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THEORETICAL REMARKS ON THE A_2 MESON*

MASTER

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Abstract

The A_2 is in no way anomalous regarding its SU(3) properties or production mechanism. Evidence regarding substructure (or lack thereof) in the peak is discussed with particular regard to any further experiments that may be planned.

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I. Introduction.

The source of recent interest in the resonance shape of the A_2 meson was the report of a two-peak structure^{1,2} observed in the missing-mass spectrum of X^- , $m(X^-) \approx 1300$ MeV, in

$$\pi^- p \rightarrow A_2^- p \quad (1)$$

for $p_{\pi^-} = 6$ and 7 GeV/c. Since 1967, when the effect was first considered to be statistically significant,² additional experiments under different conditions have given conflicting results.^{3,4} The results of the first attempt to repeat the exact experiment of ref. 2 have been announced at this conference,⁵ and they fail to confirm the effect.^{5,6}

Given the parameters that one thinks are important in (1) - the beam momentum, and the angle, multiple scattering, and final momentum of the recoil proton, etc. - as quoted in refs. 1-2 ("CERN") and 5-6 ("Northeastern-Stony Brook" or "NE-SB"), the two sets of data contradict one another.⁶ However, some models for the two-peak structure are more sensitive than others to variations in these parameters away from their quoted values. For this reason it may be useful to recall some theoretical aspects of the A_2 and its possible substructure despite the feeling on the part of many physicists that the splitting² may be irreproducible.

The role of the unsplit A_2 in our picture of the strongly interacting particles will be treated in section II, with particular regard to SU(3) properties. We shall discuss A_2 production mechanisms (sec. III) since one seems to have learned a great deal about them in recent high statistics experiments. Any discussion of possible splitting would require such information, as the effect is certainly dependent on s and probably on t if it is present at all.

Section IV deals with some experimental results on the splitting and their interpretation. We suggest that any further experiments to look for A_2 substructure reject the dipole⁷ hypothesis in favor of the (much more demanding) possibility of the interference of a broad

($\Gamma \approx 90$ MeV) state with a narrow one ($\Gamma \approx 5$ MeV). Section V contains our conclusions.

II. Features of the unsplit A_2 .

The existence of the A_2 was inferred⁸ from arguments similar to considerations of exchange degeneracy.⁹ (The K^{**} was also predicted in this manner). The broad "A" peak was clearly resolved into what we now call the A_1 and the A_2 fairly early.¹⁰ Observation of $K^- K^0$, $K_S^0 K_S^0$, and $(3\pi)^-$ modes near the same mass (1310 MeV) suggested the existence of a 2^+ resonance there, while the lower mass "A₁" effect has continued to elude unambiguous identification as a resonance.¹¹

The A_2 appears to form a nonet with the other 2^+ resonances $f_0(1260)$, $K^{**}(1420)$, and $f'(1515)$.¹² Recent experiments have been published on branching ratios of the (unsplit) A_2 ¹³ and the K^{**} .¹⁴ These new numbers have been used in a comparison of decay rates for $2^+ \rightarrow 1^- 0^-$ and $2^+ \rightarrow 0^- 0^-$ based on SU(3). From the observed partial widths, one derives reduced amplitudes^{15,16} $\tilde{D}_i^2$ based on the expression

$$\tilde{D}_i^2 = (M_i^2/p)(p_0/p)^4 \Gamma_i/C_i \quad (2)$$

where M_i is the mass of the decaying resonance, p is the 3-momentum of either decay product in the CM, p_0 is an arbitrary constant taken to be 0.5 GeV/c for normalization, Γ_i is the partial width for the decay process i , and C_i is the square of an appropriately defined Clebsch-Gordan coefficient. The values of $\tilde{D}_i^2$ are shown in Fig. 1.¹⁷ The two halves of this figure should lead to two independent consistent values of $\tilde{D}_i^2$ for couplings obeying SU(3), and to a single value of $\tilde{D}_i^2$ for various SU(6)-like models.^{16,18,19}

One sees reasonable agreement with SU(3) and some possible deviation from SU(6) in the same direction as for the coupling ratio $(1^- \rightarrow 1^- 0^-)/(1^- \rightarrow 0^- 0^-)$.²⁰ As already noted,²¹ to preserve SU(3) one would be reluctant to tamper with some of the partial widths in Fig. 1 too much without reestimating others as well. This would have been the case if (say) half of the A_2 peak had been ascribed to some

other state not related to the f , K^{**} , and f' .

The A_2 is often described as a 3P_2 state of a quark (q) and an antiquark ($\bar{q}$).²²⁻²⁶ If the $q\bar{q}$ spectrum is roughly that of a harmonic oscillator, 3P_1 , 3P_0 , and 1P_1 $q\bar{q}$ states lie somewhere nearby in mass. One has reasonable candidates for all but a handful of these states.²⁴⁻²⁶ Furthermore, there appear to be no additional $P = +$ mesons²⁷ below 1.5 GeV, indicating little distortion of the harmonic oscillator sequence^{23,24}

$$\begin{array}{ccccccc}
 n = 0 & & n = 1 & & n = 2 & & n = 3 \\
 q\bar{q}: & L = 0 & < & L = 1 & < & L = 2, 0 & < & L = 3, 1 & < \dots & (3) \\
 & P = - & & P = + & & P = - & & P = + & &
 \end{array}$$

a fact of interest in connection with mechanisms for possible A_2 splitting.²⁶ In this respect, two states bear watching:

(a) The $E(1420)$, if 0^- ,²⁸ indicates some depression of the masses of the first radially excited states ($n = 2$, $L = 0$).

(b) The J^P assignment 1^+ has not been ruled out for the $I = 1$ $F_1(1540)$,²⁹ produced in $\bar{p}p$ annihilations. This state would have to come from the $n = 3$ group in eq. (3), if the A_1 and B were to fill the similar $n = 1$ slots as is usually assumed.²⁴⁻²⁶

Claims^{4,30} for additional low-lying states with ordinary I , Y , and J^{PC} but which strain the scheme (3) would be every bit as important as the discovery of low-lying "exotic" mesons,³¹ against which there is considerable evidence.³⁰ For the time being, the $q\bar{q}$; L picture seems as good as any other for classifying the broad, easily-observed low-lying mesons.

III. Production mechanism in $\pi N \rightarrow A_2 N'$.

The A_2 can be produced in the following pion-induced reactions with a recoil nucleon:

$$\pi^- p \rightarrow A_2^- p \quad (4a)$$

$$\pi^+ p \rightarrow A_2^+ p \quad (4b)$$

$$\pi^+ n \rightarrow A_2^0 p \quad (4c)$$

$$\pi^- p \rightarrow A_2^0 n \quad (4d)$$

identical by
isospin

In the best-studied of these, process (4a), the mechanism suggested by all data above 5 GeV/c is f_0 and ρ exchange, with f_0 accounting for at least 75 per cent of the cross section at high energies, and with little coherence between the two. In addition there appears to be some evidence for B exchange at lower energies. Furthermore, present data do not rule out a small contribution from an $I = 0$ trajectory with intercept greater than $1/2$. We now discuss the evidence.

A. Isospin structure of exchanges.

One may write, summing over t -values and helicity amplitudes i ,

$$\begin{aligned} \sigma(A_2^\pm) &= \sum_i |M_i^{(0)}|^2 + \sum_i |M_i^{(1)}|^2 + \\ &\pm 2 \sum_i \text{Re}(M_i^{(0)*} M_i^{(1)}) \end{aligned} \quad (5)$$

$$\sigma(A_2^0) = 2 \sum_i |M_i^{(1)}|^2 \quad (6)$$

where the A_2 charge state refers to the appropriate reaction (4), and the superscript refers to the exchanged isospin. Then

$$\sigma_1 \equiv \sum_i |M_i^{(1)}|^2 = \sigma(A_2^0)/2 \quad (7)$$

$$\sigma_0 \equiv \sum_i |M_i^{(0)}|^2 = [\sigma(A_2^+) + \sigma(A_2^-) - \sigma(A_2^0)]/2 \quad (8)$$

Furthermore, one can define a coherence fraction f_c ($|f_c| \leq 1$) by

$$f_c \equiv \frac{\operatorname{Re} \sum_i M_i^{*(0)} M_i^{(1)}}{\sqrt{\sum_i |M_i^{(0)}|^2} \sqrt{\sum_i |M_i^{(1)}|^2}} = \frac{\sigma(A_2^+) - \sigma(A_2^-)}{4 \sqrt{\sigma_0 \sigma_1}} \quad (9)$$

The value of f_c can be close to 0 for many reasons ($I_t = 0$ and $I_t = 1$ 90° out of phase for given i ; $M_i^{(0)}$, $M_j^{(1)}$ nonzero only for $i \neq j$, etc.), so that deviations from 0 would be a stringent restriction on exchange models.

A comparison of σ_1 and σ_0 has been carried out at 5 and 7.5 GeV/c;³² from this work it is also possible to estimate f_c . The results are shown in Table I. The cross sections represent only those for $A_2 \rightarrow 3\pi$, with the $A_2^\pm$ cross sections based on $A_2^\pm \rightarrow \pi^+ \pi^\pm \pi^-$ and multiplied by 2 for the unseen $\pi^+ \pi^0 \pi^0$ modes.³³

Table I. Comparison of A_2 production cross sections at 5 and 7.5 GeV/c. (From Ref. 32). Reactions (4a-c).

	$p_L = 5$	$p_L = 7.5$
$\sigma(A_2^+, \mu b)$	146 ± 24	94 ± 16
$\sigma(A_2^-)$	124 ± 12	88 ± 12
$\sigma(A_2^0)$	112 ± 28	50 ± 15
σ_1	56 ± 14	25 ± 8
σ_0	79 ± 20	66 ± 12
$R = \sigma_0 / (\sigma_0 + \sigma_1)$	$.59 \pm .11$	$.72 \pm .10$
$\sigma(A_2^+) - \sigma(A_2^-)$	22 ± 29	6 ± 20
f_c	$.08 \pm .13$	$.04 \pm .14$

Table I shows that:

- (i) σ_0 is dominant already at 5 GeV/c and perhaps more so at 7.5.
- (ii) σ_1 is non-negligible at both momenta.
- (iii) No coherence between $I_t=0$ and $I=1$ amplitudes can be demonstrated.³² This conclusion is strengthened by the study³² of 2^+-1^+ interference, in which the $I_t=0$ amplitude for A_1 production and that for A_2 production display a large amount of coherence. No such

coherence is seen between $I_t=0(A_1)$ and $I_t=1(A_2)$.

The $I_t=0$ amplitude probably becomes even more important relative to $I_t=1$ at higher energies. A comparison of reactions (4a) and (4c) at various energies³⁴ indicates that some portion of the $I_t=1$ amplitude is probably vanishing more rapidly than one would expect from pure ρ exchange. This comparison is shown in Fig. 2. Even if one were to fit both sets of data with an effective $1/p_L$ fall-off, however, one would expect $\sigma(A_2^0) \lesssim \sigma(A_2^-)/2$, and, assuming incoherence³² to hold in general, one has $2\sigma_1 \lesssim (\sigma_1 + \sigma_0)/2$ or

$$\sigma_1 \lesssim \sigma_0/3 \quad (10)$$

the result quoted earlier.

The best fit to reaction (4c) favors a fall-off more as p_L^{-2} ,³⁴ suggestive of a large contribution from a lower-lying trajectory such as B.³⁷ If this is the case, $I=0$ exchange becomes even more dominant in reaction (4a) at high energies.

The horizontal line in Fig. 2a shows that a leveling off of the cross section for (4a) cannot be excluded. Again, this would augment the dominance of $I_t=0$ at high energies.

B. Density matrices.

Above 5 GeV/c, reactions (4a) and (4b) seem to lead to A_2 with $J_z = \pm 1$ in its rest frame (z-axis: incident pion direction $\hat{q}$; y-axis: $\hat{p}' \times \hat{q} / |\hat{p}' \times \hat{q}|$ where $\hat{p}'$ is the direction of the recoil proton.³⁹ This is the "t-channel" or "Gottfried-Jackson" helicity system.) A compilation of density matrices for these reactions, averaged over t ,⁴⁰ is shown in Table II and Fig. 3.

The figure shows a tendency toward $\rho_{11} \approx \rho_{1-1} \approx 1/2$ at high energy, a fact which favors natural parity exchange.^{58,59}

At lower energies the indications of nonzero ρ_{00} would require the exchange of some unnatural parity trajectory. If this were to be the B(as expected; see Ref. 37) a large ρ_{00} should occur in the charge-exchange reactions (4c-d) at similar energies.

Table II. A_2 density matrices (Gottfried-Jackson system) in $\pi^\pm p \rightarrow A_2^\pm p$.

π^+	p_L	Ref.	Mode	ρ_{00}	ρ_{11}	ρ_{22}	ρ_{1-1}
3.7	42		$\eta \pi^+$	$.15 \pm .12$	$.42 \pm .06$	$.00 \pm .06$	$.20 \pm .11$
5	43,41		$(3\pi)^+$	$.03 \pm .07$	$.45 \pm .06$	$.03 \pm .07$	$.16 \pm .07$
5.4	44,41		$(3\pi)^+$	$.09 \pm .06$	$.46 \pm .03$	(set=0)	$.37 \pm .07$
"8"	45,41		$(3\pi)^+$	$.04 \pm .07$	$.42 \pm .06$	$.06 \pm .06$	$.42 \pm .09$
π^-							
4.5	46		$K^- K_S^0$	$.19 \pm .05$	$.34 \pm .03$	$.06 \pm .04$	$.24 \pm .04$
5	47,41		$(3\pi)^-$	$.10 \pm .05$	$.43 \pm .04$	$.02 \pm .05$	$.36 \pm .05$
7	48,41		$(3\pi)^-$	$.12 \pm .11$	$.38 \pm .08$	$.06 \pm .10$	$.31 \pm .10$
7	49,50		$K^- K_S^0$	$\leq .07$	$.45 \pm .02$	$.05 \pm .02$	$.37 \pm .05$
7.5	47,41		$(3\pi)^-$	$.06 \pm .07$	$.44 \pm .05$	$.03 \pm .06$	$.33 \pm .07$
5-16	53		$\eta \pi^-$	$.11 \pm .13$	$.48 \pm .06$	$.03 \pm .06$	$.49 \pm .10$
11	54,41		$(3\pi)^-$	$.08 \pm .13$	$.40 \pm .11$	$.06 \pm .13$	$.34 \pm .15$
13	55,41		$(3\pi)^-$	$.05 \pm .16$	$.44 \pm .17$	$.04 \pm .20$	$.35 \pm .17$
"17"	56,41		$(3\pi)^-$	$.09 \pm .07$	$.43 \pm .06$	$.03 \pm .07$	$.38 \pm .07$
17.2	35		$K^- K_S^0$	$\simeq 0$	$\simeq .5$	$\simeq 0$	$\simeq .5$
20	55,41		$(3\pi)^-$	$.04 \pm .17$	$.41 \pm .14$	$.06 \pm .15$	$.35 \pm .15$
25	34,57,41		$(3\pi)^-$	$.07 \pm .32$	$.39 \pm .27$	$.08 \pm .33$	$.36 \pm .34$

The suppression of ρ_{22} is a feature⁵⁹ which may be understood by a simple argument based on exchange degeneracy. Consider the process

$$\pi^+ \pi^+ \rightarrow A_2^+ \pi^+ \quad (11)$$

which involves f_0 and ρ exchange in the t channel. The s channel is exotic, so one expects the imaginary parts of the f_0 and ρ exchange contributions to cancel. Consider the amplitude for (11) with A_2 t -channel helicity 2. It cannot have a ρ pole, so that the ρ residue must have a zero at $t = m_\rho^2$. By exchange degeneracy, then, so must the f_0 residue. Since no such zero is required in $\pi^+ \pi^+ \rightarrow \pi^+ \pi^+$, factorization tells us this must be a property of the π - A_2 - f_0 coupling.

In processes (4a-b), one thus has a zero of ρ_{22}/ρ_{11} at $t = m_\rho^2$. Kinematics also gives a zero of order t of this quantity at $t = 0$. Assuming any sort of smooth behavior, one thus cannot expect ρ_{22}/ρ_{11} to be very large in the range of t ($t \geq -0.6(\text{GeV}/c)^2$; see, e.g., Refs. 46, 47) for which A_2 production is appreciable.

The expectation of small ρ_{22} is confirmed in a simple model⁶⁰ for process (11) based on beta functions, in which,⁶¹ as $s \rightarrow \infty$,

$$\frac{\rho_{22}}{\rho_{11}} \rightarrow \frac{-3t}{2} \left[\frac{1-2t}{3-4t} \right]^2 \quad (12)$$

(units $2m_\rho^2 = 1$). This expression is quite small for the t range mentioned above, so that ρ_{22} would be essentially undetectable if a similar suppression held for processes (4a,b).

C. Further experimental questions.

The data discussed above, as stated at the outset, point to a picture of f_0 - ρ exchange, with $\sigma_0 \gtrsim 3\sigma_1$ at high energies and little coherence between $I_t=0$ and $I_t=1$ amplitudes. However, several interesting points could probably be cleared up with somewhat better data.

1. Magnitude of $\rho_{00} \frac{d\sigma}{dt}$: is there evidence for B exchange? The

data of Refs. 46 and 47 show some evidence for a variation in ρ_{00} with respect to t , but it is not even clear whether this quantity is larger for small $|t|$ ⁴⁶ or large $|t|$ ⁴⁷. As suggested above, a study of reactions (4c,d) is probably the best way to see the effects of B exchange, but density-matrix analyses are technically more difficult: the $K\bar{K}$ mode comes from f_0 as well as A_2^0 , while $\eta\pi^0$ is hard to identify. One must then perform an analysis of the type in Ref. 47 (combined normal-to-production-plane⁶² and Dalitz plot⁶³ analysis) on the $\pi^+\pi^-\pi^0$ mode. This presents no problem in principle; statistics would be the only limitation.

2. Possible π -B- A_2 coupling giving rise to $A_2^0 \rightarrow \omega\pi^+\pi^-$. This decay would proceed via a virtual B and/or a virtual ρ . It should be present if the π - A_2 -B coupling is of any importance.⁶⁴

3. Energy dependence of σ_1 and σ_0 . Does σ_1 continue to fall as p_L^{-2} ? Does σ_0 flatten out? (see Fig. 2). Both questions require high statistics at higher energies, and have interesting consequences. If σ_1/σ_0 continues to fall, the ρ - f_0 picture is called into question, with either (a) sizeable B contributions, or (b) an $I = 0$ exchange with $\alpha(0) > 1/2$. The latter might indicate a violation of the selection rule⁶⁵ for "diffraction dissociation":

$$P_f = P_i (-)^{J_f - J_i} \quad (13)$$

at a very small cross section level; the rule seems rather good in general.

4. Interference of f_0 and A_2 in $K_S^0 K_S^0$ and $K^+ K^-$ modes. This effect has received previous theoretical attention,⁶⁶ but a point of particular interest is the study of the mass spectrum arising from unnatural parity exchange in reactions like

$$\begin{array}{l} \pi^- p \rightarrow (f_0, A_2^0) n \\ \quad \quad \quad \begin{array}{l} \rightarrow K^+ K^- \\ \rightarrow K_S^0 K_S^0 \end{array} \end{array} \quad (14)$$

This requires good statistics, of course, but the measurement of a quantity such as

$$\rho_{00} \frac{d^2\sigma}{dt \, dm(K\bar{K})} \quad (15)$$

allows one to separate out π and B exchange in f_0 and A_2^0 production, respectively. From experience with ρ - ω interference,⁶⁷ one thinks one knows the phases with which these amplitudes will occur,^{38,68} whereas without projecting out unnatural parity exchange no such clean statement about phases seems possible.

5. Further studies of A_1 - A_2 interference. The claim³² for essentially complete coherence between $I_t=0(A_1)$ and $I_t=0(A_2)$ in $\pi^- p \rightarrow A_{1,2}^- p$ would be puzzling if only the Pomernanchuk trajectory contributed to A_1 production and only the f_0 to A_2 production. The interference should be studied at higher energies to see if this complete coherence persists.

IV. A_2 resonance shape.

Table III summarizes some present and future data of high resolution and high statistics regarding this question. Only experiments with $\gtrsim 150$ events in the A_2 peak have been included; as shown by the χ^2 probabilities $P(\chi^2)$ for Breit-Wigner (BW) or dipole (DP), not all of these are terribly significant.

The salient point of this table is the contradiction between the two boxed results. The "best-resolution" sample⁷⁸ of ref. 2 (see the review by W. Kienzle, ref. 2, fig. 7) has two peaks separated by about 24 MeV and a deep dip near 1300 MeV in between these peaks. On the other hand, with at least 10 times as many events in the peak as this sample, the NE-SB experiment sees a distribution indistinguishable from a Breit-Wigner.

A model-independent comparison of the combined data of refs. 1-2 with those of refs. 5-6 indicates a significant discrepancy, quoted as at least 7 standard deviations,⁶ which cannot be ascribed to the small

Table III. High (resolution/statistics) data regarding A_2 resonance shape in $\pi N \rightarrow A_2 N'$.

	P_L	Group	Ref.	Mode	Quoted Γ_R	Quoted p(BW)	Quoted p(DP)	Split ?
π^- : (Low)*		RHEL	69	x^-	*	*	*	*
$\downarrow$ A_2^-	2.6	CERN	70	x^-	10.4	*+	*+	Yes
	4.5	BNL	45	$K^- K_S^0$	8 ± 1	80%	13%, 19% [‡]	No
	5	NE-SB	5,6	x^-	16 ± 1	B.W.favored by	≥ 11 s.d.	No
	6	UC-WIS	71	$2\pi^-, x^-$	15-20	*	*	*
	6,7	CERN	1,2	x^-	16 ± 3 (best)	0.1%	70%	Yes
	7	CERN	72	$K^- K_S^0$	≤ 20	5%	60%	Yes
	7	NE-SB	5,6	x^-	21 ± 1	B.W.favored by	≥ 4 s.d.	No
	17.2	CERN-Munich	35	$K^- K_S^0$	13.5 ± 1.4	32%**	.4%**	No
	20	BNL	73	$K^- K_S^0$	11	55%	2.5%	No
π^+ : 3.7		LRL-G-T	74	$(3\pi)^+$	14	11%	53%	Yes
$\downarrow$ A_2^+	5	NE-SB	5,6	x^+	16 ± 1	B.W.favored by	≥ 9 s.d.	No
	7	LRL-A	75	$(3\pi)^+$	12.8	14%	0.3%	No
				$K^+ K_S^0, \pi^+$	7.2, 16.4	13%	0.3%	No
π^- : 3.2		CERN-Bologna	76	x^0	15	1%	65%	Yes
$\downarrow$ A_2^0	4.5	Illinois	77	$(3\pi)^0$	7	*	*	*

*: unknown to author at present

+: fits to data of refs. 70 and 2 combined.

‡: two χ^2 minima.

** : sample fit.

differences in quoted mass resolution between the two experiments.

The other experiments in Table III which do not see the splitting are irrelevant if the NE-SB result is correct. Not one of them contradicts a report of splitting for comparable s , t , charge state, and decay mode. On the other hand, none of these other split- A_2 results is of comparable significance to that of ref. 2, which is really the keystone of the case for A_2 substructure.

Given the present contradiction between the two most significant sets of data^{2,5,6}, the question is how to avoid a third result which may be so indecisive as to call for a fourth experiment, etc.! The only suggestion I would dare to make if such experiments are still contemplated is that one proceed as if narrow structures were involved. Such structures are considerably more sensitive to resolution than the popular dipole⁷ shape, as we shall show.

We shall not attempt a review of the many theoretical efforts^{26,79-98} to explain A_2 substructure. We discuss only the "broad-narrow"⁷⁰ model. While the degeneracy of the A_2 (broad) and its possible narrow partner remains a puzzle in such a model, the "broad-narrow" hypothesis allows very easily both for differences among A_2 experiments under various conditions (e.g. s , charge; see Table III) and for differences among members of the 2^+ nonet (A. Barbaro-Galtieri, Ref. 3).

A. Two-peak hypothesis.

Spin-parity analyses in various decay modes of the A_2 ^{3,35,43,47-49,51,75,99,100} agree that the only rapidly varying (resonant) amplitude in the $(3\pi)^\pm$ and $(K\bar{K})^\pm$ systems near m_{A_2} is that with $J^P=2^+$. In the $\rho\pi^-$ system, the 2^+ is certainly dominant^{2 53} (for both high- and low-mass halves of the peak) but the possibility of interference with a very small, rapidly varying $J^{PC}=1^{-+}$ amplitude cannot be ruled out.^{53,101} If there is any splitting, it must then occur in the 2^+ amplitude alone, to which we now restrict our discussion.

The dipole expression^{7,80} used in the fits of Table III has the advantage that it allows a quick comparison of resonance shapes in various experiments. It is a "favored" shape theoretically only by

virtue of economy of parameters, however. Once the shape depends on charge, s and possibly t , as suggested by the experiments in Table III prior to that of NE-SB,⁵⁻⁶ this economy is lost, and comparisons based on the dipole shape alone¹⁰³ lose their theoretical significance.¹⁰⁴

The dipole is a particular limit of the more general case of two 2^+ resonances which undergo mixing through real or virtual intermediate states.^{79,80,88-94} These two resonances may have different production and decay properties, so that one would expect the shape to be invariant only under certain very special circumstances. In the next subsection we apply the formalism of production, mixing, and decay of two degenerate states to a simple model of the "broad-narrow" variety.

B. Broad-narrow model.

The "usual" A_2 , with properties noted in Sections II and III, we denote by $A_2^{(1)}$ prior to mixing. Similarly, the unmixed "extra" state we denote by $A_2^{(2)}$.

Let us consider $A_2^{(1)}$ to have natural width Γ_1 , and $A_2^{(2)}$ to be quite narrow;^{79,86,96,105} we take $\Gamma_2 = 0$. One denotes production and decay processes by vectors

$$\vec{G}(r) = [G_1^{(r)}, G_2^{(r)}] \quad (\text{production}) \quad (16)$$

$$\vec{F}(n) = [F_1^{(n)}, F_2^{(n)}] \quad (\text{decay}) \quad (17)$$

where the components 1, 2, refer to $A_2^{(1,2)}$, and r, n label production and decay channels. The transition matrix is then given in the presence of mixing by

$$A^{nr} = - \vec{F}(n) \cdot \vec{D} \cdot \vec{G}(r), \quad (18)$$

where $\vec{D}$ is a 2×2 matrix propagator, with^{38,79}

$$\vec{D}^{-1} = \begin{bmatrix} E - M_0 + i \Gamma_1/2 & -V \\ -V & E - M_0 \end{bmatrix} \quad (19)$$

The off-diagonal mixing term V is assumed real, as would be the case for mixing via virtual transitions.^{90,107} We have taken the two states to have common mass M_0 for simplicity. Performing the matrix inversion, one has

$$\vec{D} = (\text{Det } D)^{-1} \begin{bmatrix} E - M_0 & V \\ V & E - M_0 + i \Gamma_1/2 \end{bmatrix} \quad (20)$$

$$\text{Det } D = D_+ D_- \quad (21)$$

$$D_{\pm} = E - M_0 + i \Gamma_{\pm}/2 \quad (22)$$

where

$$V = \sqrt{\Gamma_+ \Gamma_-} / 2, \quad \Gamma_1 = \Gamma_+ + \Gamma_- \quad (23), (24)$$

determine $\Gamma_{\pm}$.

The poles of the S-matrix behave as shown in Fig. 4a, starting out for $V = 0$ at $\Gamma_+ = \Gamma_1$, $\Gamma_- = 0$; coalescing to the dipole shape⁷⁹ at $V = \Gamma_1/4$; and spreading apart in opposite real directions for larger V . One popular type of fit to split- A_2 spectra^{70,74} is based on the pole positions illustrated in Fig. 4b. This is the class of models we shall be discussing here.

Assume that

$$\vec{F}^{(n)} = F_1^{(n)} \begin{bmatrix} 1, 0 \end{bmatrix}; \quad \vec{G}^{(r)} = G_1^{(r)} \begin{bmatrix} 1, \epsilon_r \end{bmatrix}, \quad (25), (26)$$

corresponding to the following physical situation:

- (a) $A_2^{(2)}$ has $\Gamma_2 = 0$ (no open decay channels), as mentioned above.
- (b) $A_2^{(2)}$ is produced with amplitude ϵ_r relative to $A_2^{(1)}$ in channel r .

When ϵ_r is taken to be zero, one sees from Eqs. (18) and (20) that a symmetric two-peak structure results.⁷⁹ However, we retain ϵ_r in what follows.

The shape is then given by¹⁰⁸

$$|A^{nr}|^2 = \left| \frac{F_1^{(n)} G_1^{(r)}}{D_+ D_-} \right|^2 \left[(E - M_0 + a_r)^2 + b_r^2 \right] \quad (27)$$

where

$$a_r = V \operatorname{Re} \epsilon_r; \quad b_r = V \operatorname{Im} \epsilon_r \quad (28), (29)$$

One can parametrize even a single Breit-Wigner form using Eq. (27), since for

$$a_r = 0, \quad b_r = \Gamma_- / 2 \quad (30)$$

the square-bracket term just cancels $|D_-|^2$. Hence Eq. (27) is an ideal form for quantitative comparison of resonance shapes independent of any theoretical ideas.¹⁰⁹ The two-state interpretation, however, is that ϵ_r is a relative production amplitude which may easily cause variations of any observed structure with respect to reaction, s , and t . The only relevant comparison in such a case, therefore, is that boxed in Table III.

Given a pair Γ_+ , Γ_- claimed to be necessary to fit data with a symmetric shape ($a_r=0$) and a dip to 0 ($b_r=0$),^{70,74} one may then attempt to obtain fits to A_2 shapes in other experiments.¹¹⁰ If all these fits were to cluster around the values (30), one would not have seen the effect of the "second state" at all, and could just as well omit it altogether.

Needless to say, such fits are meaningless if one has no good idea of the true values of Γ_+ and Γ_- . One needs at least one experiment with resolution smaller than Γ_- in order to be sure that such parameters are being correctly determined, and even then one must rely on the happy accident $a_r=b_r=0$.

Eq. (27) is merely another form of the two-interfering-Breit-Wigner model of Ref. 74:

$$A^{nr} \sim \frac{f \sqrt{\Gamma_+}}{D_+} + \frac{e^{i\phi} \sqrt{\Gamma_-}}{D_-} \quad (31)$$

The connection between (27) and (31) is:

$$z = f e^{-i\phi} \quad (32)$$

$$\epsilon_0 \equiv \sqrt{\Gamma_+/\Gamma_-} \quad (33)$$

$$\epsilon = i(z+\epsilon_0)/(1+z\epsilon_0) \quad (34)$$

i.e. a linear fractional transformation. It is amusing that, since ϵ and ϵ^* give the same resonance shape (see Eqs. (27)-(29)), so do $z(\epsilon)$ and $z_c \equiv z(\epsilon^*) \neq z(\epsilon)$.

The results of some fits to eq. (31) and (27) are shown in Fig. 5. Fits in Ref. 74 were constrained to have $|z| = \epsilon_0$. By comparing the closed circle with the open triangle (unconstrained fit to the same data, minus a low-statistics sample of $\eta\pi$ and $K\bar{K}^{111}$) in Fig. 5b one sees that such a constraint can be quite misleading. The best fit to the data of Ref. 75 on the basis of Eq. (27) is considerably closer to the case indistinguishable from the unsplit one ($z = \infty$, $\epsilon = i/\epsilon_0$) than the fit of Ref. 74.

Given parameters such as $\Gamma_+ = 90$, $\Gamma_- = 12$,⁷⁰ or $\Gamma_+ \cong 80$, $\Gamma_- \cong 10$,⁷⁴ the NE-SB data should lead to a fit to Eq. (27) with very small allowable deviation of a_r , b_r from the values in Eq. (30). One would then exhibit the contradiction between the CERN² and NE-SB^{5,6} data by showing that on Fig. 4b, the derived values of ϵ were outside each other's error bars, despite the claimed identical nature of the experiments.¹¹²

In the next subsection we concentrate on the case for which the resonance shape would be symmetric ($a_r=0$) with a dip to 0 ($b_r=0$) in the limit of ideal resolution, and show that for $\Gamma_- \cong 5$ MeV the resulting structure is rather sensitive to resolution.

C. Effects of resolution.¹¹³

We have folded a Gaussian resolution function $e^{-(x-y)^2/2\sigma^2}$ into resonance shapes corresponding to (i) dipole⁷ (ii) broad-narrow with maximal destructive interference and symmetric shape (eq. (27) with $a_r = b_r = 0$). The results are shown in Fig. 6. While the usual broad-narrow shape⁷⁰ with $\Gamma_- = 12$, $\Gamma_+ = 90$ is no more sensitive to resolution

than the dipole form² (cf. Figs. 6a and 6b), more sensitivity sets in as Γ is reduced (Figs. 6c, 6d). For the cases of Figs. 6a, 6b to hold true in Nature, NE-SB would have to have made a gross underestimate of its resolution (the arrows indicate by how much; author's rough estimate). As Γ is reduced, one passes through a situation for which each group would have had to misjudge its resolution by some 40%. This appears to be around $\Gamma \approx 5$ MeV. It is unlikely that CERN would have so overestimated its resolution as to have seen any substructure associated with Γ much less than this.

Other models can be invented to give considerable sensitivity to resolution. One of these is two line resonances separated by a given amount.¹¹⁴ However, such a model is at odds with the discussion of sec. II unless all the 2^+ resonances were to have this pathological structure.

V. Conclusions.

In the past couple of years the A_2 has been behaving itself remarkably well: as if it were the "usual" 3P_2 state of a quark and an antiquark. This good behavior is reflected in well-observed SU(3) relations. We seem to have learned a good deal about A_2 production by pions, and it also shows no anomalies: f_0 and ρ exchange describe it well. On the other hand, the question of possible substructure remains unsolved, with the original, most significant results in favor of a splitting² called into question.^{5,6} Doubtless CERN² has seen some sort of anomalous effect; its interpretation in terms of a second A_2 state remains problematic.

If a third experiment is contemplated we urge some restraint in case narrow structures are involved. One must probably wait until experimental resolutions drop below 5 MeV (full width at half maximum) before re-opening the question. It may be a long time before our experimental techniques get that good.¹¹⁵ Moreover, such an experiment should be capable of doing some other piece of physics should it again fail to see the splitting.

Should the A_2 substructure² turn out to have been merely an

instrumental effect or a huge statistical fluctuation one would be left with the question of why the naive quark model predicts the meson spectrum below 1500 MeV so well.²²⁻²⁶ In fact, however, not all of these predictions have been confirmed, and it might be useful to check them more systematically.¹¹⁸

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Figure Captions

Figure 1: Comparison of $2^+ \rightarrow 1^- 0^-$ and $2^+ \rightarrow 0^- 0^-$ partial widths according to Eq. (2). The value of C_i is the square of a Clebsch-Gordan coefficient (see text); the reduced widths $\tilde{D}_i^2$ should be equal separately in the two sections of Fig. 1 if SU(3) holds, and in the whole figure if SU(6) holds.

Figure 2: A_2 production cross sections from incident pions (from Ref. 34 except as noted). (a) $\pi^- p \rightarrow p A_2^- \rightarrow p \pi^+ \pi^- \pi^-$. Point (i) represents a comparable value for $\pi^+ p \rightarrow A_2^+ p$ at 5 GeV/c (Table I). Point (ii) represents an estimate based on the $K^- K_S^0$ mode of the A_2^- produced in 17.2 GeV/c $\pi^- p \rightarrow A_2^- p$.^{35,36} Diagonal dashed line: ³⁴ fit to $\sigma(A_2^-) \sim p_L^{-1}$. Horizontal dashed line: present data are consistent with a constant limiting value of $\sigma(\pi^- p \rightarrow p A_2^- \rightarrow p \pi^+ \pi^- \pi^-)$ as large as $50 \mu\text{b}$. (b) $\pi^+ d \rightarrow p_S p A_2^0 \rightarrow p_S p \pi^+ \pi^- \pi^0$. Solid line: ³⁵ fit to $\sigma(A_2^0) \sim p_L^{-2}$. Dashed line: $\sigma \sim p_L^{-1}$ also barely compatible with data.

Figure 3: Compilation of density matrices for A_2 production in $\pi^\pm p \rightarrow A_2^\pm p$ as a function of p_L . (See Table II). $\bullet$: 3π ; $\circ$: KK ; Δ : $\eta \pi$.

Figure 4: (a) Behavior of poles in S-matrix as a function of V (see Eqs. (18)-(24)). (b) Poles in "broad-narrow" solution.^{70,74}

Figure 5: (a) Fits of A_2 spectra to expressions of the form (31). (b) Fits to the expression (27), derived from those of (a) except in the case of the open triangle, where a direct fit was made. Parameters used: $\Gamma_+ = 78 \text{ MeV}$, $\Gamma_- = 9.6 \text{ MeV}$, M_0 variable (assuming calibration impossible⁷⁴). The points $z = \infty$ (Fig. 5a) and $\epsilon = i \sqrt{\Gamma_-/\Gamma_+} \equiv \infty_z$ (Fig. 5b) correspond to a single B-W, width Γ_+ .

Figure 6: Relative double peak and dip height and peak spacings when folded with Gaussian resolution functions $e^{-(x-y)^2/2\sigma^2}$. (a) Dipole with $\Gamma = 30 \text{ MeV}$.² (b) Broad-narrow form $(E - M_0)^2 / |D_+ D_-|^2$

(see eq. (22)), $\Gamma_+ = 90$ MeV, $\Gamma_- = 12$ MeV. (c), (d) same as (b) but $\Gamma_- = 8, 4$ MeV, respectively. In all cases, shaded areas correspond to ranges of σ quoted in refs. 2 (best resolution sample) and 5, 6 (7 GeV/c π^-). Arrows indicate shifts from quoted values of resolution needed to make results compatible with the given hypotheses.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

PK. SPACING
MeV

Figs. 6a,b

DIP, PEAK HEIGHT (REL. UNITS)

PK.
SPACING
MeV

Figs. 6c,d