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RANGE-MOMENTUM CORRELATION IN CARBON AND IRON
FOR MUONS OF 500-1500 MeV/c

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MASTER

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ABSTRACT

The range and straggling of positive and negative muons in a stopping medium composed principally of lead, graphite and steel has been investigated experimentally with data from a Bevatron spectrometer. It is concluded that the Bethe-Bloch formula is capable of representing the range behavior of muons between 500 MeV/c and 1500 MeV/c within $\pm 0.3\%$. Values of the mean excitation energies of 79 ± 7 eV for graphite and 287 ± 13 eV for iron are obtained.

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The measurement of range is a potentially very accurate technique for determining particle energies, at least where the energy loss is dominated by atomic excitation processes. For muons in particular this extends from below 100 MeV to tens of GeV. The use of such a technique for calibrating a beam line has recently been described¹, but range has more frequently been used in conjunction with momentum measurement as a very sensitive way to discriminate specific particles from background, for example, in branching ratio measurements.^{2,9}

The basic theory of energy loss by ionization, developed over many years, centers around the Bethe-Bloch theory³, with a correction for polarization described by Sternheimer and others.⁴

$$-\frac{1}{\rho} \frac{dE}{dx} = \frac{2\pi n e^4}{m c^2 \rho \beta^2} \left[\ln \left(\frac{2m p^2 W}{I^2 \mu^2} \right) - 2\beta^2 - \delta - U \right]$$

ρ, x = density, thickness (cms.) of stopping material

n = no. of electrons per cm^3 in material

Z, A = atomic number, atomic weight

e, m = electronic charge, mass

$\beta = v/c$ = particle velocity

p, μ = particle momentum, mass

W = maximum kinetic energy transferred to an electron at rest

I = mean excitation energy of stopping medium

δ = density effect correction⁴

U = correction for electronic shell effects⁵

The effects of nuclear excitation, bremsstrahlung and pair production are negligible for this study and will not be considered.

The theory is dependent on the parameters I which must be measured

experimentally. Values for these have in general been obtained by low energy studies of protons where the shell corrections and multiple scattering are significant. High energy measurements, which should be comparatively free of this difficulty, have been few in number ⁶, but have tended to disagree with earlier values.

Several sets of range-energy tables have been compiled based on available information⁷. Most recently the range in lead of muons of momentum 1.2 - 2.5 GeV/c has been compared with calculated values ^{8, 12}. The measurements had a stated accuracy of about 2%. Most other experiments in the high energy region have been concerned with establishing within a few percent the energy loss plateau caused by the density effect ⁴, and have used various techniques of energy loss measurement.

The data available in this study were the ranges of a spectrum of muons between about 500 and 1500 MeV/c, stopping in an absorber principally of iron, but having passed through a significant thickness of graphite. The effective ionization potentials of these materials have been evaluated.

Apparatus

The muons came from part of the large body of $K^0 \mu^3$ events collected during a search conducted at the Bevatron for the elusive decay mode $K_L^0 \rightarrow \mu^+ \mu^-$ ⁹. The apparatus has already been described in some detail. ¹⁰

The plan view in Fig. 1 illustrates the principle of the experiment which was to accept both particles from charged two body decays of transverse momentum 190-250 MeV/c, and bend them roughly parallel to the beam, one on either side of the apparatus. Hodoscopes were placed to trigger spark chambers on such occurrences.

High resolution in the kinematic reconstruction of events was very important in the experiment. For this reason the apparatus was designed to

present minimal scattering thicknesses to particles within the symmetric pair of spectrometers. Trajectories were defined on each side by five wire spark chambers, each with four coordinate planes read out magnetostrictively. The wire spacing and distance between chambers were such that angular resolution was comparable with the multiple scattering angle ~ 0.5 mr.

The line integrals of the bending fields were quite uniform. Both fields were mapped in three components on a 2 inch cubic grid, and their currents held constant to within $\pm 0.03\%$ during running. An error of 0.2% in overall calibration is considered possible and is incorporated in the uncertainties discussed below.

Particle identification was assisted by Freon 12 Cherenkov counters for electrons, and by range measurement to separate muons from pions. One of the two similar range telescopes is represented in Fig. 2. The cross-sectional dimensions were 120 cms. square. An initial thickness of 43 g cm^{-2} of lead and 160 g cm^{-2} of graphite was intended to minimize electron and pion penetration. Behind this the 17 scintillators were separated by varying numbers of steel plates so that the average range interval corresponded approximately to a 7% change in muon momentum.

DATA REDUCTION

In the analysis of the data, $K^0 \mu^3$ candidates were easily recognized by making use of a loose range-momentum correlation. For this investigation, which was motivated by the necessity to examine the range-momentum relationships of background $K_L^0 \rightarrow \mu^+ \mu^-$ candidates, some of the $K^0 \mu^3$ data were abstracted in histogram form. The two sides of the apparatus were taken independently, but muons of both signs were treated together. Since the spectrometer fields were reversed systematically during data collection, a check could

be made of the equality of ranges of μ^+ and μ^- as far as the overall uncertainties involved in this study. A charge dependent discrepancy is reported in detail elsewhere.¹³ A 0.1% correction is necessary to the Bethe-Bloch formula of opposite sign for μ^- and μ^+ . When μ^- and μ^+ are taken together the effect is averaged out. Another small effect is, however, expected to increase the apparent ranges of μ^- . The atomic cascade X-rays from μ^- absorption could cause signals in the next scintillator on about 1.2% of occasions, increasing the average apparent range by about 0.05%, negligible as far as this work is concerned.

Since discrete range intervals were employed, the investigation used the momentum spectra for events stopping in each interval. The spectra were in 2.5 MeV/c bins below a range of 10 counters -- about 900 MeV/c -- and in 5 MeV/c bins above this. As well as the muon peak there was in each case a background at higher momentum which extended into the muon region. Originating from pions, these events amounted to a 4% subtraction from events in the muon region at the lowest ranges, decreasing to, for example, 2.4% for range 8 (760 MeV/c). Subtraction was done empirically by extrapolation, and an uncertainty estimated by performing the analysis without subtraction. The possible error was taken arbitrarily to be $\pm 0.5 X$ (the difference between the two analyses).

Because the total muon spectrum had a broad peak at about 1.0 GeV/c (Fig. 5) the individual spectra for each range showed an assortment of skewed shapes. It was felt, however, that each one could be adequately represented by its mean momentum and R.M.S. width.

These parameters were then compared with theoretical values obtained from a simulation of the apparatus.

As an estimate of the errors on the mean momenta, the uncertainty from the background subtraction was combined with the statistical error. In the case of the widths, the various peaks were truncated at 10%, 20% and 30% amplitudes, and the widths of all three cases corrected to that of the full

curves, assuming them to be gaussian. The error assigned to a peak was made to cover all three cases. The R.M.S. of the errors from all peaks of approximately the same width - left and right - was calculated and applied to all estimated widths within the group.

Range Calculations

A trajectory stepping program was written to calculate the range and straggling¹¹ of muons passing through all the layers of material behind the spark chambers (See Appendix). Standard expressions were used throughout^{3,4,5} with K and L shell corrections, the density effect and the effect of multiple scattering included. Corrections to the various published density effect parameters were made graphically to correspond to the materials used. The small differences in thickness of the absorber on the two sides were allowed for by treating the two sides separately.

Apart from the effective ionization potential parameters of carbon and iron, the significant uncertainties were in the density of the graphite, determined only to $\pm 0.25\%$, a possible miscalibration of the momentum discussed below, and uncertainty of $\pm 0.1\%$ in the total thickness and density of the iron.

The stepping program was made to give the values of momenta which corresponded to stopping points exactly on the boundary between discrete range intervals. These momenta and their derivatives with respect to the uncertain parameters could be obtained for any values of these parameters.

Using the experimental momentum spectrum and the range - momentum relations from the previous step, the distribution of muons among the range intervals was simulated numerically. The momentum resolution, discussed below, and range straggling were taken into account.

The mean momentum and the R.M.S. deviation from it were evaluated for each range interval since these numbers corresponded to the experimental data available.

Comparison with Experiment

Since the corresponding range bites on each side were very closely the same, the widths of the corresponding momentum spectra could be averaged. The results of the simulation program were then compared against these as in Fig. 3. The momentum dependent theoretical straggling width determined from the stepping program was permitted to have a simple free valued factor of the form.

$$\sigma_{\text{empirical}} = \sigma_{\text{theoretical}} (1 + A).$$

It was found that a straggling width $(13.0 \pm 2.0)\%$ greater than theoretical gave the best fit to the data.

Using the simulation which fitted the widths best, the experimental values of mean momentum were compared to those from the stepping and simulation programs (Fig. 4).

A minimum χ^2 search was carried out to establish the best values of I_C and I_{Fe} , allowing for uncertainty in the density of the carbon, the density and thickness of iron, and the momentum calibration.

The values obtained were

$$I_C = 79 \pm 7 \text{ eV.}$$

$$I_{Fe} = 287 \pm 13 \text{ eV.}$$

Uncertainties

The uncertainties inherent in the analysis can be considered in several groups. The first is the question of counter performance in the range telescopes. The efficiency of these counters was continuously monitored from the data by looking for signals missing from the trail of long range muons. Inefficiencies were in the range of a small fraction

of one per cent and were insignificant. Random coincidence rates were determined continuously by several techniques and averaged about 0.9% for the data used here. This would on average increase the mean ranges by approximately 0.06% and the apparent range straggling width by 3%.

Length measurements of the various layers penetrated had an uncertainty of $\pm 0.1\%$. The densities of two of the major components were determined by samples; lead to $\pm 0.25\%$ and steel to $\pm 0.1\%$. The carbon density was determined for each block by the vendor with an uncertainty of $\pm 0.25\%$.

The other uncertainty was in the momentum measurement. For the analysis of the large numbers of events accepted in the experiment the momenta were calculated by means of a simple procedure involving interpolation in a table of line integrals determined from theoretical trajectories through the field maps.

Apart from normal surveying techniques the relative alignment of the chambers in the two spectrometers was frequently checked by straight tracks taken with the magnets off. Since the magnet currents were frequently reversed the effect of remanent fields at zero currents was eliminated.

The possibility of a small miscalibration in the current monitoring system has been noted above. There was, however, a continuous calibration from the reconstruction of $K_L^0 \rightarrow \pi^+ \pi^-$ data. Momenta, positions and direction vectors of the two pions were, of course, adequate to reconstruct the mass, trajectory and momentum of the parent particle.

For the data used in this work, the corresponding $K_L^0 \rightarrow \pi^+ \pi^-$ invariant mass peak was slightly shifted from the tabulated K_L^0 mass. This could result from errors in momentum measurement or in the opening angle caused by an error in the relative positions of the spark chambers on the two sides.

Additionally a momentum error in one magnet could be partially compensated by an error in the opposite sense in the other.

The last point was checked by reversing the magnetic fields. The resulting small shifts in the mass and mean K_L^0 direction were consistent with an irreversibility in one of the fields of 1 in 10^3 (Fig. 6).¹⁶

From the offset in the experimental invariant mass spectrum a correction of + 0.2% is necessary to the momenta. Allowing for conservative estimates of the opening angle uncertainties and the non-reversibility effect, a possible fractional uncertainty on the momenta of $\pm 0.2\%$ is assumed.

The question of momentum resolution was raised above. Because of the combined function of the front chambers in measuring opening angle and momentum, the resolution in invariant mass was not strongly dependent on the angular resolution of these chambers. Taking this into account, the observed momentum independent standard deviation of $1.45 \text{ MeV}/c^2$ in the invariant mass peak can be used to estimate the momentum resolution. An analysis indicates that the momentum resolution was $\pm 0.6\%$ changing only slightly with momentum. It is of interest that, for a sample of the data, the simple momentum calculation was replaced by step-by-step integration through the fields. The invariant mass resolution improved to $\pm 1.0 \text{ MeV}/c^2$, and the momentum resolution to $\pm 0.43\%$.

Conclusion

The range-energy relation seems capable of representing experiment to within 0.3% in the energy range 500 - 1500 MeV (Fig. 4). For carbon and iron the effective excitation energies were measured to be

$$I_C = 79 \pm 7 \text{ eV}; \quad I_{Fe} = 287 \pm 13 \text{ eV}.$$

The value recommended in a National Research Council¹⁵ report for iron is

$I_{Fe} = 286$, while comparatively high velocity measurements⁶ of I_c yield results from 77 ± 3 eV to 90 ± 5 eV. High energy measurements of iron do not seem to have been reported.

The density effect calculated by the formula of Sternheimer accounts for a 2.9% correction to range at 700 MeV/c and 5% at 1500 MeV/c and is reasonably well substantiated by these results.

On the other hand it has not been possible to account for the width of the distributions. Allowing for the widening effect of random coincidences and the $\mu^- - \mu^+$ range difference the number quoted above still indicates an observed straggling width $(7.5 \pm 3)\%$ greater than theoretically expected. This could be represented by an extra independent smearing effect in range of $\pm 1.1\%$ compared to the expected straggling of about $\pm 2.8\%$. The effect may partially be caused by non-gaussian energy loss distributions, and an underestimate of the pion background within the errors quoted above.

Acknowledgments

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C. I = 0 s wave

The I = 0 s-wave amplitude is parametrized in terms of a 2×2 M-matrix; ¹⁴ we assume that only the $\pi\pi$ and $K\bar{K}$ channels are important. As noted earlier data from other channels indicate that this is a reasonable assumption. ³

$$\text{Set } T_0^0 = \begin{pmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{pmatrix},$$

$$\begin{aligned} \text{where } T_{11} &= \pi^+ \pi^- \rightarrow \pi^+ \pi^- \text{ s-wave amplitude,} \\ T_{12} &= \pi^+ \pi^- \rightarrow K\bar{K} \text{ s-wave amplitude,} \\ T_{22} &= K\bar{K} \rightarrow K\bar{K} \text{ s-wave amplitude.} \end{aligned}$$

These amplitudes are normalized so that

$$\sigma_{ij} = 4\pi k^2 |T_{ij}|^2.$$

In terms of the M-matrix,

$$\begin{aligned} T &= k^{1/2} (M - ik)^{-1} k^{1/2}, \\ k &= \text{diagonal matrix of momenta.} \end{aligned}$$

Explicitly, T is given by

$$T = \frac{1}{D} \begin{pmatrix} k_1 (M_{22} - ik_2) & -\sqrt{k_1 k_2} M_{12} \\ -\sqrt{k_1 k_2} M_{12} & k_2 (M_{11} - ik_1) \end{pmatrix}, \quad (\text{IV.11})$$

where

$$D = (M_{11} - ik_1) (M_{22} - ik_2) - M_{12}^2,$$

$$k_1 = \pi \text{ momentum in } \pi\pi \text{ c.m. system,}$$

$$k_2 = K \text{ momentum in } K\bar{K} \text{ c.m. system.}$$

```

PROGRAM RANGER(INPUT,OUTPUT)
COMMON/REGV/RHO(10),H1W(10),V1(10),V1Q(10),COL(10),ADL(10),EMDL(10)
$ 1,X0(10),X1(10),WK(10),WL(10),RL(10),Z(10),AA(10)
000002 INTEGER Z
000002 COMMON IREG,ISYP,DIAG,PIN,JDG
000002 COMMON WEL,WELQ,W,WQ,CX
$ ,DXEZ
$ ,DIFE
000002 COMMON/SEQNC/THK(110),NSEQ(110)
000003 W=105.66
000004 WQ=W*W
000004 WEL=0.511
000006 WELQ=WEL*WEL
000006 CONST=2*3.14159*2.81794**2*0.511*1.0E-3
C READ NO. MATERIALS AND REGIONS
000013 READ 5,MAT,NODL
000022 5 FORMAT(2I4)
C DATA ON MATERIALS
000022 DO 10 K=1,MAT
000024 READ 1,KSSS,RHO(K),H1W(K),V1(K),COL(K),ADL(K),EMDL(K),X0(K),X1(K),
$ WK(K),WL(K),RL(K),Z(K),AA(K)
000063 1 FORMAT(110,F10.7,F4F10.4,F10.5)
000063 IF(KSSS.NE.K) PRINT 2,KSSS
000073 2 FORMAT(* MATERIAL *I4* OUT OF ORDER*)
000073 V1Q(K)=V1(K)**2
000075 10 CONTINUE
000100 DO 15 K=1,MAT
000101 PRINT 6,K , RHO(K),H1W(K),V1(K),COL(K),ADL(K),EMDL(K),X0(K),X1(K)
$ ,WK(K),WL(K),RL(K),Z(K),AA(K)
000140 6 FORMAT(* I/P CONSTS. FOR MATERIALS */3X,110,F10.5/3X,4F10.5,
$ /4,F10.5)
000140 15 CONTINUE
C INPUT STEP SIZE ,TYPE
000143 DO 11 IREG=1,NODL
000143 C TYPE, THICKNESS(INCHES)
000144 READ 3,NSEQ(IREG),THK(IREG)
000153 3 FORMAT(I3,F12.5)
000153 THK(IREG)=THK(IREG)*2.54
000156 11 CCNTINUE
000160 TTK=0.0
000160 WTK=0.0
000161 DO 21 J=1,NODL
000164 ITP=NSEQ(J)
000165 TTK=TTK+THK(J)
000167 WTK=WTK+THK(J)*RHO(ITP)
000171 PRINT 403,J,THK(J),NSEQ(J),RHO(ITP),RL(ITP),TTK,WTK
000213 403 FORMAT(* THK TYPE DENS R.L. TOT. THK */7,3X,13,5X,F10.4,16,4F10.4)
$ 4)
000213 21 CONTINUE
C THICKNESS, TYPE, DENS, RAD. LENGTH, TOTAL THICKNESS TO DATE IN CR, ORR.
C NO. OF MON CONSIDERED
000216 READ 7,NUP
000223 7 FORMAT(I3)
000223 PIRE=451.1 $ STRE=186.0
000226 PIRE=0.0 $ STRE=0.0
000227 DO 31 KUP=1,NUP
000231 PRINT 404
000234 404 FORMAT(1* *)

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```

C MON, (NEV), SCATT. PROJECTION PLANE (INCHES)
000234 READ 4, PIN, PLN1
000244 * FORMAT(2F10.5)
000244 PLN1=PLN1*2.54
000246 PMU=P/IN
000247 THQ=0.
000247 ZTOT=0.
000250 ZTOTR=0.
000251 SXPJQ=0
000251 SAMQ=0.0
000252 THQ=0.0
000252 TMQ1=0.0
000253 STREQ=0.0 $ STRG=0.0
000254 FOK=0.
000255 DO 101 IREG=1,NOBL

C CALC. STEP SIZE
000257 ITP=NSQI(IREG)
000260 A=THK(IREG)*.0
000262 B=THK(IREG)*RHQ1(ITP)/(RL(ITP)*0.25)
000265 KA=A+.1
000270 K1=B+.1
000273 NSTP=MAX(D1KA,K1)
000277 ZSTP1=THK1(IREG)/NSTP
000301 NSTP1=ZSTP1/RHQ1(ITP)
000303 CX=5*ZSTP1*H1N1(ITP)
000305 AMC2=11*ITP*1131(ITP)+13/AA1(ITP)
000313 AMC1=0.39*SQRT(AMC2)
000316 DO 110 ISTEP=1,NSTP
000321 ZPROJ=PMU*(1-ZSTP1)*0.5

C H.S. RANGE EFFECT USE LAST STEPS SCATT. ANGLE
000324 ZSTP=ZSTP1/(1-0.5*THQ1)
000326 NSTP=NSTP1/(1-0-0.5*THQ1)
000331 RLST=NSTP/RL(ITP)
000333 DM1=DP(PMU,ITP,NSTP)
000337 FOK=FOK*DIFE
000340 STREQ=STREQ*DM2
000342 IF(DM1-GE-PMU) GOTO 114
000345 PNEAN=PMU-DM1*0.5
000346 DM2=DP(PNEAN,ITP,NSTP)
000351 IF(DM2-GE-PMU) GOTO 114
000354 PNE=PNE-DIFE
000355 ENEM=SQRT(PNE**2+MG)
000362 OTNM=PNE/ENEM
000363 PNEAN=PMU-DM2*0.5
000366 EMEAN=SQRT(PNEAN**2+MG)
000373 BEAT=PNEAN/EMEAN
000374 ZTOT=ZTOT+ZSTP1
000376 ZTOTR=ZTOT+NSTP1
000400 AMC=AMC1+SQRT(NSTP1)/(BEAT*PMU)
000405 SNOR=ELQAT(ZTOT,ITP)
000410 AM=4.49E-3*SNOR*0.3333*SQRT(1.+3.35*SNOR**2/(137013*BEAT**2))
000425 BMQ=3.7*2.25*ALD(GTOT,AMC,AMC/YAMA*ANAT)
000436 IF(BMQ-LE-0.7) BMQ=0.7001
000440 SIG=AMC*SQRT(BMQ-G.T1/1.404)
000446 THQ1=THQ1+SIG*SIG
000450 IF(ZPROJ-LE-0.1) GOTO 111
000453 THQ=THQ+SIG*SIG

```

CHOOSE SUITABLE STOPPING
POTENTIAL ON THICKNESS OF
MATERIAL AND RADIATION LENGTH

PARAMETERS USED IN MULT. SCATT.
CALCULATION

1- CALL OF SUBROUTINE - INCL. SUBROUTINE
2- M.P. DIFFUSION
STRAGGLING
3- ESTIMATE MEAN PENETRATION IN STOP
4- EVALUATION OF ENERGY - LAST TRANSMISSION

MULT. SCATT.
CALCULATIONS

```

000455 XHQ=SIG*SIG*ZPJQ*ZPJQ
000456 SXHQ= SXHQ*XHQ
000457 THQI=225*RLST*EHEAN*2/PHEAN*4
000463 THQI=THQI*THQI
000465 XPJQ=THQI*ZPJQ**2
000466 SXPJQ= SXPJQ*XPJQ
000470 111 CONTINUE
000470 PMU=PMU*PHEW
000471 IF(PHEW-LY.30.) GOTO 112
000474 110 CONTINUE

```

```

000476 PRINT 401,I,REG,PMU
000506 601 FORMAT( ' END REGION *16* MOMENTUM *F12.3)

```

```

000506 101 CONTINUE
000511 GOTO 113
000511 114 CONTINUE
000511 PRINT 701
000515 701 FORMAT( ' OUT AT PGE30*)
000519 112 CONTINUE
C ACORNDOP CORRECTION "HODS" N.B.
  ZFIN=0.0134*PMU/15.1**3.55*EL.0*(1TP/6.0)**0.3333
  ZFIN=ZFIN*(1+ZFIN)
  ZTOT=ZTOT+ZFIN
  ZTOT=ZTOT+ZFIN
  PMU=0.0
  113 CONTINUE
  STRE=SQRT(STREQ)
  STRP=SQRT(PIN**2+MQI*STRE/PIN
  FACP=STRP/ZIN
  OPBOX=PIN-PIREN 8 DRBOX=ZTOTR-STREN
  OPSGN=POKE*SQRT(PIN**2+MQI/PIN
  PRGN=OPSGN/PIEN
  ORGN=OPSGN*DRBOX/OPBOX
  PRGN=ORGN/ZTOTR
  STRG=STRP*DRBOX/OPBOX
  PRCG=STRG/ZTOTR
  PIREN=PIN 8 STREN=ZTOTR
C PRINT INITIAL STATUS
  PRINT 401
  401 FORMAT( '///* INITIAL CONDITIONS*)
  PRINT 402, PIN
  402 FORMAT( ' INITIAL MOM *F10.4)
C PRINT FINAL STATUS
  SXPJ=SQRT(SXPJQ)
  TH=SQRT(THQ)
  TX=SQRT(THQ)
  SX=SQRT(SXQ)
  PRINT 501
  501 FORMAT( ' FINAL CONDITIONS*)
  PRINT 501,ISTP
  505 FORMAT( ' NO. STEPS IN FINAL REGION *16)
  PRINT 502,PMU,ZTOT,I,REG
  502 FORMAT( ' FINAL MOM *F10.3 * CH5. TRAVELLED *F10.3* REGION *16)
  PRINT 504, ZTOTR
  504 FORMAT( ' RMS TRAVELLED *F12.5)

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```

000655 PRINT 503,TH,SXPJ,PLNI
000667 503 FORMAT( ' SCATT. T: *F10.5* DX *F10.5* PLANE Z= *F10.4)
000667 PRINT 504,TH,SEN
000677 504 FORMAT( ' GAUSSIAN APPROX TO MOLLIERE * 2E12.4)
000677 PRINT 505,STRE,STRP,STRG
000711 505 FORMAT( ' 1 ST DEV STRAGGLE T, P, RG *3E12.4)
000711 PRINT 506,PRCP,PRCG
000721 506 FORMAT( ' FRACT. STRAGGLE P, RG *2F10.4)
000721 PRINT 2000,POKE,OPSGN,ORGN
000733 2000 FORMAT( ' HUS- DIFFES. KE, P, RGE *3E12.4)
000755 PRINT 2001,PRGN,PRGN
000743 2001 FORMAT( ' HUS- FRACT.DIFFS P, RGE * 2E12.4)
000743 31 CONTINUE
000746 STOP
000750 END

```

```

000005 FUNCTION DP,ITP,MSTP
COMMON/REGH/RND(10),M1(10),V1(10),V1Q(10),COL(10),ADL(10),ENOLF(10)
5 1,X(10),X1(10),XK(10),ML(10),RL(10),Z(10),AA(10)
000015 INTEGER Z
000005 COMMON /REG,ISTP,DIAG,PM,JOG
000005 COMMON /REL,MEL0,M2,MQ,CX
5,DK2
5,DIFF
000005 COMMON/SEQUENCE/FNK(110),NSEQ(110)
000005 WMQ=105.66
000006 E=SQRT(P**2+MQ)
000020 BET=P/E
000021 BETQ=BET*BET
000022 WMAX=P/P/(M*(M*0.5/MEL+MEL*0.5/M*E/M1)
000030 XPL=24MEL*BETQ*WMAX/(V1Q(11P)*(1-BETQ))*1.0ELZ
C K,L SMOEL CORRECTIONS
000037 ETK=1.88E4*BETQ/(12*(ITP-0.3)**2*(1-BETQ))
000045 ETL=1.88E4*BETQ/(12*(ITP-4.15)**2*(1-BETQ))
000053 CK=XK(11P)/ETK*7.32/ETK**2
000056 CL=ML(11P)/ETL*1.5/ETL**2
000062 U=2*(CK+CL)/Z(11P)
C DENSITY EFFECT --AL,FE,CU,PB TREATED SEPARATELY
000065 DEL=0.0
000067 XDL=ALOG(DP/M)
000072 IF(XDL.LT.XQ(11P)) DEL=0
000101 IF(XDL.GE.XQ(11P)) DEL=XDL-LE.X1(11P) DEL=0.606*XDL-COL(11P)
5 XDL(11P)=X1(11P)-XDL1**ENOLF(11P)
000123 IF(XDL.GT.X1(11P)) DEL=XDL-5.96*XDL-COL(11P)
000131 GOTO 14
C AL,FE,CU,PB HERE
000132 YF=P/M
000133 ***** DP
000133 IF(ITP.NE.3) GOTO 11
000135 IF(P.LT.3*M) DEL=0.1025*BETQ*(P/M*0.11**1.225
000144 IF(P.GT.3*M) DEL=DLNF+1.0/(1.8+0.00914*P/P/(M*M))
000154 IF(P.GE.3*M.AND.P.LE.30*M) DEL=DLNF+10.0**(-0.045-0.771*ALOG10
5 (P/M)-1))
000221 11 CONTINUE
000221 IF(ITP.NE.4) GOTO 12
000223 IF(P.LT.4*M) DEL=0.0443*BETQ**4**1.75
000234 DLNF=ALOG(1/Y**4)-4.3
000241 IF(P.GT.100*M) DEL=DLNF+1.0/(1.80+0.000333*Y**4)
000253 IF(P.GE.4*M.AND.P.LE.100*M) DEL=DLNF+10.0**(-0.043-C.730*ALOG10/Y
5 -1)-0.189*ALOG10/Y)-1)**2)
000305 12 CONTINUE
000305 IF(ITP.NE.7.AND.ITP.NE.10) GOTO 13
000314 IF(Y.LT.5) DEL=0.0095*BETQ*(Y**0.05)**2.33
000325 DLNF=ALOG(1/Y**4)-4.1
000332 IF(Y.GT.25) DEL=DLNF+1.0/(1.0+0.00287*Y**4)
000343 IF(Y.GE.5.AND.Y.LE.25) DEL=DLNF+10.0**(-0.024-0.866*ALOG10(Y)-1)
5 -1)-0.367*ALOG10(Y)-1)**2)
000373 13 CONTINUE
000373 IF(ITP.NE.2) GOTO 14
000375 IF(Y.LT.6.3) DEL=0.016*BETQ**4**1.83
000405 DLNF=ALOG(1/Y**4)-6.17
000412 IF(Y.GT.25) DEL=DLNF+9.88/(SQRT(Y)*(1.42+0.0063*Y))
000427 IF(Y.GE.6.3.AND.Y.LE.25) DEL=DLNF+8.1/(Y**0.508)
C 14 CONTINUE
000430 THIS IS DE/OX PER GH.
000430 DEOX=CX*(ALOG(XP1)-2*BETQ-DEL-U)/BETQ
000460 DEZ=1.022CX*(1.-0.5*BETQ)*MSTP/(11-BETQ)*1.+1.022/(WMQ*SQRT(
5 11-BETQ))
C DIAGNOSTIC CARD NO *****
000501 IF(DEX.GE.01 GOTO 101
000503 DEX=20.
000504 PRINT 100
000507 100 FORMAT(= NEGATIVE DEX=)
000511 PRINT 102,P,BET,WMAX,XPL,ETK,ETL,U,DEL,DEDX
000527 102 FORMAT(3X,F12.3)
000541 101 CONTINUE
C
000543 DIF=CX*0.314159*2*MSTP/BET/37.04
000546 DEOX=DEX*MSTP
000550 DP=DEOX/BET
000551 RETURN
000551 END

```

TO OLD JORDAN'S DENSITY EFFECT
PARAMETERIZATION, REMOVE THIS CARD

ALPHABET

IRON

COPPER/BRASS

LEAD

8.87E-06 MCM FORMULA
STRAUSS

8.87E-06 MCM FORMULA

I/P	CONSTS.	FOR MATERIALS							
	1	1.75000	3.00960	82.00000	3.13000	.48100	2.72000	-.05000	CARBON
	2.00000	2.00650	.10000	42.40000	6	12.01100			
I/P	CONSTS.	FOR MATERIALS							
	2	11.20000	2.38374	811.00000	6.18000	.34500	2.66000	.40000	LEAD
	3.00000	2.12000	2.00000	6.52000	82	207.19000			
I/P	CONSTS.	FOR MATERIALS							
	3	2.70000	2.90160	156.00000	4.12300	.05700	3.90000	.05000	ALUMINUM
	3.00000	2.08000	1.50000	24.00000	13	26.98100			
I/P	CONSTS.	FOR MATERIALS							
	4	7.84600	2.80280	290.00000	4.24000	.10140	3.41200	.01000	IRON
	3.00000	2.10000	1.85000	13.90000	26	55.84700			
I/P	CONSTS.	FOR MATERIALS							
	5	1.03000	3.26100	63.80000	3.15000	.42900	2.85000	.13000	SCINTILLATOR
	2.00000	2.00650	.10000	43.40000	6	11.80000			
I/P	CONSTS.	FOR MATERIALS							
	6	1.20000	3.24900	69.09000	3.21000	.45600	2.78000	.14000	LUCITE
	2.00000	2.00650	.10000	40.30000	6	12.10000			
I/P	CONSTS.	FOR MATERIALS							
	7	8.96000	2.74920	320.00000	4.42000	.10800	3.39000	.20000	COPPER
	3.00000	2.10000	1.85000	12.00000	29	63.54000			
I/P	CONSTS.	FOR MATERIALS							
	8	.00129	3.00930	94.00000	10.70000	.12600	3.72000	1.87000	AIR
	4.00000	2.00700	.10000	36.50000	7	14.00000			
I/P	CONSTS.	FOR MATERIALS							
	9	.00633	3.00000	131.70000	0.	0.	1.00000	1000.00000	FROM 12 (APPROX.)
	9000.00000	0.	0.	27.20000	12	24.00000			
I/P	CONSTS.	FOR MATERIALS							
	10	8.48900	2.74300	332.45000	4.49000	.11100	3.39000	.20000	BRASS
	3.00000	2.10000	1.85000	12.00000	29	63.54000			

LIST OF LAYERS OF MATERIALS IN ORDER.

THICKNESS (CMS.), TYPE (NUMBERED AS IN INPUT CONSTANTS ABOVE), DENSITY, RADIATION LENGTH, SPINDED THICKNESS

IN CMS. AND GR.

THK	TYPE	DENS	R.L.	TOT.	THK				
1		2.0574			5	1.0300	43.4000	2.0574	2.1191
THK	TYPE	DENS	R.L.	TOT.	THK				
2		15.2400			8	.0013	36.5000	17.2974	2.1388
THK	TYPE	DENS	R.L.	TOT.	THK				
3		220.9800			9	.0063	27.2000	238.2774	3.5376
THK	TYPE	DENS	R.L.	TOT.	THK				
4		.9525			6	1.2000	40.3000	239.2259	4.6806
THK	TYPE	DENS	R.L.	TOT.	THK				
5		15.2400			8	.0013	36.5000	254.4699	4.7002
THK	TYPE	DENS	R.L.	TOT.	THK				
6		3.3274			5	1.0300	43.4000	257.7973	8.1275
THK	TYPE	DENS	R.L.	TOT.	THK				
7		5.0800			8	.0013	36.5000	262.8773	8.1340
THK	TYPE	DENS	R.L.	TOT.	THK				
8		1.2700			2	11.2000	6.5200	264.1473	22.3580
THK	TYPE	DENS	R.L.	TOT.	THK				
9		29.6545			1	1.7500	42.4000	293.8018	74.2534
THK	TYPE	DENS	R.L.	TOT.	THK				
10		2.5400			2	11.2000	6.5200	296.3418	102.7014
THK	TYPE	DENS	R.L.	TOT.	THK				
11		62.8142			1	1.7500	42.4000	359.1560	212.6262
THK	TYPE	DENS	R.L.	TOT.	THK				
12		5.0800			8	.0013	36.5000	364.2360	212.6328
THK	TYPE	DENS	R.L.	TOT.	THK				
13		1.9583			5	1.0300	43.4000	366.1943	214.6499
THK	TYPE	DENS	R.L.	TOT.	THK				
14		2.6632			4	7.8460	13.9000	368.8575	235.5453
THK	TYPE	DENS	R.L.	TOT.	THK				
15		.6350			8	.0013	36.5000	369.4925	235.5461
THK	TYPE	DENS	R.L.	TOT.	THK				
16		1.9583			5	1.0300	43.4000	371.4509	237.5632
THK	TYPE	DENS	R.L.	TOT.	THK				
17		2.6010			4	7.8460	13.9000	374.0518	257.9703
THK	TYPE	DENS	R.L.	TOT.	THK				
18		.6350			8	.0013	36.5000	374.6869	257.9711
THK	TYPE	DENS	R.L.	TOT.	THK				
19		1.9583			5	1.0300	43.4000	376.6452	259.9882
THK	TYPE	DENS	R.L.	TOT.	THK				
20		2.5463			4	7.8460	13.9000	379.1915	279.9669
THK	TYPE	DENS	R.L.	TOT.	THK				
21		.6350			8	.0013	36.5000	379.8265	279.9677
THK	TYPE	DENS	R.L.	TOT.	THK				
22		1.9583			5	1.0300	43.4000	381.7849	281.9848
THK	TYPE	DENS	R.L.	TOT.	THK				
23		2.5486			4	7.8460	13.9000	384.3335	301.9814
THK	TYPE	DENS	R.L.	TOT.	THK				
24		.6350			8	.0013	36.5000	384.9685	301.9822
THK	TYPE	DENS	R.L.	TOT.	THK				
25		1.9583			5	1.0300	43.4000	386.9268	303.9993
THK	TYPE	DENS	R.L.	TOT.	THK				
26		2.5466			4	7.8460	13.9000	389.4734	323.9800
THK	TYPE	DENS	R.L.	TOT.	THK				

END REGION	1	MOMENTUM	589.324
END REGION	2	MOMENTUM	589.187
END REGION	3	MOMENTUM	586.630
END REGION	4	MOMENTUM	584.346
END REGION	5	MOMENTUM	584.309
END REGION	6	MOMENTUM	577.396
END REGION	7	MOMENTUM	577.334
END REGION	8	MOMENTUM	560.289
END REGION	9	MOMENTUM	466.200
END REGION	10	MOMENTUM	432.419
END REGION	11	MOMENTUM	224.448
END REGION	12	MOMENTUM	214.434
END REGION	13	MOMENTUM	219.746
END REGION	14	MOMENTUM	182.418
END REGION	15	MOMENTUM	182.416
END REGION	16	MOMENTUM	177.159
END REGION	17	MOMENTUM	134.127
END REGION	18	MOMENTUM	134.125
END REGION	19	MOMENTUM	127.228
END REGION	20	MOMENTUM	31.070
END REGION	21	MOMENTUM	31.029

INITIAL CONDITIONS
 INITIAL MOM 593.5000
 FINAL CONDITIONS
 NO. STEPS IN FINAL REGION 1
 FINAL MOM 0. CMS. TRAVELLED 380.071 REGION 22
 GMS TRAVELLED 280.21986
 SCATT. TH .12124 DX 7.86615 PLANE Z= 363.2200 CM. FROM START. 11
 GAUSSIAN APPROX TO MOLLIERE 1.0345E-01 6.9391E+00
 1 ST DEV STRAGGLE T, P, RG 1.2342E+01 1.2536E+01 8.0627E+00
 FRACT. STRAGGLE P, RG .0211 .0288
 MU+- DIFFS. KE. P, RGF 1.0435E+00 1.0599E+00 6.8173E-01
 MU+- FRACT. DIFFS P, RGE 1.7859E-03 2.4328E-03

NOTE- GAUSSIAN MULT. SCATT. :

15 PROTECTED SCATT. DX (cm.)

AT X

B- MOLLIERE MULT. SCATT. FOR DX
 B- STRAGGLING IN N.E., MOLLIERE, RANGE

Footnotes and References

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16. The effect shows up in Fig. 4 as an offset between the ranges on left and right sides.

Figure Captions

- Fig. 1 - Plan view of apparatus. F, R are vertical scintillation counter hodoscopes, H is a six-counter array and T are fast timing counters.
- Fig. 2 - Elevation view of Cherenkov counter and range telescope.
- Fig. 3 - Experimental and predicted R.M.S. widths of momentum distributions for events stopping in the 14 useful range intervals.
- Fig. 4 - Fractional deviation of experimental from theoretical means of the 14 useful momentum distributions on right (top) and left.
- Fig. 5 - Muon momentum spectrum after background subtraction.
- Fig. 6 - Reconstructed kaon origin (at target) and mass from dipion data for positive (top) and negative magnet currents.
The peaks of the distributions have about 5000 events per bin.

Fig. 1

XBL 702-316

Fig. 2

XBL 724-677

XBL 724-676

Fig. 4

XBL 724-675

Fig. 5

MAGNET CURRENT POSITIVE

MAGNET CURRENT NEGATIVE

XBL 724-674