (47), and (51) and substituting into Equation (3) gives the potential energy expression which is minimized with respect to the nodal displacements resulting in the following equilibrium equations written in incremental form

$$[K] \{\Delta u\} = \{\Delta F^c\}. \quad (52)$$

$\{\Delta F^c\}$ represents the fictitious incremental creep forces for the plane stress case given by

$$\{\Delta F^c\} = \int_{vol} [h]^T [g]^T [D] \{\Delta \epsilon^c\} dV.$$

Since all of the quantities under the integral sign are constant within each element including the incremental creep strains, it is possible to write Equation (52) in the form of Equation (21).

$$\{\Delta u\} = [K]^{-1} [M] \{\Delta \epsilon^c\} \quad (53)$$

where $[M] = [h]^T [g]^T [D] \int_{Area} dA$. The volume integral reduces to an area integral since the triangular element was assumed to have unit thickness.

Upon selecting the specific form of the incremental creep law to be used, it is possible to define the incremental creep strains using Equations (23), (24), and (26).

$$\begin{Bmatrix} \Delta \epsilon_{rr}^c \\ \Delta \epsilon_{zz}^c \\ \Delta \epsilon_{rz}^c \end{Bmatrix} = \frac{\Delta \epsilon_e^c}{2\sigma_e} \begin{Bmatrix} 2\sigma_{rr} - \sigma_{zz} \\ 2\sigma_{zz} - \sigma_{rr} \\ 3\sigma_{rz} \end{Bmatrix} \quad (54)$$

$$\text{where } \sigma_e = \sqrt{\sigma_{rr}^2 - \sigma_{rr}\sigma_{zz} + \sigma_{zz}^2 + 3\sigma_{rz}^2}.$$

Thus, the incremental form of Equation (47), corresponding to Equation (20), together with Equations (53) and (54) represent the incremental relationships needed to solve the plane stress creep problem.

PLANE STRAIN ANALYSIS

The plane strain analysis uses the same basic stiffness element as used for the plane stress analysis and shown in Figure 3. The difference between the plane stress and plane strain analysis concerns the specific form of the stress-strain relationship. For the plane strain case the following conditions are assumed:

$$\epsilon_{yy} = \sigma_{ry} = \sigma_{zy} = 0; \epsilon_{yy}^c = 0. \quad (55)$$

Thus, Equation (34) specialized to the plane strain case becomes in matrix form,

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{rz} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ 0 & 0 & (1-2\nu) \end{bmatrix} \begin{Bmatrix} \epsilon_{rr}^E \\ \epsilon_{zz}^E \\ \epsilon_{rz}^E \end{Bmatrix} \quad (56)$$

To satisfy the condition ($\epsilon_{yy} = 0$), the stress normal to the plane of the triangular element is given by,

$$\sigma_{yy} = \nu(\sigma_{rr} + \sigma_{zz}). \quad (57)$$

Substituting the appropriate form of Equation (33) into Equation (56) and assuming that the creep strains are incompressible ($\epsilon_{kk}^c = 0$) yields the following stress-strain relationships:

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{rz} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ 0 & 0 & (1-2\nu) \end{bmatrix} \begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{rz} \end{Bmatrix} - \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ 0 & 0 & (1-2\nu) \end{bmatrix} \begin{Bmatrix} \epsilon_{rr}^c \\ \epsilon_{zz}^c \\ \epsilon_{rz}^c \end{Bmatrix} - \frac{E\alpha\Delta T}{(1-2\nu)} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \quad (58)$$

Comparing Equations (50) and (58) it is apparent that the plane stress equations can be converted to plane strain equations through the following material property transformations:

$$E_1 = \frac{E}{(1-\nu^2)}; \quad \nu_1 = \frac{\nu}{(1-\nu)}; \quad \alpha_1 = \alpha(1+\nu) \quad (59)$$

Thus, to solve a plane strain problem the material properties are modified by Equation (59) and substituted into the plane stress solution routine. The resultant solution corresponds to the solution for the plane strain problem.

The other modifications necessary for the plane strain problem are associated with the incremental creep strains derived from Equations (23), (24), and (26). For the plane stress case, $\sigma_{yy} = 0$, but for the plane strain case σ_{yy} is given by Equation (57). Therefore, the incremental strains are given by

$$\begin{Bmatrix} \Delta \epsilon_{rr}^c \\ \Delta \epsilon_{zz}^c \\ \Delta \epsilon_{rz}^c \end{Bmatrix} = \frac{\Delta \epsilon_e^c}{2\sigma_e} \begin{Bmatrix} (2-\nu)\sigma_{rr} - (1+\nu)\sigma_{zz} \\ (2-\nu)\sigma_{zz} - (1+\nu)\sigma_{rr} \\ 3\sigma_{rz} \end{Bmatrix} \quad (60)$$

where

$$\sigma_e = \sqrt{\sigma_{rr}^2 - \sigma_{rr}\sigma_{zz} + \sigma_{zz}^2 - 2\nu(1-\nu)\sigma_{rr}\sigma_{zz} + 3\sigma_{rz}^2}.$$

Note that the value used for Poisson's ratio (ν) should be the initial value before transformation. Since the material properties input to the program for the plane strain case will be the transformed values, Equations (60) should be written in terms of the transformed value of Poisson's ratio (ν_1).

$$\begin{Bmatrix} \Delta \epsilon_{rr}^c \\ \Delta \epsilon_{zz}^c \\ \Delta \epsilon_{rz}^c \end{Bmatrix} = \frac{\Delta \epsilon_e^c}{2\sigma_e} \begin{Bmatrix} \left[\frac{(2+\nu_1)}{(1+\nu_1)} \right] \sigma_{rr} - \left[\frac{(1+2\nu_1)}{(1+\nu_1)} \right] \sigma_{zz} \\ \left[\frac{(2+\nu_1)}{(1+\nu_1)} \right] \sigma_{zz} - \left[\frac{(1+2\nu_1)}{(1+\nu_1)} \right] \sigma_{rr} \\ 3\sigma_{rz} \end{Bmatrix} \quad (61)$$

$$\text{where } \sigma_e = \sqrt{\sigma_{rr}^2 - \sigma_{rr}\sigma_{zz} + \sigma_{zz}^2 - \left[\frac{(2\nu_1)}{(1+\nu_1)} \right]^2 \sigma_{rr}\sigma_{zz} + 3\sigma_{rz}^2}.$$

Thus, to solve a plane strain problem it is necessary to replace Equation (54) in the plane stress routine with Equation (61) and modify the material properties using Equation (59). The resultant solution from the plane stress routine corresponds to the solution for the plane strain problem.

TIME INCREMENT SELECTION

The success of the incremental procedure for solving creep problems depends upon the ability to select appropriate time intervals. In turn, the time intervals selected depend upon the specific form of the creep law [Equation (27)]

that is used. Thus, it is necessary to compute internally in the computer program the desired time increments.

As an example consider the following creep law:

$$\Delta \epsilon_e^C = C 10^{MT} \sigma_e^n \Delta t \quad (62)$$

where C, M, n are constants associated with different materials,

T = temperature (°F)

t = time (hr)

Since the first step in the incremental procedure is to solve the elastic problem, the elastic values for the effective strain are known for each of the triangular finite elements

$$\epsilon_e^E = \frac{1}{E} \sigma_e \quad (63)$$

Thus, the initial time increment can be found by setting the initial incremental effective creep strain equal to a fraction (η_0) of the maximum value for the effective elastic strain.

$$\Delta \epsilon_e^C = \eta_0 (\epsilon_e^E)_{\max}. \quad (64)$$

Substituting Equations (62) and (63) into (64) gives the following values for the initial time increment.

$$\Delta t_1 = \eta_0 \sigma_e^{(1-n)} / 10^{MT} C E \quad (65)$$

Experience with the program has shown that the incremental numerical procedure becomes unstable when the maximum incremental effective creep strain for a time interval exceeds the maximum effective elastic strain. This condition limits the maximum time increment that can be used for any time interval.

The specific value used for η_0 depends upon the particular problem. The above limiting conditions restrict η_0 to fall within the following range

$$1/25 \leq \eta_0 \leq 1/10.$$

Subsequent time intervals will be computed on the basis of the fractional change in maximum effective stress allowed per time interval

$$\Delta t_{i+1} = (\Delta t_i) \eta / (\Delta \sigma_e / \sigma_e)_{\max} \quad (i = 1, 2, \dots) \quad (66)$$

where η = assumed fractional change in stress allowed for each time interval. For each problem, η must be selected keeping in mind that time increments are chosen sufficiently small such that the stresses may be assumed to be constant within each time interval. Experience has shown that η should fall within the following range.

$$0.03 \leq \eta \leq 0.10$$

One further limiting condition is placed upon the time interval selection $\eta / (\Delta \sigma_e / \sigma_e)_{\max} \leq D$ where D is a constant ranging in values from 1.2 to 2.0 depending upon the specific problem. This condition restricts the maximum increase in the time increment for any time interval and is imposed to assure convergence of the solution.

STEADY-STATE SOLUTION

For some creep problems, a steady-state condition (stress state does not vary with time) is reached after which creep strains and deformations can be extrapolated to the final time of interest without taking incremental steps. Two tests are made to establish the steady-state condition. If the maximum fractional change in effective stress is less than ρ , then the program extrapolates the results to the final time.

$$(\Delta \sigma_e / \sigma_e)_{\max} \leq \rho$$

The other test establishes that if the maximum change in effective stress per hour is less than β , then the program extrapolates the results to the final time.

$$(\Delta \sigma_e / \Delta t)_{\max} \leq \beta$$

The specific problem to be solved and the type of results desired dictate the values to be used for ρ and β .

APPLICATIONS

The finite element computer code "AXICRP" has been applied to obtain creep solutions for typical examples of the three classes of problems that it is capable of solving.

AXISYMMETRIC BODY OF REVOLUTION PROBLEM

An infinitely long, thick-walled cylinder with open ends subject to internal pressure was selected as an example for the axisymmetric body of revolution problem. For this problem the displacement along the axis of the cylinder is assumed to vanish. The closed form solution for this problem, assuming an incompressible material, has been obtained by Odqvist. Since the incompressibility condition ($\nu = 1/2$) represents a singularity in the AXICRP solution, the problem was solved for ($\nu = 0.30$). The circumferential and radial stresses for the elastic and steady state creep solution can still be checked against Odqvist's results. This is possible because the circumferential and radial stresses for the limiting conditions (elastic and steady state creep) are independent of the material properties.

Figure 4 shows the finite element idealization necessary to solve the problem. Each of the geometric subdivisions represents a triangular cross-sectional circular ring. The material properties, assumed creep law, internal pressure loading, and necessary boundary conditions are given in Figure 4.

Figure 5 gives the circumferential and radial stress distribution for the elastic and steady-state conditions. The finite element results agree quite well with Odqvist's solution. Figure 6 gives the axial and effective stress distribution for the elastic and steady-state solutions. Observe that the axial stress for the elastic solution is initially constant through the radius, but the steady-state axial stress distribution shows

Pressure: 365 psi
 Young's Modulus: 20×10^6
 Poisson's Ratio: 0.30
 Creep Law: $\epsilon_e^c = 6.4 \times 10^{-18} \sigma_e^{4.4} t.$

FIGURE 4. Finite Element Idealization of a Thick-Walled Cylinder

FIGURE 5. Circumferential and Radial Stress Distribution

FIGURE 6. Axial and Effective Stress Distribution

a marked variation through the radius. Figure 7 shows the time variation of the effective stress (σ_e) at the inside and outside surfaces.

$$\sigma_e = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{rr} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{\theta\theta})^2 + 6\tau_{rz}^2}$$

For the particular creep law used, the steady-state condition is reached after about 2 hr. The plotted points indicate the number of time intervals and the time interval size used to solve the problem. Figure 8 shows the time variation of the radial deformation on the inside and outside surface.

These results are indicative of the type of results that can be obtained using the portion of the AXICRP program which handles axisymmetric bodies of revolution. This capability could be used to perform a creep analysis of reactor nozzles considering internal pressure loading and axisymmetric temperature variation. The program is also applicable to the creep analysis of the axisymmetric two-dimensional fuel pin problem including the end caps and the associated localized stress effects.

PLANE STRAIN PROBLEM

The plane strain option has been used to analyze the creep behavior of a typical flow duct. Figure 9 shows the cross-section of a typical flow duct and the finite element geometrical idealization needed to analyze it. Symmetry makes it necessary to analyze only section A. The particular plane strain section analyzed was assumed to have an internal pressure loading of 120 psi at a temperature of 1000 °F. The particular creep law used

$$\dot{\epsilon}_e^C = (5.6 \times 10^{-54.05}) 10^{(0.0197)T} \sigma_e^{6.35}$$

is for solution treated 316 stainless steel.

Figure 10 shows the effective stress distribution through the cross-section at the corner of the flow duct for different

FIGURE 7. Effective Stress Variation with Time

FIGURE 8. Radial Deformation Variation with Time

FIGURE 9. Finite Element Idealization of Flow Duct

FIGURE 10. Effective Stress Distribution Through the Cross-Section at the Corner of the Flow Duct ($T = 1000^\circ\text{F}$)

times. Observe that the elastic solution shows significant stress concentration at the inner boundary (21.66 ksi on the inner boundary and 11.6 ksi on the outer boundary). As the stresses relax with time, the effective stress on the inner boundary decreases while the effective stress on the outer boundary increases. This redistribution of stress from the inner to the outer boundary continues until the stress concentration effect at the inner boundary is essentially eliminated. Figure 11 shows the effective stress distribution with respect to time on the inner and outer boundary. (Actual values plotted are for points 0.0071 in. inside the outer boundaries). The effective stress on the outside surface increases for the first 8000 hr, after which time the effective stress on both the inner and outer surface continues to decrease. For the particular creep law used, a steady-state condition has not been reached after 16,800 hr.

Figure 12 gives the effective creep strain variation with time at the inside corner of the flow duct. At the mid-plane of the flat the maximum effective stress occurs at the outside surface as shown in Figure 13. Figure 14 shows the flat mid-point displacement as a function of time. The initial elastic displacement was 1.84 mils and after 20,000 hr the total deformation is only 2.15 mils. Thus, for this particular creep law at 1000 °F, the flat-to-flat increase due to creep would only be 0.6 mils in 20,000 hr. However, Figure 15 shows that this same creep law at 1200 °F would result in a flat-to-flat increase due to creep of 8.82 mils in only 100 hr. Figure 16 gives the effective stress variation at the inside and outside corners for the higher temperature. Note that a steady-state condition is being approached in about 80 hr at this temperature. To establish the numerical stability of the solution procedure, time intervals were selected on the basis of a 5 and 10% stress change per time

FIGURE 11. Effective Stress at the Corners of the Flow Duct ($T = 1000\text{ }^{\circ}\text{F}$)

FIGURE 12. Effective Creep Strain at the Corner of the Flow Duct ($T = 1000\text{ }^{\circ}\text{F}$)

FIGURE 13. *Effective Stress Distribution Through the Mid-Plane of the Flat ($T = 1,000^\circ\text{F}$)*

FIGURE 14. Flat Mid-Point Displacement ($T = 1,000^{\circ}F$)

FIGURE 15. Flat Mid-Point Displacement ($T = 1,200\text{ }^{\circ}\text{F}$)

FIGURE 16. Effective Stress at the Corners of the Flow Duct ($T = 1,200^{\circ}\text{F}$)

interval. These results are shown in Figure 16 and indicate that the numerical procedure, within limits, is quite stable. Those limits are discussed in the section on time increment selection.

PLANE STRESS PROBLEM

A simply supported beam under a concentrated center load was selected as the example of a plane stress problem. Figure 17 shows the geometrical properties of the beam and the finite element idealization and boundary conditions used to solve the problem. For this particular example, the node numbers and structural elements (circled numbers) which must be input as data for the program are designated. All other node numbers and structural elements are generated internally by the program.

Figure 18 shows the redistribution with time of the longitudinal stresses through a cross-section of the beam ($r = 9.8$). For the particular creep law used ($\epsilon_e^C = 6.4 \times 10^{-18} \sigma_e^{4.4} t$) and the assumed concentrated load ($P = 333 \text{ lb}$), the stresses redistribute themselves very rapidly as shown by the curve for $t = 0.01 \text{ hr}$. Figure 19 shows the redistribution of shear stress for two different cross sections ($r = 4.6$ and 9.8). Note the large increase in shear stress along the neutral axis for the cross-section ($r = 9.8$). Figure 20 shows the effective stress distribution for the same cross-sections, where

$$\sigma_e = \sqrt{\sigma_{rr}^2 - \sigma_{rr}\sigma_{zz} + \sigma_{zz}^2 + 3\sigma_{rz}^2}.$$

Figure 21 indicates the deformed shape of the neutral axis at different times. Observe that the maximum elastic displacement was 0.029 in. and after 0.01 hr, the maximum displacement was 0.124 in. Thus, creep deformation of a beam can be very significant depending upon the magnitude of the loading, creep law for the material, and the environmental conditions under which the system operates.

FIGURE 17. Finite Element Idealization of a Simply-Supported Beam

FIGURE 18. Redistribution of Beam Longitudinal Stress

FIGURE 19. Redistribution of Beam Shear Stress

FIGURE 20. Redistribution of Effective Stress

FIGURE 21. Creep Deformation of a Beam

The plane stress idealization of this beam problem underestimates the maximum elastic longitudinal stress and maximum elastic displacement predicted by conventional beam theory by about 10%. This deviation from beam theory should be kept in mind when solving beam problems using the plane stress option.

These examples and plotted results are only indicative of the variety of problems and detailed creep results that can be obtained by application of the AXICRP program.

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APPENDIX A
INPUT/OUTPUT DESCRIPTION

PROGRAM INPUT

The first step in a finite element analysis is to obtain a satisfactory geometrical idealization of the structure to be analyzed. This idealization is achieved by breaking the structure into a number of discrete elements. Small elements are necessary where the spatial variation of the stresses is rapid and larger elements may be used where the spatial stress variation is small. The basic finite element used in this program for idealization is the quadrilateral element. The triangular elements are used to bridge the gap between coarse and fine structural idealizations. The program has automatic data generation capability associated with the quadrilateral element, and this will be elaborated upon in the section dealing with element data.

For each creep problem it is necessary to select an appropriate incremental creep law corresponding to Equation (22). Based upon this creep law it is then necessary to make the following modifications in the program:

1. The first time increment calculation must be modified to incorporate the new creep law.
2. The incremental effective creep strain must be calculated from the new creep law.

More details concerning these changes are given in the following discussion of the control cards.

Control cards for a standard AXICRP problem should be as follows (^ denotes 1 or more blank spaces)

Column 1	
1.	∇ Run card as desired
2.	$\nabla^{\wedge} \text{ASG}^{\wedge} \text{A} = \text{U2399 (AXICRP)}$
3.	$\nabla^{\wedge} \text{XQT}^{\wedge} \text{CUR}$
4.	$\text{IN}^{\wedge} \text{A}$
5.	$\text{TRI}^{\wedge} \text{A}$
6.	$\nabla^{\wedge} \text{XQT}, \lambda^{\wedge} \text{AXICRP}$
	(where λ is A, B, C, . . . according to table below)
	DATA
	$\nabla^{\wedge} \text{FIN}$

Three versions of STRESS, each of which corresponds to a different creep law, are currently on tape U2399; more can be added by updating and renaming one of the present elements. The following cards might be used to create a version D. (This version can be added to the user tape without affecting versions A, B, and C.)

Column 1
$\nabla \text{WITH}^{\wedge} \text{FR5}, *^{\wedge} \text{STRSSA}, \text{STRSSD}, \text{STRSSD}/\text{CODE}(\text{D})$
-173, 173
(Fortran card or cards for DT)
-242, 243
(Fortran cards for output print of new formula)
-304, 304
(Fortran card)(s) for $\text{ESTRN}(\text{N}, \text{I})$
Card 6 above would then read
$\nabla^{\wedge} \text{XQT}, \text{D}^{\wedge} \text{AXICRP}$

The three versions presently available are

Version	Creep law components
A	$DT = ESMAX^{**}(1.-4.4)/(10.*6.4E-18*ERMAX)$ $ESTRN(N,I) = 6.4E-18*[STRS(N,I,5)**4.4]*DT$
B	$DT = ESMAX^{**}(-5.35)/(10.*AC*10^{**}(0.0197*TEMM)$ $*ERMAX *10^{**}(-19.05)$ <p style="text-align: center;">where AC = 5.6E-35</p> $ESTRN(N,I) = 5.6E-35*10^{**}(0.0197*TEM)*STRS(N,I,5)$ $**6.35*10^{**}(-19.05)*DT$
C	$DT = (\text{same as version B}) *10^{**}(0.00394*TEMM)$ $ESTRN(N,I) = (\text{same as version B}) *10^{**}(0.00394*TEM)$ <p>This has the effect of creep at a temperature which is 1.2 times the input temperature</p>

The symbols above are:

DT - Initial time increment
 ESMAX - Maximum effective stress
 ERMAX - Modulus of elasticity for the element where
 ESMAX occurs
 ESTRN (N,I) - Effective creep strain for the I-th
 triangle in element N
 TEM - Temperature of element N, triangle I
 TEMM - Temperature of element with greatest stress

The following group of data cards is listed in the order in which they should appear in the program deck.

1. IDENTIFICATION CARD - (72H)
 Columns 1 - 72 Identify the particular problem to be solved
2. CONTROL CARD - (4I5, 20X1F10.2, 5X2I5)
 Columns 1 - 5 Number of Nodal points (550 maximum)
 6 - 10 Number of elements (500 maximum)

- 11 - 15 Number of different materials
(12 maximum)
- 16 - 20 Number of boundary pressure cards
(200 maximum)
- 41 - 50 Reference temperature
- 56 - 60 = 0 Axisymmetric analysis
= 1 Plane stress analysis
= 2 Plane strain analysis
- 61 - 65 Number of elements (NPRTTEL) to be printed
at each time step ($0 < \text{NPRTTEL} \leq \text{number of elements}$)

2a. PRINT INFORMATION - (16I5)

Columns 1 - 5 } Element numbers for which the results
6 - 10 } are to be printed for each time interval.

3. CREEP SOLUTION TIME INFORMATION - (8F10.0)

Columns 1 - 10 Maximum number of time iterations

11 - 20 Maximum time span

21 - 30 Minimum fractional change in effective stress allowed for each time interval.

If the change is below this value the program will extrapolate to final time.

31 - 40 Maximum fractional change in effective stress allowed for each time interval.

41 - 50 Maximum change in effective stress allowed between print intervals

51 - 60 = 0 No printing of initial triangular stresses
= 1 Initial stresses for each triangle are printed.

61 - 70 Maximum relative increase in time interval allowed.

71 - 80 Minimum change of effective stress per hour.
If the change is below this value the program will extrapolate to final time.

For further clarification of these input parameters consult the section on time interval selection.

4. MATERIAL PROPERTY INFORMATION

The following group of cards must be supplied for each different material. Note the elastic part of the program can handle anisotropic material behavior. The creep program only considers isotropic material behavior and requires that Young's Modulus (E), Poisson's ratio (ν), and the coefficient of thermal expansion (α) each have a single value. These values should be inserted into the data card wherever a value for the material property is called for.

Columns 1 - 5 Material identification (any number from 1 to 12)

6 - 10 Number of different temperatures for which properties are given (8 maximum)

Following Cards - One card for each temperature and the specific format depends upon the type of problem (axisymmetric, plane stress, plane strain analysis).

If Axisymmetric Analysis: (7F10.0)

Columns 1 - 10 Temperature

11 - 20 Modulus of elasticity, E_r and E_z

21 - 30 Poisson's ratio, ν_{rz}

31 - 40 Modulus of elasticity, E_θ

41 - 50 Poisson's ratio, $\nu_{\theta r}$ and $\nu_{\theta z}$

51 - 60 Coefficient of thermal expansion, α_r and α_z

61 - 70 Coefficient of thermal expansion, α_θ

If Plane Stress Analysis: (4F10.0, 10X1F10.0)

Columns 1 - 10 Temperature

11 - 20 Modulus of elasticity, E_r

21 - 30 Poisson's ratio, ν

31 - 40 Modulus of elasticity, E_z

51 - 60 Coefficient of thermal expansion, α

If Plane Strain Analysis: (4F10.0, 10X1F10.0)

Columns 1 - 10 Temperature

11 - 20 Modulus of elasticity, $(E_r)_1 = E_r/(1-\nu^2)$

21 - 30 Poisson's ratio, $\nu_1 = \nu/(1-\nu)$

31 - 40 Modulus of elasticity, $(E_z)_1 = E_z/(1-\nu^2)$

51 - 60 Coefficient of thermal expansion, $\alpha_1 = \alpha(1+\nu)$

5. NODAL POINT CARDS - (I5, F5.0, 5F10.0)

One card for each nodal point with the following information:

Columns 1 - 5 Nodal point number

6 - 10 Number which indicates if displacements
or forces are to be specified.

(Code - 0, 1, 2, or 3 - see below)

11 - 20 r - ordinate

21 - 30 z - ordinate

31 - 40 XR

41 - 50 XZ

51 - 60 Temperature

The significance of XR and XZ depends upon the number in column 10.

If the number in column 10 is

0 XR is the specified r-load and
XZ is the specified z-load

1 XR is the specified r-displacement and
XZ is the specified z-load

2 XR is the specified r-load and
XZ is the specified z-displacement

3 XR is the specified r-displacement and
XZ is the specified z-displacement

If the number in columns 6-10 of the nodal point cards is other than 0, 1, 2 or 3 it is interpreted as the magnitude of an angle in degrees. This angle is shown in the following figure.

The angle θ must always be input as a negative angle and may range from -0.001 to -180 degrees.

If the number in column 6-10 is

θ XR is the specified load in the s-direction and

XZ is the specified displacement in the n-direction.

The displacements of these nodal points have the following significance:

$XR \sim u_r$ = displacement in the s-direction

$XZ \sim u_z$ = displacement in the n-direction.

All loads are considered to be total forces acting on a one radian segment (or unit thickness in the case of plane stress or plane strain analysis).

Automatic Nodal Data Generation

Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the specified nodal points. For the automatically generated nodal points the necessary temperatures are determined by linear interpolation. The boundary code (column 10) is set equal to zero, and the specific values for XR and XZ are set equal to zero. If these automatically generated boundary conditions are not appropriate, then it will be necessary to insert a nodal point card for each nodal point along a boundary.

6. ELEMENT CARDS - (6I5)

One card for each element

Columns 1 - 5 Element number

6 - 10 Nodal point I

11 - 15 Nodal point J

16 - 20 Nodal point K

21 - 25 Nodal point L

26 - 30 Material identification

Triangular elements are permissible and are identified by repeating the last nodal point numbers (I,J,K,K). For both triangular and quadrilateral elements it is necessary to order the nodal points counter-clockwise around the element.

Automatic Element Data Generation

Element cards must be in element number sequence. If element cards are omitted the program automatically generates the omitted information by incrementing by one the preceding I,J,K,L. The material identification code for the program generated cards is set equal to the value given on the last card. The last element card must always be supplied. The following example illustrates the automatic data generation capability of the program.

Using the automatic data generation capabilities it is only necessary to provide nodal point cards for nodes 1,4,5,8, 9, and 12. Likewise, element data cards are necessary only for elements (1), (4) and (6). For element (1) the ordering

of the nodal points around the element should be (1,2,6,5) corresponding to (I,J,K,L). All other input elements should be ordered in the same manner. Element (2) ordering of nodal points can be obtained by incrementing by one the (I,J,K,L) results for element (1). Notice that the particular sequence of nodal point and element numbering is essential for successful automatic data generation.

An element card must be submitted for each triangular element since no automatic data generation capability exists for triangular elements.

7. PRESSURE CARDS - (2I5, F10.0)

One card for each boundary element which is subjected to a normal pressure.

Columns 1 - 5 Nodal point I

6 - 10 Nodal point J

11 - 20 Normal pressure

By convention, the boundary element must be on the left as one progresses from nodal point I to J. Surface tensile force is input as negative pressure.

OUTPUT DESCRIPTION

The program output is a complete listing of all the input data, including the internally generated nodal point and structural element data. This permits a check to be made of all input and internally generated data.

For the initial elastic solution the stresses and effective stress are printed for each element. The displacements are printed for each nodal point. The stresses for the quadrilateral elements represent the average of the stresses from four triangles making up the quadrilateral. If individual triangular stresses are desired, then a specific parameter designated in the input data results in a print of the initial stresses for each of the triangles. The maximum effective stress within the structure and the element in which this maximum occurs are printed for the initial elastic solution and all subsequent solutions at a later time.

After the initial elastic solution, it is possible to select the specific elements for which the time-dependent creep results are printed. This option is provided to limit the size of the output report for each time interval. At the end of each time interval, the stresses, effective stress, effective creep strain, and total displacements for each nodal point are printed for each of the specified elements. For each time interval, the maximum change in effective stress per hour and the maximum value of $\Delta\sigma_e/\sigma_e$ is printed, and the elements in which these maximums occur are designated.

The end result of the AXICRP analysis is a complete time history for the stresses, effective stresses, effective creep strains, and deformations.

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