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PHENOMENOLOGICAL DYNAMICS FOR CP VIOLATION*

by

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ABSTRACT

A fairly general current-current form for the CP violating interaction is investigated in the phenomenological Lagrangian framework. Special attention is given to the computation of both the CP conserving and CP violating self masses in the K_L - K_S system. In addition the relevant amplitudes appearing in the neutral kaon decay matrix are calculated. We find that several kinds of models naturally give the same results as the super-weak theory, within present experimental accuracy.

I. INTRODUCTION

Although the mystery of CP violation has been with us for some years now there have not been a very large number of attempts to make dynamical calculations. Some work has been done⁽¹⁾ using the current-algebra technique; this has centered around relating the CP violating nonleptonic amplitudes to ~~semi-leptonic~~ amplitudes in certain models. However it is becoming more and more clear experimentally that the CP violation tends to behave as if it were in the self-mass part of the K_L - K_S system rather than in the amplitudes. Thus a dynamical calculation necessarily must confront head on the problem of calculating the K_L - K_S mass difference and the CP violating transition mass. (Of course, if one adopts the super weak model⁽²⁾ there is no calculation to be done but then one also has the feeling that the problem has been "swept under the rug" for future generations to puzzle out.)

While the computations of nonleptonic amplitudes by the current-algebra or phenomenological Lagrangian methods have had a certain amount of success, the calculation⁽³⁾ of the K_L - K_S mass difference is considered fairly controversial. We will attack this problem first and show that the simplest application of the phenomenological Lagrangian method does in fact lead to a reasonable answer for this quantity. (However, there is a delicate cancellation which may partially account for the lack of agreement as to the validity of this approach.) Then we shall assume a current-current form for the CP violation term and use the same method to calculate the CP violating transition mass. The amplitude CP violation will also be calculated and the possible kinds of current-current models which give agreement

with experiment will be discussed.

If one constructs a $|\Delta S|=1$ CP violating Lagrangian density of the form (octet current) $\times$ (octet current) out of the usual vector and pseudovector currents there are twelve possible terms⁽⁴⁾ i.e.,

$$\begin{aligned} \mathcal{L}_{\text{viol.}} = & iG \{ a(P_2^1 P_1^3 - P_3^1 P_1^2) + (P_2^3 - P_3^2) [b(P_1^1 - P_2^2) + c(P_1^1 + P_2^2 - 2P_3^3)] \\ & + a'(V_2^1 V_1^3 - V_3^1 V_1^2) + (V_2^3 - V_3^2) [b'(V_1^1 - V_2^2) + c'(V_1^1 + V_2^2 - 2V_3^3)] \\ & + \alpha(P_2^1 V_1^3 - V_3^1 P_1^2) + (V_2^3 - V_3^2) [\beta(P_1^1 - P_2^2) + \gamma(P_1^1 + P_2^2 - 2P_3^3)] \\ & + \alpha'(V_2^1 P_1^3 - P_3^1 V_1^2) + (P_2^3 - P_3^2) [\beta'(V_1^1 - V_2^2) + \gamma'(V_1^1 + V_2^2 - 2V_3^3)] \} \end{aligned} \quad (1.1)$$

where $G \approx 1.02 \times 10^{-5}/M_p^2$ is the Fermi constant and our normalizations and conventions for the vector and pseudovector currents are revealed by noting that in the three-quark model:

$$\begin{aligned} V_a^b &= i \bar{q}_b \gamma q_a \\ P_a^b &= i \bar{q}_b \gamma \gamma_5 q_a \end{aligned} \quad (1.2)$$

In (1.1) and (1.2) the Lorentz indices have been deleted for compactness of writing.

Since nature apparently prefers the matrix elements of the non-octet parts of current-current products to be drastically suppressed we shall work with a simplified model in which only pure octets appear in (1.1). The twelve real constants in (1.1) will thus be taken to satisfy the following relations:

$$\begin{aligned} -\frac{a}{2} = b = 3c, \quad -\frac{a'}{2} = b' = 3c' \\ -\frac{\alpha}{2} = \beta = 3\gamma, \quad -\frac{\alpha'}{2} = \beta' = 3\gamma' \end{aligned} \quad (1.3)$$

Thus our CP violating term will depend on the four parameters:

a , a' , α , and α' .

We shall also need a phenomenological Lagrangian for the CP conserving processes. It has been found⁽⁵⁾ that the pure octet Lagrangian density

$$\mathcal{L} = \frac{G}{\sqrt{2}} \bar{X} (J_2^C J_C^3 + J_3^C J_C^2) \quad , \quad (1.4)$$

where

$$J_a^b = V_a^b + P_a^b - \frac{1}{3} \delta_a^b (V_C^C + P_C^C) \quad (1.5)$$

and $\bar{X}$ is a numerical factor whose magnitude is about 1.10 adequately explains the $\Delta I = \frac{1}{2} K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ amplitudes.

The plan of this paper is as follows. The computation of the (CP conserving) K_1-K_2 mass difference is discussed at some length in Section II while the same method is applied to the (CP violating) K_1-K_2 transition mass in Section III. The $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ decay amplitudes are calculated in Section IV. In Section V the decay matrix and observable $K \rightarrow 2\pi$ parameters are calculated. Section VI contains a brief summary (which may be read first for orientation) and several detailed models. Finally Section VII contains conclusions and some additional discussion.

II. K_L-K_S MASS DIFFERENCE

In this section we shall work in the limit of CP conservation and apply (1.4) to the computation of the K_L-K_S (or K_1-K_2) mass difference. We use the approximation in which the currents of (1.2) are dominated by the appropriate spin 0 and spin 1 particles.

Specifically, we write (∂ stands for $\frac{\partial}{\partial x_\mu}$):

$$\begin{aligned}
 P_2^3 &= F_K \partial K^0 + \dots, \quad P_3^2 = F_K \partial \bar{K}^0 + \dots \\
 \frac{1}{\sqrt{2}}(P_1^1 - P_2^2) &= F_\pi \partial \pi^0 + F_{A_1} A_1^0 \\
 \frac{1}{\sqrt{6}}(P_1^1 + P_2^2 - 2P_3^3) &= F_\eta \partial \eta + F_{\eta'} \partial \eta' + F_D D + F_E E \\
 \frac{1}{\sqrt{2}}(V_1^1 - V_2^2) &= F_\rho \rho^0 \\
 \frac{1}{\sqrt{6}}(V_1^1 + V_2^2 - 2V_3^3) &= F_\omega \omega + F_\phi \phi
 \end{aligned} \tag{2.1}$$

In (2.1) A_1^0 is a member of an axial vector isotriplet, and D and E are axial vector isosinglets. The other particles are the usual ones and the F's are the "decay constants". Before proceeding further we define the all important objects K_1 and K_2 :

$$\begin{aligned}
 K_1 &= \frac{1}{\sqrt{2}i} (\bar{K}^0 - K^0) \\
 K_2 &= \frac{1}{\sqrt{2}} (\bar{K}^0 + K^0)
 \end{aligned} \tag{2.2}$$

Note that K_1 and K_2 are both hermitean and that, for example, K^0 is an operator which destroys a K^0 particle. Also $\bar{K}_0 \equiv U_c K_0 U_c^{-1}$, where U_c is the charge conjugation operator.

With the substitutions (2.1), the interaction (1.4) will lead in second order perturbation to self-mass transitions of the form $K_2 \rightarrow \text{virtual meson} \rightarrow K_2$. It is easily seen that there are no analogous transitions for K_1 and hence we find a K_1 - K_2 mass difference⁽⁶⁾ to second order in the weak interaction. The formula is:

$$\begin{aligned}
 m(K_2) - m(K_1) = \frac{1}{4} G^2 \bar{X}^2 F_K^2 m(K) & \left[\frac{F_\pi^2 m^2(K)}{m^2(K) - m^2(\pi)} - \frac{1}{3} \frac{F_\eta^2 m^2(K)}{m^2(\eta) - m^2(K)} \right. \\
 - \frac{1}{3} \frac{F_{\eta'}^2 m^2(K)}{m^2(\eta') - m^2(K)} & + \frac{F_\rho^2}{m^2(\rho)} + \frac{1}{3} \frac{F_\omega^2}{m^2(\omega)} + \frac{1}{3} \frac{F_\phi^2}{m^2(\phi)} \\
 \left. + \frac{F_{A_1}^2}{m^2(A_1^0)} + \frac{1}{3} \frac{F_D^2}{m^2(D)} + \frac{1}{3} \frac{F_E^2}{m^2(E)} \right] & \quad (2.3)
 \end{aligned}$$

In (2.3) each term represents the contribution from the appropriate virtual meson. Note that all terms are positive definite except for the η and η' terms which are negative definite. We now must give some thought to the values of the decay constants involved. For the pseudoscalar mesons

$$F_\pi \approx 1.01 m(\pi^0) \quad , \quad F_K \approx 1.28 F_\pi \quad (2.4)$$

can be gotten from leptonic decays (F_K is not critical for our purposes) while the small η - η' mixing angle implies that η is essentially a pure SU(3) singlet so that

$$F_{\eta'} \approx 0 \quad (2.5)$$

is reasonable for our purposes. $\frac{F_\eta}{F_\pi}$ should be around unity but its exact value is fairly critical so we will consider it to be a parameter. For the ρ -meson the field-current identity and the KSRF relation⁽⁷⁾ give

$$F_\rho = \sqrt{2} m_\rho F_\pi = (.1475)(\text{GeV})^2 \quad (2.6)$$

The "decay constants" for the ω and ϕ mesons can be related to F_ρ by SU(3). Taking the canonical ω - ϕ mixing⁽⁸⁾ into account we get:

$$F_\phi = \sqrt{2} F_\omega = \sqrt{\frac{2}{3}} F_\rho \quad (2.7)$$

Finally, if the axial vector mesons exist one expects by chiral symmetry their decay constants to be about the same as F_ρ . However their masses would be larger than the vector meson masses; (2.3) then shows that their contributions would be suppressed because of this. Thus, considering their dubious status⁽⁹⁾ at present we shall neglect them. Putting all the above numbers into (2.3) and using⁽¹⁰⁾

$$\Delta \equiv [m(K_2) - m(K_1)]_{\text{exp.}} = (3.56 \pm .03) \times 10^{-12} \text{ MeV} \quad (2.8)$$

then enables us to compute

$$F_\eta = 1.46 F_\pi \quad (2.9)$$

which is not far from what we expect.⁽¹¹⁾

Now let us examine the contribution of each term in (2.3) to the total. The following chart describes the situation:

<u>intermediate meson</u>	<u>contribution to Δ</u>
π	2.4 Δ
η	-8.4 Δ
ρ	4.9 Δ
ω	.5 Δ
ϕ	1.6 Δ

What is happening is clear - a vector meson contribution which is many times the correct value is being almost completely cancelled by the pseudoscalar meson contribution. The η term is the dominant one since the small difference $m^2(\eta) - m^2(K)$ appears in the denominator of this term in (2.3). Note that small changes in the decay constants or the inclusion of the axial vector mesons will not change the basic situation even though they might easily cause the sign of

the computed value of Δ to switch. Thus, it cannot be said that we can really predict Δ . On the other hand it seems fair to say that we can understand Δ within the framework of the present calculation. In particular, it would seem reasonable to use the above method of calculation for computing the CP violating transition mass, where a delicate cancellation does not in general take place.

Before going to the CP violating case we briefly comment on a possible modification of the CP conserving interaction (1.4). If one requires the phenomenological nonleptonic current-current interaction to satisfy the $\Delta I = 1/2$ rule but not necessarily to be a pure octet (1.4) is replaced by the more general form

$$\mathcal{L} = \frac{-G}{\sqrt{2}} \bar{\chi} \{ J_2^C J_C^3 + J_3^C J_C^2 + \epsilon J_3^3 (J_2^3 + J_3^2) \} \quad (2.10)$$

where ϵ is an arbitrary dimensionless parameter. If (2.10) is adopted, the formula (2.3) for the K_1 - K_2 mass difference is simply modified by substitution for each factor $\frac{1}{3}$ the quantity $\frac{1}{3}(1+2\epsilon)^2$. Moshe and P. Singer⁽¹²⁾ have suggested the adoption of a phenomenological interaction of the form (2.10) but with the value $\epsilon = -1/2$. Eq. (2.3) then shows that the η contribution will vanish so that the above cancellation would not take place. Then there would be no hope for getting the right order of magnitude for Δ in the present method of calculation. On the other hand, a small value of ϵ is not ruled out.

III. K_1-K_2 TRANSITION MASS

Here we compute the K_1-K_2 transition mass by the method of the last section. This transition arises in second order perturbation wherein K_1 first goes to an intermediate meson with the CP violating term (1.1), which then goes to K_2 with the CP conserving term (1.4). Computation with ordinary covariant perturbation theory then gives an S-amplitude for the K_1-K_2 transition which is the same as the one that would arise from the following effective Hamiltonian density:

$$\mathcal{H}_{\text{eff}}(x) = 2m(K)M_{12}K_1(x)K_2(x) \quad , \quad (3.1)$$

where

$$\begin{aligned} M_{12} = & \frac{1}{2\sqrt{2}} G^2 \bar{X} F_K^2 m(\bar{K}) \times \\ & \times \left\{ a \left[\frac{F_\pi^2 m^2(K)}{m^2(K) - m^2(\pi)} - \frac{1}{3} \frac{F_\eta^2 m^2(K)}{m^2(\eta) - m^2(K)} - \frac{1}{3} \frac{F_{\eta'}^2 m^2(K)}{m^2(\eta') - m^2(K)} \right. \right. \\ & + \left. \frac{F_{A_1}^2}{m^2(A_1^0)} + \frac{1}{3} \frac{F_D^2}{m^2(D)} + \frac{1}{3} \frac{F_E^2}{m^2(E)} \right] \\ & + \alpha' \left[\frac{F_\rho^2}{m^2(\rho)} + \frac{1}{3} \frac{F_\omega^2}{m^2(\omega)} + \frac{1}{3} \frac{F_\phi^2}{m^2(\phi)} \right] \left. \right\} \quad (3.2) \end{aligned}$$

The numerical values of the decay constants appearing in (3.2) should be taken from the last section. Hence (3.2) only depends on the parameters a and α' [see (1.1) and (1.3)].

Note that the nonappearance of the parameters a' and α is due to the fact that we are working in the approximation where currents are dominated by single particles. Finally note that the η term

multiplies a while the vector meson terms multiply α' . Thus there will not be any delicate cancellations unless these two are about equal. However in that event comparison of (3.2) and (2.3) gives the simplified formula

$$M_{12} = \frac{\sqrt{2}a}{\underline{X}} \Delta \quad (\text{For } a=\alpha') \quad (3.3)$$

IV. $K \rightarrow 2\pi$ AND $K \rightarrow 3\pi$ AMPLITUDES

In this section we give both the CP conserving and CP violating $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ amplitudes computed from (1.4) and (1.1) [assuming (1.3)]. These are of course needed for a complete description of CP violation in the K-meson system. The quickest method of calculation involves the use of Cronin's nonlinear model⁽¹³⁾ of pseudoscalar mesons. This gives the "current-algebra" results for the CP conserving $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ amplitudes which are in reasonable agreement with experiment. Thus we expect the model to be adequate for our purposes even though it doesn't include the effects of spin 1 mesons which were needed for the computation of Δ in Section II. In this connection we point out⁽¹⁴⁾ that for the case of the π -N s-wave scattering lengths (which like the $K \rightarrow 2\pi, 3\pi$ amplitudes is another standard chiral symmetry result) the inclusion of spin 1 mesons has no effect at all.

In the nonlinear model the vector and pseudovector currents are represented by infinite series in the traceless pseudoscalar octet field ϕ_a^b . Instead of (2.1) we now take:⁽¹⁵⁾

$$\begin{aligned}
 V_a^b - \frac{1}{3} \delta_a^b V_c^c &= i(\phi_a^d \phi_d^b) + \dots \\
 P_a^b - \frac{1}{3} \delta_a^b P_c^c &= F_\pi \partial \phi_a^b - \frac{2}{3} \frac{1}{F_\pi} (\phi_a^d \phi_d^e \partial \phi_e^b + \partial \phi_a^d \phi_d^e \phi_e^b) \\
 &\quad + \frac{4}{3} \frac{1}{F_\pi} \phi_a^d \partial \phi_d^e \phi_e^b + \dots
 \end{aligned} \tag{4.1}$$

where higher order terms which don't contribute to $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ in the tree approximation have been deleted. Using (4.1) in the octet form of (1.1) and in (1.4) gives the four $\Delta I = 1/2$ $K \rightarrow 2\pi$ amplitudes:

$$\begin{aligned}
 T(K_1 \rightarrow \pi^+ \pi^-) &= T(K_1 \rightarrow \pi^0 \pi^0) = F_\pi G_X [m^2(K) - m^2(\pi)] \\
 T(K_2 \rightarrow \pi^+ \pi^-) &= T(K_2 \rightarrow \pi^0 \pi^0) = -\sqrt{2} \alpha F_\pi G [m^2(K) - m^2(\pi)].
 \end{aligned} \tag{4.2}$$

The T-amplitudes in (4.2) are related to the widths and lifetimes by

$$\begin{aligned}
 \Gamma(K \rightarrow 2\pi) &\equiv \tau^{-1}(K \rightarrow 2\pi) = \frac{1}{N!} \rho_{2\pi} |T|^2 \\
 \rho_{2\pi} &= \frac{1}{16\pi m(K)} \left[1 - 4 \frac{m^2(\pi)}{m^2(K)} \right]^{1/2}
 \end{aligned} \tag{4.3}$$

where N is the number of identical final state pions.

For the computation of $K \rightarrow 3\pi$ in the tree approximation one needs in addition to (4.1) the strong 4-point pseudoscalar meson vertex function. This is given by eq. (33) of Cronin's paper⁽¹³⁾ with the choice $a_3 = \frac{4}{3}$. Carrying out the calculation yields a result which is linear in the energy of the "odd pion" and which by assumption satisfies the $\Delta I = 1/2$ rule. Just these two statements are sufficient to let us parametrize all eight $K \rightarrow 3\pi$ amplitudes in terms of two complex numbers, A and B . We may then write on general grounds:

$$\begin{aligned}
T(K^+ \rightarrow \pi^+ \pi^0 \pi^0) &= A + B(K \cdot P_+) \\
T(K^- \rightarrow \pi^- \pi^0 \pi^0) &= A^* + B^*(K \cdot P_-) \\
T(K^+ \rightarrow \pi^+ \pi^+ \pi^-) &= 2A - Bm^2(K) - B(K \cdot P_-) \\
T(K^- \rightarrow \pi^- \pi^- \pi^+) &= 2A^* - B^*m^2(K) - B^*(K \cdot P_+) \\
T(K_2^+ \rightarrow \pi^+ \pi^- \pi^0) &= -\frac{1}{2}(A+A^*) - \frac{1}{2}(B+B^*)(K \cdot P_0) \\
T(K_1^+ \rightarrow \pi^+ \pi^- \pi^0) &= \frac{i}{2}(A-A^*) + \frac{i}{2}(B-B^*)(K \cdot P_0) \\
T(K_2^+ \rightarrow \pi^0 \pi^0 \pi^0) &= -\frac{3}{2}(A+A^*) + \frac{1}{2}(B+B^*)m^2(K) \\
T(K_1^+ \rightarrow \pi^0 \pi^0 \pi^0) &= \frac{3i}{2}(A-A^*) - \frac{i}{2}(B-B^*)m^2(K)
\end{aligned} \tag{4.4}$$

where K is the K -meson four-momentum and each P is the appropriate "odd pion" four momentum. (Eq.(4.4)) can be simply derived by writing out the most general $\Delta I = 1/2$ effective Lagrangian for $K \rightarrow 3\pi$ allowing two derivatives.) The explicit phenomenological Lagrangian calculation gives for A and B :

$$\begin{aligned}
A &= \left(-\frac{\bar{X}}{\sqrt{2}} + ia - 2ia' \right) Gm^2(K) \\
B &= \left(-\frac{2}{\sqrt{2}} X + 4ia - 6ia' \right) G
\end{aligned} \tag{4.5}$$

Note that A and B would be real in the limit of CP conservation. The T -amplitudes in (4.4) are related to the widths and lifetimes by

$$\Gamma(K \rightarrow 3\pi) = \tau^{-1}(K \rightarrow 3\pi) = \frac{1}{N!} \frac{1}{64\pi^3 m(K)} \int \int d\omega_1 d\omega_2 |T|^2 \tag{4.6}$$

where ω_1 and ω_2 are the energies of two of the three final pions and N is the number of identical final pions.

In the calculations of this section we have not yet taken into account the modifications of the amplitudes due to strong final state

interactions. Since we are using a nonleptonic interaction which satisfies the $\Delta I = 1/2$ rule the $K \rightarrow 2\pi$ amplitudes of (4.2) are simply modified according to the final state interaction theorem by multiplying them by $e^{i\delta_0}$, where δ_0 is the $I=0$ s-wave $\pi\pi$ phase shift evaluated at the CM energy equal to $m(K)$. A realistic treatment of final state effects⁽¹⁶⁾ in the $K \rightarrow 3\pi$ processes is far more complicated. Thus we shall rely on the phenomenological prescription of Barrett and Truong⁽¹⁷⁾ which tells us to multiply the $I=1$ symmetric part of the three pion momentum space wave function by $e^{i\bar{\delta}_0}$ where⁽¹⁸⁾ $\bar{\delta}_0 \approx 20^\circ$ represents some average value of δ_0 in the appropriate energy range. The decay modes for which CP violating effects in our model can most readily be checked are the $\tau^\pm(K^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp)$ and $\tau'^\pm(K^\pm \rightarrow \pi^\pm \pi^\pm \pi^0)$. The appropriate modifications of (4.4) for these modes are as follows:

$$\begin{aligned}
 T(K^+ \rightarrow \pi^+ \pi^0 \pi^0) &= [A - \frac{1}{3}Bm^2(K)]e^{i\delta_0} + \frac{1}{2}B(s-s_0) \\
 T(K^- \rightarrow \pi^- \pi^0 \pi^0) &= [A^* - \frac{1}{3}B^*m^2(K)]e^{i\bar{\delta}_0} + \frac{1}{2}B^*(s-s_0) \\
 T(K^+ \rightarrow \pi^+ \pi^+ \pi^-) &= [2A - \frac{2}{3}Bm^2(K)]e^{i\delta_0} - \frac{1}{2}B(s-s_0) \\
 T(K^- \rightarrow \pi^- \pi^- \pi^+) &= [2A^* - \frac{2}{3}B^*m^2(K)]e^{i\bar{\delta}_0} - \frac{1}{2}B^*(s-s_0)
 \end{aligned} \tag{4.7}$$

where $(s-s_0) = \frac{2}{3}m^2(K) + 2K \cdot P$ with the proper P for each case. Note that if the factor $e^{i\delta_0}$ were not present the quantities $|T|^2$ would not differ between the K^+ and K^- modes so there would be no detectable CP violating effects. The relevant experimental parameters for checking CP violation^(16,17) are the slope asymmetries, the slope in each case being given by the coefficient λ of the Dalitz parameter

$y[y = \frac{3t-Q}{Q}]$, t = odd pion kinetic energy and $Q = m(K) - 3m(\pi)$ in the expression:

$$|T|^2 = \lambda_0(1 + \lambda y + \dots) \quad (4.8)$$

From (4.7) and (4.5) we find for the slope asymmetries in the $\tau^\pm$ and $\tau'^\pm$ modes:

$$\frac{\lambda_+ - \lambda_-}{\lambda_+ + \lambda_-} = \frac{\lambda'_+ - \lambda'_-}{\lambda'_+ + \lambda'_-} = \frac{3\sqrt{2}}{\bar{X}} (a' - a) \tan \bar{\delta}_0 \quad (4.9)$$

It is amusing to note that the asymmetry given by (4.9) would vanish when the parity conserving, CP violating interaction is of the form (PP+VV), since then $a = a'$.

V. DECAY MATRIX AND OBSERVABLE $K \rightarrow 2\pi$ PARAMETERS

To make comparison with experiment for the neutral Kaon decays we must of course assemble the amplitudes and self-masses computed in the last three section to form the well known decay matrix.⁽¹⁹⁾ It is convenient for us to use the phase conventions given in (2.2) and the K_1 - K_2 basis rather than the K^0 - $\bar{K}^0$ basis and Wu-Yang⁽¹⁹⁾ phase conventions which are often used, but which are not really as well suited for model calculations. In the K_1 - K_2 space we then write the decay matrix as

$$\begin{pmatrix} M_{11} - \frac{i}{2} \Gamma_{11} & , & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12} - \frac{i}{2} \Gamma_{12} & , & M_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix} \quad (5.1)$$

where

$$M_{11} = m(K)$$

$$M_{22} = m(K) + \Delta, \quad [\text{see (2.8)}]$$

$$M_{12} = \text{Eq. (3.2)}$$

$$\Gamma_{11} = \rho_{2\pi} |T(K_1 \rightarrow \pi^+ \pi^-)|^2 + \frac{1}{2} \rho_{2\pi} |T(K_1 \rightarrow \pi^0 \pi^0)|^2 + \dots [\text{see (4.3)}]$$

$$\Gamma_{22} = \frac{1}{64\pi^3 m(K)} \iint d\omega_+ d\omega_- |T(K_2 \rightarrow \pi^+ \pi^- \pi^0)|^2 + \dots [\text{see (4.6)}]$$

$$\begin{aligned} \Gamma_{12} = & \rho_{2\pi} [T(K_2 \rightarrow \pi^+ \pi^-) T^*(K_1 \rightarrow \pi^+ \pi^-) + \frac{1}{2} T(K_2 \rightarrow \pi^0 \pi^0) T^*(K_1 \rightarrow \pi^0 \pi^0)] \\ & + \frac{1}{64\pi^3 m(K)} \iint d\omega_+ d\omega_- [T(K_2 \rightarrow \pi^+ \pi^- \pi^0) T^*(K_1 \rightarrow \pi^+ \pi^- \pi^0) + \\ & + \frac{1}{6} T(K_2 \rightarrow \pi^0 \pi^0 \pi^0) T^*(K_1 \rightarrow \pi^0 \pi^0 \pi^0)] + \dots \end{aligned}$$

Note that (5.1) is a symmetric matrix and that the off-diagonal elements would be zero in the limit of CP conservation. Hence we will work in the approximation where the off-diagonal elements are first order in smallness compared to the diagonal ones. Then it is an easy matter to find the eigenstates of (5.1), corresponding to particles of definite lifetime:

$$\begin{aligned} |K_S\rangle &= |K_1\rangle + \omega |K_2\rangle \\ |K_L\rangle &= -\omega |K_1\rangle + |K_2\rangle, \end{aligned} \quad (5.2)$$

where the small but crucial quantity ω is given by

$$\omega = \frac{M_{12} - \frac{i}{2} \Gamma_{12}}{(M_{11} - M_{22}) - \frac{i}{2} (\Gamma_{11} - \Gamma_{22})} \quad (5.3)$$

Noting that $\Gamma_{11} \approx \Gamma_S \gg \Gamma_L \approx \Gamma_{22}$ we can, for all practical purposes, approximate (5.3) by

$$\omega \approx \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{-\Delta - \frac{i}{2}\Gamma_S} \quad (5.4)$$

The denominator of (5.4) involves the experimental quantities Δ and Γ_S ⁽¹⁰⁾ $\Gamma_S = (7.63 \pm .06) \times 10^{-15}$ GeV, while the objects appearing in the numerator are to be computed from our model. We are here assuming for simplicity that the CP violation in the K_L - K_S amplitudes involves only the 2π and 3π modes. Thus it will be convenient to resolve Γ_{12} into its 2π and 3π parts:

$$\Gamma_{12} \approx \Gamma_{12}^{(2\pi)} + \Gamma_{12}^{(3\pi)} \quad (5.5)$$

We now discuss the experimental CP violation parameters η_{+-} and η_{00} . Using (5.2) we may write

$$\eta_{+-} \equiv -i \frac{\langle \pi^+ \pi^- | T | K_L \rangle}{\langle \pi^+ \pi^- | T | K_S \rangle} \approx -i \left[\frac{\langle \pi^+ \pi^- | T | K_2 \rangle}{\langle \pi^+ \pi^- | T | K_1 \rangle} - \omega \right] \quad (5.6)$$

Noting that K_S decays almost entirely into the 2π modes and taking (4.2) into account gives

$$\frac{\langle \pi^+ \pi^- | T | K_2 \rangle}{\langle \pi^+ \pi^- | T | K_1 \rangle} \approx \frac{\Gamma_{12}^{(2\pi)}}{\Gamma_S} \quad (5.7)$$

Substituting this result as well as (5.4) and (5.5) into (5.6) then gives:

$$\eta_{+-} \approx \frac{-i}{\Delta + i\Gamma_S/2} \left[M_{12} + \Delta \frac{\Gamma_{12}^{(2\pi)}}{\Gamma_S} - \frac{i}{2} \Gamma_{12}^{(3\pi)} \right] \quad (5.8)$$

In an entirely similar way we find

$$\eta_{00} \equiv -i \frac{\langle \pi^0 \pi^0 | T | K_L \rangle}{\langle \pi^0 \pi^0 | T | K_S \rangle} \approx \eta_{+-} \quad (5.9)$$

The equality of η_{+-} and η_{00} as displayed in (5.8) and (5.9) is to be expected⁽²⁰⁾ since our nonleptonic interaction satisfies the $\Delta I = 1/2$ rule [this fact is essentially contained in (5.7)]. The deviation of the measured phase of η_{+-} from the phase of $-i/(\Delta + i\Gamma_S/2)$ gives an upper limit to the size of $\Gamma_{12}^{(3\pi)}$.

It is convenient to write here our theoretical predictions for the quantities $\Gamma_{12}^{(2\pi)}$ and $\Gamma_{12}^{(3\pi)}$ which appear in (5.8). From (5.1) and (4.2) we immediately find

$$\Gamma_{12}^{(2\pi)} = - \frac{3}{\sqrt{2}} \alpha F_\pi^2 G^2 \bar{X} [m^2(K) - m^2(\pi)]^2 \rho_{2\pi}, \quad (5.10)$$

where the 2-particle phase space factor $\rho_{2\pi}$ is given in (4.3). For the 3π contribution $\Gamma_{12}^{(3\pi)}$ we use the $K_{1,2} \rightarrow 3\pi$ amplitudes of (4.4) and (4.5) in the reasonable approximation where the y dependent [$y = \frac{3t-Q}{Q}$] terms are neglected. We also evaluate the phase space integral non-relativistically:

$$\iint d\omega_1 d\omega_2 \approx \frac{\sqrt{3}\pi Q^2}{18} \quad (5.11)$$

and finally obtain;

$$\Gamma_{12}^{(3\pi)} \approx \frac{5\sqrt{6}}{41,472\pi^2} G^2 \bar{X} m^3(K) Q^2 a \quad (5.12)$$

VI. MODELS OF CP VIOLATION

In the preceding sections we have considered the CP violating Lagrangian density [(1.1) with (1.3) imposed],

$$\mathcal{L}_{\text{viol.}} = iG \{ a P_2^c P_c^3 + a' \tilde{V}_2^c \tilde{V}_c^3 + \alpha \tilde{P}_2^c \tilde{V}_c^3 + \alpha' \tilde{V}_2^c \tilde{P}_c^3 \} + \text{h.c.} \quad (6.1)$$

$$\tilde{V}_a^b = V_a^b - \frac{1}{3} \delta_a^b V_c^c$$

$$\tilde{P}_a^b = P_a^b - \frac{1}{3} \delta_a^b P_c^c, \quad ,$$

and have given theoretical predictions for the experimentally accessible quantities

$$\omega \quad [\text{eq. (5.4)}]$$

$$\eta_{+-} \quad [\text{eqs. (5.8), (5.10), (5.12), and (3.2)}]$$

$$\eta_{00} \quad [\text{eq. (5.9)}]$$

$$\tau^{\pm} \text{ slope asymmetry } [\text{eq. 4.9}]$$

in terms of the four real parameters a , a' , α , and α' appearing in (6.1). All other parameters appearing in our expressions are to be considered known. The experimental information on ω comes from the $K\ell 3$ decays where we have (assuming the $\Delta I=1/2$ rule):

$$\begin{aligned} \text{Im } \omega &= -\frac{1}{2} \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \\ \Gamma_{\pm} &= \Gamma(K_L \rightarrow \ell^{\pm} \pi^{\mp} \nu) \end{aligned} \quad (6.2)$$

Many other decay modes could be calculated with our model but there is not enough experimental information yet... To proceed one could in principle equate our theoretical expressions to the appropriate experimental results and solve for the parameters a , a' , α , and α' . However, the experimental uncertainties are too large for this to be done in any meaningful way. Therefore to get a feeling for what kind of models are suitable we shall consider four cases in which each of a , a' , α , and α' is separately non-zero.

Case 1, only $a \neq 0$

This is a parity conserving interaction so $\Gamma_{12}^{(2\pi)} = 0$. Then from (5.8) and (5.9) we get

$$\begin{aligned}\eta_{+-} = \eta_{00} &= \frac{-i}{\Delta + i\Gamma_S/2} [M_{12} + \frac{i}{2} \Gamma_{12}^{(3\pi)}] \\ &\approx - (1.92 \times 10^{-14} \text{ GeV}^{-1}) e^{i43^\circ} [M_{12} + \frac{i}{2} \Gamma_{12}^{(3\pi)}].\end{aligned}\quad (6.3)$$

Noting that the experimental values ⁽¹⁰⁾ are

$$\begin{aligned}\eta_{+-} &= (1.96 \pm .03) \times 10^{-3} e^{i(43 \pm 3)^\circ} \\ \eta_{00} &= (2.09 \pm .12) \times 10^{-3} e^{i(43 \pm 19)^\circ}\end{aligned}\quad (6.4)$$

we see that the predicted phase will be correct within experimental error provided that $\Gamma_{12}^{(3\pi)}/M_{12}$ is negligible. This is in fact the case since substituting numbers into the ratio of (5.12) and (3.2) gives the prediction

$$\Gamma_{12}^{(3\pi)}/M_{12} \approx 7 \times 10^{-5} \quad (6.5)$$

Then the correct value of η_{+-} will be obtained by adjusting M_{12} which [see (3.2)] implies

$$a = 3.48 \times 10^{-4} \quad (6.6)$$

Note that this value of the dimensionless quantity a is about 1/5 the magnitude of η_{+-} . The extra suppression comes about because there is no large cancellation in our calculation of M_{12} while there is a large cancellation in our calculation of Δ . From (5.4) we predict

$$\text{Im } \omega \approx -\text{Re } \eta_{+-} = -1.43 \times 10^{-3} \quad (6.7)$$

which may be compared with the experimental result ⁽²¹⁾ from $K_{\mu 3}$ decays [see (6.2)]

$$\text{Im } \omega = (-1.39 \pm 0.25) \times 10^{-3} \quad (6.8)$$

The equality of (6.7) and (6.8) is not surprising since our formula for ω [eq. (5.4)] in the present case is the same as the one in the superweak theory. Finally the $\tau^\pm$ slope asymmetry is predicted to be [eq. (4.9)]

$$\frac{\lambda_+ - \lambda_-}{\lambda_+ + \lambda_-} \approx -(1.48 \times 10^{-3}) \tan \bar{\delta}_0 \approx -(5.4 \times 10^{-4}) . \quad (6.9)$$

The experimental value⁽²²⁾ is.

$$\frac{\lambda_+ - \lambda_-}{\lambda_+ + \lambda_-} = (-7.0 \pm 5.3) \times 10^{-3} \quad (6.10)$$

The experimental uncertainty associated with (6.10) is still too large to permit any firm conclusion to be drawn; in particular there is not a "three standard deviation" from zero which is a rule of thumb for concluding that there is any effect at all. Our predicted asymmetry is in fact 1/10 the experimental uncertainty. Improvement of the experimental accuracy here (a difficult job) would be an interesting test of case 1. In sum, the model of case 1 agrees with present experimental data to the same extent as the superweak theory.

Case 2, only $a' \neq 0$

Eq. (3.2) and (5.12) show that this term neither contributes to M_{12} nor significantly to $\Gamma_{12}^{(3\pi)}$. Then (6.3) shows that $\eta_{+-} = \eta_{00} = 0$. Thus this case is not suitable by itself. It does however contribute to the $\tau^\pm$ slope asymmetry [see the discussion after (4.9)] and can thus modify this quantity without other effects if used in conjunction with case 1.

Case 3, only $\alpha \neq 0$

This is a parity violating term so $\Gamma_{12}^{(3\pi)} = 0$ and there is no predicted $\tau^\pm$ slope asymmetry. Furthermore (3.2) shows that $M_{12} = 0$ so (5.8) becomes

$$\begin{aligned} \eta_{+-} = \eta_{00} &= \frac{-i}{\Delta + i\Gamma_S/2} \frac{\Delta\Gamma_{12}^{(2\pi)}}{\Gamma_S} \\ &\approx -(1.92 \times 10^{14} \text{ GeV}^{-1}) e^{i43^\circ} \frac{\Delta\Gamma_{12}^{(2\pi)}}{\Gamma_S} \end{aligned} \quad (6.11)$$

Reference to (6.4) shows that the predicted phase angle is correct. The magnitude of η_{+-} will be correct if $\Gamma_{12}^{(2\pi)}$ is properly adjusted. This requires [see eq. (5.10)]:

$$\alpha \approx 2.39 \times 10^{-3} \quad (6.12)$$

which has about the same magnitude as η_{+-} . Finally (5.4) predicts,

$$\begin{aligned} \text{Im } \omega &= -\frac{\Gamma_S}{2\Delta} \text{Im } \eta_{+-} \\ &= -1.43 \times 10^{-3} \end{aligned} \quad (6.13)$$

which agrees with (6.8). Note however that the form of (6.13) differs from the form of (6.7). Thus the present case also gives an acceptable model for CP violation. It is amusing that it gives the same results as the superweak theory even though all CP violating effects are in the amplitude rather than the mass matrix.

Case 4, only $\alpha' \neq 0$

In this case $\Gamma_{12}^{(2\pi)} = \Gamma_{12}^{(3\pi)} = 0$ so we have, for the processes being considered here, a perfect imitation of the superweak theory.

Specifically, there is no $\tau^\pm$ slope symmetry and

$$\eta_{+-} = \eta_{00} = \frac{-iM_{12}}{\Delta + i\Gamma_S/2} \quad (6.14)$$

$$\text{Im } \omega = -\text{Re } \eta_{+-}$$

Adjusting M_{12} to give the proper magnitude of η_{+-} yields [see (3.2)]:

$$\alpha' = -2.97 \times 10^{-4} \quad (6.15)$$

This value is about 1/10 the magnitude of η_{+-} and the reason for this extra suppression is discussed after (6.6).

VII. DISCUSSION

i. It is interesting that three out of the four models we have considered reproduce the satisfactory results of the superweak theory within experimental uncertainty. Thus a current-current form for the CP violating weak interaction seems fairly plausible. In fact, improvement of the experimental accuracy in τ -decay measurements by one order of magnitude and high precision measurements of other weak decays seem like exciting prospects for testing such theories.

ii. Although we have for simplicity considered four special models our formulas can be applied to any $\Delta S = 1$ octet current-octet current model. To make a specific correspondence of a particular model with our Lagrangian in (6.1) it is only required to resolve the current-current product of that model into its irreducible SU(3) representations and to pick up the octet part. (Hence assuming the non-octet part of the current-current product to be dynamically suppressed.) Thus the present paper may be useful now that there is renewed interest in weak interactions because of the unified gauge schemes.

iii. In the present paper we have only taken nonleptonic CP violation into account (except for making use of some experimental information on $K_{\ell 3}$ decays). This is because there is as yet no hard evidence for CP violation in the leptonic and semi-leptonic modes apart from that due to K_1 - K_2 mixing effects. In the papers of ref. 1, the hadronic CP violating terms were considered to be proportional to certain phases appearing⁽²³⁾ in semi-leptonic decays. These phases can of course be estimated in the present approach; they will be of the order 10^{-4} to 10^{-3} [unless some large cancellations are postulated] and hence about one order of magnitude below the present accuracy of β -decay experiments.

iv. The use of a pure octet form for $\mathcal{L}_{\text{viol}}$ is reasonable both because dynamical suppression of non-octet components appears to exist and because it enforces the experimentally accurate result $\eta_{+-} = \eta_{00}$ [see the discussion leading to (5.9)]. One might attempt to use the present phenomenological Lagrangian approach to perform a calculation in which octet dominance is not assumed. However, we do not feel that such an attempt is plausible by the following argument. One often considers the CP conserving nonleptonic interaction to be of the form $(J_1^3 J_2^1 + \text{h.c.})$ [see (1.5)], and this interaction yields a nonvanishing amplitude for $K_1 \rightarrow \pi^0 \pi^0$ by the current algebra method. However if we consider the approximation in which the J 's are given either by (2.1) or (4.1) we will get zero for this amplitude in the tree approximation. This is because J_1^3 only contains terms like K^+ or $K^0 \pi^+$ and no terms like K^0 or $K^0 \pi^0$. On the other hand $K_1 \rightarrow \pi^+ \pi^-$ will be present. The addition of the terms $(J_2^3 J_2^2 + J_3^3 J_2^3 + \text{h.c.})$ for example to the above interaction

converts it into a pure octet and makes available the needed field operators to give $K_1 \rightarrow \pi^0 \pi^0$. Since the $\Delta I = 1/2$ rule requires equal amplitudes for $K_1 \rightarrow \pi^+ \pi^-$ and $K_1 \rightarrow \pi^0 \pi^0$, it would be grossly violated for the CP conserving modes in the present method of calculation unless we assume it a priori. That is another reason why we restricted ourselves to the octet form.

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