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REACTOR CONTROL SYSTEMS BASED ON COUNTING AND CAMPBELLING TECHNIQUES

FULL - RANGE INSTRUMENTATION DEVELOPMENT PROGRAM

Final Progress Report

By

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July 1965

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INTRODUCTION

The objective of Project Agreement 22 was to determine the feasibility of covering the complete reactor neutron flux range from 1×10^3 to 5×10^{13} nv in-core by using in-core or out-of-core ion chambers.

This report describes the analytical predictions and the experimental results of a nuclear instrumentation development program undertaken by General Electric to determine the feasibility of this approach. The program has been concerned with the development of a full-range reactor control system incorporating either in-core or out-of-core ion chambers, and the purpose of the program has been satisfied by the development of a full-range instrumentation system consisting of two electronics subsystems. Counting techniques are utilized for source and lower intermediate range neutron fluxes, and Campbell - or mean-square voltage - techniques are utilized for intermediate and power range fluxes.

Historically, power reactors have used out-of-core neutron detectors for source and intermediate range coverage during a plant startup. In present power reactors, an in-core neutron source producing about 3×10^4 nv is attenuated by the water annulus surrounding the core so that the incident flux at the detector is approximately 0.5 to 3.0 nv. Larger power plants, especially those with internal steam separation, provide for much larger water annuli and neutron source attenuations of perhaps 10^{-9} have been projected. Additionally, there is considerable interest in the in-core flux as opposed to the out-of-core flux, and consequently, it has become necessary to develop suitable in-core detectors for the source and intermediate range fluxes.

The development program has investigated the problems associated with the in-core counting detector, the in-core Campbelling detector, the in-core transmission lines, as well as those unique problems of a two-channel, full-range instrumentation system. In addition, the effort has been expanded to ensure a compatible system for either in-core or out-of-core applications. In this manner, flexibility of the developed hardware for a wide range of applications has been assured.

SECTION I

GENERAL

1.1 SUMMARY OF PERFORMANCE

Two prototype full-range instrumentation systems have been developed, each capable of covering about 10 decades of reactor power.

The in-core system contains two detectors. The first of these is a U^{235} -lined fission counter that covers the range 10^3 nv to 1.5×10^9 nv; at 10^3 nv the counting rate is 1.5 cps, and at 1.5×10^9 nv (and 2.5×10^7 R/h) the counting loss due to the electronic subsystem is 23 percent. The second is a U^{235} -lined fission chamber, operating in the MSV mode, that covers the range 2.2×10^8 nv to 5×10^{13} nv; at 2.2×10^8 nv the neutron-produced signal equals that produced by 2.5×10^7 R/h of gammas, and at 5×10^{13} nv the signal is still proportional to flux (indicating that even higher fluxes could be measured).

The out-of-core system contains one detector, a U^{235} -lined fission counter-chamber. In the counting mode, this chamber covers the range 1.4 nv to 1.4×10^6 nv; at 1.4 nv, the counting rate is 1.0 cps, and at 1.4×10^6 nv the counting loss due to the electronics subsystem is 23 percent. In the MSV mode, this chamber covers the range 10^4 nv to 10^{10} nv; the lower limit may be somewhat greater than 10^4 nv, depending on the gamma field in the vicinity of the detector, and the higher limit may be greater than 10^{10} nv for the signal is still proportional to flux at that level.

1.2 SYSTEM ALTERNATIVES

The various alternatives offered by the development instrumentation system are illustrated in Figure 1-1. On the left, an application using an in-core detector (ion-chamber) would use the counting channel to monitor from 10^{-7} to 10^{-3} of rated PERCENT POWER, and by successive partial withdrawal movements of the detector could continue to monitor well into the intermediate range. The in-core Campbell system would overlap the counting channel at approximately 10^{-4} of rated PERCENT POWER, and provide coverage into the power range without withdrawal of the detector. Each counting and Campbell channel has an associated mechanical drive subsystem to position each detector at any desired point between the initial in-core elevation and the storage elevation beneath the core.

On the other hand, those applications not requiring the use of in-core detectors may attain similar performance using larger out-of-core detectors, as shown on the right in Figure 1-1.

Figure 1-1. System Alternatives

The counting channel would function from 10^{-7} to 10^{-3} of rated PERCENT POWER, and the Campbell channel would overlap and continue coverage through full-rated output of the plant. These detectors remain in a fixed position at all times.

Thus, the development instrumentation system offers the following significant advantages relative to conventional instrumentation:

- a. The power plant designer may choose between in-core and out-of-core detectors for his application, and yet retain a consistent electronics system.
- b. Only two channels of electronics are required to monitor from source level through rated plant output power.
- c. The Campbell system provides for improved discrimination against gamma flux, and enables monitoring under certain conditions where conventional instrumentation would be inoperable.

1.3 IN-CORE SYSTEM DESCRIPTION

The essential elements of the development in-core reactor instrumentation system are shown in Figure 1-2. Dry tubes are provided in the reactor core, and the detectors are positioned vertically within the tubes by the mechanical drive hardware. At startup of the power plant, the counting and Campbell detectors are located at an initial in-core position and are connected to their respective electronics assemblies by means of specially shielded cables. The counting electronics provides a seven-decade log count-rate meter indication, an analog recorder output voltage for the log count rate (level), and a meter indication and analog recorder output voltage for the reactor period. The Campbell electronics provides a recorder and meter indication output in linear switched increments over six decades. Auxiliary functions such as trip signals are also provided by the Campbell system. As the reactor is brought to full-power conditions, the in-core detectors are retracted from the high-flux region and ultimately stored several feet below the core bottom.

1.4 OUT-OF-CORE SYSTEM DESCRIPTION

The out-of-core system block diagram is shown in Figure 1-3; it closely parallels the in-core system in many respects. The out-of-core detectors are much larger than their in-core counterparts, and are located in a fixed position near the core. The cable and electronics portions of the out-of-core system are essentially identical, and operate in an analogous fashion to the in-core system.

Figure 1-2. In-Core Log Count Rate Monitors and Multi-range Mean Square Voltage Monitor Systems

Figure 1-3. Out-of-Core Log Count Rate Monitor and Multi-range Mean Square Voltage Monitor Systems

1.5 DEVELOPMENT SYSTEM HARDWARE

Specific portions of the in-core and out-of-core systems were constructed on a prototype basis to confirm system feasibility, to correlate predicted performance with experimental results, and to establish the pertinent operating limitations of each system. The following tabulation provides a means of identification for the test hardware referenced in this report:

<u>Identification</u>	<u>Description</u>
Detector Assembly 1	In-core fission chamber (counter)
Detector Assembly 2	In-core fission chamber (counter)
Detector Assembly 3	In-core MSV fission chamber (Campbeller)
Detector Assembly 4	In-core MSV fission chamber (Campbeller)
Detector Assembly 5	Out-of-core MSV fission chamber (Campbeller)
Detector Assembly 6	Out-of-core fission chamber (counter)
Detector Assembly 7	Out-of-core fission chamber (counter)
Log Count Rate Monitor	Counting electronics main chassis and remote amplifier
Multi-range MSV Monitor	Campbell mean square voltage electronics main chassis and remote amplifier
Modified RG-6A/U Cable	Counting electronics signal cable with added shielding layers
Modified RG-114A/U Cable	Campbell electronics signal cable with added shielding layers
RG-59A/U Cable	Used for many interconnection functions
Control Cable	Ten-wire control cable between the Campbell remote amplifier and the main Campbell chassis.

1.6 TEST FACILITIES

The development program utilized various test sites for evaluation of the prototype hardware. These test sites are described as follows:

a. NEPS - Building W. Neutron Source Site

This facility contains a small neutron source from which a neutron flux of approximately 1.5×10^4 nv is available. This flux was used to determine initial operability of the counting electronics hardware. In addition, measurements of counting system noise and in-core detector and cable temperature effects were conducted at this site.

b. Vallecitos - Cobalt-60 Facility

The effect of gamma flux, up to a maximum value of 3×10^6 R/h, on the counting and Campbell systems was investigated under ambient temperature conditions. Additional tests were performed at various gamma flux levels for both systems, using a small neutron source of approximately 3×10^4 nv in close proximity to the detectors.

c. Vallecitos - Nuclear Test Reactor (NTR)

This facility was used to determine counting system range, linearity and approximate system response during mechanical withdrawal movements of the detector, and linearity of the Campbell system over the available flux range at NTR. This reactor is easily controlled from a source level flux of approximately 10^4 nv to a maximum flux of 6.6×10^{11} nv. The maximum gamma dose rate at full power is approximately 2×10^6 R/h for this reactor. A number of response measurements were also made for both counting and Campbell systems as a function of time following a reactor scram.

d. Vallecitos - General Electric Test Reactor (GETR)

A measurement of the Campbell system linearity up to a peak flux of approximately 10^{14} nv was made in the Z trail cable. Campbell and counting system performances as a function of time following shutdown were recorded in order to make the sensitivity determination.

SECTION II

COUNTING STATISTICS

This section contains a discussion on some of the properties of a counting channel comprising a detector, logarithmic count-rate meter, and period meter. The properties considered are:

- a. Variance of the count-rate indication,
- b. Probability of false trip from the count-rate indication,
- c. Variance of the period indication, and
- d. Probability of false trip from the period indication.

The derivations are based on the assumption of constant average counting rate (infinite period), but the results should be valid for finite periods greater than the longest time-constant in the system.

2.1 VARIANCE OF THE COUNT-RATE INDICATION

The logarithmic count-rate meter considered is of the multiple-diode pump type (Cooke-Yarborough), of which one section is delineated in Figure 2-1, and in which R is the same for

Figure 2-1. One Section of Multiple-Diode Pump Type of Log Count-Rate Meter

each section, C_f/C_τ is the same for each section, and C_f of any section is $\frac{1}{10} C_f$ of the preceding section. That is,

$$(RC_f)_k = 10(RC_f)_{(k+1)}, \quad (2-1)$$

and

$$(RC_\tau)_k = 10(RC_\tau)_{(k+1)}, \quad (2-2)$$

where k is the section number and starts at 1 for the section with the lowest frequency break-point. All sections of the multiple pump are driven from the output of a scale-of-two circuit.

Let the average counting rate, r , at the input of the scale-of-two be such that the output of the k^{th} diode pump is one-half of its saturation value; i. e.,

$$\frac{r}{2} = \frac{1}{(RC_f)_k} \quad (2-3)$$

Then individual pulses from the $(k-1)^{\text{th}}$, k^{th} , and $(k+1)^{\text{th}}$ pumps are

$$v_{(k-1)} = \frac{2V}{r\tau_{(k-1)}} e^{-t/\tau_{(k-1)}}, \quad (2-4)$$

$$v_k = \frac{VT_k}{2\tau_k} e^{-t/\tau_k}, \quad (2-5)$$

and

$$v_{(k+1)} = \frac{VT_{(k+1)}}{\tau_{(k+1)}} e^{-t/\tau_{(k+1)}}, \quad (2-6)$$

respectively, where $T_k = (RC_f)_k$, $\tau_k = (RC_\tau)_k$, and V is the voltage swing of the output of the scale-of-two. To obtain these expressions, note that the charge delivered to C_τ , each time the scale-of-two output swings in the positive direction, is

$$\frac{VT}{R(1 + 0.5 rT)},$$

and this charge leaks off through R with a time-constant, τ ; hence,

$$v = \frac{VT}{\tau (1 + 0.5 r T)} e^{-t/\tau} \quad (2-7)$$

By combining Equation (2-7) with the relations,

$$0.5 r T_{(k-1)} \gg 1, \quad (2-8)$$

$$0.5 r T_k = 1, \quad (2-3a)$$

$$0.5 r T_{(k+1)} \ll 1, \quad (2-9)$$

Equations (2-4), (2-5), and (2-6) are obtained.

Now rewrite Equations (2-4), (2-5), and (2-6) as

$$v_{(k-1)} = \frac{V}{10} \left(\frac{T_k}{\tau_k} \right) e^{-t/10\tau_k}, \quad (2-10)$$

$$v_k = \frac{V}{2} \left(\frac{T_k}{\tau_k} \right) e^{-t/\tau_k}, \text{ and} \quad (2-11)$$

$$v_{(k+1)} = V \frac{T_k}{\tau_k} e^{-10t/\tau_k}, \quad (2-12)$$

by using the assumed relations between the various T 's and τ 's; viz.,

$$\frac{T_{(k-1)}}{\tau_{(k-1)}} = \frac{T_k}{\tau_k} = \frac{T_{(k+1)}}{\tau_{(k+1)}},$$

$$0.1 T_{(k-1)} = T_k = 10 T_{(k+1)}, \text{ and}$$

$$0.5 r T_k = 1.$$

An individual pulse from the log count-rate meter is a fraction, A , of the sum of these three pulses:

$$v = \frac{AV}{10} \left(\frac{T_k}{\tau_k} \right) e^{-t/(10\tau_k)} + \frac{AV}{2} \left(\frac{T_k}{\tau_k} \right) e^{-t/\tau_k} + AV \left(\frac{T_k}{\tau_k} \right) e^{-10t/\tau_k}. \quad (2-13)$$

(Actually, the total pulse is a fraction of the sum of the outputs of all of the individual sections; however, the terms below $(k-1)$ and above $(k+1)$ contribute an insignificant amount of energy to the total pulse and can be neglected.)

Now the variance, σ_S^2 , of a voltage, S , that consists of a linear superposition of randomly arriving pulses of the form $v(t)$ and an average arrival rate of $0.5 r$ is given by⁽¹⁾

$$\sigma_S^2 = 0.5 r \int_{-\infty}^{\infty} v(t)^2 dt. \quad (2-14)$$

(Equation (2-14) is true for pulses whose arrival satisfies Poisson's distribution; pulses from a scale-of-two do not. However, it is shown in Appendix A that the variance obtained by the use of Equation (2-14) is in error by less than a factor of 2 and is on the large side.) Substitution of Equation (2-13) into Equation (2-14) yields the variance,

$$\sigma_S^2 = 0.5 r \left(AV \frac{T_k}{\tau_k} \right)^2 \left(0.43 \tau_k \right),$$

and this can be combined with Equation (2-3a) to obtain

$$\sigma_S^2 = 0.43 A^2 V^2 \left(\frac{T_k}{\tau_k} \right). \quad (2-15)$$

The average voltage from one diode pump is given by

$$\bar{s}_i = \frac{0.5 r V T_i}{1 + 0.5 r T_i},$$

and the average total output voltage is a fraction, A , of the sum of these individual voltages:

$$\bar{S} = A \sum_{i=1}^N \frac{0.5 r V T_i}{1 + 0.5 r T_i} \quad (2-16)$$

Combining Equation (2-16) with Equations (2-3a), (2-8) and (2-9) yields

$$\bar{S} = A \left[(k - 1) V + \frac{V}{2} + \left(\text{terms} \leq \frac{V}{10} \right) \right],$$

or, neglecting the terms equal to or less than $\frac{V}{10}$,

$$\bar{S} = A \left(k - \frac{1}{2} \right) V \quad (2-17)$$

Hence, the fractional standard deviation, $\sigma_S / \bar{S}$, of the voltage that indicates the counting rate is, from Equations (2-15) and (2-17),

$$\frac{\sigma_S}{\bar{S}} = \frac{0.66}{k - \frac{1}{2}} \sqrt{\frac{T_k}{\tau_k}} \quad (2-18)$$

It should be remembered that the manner in which Equation (2-18) was obtained imposes the following restrictions:

- It is valid when the counting rate is such that the k^{th} pump is half saturated,
- It is approximately valid for noninteger values of k ,
- It is too large by a factor less than $\sqrt{2}$ because of the slight regularity in the output of the scale-of-two, and
- It is invalid for $k = 1$ or $k = N$ (where N is the number of pumps in the circuit).

For $k = N$ (high count rate, N^{th} pump half saturated, all other pumps saturated), the standard deviation is

$$\sigma_S = 0.52 A V \sqrt{\frac{T_N}{\tau_N}} \quad (2-19)$$

The average output voltage is

$$\bar{S} = A \left(N - \frac{1}{2} \right) V, \quad (2-20)$$

and the fractional standard deviation is

$$\frac{\sigma_S}{\bar{S}} = \frac{0.52}{N - \frac{1}{2}} \sqrt{\frac{T_N}{\tau_N}}. \quad (2-21)$$

This expression is subject to restriction c. of the preceding paragraph.

2.2 PROBABILITY OF FALSE TRIP FROM THE COUNT-RATE INDICATION

For a signal composed of the linear superposition of randomly-arriving pulses, as the product of count rate and pulse width becomes large with respect to unity, the amplitude distribution of the signal approaches a normal distribution; also, the average number of times per second that the signal crosses the value S_T with positive slope is given⁽¹⁾ approximately by

$$r_T = 0.58 f_2 e^{-\left(S_T - \bar{S} \right)^2 / \left(2 \sigma_S^2 \right)}, \quad (2-22)$$

where f_2 is the frequency of the upper half-power point of the power spectral density of the signal.

For the system considered in this section, the pulse width is of the order of τ_k , and the counting rate is of the order of $1/T_k$; so the product is τ_k/T_k , and this product is much greater than unity. Also, a single pulse is described approximately by

$$v(t) = \frac{A V}{2} \left(\frac{T_k}{\tau_k} \right) e^{-t/\tau_k}, \quad (2-23)$$

so the upper cutoff frequency is, approximately,

$$f_2 = \frac{1}{2 \pi \tau_k}. \quad (2-24)$$

The false trip rate is obtained by combining Equation (2-22) with Equation (2-24):

$$r_T = \frac{0.09}{\tau_k} e^{-\left(\frac{S_T - \bar{S}}{2\sigma_S^2}\right)^2} \quad (2-25)$$

Values of $\bar{S}$ and σ_S , for use in this equation, can be obtained by plotting $\bar{S}$ and σ_S versus k , using Equations (2-15), (2-17), and (2-19), and then transforming the linear k scale of the plot to a logarithmic count-rate scale. Note that for this purpose the values of A and V do not have to be known, for they divide out in the exponent.

2.3 VARIANCE OF THE PERIOD INDICATION

The variance of the period indication (the voltage proportional to growth rate) can be obtained by using Equation (2-14) with the expression for an individual pulse at the period-meter output, or by integrating the power spectral density of the period indication. The latter method is used here because the power spectral density is also needed to calculate the false trip probability.

For a signal composed of the linear superposition of randomly-arriving pulses, the distributed portion of the power spectral density is given⁽¹⁾ by

$$G(\omega) = \frac{2}{\pi} \int_0^\infty \psi(\delta) \cos \omega \delta \, d\delta \quad (2-26)$$

where

$$\psi(\delta) = 0.5 r \int_0^\infty v(t) v(t + \delta) \, dt \quad (2-27)$$

In this last expression, $0.5 r$ is the average arrival rate of the pulses and $v(t)$ is the shape of one pulse that starts at $t = 0$. But $v(t)$ at the output of the log count-rate meter is given by Equation (2-13). Substitution of Equation (2-13) into Equations (2-27) and (2-26) gives the power spectral density there as

$$G_1(\omega) = \frac{1.18}{\pi} r (AVT_k)^2 \frac{(1 + 0.05 \tau_k^2 \omega^2)(1 + 20 \tau_k^2 \omega^2)}{(1 + 100 \tau_k^2 \omega^2)(1 + \tau_k^2 \omega^2)(1 + 0.01 \tau_k^2 \omega^2)} \quad (2-28)$$

The transfer function of the rest of the system (period meter) is

$$H(\omega) = H_A \frac{j \omega R_2 C_1}{(1 + j \omega R_1 C_1) (1 + j \omega R_2 C_2)} \quad (2-29)$$

where R_1 , R_2 , C_1 , and C_2 are identified in the schematic (Figure 2-2) of the period meter.

Figure 2-2. Schematic Diagram of Period Meter

So the power spectral density at the output of the period meter is

$$G_2(\omega) = \frac{1.18}{\pi} r (AVT_k)^2 H_A^2 (R_2 C_1)^2 \quad (2-30)$$

$$\times \frac{\omega^2 (1 + 0.05 \tau_k^2 \omega^2) (1 + 20 \tau_k^2 \omega^2)}{(1 + 100 \tau_k^2 \omega^2) (1 + \tau_k^2 \omega^2) (1 + 0.01 \tau_k^2 \omega^2) (1 + R_1^2 C_1^2 \omega^2) (1 + R_2^2 C_2^2 \omega^2)}$$

in (volts)²/(radian sec⁻¹).

To normalize this expression, note that at a period of 1 second per neper the flux changes by a factor of 10 in 2.303 seconds. This corresponds to a change of unity in k or, from Equation (2-17), a change of AV volts at the output of the log count-rate meter. Hence, at 1-second period,

$$\frac{dS}{dt} = \frac{AV}{2.303}$$

At the input to the period meter, the rate of change of voltage is

$$\frac{\Delta V}{2.303} H_A,$$

and this produces a voltage at the output of the period meter of

$$\frac{\Delta V}{2.303} H_A R_2 C_1 \frac{\text{volts}}{\text{neper sec}^{-1}} \quad (2-31)$$

The power spectral density at the output of the period meter, in $(\text{nepers sec}^{-1})^2 / (\text{radian sec}^{-1})$, is obtained by dividing the right-hand side of Equation (2-30) by the square of Equation (2-31). The result is

$$G_2'(\omega) = \frac{4 T_k \omega^2 (1 + 0.05 \tau_k^2 \omega^2) (1 + 20 \tau_k^2 \omega^2)}{(1 + 100 \tau_k^2 \omega^2) (1 + \tau_k^2 \omega^2) (1 + 0.01 \tau_k^2 \omega^2) (1 + R_1^2 C_1^2 \omega^2) (1 + R_2^2 C_2^2 \omega^2)} \quad (2-32)$$

(Equation (2-3a) was also used in obtaining this expression.)

The variance of the period indication is obtained by integrating Equation (2-32) over all frequencies; i. e.,

$$\sigma_P^2 = \int_0^\infty G_2'(\omega) d\omega \text{ nepers/sec.} \quad (2-33)$$

2.4 PROBABILITY OF FALSE TRIP FROM THE PERIOD INDICATION

The false trip rate is obtained from Equation (2-22), rewritten for the conditions at the output of the period meter. The average number of times per second that the signal crosses a voltage (with positive slope) corresponding to a period P_T is given by

$$r_{T'} = 0.58 f_b' e^{-\left(\frac{1}{P_T} - \frac{1}{\bar{P}}\right)^2 / (2 \sigma_P^2)} \quad (2-34)$$

where $\bar{P}$ is the average period being presented at the time considered. f_b' is the frequency of the upper half-power point of the power spectral density of the signal (Equation (2-32)), and σ_P^2 is the variance (Equation (2-33)).

2.5 SPECIFIC RESULTS: COUNT-RATE INDICATION

Consider a channel which contains a logarithmic integrator of the Cooke-Yarborough type with a basic section as shown in Figure 2-1, containing eight sections with the frequency break-points spaced one decade apart and the time constants one decade apart.

If the ratio of $\frac{T_k}{\tau_k}$ is constant over all sections and equal to 0.0256, and the frequency break points are established to provide 0 to 10 volts for a range of 0.1 to 10^6 counts/sec input, Table 2-1 is a tabulation of the fractional standard deviation and average output voltage as a function of the input count rate obtained by the use of Equations (2-17) and (2-18).

TABLE 2-1

FRACTIONAL STANDARD DEVIATION AND AVERAGE OUTPUT VOLTAGE
AS A FUNCTION OF INPUT COUNT RATE

Count Rate (sec^{-1})	k	$\sigma_S/\bar{S}$	$\bar{S}$ (Average output voltage at rate specified)
0.5	1	0.180*	1.0 volts
5.0	2	0.0705	2.40
50	3	0.0423	3.85
5×10^2	4	0.0300	5.25
5×10^3	5	0.0234	6.70
5×10^4	6	0.0192	8.10
5×10^5	7	0.0162	9.55

*Based on the single diode pump relationship $\frac{\sigma_S}{\bar{S}} = \sqrt{\frac{T_k}{8 \tau_k}}$ since Equation (2-18) is inaccurate at the end point.

If a trip or alarm is provided at 5×10^5 counts/sec then the number of σ_S 's between the trip or alarm point and the average output voltage is given by

$$n = \frac{S_T - \bar{S}}{\sigma_S}$$

where

n = number of σ_S 's between S_T and $\bar{S}$, and

S_T = trip or alarm point.

A tabulation of n as a function of count rate is given in Table 2-2.

TABLE 2-2

TABULATION OF n AS A FUNCTION OF COUNT RATE

<u>Count Rate (sec⁻¹)</u>	<u>$\frac{\bar{S}}{\sigma_S}$</u>	<u>S_T</u>	<u>$\bar{S}$</u>	<u>n</u>
0.5	5.56	9.55	1.00	47.54
5.0	14.20	9.55	2.40	42.31
50	24.8	9.55	3.85	36.70
5×10^2	33.3	9.55	5.25	27.27
5×10^3	42.7	9.55	6.70	18.15
5×10^4	52.2	9.55	8.10	9.344
1.0×10^5	55.1	9.55	8.53	6.557
1.1×10^5	55.5	9.55	8.59	6.161
1.2×10^5	55.9	9.55	8.65	5.814
1.3×10^5	56.2	9.55	8.70	5.491
1.4×10^5	56.5	9.55	8.74	5.238
1.5×10^5	56.8	9.55	8.79	4.913
1.6×10^5	57.1	9.55	8.83	4.654
1.7×10^5	57.3	9.55	8.87	4.395
1.8×10^5	57.5	9.55	8.90	4.198
1.9×10^5	57.8	9.55	8.94	3.942
2.0×10^5	58.0	9.55	8.97	3.753
2.2×10^5	58.4	9.55	9.03	3.364
2.4×10^5	58.7	9.55	9.09	2.970
2.6×10^5	59.1	9.55	9.13	2.719
2.8×10^5	59.4	9.55	9.18	2.394
3.0×10^5	59.7	9.55	9.22	2.137
5×10^5	61.8	9.55	9.55	0

These values of $n \left(= \frac{S_T - \bar{S}}{\sigma_S} \right)$ and the values of τ_k for the log integrator being considered

can be substituted into Equation (2-25) to produce the tabulation (Table 2-3) of the average rate of crossings over the trip level, S_T .

TABLE 2-3

AVERAGE NUMBER OF CROSSINGS PER SECOND AS A FUNCTION OF COUNT RATE

Count Rate (sec^{-1})	n^*	$\frac{0.09}{\tau_k}$	$\frac{n^2}{2}$	$e^{-\frac{n^2}{2}}$	Average Number Crossings per Second
0.5	47.54	0.0012	1130	1×10^{-491}	1.2×10^{-494}
5.0	42.31	0.012	895	1×10^{-389}	1.2×10^{-391}
5×10	36.70	0.12	674	1×10^{-293}	1.2×10^{-294}
5×10^2	27.27	1.2	372	1×10^{-162}	1.2×10^{-162}
5×10^3	18.15	12	165	1×10^{-72}	1.2×10^{-71}
5×10^4	9.344	120	43.66	9×10^{-20}	1.1×10^{-17}
1.0×10^5	6.557	280	21.50	4.675×10^{-10}	1.3×10^{-7}
1.1×10^5	6.161	310	18.98	5.61×10^{-4}	1.7×10^{-6}
1.2×10^5	5.814	340	16.90	4.6×10^{-8}	1.6×10^{-5}
1.3×10^5	5.491	370	15.08	2.81×10^{-7}	1.0×10^{-4}
1.4×10^5	5.238	400	13.72	1.12×10^{-6}	4.4×10^{-4}
1.5×10^5	4.913	420	12.07	5.52×10^{-6}	2.3×10^{-3}
1.6×10^5	4.654	460	10.83	2.0×10^{-5}	0.91×10^{-2}
1.7×10^5	4.395	480	9.658	6.44×10^{-5}	3.1×10^{-2}
1.8×10^5	4.198	510	8.812	1.50×10^{-4}	0.76×10^{-1}
1.9×10^5	3.942	550	7.770	4.30×10^{-4}	2.4×10^{-1}
2.0×10^5	3.753	570	7.043	8.67×10^{-4}	4.9×10^{-1}
2.2×10^5	3.364	630	5.658	3.51×10^{-3}	2.2
2.4×10^5	2.970	700	4.411	1.23×10^{-2}	0.86×10^1
2.6×10^5	2.719	760	3.697	2.47×10^{-2}	1.9×10^1
2.8×10^5	2.394	820	2.866	5.70×10^{-2}	4.6×10^1
3.0×10^5	2.137	890	2.283	1.02×10^{-1}	0.91×10^2
5.0×10^5	0	1200	0	1.0	1.2×10^3

*Trip point at 5×10^5 counts/sec

If the counting channel is to be operated in the region near 1×10^5 cps with a trip setting at 5×10^5 cps, the digest of Table 2-3 (shown in Table 2-4) illustrates the average number of spurious trips possible in this region.

TABLE 2-4

AVERAGE TIME BETWEEN TRIPS AS A FUNCTION OF COUNT RATE

<u>Count Rate (sec⁻¹)</u>	<u>Average Time Between Trips</u>
1×10^5	2.9 months
1.1×10^5	6.6 days
1.2×10^5	18 hours
1.3×10^5	2.7 hours
1.4×10^5	44 minutes
1.5×10^5	7.1 minutes
1.6×10^5	1.8 minutes
1.7×10^5	32 seconds
1.8×10^5	13 seconds
1.9×10^5	4.1 seconds
2.0×10^5	2.0 seconds

It is shown in Table 2-4 that the average time to trip changes very rapidly for a very small increment of change in the reading, especially since the scale is logarithmic and covers seven decades.

In order that a smaller operating range not be required, the response time of the logarithmic integrator may be made slower and the average time between trips at the region of 1×10^5 counts/sec can be made much more tolerable.

In Equation (2-25) the part of the function which varies rapidly is the exponential, which is a function of the number of root mean-square fluctuations between the operating point and the trip or alarm point. From Equation (2-18) it is seen that the fractional standard deviation $\frac{\sigma_S}{S}$ is directly proportional to

$$\sqrt{\frac{T_k}{\tau_k}}$$

where

T_k is the frequency break, and

τ_k is the time constant break.

For the counting channel considered in the previous analysis, the time constant breaks and response time (measured) as a function of k and rate are listed in Table 2-5.

TABLE 2-5

RESPONSE TIME AS A FUNCTION OF COUNT RATE

<u>Count Rate (sec⁻¹)</u>	<u>k</u>	<u>τ_k</u>	<u>Measured (0-63%) τ_k</u>
0.05	--	781 sec	-----
0.5	1	78.1 sec	34 sec
5.0	2	7.81 sec	5.4 sec
5×10^1	3	0.781 sec	0.54 sec
5×10^2	4	78.1 msec	54 msec
5×10^3	5	7.81 msec	5.4 msec
5×10^4	6	0.781 msec	0.54 msec
5×10^5	7	78.1 μ sec	54 μ sec

If the measured response time at 5×10^4 counts/sec is established to be 40 msec, then the ratio of $\frac{\tau_k}{\tau_k}$ will change from $\frac{1}{39}$ to approximately $\frac{1}{3900}$; therefore, the standard deviation will be decreased by a factor of 10. and Table 2-1 is amended as shown in Table 2-1a.

TABLE 2-1a

FRACTIONAL STANDARD DEVIATION AND AVERAGE OUTPUT VOLTAGE
AS A FUNCTION OF INPUT COUNT RATE

<u>k</u>	<u>$\sigma_s / \bar{S}$</u>	<u>σ_s</u>	<u>$\bar{S}$</u>
1	0.0180	0.0180	1.00
2	0.00705	0.0169	2.40
3	0.00423	0.0163	3.85
4	0.00300	0.0158	5.25
5	0.00234	0.0157	6.70
6	0.00192	0.0156	8.10
7	0.00162	0.0155	9.55

Table 2-2 is then amended as shown in Table 2-2a.

TABLE 2-2a

TABULATION OF n AS A FUNCTION OF COUNT RATE

<u>Count Rate (sec⁻¹)</u>	<u>$\frac{\bar{S}}{\sigma_s}$</u>	<u>S_T</u>	<u>$\bar{S}$</u>	<u>n</u>
0.5	55.6	9.55	1.00	475.3
5.0	142.0	9.55	2.40	423.2
50	248	9.55	3.85	367.0
5×10^2	333	9.55	5.25	272.7
5×10^3	427	9.55	6.70	181.5
5×10^4	522	9.55	8.10	93.43
1.0×10^5	551	9.55	8.53	65.57
1.1×10^5	555	9.55	8.59	61.61
1.2×10^5	559	9.55	8.65	58.14
1.3×10^5	562	9.55	8.70	54.91
1.4×10^5	565	9.55	8.74	52.38
1.5×10^5	568	9.55	8.79	49.13
1.6×10^5	571	9.55	8.83	46.54
1.7×10^5	573	9.55	8.87	43.95
1.8×10^5	575	9.55	8.90	41.98
1.9×10^5	578	9.55	8.94	39.42
2.0×10^5	580	9.55	8.97	37.53
2.2×10^5	584	9.55	9.03	33.64
2.4×10^5	587	9.55	9.09	29.70
2.6×10^5	591	9.55	9.13	27.19
2.8×10^5	594	9.55	9.18	23.94
3.0×10^5	597	9.55	9.22	21.37
5×10^5	618	9.55	9.55	0

From Table 2-2a and the proposed log integrator circuit constants, the average number of crossing-per-unit time over the trip level is tabulated in Table 2-3a.

TABLE 2-3a

AVERAGE NUMBER OF CROSSINGS PER SECOND AS A FUNCTION OF COUNT RATE

Count Rate (sec ⁻¹)	n	$\frac{0.09}{\tau_k}$	$\frac{n^2}{2}$	$e^{-\frac{n^2}{2}}$	Average Number Crossings per Second
0.5	475.3	1.2×10^{-5}	1.13×10^5	1×10^{-49000}	1.2×10^{-49005}
5.0	423.2	1.2×10^{-4}	8.95×10^4	1×10^{-38840}	1.2×10^{-38844}
5×10	367.0	1.2×10^{-3}	6.73×10^4	1×10^{-29210}	1.2×10^{-29213}
5×10^2	272.7	1.2×10^{-2}	3.72×10^4	1×10^{-16150}	1.2×10^{-16152}
5×10^3	181.5	1.2×10^{-1}	1.65×10^4	1×10^{-7161}	1.2×10^{-7162}
5×10^4	93.43	1.2	4.37×10^3	1×10^{-1897}	1.2×10^{-1897}
1.0×10^5	65.57	2.8	2.15×10^3	7.94×10^{-934}	2.2×10^{-933}
1.1×10^5	61.61	3.1	1.90×10^3	3.96×10^{-824}	1.2×10^{-823}
1.2×10^5	58.14	3.4	1.69×10^3	3.16×10^{-733}	1.1×10^{-732}
1.3×10^5	54.91	3.7	1.51×10^3	2.0×10^{-655}	0.74×10^{-654}
1.4×10^5	52.38	4.0	1.37×10^3	3.96×10^{-594}	1.6×10^{-593}
1.5×10^5	49.13	4.2	1.21×10^3	1.2×10^{-525}	0.54×10^{-524}
1.6×10^5	46.54	4.6	1.08×10^3	6.3×10^{-468}	2.9×10^{-467}
1.7×10^5	43.95	4.8	9.66×10^2	1.59×10^{-419}	0.72×10^{-418}
1.8×10^5	41.98	5.1	8.81×10^2	2.5×10^{-382}	1.3×10^{-381}
1.9×10^5	39.42	5.5	7.77×10^2	1.59×10^{-337}	0.88×10^{-336}
2.0×10^5	37.53	5.7	7.04×10^2	3.14×10^{-305}	1.8×10^{-304}
2.2×10^5	33.64	6.3	5.66×10^2	4.0×10^{-245}	2.5×10^{-244}
2.4×10^5	29.70	7.0	4.41×10^2	2.5×10^{-191}	1.8×10^{-190}
2.6×10^5	27.19	7.6	3.70×10^2	3.94×10^{-160}	3.0×10^{-159}
2.8×10^5	23.94	8.2	2.87×10^2	3.94×10^{-124}	3.2×10^{-123}
3.0×10^5	21.37	8.9	2.27×10^2	9×10^{-98}	0.80×10^{-97}
5.0×10^5	0	12	0	1	12

It is shown in Table 2-3a that for the worst overrun calculated (3×10^5 counts/sec), the average time between spurious trips would be 1.2×10^{97} seconds or 3.8×10^{89} years, as compared with 1.1×10^{-2} seconds with the faster time response logarithmic integrator. The counting channel with the slower time response as indicated will have on the average no spurious trips or alarms during the life of the plant in which it is used.

2.6 SPECIFIC RESULTS: PERIOD INDICATION

Standard deviations and trip rates were calculated for the period indication (voltage proportional to growth rate) for a system consisting of eight diode pumps satisfying Equations (2-1) and (2-2), and for which $T_2 = 2$ seconds, $\tau_2 = 80$ seconds, and $R_1 C_1 = R_2 C_2 = 2$ seconds.

The standard deviations at various average counting rates are given in Table 2-6.

TABLE 2-6

STANDARD DEVIATION, σ_P , OF PERIOD INDICATION VERSUS COUNTING RATE

<u>Count Rate (sec⁻¹)</u>	<u>σ_P (nepers sec⁻¹)</u>
0.5	2.6×10^{-2}
5	4.1×10^{-2}
5×10^1	4.8×10^{-2}
5×10^2	3.3×10^{-2}
5×10^3	1.2×10^{-2}
5×10^4	0.36×10^{-2}

Then, using the largest σ_P listed in Table 2-6 (4.8×10^{-2} nepers/sec), the average rate at which the period indication exceeds the true growth rate $(1/P)_T$ by an amount $(1/P)_E$ is shown in Table 2-7.

TABLE 2-7

RATE AT WHICH PERIOD INDICATION EXCEEDS TRUE GROWTH RATE, $(1/P)_T$
BY AN AMOUNT $(1/P)_E$

<u>Growth-Rate Error, $(1/P)_E$</u>	<u>Rate of Exceeding $(1/P)_T + (1/P)_E$</u>
0.050 nepers sec ⁻¹	0.048 sec ⁻¹
0.060 nepers sec ⁻¹	0.026 sec ⁻¹
0.100 nepers sec ⁻¹	0.011 sec ⁻¹
0.125 nepers sec ⁻¹	0.0035 sec ⁻¹
0.150 nepers sec ⁻¹	0.00088 sec ⁻¹
0.175 nepers sec ⁻¹	0.00018 sec ⁻¹
0.200 nepers sec ⁻¹	0.000027 sec ⁻¹

SECTION III

THEORY OF CAMPBELL SYSTEM OF REACTOR INSTRUMENTATION

This section derives the equations that can be used to design a Campbell (MMSV) monitor for radiation measurement. The section is divided into four parts. The first part contains the derivations and results in the most general form; these results can be applied to any system. The second part comprises the simpler results for a system that contains one high-pass filter and one low-pass filter. The third part consists of the derivation of the signal contribution due to reactor noise. The fourth part covers the derivation of the probability of false trips, with several optional maximum allowed average signal levels and a particular trip setting; a calculation of trip probability as a function of operating level with a fixed trip point setting is also included.

3.1 MATHEMATICAL DERIVATIONS

Consider the system shown in Figure 3-1. It consists of four components: a detector, an amplifier, a squaring circuit, and an averaging (or integrating) circuit of time-constant τ . The detector and amplifier are linear, and somewhere in the system before the squaring circuit (i. e., in the linear part of the system) there is a component to remove the d-c, such as a series capacitor or a shunt inductor. The time-varying voltage at the output of the amplifier is designated as $V(t)$. At the output of the squaring circuit the voltage is $AV^2(t)$, where A is the squaring-circuit constant; i. e., the ratio between its output voltage and the square of its input voltage. The voltage at the output of the integrating circuit, which will be called the signal, is $S(t)$, and is related to the output of the squaring circuit by the differential equation of an integrating circuit of time-constant τ

$$\frac{dS(t)}{dt} + \frac{S(t)}{\tau} = \frac{AV^2(t)}{\tau} \quad (3-1)$$

Due to the random nature of the input, the value of the voltage or current at any point in the system at time t cannot be predicted or expressed mathematically; however, if simultaneous observations were made at the same point and at the same time, t , in a large number of identical systems, the results could be averaged. This ensemble average is called the expected value of that voltage or current at time t , and can be predicted and expressed mathematically. Also, the square of the difference between each result and the expected value could be averaged to obtain the variance of that voltage or current at time t ; this variance can also be predicted and expressed mathematically.

Figure 3-1. Basic Elements of a Campbell System

We will use the brackets $\langle \rangle$ to indicate the expected value of a voltage or signal.

It should be noted that Equation (3-1) applies to the expected values of $S(t)$ and $AV^2(t)$ as well as to their instantaneous values; i. e.,

$$\frac{d \langle S(t) \rangle}{dt} + \frac{\langle S(t) \rangle}{\tau} = \frac{\langle AV^2(t) \rangle}{\tau} \quad (3-2)$$

Since the system is linear up to the input of the squaring circuit (output of the amplifier), the total effect at any point in this part of the circuit is just the sum of the effects caused by the detection of each neutron. Hence, if the detection of a neutron at time zero produces a voltage pulse $v(t)$ at the output of the amplifier, then the voltage at time t due to the detection of a neutron at time t_k is $v(t-t_k)$, and the total voltage is the sum of the individual voltages produced by all previously-detected neutrons.

$$V(t) = \sum_{t_k = -\infty}^t v(t-t_k),$$

where the k^{th} particle arrives at t_k .

This equation can be written in a form suitable for statistical analysis in the following manner. Divide the time axis into small equal intervals of length Δt , then the detection of a neutron during the n^{th} interval of time will produce a voltage at time t of $v(t-n \Delta t)$, and the total voltage due to all of the previously-detected neutrons will be:

$$V(t) = \sum_n v(t-n \Delta t),$$

where the summation includes only those intervals during which a neutron is detected. If we define a random variable η_n that equals one if a neutron is detected during the n^{th} time interval and equals zero if no neutron is detected during the n^{th} time interval, then the total voltage at the output of the amplifier can be written as

$$V(t) = \sum_{n=-\infty}^{t/\Delta t} \eta_n v(t-n\Delta t), \quad (3-3)$$

where the summation now includes all the time intervals that precede t .

3.1.1 Expected Value of the Signal for Constant Flux

Make the substitution

$$a = \frac{t}{\Delta t} - n$$

in Equation (3-3); then

$$V(t) = \sum_{a=0}^{\infty} \frac{\eta_{t/\Delta t - a}}{\Delta t} v(a\Delta t),$$

and

$$V^2(t) = \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \frac{\eta_{t/\Delta t - a}}{\Delta t} \frac{\eta_{t/\Delta t - b}}{\Delta t} v(a\Delta t) v(b\Delta t). \quad (3-4)$$

Equation (3-4) can be written as

$$\begin{aligned} V^2(t) = & \sum_{a=0}^{\infty} \left(\frac{\eta_{t/\Delta t - a}}{\Delta t} \right)^2 v^2(a\Delta t) \\ & + \sum_{a=0}^{\infty} \sum_{\substack{b=0 \\ a \neq b}}^{\infty} \frac{\eta_{t/\Delta t - a}}{\Delta t} \frac{\eta_{t/\Delta t - b}}{\Delta t} v(a\Delta t) v(b\Delta t) \end{aligned}$$

by separating the terms in which $a = b$ from those in which $a \neq b$, and the expected value of $V^2(t)$ is

$$\begin{aligned} \langle V^2(t) \rangle &= \sum_{a=0}^{\infty} \langle \left(\frac{\eta_t}{\Delta t} - a \right)^2 \rangle v^2(a\Delta t) \\ &+ \sum_{a=0}^{\infty} \sum_{\substack{b=0 \\ a \neq b}}^{\infty} \langle \frac{\eta_t}{\Delta t} - a \quad \frac{\eta_t}{\Delta t} - b \rangle v(a\Delta t) v(b\Delta t), \quad (3-5) \end{aligned}$$

since the expectation of the sum of a number of random variables is equal to the sum of their expectations.

If the average flux of neutrons is Φ and the counting efficiency of the detector is k , then the probability of detecting a neutron during the n^{th} time interval is $k\Phi\Delta t$, the probability of detecting no neutron during the n^{th} time interval is $1-k\Phi\Delta t$, and the expected values of the η combinations are

$$\begin{aligned} \langle \left(\frac{\eta_t}{\Delta t} - a \right)^2 \rangle &= [1^2 \times k\Phi\Delta t] + [0^2 \times (1-k\Phi\Delta t)] \\ &= k\Phi\Delta t, \\ \langle \frac{\eta_t}{\Delta t} - a \quad \frac{\eta_t}{\Delta t} - b \rangle &= [1^2 \times (k\Phi\Delta t)^2] + [1 \times k\Phi\Delta t \times 0 \times (1-k\Phi\Delta t)] \\ &+ [0 \times (1-k\Phi\Delta t) \times 1 \times k\Phi\Delta t] + [0^2 \times (1-k\Phi\Delta t)^2] \\ &= (k\Phi\Delta t)^2, \quad (3-6) \end{aligned}$$

so

$$\langle V^2(t) \rangle = \sum_{a=0}^{\infty} k\Phi\Delta t v^2(a\Delta t) + \sum_{a=0}^{\infty} \sum_{\substack{b=0 \\ a \neq b}}^{\infty} (k\Phi\Delta t)^2 v(a\Delta t) v(b\Delta t), \quad (3-7)$$

and this can be written as

$$\begin{aligned} \langle V^2(t) \rangle = & \sum_{a=0}^{\infty} k\Phi \Delta t v^2(a\Delta t) + \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} (k\Phi \Delta t)^2 v(a\Delta t) v(b\Delta t) \\ & - \sum_{a=0}^{\infty} (k\Phi \Delta t)^2 v^2(a\Delta t) \end{aligned} \quad (3-8)$$

by adding and subtracting the terms in which $a = b$.

If we let

$$x_a = a\Delta t, \text{ and}$$

$$y_b = b\Delta t,$$

then

$$\Delta x = \Delta a \Delta t = \Delta t, \text{ and}$$

$$\Delta y = \Delta b \Delta t = \Delta t,$$

since $\Delta a = \Delta b = 1$. Then we can write

$$\begin{aligned} \langle V^2(t) \rangle = & k\Phi \sum_{a=0}^{\infty} v^2(x_a) \Delta x + (k\Phi)^2 \sum_{a=0}^{\infty} v(x_a) \Delta x \sum_{b=0}^{\infty} v(y_b) \Delta y \\ & - (k\Phi)^2 \Delta t \sum_{a=0}^{\infty} v^2(x) \Delta x, \end{aligned} \quad (3-9)$$

which becomes, as $\Delta t \rightarrow 0$,

$$\langle V^2(t) \rangle = k\Phi \int_0^{\infty} v^2(x) dx + \left[k\Phi \int_0^{\infty} v(x) dx \right]^2. \quad (3-10)$$

We have postulated a d-c removing component, such as a series capacitor or a shunt inductor, in the linear part of the circuit and it is easy to show that under this condition

$$\int_0^{\infty} v(x) dx = 0. \quad (3-11)$$

So the expected value of $V^2(t)$ is

$$\langle V^2(t) \rangle = k\Phi \int_0^{\infty} v^2(x) dx. \quad (3-12)$$

Referring to Figure 3-1, the output of the squaring circuit is $AV^2(t)$, so

$$\begin{aligned}\langle AV^2(t) \rangle &= A \langle V^2(t) \rangle \\ &= Ak\Phi \int_0^\infty v^2(x) dx.\end{aligned}\quad (3-13)$$

Also, the output of the integrating circuit, τ , satisfies the differential equations

$$\frac{dS(t)}{dt} + \frac{S(t)}{\tau} = \frac{AV^2(t)}{\tau}, \quad (3-1)$$

$$\frac{d\langle S(t) \rangle}{dt} + \frac{\langle S(t) \rangle}{\tau} = \frac{\langle AV^2(t) \rangle}{\tau}, \quad (3-2)$$

and since $\langle AV^2(t) \rangle$ is constant in time, then

$$\langle S(t) \rangle = \langle AV^2(t) \rangle = Ak\Phi \int_0^\infty v^2(x) dx. \quad (3-14)$$

If there are two different types of particles present, arguments similar to the above will yield

$$\begin{aligned}\langle S(t) \rangle &= Ak_1\Phi_1 \int_0^\infty v_1^2(x) dx + Ak_2\Phi_2 \int_0^\infty v_2^2(x) dx \\ &\quad + A \left[k_1\Phi_1 \int_0^\infty v_1(x) dx + k_2\Phi_2 \int_0^\infty v_2(x) dx \right]^2,\end{aligned}\quad (3-15)$$

where the subscripts 1 and 2 refer to the two types of particles. Because of Equation (3-11), this becomes

$$\langle S(t) \rangle = Ak_1\Phi_1 \int_0^\infty v_1^2(x) dx + Ak_2\Phi_2 \int_0^\infty v_2^2(x) dx. \quad (3-16)$$

3.1.2 Variance of the Signal

Consider the integrating circuit of time-constant τ , labelled τ in Figure 3-1, and consisting of a series resistor and a capacitor to ground. If a pulse of voltage of amplitude E and short time duration compared to τ is applied to its input, its output is

$$s(t) = B e^{-t/\tau}. \quad (3-17)$$

and the time-integral of this voltage must equal the time-integral of the input pulse; i.e.,

$$\int_0^{\infty} B e^{-t/\tau} dt = E\Delta t, \quad (3-18)$$

so

$$B = \frac{E\Delta t}{\tau}, \quad (3-19)$$

and

$$s(t) = \frac{E\Delta t}{\tau} e^{-t/\tau}. \quad (3-20)$$

Also, if the circuit is supplied with a succession of short voltage pulses, not necessarily equal, its output is a linear superposition of terms like Equation (3-20)

$$S(t) = \sum_{i=-\infty}^{t/\Delta t} s_i(t-i\Delta t), \quad (3-21)$$

or

$$S(t) = \sum_{i=-\infty}^{t/\Delta t} \frac{E_i\Delta t}{\tau} e^{-(t-i\Delta t)/\tau}. \quad (3-22)$$

Now the continuously-varying voltage, $AV^2(t)$, that is being applied across this circuit can be considered as supplying a succession of short voltage pulses by dividing the time axis into short equal intervals of length Δt . The amplitude of the i^{th} pulse is

$$E_i = AV^2(i\Delta t), \quad (3-23)$$

so Equation (3-22) becomes

$$S(t) = \frac{A}{\tau} \sum_{i=-\infty}^{t/\Delta t} V^2(i\Delta t)\Delta t e^{-(t-i\Delta t)/\tau}. \quad (3-24)$$

Equation (3-4) shows that

$$V^2(i\Delta t) = \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \eta_{i-a} \eta_{i-b} v(a\Delta t)v(b\Delta t). \quad (3-25)$$

so Equation (3-24) can be written as

$$S(t) = \frac{A}{\tau} \sum_{i=-\infty}^{t/\Delta t} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \eta_{i-a} \eta_{i-b} v(a\Delta t) v(b\Delta t) \Delta t e^{-(t-i\Delta t)/\tau} \quad (3-26)$$

If we let

$$c = \frac{t}{\Delta t} - i,$$

then Equation (3-26) becomes

$$S(t) = \frac{A}{\tau} \sum_{c=0}^{\infty} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \eta_{\frac{t}{\Delta t}-c-a} \eta_{\frac{t}{\Delta t}-c-b} v(a\Delta t) v(b\Delta t) \Delta t e^{-c\Delta t/\tau} \quad (3-27)$$

Now the variance is obtained from

$$\sigma_S^2 = \langle S^2 \rangle - \langle S \rangle^2 \quad (3-28)$$

so we will compute $\langle S^2 \rangle$. From Equation (3-27)

$$S^2(t) = \left(\frac{A}{\tau}\right)^2 \sum_{c=0}^{\infty} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \sum_{h=0}^{\infty} \sum_{f=0}^{\infty} \sum_{g=0}^{\infty} \left[\eta_{\frac{t}{\Delta t}-c-a} \eta_{\frac{t}{\Delta t}-c-b} \eta_{\frac{t}{\Delta t}-h-f} \right. \\ \left. \eta_{\frac{t}{\Delta t}-h-g} v(a\Delta t) v(b\Delta t) v(f\Delta t) v(g\Delta t) (\Delta t)^2 e^{-(c\Delta t + h\Delta t)/\tau} \right] \quad (3-29)$$

and this can be written as

$$\begin{aligned}
 S^2(t) = & \left(\frac{A}{\tau}\right)^2 \sum_{c=0}^{\infty} \left[\sum_{b=0}^{\infty} \sum_{h=0}^c \eta_{\Delta t}^4 \frac{v^2(b\Delta t)v^2(b\Delta t + c\Delta t - h\Delta t)}{\Delta t^{-c-b}} \right. \\
 & \cdot e^{\frac{-c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^2 + \sum_{g=0}^{\infty} \sum_{h=c}^{\infty} \eta_{\Delta t}^4 \frac{v^2(g\Delta t)v^2(g\Delta t + h\Delta t - c\Delta t)}{\Delta t^{-h-g}} \\
 & \cdot e^{\frac{-c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^2 \left. \right] + 2\left(\frac{A}{\tau}\right)^2 \sum_{c=0}^{\infty} \left[\sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \sum_{h=0}^c \eta_{\Delta t}^2 \frac{v(a\Delta t)v(b\Delta t)v(a\Delta t + c\Delta t - h\Delta t)v(b\Delta t + c\Delta t - h\Delta t)}{\Delta t^{-c-a} \Delta t^{-c-b}} \right. \\
 & \cdot e^{\frac{-c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^2 \\
 & + \sum_{f=0}^{\infty} \sum_{g=0}^{\infty} \sum_{h=c}^{\infty} \eta_{\Delta t}^2 \frac{v(f\Delta t)v(g\Delta t)v(f\Delta t + h\Delta t - c\Delta t)}{\Delta t^{-h-f} \Delta t^{-h-g}} \\
 & \cdot v(g\Delta t + h\Delta t - c\Delta t) e^{\frac{-c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^2 \left. \right] + \left(\frac{A}{\tau}\right)^2 \sum_{c=0}^{\infty} \sum_{b=0}^{\infty} \sum_{h=0}^{\infty} \sum_{g=0}^{\infty} \\
 & \cdot \eta_{\Delta t}^2 \frac{v^2(b\Delta t)v^2(g\Delta t)}{\Delta t^{-c-b} \Delta t^{-h-g}} e^{\frac{-c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^2, \quad (3-30)
 \end{aligned}$$

where it is understood that no two subscripts of η in any summation are equal.

Equation (3-30) is obtained from Equation (3-29) by performing the following operations:

- Equation (3-29) is written as the sum of five separate summations. The first has the subscripts of all four η 's equal, i.e., four of a kind. The second has the subscripts of three of the η 's equal, i.e., three of a kind. The third has two pairs, the fourth has one pair, and the fifth has none equal. The second, fourth, and fifth summations will go to zero because of Equation (3-11), leaving only the first and third.
- All four subscripts are equal only if $a = b$, $f = g$, and $c + b = h + g$. For those terms in which $c > h$, we write b for a and $b + c - h$ for f and g , and for those terms in which $c < h$, we write g for f and $g + h - c$ for a and b ; the result is the first summation in Equation (3-30).
- There are three ways in which there can be two pairs. The first is for $a = b$ and $f = g$. Then, if we write b for a and g for f , the result is the third summation in Equation (3-30).

- d. The second way to obtain two pairs is for $c + a = h + f$ and $c + b = h + g$, and the third way is for $c + a = h + g$ and $c + b = h + f$. For those terms in which $c > h$ we write $a + c - h$ for f and $b + c - h$ for g to take care of the second way, and we write $a + c - h$ for g and $b + c - h$ for f to take care of the third way; for those terms in which $c < h$ we write $f + h - c$ for a and $g + h - c$ for b to take care of the second way, and we write $f + h - c$ for b and $g + h - c$ for a to take care of the third way. The result is the second summation in Equation (3-30).
- e. It is necessary to separate the terms in which $c > h$ from those in which $c < h$ to insure that the argument of v is always positive [cf. Equation (3-29)].

If the average flux of neutrons is Φ and the counting efficiency of the detector is k , then the expected values of the η combinations are

$$\begin{aligned}\langle (\eta)^4 \rangle &= k\Phi \Delta t \\ \langle (\eta)^2 (\eta)^2 \rangle &= (k\Phi \Delta t)^2\end{aligned}\quad (3-31)$$

and the expected value of $S^2(t)$ is

$$\begin{aligned}\langle S^2(t) \rangle &= k\Phi \left(\frac{\Delta}{\tau}\right)^2 \sum_{c=0}^{\infty} \left[\sum_{b=0}^{\infty} \sum_{h=0}^c v^2(b\Delta t) v^2(b\Delta t + c\Delta t - h\Delta t) \cdot e^{-\frac{c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^3 \right. \\ &\quad \left. + \sum_{g=0}^{\infty} \sum_{h=c}^{\infty} v^2(g\Delta t) v^2(g\Delta t + h\Delta t - c\Delta t) e^{-\frac{c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^3 \right] \\ &\quad + 2 \left(k\Phi \frac{\Delta}{\tau} \right)^2 \sum_{c=0}^{\infty} \left[\sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \sum_{h=0}^c v(a\Delta t) v(b\Delta t) v(a\Delta t + c\Delta t - h\Delta t) \right. \\ &\quad \left. a \neq b \right. \\ &\quad \cdot v(b\Delta t + c\Delta t - h\Delta t) e^{-\frac{c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^4 \\ &\quad + \sum_{f=0}^{\infty} \sum_{g=0}^{\infty} \sum_{h=c}^{\infty} v(f\Delta t) v(g\Delta t) v(f\Delta t + h\Delta t - c\Delta t) \\ &\quad \left. f \neq g \right. \\ &\quad \cdot v(g\Delta t + h\Delta t - c\Delta t) e^{-\frac{c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^4 \left. \right] \\ &\quad + \left(k\Phi \frac{\Delta}{\tau} \right)^2 \sum_{c=0}^{\infty} \sum_{b=0}^{\infty} \sum_{h=0}^{\infty} \sum_{g=0}^{\infty} v^2(b\Delta t) v^2(g\Delta t) e^{-\frac{c\Delta t}{\tau} - \frac{h\Delta t}{\tau}} (\Delta t)^4. \quad (3-32) \\ &\quad g \neq b + c - h\end{aligned}$$

We should now remove the conditions $a \neq b$, $f \neq g$, and $g \neq b + c - h$ by adding and subtracting the appropriate terms; however, the additional summations so created will go to zero in the limit as $\Delta t \rightarrow 0$ because they will contain one more Δt than summation signs [cf. Equations (3-7), (3-8), and (3-9)], so we will not write them. As $\Delta t \rightarrow 0$, Equation (3-32) (with the conditions $a \neq b$, $f \neq g$, and $g \neq b + c - h$ removed) becomes

$$\begin{aligned}
 \langle S^2(t) \rangle &= k\Phi \left(\frac{A}{\tau} \right)^2 \int_0^\infty \int_0^\infty v^2(y) e^{-x/\tau} \\
 &\quad \cdot \left[\int_0^x e^{-z/\tau} v^2(y+x-z) dz + \int_x^\infty e^{-z/\tau} v^2(y+z-x) dz \right] dx dy \\
 &+ 2 \left(\frac{k\Phi A}{\tau} \right)^2 \int_0^\infty \int_0^\infty \int_0^\infty v(\xi)v(y) e^{-x/\tau} \\
 &\quad \cdot \left[\int_0^x e^{-z/\tau} v(y+x-z)v(\xi+x-z) dz + \int_x^\infty e^{-z/\tau} v(y+z-x)v(\xi+z-x) dz \right] dx dy d\xi \\
 &+ \left[k\Phi A \int_0^\infty v^2(x) dx \right]^2 \quad (3-33)
 \end{aligned}$$

By combining Equations (3-14), (3-28), and (3-33), we obtain the variance of the signal

$$\begin{aligned}
 \sigma_S^2 &= \left(\frac{A}{\tau} \right)^2 k\Phi \int_0^\infty \int_0^\infty v^2(y) e^{-x/\tau} \\
 &\quad \cdot \left[\int_0^x e^{-z/\tau} v^2(y+x-z) dz + \int_x^\infty e^{-z/\tau} v^2(y+z-x) dz \right] dx dy \\
 &+ 2 \left(\frac{A k \Phi}{\tau} \right)^2 \int_0^\infty \int_0^\infty \int_0^\infty v(\xi)v(y) e^{-x/\tau} \\
 &\quad \cdot \left[\int_0^x e^{-z/\tau} v(y+x-z)v(\xi+x-z) dz \right. \\
 &\quad \left. + \int_x^\infty e^{-z/\tau} v(y+z-x)v(\xi+z-x) dz \right] dx dy d\xi \quad (3-34)
 \end{aligned}$$

3.1.3 Expected Value of the Signal for Time-Varying Flux

An expression for the signal at the output of an a-c system is given by Equation (3-27)

$$S(t) = \frac{A}{\tau} \sum_{c=0}^{\infty} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \eta_{t-\Delta t-c-a} \eta_{t-\Delta t-c-b} v(a\Delta t)v(b\Delta t)\Delta t e^{-c\Delta t/\tau} \quad (3-27)$$

and the expected value of this signal is

$$\langle S(t) \rangle = \frac{A}{\tau} \sum_{c=0}^{\infty} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \langle \eta_{\frac{t}{\Delta t} - c - a} \eta_{\frac{t}{\Delta t} - c - b} \rangle v(a\Delta t) v(b\Delta t) \Delta t e^{-c\Delta t/\tau} \quad (3-35)$$

Since

$$\begin{aligned} \langle \eta_{\frac{t}{\Delta t} - c - a} \eta_{\frac{t}{\Delta t} - c - b} \rangle &= k\Phi(t - c\Delta t - a\Delta t)\Delta t && \text{for } a = b \\ &= k^2\Phi(t - c\Delta t - a\Delta t)\Phi(t - c\Delta t - b\Delta t)(\Delta t)^2 && \text{for } a \neq b, \end{aligned} \quad (3-36)$$

then

$$\begin{aligned} \langle S(t) \rangle &= \frac{Ak}{\tau} \sum_{c=0}^{\infty} \sum_{a=0}^{\infty} \Phi(t - c\Delta t - a\Delta t) v^2(a\Delta t) e^{-c\Delta t/\tau} (\Delta t)^2 \\ &+ \frac{Ak^2}{\tau} \sum_{c=0}^{\infty} \sum_{a=0}^{\infty} \sum_{\substack{b=0 \\ a \neq b}}^{\infty} \Phi(t - c\Delta t - a\Delta t) \Phi(t - c\Delta t - b\Delta t) \\ &\quad v(a\Delta t) v(b\Delta t) e^{-c\Delta t/\tau} (\Delta t)^3, \end{aligned} \quad (3-37)$$

and this becomes, as $\Delta t \rightarrow 0$,

$$\begin{aligned} \langle S(t) \rangle &= \frac{Ak}{\tau} \int_0^{\infty} e^{-z/\tau} \left[\int_0^{\infty} \Phi(t - z - x) v^2(x) dx \right] dz \\ &+ \frac{Ak^2}{\tau} \int_0^{\infty} e^{-z/\tau} \left[\int_0^{\infty} \Phi(t - z - x) v(x) dx \right]^2 dz. \end{aligned} \quad (3-38)$$

The expected value of the voltage at the output of the squaring circuit, $\langle AV^2(t) \rangle$, for a time-varying flux is most easily obtained by letting $\tau \rightarrow 0$ in Equation (3-38). For a very small τ , $\frac{1}{\tau} e^{-z/\tau}$ is one form of the Dirac delta function, $\delta(z-0)$, which has the property

$$\int_{-\infty}^{\infty} F(z) \delta(z-0) dz = F(0),$$

so

$$\langle AV^2(t) \rangle = Ak \int_0^{\infty} \Phi(t-x) v^2(x) dx + A \left[k \int_0^{\infty} \Phi(t-x) v(x) dx \right]^2 \quad (3-39)$$

3.1.4 Value of the Signal for Flux with Sinusoidal Component

If the expected value of flux at time t is given by

$$\Phi(t) = \bar{\Phi} (1 + f \sin \omega t), \quad (3-40)$$

where $\bar{\Phi}$ is the time-averaged value of the flux and f is a number less than unity, then the expected value, $\langle AV^2(t) \rangle$, of the voltage at the input to the integrating circuit is, from Equations (3-39) and (3-40),

$$\begin{aligned} \langle AV^2(t) \rangle = & Ak\bar{\Phi} \int_0^\infty v^2(x) dx \\ & + \frac{A(k\bar{\Phi}f)^2}{2} \left[\left(\int_0^\infty v(x) \cos \omega x dx \right)^2 + \left(\int_0^\infty v(x) \sin \omega x dx \right)^2 \right] \\ & + Ak\bar{\Phi}f \left[\left(\int_0^\infty v^2(x) \cos \omega x dx \right)^2 + \left(\int_0^\infty v^2(x) \sin \omega x dx \right)^2 \right]^{1/2} \sin(\omega t - \delta_1) \\ & - \frac{A(k\bar{\Phi}f)^2}{2} \left[\left(\int_0^\infty v(x) \cos \omega x dx \right)^2 + \left(\int_0^\infty v(x) \sin \omega x dx \right)^2 \right] \cos 2(\omega t - \delta_2). \quad (3-41) \end{aligned}$$

When averaged over a period of time long compared to one cycle, the last two terms drop out, and we are left with the following expression for the time-averaged value of $AV^2(t)$ [which equals the time averaged value of the signal, $S(t)$]

$$\begin{aligned} \overline{S(t)} = & Ak\bar{\Phi} \int_0^\infty v^2(x) dx \\ & + \frac{A(k\bar{\Phi}f)^2}{2} \left[\left(\int_0^\infty v(x) \cos \omega x dx \right)^2 + \left(\int_0^\infty v(x) \sin \omega x dx \right)^2 \right]. \quad (3-42) \end{aligned}$$

This, of course, is independent of the value of τ .

3.1.5 Value of the Signal for Flux with Continuous Spectrum

Here it is desired to obtain the value of $\langle S(t) \rangle$ for a flux whose expected value at time t is given by

$$\Phi(t) = \bar{\Phi} + \phi(t), \quad (3-43)$$

where $\bar{\Phi}$ is the time-averaged value and $\phi(t)$ has a power spectral density of $G(\omega)$.

Since the variance (power) of a sine wave is one half the square of its amplitude, Equation (3-42) can be written as

$$\begin{aligned} \overline{S(t)} = & Ak\bar{\Phi} \int_0^{\infty} v^2(x) dx \\ & + Ak^2 \sigma_{\Phi}^2 \left[\left(\int_0^{\infty} v(x) \cos \omega x dx \right)^2 + \left(\int_0^{\infty} v(x) \sin \omega x dx \right)^2 \right], \end{aligned} \quad (3-44)$$

which says that the time-averaged signal has two components, one proportional to the time-averaged value of flux and one proportional to the power, σ_{Φ}^2 , in the sinusoidal component of angular frequency ω

For the case here considered (continuous power spectrum), the power in the frequency range ω to $\omega + d\omega$ is $G(\omega) d\omega$, and the signal due to this much of the spectrum is

$$d \overline{S(t)} = Ak^2 G(\omega) d\omega \left[\left(\int_0^{\infty} v(x) \cos \omega x dx \right)^2 + \left(\int_0^{\infty} v(x) \sin \omega x dx \right)^2 \right]. \quad (3-45)$$

Hence the total signal is given by

$$\begin{aligned} S(t) = & Ak\bar{\Phi} \int_0^{\infty} v^2(x) dx \\ & + Ak^2 \int_0^{\infty} G(\omega) \left[\left(\int_0^{\infty} v(x) \cos \omega x dx \right)^2 + \left(\int_0^{\infty} v(x) \sin \omega x dx \right)^2 \right] d\omega. \end{aligned} \quad (3-46)$$

3. 1. 6 Value of the Signal for Transient in the Flux

If the flux is constant for $t < 0$ and differs from this constant value by $\phi(t)$ for $t > 0$; i.e.,

$$\begin{aligned} \Phi(t) &= \Phi_0 & \text{for } t < 0 \\ \Phi(t) &= \Phi_0 + \phi(t) & \text{for } t > 0, \end{aligned} \quad (3-47)$$

then the expected value of the signal from an a-c system is, from Equations (3-38) and (3-47),

$$\begin{aligned}
 \langle S(t) \rangle = & Ak\Phi_0 \int_0^\infty v^2(x) dx \\
 & + \frac{Ak}{\tau} \int_0^t e^{-z/\tau} \left[\int_0^{t-z} \phi(t-z-x) v^2(x) dx \right] dz \\
 & + \frac{Ak^2}{\tau} \int_0^t e^{-z/\tau} \left[\int_0^{t-z} \phi(t-z-x) v(x) dx \right]^2 dz .
 \end{aligned} \tag{3-48}$$

At the input to the integrating circuit the expected value of voltage is, from Equations (3-39) and (3-47),

$$\begin{aligned}
 \langle AV^2(t) \rangle = & Ak\Phi_0 \int_0^\infty v^2(x) dx + kA \int_0^t \phi(t-x) v^2(x) dx \\
 & + A \left[k \int_0^t \phi(t-x) v(x) dx \right]^2 .
 \end{aligned} \tag{3-49}$$

The application of Equation (3-48) to an actual case is usually quite difficult, leading to a large quantity of algebraic manipulation during which mistakes are very probable. An alternative method, that lends itself to some approximation and considerable simplification, is as follows.

Equation (3-49) is used to obtain the expected value of voltage at the input to the integrating circuit. This expression is replaced by a simpler one that should give essentially the same output; a knowledge of the properties of integrating circuits is useful here. This simpler expression is then used as the driving function in the differential Equation (3-2) of the integrating circuit to obtain the expected value of the signal, $\langle S(t) \rangle$. Examples of this procedure are included in Subsection 3.2. If an exact answer is desired, use either Equation (3-48), or Equations (3-49) and (3-2) with no approximations.

3.2 SPECIFIC RESULTS

To apply the above general results, it is necessary to obtain an expression for the pulse generated at the amplifier output by the detection of a single neutron, and this can be derived from the energy spectral density of the pulse or from a knowledge of the time constants of the circuit.

For example, consider a system whose linear portion (cable plus amplifier) has a single low-frequency break at ω_L , a single high-frequency break at ω_H , and a mid-frequency transfer impedance of Z_{12} , and whose detector is a neutron-sensitive ion chamber. The energy

spectral density of a current pulse from the detector is flat and equal to Q_e^2/π between the angular frequencies $2\pi/T_i$ and π/T_e , where Q_e is the product of the fraction of the chamber potential through which the electrons travel and the total charge of one sign released in the chamber by the detection of one neutron, T_i is the ion collection time, and T_e is the electron collection time. So if $\omega_L > 2\pi/T_i$ and $\omega_H < \pi/T_e$ (see Figure 3-2), the energy spectral density of $v(t)$ is the product of Q_e^2/π and the square of the absolute value of the transfer function of the linear portion of the system:

$$E(\omega, v) = \frac{Q_e^2 Z_{12}^2}{\pi} \frac{\omega^2 / \omega_L^2}{\left(1 + \frac{\omega^2}{\omega_L^2}\right) \left(1 + \frac{\omega^2}{\omega_H^2}\right)}, \quad (3-50)$$

and the pulse itself is given by

$$v(t) = Q_e Z_{12} \frac{\omega_H}{\omega_H - \omega_L} \left(\omega_H e^{-\omega_H t} - \omega_L e^{-\omega_L t} \right). \quad (3-51)$$

This expression for $v(t)$ can be used in any of the preceding equations.

3.2.1 Value of the Signal for Constant Flux

Substitution of Equation (3-51) into Equation (3-14) yields the following expression for the expected value of the signal:

$$\langle S(t) \rangle = \frac{Ak\Phi Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \quad (3-52)$$

If both neutrons and gamma photons are being detected, this expression becomes

$$\langle S(t) \rangle = \frac{Ak_n \Phi_n Q_{en}^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} + \frac{Ak_g \Phi_g Q_{eg}^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \quad (3-53)$$

Note that the total signal is the sum of one part due to neutrons and one due to photons.

A measure of the gamma-discrimination ability of a system is the ratio of the flux of gammas to the flux of neutrons when both are producing the same output. This ratio can be obtained by equating the two terms in the right-hand side of Equation (3-53):

$$\frac{\Phi_g}{\Phi_n} = \frac{k_n}{k_g} \left(\frac{Q_{en}}{Q_{eg}} \right)^2 \quad (3-54)$$

Figure 3-2. Frequency Considerations

3.2.2 Variance of the Signal

The fractional standard deviation, defined as the ratio of the square-root of the variance of the signal to the expected value of the signal, is a more useful figure than the variance itself. Using Equation (3-51) for $v(t)$, Equation (3-34) for σ_S^2 , and Equation (3-14) for $\langle S(t) \rangle$, this ratio is

$$\frac{\sigma_S}{\langle S(t) \rangle} = \left(\frac{1}{\tau (\omega_H + \omega_L)} + \frac{1}{2k\Phi\tau} \right)^{1/2}, \quad (3-55)$$

where it has been assumed that $\tau \gg 1/\omega_L$ to simplify the result. (The unsimplified expression shows that the condition $\tau \gg 1/\omega_L$ must be met in order to make the fractional standard deviation practically small. It also shows that Equation (3-55) is accurate to about 5 percent when $\tau = 10/\omega_L$.)

Note that for large fluxes [$k\Phi \gg (\omega_H + \omega_L)/2$], the fractional standard deviation approaches a constant value

$$\frac{\sigma_S}{\langle S(t) \rangle} = \left(\frac{1}{\tau (\omega_H + \omega_L)} \right)^{1/2}. \quad (3-56)$$

3.2.3 Value of the Signal for Flux with Sinusoidal Component

If the expected value of flux at time t is given by

$$\Phi(t) = \bar{\Phi} (1 + f \sin \omega t), \quad (3-40)$$

then the time-averaged value of the a-c signal, using Equation (3-51) for $v(t)$ and Equation (3-42) for $S(t)$, is

$$\frac{\overline{S(t)}}{\langle S(t) \rangle} = \frac{Ak\bar{\Phi}Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} + \frac{A(k\bar{\Phi}fQ_e Z_{12})^2}{2\omega_L^2} \frac{\omega^2}{\left(1 + \frac{\omega^2}{\omega_L^2}\right) \left(1 + \frac{\omega^2}{\omega_H^2}\right)}. \quad (3-57)$$

Since the first term on the right is the output due to the time-averaged flux, it is the desired value, $\overline{S(t)}_c$, of the average signal. The second term represents the error, E_s , due to the fluctuations of the flux. The fractional error is the ratio of these two terms:

$$\frac{E_s}{\overline{S(t)}_c} = k\bar{\Phi}f^2 \frac{\omega_H + \omega_L}{(\omega_H \omega_L)^2} \frac{\omega^2}{\left(1 + \frac{\omega^2}{\omega_L^2}\right) \left(1 + \frac{\omega^2}{\omega_H^2}\right)}. \quad (3-58)$$

3.2.4 Value of the Signal for Flux with Continuous Spectrum

If the expected value of flux at time t is given by

$$\Phi(t) = \bar{\Phi} + \phi(t), \quad (3-43)$$

where $\bar{\Phi}$ is the time-averaged value and $\phi(t)$ has a power spectral density of $G(\omega)$, then the time-averaged value of the signal from an a-c system, using Equation (3-51) for $v(t)$ and Equation (3-46) for $S(t)$, is

$$\begin{aligned} \overline{S(t)} = & \frac{Ak\bar{\Phi}Q_e^2Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \\ & + \frac{Ak^2Q_e^2Z_{12}^2}{\omega_L^2} \int_0^\infty \frac{G(\omega) \omega^2 d\omega}{\left(1 + \frac{\omega^2}{\omega_L^2}\right) \left(1 + \frac{\omega^2}{\omega_H^2}\right)}. \end{aligned} \quad (3-59)$$

Since the first term on the right is the output due to the time-averaged flux, the second term represents the error due to the fluctuations of the flux. The fractional error is the ratio of these two terms:

$$\frac{E_s}{\overline{S(t)}_c} = \frac{2k}{\bar{\Phi}} \frac{\omega_H + \omega_L}{(\omega_H \omega_L)^2} \int_0^\infty \frac{G(\omega) \omega^2 d\omega}{\left(1 + \frac{\omega^2}{\omega_L^2}\right) \left(1 + \frac{\omega^2}{\omega_H^2}\right)}. \quad (3-60)$$

3.2.5 Value of the Signal for Transient in the Flux

In this section the expected value of the signal will be calculated for the case in which $v(t)$ is given by Equation (3-51) and for three different transients in the flux.

The procedure that will be followed is:

- Substitution of $v(t)$ [Equation (3-51)] and the appropriate expression for $\phi(t)$ into Equation (3-49) to obtain $\langle AV^2(t) \rangle$;
- Simplification of this expression for $\langle AV^2(t) \rangle$;
- Substitution of the simplified expression for $\langle AV^2(t) \rangle$ and the proper initial condition into the differential Equation (3-2) of the integrating circuit to obtain the expected value of the signal, $\langle S(t) \rangle$; and
- Simplification of $\langle S(t) \rangle$.

Case 1. The flux has one constant value for $t < 0$ and a second constant value for $t > 0$; i.e.,

$$\begin{aligned}\Phi(t) &= \Phi_0 && \text{for } t < 0, \text{ and} \\ \Phi(t) &= \Phi_0 + \Delta \Phi && \text{for } t > 0.\end{aligned}$$

Then $\phi(t) = \Delta \Phi$, $\phi(t-x) = \Delta \Phi$, and the expected value of $AV^2(t)$ is

$$\begin{aligned}\langle AV^2(t) \rangle &= \frac{Ak(\Phi_0 + \Delta \Phi) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{(\omega_H + \omega_L)} \\ &\quad - Ak\Delta \Phi Q_e^2 Z_{12}^2 \left(\frac{\omega_H}{\omega_H - \omega_L} \right)^2 \left(\frac{\omega_H e^{-2\omega_H t}}{2} - \frac{2\omega_H \omega_L e^{-(\omega_H + \omega_L)t}}{\omega_H + \omega_L} \right. \\ &\quad \left. + \frac{\omega_L e^{-2\omega_L t}}{2} \right) \\ &\quad + A \left[k\Delta \Phi Q_e Z_{12} \frac{\omega_H}{\omega_H - \omega_L} \left(e^{-\omega_H t} - e^{-\omega_L t} \right) \right]^2.\end{aligned}\quad (3-61)$$

If $\tau \gg 1/\omega_L$, Equation (3-61) can be simplified for use in the differential Equation (3-2) by noting that the first two terms approach, with a time constant of about $1/\omega_L$, the value

$$\frac{Ak(\Phi_0 + \Delta \Phi) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L},$$

and that the third term has a total width of about $1/\omega_L$ so it may be replaced by a Dirac delta function having the same area:

$$\frac{A(k\Delta \Phi Q_e Z_{12})^2}{2} \frac{\omega_H}{\omega_L (\omega_H + \omega_L)} \delta(t-0).$$

So Equation (3-61) becomes

$$\begin{aligned}\langle AV^2(t) \rangle &= \frac{Ak(\Phi_0 + \Delta \Phi) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{(\omega_H + \omega_L)} \\ &\quad + \frac{A(k\Delta \Phi Q_e Z_{12})^2}{2} \frac{\omega_H}{\omega_L (\omega_H + \omega_L)} \delta(t-0).\end{aligned}\quad (3-62)$$

Substitution of this expression into Equation (3-2), with the initial condition

$$\langle S(0) \rangle = \frac{k\Phi_0 A Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{(\omega_H + \omega_L)}$$

yields the expected value of the signal

$$\begin{aligned} \langle S(t) \rangle = & \frac{A k \Phi_0 Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \\ & + \frac{A k \Delta \Phi Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \left(1 - e^{-t/\tau} \right) \\ & + \frac{A (k \Delta \Phi Q_e Z_{12})^2}{2\tau} \frac{\omega_H}{\omega_L (\omega_H + \omega_L)} e^{-t/\tau} \end{aligned} \quad (3-63)$$

Since the correct value is

$$\langle S(t) \rangle_c = \frac{A k (\Phi_0 + \Delta \Phi) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L}, \quad (3-64)$$

the error is

$$\begin{aligned} E(t) = & - \frac{A k \Delta \Phi Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} e^{-t/\tau} \\ & + \frac{A (k \Delta \Phi Q_e Z_{12})^2}{2\tau} \frac{\omega_H}{\omega_L (\omega_H + \omega_L)} e^{-t/\tau}, \end{aligned} \quad (3-65)$$

and the fractional error is

$$\frac{E(t)}{\langle S(t) \rangle_c} = - \frac{\Delta \Phi e^{-t/\tau}}{\Phi_0 + \Delta \Phi} + \frac{k (\Delta \Phi)^2}{(\Phi_0 + \Delta \Phi) \tau \omega_H \omega_L} e^{-t/\tau} \quad (3-66)$$

Case 2. The flux is constant for $t < 0$ and increases at a constant rate, S , for $t > 0$; i.e.,

$$\Phi(t) = \Phi_0 \quad \text{for } t < 0, \text{ and}$$

$$\Phi(t) = \Phi_0 + St \quad \text{for } t > 0,$$

then $\phi(t) = St$, $\phi(t-x) = St - Sx$, and the expected value of $AV^2(t)$ is

$$\begin{aligned}
\langle AV^2(t) \rangle = & \frac{A k(\Phi_o + St) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \\
& - AkSQ_e^2 Z_{12}^2 \left(\frac{\omega_H}{\omega_H - \omega_L} \right)^2 \left[\frac{1 - e^{-2\omega_H t}}{4} - \frac{2\omega_H \omega_L}{(\omega_H + \omega_L)^2} \left(1 - e^{-(\omega_H + \omega_L)t} \right) \right. \\
& \quad \left. + \frac{1 - e^{-2\omega_L t}}{4} \right] \\
& + A \left(kSQ_e Z_{12} \frac{\omega_H}{\omega_H - \omega_L} \right)^2 \left[\frac{1 - e^{-\omega_L t}}{\omega_L} - \frac{1 - e^{-\omega_H t}}{\omega_H} \right]^2. \quad (3-67)
\end{aligned}$$

This expression approaches, with a time-constant of about $1/\omega_L$, the simpler one

$$\begin{aligned}
\langle AV^2(t) \rangle = & \frac{Ak(\Phi_o + St) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \\
& - \frac{AkSQ_e^2 Z_{12}^2}{2} \left(\frac{\omega_H}{\omega_H + \omega_L} \right)^2 + A \left(\frac{kSQ_e Z_{12}}{\omega_L} \right)^2, \quad (3-68)
\end{aligned}$$

and, if $\tau \gg 1/\omega_L$, this simpler expression may be substituted into Equation (3-2) along with the initial condition.

$$\langle S(o) \rangle = \frac{Ak\Phi_o Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L}$$

to obtain the expected value of the signal

$$\begin{aligned}
\langle S(t) \rangle = & \frac{Ak(\Phi_o + St) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \\
& - \left[\frac{AkS\tau Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} + \frac{AkSQ_e^2 Z_{12}^2}{2} \frac{\omega_H}{\omega_H + \omega_L} \right] \left(1 - e^{-t/\tau} \right) \\
& + A \left(\frac{kSQ_e Z_{12}}{\omega_L} \right)^2 \left(1 - e^{-t/\tau} \right). \quad (3-69)
\end{aligned}$$

Since the correct value is

$$\langle S(t) \rangle_c = \frac{Ak(\Phi_0 + St) Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L}, \quad (3-70)$$

the error is

$$E(t) = - \left[\frac{AkS\tau Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} + \frac{AkSQ_e^2 Z_{12}^2}{2} \frac{\omega_H}{\omega_H + \omega_L} \right] \cdot \left(1 - e^{-t/\tau} \right) + A \left(\frac{kS Q_e Z_{12}}{\omega_L} \right)^2 \left(1 - e^{-t/\tau} \right), \quad (3-71)$$

and the fractional error is

$$\begin{aligned} \frac{E(t)}{\langle S(t) \rangle_c} = & - \frac{S\tau + \frac{S}{\omega_H + \omega_L}}{\Phi_0 + St} \left(1 - e^{-t/\tau} \right) \\ & + \frac{2kS^2 \frac{\omega_H + \omega_L}{(\omega_H \omega_L)^2}}{\Phi_0 + St} \left(1 - e^{-t/\tau} \right). \end{aligned} \quad (3-72)$$

Since we have assumed that $\tau \gg 1/\omega_L$, this may be simplified to

$$\frac{E(t)}{\langle S(t) \rangle_c} = \left[- \frac{S\tau}{\Phi_0 + St} + \frac{2kS^2(\omega_H + \omega_L)}{(\Phi_0 + St)(\omega_H \omega_L)^2} \right] \left(1 - e^{-t/\tau} \right). \quad (3-73)$$

Case 3. The flux is constant for $t < 0$ and increases exponentially (constant period, P) for $t > 0$; i.e.,

$$\Phi(t) = \Phi_0 \quad \text{for } t < 0, \text{ and}$$

$$\Phi(t) = \Phi_0 e^{t/P} \quad \text{for } t > 0,$$

then $\phi(t) = \Phi_0(e^{t/P} - 1)$, $\phi(t-x) = \Phi_0(e^{(t-x)/P} - 1)$, and the expected value of $AV^2(t)$ is

$$\begin{aligned} \langle AV^2(t) \rangle = & \frac{Ak\Phi_o Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \frac{2P(\omega_H + \omega_L)(2P^2\omega_H\omega_L + P\omega_H + P\omega_L + 1)}{(2P\omega_H + 1)(2P\omega_L + 1)(P\omega_H + P\omega_L + 1)} e^{t/P} \\ & + A \left[k\Phi_o Q_e Z_{12} \frac{P\omega_H}{(P\omega_H + 1)(P\omega_L + 1)} \right]^2 e^{2t/P}. \end{aligned} \quad (3-74)$$

This is really the simplified expression for $\langle AV^2(t) \rangle$. The actual expression approaches this simpler one with a time constant of about $1/\omega_L$. So, if $|P| \gg 1/\omega_L$ and $\tau \gg 1/\omega_L$, this simpler Equation (3-74) may be substituted into the differential Equation (3-2) along with the initial condition

$$\langle S(0) \rangle = \frac{Ak\Phi_o Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L}$$

to obtain the expected value of the a-c system signal

$$\langle S(t) \rangle = GHe^{t/P} + Je^{2t/P} - [G(H-1) + J] e^{-t/\tau}, \quad (3-75)$$

where

$$\begin{aligned} G &= \frac{Ak\Phi_o Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{(\omega_H + \omega_L)}, \\ H &= \frac{2P^2(\omega_H + \omega_L)(2P^2\omega_H\omega_L + P\omega_H + P\omega_L + 1)}{(P + \tau)(2P\omega_H + 1)(2P\omega_L + 1)(P\omega_H + P\omega_L + 1)}, \end{aligned}$$

and

$$J = A \left[\frac{k\Phi_o Q_e Z_{12} P\omega_H}{(P\omega_H + 1)(P\omega_L + 1)} \right]^2 \frac{P}{P + 2\tau}.$$

Since the correct value is

$$\langle S(t) \rangle_c = Ge^{t/P}, \quad (3-76)$$

the error is

$$E(t) = G(H-1)(e^{t/P} - e^{-t/\tau}) + J(e^{2t/P} - e^{-t/\tau}), \quad (3-77)$$

and the fractional error is

$$\frac{E(t)}{\langle S(t) \rangle_c} = (H-1) \left(1 - e^{-t \left(\frac{1}{\tau} + \frac{1}{P} \right)} \right) + \frac{J}{G} \left(e^{t/P} - e^{-t \left(\frac{1}{\tau} + \frac{1}{P} \right)} \right) \quad (3-78)$$

or

$$\frac{E(t)}{\langle S(t) \rangle_c} = \left[\frac{2P^2(\omega_H + \omega_L) (2P^2\omega_H\omega_L + P\omega_H + P\omega_L + 1)}{(P+\tau) (2P\omega_H+1) (2P\omega_L+1)(P\omega_H+P\omega_L+1)} - 1 \right] \left(1 - e^{-t\left(\frac{1}{\tau} + \frac{1}{P}\right)} \right) + 2k\Phi_0 \frac{P^3 (\omega_H + \omega_L)}{(P+2\tau)(P\omega_H+1)^2 (P\omega_L+1)^2} \left(e^{t/P} - e^{-t\left(\frac{1}{\tau} + \frac{1}{P}\right)} \right) \quad (3-79)$$

Since we have assumed that $\tau \gg 1/\omega_L$ and $|P| \gg 1/\omega_L$, this may be simplified to

$$\frac{E(t)}{\langle S(t) \rangle_c} = - \frac{\tau}{P + \tau} \left(1 - e^{-t\left(\frac{1}{\tau} + \frac{1}{P}\right)} \right) + 2k\Phi_0 \frac{\omega_H + \omega_L}{P(P+2\tau) \cdot (\omega_H\omega_L)^2} \left(e^{t/P} - e^{-t\left(\frac{1}{\tau} + \frac{1}{P}\right)} \right) \quad (3-80)$$

3.3 RESPONSE TO REACTOR NOISE

Since the signal from a Campbell system is due to noise caused by the random nature of the detection process, it is reasonable to expect that reactor noise could increase the size of the signal. This additional output is not an error, for it is shown on the following pages that it, too, is proportional to the reactor flux (except for some very low-frequency noises caused by mechanical disturbances; these very low-frequency noises are filtered out by the relatively high-frequency bandpass of the amplifier). The magnitude of the additional flux-proportional signal, caused by reactor noise, is also very small due to low detection efficiency and the low frequency of the reactor noise coupled with the relatively high-frequency bandpass of the amplifier. The magnitude of additional signal caused by reactor noise is obtained in two ways.

3.3.1 Derivation in Time Domain

Since the voltage at the output of the amplifier is given by

$$V(t) = \sum_{n=-\infty}^{t/\Delta t} \eta_n v(t - n\Delta t) \quad (3-3)$$

then

$$V^2(t) = \sum_{m=-\infty}^{t/\Delta t} \sum_{n=-\infty}^{t/\Delta t} \eta_m \eta_n v(t - m\Delta t) v(t - n\Delta t) \quad (3-81)$$

Equation (3-81) can be written as

$$\begin{aligned}
 v^2(t) &= \sum_{m=-\infty}^{t/\Delta t} \left(\eta_m \right)^2 v^2(t - m\Delta t) \\
 &+ \sum_{m=-\infty}^{t/\Delta t} \sum_{\substack{n=-\infty \\ m \neq n}}^{t/\Delta t} \eta_m \eta_n v(t - m\Delta t) v(t - n\Delta t)
 \end{aligned} \tag{3-82}$$

by separating the terms in which $m = n$ from those in which $m \neq n$. The expected value of $v^2(t)$ is

$$\begin{aligned}
 \langle v^2(t) \rangle &= \sum_{m=-\infty}^{t/\Delta t} \langle \left(\eta_m \right)^2 \rangle v^2(t - m\Delta t) \\
 &+ \sum_{m=-\infty}^{t/\Delta t} \sum_{\substack{n=-\infty \\ m \neq n}}^{t/\Delta t} \langle \eta_m \eta_n \rangle v(t - m\Delta t) v(t - n\Delta t).
 \end{aligned} \tag{3-83}$$

since the expectation of the sum of a number of random variables is equal to the sum of their expectations.

If the average fission rate in the reactor is F and the counting efficiency of the detector is ϵ counts per fission, then the probability of detecting a neutron during the n^{th} time interval is $F\epsilon\Delta t$, and the probability of detecting no neutron during the n^{th} time interval is $1 - F\epsilon\Delta t$. So the expected value of $\left(\eta_m \right)^2$ is

$$\langle \left(\eta_m \right)^2 \rangle = \left[1^2 \times F\epsilon\Delta t \right] + \left[0^2 \times (1 - F\epsilon\Delta t) \right] = F\epsilon\Delta t. \tag{3-84}$$

The expected value of $\eta_m \eta_n$ has two terms: one due to accidental pairs, and one due to coupled pairs. An accidental pair is caused by a count registering in the m^{th} time interval due to a neutron in a family from one primary source neutron, and a count registering in the n^{th} time interval due to a neutron in an entirely separate family from another independent primary neutron. A coupled pair is caused by counts in the m^{th} and n^{th} time intervals due to neutrons that can be traced back to some common ancestor; i. e., one fission (refer to de Hoffman⁽²⁾ for a discussion of accidental and coupled pairs). So the expected value of $\eta_m \eta_n$ is

$$\eta_m \eta_n = 1 \times P(\text{count in } m) \times 1 \times \left[P_A(\text{count in } n) + P_C(\text{count in } n) \right] \tag{3-85}$$

where

P (count in m) = probability of registering a count in the m^{th} time interval.

P_A (count in n) = conditional probability that if a count is registered in the m^{th} time interval an accidental count will be registered in the n^{th} time interval, and

P_C (count in n) = conditional probability that if a count is registered in the m^{th} time interval a coupled count will be registered in the n^{th} time interval.

It is shown by de Hoffmann⁽²⁾ that

$$\begin{aligned}
 P(\text{count in } m) [P_A(\text{count in } n) + P_C(\text{count in } n)] \\
 = (F \epsilon \Delta t)^2 + \frac{F \epsilon^2 \bar{\nu} (\bar{\nu} - 1)}{2 \alpha \tau_f^2} e^{-\alpha (t_2 - t_1)} (\Delta t)^2,
 \end{aligned} \tag{3-86}$$

where

$\bar{\nu}$ = number of neutrons emitted per fission.

α = Rossi alpha = $\bar{\nu} (1 - K_P) / \tau_f$.

$t_2 - t_1$ = time between m^{th} and n^{th} time intervals.

K_P = prompt neutron multiplication factor, and

τ_f = mean life of neutron for fission;

and Orndoff⁽³⁾ gives

$$\begin{aligned}
 P(\text{count in } m) [P_A(\text{count in } n) + P_C(\text{count in } n)] \\
 = F \epsilon \Delta t \left[F \epsilon \Delta t + \frac{\bar{\nu} (\bar{\nu} - 1) K_P^2}{2 \bar{\nu}^2 (1 - K_P) \tau_0} e^{-\alpha t \Delta t} \right]
 \end{aligned} \tag{3-87}$$

where

t = time between m^{th} and n^{th} time intervals.

τ_0 = mean life of neutron including absorption and leakage, and

α = Rossi alpha = $(1 - K_P) / \tau_0$

But since⁽³⁾

$$\tau_f = \frac{\bar{\nu} \tau_0}{K_P}, \quad (3-88)$$

these two expressions are equivalent within a factor of K_P (which is essentially unity in a critical reactor). We will use Equation (3-87) and write, by combining Equations (3-85) with (3-87),

$$\begin{aligned} \langle \eta_m \eta_t \rangle &= F \epsilon \Delta t \left[F \epsilon \Delta t + A \epsilon e^{-\alpha (m \Delta t - n \Delta t)} \Delta t \right], \\ &\text{for } m > n \\ &= F \epsilon \Delta t \left[F \epsilon \Delta t + A \epsilon e^{-\alpha (n \Delta t - m \Delta t)} \Delta t \right], \\ &\text{for } m < n, \end{aligned} \quad (3-89)$$

where

$$A = \frac{\bar{\nu} (\bar{\nu} - 1) K_P^2}{2 \bar{\nu}^2 (1 - K_P) \tau_0} \quad (3-90)$$

So, substitution of (3-84) and (3-89) into (3-83) yields

$$\begin{aligned} \langle V^2(t) \rangle &= \sum_{m=-\infty}^{t/\Delta t} F \epsilon \Delta t \bar{\nu}^2 (t - m \Delta t) \\ &+ \sum_{m=-\infty}^{t/\Delta t} \sum_{n=-\infty}^m \left[(F \epsilon \Delta t)^2 + F \epsilon^2 A e^{-\alpha (m \Delta t - n \Delta t)} (\Delta t)^2 \right] \\ &\times v(t - m \Delta t) v(t - n \Delta t) \\ &+ \sum_{m=-\infty}^{t/\Delta t} \sum_{n=m}^{t/\Delta t} \left[(F \epsilon \Delta t)^2 + F \epsilon^2 A e^{-\alpha (n \Delta t - m \Delta t)} (\Delta t)^2 \right] \\ &\times v(t - m \Delta t) v(t - n \Delta t). \end{aligned} \quad (3-91)$$

Note that we have dropped the requirement $m \neq n$ from the second and third summations; the argument for the validity of this step is given earlier in this section.

Now let

$$a = \frac{t}{\Delta t} - m,$$

$$b = \frac{t}{\Delta t} - n$$

in Equation (3-91); then

$$\begin{aligned} \langle V^2(t) \rangle = & \sum_{a=0}^{\infty} F \epsilon \Delta t v^2(a \Delta t) \\ & + \sum_{a=0}^{\infty} \sum_{b=0}^a \left[(F \epsilon \Delta t)^2 + F \epsilon^2 A e^{-\alpha(a \Delta t - b \Delta t)} (\Delta t)^2 \right] \\ & \times v(a \Delta t) v(b \Delta t) \\ & + \sum_{a=0}^{\infty} \sum_{b=a}^{\infty} \left[(F \epsilon \Delta t)^2 + F \epsilon^2 A e^{-\alpha(b \Delta t - a \Delta t)} (\Delta t)^2 \right] \\ & \times v(a \Delta t) v(b \Delta t), \end{aligned} \quad (3-92)$$

which becomes, as $\Delta t \rightarrow 0$,

$$\begin{aligned} \langle V^2(t) \rangle = & \int_0^{\infty} F \epsilon v^2(x) dx \\ & + \int_0^{\infty} \left[\int_0^{\infty} (F \epsilon)^2 v(x) v(y) dy \right] dx \\ & + \int_0^{\infty} \left[\int_0^x F \epsilon^2 A e^{-\alpha(x-y)} v(x) v(y) dy \right] dx \\ & + \int_0^{\infty} \left[\int_x^{\infty} F \epsilon^2 A e^{-\alpha(y-x)} v(x) v(y) dy \right] dx. \end{aligned} \quad (3-93)$$

Because of Equation (3-11), the second term in Equation (3-93) is equal to zero; hence the expected value of $V^2(t)$ is

$$\begin{aligned}
 \langle V^2(t) \rangle &= F \epsilon \int_0^\infty v^2(x) dx \\
 &+ F \epsilon^2 A \int_0^\infty e^{-\alpha x} v(x) \left[\int_0^x e^{\alpha y} v(y) dy \right] dx \\
 &+ F \epsilon^2 A \int_0^\infty e^{\alpha x} v(x) \left[\int_x^\infty e^{-\alpha y} v(y) dy \right] dx.
 \end{aligned} \tag{3-94}$$

In the particular case considered above, in which

$$v(t) = Q_e Z_{12} \frac{\omega_H}{\omega_H - \omega_L} \left(\omega_H e^{-\omega_H t} - \omega_L e^{-\omega_L t} \right), \tag{3-51}$$

substitution of (3-51) into (3-94) yields

$$\begin{aligned}
 \langle V^2(t) \rangle &= \frac{F \epsilon Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L} \\
 &+ F \epsilon^2 A Q_e^2 Z_{12}^2 \frac{\omega_H^2 \alpha}{(\omega_H + \alpha)(\omega_L + \alpha)(\omega_H + \omega_L)}.
 \end{aligned} \tag{3-95}$$

But the first term on the right, above, is the value of the signal without reactor noise, so the second term is the contribution made by the reactor noise. Hence, the reactor-noise contribution expressed as a fraction of the "correct" signal is the ratio of the second to the first term; i. e.,

$$\frac{\Delta \langle V^2(t) \rangle}{\langle V^2(t) \rangle_C} = \frac{2 \epsilon A \alpha}{(\omega_H + \alpha)(\omega_L + \alpha)}. \tag{3-96}$$

The product $A \alpha$ is

$$A \alpha = \frac{\nu(\nu - 1) K_P^2}{2 \nu^2 \tau_0^2}. \tag{3-97}$$

so

$$\frac{\Delta \langle V^2(t) \rangle}{\langle V^2(t) \rangle_C} = \frac{\overline{\epsilon \nu (\nu - 1)} K_P^2}{\bar{\nu}^2 \tau_o^2 (\omega_H + \alpha) (\omega_L + \alpha)} \quad (3-98)$$

And finally, since

$$\alpha \approx \beta / \tau_o, \quad (3-99)$$

where β is the delayed-neutron fraction, then

$$\frac{\Delta \langle V^2(t) \rangle}{\langle V^2(t) \rangle_C} = \frac{\overline{\epsilon \nu (\nu - 1)} K_P^2}{\bar{\nu}^2 \tau_o^2 \left(\omega_H + \frac{\beta}{\tau_o} \right) \left(\omega_L + \frac{\beta}{\tau_o} \right)} \quad (3-100)$$

3.3.2 Derivation in Frequency Domain

The power spectral density of the current from a detector in a reactor is given by Cohn⁽⁴⁾ as

$$\langle |I(\omega)|^2 \rangle = \frac{Q^2 \epsilon' n}{\pi \tau_o} \left[1 + \frac{\epsilon'}{(\omega L)^2 + B^2} \left(\frac{\bar{\nu}^2 - \bar{\nu}}{\bar{\nu}} \right) \right], \quad (3-101)$$

where

Q = charge transferred per neutron detected.

ϵ' = detector efficiency in counts/(neutron absorbed in the reactor in whatever manner).

n = total number of neutrons in the reactor,

τ_o = neutron lifetime.

$$L = \tau_o + \frac{\lambda \beta}{\omega^2 + \lambda^2},$$

$$B = \beta - \frac{\lambda^2 \beta}{\omega^2 + \lambda^2}, \text{ and}$$

λ = equivalent one-group delayed-neutron-emitter's decay constant.

Equation (3-101) can be rearranged into the canonical form:

$$\langle |I(\omega)|^2 \rangle = \frac{Q^2 \epsilon' n}{\pi \tau_o} \left[1 + \frac{\frac{\epsilon' \lambda^2}{\beta^2} \left(\frac{\bar{\nu}^2 - \bar{\nu}}{\bar{\nu}} \right) \left(1 + \frac{\omega^2}{\lambda^2} \right)}{\omega^2 \left(1 + \frac{\omega^2 \tau_o^2}{\beta^2} \right)} \right] \quad (3-102)$$

(Equation (3-102) has been somewhat simplified by using the assumption that $\beta \gg \tau_o \lambda$)

But the efficiency ϵ' , in terms of counts per neutrons lost, is related to the efficiency ϵ in terms of counts per fission by

$$\epsilon' = \epsilon \frac{\tau_o}{\tau_f} = \epsilon \frac{K_P}{\bar{\nu}} \quad (3-103)$$

So Equation (3-102) becomes

$$\langle |I(\omega)|^2 \rangle = \frac{Q^2 \epsilon K_P n}{\pi \tau_o \bar{\nu}} \left[1 + \frac{\frac{\epsilon K_P \lambda^2}{\beta^2} \left(\frac{\bar{\nu}(\bar{\nu} - 1)}{\bar{\nu}^2} \right) \left(1 + \frac{\omega^2}{\lambda^2} \right)}{\omega^2 \left(1 + \frac{\omega^2 \tau_o^2}{\beta^2} \right)} \right] \quad (3-104)$$

Since the low-frequency, break point of the system is at a frequency much greater than λ , we will disregard that part of the spectrum below $\omega = \lambda$ and write

$$\langle |I(\omega)|^2 \rangle = \frac{Q^2 \epsilon K_P n}{\pi \tau_o \bar{\nu}} \left[1 + \frac{\frac{\epsilon K_P}{\beta^2} \left(\frac{\bar{\nu}(\bar{\nu} - 1)}{\bar{\nu}^2} \right)}{1 + \frac{\omega^2 \tau_o^2}{\beta^2}} \right], \quad (3-105)$$

which is equivalent to Equation (3-104) for $\omega > \lambda$.

The transfer function of the linear part of the system (cable plus amplifier) is given by

$$g(\omega) = Z_{12} \frac{j \frac{\omega}{\omega_L}}{\left(1 + j \frac{\omega}{\omega_L}\right) \left(1 + j \frac{\omega}{\omega_H}\right)} \quad (3-106)$$

and the ratio of reactor-noise contribution to the mean square voltage at the amplifier output to the "correct" value is given by

$$\frac{\Delta \langle V^2(t) \rangle}{\langle V^2(t) \rangle_C} = \frac{\int_0^\infty \frac{\epsilon K_P}{\beta^2} \left(\frac{\nu(\nu-1)}{\bar{\nu}^2} \right) \frac{g(\omega)^2 d\omega}{1 + \frac{\omega^2 \tau_o^2}{\beta^2}}}{\int_0^\infty 1 \times |g(\omega)|^2 d\omega} \quad (3-107)$$

or

$$\frac{\Delta \langle V^2(t) \rangle}{\langle V^2(t) \rangle_C} = \frac{\epsilon \nu(\nu-1) K_P}{\bar{\nu}^2 \tau_o^2 \left(\omega_H + \frac{\beta}{\tau_o} \right) \left(\omega_L + \frac{\beta}{\tau_o} \right)} \quad (3-108)$$

Comparison of Equations (3-100) and (3-108) shows them to be equal within a factor of K_P .

3.4 PROBABILITY OF FALSE TRIP FROM MEAN SQUARE VOLTAGE

If noise having a Gaussian amplitude distribution is passed through a nonlinear device (such as a squaring circuit), and if the resulting voltage or current is passed through a relatively narrow bandpass filter, the statistical properties of the output voltage or current approximate those of a random noise voltage or current⁽¹⁾. These conditions are satisfied by a Campbell system operating at high reactor fluxes, for then the average pulse spacing is much less than the pulse width, ensuring a Gaussian amplitude distribution at the squaring-circuit input; also, since τ , the time constant of the averaging circuit after squaring, satisfies the relation

$$\frac{1}{\tau} \ll \omega_L < \omega_H, \quad (3-109)$$

the filter after squaring has a relatively narrow bandpass. Hence we can use the expression for the frequency with which a random-noise voltage crosses a given level. This rate of crossing is given⁽¹⁾ approximately by

$$r_T = 0.58 f_2 e^{-\left(S_T - \langle S \rangle\right)^2 / (2 \sigma_S^2)} \quad (3-110)$$

where

r_T = rate, per second, at which signal crosses the value S_T with positive slope,

f_2 = $1/(2 \pi \tau)$.

$\langle S \rangle$ = expected value of signal

$$= \frac{Ak \Phi Q_e^2 Z_{12}^2}{2} \frac{\omega_H^2}{\omega_H + \omega_L}$$

S_T = trip level setting, and

σ_S^2 = variance of signal

$$= \frac{(Ak \Phi)^2 \left(Q_e Z_{12}\right)^4 \omega_H^4}{4 \tau (\omega_H + \omega_L)^3}$$

Neglected in this analysis are the effects of internally generated noise within the subsystem or system components, and the susceptibility of these components to externally generated noise. These effects tend to increase the probability of false trips and, therefore, the results of this analysis will relate to a more stable subsystem or system than can generally be achieved in practice.

The Campbell subsystem covers its range with a linear scale in PERCENT POWER and a 10-position range switch. If the following operating conditions apply:

Scale Reading (Percent Power)	Equivalent Percent of Full Scale	System Function Required
125	100	(For reference only)
120	96	Scram trip
115	92	Alarm indication
100	80	Maximum operating point

and if the level is allowed to reach 67 percent of the full-scale value, the false trip probability, r_T , will be as tabulated in Table 3-1 for the various ranges (the equivalent neutron flux is based on an assumed subsystem sensitivity of 8.1×10^{-19} volts²/nv). If the level is allowed to reach only 63 percent of the full-scale value, the false trip probability will be as tabulated in Table 3-2 for the various ranges. From the value of r_T (Events/year) in Tables 3-1 and 3-2, it can be seen that the probability of false trip for either operating option is very small. A determination of the variation of r_T as a function of operating level for the most sensitive range and a trip setting of 96 percent of full scale has been made as follows:

<u>Operating Point (Max) (nv equiv.)</u>	<u>Upper Trip Point (nv equiv.)</u>	<u>Scale Reading (125 = full scale)</u>	<u>r_T (Events/year)</u>
6.67×10^8	1×10^9	80	2.89×10^{-136}
8.33×10^8	1×10^9	100	$2.89 \times 10^{-15.9}$
8.75×10^8	1×10^9	105	$2.89 \times 10^{-4.6}$
9.17×10^8	1×10^9	110	$2.89 \times 10^{+3.74}$
9.58×10^8	1×10^9	115	$2.89 \times 10^{+5.90}$

Thus, the "one false scram per year" yardstick occurs when the MMSVM is operated at an approximate scale reading of 108 on its most sensitive range.

TABLE 3-1

OPTION 1 ANALYSIS
 $\langle S \rangle_{\max} = 67 \text{ PERCENT OF FULL SCALE}$

Operating Range (nv equiv.)	Upper Trip Point (nv equiv.)	$\frac{\sigma_S}{\langle S \rangle}$ %	$\langle S \rangle_{\max}$ $10^{-10} v^2$	$(S_T - \langle S \rangle)$ Trip $10^{-10} v^2$	σ_S $10^{-12} v^2$	$-\frac{(S_T - \langle S \rangle)^2}{2 \sigma_S^2}$	$\log_{10} X =$ n. $\log_{10} e$	$r_T = A \cdot X$ (Events/year)
$1 \times 10^8 - 2.12 \times 10^8$	None							Not Applicable
$2.12 \times 10^8 - 6.67 \times 10^8$	1.0×10^9	1.95	5.4	2.7	10.5	-330	-143	2.89×10^{-136}
$6.67 \times 10^8 - 2.12 \times 10^9$	3.16×10^9	0.99	17.2	8.4	17.0	-1220	-529	2.89×10^{-522}
$2.12 \times 10^9 - 6.67 \times 10^9$	1.0×10^{10}	0.99	54.0	27.0	53.5	-1280	-552	2.89×10^{-545}
$6.67 \times 10^9 - 2.12 \times 10^{10}$	3.16×10^{10}	0.835	172	84	144	-1700	-735	2.89×10^{-728}
$2.12 \times 10^{10} - 6.67 \times 10^{10}$	1.0×10^{11}	0.835	540	270	451	-1790	-775	2.89×10^{-768}
$6.67 \times 10^{10} - 2.12 \times 10^{11}$	3.16×10^{11}	0.45	89	44	40	-6050	-2620	2.89×10^{-2612}
$2.12 \times 10^{11} - 6.67 \times 10^{11}$	1.0×10^{12}	0.45	281	139	126	-6100	-2640	2.89×10^{-2632}
$2.67 \times 10^{11} - 2.12 \times 10^{12}$	3.16×10^{12}	0.425	890	440	377	-6800	-2950	2.89×10^{-2942}
$2.12 \times 10^{12} - 6.67 \times 10^{12}$	1.0×10^{13}	0.425	2810	1390	1195	-6800	-2950	2.89×10^{-2942}

TABLE 3-2

OPTION 2 ANALYSIS

 $\langle S \rangle_{\max} = 63 \text{ PERCENT OF FULL SCALE}$

Operating Range (nv equiv.)	Upper Trip Point (nv equiv.)	$\frac{\sigma_S}{\langle S \rangle \%}$	$\langle S \rangle_{\max}$ 10^{-10} v^2	$(S_T - \langle S \rangle)$ Trip 10^{-10} v^2	σ_S 10^{-12} v^2	$-\frac{(S_T - \langle S \rangle)^2}{2\sigma_S^2}$	$\log_{10} X =$ n. $\log_{10} e$	$r_T = A \cdot X$ (Events/year)
$1 \times 10^8 - 2.64 \times 10^8$	None							Not Applicable
$2.64 \times 10^8 - 8.33 \times 10^8$	1×10^9	1.95	6.75	1.35	13.15	-52.9	-22.9	$2.89 \times 10^{-15.9}$
$8.33 \times 10^8 - 2.64 \times 10^9$	3.16×10^9	0.99	21.4	4.2	21.1	-195.5	-84.6	$2.89 \times 10^{-77.6}$
$2.64 \times 10^9 - 8.33 \times 10^9$	1×10^{10}	0.99	67.5	13.5	66.9	-205	-88.8	$2.89 \times 10^{-81.8}$
$8.33 \times 10^9 - 2.64 \times 10^{10}$	3.16×10^{10}	0.835	214	42	179	-272	-118	2.89×10^{-111}
$2.64 \times 10^{10} - 8.33 \times 10^{10}$	1×10^{11}	0.835	675	135	563	-286	-124	2.89×10^{-117}
$8.33 \times 10^{10} - 2.64 \times 10^{11}$	3.16×10^{11}	0.45	111	22	49.9	-972	-422	2.89×10^{-414}
$2.64 \times 10^{11} - 8.33 \times 10^{11}$	1×10^{12}	0.45	350	70	158	-983	-427	2.89×10^{-419}
$8.33 \times 10^{11} - 2.64 \times 10^{12}$	3.16×10^{12}	0.425	1110	220	470	-1100	-476	2.89×10^{-468}
$2.64 \times 10^{12} - 8.33 \times 10^{12}$	1×10^{13}	0.425	3500	700	1490	-1103	-476	2.89×10^{-468}

SECTION IV

DETECTORS

4.1 DESCRIPTION

4.1.1 In-Core Detector Assemblies

The in-core detector assemblies are designed for operation within power reactor cores in either a water or air ambience. Both the in-core Campbell and the counter assemblies consist of a detector enclosure which has a 1/4-inch o. d. $\times$ 2-5/8-inch length, a 3/16-inch-o. d. radiation- and temperature-resistant cable, and a ceramic-to-metal cable seal termination. An adapter is mounted on the cable seal which mates with a standard HN connector. Only Type-304 stainless steel is exposed to the reactor ambience. All exposed joints are inert-arc-welded.

The detectors proper are integral units 0.160 o. d. $\times$ 1-3/4 inches long, contained within the detector enclosure of the probe. Chamber materials are titanium, fosterite ceramic, and argon filling gas. A uranium coating is electro-deposited on the inside diameter of the chamber shell. Careful attention is given to the outgassing and backfilling processes to prevent impurity contamination of the fill gas. All materials and processes used in the fabrication of these devices are similar to those used in the ceramic-metal electronic tube industry. The Campbell chamber differs from the counter in that the gas pressure-gap length product is smaller, and the chamber contains less uranium.

The coaxial cable consists of a Type-304 stainless steel center conductor, quartz fiber insulation, a stainless steel braid, a double braid of copper, and a Type-304 stainless steel sheath tube. The materials used in the cable were chosen to meet the requirements of signal transmission, physical compatibility, and tolerance to elevated temperature and radiation levels.

An HN type termination for the cable connector provides a high-quality, moisture-proof connection to the interconnecting cable.

4.1.2 Out-of-Core Detectors

The out-of-core detector used in the tests was a standard General Electric fission counter, NA04, constructed mainly of aluminum and alumina ceramic. The argon gas gap length is 1/8 inch. External dimensions are 3 o. d. $\times$ 13-1/2 length. An HN termination provides connection to the standard cable. Both the inner and outer surfaces of the active volume are coated with uranium.

A brief description of the pulse formation process in a fission ion chamber is contained in the following paragraphs of this section.

4.2 PULSE FORMATION

When the detector is exposed to neutron flux, fission is induced in the uranium coating, $U^{235}(n, F)fp$. Two energetic, highly-charged fission particles travel in opposite directions from their original site. Since the uranium coating is extremely thin, it is highly probable that one of the fission particles will traverse the gas gap in the chamber.

The shape of both the ionic and electronic current pulses from a well saturated chamber have the form:

$$i = \frac{\rho_o d}{T_e} \left(1 - \frac{t}{T_e} \right) \quad t \leq T_e \quad (4-1)$$

where

ρ_o = the charge density along the fission particle track,

d = electrode spacing,

t = time from instant of track formation, and

T_e = transit time of electrons (or ions) across gas gap.

Equation (4-1) for the electronic pulse is derived as follows (the circuit is shown in Figure 4-1):

Figure 4-1. Schematic Diagram of Chamber and Circuitry

The typical charged particle track is one that extends across the entire gas gap in the chamber since the electrode spacing is small relative to the range of the fission particle. After passage of a fission particle, the resultant ions and electrons drift respectively toward the electrode of opposite polarity.

Assume that the rise of the pulse is instantaneous since it is associated with the transit time of the fission particle and the time required for the charged particle to reach terminal velocity. Then, the extent in space of the electrons due to the passage of a single charged particle at time, t , after pulse initiation is as shown in Figure 4-1.

An energy balance equation is written relating the work done by the battery or power supply to work done on gas by the movement of the charged particles. The work required to collect the remainder of the pulse is

$$\int_a^d [\phi(d) - \phi(x)] \rho_0 dx = V_p q, \quad (4-2)$$

where V_p is the battery potential and q is the charge yet to flow at time, t .

But

$$\phi(d) = V_p, \quad (4-3)$$

$$\phi(x) = \frac{V_p}{d} x, \quad (4-4)$$

and

$$a = vt = \mu_e Et = \mu_e \frac{V_p}{d} t, \quad (4-5)$$

where μ_e is the electron mobility (assumed to be a linear function of electric field).

Therefore,

$$\begin{aligned} V_p q &= \int_{\frac{\mu_e V_p t}{d}}^d \left(V_p - \frac{V_p}{d} x \right) \rho_0 dx \\ &= V_p \rho_0 \left(x - \frac{x^2}{2d} \right) \Big|_{\frac{\mu_e V_p t}{d}}^d = V_p \rho_0 \left[\left(d - \frac{d^2}{2d} \right) - \left(\mu \frac{V_p}{d} t - \frac{\mu^2 V_p^2 t^2}{2d^3} \right) \right]. \quad (4-6) \end{aligned}$$

The charge collected, q' , is given as

$$q' = q_T - q = q_T - \rho_0 \left[\frac{d}{2} - \left(\mu_e \frac{V_p}{d} t - \frac{\mu_e^2 V_p^2 t^2}{2d^3} \right) \right] \quad (4-7)$$

At $t = 0$, $q' = 0$, hence

$$q' = q_T - \rho_0 \left[\frac{d}{2} \right]$$

$$q_T = \frac{\rho_0 d}{2}$$

Differentiate q' with respect to time to obtain the instantaneous current:

$$\begin{aligned} i &= \frac{dq'}{dt} = \rho_0 \left[\mu_e \frac{V_p}{d} - \frac{\mu_e^2 V_p^2 t}{d^3} \right] \\ &= \rho_0 \mu_e \frac{V_p}{d} \left(1 - \frac{V_p \mu_e t}{d^2} \right) \end{aligned} \quad (4-8)$$

The total time interval of the pulse due to the electrons is equal to the transit time of the electrons and is given by

$$T_e = \frac{d^2}{\mu_e V_p} \quad (4-9)$$

Substitution into Equation (4-8) yields

$$i = \frac{\rho_0 d}{T_e} \left(1 - \frac{t}{T_e} \right), \quad (4-1)$$

which is the expression for the current pulse. The time interval of the pulse was measured to be $0.1 \mu\text{sec}$.

4.3 OPERATING RANGE

4.3.1 Counter

The operating range can be considered as that range of the measured parameter throughout which the system indication bears a stated relationship to the true value of the measured parameter, within the boundaries of other constraints on the system such as response time and allowable fractional variation.

The upper end of the range of the counting channel is limited by counting losses in the electronic subsystem at high count rates. The properties of the electronic subsystem are discussed in Section VI.

The lower end of the range of the counting channel is limited by the allowable error due to counting statistics coupled with the counting sensitivity of the chamber. For example, if a minimum of 1 count per second is required and the counting sensitivity is 1×10^{-3} counts per second per nv, then the lower end of the range is 10^3 nv.

There are other effects that could, in principle, limit the lower end of the range to a different value than that determined by statistics and counting efficiency, but they are not realized in practice. At higher gamma levels than those normally encountered at startup, gamma-pulse pile-up would be more severe, requiring a higher discriminator setting and hence resulting in a lower counting sensitivity than at the normal gamma level. Some examples of this effect are worked out below. The rate, r_D , that a pulse pile-up will exceed a certain amplitude, per second, is:

$$r_D = \sqrt{f^2} \exp \left[- \frac{V_D^2}{2 \sigma^2} \right] \quad (4-10)$$

$$\sigma^2 = n \int_0^\infty v^2 dt, \quad (4-11)$$

where

- r_D = rate that pulse pile-up exceeds the level V_D ,
- V_D = critical voltage level (proportional to discriminator setting),
- v = voltage profile of a single pulse at point of interest in system,

n = pulse rate,

$\bar{f}^2$ = mean square of spectral distribution of pulse power at point of interest in system, and

σ^2 = the variance of the signal.

The above expression assumes background pulses of uniform size which is an approximation, of course; however, an appreciation of the functional dependence is afforded by the above expression.

If the logarithm is taken of Equation (4-10) for the number of positive crossings due to pulse pile-up, the following is obtained:

$$\log_{10} r_D = \log_{10} \sqrt{\bar{f}^2} - \frac{V_D^2}{2\sigma^2} \log_{10} e = \log_{10} \sqrt{\bar{f}^2} - \frac{0.2171 V_D^2}{\sigma^2} \quad (4-12)$$

This expression can be applied to the integral bias curve to obtain σ^2 .

Figure 4-2 is a plot of integral bias data taken at different gamma levels. If the equivalent count rate is plotted on semi-log paper versus the square of the discriminator setting, Figure 4-3 results. From this figure the following is obtained:

$$\text{The slope of each curve, } S_i = - \frac{0.2171}{\sigma_i^2},$$

or

$$\frac{1}{S_i} = - 4.61 \sigma_i^2, \quad (4-13)$$

where

$$\sigma_i^2 = \sigma_e^2 + \sigma_{\gamma i}^2, \quad (4-14)$$

and

σ_e^2 = the variance due to electronic circuit effect.

$\sigma_{\gamma i}^2$ = the variance due to gamma pulse pile-up.

Figure 4-2. Integral Bias Curves, Detector Assembly No. 1 (Counter) Versus Gamma Flux

The terms σ_e^2 and σ_{γ}^2 are obtained from the slopes S_i below as:

$$\frac{1}{S_1 (4 \times 10^5 \text{ R/h})} = \frac{1}{-1.96} = -4.61 (\sigma_e^2 + \sigma_{\gamma 1}^2),$$

$$\frac{1}{S_2 (3 \times 10^6 \text{ R/h})} = \frac{1}{-1.83} = -4.61 (\sigma_e^2 + \sigma_{\gamma 2}^2).$$

But since

$$\frac{\sigma_{\gamma 1}^2}{\sigma_{\gamma 2}^2} = \frac{(R/h)_1}{(R/h)_2} \quad (4-15)$$

by the relationship of $\sigma^2 = n \int_0^\infty v^2 dt$,

then,

$$0.111 = \sigma_e^2 + \sigma_{\gamma 1}^2$$

and

$$0.119 = \sigma_e^2 + \frac{30}{4} \sigma_{\gamma 1}^2,$$

or subtracting

$$0.008 = \frac{26}{4} \sigma_{\gamma 1}^2,$$

or

$$\sigma_{\gamma 1}^2 = 1.2 \times 10^{-3}$$

Thus,

$$\sigma_{\gamma i}^2 = \frac{1.2 \times 10^{-3}}{4 \times 10^5} (R/h)_i = 3.0 \times 10^{-9} (R/h)_i$$

and

$$\sigma_e^2 = 0.111 - 0.0012 = 0.110$$

Figure 4-3. Count Rate Versus Square of Critical Voltage Level

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Several gamma background situations are now considered and plotted:

Case I: $\Phi_g = 10^7 \text{ R/h.}$

$$\sigma^2 = 0.110 + (3.0 \times 10^{-9}) (10^7 \text{ R/h}) = 0.140,$$

$$S (10^7) = - \frac{0.2171}{0.140} = -1.55$$

and it can be seen from the graph that the count rate is less than 0.01 sec^{-1} at a discriminator setting of 2.5 volts,

Case II: At a count rate of 1 count/sec due to gamma, with discriminator set at 2.5 volts,

$$S = \frac{-5}{4.75},$$

or

$$\frac{1}{S} = \frac{4.75}{-5} = -4.61 (0.110 + \sigma_\gamma^2)$$

or

$$\sigma_\gamma^2 (\text{at } 1 \text{ count/sec}) = 0.206 - 0.110 = 0.096$$

and

$$\Phi_g = \frac{0.096}{3.0 \times 10^{-9}} = 3.2 \times 10^7 \text{ R/h.}$$

A plot of count-rate indication versus discriminator setting, at $1.45 \times 10^4 \text{ nv}$ neutron flux and with both zero gamma and $2.5 \times 10^7 \text{ R/h}$ gamma, is shown in Figure 6-9 in Section VI.

Case III: Conversely, if the area of the counter electrodes were larger, the count rate due to gamma would increase. For a counter with 10 times the present electrode area the variance is found to be at 10^7 R/h ,

$$\sigma_\gamma^2 (10^7 \text{ R/h}) = 0.30,$$

thus

$$\sigma^2 = 0.110 + 0.30 = 0.41$$

or

$$S = \frac{0.2171}{0.41} = 0.53.$$

The large increase in pile-up count rate with increased electrode area shows that a larger counter would not be desirable.

The second possible effect that could limit the low end of the range is the electric potential distribution change in the gas volume of the chamber (space charge effects). When the flux levels are high enough, the electric field will go to zero in the vicinity of the positive electrode, and recombination will commence. As a result both the pulse height and the average current will reduce in magnitude. This effect, however, does not occur at flux levels that will be encountered by the in-core counter; for example, it would commence for Counter No. 1 at a gamma level of 4×10^8 R/h.

4.3.2 Campbeller

Though the spurious signals arising from gamma and alpha pulses are not rejected by amplitude discrimination, they are effectively suppressed, since the response is proportional to the square of the pulse amplitude. However, the MSV gamma signal does exceed the electronic noise level, and is the predominant source of spurious signal in the system at high gamma levels. Shown in Figure 4-4 is the gamma contribution to the MSV signal level at a measured gamma flux level of 3×10^6 R/h and the projected level at 2.5×10^7 R/h. Also shown in this figure is the counting channel output at a counting sensitivity of $0.67 \times 10^{-3} \text{ sec}^{-1} \text{ nv}^{-1}$.

Figure 4-4. MMSVM Neutron Flux Threshold

4.4 EXPERIMENTAL OUTPUT LEVELS

4.4.1 Measured Sensitivities

The following sensitivities are obtained directly from measured signal and flux levels. The units for d-c sensitivity are the usual. The counting sensitivity is given for three values of gamma flux background.

a. D-C Sensitivity

	<u>D-C Sensitivity (Neutron)</u>	<u>D-C Sensitivity (Gamma)</u>
Counter No. 1	$\frac{1.9 \times 10^{-6} \text{ a}}{6.6 \times 10^9 \text{ nv}} = 2.9 \times 10^{-16} \text{ a/nv}$	$\frac{9.4 \times 10^{-7} \text{ a}}{2.9 \times 10^6 \text{ R/h}} = 3.2 \times 10^{-13} \text{ a/R/h}$
Counter No. 2	$\frac{1.9 \times 10^{-6} \text{ a}}{6.6 \times 10^9 \text{ nv}} = 2.9 \times 10^{-16} \text{ a/nv}$	$\frac{1.2 \times 10^{-6} \text{ a}}{2.9 \times 10^6 \text{ R/h}} = 4.1 \times 10^{-13} \text{ a/R/h}$
No. 3 (MSV Chamber)	$\frac{3.9 \times 10^{-6} \text{ a}}{6.6 \times 10^{11} \text{ nv}} = 5.9 \times 10^{-18} \text{ a/nv}$	Erroneous reading
No. 4 (MSV Chamber)	$\frac{4.7 \times 10^{-6} \text{ a}}{6.6 \times 10^{-11} \text{ nv}} = 6.8 \times 10^{-18} \text{ a/nv}$	$\frac{1.5 \times 10^{-7} \text{ a}}{2.9 \times 10^6 \text{ R/h}} = 5.2 \times 10^{-14} \text{ a/R/h}$
Out-of-Core Chamber NA04	$1.4 \times 10^{-13} \text{ a/nv}$	$2.2 \times 10^{-11} \text{ a/R/h}$

b. Counting Sensitivities

In-Core

	<u>cps/nv at $\sim 10^5 \text{ R/h}$</u>	<u>cps/nv at $2.5 \times 10^7 \text{ R/h}$</u>
Counter No. 1	$\frac{1.00 \times 10^4 \text{ cps}}{6.6 \times 10^6 \text{ nv}} = 1.52 \times 10^{-3}$	$\frac{1.22 \times 10^6 \text{ cps}}{1.80 \times 10^9 \text{ nv}} = 0.68 \times 10^{-3}$

Discriminator setting = 3.5 (arbitrary scale), 40 feet of 50-ohm in-core cable, 85 feet of standard 75-ohm cable (RG-6A/U) terminated in 75 ohms.

c. Mean Square Outputs of Detector Assemblies

Counter No. 1	$S_{\langle V^2 \rangle \gamma}$	$S_{\langle V^2 \rangle n}$
	$9.2 \times 10^{-18*}$	$5.6 \times 10^{-18*}$
Counter No. 2	not tested	not tested
MSV No. 3 Chamber	$6.4 \times 10^{-18**}$	$1.1 \times 10^{-19**}$
MSV No. 4 Chamber	$1.3 \times 10^{-18**}$	$1.7 \times 10^{-19**}$
Out-Of- Core Chamber NA04	not tested	$3.4 \times 10^{-14***}$

NOTE 1. The mean square output/cycle below breakpoint due to ionic mobility in the $\langle V^2 \rangle$ distribution curve of the ion produced signal is four-times that above the breakpoint. All sensitivities quoted are below this breakpoint.

NOTE 2. The fraction of the applied field through which the charge falls is about one-half if only electron motion is contributing to the signal; i. e., if the bandpass of the circuit lies above the breakpoint due to ion transit time in the power spectral density curve. It is unity if both electron and ion motion contribute to the signal; i. e., if the bandpass of the circuit lies below the breakpoint due to ion transit time in the power spectral density curve.

* 40 feet of 50-ohm cable, 125 feet of 75-ohm (RG-6) cable terminated in 5000-ohms, electronic system frequency breakpoint at 5 and 9 kc, 100 volts applied.

** 40 feet of 75-ohm cable, 85 feet of 185-ohm (RG-114) cable terminated in 5000 ohms, electronic system frequency breakpoints at 8 and 18 kc, 100 volts applied.

*** 85 feet of 185-ohm cable (RG-114) cable terminated in 5000 ohms, electronic system frequency breakpoints at 8 and 60 kc, 400 volts applied.

4.4.2 Comparison of Calculated and Measured MSV Output

To compare the measured output of a detector in the Campbell mode with the predicted value, experimental values were applied in the following expression:

$$\frac{\langle V^2 \rangle}{\phi} = \frac{1}{2} k Z_{12}^2 \left(\frac{\overline{Q^2} \omega_{1H}^2}{\omega_L + \omega_{1H}} \right) \quad (4-24)$$

where

- k = detector counting efficiency (with discriminator setting of zero),
- Z_{12} = mid-bandpass transfer impedance,
- $\overline{Q^2}$ = [the product of charge of either sign per pulse and fraction of applied field through which it falls]² average,
- ω_{1H} = upper break frequency of bandpass of system, and
- ω_L = lower break frequency of bandpass of system.

An in-core counter (No. 1 assembly) was chosen for comparison because (a) the load impedance seen by the chamber can be expressed simply, being 5000 ohms; (b) a pulse height spectrum for determination of $\overline{Q^2}$ could be obtained from the detector in its actual operating condition (the Campbell detectors, having low internal gas pressure, do not provide pulses of adequate amplitude). For this counter, the ion collection time (0.56×10^{-4} sec) and the bandpass of the system (8 to 10 kHz) result in an equivalent fraction of applied field through which the charge falls, of 3/4.

The experimentally determined values are:

$$k = 2.54 \times 10^{-3} \text{ cps/nv, and}$$

$$\overline{Q^2} = 1.07 \times 10^{-26} \text{ coul}^2 = \left[q \times \frac{3}{4} \right]^2 \quad (\text{See Note 2 on previous page})$$

$\overline{Q^2}$ is determined in the following manner. On a differential pulse height spectrum, the charge in a given pulse, q_i , is related to the indicated pulse height, P_i , by a constant C.

$$q_i = C P_i$$

$$\overline{q^2} = \frac{C^2 \sum_i^n n_i P_i^2}{N} \quad (4-25)$$

where N is total number of pulses in the counting interval, and n is the number of pulse in the interval of average height P_i .

The constant, C , can be evaluated from d-c sensitivity of the chamber.

$$\bar{q} = \frac{S_{DC}}{k} = \frac{C \sum_i^n n_i P_i}{N},$$

where S_{DC} is the d-c sensitivity of the chamber.

Hence,

$$C = \frac{S_{DC}}{k} \frac{N}{\sum_i^n n_i P_i}, \quad (4-26)$$

$$C = 0.373 \times 10^{-13} \text{ coul/unit of pulse height,}$$

$$\overline{q^2} = 1.90 \times 10^{-26} \text{ coul}^2,$$

$$\omega_{1H} = 0.59 \times 10^5 \text{ radians per sec,}$$

$$\omega_L = 0.5 \times 10^5 \text{ radians per sec, and}$$

$$Z_{12} = 5 \times 10^3 \text{ ohms.}$$

Substituting the original expression

$$\begin{aligned} S < V^2 > &= \frac{1}{2} k Z_{12}^2 \frac{\overline{Q^2} \omega_{1H}^2}{\omega_L + \omega_{1H}} \\ &= \frac{1}{2} \left(2.54 \times 10^{-3} \right) \left(25 \times 10^6 \right) \frac{(1.07 \times 10^{-26}) (0.348 \times 10^{10})}{1.09 \times 10^5} \\ &= 10.8 \times 10^{-18} \text{ v}^2/\text{nv} \end{aligned}$$

This compares with a measured value of $5.6 \times 10^{-18} \text{ v}^2/\text{nv}$.

4.5 TEMPERATURE EFFECTS ON THE IN-CORE DETECTORS

Tests were conducted to determine what effects temperature might have on the in-core detectors and cables. Detector assemblies 1 and 3 were used for these tests.

4.5.1 Temperature Effect On Breakdown

The first test was to determine what effect temperature might have on breakdown. For this test, both detector assemblies were placed in an oven and then slowly heated to 700°F. The breakdown was monitored on an oscilloscope connected to the output of the pulse amplifier. Figure 4-5 shows the results of this test. There appears to be no significant change in breakdown voltage over the temperature range tested.

Previous tests indicated that breakdown voltage would decrease as the temperature was raised. However, in these previous tests only part of the detector assembly had been heated, the result being that the density of gas in the heated portion decreased as predicted by the ideal gas law (Equation 4-27):

$$N_1 = \frac{N}{1 + \frac{T_1 V_2}{T_2 V}}, \quad (4-27)$$

where

- N = number of gas molecules in whole cable,
- N_1 = number of gas molecules in heated section of cable,
- T_1 = temperature of heated section of cable,
- T_2 = temperature of unheated section of cable,
- V_1 = volume of heated section of cable, and
- V_2 = volume of unheated section of cable.

A decrease in gas density can also be achieved by lowering the pressure while maintaining constant temperature, and under such conditions it is known that the breakdown voltage decreases. Therefore, it seems reasonable that breakdown voltage should decrease when an in-core detector assembly is heated nonuniformly, thus making the two experimental results consistent with each other and with theoretical predictions.

Figure 4-5. Temperature Effect on a-c Breakdown for In-Core Detector Assemblies 1 and 3

4.5.2 Temperature Effect on Neutron Pulse-Height Spectrum

The second test was to determine what effect long-term temperature exposure might have on the neutron pulse height spectrum. For this test the detector and about 5 feet of the attached in-core cable were heated to about 700°F for approximately 300 hours. Curves of the chamber's neutron pulse height spectrum were recorded at ambient and 700°F at the start and finish of the 300-hour test. These curves are shown in Figures 4-6, 4-7, 4-8, and 4-9. It can be noted that there is no visibly significant difference between the curves taken at the start of the temperature run and those taken at the end.

This test brought to light another temperature effect. By comparing Figures 4-6, 4-8, and 4-10 it can be seen that the pulse height spectrum taken at 710°F and a polarizing potential of 250 volts (Figure 4-8) lies between the pulse height spectrum taken at 75°F and polarizing potentials of 300 (Figure 4-10) and 400 volts (Figure 4-6). This increase in pulse height with increased temperature is probably caused by either an increase in electron mobility or a decrease in recombination rate, or both.

Although the change appears to be of no great significance, the increase of pulse height from the chamber might well compensate for the increase in cable attenuation as the temperature is raised.

4.6 IN-CORE CABLE DATA

Manufacturer: New England Electric Wire Corporation

Configuration: Triax; stainless steel center conductor, quartz insulation, stainless steel guard, quartz insulation, double braid copper shield.

Preparation at Manufacturer: Starch and oil impregnated, heat cleaned.

NEPS Preparation: Baked out in vacuum at 700°F, back-filled with Argon at 200 psig.

4.7 EFFECT OF NEUTRON EXPOSURE (nvt)

Tests were conducted at the General Electric Test Reactor (GETR) to determine the effects of nvt on the pulse height spectrum of an in-core detector. Detector assembly No. 2 was used for these tests. During the data-taking periods before and after high flux irradiation, the detector assembly was in a neutron flux of approximately 8×10^8 nv and a gamma flux of 4.3×10^6 R/h. The peak flux during irradiation was about 8×10^{13} nv. The detector assembly was in this flux for a period of 29 days, which resulted in an integrated exposure of 2×10^{20} nvt. Compared in Figures 4-11 and 4-12 are the pulse height spectrums before and after the irradiation period. No conclusions regarding changes in sensitivity can be drawn from this data since the exact values of neutron and gamma fluxes were not known. However, it appears that the high integrated flux had little effect on the pulse height spectrum.

Figure 4-6. Neutron Differential Pulse Height Spectrum Before Temperature Run - Detector Assembly No. 1 (Counter) at 400 V, 75°F

Figure 4-7. Neutron Differential Pulse Height Spectrum After Approximately 300 Hours at 700°F - Detector Assembly No. 1 (Counter) at 400 V, 100°F

Figure 4-8. Neutron Differential Pulse Height Spectrum, Beginning of Temperature Run - Detector Assembly No. 1 (Counter) at 250 V, 710°F

Figure 4-9. Neutron Differential Pulse Height Spectrum, End of 300-Hour Temperature Run at 700°F - Detector Assembly No. 1 (Counter) at 250 V, 730°F

Figure 4-10. Neutron Differential Pulse Height Spectrum Before Temperature Run, Detector Assembly No. 1 (Counter) at 300 V, 75°F

Figure 4-11. Neutron Differential Pulse Height Spectrum Before High nvt Exposure

Figure 4-12. Neutron Differential Pulse Height Spectrum After Exposure to 2×10^{20} nvt

The irradiation did cause a change in breakdown voltage from 850 V to about 400 V, but since a short portion of the in-core cable was also in the high flux region during the tests it is difficult to state accurately whether or not the breakdown actually occurred in the chamber or in the cable of the in-core detector assembly.

SECTION V

CABLE

This section is concerned with the transmission of information from a detector (ionization or fission chamber) inside a reactor vessel to the electronic equipment outside the vessel.

At the present time, coaxial cable is used for this purpose; hence, the transmission of information through a coaxial cable is considered. Three modes of information transmission are studied:

- a. The conventional mean-value system of instrumentation. This mode uses only the d-c component of the current from the detector. For this system, only a theoretical treatment of the transmission of direct current is considered.
- b. The counting system of instrumentation. This mode uses each individual pulse generated by the detector. For this system, the transmission of pulses is studied theoretically, the theoretical pulse transmission data are compared with experimental data, the experimental frequency response and transient pulse characteristics of two of the in-core cables are studied, and experimental temperature and radiation effects are discussed.
- c. The Campbell system of instrumentation. This mode uses the a-c component of current from the detector. Here the transmission of alternating current is considered.

5.1 DIRECT-CURRENT TRANSMISSION CHARACTERISTICS OF IN-CORE CABLE

For direct current, the transmission line equations⁽⁵⁾ are:

$$I = I_r \cosh \sqrt{rg} x + V_r \sqrt{\frac{g}{r}} \sinh \sqrt{rg} x, \quad (5-1)$$

and

$$V = V_r \cosh \sqrt{rg} x + I_r \sqrt{\frac{r}{g}} \sinh \sqrt{rg} x. \quad (5-2)$$

where I = current at sending end,

I_r = current at receiving end.

V = voltage at sending end.

V_r = voltage at receiving end.

r = resistance, per ft. of conductors,

g = conductance, per ft. of insulation, and

x = length of cable, in feet.

The term V_r can be eliminated between these two, yielding

$$I = I_r \left(\cosh \sqrt{rg} x - \sinh \sqrt{rg} x \tanh \sqrt{rg} x \right) + V \sqrt{\frac{g}{r}} \tanh \sqrt{rg} x. \quad (5-3)$$

When practical values of r , g , and x are inserted in the above, it will be noted that the second term is small compared to the first, resulting in

$$I = I_r \cosh \sqrt{rg} x + V \sqrt{\frac{g}{r}} \tanh \sqrt{rg} x, \quad (5-4)$$

and that this can be further simplified to

$$I = I_r + Vgx. \quad (5-5)$$

Since I_r is the detector current and hence the signal, and I is the meter reading, then Vgx is the error. But Vgx is just the leakage current through the insulation due to the application of the voltage V , and this is the only source of error introduced by the cable. This error is often minimized by using triaxial cable and maintaining a minimum potential difference between the center conductor and the inner shield - in other words, by using standard guard techniques.

It should be cautioned that the conductance per foot of the insulation is a function of temperature, radiation field, and total dose, and is always greater than the value obtained at room temperature at zero radiation field.

5.2 PULSE TRANSMISSION CHARACTERISTICS OF IN-CORE CABLE

5.2.1 Theoretical Treatment

For pulse counting, the form of the transmission line equation⁽⁵⁾ is

$$V = V_r e^{\alpha x} e^{j\beta x}, \quad (5-6)$$

$$\text{where } \alpha = \text{attenuation constant} = \frac{\sqrt{\rho f} \left(1 + \frac{b}{a}\right)}{2012 \pi b Z_o},$$

$$\beta = \text{phase constant} = 2 \pi f c Z_o,$$

$$\rho = \text{resistivity of conductor material, in ohm-meters,}$$

$$f = \text{frequency, in cycles per second,}$$

$$b = \text{inner radius of outer conductor, in feet,}$$

$$a = \text{outer radius of inner conductor, in feet,}$$

$$Z_o = \text{characteristic impedance of cable, in ohms,}$$

$$c = \text{capacitance, per foot, of cable,}$$

$$x = \text{length of cable, in feet,}$$

$$V = \text{voltage at sending end, and}$$

$$V_r = \text{voltage at receiving end.}$$

Since we are interested in only the magnitude of the pulse, this can be written as

$$|V| = |V_r| e^{\alpha x}. \quad (5-7)$$

This equation is valid only if the cable is terminated in its characteristic impedance. To make it applicable for any termination, a term must be added to allow for the reflection factor. The complete equation, solved for $|V_r|$, is

$$|V_r| = |V| \left(1 + \frac{Z - Z_o}{Z + Z_o} \right) e^{-\alpha x}, \quad (5-8)$$

where Z = termination impedance, in ohms.

5.2.2 A Comparison of Calculated and Experimental Attenuations

The frequency to use in determining the attenuation constant, α , can be chosen fairly well from the pulse width. For example, a rectangular pulse of width T seconds has an energy spectrum that has an upper half-power point at $f = 1/(2.2 T)$, goes to zero at $f = 1/T$, and has a maximum energy per decade of frequency at $f = 1/(2.5 T)$. So an equivalent frequency of $1/(2T)$ is a reasonable compromise, and this value of frequency produces good agreement with experimental values.

Table 5-1 shows calculated and experimental values of attenuation ($|V_r|/|V|$) for various cables.

TABLE 5-1

CALCULATED AND EXPERIMENTAL ATTENUATIONS

Cable	Z_0	Termination	Attenuation ($ V_r / V $)	
			Calculated	Experimental
Copper - Q 103 ft	56 Ω	56 Ω	0.712	0.625
		100 Ω	0.915	0.775
Stainless Steel - MI 30 ft	36 Ω	36 Ω	0.298	0.262
		100 Ω	0.438	0.400
Molybdenum - Q 102 ft	56 Ω	56 Ω	0.646	0.601
		100 Ω	0.829	0.772
Stainless Steel - Q 100 ft	75 Ω	75 Ω	0.250	0.225
		100 Ω	0.285	0.250

It is useful to express the attenuation constant, α , in terms of independently variable quantities to facilitate its minimization:

$$\alpha = \frac{\sqrt{\rho f k} \left(1 + \frac{b}{a} \right)}{2.78 \times 10^5 \pi b \log_{10} (b/a)}$$

5.2.3 Frequency Response and Transient Pulse Characteristics of Two Types of In-Core Cable

The characteristics of the two cables are compared with each other, and with some representative types of standard coaxial cables.

5.2.3.1 Nickel-Clad Copper - Quartz - Stainless Steel Cable

With a 21-foot length of this cable, a determination of the attenuation characteristics versus frequency was made (see Figure 5-1). It is shown in this illustration that the cable starts to attenuate at approximately 200 kc/sec, and exhibits a 3 dB attenuation at a frequency of approximately 1.4 Mc/sec. The attenuation then falls with a continually increasing negative slope, and has an attenuation factor of 57 dB at a frequency of 100 Mc/sec.

The transient pulse characteristics of this cable were investigated by inserting a pulse with a 1 μ sec width at the sending end of the cable, and determining the rise time of the pulse at the receiving end of the cable. Figure 5-2 shows these response characteristics. Note that the rise time of the pulse at the receiving end of this cable is 0.312 μ sec, with a sending end pulse rise time of 14 nsec (uncorrected)* or 7.7 nsec (corrected). If the cable is considered a single time constant system, the upper 3 dB frequency is determined as

$$f_2 = \frac{0.35}{\tau_r} = \frac{0.35}{0.31} \times 10^6 = 1.13 \text{ Mc/sec.}$$

5.2.3.2 Stainless Steel - Quartz - Stainless Steel - Double Copper Braid Prototype Cable

With a 42-foot length of this cable, a determination of the attenuation characteristics versus frequency was also made (see Figure 5-3). It is shown in this illustration that the cable starts to attenuate at approximately 5 Mc/sec, and exhibits a 3 dB attenuation at approximately 20 Mc/sec. The attenuation then falls with a continually increasing negative slope, and has an attenuation factor of 16 dB at a frequency of 100 Mc/sec.

The transient pulse characteristics for this cable are shown in Figure 5-4. Note that the rise time of the pulse at the receiving end of this cable is 25 nsec (uncorrected), or 22 nsec (corrected), for the same sending end pulse rise time of 7.7 nsec (corrected). If this cable is considered as a single time constant system, the upper 3 dB frequency is calculated to be:

$$f_2 = \frac{0.35}{\tau_r} = \frac{0.35}{22 \times 10^{-9}} = 16 \text{ Mc/sec.}$$

*The rise time of the measuring device was 11.7 nsec.

Figure 5-1. Gain Versus Frequency - Nickel-Clad Copper - Quartz Cable

(1) INPUT PULSE
SWEEP SPEED: $0.2\mu\text{sec}/\text{cm}$

INPUT PULSE
SWEEP SPEED: $20\text{nsec}/\text{cm}$

(3) OUTPUT PULSE
SWEEP SPEED: $0.2\mu\text{sec}/\text{cm}$

(4) OUTPUT PULSE
SWEEP SPEED: $20\text{nsec}/\text{cm}$

Figure 5-2. Transient Response Characteristics, Nickel-Clad Copper - Quartz -
Stainless Steel Cable - All Amplitudes Normalized

Figure 5-3. Gain Versus Frequency - Prototype Cable

(1) INPUT PULSE
SWEEP SPEED: $0.2\mu\text{sec/cm}$

(2) INPUT PULSE
SWEEP SPEED: 20nsec/cm

(3) OUTPUT PULSE
SWEEP SPEED: $0.2\mu\text{sec/cm}$

(4) OUTPUT PULSE
SWEEP SPEED: 20nsec/cm

Figure 5-4. Transient Response Characteristics of Prototype Cable - All Amplitudes Normalized

5.2.3.3 RG-6A/U And RG-114A/U Coaxial Cables

For comparison, these cables show the following attenuation characteristics*:

RG-6A/U: Down approximately 3 dB for 100 feet at a frequency of 100 Mc/sec.

RG-114A/U: Down approximately 3 dB for 100 feet at a frequency of 100 Mc/sec.

5.2.4 The Effect of Temperature on Pulse-Height Attenuation and D-C Resistance

A variety of cables were tested for pulse-height attenuation at several temperatures. A description of the cables is given in Table 5-2, and the results of the tests are shown in Figure 5-5.

The main contributor to variation of attenuation with temperature is variation of the resistivity, ρ , of the conductor. A plot of measured resistance of several materials, normalized to 20 mils diameter by 100 feet long, versus temperature is given in Figure 5-6.

Note that the theoretical data do not include calculations for the cables using nickel or nickel clad copper as conductors, because the attenuation equations are not valid for cables having conductors with magnetic properties.

Note, also, that the above procedure is valid only for cables in which the attenuation at $f = 1/(2T)$ is not much greater than that at low frequencies. If these two attenuations are significantly different, the shape of the pulse changes in traveling along the cable and the method of calculating output pulse height would be more involved than the approximate method outlined above. The ratio of high-frequency to low-frequency attenuations is given by

$$\frac{|V_r|_{l.f.}}{|V_r|_{h.f.}} = \exp \left[\frac{\sqrt{\rho f} \left(1 + \frac{b}{a} \right) x}{2012 \pi b Z_0} - \frac{\rho x}{0.61 \pi a^2 Z_0} \right] \quad (5-9)$$

where ρ is in ohm-meters and a , b , and x are in feet.

*Catalog W2, American Phenolic Corporation.

TABLE 5-2

CABLE DESCRIPTION

Cable	Insulation	Center Conductor o. d.	Guard o. d.	Shield o. d.	Length (ft)	Capacitance (pf/ft)	Characteristic Impedance
Copper	Quartz Fiber	7 - 5-mil strands	75 mils	110 mils	103	36.5	56 Ω
Nickel Clad Copper	Quartz Fiber	20 mils	95 mils	135 mils	100	26.7	82 Ω
Stainless Steel	Alumina	17 mils	62 mils	125 mils	30	124	36 Ω
Molybdenum	Quartz Fiber	20 mils	95 mils	135 mils	102	32.2	56 Ω
Nickel	Quartz Fiber	10 mils	95 mils	135 mils	32.25	22.6	
Stainless Steel	Quartz Fiber	20 mils	95 mils	135 mils	100	27	75 Ω

NOTE: Quartz fiber insulated cables have braided guards and shields of 4-mil wire.

Figure 5-5. Attenuation Variation With Temperature

Figure 5-6. D-C Resistance Variation With Temperature - Normalized to 20 Mils Diameter

5.3 ALTERNATING-CURRENT TRANSMISSION CHARACTERISTICS OF IN-CORE CABLE

5.3.1 Cable Terminated In Z_0

For alternating current, the transmission line equation⁽⁵⁾ is also given by

$$V = V_r e^{\alpha x} e^{j\beta x} \quad (5-6)$$

when the line is terminated in its characteristic impedance. Power transmission as a function of frequency is, then,

$$\left| \frac{V_r}{V} \right|^2 = e^{-2x\alpha} \quad (5-10)$$

where α is a function of frequency. The term α is given by the formidable expression

$$\alpha = \sqrt{\frac{1}{2} \left[\sqrt{(r^2 + \omega^2 \ell^2)} (g^2 + \omega^2 c^2) + rg - \omega^2 \ell c \right]} \quad (5-11)$$

However, an examination of this equation for the case under consideration often leads to considerable simplification. For example, a stainless-steel quartz-fibre cable used for Campbell operation in the frequency range 0.3 to 0.6 Mc/sec has the following parameters:

$$\begin{aligned} c &= 30 \times 10^{-12} \text{ farad/ft} \\ Z_0 &= 75 \text{ ohms} \\ \ell &= Z_0^2 c = 1.69 \times 10^{-7} \text{ h/ft} \\ r &= 1.9 \text{ ohms/ft} \\ g &= 3 \times 10^{-10} \text{ mho/ft} \end{aligned}$$

An examination of the terms in Equation (5-11), using these values, results in

$$\begin{aligned} r^2 &\gg \omega^2 \ell^2 \\ \omega^2 c^2 &\gg g^2 \\ (\omega r c - \omega^2 \ell c) &\gg rg \end{aligned}$$

so that for this case, Equation (5-11) may be written as

$$\alpha = \sqrt{\frac{1}{2} \left(\omega r c - \omega^2 Z_o^2 c^2 \right)} \quad (5-12)$$

and the power transmission is obtained by combining Equations (5-10) and (5-12):

$$\left| \frac{V_r}{V} \right|^2 = e^{-x} \sqrt{2 \left(\omega r c - \omega^2 Z_o^2 c^2 \right)} \quad (5-13)$$

Hence, if the power spectral density of current from the detector is $G_1(\omega)$ amp² sec and the frequency response function of the amplifier fed by the cable is $Y(j\omega)$, then the power spectral density of voltage at the output of the cable is

$$G_2(\omega) = Z_o^2 G_1(\omega) e^{-x} \sqrt{2 \left(\omega r c - \omega^2 Z_o^2 c^2 \right)} \quad (5-14)$$

volts² sec, in the range of ω over which the approximation (12) is valid. The power spectral density of voltage at the output of the amplifier is

$$G_3(\omega) = Z_o^2 G_1(\omega) e^{-x} \sqrt{2 \left(\omega r c - \omega^2 Z_o^2 c^2 \right)} \left| Y(j\omega) \right|^2 \quad (5-15)$$

volts² sec, and the mean-square voltage at the amplifier output is

$$\overline{V_3(t)^2} = \int_0^\infty G_3(\omega) d\omega \quad (5-16)$$

No range of validity of ω was considered in Equations (5-15) and (5-16) because it is assumed that $|Y(j\omega)|^2$ falls off rapidly, in this case, below 0.3 Mc/sec and above 0.6 Mc/sec.

5.3.2 Cable Not Terminated In Z_0

The voltage at the amplifier output in a Campbell system consists of a linear superposition of discrete pulses, and the mean-square voltage there can be determined from the shape of one of these pulses by means of the equation⁽⁶⁾

$$V(t)^2 = n \int_0^{\infty} v^2(t) dt \quad (5-17)$$

where n = pulse rate,

$V(t)$ = voltage at amplifier output, and

$v(t)$ = voltage amplitude of an individual pulse starting at $t = 0$, at amplifier output.

So the problem is to determine the pulse shape, $v(t)$, at the amplifier output.

The conditions under which an improperly terminated cable will be used are:

- a. Relatively lossy cable with characteristic impedance of about 75 ohms,
- b. Terminating resistance of 5000 ohms, and
- c. Amplifier with bandpass of 8 kc/sec to 60 kc/sec.

Photographs were taken at the input and output of an amplifier having an input impedance of 5000 ohms and a bandpass of 10 kc/sec to 60 kc/sec. It was driven by either 200 feet of RG-59/U or 100 feet of nickel-clad copper, quartz insulated cable; the cable, in turn, was driven by 5×10^{-8} sec current pulses. The results of these tests are shown in Figures 5-7 through 5-10.

Figure 5-7* shows the amplifier input when driven by the low-loss RG-59/U cable. It has two distinguishing features or components: (a) the original pulse and its reflections, and (b) an exponentially decaying tail. The first component contains a considerable amount of energy, but this energy is nearly uniformly distributed among frequencies from zero to $1/T$, where T is the width of one pulse, so very little energy lies in the narrow pass band of the amplifier. The second component decays with a time-constant of RC , where R is the input impedance of the amplifier and C is the total capacitance of the cable; furthermore, if it is extrapolated back to zero its amplitude is Q_p/C , where Q_p is the charge in the pulse. Hence, for this case, the pulse at the amplifier output is essentially due to an input pulse of the form

$$\frac{Q_p}{C} e^{-t/RC}$$

*See footnote at the bottom of Page 5-19.

Figure 5-7. Amplifier Input Driven by Unterminated Low-Loss Cable.
5 μ sec and 0.05 volt per division.

Figure 5-8. Amplifier Output Driven by Unterminated Low-Loss Cable.
10 μ sec and 1.0 volt per division.

Figure 5-9. Amplifier Input Driven by Unterminated High-Loss Cable.
2 μ sec and 0.05 volt per division.

Figure 5-10. Amplifier Output Driven by Unterminated High-Loss Cable.
20 μ sec and 1.0 volt per division.

The amplifier output is shown in Figure 5-8. Note that the original pulse and its reflections are highly suppressed and that the energy lies mainly in the slower components, as predicted in the above paragraph.

Figure 5-9* shows the amplifier input when driven by the high-loss nickel-clad copper, quartz insulated cable. The original pulse and its reflections contain very little energy, again distributed over a wide frequency band, so a negligible amount lies in the narrow pass band of the amplifier. The slow component decays with a time-constant of RC , and, extrapolated back to zero, has an amplitude of Q_p/C . Hence, for this case, the assumption that the pulse at the amplifier output is essentially due to an input pulse of the form

$$\frac{Q_p}{C} e^{-t/RC}$$

is an even better approximation than for the case of low-loss cable drive. The amplifier output is shown in Figure 5-10. Note that it has practically no fine structure due to the original pulse and its reflections.

5.4 RADIATION EFFECTS ON IN-CORE CABLE

To qualify the in-core cable for use in the in-core startup systems, cable samples were tested in an operating reactor. D-c electrical characteristics were measured before insertion and at intervals after insertion.

5.4.1 Cable Samples And Processing

A single sample of each of two types of cable was irradiated. Both short-term and long-term effects were noted. One cable consisted of a 0.018-inch-diameter center wire of stainless-steel-clad copper (Sylvania Oxalloy 28). GE quartz fiber, 150 2/2, initially manufactured with a Dow-Corning 1053 binder (a silicone compatible binder), and an outer braid of 0.004-diameter Type-304 stainless steel wire. The braid outer diameter was 0.070 to 0.076 inch. The second cable was identical to the first except that Owens-Corning S-994 glass (S-glass) fiber, 150 1/2 with silicone compatible binder, was used instead of quartz fiber.

The binder is coated on the quartz and S-glass fibers at the time of their manufacture. The coating, by providing lubrication, increases the strength and flexing tolerance of the fibers. It would be virtually impossible to otherwise fabricate the cable; however, the binder must be completely removed before the cables can be used at high temperature and/or in intense radiation fields.

*The photos at the amplifier input were of poor quality for reproduction, so photos taken with a terminating resistance of 2700 ohms were substituted for them. This results in a pulse whose slow component has a faster decay time but which otherwise has the same general appearance as that at the 5000-ohm amplifier input resistance.

The binder, being an organic material in nature, decomposes when heated to a high temperature and/or exposed to high radiation. Some of the decomposition products are carbonaceous and can degrade the insulation resistance to the point where the cable is unusable.

The cables are cleaned by the cable manufacturer by baking them in air at 850°F for 8 hours. This treatment removes all the organic components that could cause difficulty in reactor service.

For purposes of this test, the cable samples were drawn into 1/8-inch-o.d., Type-304 stainless steel tubes. The tubes were seal-welded on the end to be irradiated, and terminated with a hermetic electrical connector on the other end. Before the final seal weld was made, the tubes were evacuated to 0.1 micron while at a temperature of 800°F. The tubes were then backfilled with reactor grade argon at 1 atmosphere, and sealed.

5.4.2 Test Conditions

The test facility consists of dry thimbles in an operating reactor core. The total length of the thimbles is about 30 feet. Half of this length is outside the pressure vessel; of the remaining half, the last 5 feet is actually in the core. The cable temperature external to the pressure vessel is about 80°F. Inside the pressure vessel but not in the core, the cable temperature is about 550°F. In the core, the cable temperature due to gamma heating, as well as ambient temperature, is about 650°F.

The profile of both the gamma flux level and the neutron flux level is roughly a cosine curve with a zero intercept 1 foot beyond the end of the thimbles and 5 feet along the thimble. The peak neutron flux was about 5×10^{12} nv during the first portion of the test and 7.5×10^{12} nv during the latter portion of the test.

5.4.3 Test Data

Figure 5-11 is a plot of the d-c leakage versus applied potential across the cable insulation for various levels of nvt. The d-c leakage of the quartz did not increase with increasing nvt. This was illustrated by measurements of d-c leakage at 100 volts applied potential for various nvt values (see Figure 5-12). On the other hand, the leakage currents through the S-glass insulation generally increased with increasing nvt.

5.4.4 Discussion of Cable Insulation Data

The primary requirements on the cable insulation are that, at detector operating voltages, the d-c current drain is not excessive, and that the noise generated by the cable with potential applied is not large enough to cause spurious counts in the counting channel.

Figure 5-11. Irradiation Effects on the Electrical Properties of Fiber Insulated Cable

Figure 5-12. D-C Leakage Current Versus Neutron Exposure

The data demonstrate that the former requirement is met. Regarding the latter requirement, it has been found that there is a correspondence between the amount of d-c leakage and the a-c noise generated. With another in-reactor test unit, which included a quartz insulated cable and an in-core counter, a-c noise was not a problem (see Section IV).

5.5 BREAKDOWN VOLTAGE OF CABLE

Because of the integral nature of the cable and the chamber in the form of a detector assembly, the experimental determination of the breakdown characteristics was made on the entire assembly. The results of these tests have been presented in Section IV.

5.6 SUSCEPTIBILITY TO EXTRANEEOUS SIGNALS

Because of the cable processing requirements mentioned earlier in this section, the transmission line that provided the best over-all performance was the stainless steel - quartz - stainless steel configuration, listed as the last item of Table 5-1. Of considerable concern in information transmission systems, especially those information transmission systems associated with the transmission of information about a nuclear device, is the susceptibility of the transmission system to extraneous signals and noise. In an attempt to minimize the susceptibility of the transmission system to extraneous electric, magnetic, and electromagnetic fields, the basic cable configuration mentioned above was evaluated with and without additional shields of various materials and in various modes of connection. The results obtained on a small sample indicated that the configuration for the best over-all performance was a stainless steel - quartz - stainless steel cable, to which was added two additional copper braids of about 20 mils total thickness. This cable is then placed in the usual solid-sheath, stainless steel jacket and the resultant cable is called the "prototype" cable. This cable was mentioned previously in Subsection 5.2.

SECTION VI

COUNTING CHANNEL ELECTRONIC SUBSYSTEM

6.1 GENERAL DESCRIPTION

The counting channel electronic subsystem is composed of the following parts: a remotely-mounted pulse amplifier, a locally-mounted monitor, a cable to connect the remote amplifier to the detector assembly (or the detector, in the case of the out-of-core system), and a set of three cables - one each for chamber polarizing voltage, signal, and power connections to the remote amplifier from the monitor.

6.1.1 Remote Amplifier

The remote amplifier is a radio frequency pulse amplifier operating in the current amplification mode. The amplifier contains the following stages: an input common base stage, a low level doublet amplifier, an attenuator, a buffer stage, a common collector amplifier stage, and a high gain stage. These stages function in the following manner: The input common base stage provides a low input impedance for cable termination; the low level complementary doublet amplifier stage provides a current gain of approximately 40; the attenuator network provides attenuation of 1:1, 2:1, 4:1, and 8:1; the common base buffer stage acts as a buffer between low and high level stages; the common collector stage provides current gain of approximately 40 without phase inversion; and the high-gain complementary doublet provides a current gain of approximately 100. In addition to the amplification stages, the remote amplifier includes filters for the chamber polarizing voltage from the monitor and the power voltage from the monitor.

6.1.2 The Monitor

The monitor consists of a modularized discriminator, logarithmic integrator, level amplifier, period amplifier, and chamber polarizing voltage supply. In addition, there are trip circuits, power supplies, and filters. Modules are mounted in the chassis. These modules function as follows:

- a. The discriminator comprises an input common base stage for cable termination, an integral pulse height discriminator, a shaper, a binary, and a driver.
- b. The logarithmic-integrator is a passive integrator of the Cooke-Yarborough type. The breakpoints of the networks are spaced one decade apart. The time constant over the range is a function of the point on the range and follows a smooth curve. The analytic expression for the analog out is:

$$V_o \text{ (TP1)} = \frac{10}{7} [\text{Log (count rate)} + 1]$$

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- c. The level amplifier is a d-c amplifier which has a current gain of approximately 3.5 amps/amp.
- d. The period amplifier is a d-c amplifier with appropriate feedback components to provide a gain of approximately 200 volt/volt/sec. This corresponds to a full-scale period indication of 10 seconds.
- e. The trip circuits are d-c difference amplifiers with two-state outputs.
- f. The chamber polarizing voltage supply is a d-c to d-c converter which provides +100 to +400 volts d-c to polarize the detector.

6.2 CHARACTERISTICS OF THE ELECTRONIC SUBSYSTEM

6.2.1 Required Bandwidth

The required bandwidth of the subsystem is dependent upon the bandwidth of the information to be processed. The shape of the information pulse that is available from both the in-core and the out-of-core detectors has been described previously. The expression given was:

$$i = \frac{\rho_o d}{T} \left(1 - \frac{t}{T} \right) \text{ amperes,}$$

where

$$\rho_o = \text{charge of either sign/unit length of track in the gas } \left(\frac{\text{coul}}{\text{cm}} \right),$$

$$d = \text{gas gap (cm),}$$

$$T = \text{transit time of the electron or positive ion across the gas gap (seconds), and}$$

$$t = \text{time after pulse initiation (seconds).}$$

In the above expression, the time required to accelerate the ion pairs to their average drift velocity is assumed to be negligible. This information pulse is applied to the interconnecting cable for transmission to the remote amplifier input.

In the out-of-core subsystem, the interconnecting cable is modified RG-6A/U coaxial cable. Because of the almost ideal transmission characteristics of the RG-6A/U cable (attenuation/100 ft = 2.9 dB at 100 Mc), the information pulse is transmitted to the amplifier input essentially unchanged in shape. For the in-core subsystem, the interconnecting cable is 40 feet of stainless steel - quartz - stainless steel prototype in-core cable in addition to the RG-6A/U mentioned above. The transmission characteristics of the prototype in-core cable are far from ideal. * The attenuation

*For an experimental comparison of these cables, see Section V.

per 100 feet of the prototype in-core cable is, typically, 6 dB/100 ft at 22 Mc. This high frequency attenuation causes an increase in the rise time of the information pulse from on the order of 10 nsec to 0.15 μ sec. The upper and lower frequency breakpoints of the electronic subsystem are made to conform to the requirements of this information pulse.

6.2.2 Theoretical Bandwidth

The bandwidth of the electronic subsystem is a function of the circuit components, and is defined in the following manner: The lower half-power frequency is defined as the lower frequency at which the current gain of the subsystem is down to 70.7 percent of the midband value; the upper half-power frequency is defined as the upper frequency at which the current gain of the subsystem is down to 70.7 percent of the midband value. The bandwidth is defined as the difference of the upper half-power frequency and the lower half-power frequency. The lower half-power frequency is determined by the equation:

$$f_1 = \frac{1}{2\pi R_L C_C}$$

where

f_1 = lower half-power frequency in cps

R_L = load resistance of a stage in ohms, and

C_C = coupling capacitance to this load resistance in farads.

These components are the same components as those which provide the appropriate RC clipping. The minimum RC clipping time that can be used is determined by the characteristics of the leading edge of the information pulse. In general, in order to restrict the losses of pulse height in an RC clipping circuit to the order of 25 percent of the peak input pulse or less, the clipping time must be at least ten times the rise time of the information pulse. As described earlier, the rise time of the slowest information pulse (the pulse from the in-core detector assembly) is approximately 0.15 μ sec. It is therefore necessary that the clipping time be not less than 1.5 μ sec to keep the loss in height to 25 percent. A clipping time of 2 μ sec was selected. This results in a calculated lower half-power frequency of approximately 21 kc/sec in the first stage of the remote amplifier and in the first stage of the discriminator.

The upper half-power frequency is determined by the high-frequency components or characteristics of the circuitry. These components and characteristics are, in general, the parallel interstage resistance, the shunt capacitance, and the frequency characteristics of the active devices used. On the basis of the parallel resistance and shunt capacitance, including the stray capacity, the upper half-power point will be in the region of 15 Mc/sec to 30 Mc/sec.

Considering a single stage with the type of transistor used in the amplifier stage of the subsystem, the upper half-power frequency due to beta fall-off will be 7 Mc/sec minimum. With five similar stages, the upper half-power point will be reduced by the additional breaks, and as described in Petit and McWhorter, ⁽⁷⁾ the new upper half-power frequency f'_2 is given by

$$f'_2 \cong \frac{0.833 f_2}{\sqrt{n}}$$

where

f_2 = single-stage upper power half-frequency, and

n = number of identical stages,

and

$$f'_2 \cong \frac{(0.833)(7)}{2.29} = 2.5 \text{ Mc/sec}$$

6.2.3 Actual Measured Bandwidth

An experimental determination of the frequency response characteristics of the amplifier is shown in Figure 6-1. This response curve indicates an experimental lower half-power frequency of 35 kc/sec and an experimental upper half-power frequency of 3.1 Mc/sec.

6.3 ELECTRONIC SUBSYSTEM SENSITIVITY REQUIREMENTS

The basic requirements for subsystem sensitivity are derived from the system requirements on measurable neutron flux, detector locations possible, and types of detectors being used. The system requirements are covered in other sections, along with the detector locations possible.

6.3.1 In-Core System

In the in-core system, the counting chamber is a fission chamber in which the U^{235} is diffused into a base metal. The differential and integral spectra for neutrons from a Pu-Be source for this chamber are shown in Figures 6-2 and 6-3, respectively. * Note that the distribution of pulse heights of the chamber shows a peak at small relative energies. The equivalent input charge is defined as that amount of charge which, when introduced into the input of the subsystem or device, will produce a given output level. The charge is introduced by applying a negative step voltage of V volts to the input of the device through a General Radio 10 pf standard air capacitor. The value of the charge is equal to the value of the step multiplied by 10^{-11} , the value of the capacitance.

*The sample chamber is chamber No. 17, which was later made a part of detector assembly No. 2.

Figure 6-1. Amplifier Frequency Response Characteristics

Figure 6-2. Differential Bias Curve - Chamber No. 17

Figure 6-3. Integral Bias Curve - Chamber No. 17

The integral pulse height curve for the sample chamber shows that with a system sensitivity of 2×10^{-14} coulombs equivalent charge, the total number of counts per unit time is 28 counts/sec in a neutron flux of 1.45×10^4 nv.

6.3.2 Out-Of-Core System

In the out-of-core system, the chamber is one in which the U^{235} is coated on a base metal. The differential and integral spectra for out-of-core detector No. 6 are shown in Figures 6-4 and 6-5, respectively.

6.4 THE EFFECT OF IN-CORE CABLE ON THE NEUTRON SPECTRA

As described previously, the in-core cable attenuates the magnitude of the signal available at the chamber. The theoretical attenuation varies as the inverse exponential of the attenuation factor and the length of cable, and is given by Equation (5-7):

$$\left| \frac{V_r}{V} \right| = e^{-\alpha x},$$

$$\alpha = \frac{\sqrt{\rho f k} \left(1 + \frac{b}{a}\right)}{2.78 \times 10^5 \pi b \log_{10} (b/a)},$$

where

ρ = resistivity of conductor material, in ohmmeters,

f = frequency in cycles per second,

k = relative dielectric constant of insulation,

b = inner radius of outer conductor, in feet,

a = outer radius of inner conductor, in feet,

x = length of cable, in feet,

V = magnitude of voltage at sending end, and

V_r = magnitude of voltage at receiving end.

Figure 6-4. Differential Bias Curve - Chamber No. 6

Figure 6-5. Integral Bias Curve - Chamber No. 6

As a result of this attenuation, there is a reduction in the pulse amplitudes and an increase in the pulse rise time. The differential and integral pulse height spectra for the chamber with in-core cable attached (referred to as "detector assembly No. 2") are shown in Figures 6-6 and 6-7, respectively. (It should be noted that these spectra were obtained with a 40-foot length of cable. Since the attenuation is also a function of cable length, additional in-core cable length will result in additional signal attenuation.) The peak on the differential pulse height distribution has moved toward the noise portion of the spectrum, and is positioned at approximately 13×10^{-14} coulombs equivalent charge. Without the in-core cable, it was positioned at 16.6×10^{-14} coulombs equivalent charge.

The electronic subsystem has a sensitivity of 6.5×10^{-14} coulombs at a discriminator setting of 2.6, and therefore is counting the major portion of the pulses in the peak distribution in both cases. Figures 6-8 and 6-9 are integral pulse height spectra for detector assemblies No. 2 and No. 1, respectively, with the complete subsystem in place. Figure 6-10 is a plot which relates equivalent charge input required to trigger to discriminator dial setting.

In the out-of-core subsystem, regular coaxial cables are used. These cables typically have 3 dB frequency attenuations at several hundreds of megacycles. The out-of-core information pulse is therefore essentially unchanged in its frequency content by the out-of-core interconnecting cable. Integral bias curves for out-of-core chambers No. 6 and No. 7 are shown in Figures 6-11 and 6-12, respectively. As before, Figure 6-10 provides the relationships of discriminator dial setting to equivalent input charge.

6.5 MAXIMUM SENSITIVITY

The maximum sensitivity of the electronic subsystem is limited by amplifier and other noise present at the input. For discriminator settings in which there is less than 0.1 cps due to noise, the electronic subsystem maximum sensitivity is approximately 5.2×10^{-14} coulomb equivalent charge at the input for the in-core subsystem, and approximately 7×10^{-14} coulomb equivalent charge at the input for the out-of-core subsystem with detector No. 6 (sensitivity ≈ 0.7 cps/nv).

Figure 6-6. Differential Bias Curve - Detector Assembly No. 2

Figure 6-7. Integral Bias Curve - Detector Assembly No. 2

Figure 6-8. Integral Bias Curves, Detector Assembly No. 2

Figure 6-9. Integral Bias Curves - Detector Assembly No. 1

Figure 6-10. Input Charge Required to Trigger Versus Discriminator Dial Setting

Figure 6-11. Integral Bias Curve - Chamber No. 6 (also termed as 3-inch fission chamber)

Figure 6-12. Integral Bias Curves - Chamber No. 7 (also termed "AT" type chamber)

6.6 RANGE OF ELECTRONIC SUBSYSTEM

The electronic subsystem has a range of 10^{-1} to 10^6 counts/sec true random input. Because of the random nature of the input signal, the pulse width of the incoming signal should be minimized in order to minimize counting loss due to random coincidences of pulses. With the in-core system, the pulse width - limited by rise time and collection time - is approximately $0.3 \mu\text{sec}$. Thus, at a true random input of 1×10^6 cps there will be a counting loss of 23 percent of true count rate. This is equivalent to 1.63 percent of full-scale analog output current or voltage, and is well within the allowed tolerance for a seven-decade logarithmic instrument. An experimental determination of the counting loss for the in-core system is shown in Figure 6-13. The counting loss of the out-of-core system is shown in Figures 6-14 and 6-15. These curves compare well with the theoretical loss calculated. Improvements in the speed of the subsystem are possible, and can be made where upper half-power frequency of the information pulse will allow it.

6.7 LINEARITY OF THE ELECTRONIC SUBSYSTEM

The electronic subsystem output does not deviate from the true random analog output current or voltage by more than a span of 10 percent of the equivalent linear full-scale output current or voltage over the top six decades, nor by more than 20 percent of the equivalent linear full-scale output current or voltage over the bottom decade, under the following conditions:

Temperature	-	5 to 50°C ,
Humidity	-	20 to 90 percent R. H.,
Line	-	19 Vdc to 29.5 Vdc.

6.8 TIME CONSTANTS - ANALOG CIRCUIT

The time constants of the analog outputs (level and period) of the electronic subsystem are established on the basis of a compromise between speed of response to changing input rates and fluctuations at the analog outputs. Fluctuations at the analog outputs are of special interest in the case where these outputs are coupled into reactor safety systems.

6.9 LEVEL (COUNT RATE) RESPONSE

For a time constant versus input rate curve, as shown in Figure 6-16, there is a specific probability of obtaining a false safety signal for any particular trip point setting, the number of occurrences of the false signal being related to the time constant curve of the circuitry. * Table 2-3a shows the interrelationship between the point of operation (i. e., the particular input rate), a trip point count rate which has been set, and the resultant average number of spurious trips per unit time for this subsystem. Note that the functional variation is very rapid and therefore relatively small changes in time constant can greatly increase the average number of spurious trips. For this subsystem, operation up to an input rate of 3×10^5 cps with a trip setting of 5×10^5 will probably have no spurious trips from this cause during the life of a plant.

*See Section II for a derivation of these relationships.

Figure 6-13. In-Core Startup Monitor Output Versus Calculated True Random Input

Figure 6-14. Out-Of-Core Startup Monitor Output Versus Calculated True Random Input

Figure 6-15. Out-Of-Core Startup Monitor Output Versus Calculated True Random Input

Figure 6-16. Time Constant Versus Counting Rate

6-10 PERIOD RESPONSE

Because the derivative of the level signal is being taken, the fluctuations on the period circuitry are very severe. For period circuitry of the form shown below,

the gain expression, assuming $(Z_{in})_{\text{ampl.}} = \infty$, is

$$\frac{E_o(S)}{E_{in}(S)} = - \frac{1}{\left[\frac{1}{A} + \frac{1+A}{A} \left(\frac{R_2 C_2 S + 1}{R_2} \right) \left(\frac{R_1 C_1 S + 1}{C_1 S} \right) \right]} \quad (6-1)$$

And, if $A \gg 1$, then Equation (6-1) reduces to

$$\frac{E_o(S)}{E_{in}(S)} = - \frac{R_2 C_1 S}{(R_1 C_1 S + 1) (R_2 C_2 S + 1)} \quad (6-2)$$

For the period portion of the electronic subsystem,

$$\begin{aligned} R_1 &= 3.9 \times 10^6 \text{ ohm,} \\ R_2 &= 4.3 \times 10^8 \text{ ohm,} \\ C_1 &= 0.51 \times 10^{-6} \text{ farads, and} \\ C_2 &= 0.0047 \times 10^{-6} \text{ farads.} \end{aligned}$$

Therefore,

$$R_1 C_1 \approx 2 \text{ seconds,}$$

$$R_2 C_2 = 2 \text{ seconds, and}$$

$$R_2 C_1 = 220 \text{ seconds.}$$

$$\text{For } S \ll \frac{1}{R_1 C_1}, \frac{1}{R_2 C_2},$$

$$\frac{E_o(S)}{E_{in}(S)} = A(S) = -R_2 C_1 S,$$

$$\text{or } e_o(t) = -R_2 C_1 \frac{d}{dt} [e_{in}(t)] \quad (6-3)$$

$$\text{or } e_o(t) = -220 \frac{d}{dt} [e_{in}(t)] \quad (6-4)$$

Although an exact analysis of the fluctuation on the period meter is not included in this report, it is seen from Equation (6-4) that the period output will change approximately 200 times the rate of change of the level output.

6.11 SUSCEPTIBILITY

Both the in-core and out-of-core subsystems are identical regarding protection from electric, magnetic, and e-m fields. The signal cable from the detector or detector assembly to the remote amplifier is constructed with additional shields and guards. The remote amplifier itself is packaged in a manner consistent with a minimally susceptible subsystem; i. e., a shielded, gasketed, and decoupled enclosure. and the signal cable connecting the remote amplifier to the monitor is double shielded and guarded.

For the in-core subsystem, the in-core cable must be protected against these fields. Copper braids have been introduced on top of the stainless steel - quartz - stainless steel structure to provide this protection.

Minimalization of susceptibility to line-conducted interference is accomplished in both subsystems by filtering and shielding the input power leads to the monitor and to the remote amplifier.

SECTION VII

CAMPBELL CHANNEL ELECTRONIC SUBSYSTEM

7.1 GENERAL DESCRIPTION

The electronics for the Campbell subsystem* consist basically of five components: remote amplifier, local amplifier, inverter, mean square analog, and d-c amplifier.

7.1.1 Remote Amplifier

The remote amplifier is constructed in such a manner that it can be located as near the reactor as possible while staying outside the biological shielding, the purpose of which is to minimize the distance between the remote amplifier and the detector. It receives its power from the main chassis, and acts as a junction between the detector and the detector polarizing supply located in the main chassis.

The remote amplifier is, in reality, two separate amplifiers housed in the same box, which differ in gain, input impedance, and frequency response. One is referred to as the low-range amplifier and the other as the high-range amplifier. The need for the two different ranges of amplifiers is dictated by the desire to monitor six decades of flux with one chamber.

- a. The low-range amplifier consists of a high-gain common emitter stage and two complementary doublet stages. The over-all gain is a nominal 1200 before application of frequency breaks.

At the lower flux levels covered by the MMSVM, it is necessary to terminate the signal input cable in the high impedance in order to obtain sufficient usable signal, and to provide adequate pulse overlap so that the fractional standard deviation will fall within acceptable limits while maintaining a minimum averaging time constant. For these reasons, an input impedance of 5,000 ohms was chosen for the low-range amplifier. Although not an extremely high input impedance, it is a factor of 27 greater than the 185-ohm characteristic impedance of the RG-114/U cable used to couple the in-core assembly to the amplifier, and it is sufficiently high that the cable appears to be nearly open circuited.

When operating a cable in an unterminated mode, the cable capacity acts to degrade the signal. The cable capacity must therefore be kept to a minimum. It was for this reason that RG-114/U was chosen for the out-of-core signal cable since its capacity is only 6.5 picofarads per foot.

*Also called a multi-range mean square voltage monitor (MMSVM).

The frequency breaks on the low-range amplifier were chosen to make best use of the signals from the unterminated cable and still maintain the response necessary for reactor control. The low-frequency break is 8 kc and the high frequency break is at 60 kc. With a detector and cable assembly of 2000 picofarads capacity connected to the input, the high frequency break is lowered to 16 kc.

- b. The high-range amplifier consists of a common base input stage and two complementary doublet stages. The gain is adjustable from about 115 to 510 before application of frequency breaks. The adjustable gain makes it possible to align the two scales involved when switching from the low- to high-range amplifier.

At high flux levels, faster system response is necessary. For this reason, the bandwidth of the high-range amplifier is designed to be from 300 kc to 600 kc, and the averaging time constant is reduced by a factor of 10. It is also necessary to shorten the pulses into the amplifier to a length that will be compatible with the higher frequency response. To accomplish this, the input impedance of the high-range amplifier is matched to the characteristic impedance of the signal cable. This reduces the pulse width to a fraction of a microsecond, but pulse pile-up is adequate since the pulse count rate from the detector is high.

7.1.2 Local Amplifier

The local amplifier serves three functions:

- a. It provides a characteristic impedance termination for the signal cable from the remote amplifier,
- b. It provides additional gain for driving the squaring circuit, and
- c. It contains the attenuation networks necessary to divide the six decades of flux coverage into twelve ranges. Its gain is nominally 50, but it is adjustable over a ± 30 percent range from its nominal value.

7.1.3 Inverter

The inverter is a unity gain amplifier whose function is to receive the signal from the local amplifier and provide two outputs to the mean square analog, one output 180 degrees out of phase with the other.

7.1.4 Mean Square Analog

The mean square analog is a piece-wise linear circuit whose output current varies as the square of the input voltage. By passing the current through a 10,000-ohm resistor, a voltage is obtained that is fed to a unity gain d-c amplifier which provides the readout signal and operates the trips. A capacitor in parallel with the 10,000-ohm resistor controls the averaging time constant of the system.

7.2 DESIGN CONSIDERATIONS

When this system was designed, it was desired that the controlling frequency breaks should be those of the remote amplifier. For this reason, the frequency breaks of the local amplifier, inverter, and mean square analog were designed to be approximately a decade below and above the extremes set by the remote amplifier, or about 800 cps to 6 Mc.

7.2.1 Selection of System Parameters

The critical system parameters (viz., input impedance Z , lower and upper half-power points ω_L and ω_H , and averaging time-constant τ) must be so chosen that the system performs satisfactorily with respect to the following properties:

- a. Signal to noise ratio,
- b. Dynamic range of the electronics subsystem,
- c. Standard deviation of the signal (steadiness of the reactor power indication),
- d. Probability of false trip or level alarm,
- e. Extraneous signal due to reactor noise, and
- f. Transient response.

The choice of these parameters requires the use of Equations (3-51), (3-52), (3-55), (3-66), (3-100) or (3-108), and (3-110), together with cognizance of the following facts:

- a. For the high range, the input impedance of the system, Z , is fixed to match the cable impedance. For the low range, increasing Z increases the value of the signal and improves the signal-to-noise ratio; but it also decreases ω_H , for ω_H is determined by Z and the cable capacitance.

The half-power points, ω_L and ω_H , must be large enough to ensure satisfactory values of standard deviation of the signal, false trip rate, response to reactor noise, and error due to flux transients; at the same time they must be small enough to ensure sufficient pulse pile-up and hence a realizable dynamic range of the electronics subsystem.

- c. The averaging time-constant, τ , must be large enough to ensure satisfactory values of standard deviation of the signal, false trip rate, and error due to flux transients; at the same time it must be small enough to ensure sufficiently rapid response to flux transients.

The values of these system parameters that were chosen, and the resulting values of the system properties at various flux levels, are listed in Table 7-1.

7.3 NOISE LEVEL

The maximum noise level (input open) referred to the input of the remote amplifier is $3 \mu\text{V rms}$, and the minimum (input shorted) is $1.6 \mu\text{V rms}$. This means that to ensure a signal-to-noise ratio of 10 to 1 mean square, the input signal should be about $10 \mu\text{V rms}$. In experiments with 2000 picofarads of detector and cable capacity attached to the input, the noise level is about $2.2 \mu\text{V rms}$.

7.4 RANGE

The MMSVM was designed to monitor flux over the six-decade range of 10^8 to 10^{14} nv (in-core detector). Assuming a linear chamber, the electronics are capable of ± 5 percent linearity operating over 6.5 decades.

7.5 EXPERIMENTAL RESULTS

The experimental results of system linearity for an in-core and out-of-core fission chamber are shown in Figures 7-1 and 7-2, respectively. In both cases, the system exhibits ± 5 percent linearity over about 5.5 decades. For the in-core detector, there is no noticeable roll-off at the maximum flux tested so it is very possible that its range can be extended. The range of the out-of-core detector can possibly be extended a little by operation at a higher voltage.

TABLE 7-1

VALUES OF SYSTEM PARAMETERS AND
RESULTING SYSTEM PROPERTIES

Flux, ϕ (nv)	10^8	10^9	10^{10}	10^{11}	10^{11}	10^{12}	10^{13}	10^{14}
Count Rate, $k\phi$ (sec ⁻¹) (1)	1.6×10^4	1.6×10^5	1.6×10^6	1.6×10^7	1.6×10^7	1.6×10^8	1.6×10^9	1.6×10^{10}
Input Impedance, Z(ohms)	5000	5000	5000	5000	185	185	185	185
ω_H (rad sec ⁻¹)	10^5	10^5	10^5	10^5	3.8×10^6	3.8×10^6	3.8×10^6	3.8×10^6
ω_L (rad sec ⁻¹)	0.5×10^5	0.5×10^5	0.5×10^5	0.5×10^5	1.9×10^6	1.9×10^6	1.9×10^6	1.9×10^6
τ (sec)	10^{-1}	10^{-1}	10^{-1}	10^{-1}	10^{-2}	10^{-2}	10^{-2}	10^{-2}
Pile-up Factor (2)	0.2	2	20	2×10^2	6	0.6×10^2	0.6×10^3	0.6×10^4
Fractional Std. Dev. (3)	1.9×10^{-2}	1×10^{-2}	0.8×10^{-2}	0.8×10^{-2}	0.4×10^{-2}	0.4×10^{-2}	0.4×10^{-2}	0.4×10^{-2}
False Trip Rate(sec ⁻¹) (4)								
Reactor Noise Contribution (5)	1×10^{-13}	1×10^{-13}	1×10^{-13}	1×10^{-13}	0.8×10^{-16}	0.8×10^{-16}	0.8×10^{-16}	0.8×10^{-16}
Flux Transient Error (6)	1.6×10^{-5}	1.6×10^{-4}	1.6×10^{-3}	1.6×10^{-2}	1.1×10^{-4}	1.1×10^{-3}	1.1×10^{-2}	0.11

NOTES:

- (1) Count rate is based on a calculated counting sensitivity of 1.6×10^{-4} sec⁻¹ nv⁻¹.
- (2) The pile-up factor is given by $2k\phi/(\omega_H + \omega_L)$ and is a measure of the ratio of pulse width to average interval between pulses.
- (3) The fractional standard deviation is given by Equation (3-55).
- (4) The false trip rate is given by Equation (3-110). Tabulations of values of this property for various operating conditions are shown in subsection 3.4.
- (5) The fraction of the signal contributed to by reactor noise is given by Equation (3-100) or (3-108). These values are based on the following assumptions:
 - (a) Mass of U²³⁵ in detector is 0.6×10^{-3} g,
 - (b) Mass of U²³⁵ in reactor is 2×10^6 g,
 - (c) $\Sigma \sigma_f$ is 3×10^{-10} ,
 - (d) $\nu(\nu-1)/\bar{\nu}^2$ is 0.795,
 - (e) Kp is unity,
 - (f) τ_0 is 0.65×10^{-3} sec, and
 - (g) β is 0.65×10^{-2} .
- (6) The flux transient error tabulated is the value of the second term in Equation (3-66) for a +100 percent step in flux.

Figure 7-1. In-Core Fission Chamber Response

Figure 7-2. Out-Of-Core Fission Chamber Response

SECTION VIII

DEVELOPMENTAL SYSTEM PERFORMANCE

8.1 IN-CORE COUNTING SYSTEM

The range of the in-core counting system is from approximately 10^3 nv to 10^9 nv (see Figure 1-1 in Section I). Since the counter has a zero gamma sensitivity of approximately 1×10^{-3} counts/sec per nv, 100 counts/sec are available at a zero gamma startup,* which provides excellent statistics and a three-decade upscale indication. Pulse overlap becomes large above 10^6 counts/sec, thus limiting the range of the system at this level. The detector can be retracted at this point to lower fluxes to continue power coverage, but since adequate overlap with the Campbell channel exists, this is not necessary. An integral bias curve for the system at zero gamma is shown in Figure 6-8 of Section VI.

8.1.1 Performance in the Presence of Gamma Flux

Tests have been performed in the GETR to determine the effect of gamma flux on the counter's sensitivity. A bundle of three detectors was lowered into the Z trail cable with the reactor at full power. The detectors in the bundle were Counter No. 1, Campbell No. 3, and an unlined 1/4-inch-diameter ion chamber. The reactor was then scrammed and the outputs of the three detectors recorded as a function of time.

The gamma flux at any time was determined by the reading of the unlined ion chamber. The neutron flux was determined by the reading of the Campbell channel using a neutron sensitivity of 1.89×10^8 nv/div, after correcting the reading for gamma contribution using a gamma sensitivity of 3.23×10^6 R hr⁻¹/div. The counting rate was determined by the reading of Counter No. 1 after correcting for counting losses by using the data of Figure 6-13.

The results are shown in the following graphs. Figure 8-1 shows the gamma flux, Figure 8-2 shows the total reading of the Campbell (MSV) channel and those parts of this reading that are caused by gammas and by neutrons, Figure 8-3 shows the observed and corrected counting rates, and Figure 8-4 shows the calculated counting sensitivity. There is considerable spread in the counting sensitivity before 19 minutes, when the correction for counting loss is large, but after that time the correction is smaller and the spread in the counting sensitivity is quite small.

The average counting sensitivity, after 18 minutes from scram, is 0.67×10^{-3} sec⁻¹ nv⁻¹ in an average gamma flux of 2.2×10^7 R/h⁻¹. This is to be compared with a counting sensitivity of 1.52×10^{-3} sec⁻¹ nv⁻¹ at gamma flux of about 10^5 R/h⁻¹.

*Assuming sources provide 10^5 nv to the detector at startup.

Figure 8-1. GETR Scram. Gamma Flux
versus Time after Scram

Figure 8-2. GETR Scram. MSV Channel Reading versus Time after Scram

Figure 8-3. GETR Scram. Count Rate versus Time after Scram

Figure 8-4. GETR Scram. Counting Sensitivity versus Time after Scram

8.1.2 Response to Scrams, Withdrawals, and Insertions

The operability of the in-core counting subsystem was investigated at the NTR in several ways:

- a. Scrams were initiated at 30 kW of reactor power after various lengths of time at this power level, and the output of the subsystem was monitored to determine if there were any deleterious effects. Detector assembly No. 1 was operated at the core centerline $\Phi = 7 \times 10^{11}$ nv, $\Phi_g = 2 \times 10^6$ R/h for a period of 5 minutes, after which the reactor was scrammed. Shown in Figure 8-5 is the decay curve with a 1/4-inch-thick lead shield on the detector. This curve has a final slope of 84 sec/ecade (193 sec/decade) at 4 minutes after the scram. Shown in Figure 8-6 is the decay curve after a similar time at the same power level but without the lead shield on the detector. This curve also exhibits a final slope of 84 sec/ecade (193 sec/decade).

Counter No. 2 was operated in the same flux conditions without lead shielding for approximately 1 hour. The final slope of the decay curve after scram in this instance was 82.2 sec/ecade (189 sec/decade) (see Figure 8-7).

After fissioning has ceased, the decay curve of the delayed neutrons from the fission process contains five major groups of neutrons. The longest-lived group is from Krypton 87 (whose precursor is Bromine 87) with a 55.6-second half-life. The 55.6-second half-life corresponds to a 80.3 sec/ecade (185 sec/decade) slope on the decay curve. The close agreement of the experimentally determined slopes with the calculated slope demonstrates a proportionality between the neutron flux in the core and the output of the counting subsystem.

- b. In addition, counter No. 1 was operated in its normal operation region prior to scram, and was immediately inserted to the core centerline upon indication of the scram (see Figure 8-8). In this case, the indicated count rate followed the insertion and then continued to decay in a manner similar to that previously described. The slope of this decay curve, defined at 3 minutes after the scram, is approximately 65 sec/ecade (150 sec/decade). Agreement of the slopes is not as good in this case as in the previous case. This is probably due to data not being available at long enough times after scram. However, the ability of the subsystem to follow the insertion is demonstrated.
- c. A retract and insertion test was performed using detector No. 2 at the NTR facility. The detector assembly was manually retracted from the core centerline at a reactor power level of 3 watts (7×10^7 nv at the detector) and then reinserted to the core centerline. The velocity of withdrawal and insertion was approximately 3 ft/min. The distance withdrawn was approximately 64 inches. No spurious counts or noise transients were evident throughout the course of the experiment. The counting subsystem output as a function of time during this experiment is shown in Figure 8-9.

Figure 8-5 Count Rate Versus Time After Scram. Detector Assembly No. 1 (Counter) With 1/4-inch-thick Lead Shield.

Figure 8-6 Count Rate Versus Time After Scram. Detector Assembly No. 1 (Counter) Without Lead Shield.

Figure 8-7 Count Rate Versus Time After Scram. Detector Assembly No. 2 (Counter)

Figure 8-8 Count Rate Versus Time After Scram from 30 Watts.
Counter No. 1 With Insertion to Core Centerline at Time of Scram

Figure 8-9 Retract and Insert Test. Detector Assembly No. 2

8.1.3 Temperature, Neutron Exposure, and Other Effects.

In extensive tests that have been run up to 700°F (see Sections IV and V), temperature was found not to appreciably affect the breakdown voltage of the detector assembly nor the pulse height spectrum of the counter. An in-core counter has been successfully tested to 10^{20} nvt, and further tests are in progress. The specified life at this time is 10^{19} nvt. The system is very well shielded and filtered to minimize noise susceptibility. Audio-frequency, radio-frequency, and transient-pulse-conducted and relay-radiated-interference tests have been conducted (less than 0.3 counts/sec due to these sources).

The detector probe is retracted at 5×10^{12} nv to avoid excessive neutron and gamma exposure. The calculated worst case temperature in the dry tube is caused by neutron and gamma heating at 1×10^{14} nv, and calculations based on the physical properties of the probe material show that this temperature will not destroy the integrity of the probe.

8.2 IN-CORE CAMPBELLING SYSTEM

The range of the in-core Campbelling system is approximately 2×10^8 to 3×10^{13} nv (see Figure 8-10). This assumes a constant gamma level of 2.5×10^7 up to 3×10^{11} nv, above which the level of the gamma flux increases as the neutron flux increases. The gamma flux reaches 1×10^9 R/h when the neutron flux is 1×10^{14} nv. The lower limit of the operating range with no gamma flux is determined by amplifier noise; the lower limit of the operative range for a hot restart is determined by the gamma flux and amplifier noise. The neutron/gamma output signal ratios can be found from the product of the neutron sensitivity/gamma sensitivity ratio and the ratio of neutron flux to gamma flux.

The upper limit of measurable neutron flux for this system is determined by ion chamber saturation. The detectors have passed breakdown tests at 700°F (Figure 4-5). The results of the in-core counter life test mentioned in Section IV demonstrate that the Campbell chamber has a life of at least 10^{20} nvt. At this time the specified life is 10^{19} nvt. Life tests are continuing.

The system is well shielded and filtered to minimize noise susceptibility. Audio-frequency, radio-frequency, and transient-pulse-conducted and relay-radiated-interference tests have been conducted (< 1 percent added to signal due to these sources). The detector is retracted at 5×10^{12} nv to avoid excessive neutron and gamma exposures and a large temperature rise in a dry tube.

8.3 OUT-OF-CORE COUNTING SYSTEM

The range of the out-of-core counting system is 1.4 nv to 1.4×10^6 nv (see Figure 8-11). This system provides 1 count/sec at startup and 10^6 counts/sec at the upper end of the range. Pulse overlap becomes large above 10^6 counts/sec, thus limiting the range of the system at this level.

Figure 8-10. In-Core Campbell Response

Figure 8-11. Out-of-Core Counting Response

This particular chamber can be used up to 300°C (aluminum limits the temperature). A titanium fission counter of a similar design, with a specified life of 10^{19} nvt, can be used up to 1000°F. The system uses the identical electronics as is used in the in-core system, except that a higher voltage polarizing power supply may be used if desired.

8.3.1 Response to Scrams and Withdrawals

The out-of-core counting subsystem was tested for response to scrams from high power levels (900 watts and 300 watts), using both types of out-of-core fission chambers; i. e., the large 3-inch-diameter chamber (chamber No. 6) and the small 1-inch-diameter chamber (chamber No. 7). The output of the subsystem was monitored before and after the scram. Shown in Figure 8-12 is the decay curve for chamber No. 7 (sensitivity ~ 0.1 counts/sec per nv). This curve indicates a constant slope after $t = +4$ minutes of 81 sec/ecade (186 sec/decade).

Shown in Figure 8-13 is the decay curve for chamber No. 6 (sensitivity ~ 0.7 counts/sec per nv). This curve shows a constant slope after $t = +3.5$ minutes of 78.3 sec/ecade (180 sec/decade). The response of the counting subsystem to rapid mechanical withdrawal at intermediate power levels (30 watts and 300 watts) was determined. The flux level at the detector was equivalent to an approximate full-scale reading on the subsystem. At this level the subsystem electronic response time is considered very short compared to the mechanical withdrawal time.

Chamber No. 7 was withdrawn four times at a power level of 30 watts (see Figure 8-14 through 8-17). The average mechanical withdrawal time was 1.5 seconds. The average time between the completion of the withdrawal and the time when the counting subsystem output reached its final value was 0.75 second. Two withdrawals of chamber No. 7 were made at a power level of 300 watts (see Figures 8-18 and 8-19). The average mechanical withdrawal time for these withdrawals was 1.22 seconds. The average time between the completion of the withdrawal and the time when the counting subsystem reached its final value in this case was 1.42 seconds. These times are not known with great precision; the possible error includes a 1-second scaler integrating time.

Chamber No. 6 was withdrawn once at a power level of 100 watts (chamber No. 6 could not be reinserted with the reactor at power). The withdrawal time for this test (see Figure 8-20) was approximately 1 second, and the counting subsystem reached its final value after a delay of 1 second from the time that the withdrawal was completed.

In all cases above, no noise transient phenomena were evident, either on the withdrawals or the subsequent reinsertion.

Figure 8-12. Count Rate Versus Time After Scram. Chamber No. 7

Figure 8-13. Count Rate Versus Time After Scram. Chamber No. 6

Figure 8-14. Count Rate Versus Time. Withdrawal of Chamber No. 7 at 30 Watts Reactor Power - Run No. 1

Figure 8-15. Count Rate Versus Time. Withdrawal of Chamber No. 7 at 30 Watts Reactor Power - Run No. 2

Figure 8-16. Count Rate Versus Time. Withdrawal of Chamber No. 7 at 30 Watts Reactor Power - Run No. 3

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Figure 8-17. Count Rate Versus Time. Withdrawal of Chamber No. 7 at 30 Watts Reactor Power - Run No. 4

Figure 8-18. Count Rate Versus Time. Withdrawal of Chamber No. 7 at 30 Watts Reactor Power - Run No. 1

Figure 8-19. Count Rate Versus Time. Withdrawal of Chamber No. 7 at 30 Watts Reactor Power - Run No. 2

Figure 8-20. Count Rate Versus Time. Withdrawal of Chamber No. 6 at 100 Watts Reactor Power

8.4 OUT-OF-CORE CAMPBELL SYSTEM

The range of the out-of-core Campbell system is $< 1 \times 10^4$ to 1×10^{10} nv (see Figure 8-21). Gamma may limit the minimum measurable neutron flux in some applications. The detector can be used to 300°C, and is limited because it is made with aluminum. A titanium detector of similar design could be used; it would operate up to 1000°F (tested) and beyond (untested). The specified life is 10^{19} nvt. The same Campbell channel electronics is used as in the in-core system, with the exception of the polarizing voltage supply (the larger detector requires higher voltage).

Figure 8-21. Out-of-Core Campbell Response

APPENDIX

VARIANCE OF LOGARITHMIC COUNT-RATE METER DRIVEN BY A SCALE-OF-TWO

The purpose of this appendix is to determine the effect of the scale-of-two on the variance of the count-rate indication. The method used consists of determining the distributed portion of the power spectral density of this indication and comparing it with the power spectral density of the indication that would be obtained without a scale-of-two. Since the variance is the integral of the power spectral density, a comparison of the variances can be obtained in this manner.

In this derivation it is assumed that the average counting rate is constant. The power spectral density of a signal, $S(t)$, is given by

$$G(\omega) = \frac{2}{\pi} \int_0^{\infty} \psi(\delta) \cos \omega \delta d\delta, \quad (\text{A-1})$$

where $\psi(\delta)$ is the correlation function of $S(t)$ and is defined as

$$\psi(\delta) = \text{ave} [S(t) S(t + \delta)] \quad (\text{A-2})$$

Now the signal, $S(t)$, can be considered as consisting of two parts: a d-c component generated by those diode pumps that are saturated, and a second part composed of a linear superposition of pulses from the unsaturated diode pumps. We will neglect the first part since it only contributes an impulse function at $f = 0$, representing a d-c component of $S(t)$.

The second part can be written as

$$S_2(t) = \sum_{t_k = -\infty}^{\infty} v(t - t_k), \quad (\text{A-3})$$

where the k^{th} pulse starts at t_k and where $v(t)$ describes a pulse that starts at $t = 0$. This equation can be written in a form suitable for statistical analysis in the following manner. Divide the time axis into small equal intervals of length Δt ; then the onset of a pulse during the n^{th} interval will produce a signal at time t of $v(t - n\Delta t)$, and the total signal due to all of the

previous pulses will be

$$S_2(t) = \sum_n v(t - n \Delta t), \quad (A-4)$$

where the summation includes only those intervals during which a pulse starts. If we define a random variable η_n , which equals one if a pulse starts during the n^{th} time interval and equals zero if no pulse starts during the n^{th} time interval, then the total signal can be written as

$$S_2(t) = \sum_{n=-\infty}^{t/\Delta t} \eta_n v(t - n \Delta t), \quad (A-5)$$

where the summation now includes all the time intervals that precede t .

Likewise,

$$S_2(t + \delta) = \sum_{m=-\infty}^{(t+\delta)/\Delta t} \eta_m v(t + \delta - m \Delta t), \quad (A-6)$$

and hence

$$S_2(t) S_2(t + \delta) = \sum_{n=-\infty}^{t/\Delta t} \sum_{m=-\infty}^{(t+\delta)/\Delta t} \eta_n \eta_m v(t - n \Delta t) v(t + \delta - m \Delta t). \quad (A-7)$$

For a stationary random process (constant average counting rate in this case), the time average of a function is equal to the ensemble average, so we can write

$$\psi(\delta) = \langle S_2(t) S_2(t + \delta) \rangle, \quad (A-8)$$

where the brackets $\langle \rangle$ indicate expected value at time t , or ensemble average. Hence,

$$\psi(\delta) = \sum_{n=-\infty}^{t/\Delta t} \sum_{m=-\infty}^{(t+\delta)/\Delta t} \langle \eta_n \eta_m \rangle v(t - n \Delta t) v(t + \delta - m \Delta t) \quad (A-9)$$

since the average of a sum of random variables is equal to the sum of the averages.

It is convenient at this point to make a change of variables.

Let

$$\begin{aligned} t - n\Delta t &= a\Delta t, \\ t + \delta - m\Delta t &= b\Delta t; \end{aligned} \quad (\text{A-10})$$

then

$$\psi(\delta) = \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left\langle \eta\left(\frac{t}{\Delta t} - a\right) \eta\left(\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b\right) \right\rangle v(a\Delta t) v(b\Delta t) \quad (\text{A-11})$$

We now must evaluate $\left\langle \eta\left(\frac{t}{\Delta t} - a\right) \eta\left(\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b\right) \right\rangle$, and we use the fact that the

expected value of a random variable is the sum of the products of each value the variable can assume and the probability of the variable assuming that value. Since η or any product of η 's can assume only the values one or zero, and the zeros will not contribute to the expected value, we need consider only the value unity and the probability of assuming this value.

The first case to consider is $\frac{t}{\Delta t} - a = \frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b$; i. e., both time intervals are one and the same. A pulse will start in the $\left(\frac{t}{\Delta t} - a\right)^{\text{th}}$ interval if a neutron is detected during this interval and if the scale-of-two output is in its less positive state during the preceding interval. Hence,

$$\left\langle \eta\left(\frac{t}{\Delta t} - a\right) \eta\left(\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b\right) \right\rangle = (0.5) \times (k\Phi \Delta t) \quad (\text{A-12})$$

for $a = b - \frac{\delta}{\Delta t}$. The first factor on the right-hand side of Equation (A-12) is the probability

that the scale-of-two output is in its less positive state, and the second factor is the probability of detecting a neutron during the $\left(\frac{t}{\Delta t} - a\right)^{\text{th}}$ interval; k is the counting efficiency, Φ is the flux, and hence $k\Phi$ is the counting rate at the input of the scale-of-two.

If $\frac{t}{\Delta t} - a$ precedes $\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b$ (i.e., if $a > b - \delta/\Delta t$), then it is necessary that a pulse start in the first of these two intervals, an odd number of neutron detections occur between them, and a neutron detection occur in the second of these two intervals. Hence,

$$\begin{aligned}
 & \left\langle \eta\left(\frac{t}{\Delta t} - a\right) \eta\left(\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b\right) \right\rangle = (0.5 k \Phi \Delta t) \\
 & \times \sum_{i=0}^{\infty} e^{-k \Phi \left(a - b + \frac{\delta}{\Delta t} - 1\right) \Delta t} \frac{\left[k \Phi \left(a - b + \frac{\delta}{\Delta t} - 1\right) \Delta t \right]^{(2i+1)}}{(2i+1)!} \\
 & \times k \Phi \Delta t
 \end{aligned} \tag{A-13}$$

for $a > b - \frac{\delta}{\Delta t}$. The first factor on the right-hand side of Equation (A-13) is the probability that a pulse starts in the first of these two intervals, the second factor is the probability of an odd number of neutron detections occurring between them, and the third factor is the probability of detecting a neutron in the second of these two intervals. This can be written as

$$\left\langle \eta\left(\frac{t}{\Delta t} - a\right) \eta\left(\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b\right) \right\rangle = 0.5 (k \Phi \Delta t)^2 \frac{1 - e^{-2k\Phi \left(a - b + \frac{\delta}{\Delta t} - 1\right) \Delta t}}{2} \tag{A-14}$$

Likewise, if $\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b$ precedes $\frac{t}{\Delta t} - a$ (i.e., if $a < b - \delta/\Delta t$), then

$$\left\langle \eta\left(\frac{t}{\Delta t} - a\right) \eta\left(\frac{t}{\Delta t} + \frac{\delta}{\Delta t} - b\right) \right\rangle = 0.5 (k \Phi \Delta t)^2 \frac{1 - e^{-2k\Phi \left(b - a - \frac{\delta}{\Delta t} - 1\right) \Delta t}}{2} \tag{A-15}$$

Substitution of Equations (A-12), (A-14), and (A-15) into (A-11) yields

$$\begin{aligned}
 \psi(\delta) &= \sum_{a=0}^{\infty} 0.5 k \Phi \Delta t v(a \Delta t) v(a \Delta t + \delta) \\
 &+ \sum_{a=0}^{\infty} \sum_{b=0}^{a+\delta/\Delta t} 0.5 (k \Phi \Delta t)^2 \frac{1 - e^{-2k\Phi \left(a - b + \frac{\delta}{\Delta t} - 1\right) \Delta t}}{2} v(a \Delta t) v(b \Delta t) \\
 &+ \sum_{a=0}^{\infty} \sum_{b=a+\frac{\delta}{\Delta t}}^{\infty} 0.5 (k \Phi \Delta t)^2 \frac{1 - e^{-2k\Phi \left(b - a - \frac{\delta}{\Delta t} - 1\right) \Delta t}}{2} v(a \Delta t) v(b \Delta t) .
 \end{aligned} \tag{A-16}$$

If we let

$$x_a = a \Delta t, \quad y_b = b \Delta t, \tag{A-17}$$

then

$$\Delta x = \Delta a \Delta t = \Delta t, \text{ and}$$

$$\Delta y = \Delta b \Delta t = \Delta t, \tag{A-18}$$

since $\Delta a = \Delta b = 1$. Then we can write

$$\begin{aligned}
 \psi(\delta) &= 0.5 k \Phi \sum_{a=0}^{\infty} v(x_a) v(x_a + \delta) \Delta x \\
 &+ 0.5 (k \Phi)^2 \sum_{a=0}^{\infty} \sum_{b=0}^{a+\delta/\Delta t} \frac{1 - e^{-2k\Phi (x_a - y_b + \delta - \Delta t)}}{2} v(x_a) v(y_b) \Delta x \Delta y \\
 &+ 0.5 (k \Phi)^2 \sum_{a=0}^{\infty} \sum_{b=a+\frac{\delta}{\Delta t}}^{\infty} \frac{1 - e^{-2k\Phi (y_b - x_a - \delta - \Delta t)}}{2} v(x_a) v(y_b) \Delta x \Delta y
 \end{aligned} \tag{A-19}$$

which becomes, as $\Delta t \rightarrow 0$,

$$\begin{aligned}
 \psi(\delta) &= 0.5k\Phi \int_0^{\infty} v(x) v(x + \delta) dx \\
 &+ 0.5(k\Phi)^2 \int_0^{\infty} \int_0^{x+\delta} \frac{1 - e^{-2k\Phi(x-y+\delta)}}{2} v(x) v(y) dy dx \\
 &+ 0.5(k\Phi)^2 \int_0^{\infty} \int_{x+\delta}^{\infty} \frac{1 - e^{-2k\Phi(y-x-\delta)}}{2} v(x) v(y) dy dx. \quad (A-20)
 \end{aligned}$$

This last Equation (A-20) can be rearranged to

$$\begin{aligned}
 \psi(\delta) &= 0.5k\Phi \int_0^{\infty} v(x) v(x + \delta) dx \\
 &+ \left[0.5k\Phi \int_0^{\infty} v(x) dx \right]^2 \\
 &- (0.5k\Phi)^2 \left\{ \int_0^{\infty} e^{-2k\Phi(x+\delta)} \left[\int_0^{x+\delta} e^{2k\Phi y} v(y) dy \right] dx \right. \\
 &\left. + \int_0^{\infty} e^{2k\Phi(x+\delta)} \left[\int_{x+\delta}^{\infty} e^{-2k\Phi y} v(y) dy \right] dx \right\}. \quad (A-21)
 \end{aligned}$$

The second term on the right-hand side of Equation (A-21) contributes to the power spectral density an impulse function at $f = 0$. (It represents the d-c component of the pulses that make up $S(t)$). We will neglect this term and write the following as the correlation function of the a-c component of the signal:

$$\begin{aligned}
\psi(\delta) &= 0.5k\Phi \int_0^\infty v(x) v(x+\delta) dx \\
&- (0.5k\Phi)^2 \left\{ \int_0^\infty e^{-2k\Phi(x+\delta)} \left[\int_0^{x+\delta} e^{2k\Phi y} v(y) dy \right] dx \right. \\
&+ \left. \int_0^\infty e^{2k\Phi(x+\delta)} \left[\int_{x+\delta}^\infty e^{-2k\Phi y} v(y) dy \right] dx \right\}. \quad (A-22)
\end{aligned}$$

The first term on the right-hand side of Equation (A-22) is⁽¹⁾ the correlation function of the a-c component of a signal composed of a linear superposition of pulses of the form $v(t)$, arriving randomly at an average rate of $0.5k\Phi$; the second term, therefore, is the contribution to the spectrum resulting from the regularizing action of the scale-of-two.

To illustrate the magnitude of this effect, let

$$v(t) = v_0 e^{-\omega_0 t} \quad (A-23)$$

and calculate: (a) the power spectral density of the signal obtained with the scale-of-two present, and (b) the power spectral density of the signal obtained without the scale-of-two present and with half the average counting rate. The first calculation is performed by combining Equations (A-1), (A-23), and (A-22); the second by combining Equations (A-1), (A-23), and the first term of (A-22). The results are:

$$G_a(\omega) = \frac{0.5}{\pi} (0.5k\Phi) \left(\frac{v_0}{\omega_0} \right)^2 \frac{1 + \frac{\omega^2}{8(0.25k^2\Phi^2)}}{\left(1 + \frac{\omega^2}{16(0.25k^2\Phi^2)} \right) \left(1 + \frac{\omega^2}{\omega_0^2} \right)} \quad (A-24)$$

with the scale-of-two present, and

$$G_b(\omega) = \frac{0.5k\Phi}{\pi} \left(\frac{v_0}{\omega_0} \right)^2 \frac{1}{1 + \frac{\omega^2}{\omega_0^2}} \quad (A-25)$$

without the scale-of-two.

A comparison of these two equations shows: (a) $G_a(\omega)$ is one-half $G_b(\omega)$ between the frequencies zero and $\sqrt{2} k \Phi$, and (b) $G_a(\omega)$ equals $G_b(\omega)$ between the frequencies $2k\Phi$ and infinity. Hence, the integral of $G_b(\omega)$ is less than twice the integral of $G_a(\omega)$, and the variance obtained by neglecting the scale-of-two is too large by less than a factor of 2.

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