STRUCTURAL ANALYSIS OF FARET CORE SUPPORTS
UNDER MECHANICAL AND THERMAL LOADINGS

by

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STRUCTURAL ANALYSIS OF FARET CORE SUPPORTS UNDER MECHANICAL AND THERMAL LOADINGS

by

M. M. Chen

ABSTRACT

A structural analysis was made (Sept 1963) of the core support grid of the Fast Reactor Test Facility (FARET). This structure consists of three circular plates which form high- and low-pressure plenums for achieving a coolant flow distribution. These plates are perforated for insertion of the fuel rods. The weight of the fuel assemblies and the coolant pressures in the plenums constitute the principal mechanical loading of the core support grid. In addition, thermal loadings arise because of the temperature distributions in the grid.

A general solution has been developed for the stress analysis of a core support plate which is subjected to these mechanical and thermal loadings for any rotationally restrained edge conditions. The resulting deflections and stresses were computed for various loading conditions by use of the IBM-704 computer. For the thermal analysis, a rotationally symmetric temperature distribution was assumed, varying linearly through the thickness of the plate. The analysis was based on classical small-deflection theory and the concept of the "equivalent solid plate" of O'Donnell and Langer. The solution was applied to a model which simulates the FARET core support subjected to a combined mechanical and thermal loading.

I. INTRODUCTION

The Fast Reactor Test (FARET) program is an experimental program directed toward the demonstration of breeder reactors which are essential to the long-range goal of utilizing fertile fuels. The facility is planned and designed for carrying out experimental work involving both reactor engineering performance, and reactor dynamics and safety. The areas of investigation in engineering performance will be those involving fuel and fuel-clad concepts and designs, high power densities and high temperatures, high fission atom burnup, and system operating experience expected in an actual power breeder. The dynamics and safety investigations
will include the determination of nuclear characteristics in connection with the stability and safety of large dilute systems, and the observation of reactivity effects due to the change in temperature of fuel, sodium, and clad. The temperature changes will simulate the actual operating conditions of a fast breeder. Detailed descriptions of some well-planned experiments are given in Ref. 1.

Because of the particular objectives of the experimental programs, three separate cores are needed for the FARET reactor, namely, (1) a zoned core, (2) a medium core, and (3) a small core. The basic plan of the reactor core is shown in Fig. 1, and the possible core sizes, excluding the reflectors, are shown in Table 1. Inside each core three types of subassemblies (fuel, control and reflector) comprise the static loading, each of which conforms to a common design subassembly. Each subassembly is hexagonal in shape; the outer dimension is 2.29 in. across flats (see Fig. 2). The weight of each subassembly is 150 lb, and the total static load of a full complement of subassemblies is about 90,000 lb. All the subassemblies are maintained in a lattice spacing of 2.32 in. (see Fig. 3). The holes are distributed symmetrically with respect to the center of the plate and are arranged in an equilateral pattern.

![Fig. 1 Core Section](image)

**Table 1**

**CORE GEOMETRIES**
(Approximate)

<table>
<thead>
<tr>
<th>Core Parameter</th>
<th>Small Core</th>
<th>Medium Core</th>
<th>Zoned Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (liters)</td>
<td>40</td>
<td>400</td>
<td>1400</td>
</tr>
<tr>
<td>Diameter (cm)</td>
<td>44</td>
<td>95</td>
<td>150</td>
</tr>
<tr>
<td>Length (cm)</td>
<td>33</td>
<td>70</td>
<td>120</td>
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</table>
Fig. 2
Core Support Grid and Cylinder

Fig. 3
Notations for a Perforated Plate
The core support grid is formed by three perforated circular plates. In addition to supporting the fuel, control, and reflector subassemblies, it forms the high- and low-pressure plenums from which liquid sodium flows into the subassemblies to obtain a cooling effect. The grid structure itself is supported within the reactor vessel by a core support cylinder (see Fig. 2). The cylindrical liner, in turn, is supported by a flange at the reactor vessel wall, shown in Fig. 4. The grid-structure models chosen for the analysis which characterize the various possible designs are shown in Fig. 5.
The primary coolant enters into the lower (high-pressure) plenum through the annular space between the vessel and the core support cylinder. The upper (low-pressure) plenum receives the coolant from the lower plenum through orificed passages in some of the outer subassembly adaptors. The inlet pressures caused by the pumps may vary from a few psi to 85 psi in the lower plenum, and from a few psi to approximately 45 psi in the upper plenum. The maximum design pressure imposed by the planned experiments is 85 psi (see Table 2).

Table 2

PRELIMINARY DESIGN REQUIREMENTS OF FARET REACTOR

<table>
<thead>
<tr>
<th>Core</th>
<th>Flow Rate (gpm @1000°F)</th>
<th>Inlet and Outlet Pressure Change (psi @6000 gpm)</th>
<th>Inlet Temperature (°F)</th>
<th>Core Temperature Increase (°F)</th>
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<tr>
<td>Small</td>
<td>600-6000</td>
<td>85</td>
<td>600-1000</td>
<td>0-240</td>
</tr>
<tr>
<td>Medium</td>
<td>600-6000</td>
<td>85</td>
<td>300-400</td>
<td>0-100</td>
</tr>
<tr>
<td>Zoned</td>
<td>600-6000</td>
<td>85</td>
<td>300-400</td>
<td>0-100</td>
</tr>
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</table>
The inlet and outlet temperatures of the coolant vary according to flow rates and types of operation. For normal operation, with a heat removal capacity of 50 MW, the inlet and outlet temperatures are considered to be 600°F and 840°F, respectively. For the highest mode of operation, the inlet temperature may be 1050°F and occasionally rise as high as 1200°F. The maximum temperature increase may be as much as 300°F. A temperature of 1200°F is currently considered as the upper limit for the construction materials of interest, such as stainless steels. The physical properties of these materials have been used in the numerical examples of this study.

The temperature requirements imposed by experiments for various cores are also shown in Table 2. The changes in temperatures in the core support region, however, are expected to be transient in nature. This may be caused by a mismatch of the reactor power with the coolant flow rates in the primary and secondary systems. Typical temperature variations for the plates of the FARET model considered in this report will be shown in Section III.

In Section II, a bending analysis is made of the perforated plates of a core support based on the design requirements of static, internal pressure, and thermal loading conditions discussed above.
II. ANALYSIS

Nomenclature

\[ W \] lateral deflection of plate midsurface
\[ h \] plate thickness
\[ a \] plate radius
\[ p \] laterally applied pressure
\[ T \] temperature distribution, Eq. (3)
\[ \alpha \] coefficient of linear thermal expansion

\[ E, E^* \] Young's modulus and effective Young's modulus of the perforated plate depending on ligament efficiency,\(^{(3)}\) respectively

\[ \nu, \nu^* \] Poisson's ratio and effective Poisson's ratio of the perforated plate depending on ligament efficiency,\(^{(3)}\) respectively

\[ D^* = \frac{E^*h^3}{12[1 - \nu^*^2]} \] flexural rigidity of the perforated plate\(^{(3)}\)

\[ \sigma_r \] Radial stress
\[ \sigma_\theta \] Circumferential stress
\[ \sigma_r^\prime \] Radial membrane stress
\[ \sigma_\theta^\prime \] Circumferential membrane stress

\[ M_r \] Radial bending moment per unit circumferential length
\[ M_\theta \] Circumferential bending moment per unit circumferential length
\[ M_t \] Thermal moment per unit circumferential length

\[ k \] Rotational restraint factor

\[ A_n, B_n \] Temperature constants (where \( n = 0, 1, \text{ etc.} \))

\[ \nabla^2 \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}, \text{ Laplacian operator} \]

\[ r, z \] radial and axial coordinates, respectively

The calculation of stresses and deflections of a reactor core support system is one of the major undertakings during the preliminary stage of a reactor design. The three perforated plates of the core support structure considered in this report will be designated as the top, middle, and bottom grid plates (see Fig. 6). The perforations of these plates are in an equilateral triangular pattern so that the "equivalent plate theory" of O'Donnell and Langer\(^{(3)}\) is applicable.
The static load due to the various subassemblies is applied only to the top plate, whereas internal pressures and thermal gradients are exerted on all three plates. The boundary conditions are such that the deflections must vanish and that the angles of rotation may vary on the boundary in an arbitrary manner. In the following, a bending analysis for a perforated circular plate under prescribed loads is developed. The classical linear theory is used.

The governing differential equation\(^{(4)}\) for the bending of circular plates made of an isotropic material of uniform thickness subjected to a temperature gradient and a lateral pressure is extended to the perforated case as follows:

\[
D^* \nabla^2 \nabla^2 W = p - \frac{1}{1 - \nu^*} \nabla^2 M_t
\]

(1)

where

\[
M_t = \alpha E^* \int_{-h/2}^{h/2} T_z \, dz.
\]

(2)

The coordinate system used is shown in Fig. 6.

The temperature is assumed to vary linearly through the thickness of the plate and to be axially symmetric. It is assumed that the temperature distribution can be expressed by a convergent power series:
where the $A_n$'s are constant and $A_n r^n$ has the same dimension as temperature. The radial temperature distribution at the midsurface of the plate is given by $T_0(r)$. It is seen from Eq. (3) that the temperature decreases as $z$ increases if the $A_n$ have positive values. The terms containing $A_n r^n$ represent the temperature difference between the upper and lower faces for various values of $r$ (Fig. 7). It is to be noted from Eq. (2) that the thermal moment vanishes when temperature variations with $z$ are symmetrical with respect to the midsurface of the plate. The solution for membrane stresses due to thermal forces has been found previously. (4, 5) The determination of the combined bending and membrane stresses for the core support model will be given in Section III.

The mathematical boundary conditions for a simply supported or a clamped plate are generally expressed by the following equations:
\[
\frac{dW(0)}{dr} = 0; \quad W(a) = 0; \quad M_r(a) = 0 \quad (4a, 4b, 4c)
\]

for simply supported plates, and

\[
\frac{dW(0)}{dr} = 0; \quad W(a) = 0; \quad \frac{dW(a)}{dr} = 0 \quad (5a, 5b, 5c)
\]

for clamped plates. However, neither of the limiting conditions, Eq. (4c) or Eq. (5c), is encountered in practice. For example, the clamped condition at the edge is rarely realized because no practical mounting can be built for a plate support with infinite rigidity. The question as to whether the plate is simply supported or clamped depends upon the relative stiffness of the plate and its support, and the means of joining. The exact degree of restraint requires experimental verification.

In this report an edge rotation restraint factor, \(k\), is introduced such that the angular displacement at the edge of the plate can be taken to be anywhere between the limiting cases of Eqs. (4c) and (5c). This restraint factor along with the energy method has been used in Ref. 6 to analyze the EBR-II reactor grid structure. The boundary conditions for a rotationally restrained plate can be stated as follows:

\[
\frac{dW(0)}{dr} = 0; \quad W(a) = 0 \quad (4a, 4b)
\]

for all edge conditions, and

\[
\frac{dW(a)}{dr} = 0 \quad \text{for } k = 1; \quad M_r(a) = 0 \quad \text{for } k = 0, \quad (6)
\]

where \(0 \leq k \leq 1\). For the simply supported case \(k = 0\), and for the fully restrained edge \(k = 1\).

Solution of Eqs. (1), (2), (3), (4a), (4b), and (6) yields

\[
\frac{W}{h} = \frac{3}{16} \frac{p}{E^*} \left(\frac{a}{h}\right)^4 (1 - \nu^2) \left\{ \left[ 1 + \frac{4(1-k)}{1+\nu^2} \right] - 2 \left[ 1 + \frac{2(1-k)}{1+\nu^2} \right] \left(\frac{r}{a}\right)^2 + \left(\frac{r}{a}\right)^4 \right\} + \frac{(1+\nu^2)}{2} \left(\frac{a}{h}\right)^2 \sum_{n=0}^{\infty} \frac{A_n a_n}{(n+2)^2} \left\{ n - \frac{2(n+2)(1-k)}{1+\nu^2} \right\} - (n+2) \left[ 1 - \frac{2(1-k)}{1+\nu^2} \right] \left(\frac{r}{a}\right)^2 + 2 \left(\frac{r}{a}\right)^{n+2}, \quad (7)
\]
provided the pressure is uniform. For a nonuniform lateral pressure, 
$p = p(r)$, a particular solution, $W_p$, may be obtained through the successive 
integration:

$$W_p(r) = \int^{r_1} \left( \int^{r_4} \left( \int^{r_3} \left( \int^{r_2} \frac{p(r_1)}{D*} dr_1 dr_2 dr_3 dr_4 \right) \right) \right). \quad (8)$$

The arbitrary constants for the general solution must be determined from 
the boundary conditions, Eqs. (4a), (4b), and (6).

If there is no thermal gradient, the solution of Eq. (7) for $k = 0$, and 
$k = 1$, reduces the deflections due to the mechanical load obtained in Ref. 7. 
Similarly, for the thermal gradient only, and for $k = 0$ and $k = 1$, Eq. (7) 
reduces to the solution obtained in Ref. 8.

The radial and circumferential bending moments are related to the 
lateral deflection and thermal moment as follows:

$$M_r = -D* \left( \frac{d^2W}{dr^2} + \frac{\nu*}{r} \frac{dW}{dr} \right) - \frac{M_t}{1 - \nu*}, \quad (9)$$

$$M_p = -D* \left( \frac{1}{r} \frac{dW}{dr} + \nu* \frac{d^2W}{dr^2} \right) - \frac{M_t}{1 - \nu*}. \quad (10)$$

The radial and circumferential stresses at the upper and lower faces of the 
plate are related to the bending moments:

$$\sigma_r = 6M_r/h^2; \quad (11)$$

$$\sigma_\theta = 6M_\theta/h^2. \quad (12)$$

The sign conventions of these quantities are indicated in Fig. 6.

Substitution of Eq. (7) into Eqs. (9) and (10) yields, respectively,

$$\overline{M_r} = \frac{1}{16} \left( \frac{P}{E*} \right) \left( \frac{a}{h} \right)^2 \left[ \frac{(3 + \nu* - 2k)}{(3 + \nu*)} \right]$$

$$- \frac{\nu}{12(1 - \nu*)} \sum_{n=0}^{\infty} \frac{A_n a^n}{(n + 2)} \left[ \frac{(1 - \nu* - 2k)}{(1 - \nu*)} \right] \left( \frac{a}{r} \right)^n; \quad (13)$$
\[ \overline{M_r} = \frac{1}{16} \left( \frac{p}{E^*} \right) \left( \frac{a}{h} \right)^2 \left[ (3 + \nu^* - 2k) - (1 + 3\nu^*) \left( \frac{r}{a} \right)^2 \right] \]

\[ - \frac{\alpha}{12(1 - \nu^*)} \sum_{n=0}^{\infty} \frac{A_n a^n}{(n+2)} \left[ (1 - \nu^* - 2k) - (n+1) (1 - \nu^*) \left( \frac{r}{a} \right)^n \right], \quad (14) \]

where \( \overline{M_r} \) and \( \overline{M_\theta} \) are nondimensional quantities:

\[ \overline{M_r} = M_r/(E^*h^2); \quad \overline{M_\theta} = M_\theta/(E^*h^2). \]

The temperature variation \( \Delta T \) is defined as \( A_n r^n \), and its graphical representation for \( n = 0 \) to \( n = 5 \) is shown in Fig. 7. The bending stresses can be obtained from Eqs. (11) to (14) for a given \( \Delta T \). The application of the solution obtained to the preliminary design of the FARET reactor grid plates is shown in the next section.
III. APPLICATION TO FARET REACTOR GRID PLATES

A. Core Model and Loading Conditions

A typical model of the FARET grid plates is shown in Fig. 8. The mid-surface is taken at \( z = 0 \) for each plate, and the same symbols as used in Section II are used for all three plates. The loading conditions due to the static load, internal pressure, and temperature variations are shown in Table 3.

![Figure 8](image-url)

**Fig. 8**

Coordinate System and Mechanical Loads for Model of FARET Grid Plates

<table>
<thead>
<tr>
<th>Plates</th>
<th>Static Load (lb)</th>
<th>( \Delta p ) (psi)</th>
<th>Temperature Distribution** (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>90 000*</td>
<td>-10</td>
<td>( B_0 ) \hline ( B_{1a} ) \hline ( A_0 ) \hline ( A_{1a} )</td>
</tr>
<tr>
<td>Middle</td>
<td>0</td>
<td>-75</td>
<td>25 \hline 30 \hline -50 \hline 10</td>
</tr>
<tr>
<td>Bottom</td>
<td>0</td>
<td>85</td>
<td>25 \hline 10 \hline -50 \hline 0</td>
</tr>
</tbody>
</table>

* The static load is assumed to be uniformly distributed over the entire plate.

** The temperature distribution is expressible as follows:

\[
T(r, z) = B_0 + B_1 r - \frac{z}{h} (A_0 + A_1 r) \tag{15}
\]

and in Figs. 8 and 9. It is seen from Table 3 that when \( n = 0 \) and \( n = 1 \), the term \( T_0(r) \) in Eq. (3) takes the following form:

\[
T_0(r) = B_0 + B_1 r, \tag{16}
\]

where \( B_0 \) and \( B_1 \) are constants, and \( B_1 r \) has the same dimension as temperature. The above postulated temperature distributions in the grid plates...
are based on the assumption that the coolant temperature at the inlet to the structure changes occasionally as much as 50°F during a fraction of a minute. Such a change in temperature might be caused by a mismatch of the reactor power with the coolant flow rates in the primary and/or secondary systems.

Fig. 9. Temperature Distributions for FARET Model

Assuming this rapid 50°F change, it was estimated that the thermal response of the grid plates would be slow enough to produce the z-direction temperature gradients used. The temperature gradient in the r-direction is based on the effects of coolant-transport time as well as energy losses from the coolant as it passes the structure, which is at a lower temperature. These temperature considerations are based on the small-zone core configurations; on the other hand, the static load is calculated for a full complement of 595 fuel elements acting on the top plate of the zone core. This corresponds to the highest thermal and static stresses, respectively.

B. Deformation of Grid Plates

It is seen from Eq. (7) that the lateral deflection due to a uniform temperature distribution across the plate, i.e., \( n = 0 \), and linear temperature distribution, i.e., \( n = 1 \), is determined by

\[
\frac{W}{h} = \frac{3}{16} \frac{p}{E^*} \left( \frac{a}{h} \right)^4 \left( 1 - \nu^2 \right) \left\{ 1 + \frac{4(1-k)}{1 + \nu^*} \right\} - 2 \left[ 1 + \frac{2(1-k)}{1 + \nu^*} \right] \left( \frac{r}{a} \right)^2 + \left( \frac{r}{a} \right)^4
\]

\[
- \frac{a_0 (1-k)}{2} \left[ 1 - \left( \frac{r}{a} \right)^2 \right]
\]

\[
+ \frac{(1+\nu^*)\alpha}{18} \left( \frac{a}{h} \right)^2 A_1 \left[ 1 - \frac{6(1-k)}{1 + \nu^*} \right] - \left[ 3 - \frac{6(1-k)}{1 + \nu^*} \right] \left( \frac{r}{a} \right)^2 + 2 \left( \frac{r}{a} \right)^3
\]

(17)
The maximum deflection occurs at the center \((r = 0)\) of the plate. The distribution of deflections due to the loadings as described in Table 3, for \(k = 0, 0.25, 0.5, 0.75,\) and \(1.0\), are plotted in Figs. 10 to 12 for the top plate only. It is to be noted that the lateral deflection of the bottom plate is caused by the internal pressure and \(A_0\) only, since \(A_1 = 0\) (See Table 3). This deflection can be obtained through interpolation by using the numerical results for the middle plate.

Fig. 10
Deflection Distributions for Top Plate, \(P = 30\) psi

Fig. 11
Deflection Distributions for Top Plate, \(\Delta T = -25^\circ F\)
In the calculations, the effective modulus of elasticity, $E^*$, was taken as $2.014 \times 10^6$ psi and the effective Poisson's ratio, $\nu^*$, as 0.605. These values correspond to Type 316 stainless steel at a service temperature of 1000°F, and were obtained from Fig. 21 (in Section C, below) for the given perforation of the plate. The coefficient of linear thermal expansion, $\alpha$, was taken as $10.36 \times 10^{-6}$ in./(in.)(°F). The internal pressure of 10 psi acting upward on the top plate (See Table 3) was not included in the computation. In the calculation of stress and displacement distributions, the thickness of the top, middle, and bottom plates were taken as 6, 4, and 4 in., respectively.

The total deflections for the top plate, including $A_0$, are plotted in Figs. 13 to 15. The deflections due to pressure and temperature were computed separately (see Figs. 10, 11, 13, and 14). The deflections due to the combined loadings are shown in Figs. 12 and 15. The maximum deflections of the grid plates are summarized in Table 4 for three types of support, namely, $k = 0$, 0.5, and 1.0.
Fig. 13. Maximum Deflection vs. Plate Thickness, 
$P = 30$ psi

Fig. 14. Maximum Deflection vs. Plate Thickness, 
$\Delta T = -25^\circ F$

Fig. 15

Maximum Deflection vs. Plate Thickness, Combined Pressure and Temperature Effects
Table 4

MAXIMUM DEFLECTIONS (in.) FOR GRID PLATES

<table>
<thead>
<tr>
<th>Plates</th>
<th>Type of Support</th>
<th>k = 0</th>
<th>k = 0.5</th>
<th>k = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{max}$</td>
<td>$h$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top (h = 6 in.)</td>
<td>$k = 0$</td>
<td>$7.57 \times 10^{-3}$</td>
<td>$4.25 \times 10^{-3}$</td>
<td>$9.22 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$k = 0.5$</td>
<td>$-5.57 \times 10^{-2}$</td>
<td>$-3.78 \times 10^{-2}$</td>
<td>$-1.99 \times 10^{-2}$</td>
</tr>
<tr>
<td>Bottom (h = 6 in.)</td>
<td>$k = 1.0$</td>
<td>$6.35 \times 10^{-2}$</td>
<td>$4.34 \times 10^{-2}$</td>
<td>$2.32 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

C. Stress Components

1. Bending Stresses

The radial and circumferential stress components can be found by substituting Eq. (15) into Eqs. (13) and (14) and then into Eqs. (11) and (12):

$$
\overline{\sigma}_r = \frac{3}{8} \frac{p}{E^*} \left( \frac{a}{h} \right)^2 \left[ (3 + \nu^* - 2k) - (3 + \nu^*) \left( \frac{r}{a} \right)^2 \right]
$$

$$
+ \frac{A_0 \zeta_k}{2(1 - \nu^*)} + \frac{\zeta A_1 a^2}{6} \left[ \frac{-1 + \nu^* + 2k}{(1 - \nu^*)} + \left( \frac{r}{a} \right)^2 \right]
$$

(18)

and

$$
\overline{\sigma}_\theta = \frac{3}{8} \frac{p}{E^*} \frac{a^2}{h} (3 + \nu^* - 2k) - (1 + 3\nu^*) \left( \frac{r}{a} \right)^2
$$

$$
+ \frac{A_0 \zeta_k}{2(1 - \nu^*)} + \frac{\zeta A_1 a^2}{6} \left[ \frac{-1 + \nu^* + 2k}{(1 - \nu^*)} + 2 \left( \frac{r}{a} \right)^2 \right],
$$

(19)

where $\overline{\sigma}_r = \sigma_r/E^*$ and $\overline{\sigma}_\theta = \sigma_\theta/E^*$ are nondimensional radial and circumferential stress components due to bending.

It is seen from Eqs. (18) and (19) that the constant thermal-stress term $A_0 \zeta_k/[2(1 - \nu^*)]$ depends on the uniform temperature $A_0$ only. In Figs. 16 and 17, the total distributions of the radial and circumferential stress components including the constant term are plotted for the top plate. In Figs. 18 to 20, the stresses at the center of the plate and the radial circumferential stresses at the plate edge are plotted versus plate thickness for $p = 30$ psi and $\Delta T = -25^\circ F$. 
Fig. 16
Radial Stress Distributions, Top Plate

Fig. 17
Circumferential Stress Distributions, Top Plate
Fig. 18. Stress at Center vs. Plate Thickness

Fig. 19. Radial Stress at Edge vs. Plate Thickness

Fig. 20
Circumferential Stress at Edge vs. Plate Thickness
2. Membrane Stress Components

The radial and circumferential membrane stresses are due only to the temperature distribution $T_0(r)$, or $BQ$ and $B_1r$, for the present case. Since there is no external force applied to the plane at the boundary, the membrane stress components are:

$$
\sigma_r' = \alpha E^* \left( \frac{1}{a^2} \int_0^a T_0 r \, dr - \frac{1}{r} \int_0^r T_0 r \, dr \right) \tag{20}
$$

and

$$
\sigma_\theta' = \alpha E^* \left( - T_0 + \frac{1}{a^2} \int_0^a T_0 r \, dr + \frac{1}{r} \int_0^r T_0 r \, dr \right), \tag{21}
$$

where primes are used to denote the membrane stress. Substitution of Eq. (16) into Eqs. (20) and (21) yields

$$
\sigma_r' = \frac{\alpha B_1 a}{3} \left( 1 - \frac{r}{a} \right) \tag{22}
$$

and

$$
\sigma_\theta' = \frac{\alpha B_1 a}{3} \left[ 1 - 2 \left( \frac{r}{a} \right) \right], \tag{23}
$$

where $\sigma_r' / E^*$ and $\sigma_\theta' / E^*$ are nondimensional radial and circumferential membrane stresses. The numerical results for typical loadings are shown in Tables 5 and 6. It is seen from these tables that the membrane stresses are comparable with the bending stresses in the top.

<table>
<thead>
<tr>
<th>Table 5</th>
<th>RADIAL STRESS DISTRIBUTION FOR A TYPICAL LOADING CONDITION (k 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r/a$ 0.2 0.4 0.6 0.8 1.0</td>
</tr>
<tr>
<td>A. Top Plate (h = 6 in.)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_r' \times 10^4$</td>
<td>1.293 1.094 0.777 0.518 0.259 0 0</td>
</tr>
<tr>
<td>$\sigma_\theta' \times 10^4$</td>
<td>6.523 6.193 5.376 4.031 2.576 2.204 0</td>
</tr>
<tr>
<td>Sum x 10^4</td>
<td>7.815 7.287 6.153 4.589 2.835 2.770 0</td>
</tr>
<tr>
<td>B. Middle Plate (h = 4 in.)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_r' \times 10^4$</td>
<td>1.036 0.829 0.622 0.414 0.207 0 0</td>
</tr>
<tr>
<td>$\sigma_\theta' \times 10^4$</td>
<td>-3.444 -3.303 -2.899 -2.330 -1.737 0 0</td>
</tr>
<tr>
<td>Sum x 10^4</td>
<td>-3.340 -3.350 -2.979 -2.530 -1.706 0 0</td>
</tr>
<tr>
<td>C. Bottom Plate (h = 4 in.)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_r' \times 10^4$</td>
<td>0.345 0.276 0.207 0.138 0.069 0 0</td>
</tr>
<tr>
<td>$\sigma_\theta' \times 10^4$</td>
<td>3.883 3.728 3.262 2.405 1.398 0 0</td>
</tr>
<tr>
<td>Sum x 10^4</td>
<td>3.918 3.756 3.263 2.541 1.467 0 0</td>
</tr>
</tbody>
</table>

* $\delta_r'$ denotes radial membrane stress, Eq. (22).
** $\delta_\theta'$ denotes radial bending stress due to pressure and temperature constants, $A_q$ and $A_1$, Eq. (16).

<table>
<thead>
<tr>
<th>Table 6</th>
<th>CIRCUMFERENTIAL STRESS DISTRIBUTION FOR A TYPICAL LOADING CONDITION (k 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r/a$ 0.2 0.4 0.6 0.8 1.0</td>
</tr>
<tr>
<td>A. Top Plate (h = 6 in.)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\theta' \times 10^4$</td>
<td>1.293 0.777 0.259 -0.259 -0.177 -1.297 0</td>
</tr>
<tr>
<td>$\sigma_r' \times 10^4$</td>
<td>6.523 6.169 5.417 4.295 3.788 0.913 0</td>
</tr>
<tr>
<td>Sum x 10^4</td>
<td>7.818 6.937 5.606 4.084 3.701 -0.392 0</td>
</tr>
<tr>
<td>B. Middle Plate (h = 4 in.)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\theta' \times 10^4$</td>
<td>1.036 0.622 0.259 -0.207 -0.177 -1.036 0</td>
</tr>
<tr>
<td>$\sigma_r' \times 10^4$</td>
<td>-3.444 -3.303 -2.899 -2.330 -2.106 -1.036 0</td>
</tr>
<tr>
<td>Sum x 10^4</td>
<td>-3.340 -3.350 -2.979 -2.530 -2.481 -1.036 0</td>
</tr>
<tr>
<td>C. Bottom Plate (h = 4 in.)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\theta' \times 10^4$</td>
<td>0.345 0.276 0.207 0.138 0.069 0 0</td>
</tr>
<tr>
<td>$\sigma_r' \times 10^4$</td>
<td>3.883 3.728 3.262 2.405 1.398 0 0</td>
</tr>
<tr>
<td>Sum x 10^4</td>
<td>3.918 3.756 3.263 2.541 1.467 0 0</td>
</tr>
</tbody>
</table>

* $\sigma_\theta'$ denotes circumferential membrane stress, Eq. (23).
** $\sigma_r'$ denotes circumferential bending stress due to pressure and temperature constants, $A_q$ and $A_1$, Eq. (16).
plate, which is thicker than the others, and that the membrane stresses in remaining plates can be neglected. The underlined quantities are the maximum stresses for each plate.

D. Ligament Efficiency

The method of calculating stresses and deflections in a perforated plate with a triangular penetration pattern has been given by O'Donnell and Langer. They reduced the perforated plate to an "equivalent solid plate" problem by reassigning values to the elastic constants. The effective elastic constants $E^*$ and $\nu^*$ of the "equivalent solid plate" having the same dimensions of the perforated plate are obtained in Fig. 21. The quantities $E^*$ and $\nu^*$ depend on the ligament efficiency (ligament efficiency = $H/R$, see Figs. 3 and 21), and their variations versus ligament efficiency for $H/R > 4$ is shown in Fig. 21. If $E^*$ and $\nu^*$ are known, the numerical bending and membrane stresses and deflections of an equivalent plate can be calculated from the equations derived in the previous sections. Deflections of the perforated plates are the same as the deflections of the equivalent solid plates. In the FARET core currently considered, $2R = 2.32$ in. and $2H = 0.375$ in. (see Fig. 3). The ligament efficiency is found to be 0.162.

![Fig. 21](image)

**Fig. 21**

Effective Elastic Constants for Perforated Plates

E. Limit Stresses for Design of Perforated Plates

In order to design a perforated plate under combined mechanical and thermal loadings, it is proposed in Ref. 3 that ligament stress intensity and peak stress intensity should be the limiting stresses. The ligament stress intensity $\sigma_{\text{eff}}$, whose calculation is based on the maximum-shear theory of failure, is the average stress across the minimum ligament section, but not through the thickness of the plate. The peak stress intensity $\sigma_{\text{max}}$ is the maximum local stress due to mechanical and thermal loads. However, the peak stress intensity, being localized, is not considered to be the most significant. It is of interest only when it is repeated often.
enough to produce fatigue. Reference 3 states that for a perforated plate under combined mechanical and thermal loadings:

1. The ligament stress intensity $\sigma_{\text{eff}}$ should be limited to $3 S_m$, where $S_m$ is the allowable stress which, according to the 1959 ASME Boiler Code, should not exceed $5/8$ of the yield strength of a ferrous material.

2. The peak stress intensities $\sigma_{\text{max}}$ due to any loading should be limited by the cumulative fatigue damage, as described in Refs. 9, 10, and 11.

In the following a sample calculation of the ligament stress intensity $\sigma_{\text{eff}}$ and peak stress $\sigma_{\text{max}}$ is given for the top plate. From Table 5, it is seen that the maximum stress occurs at the center of the plate, and that both the radial and circumferential stresses are the same, i.e., $\bar{\sigma}_r = \bar{\sigma}_\theta = 7.8 \times 10^{-4}$. The quantity $\bar{\sigma}_1$, by definition, equals the larger of $|\bar{\sigma}_r|$ and $|\bar{\sigma}_\theta|$ (see Ref. 3).

Therefore,

$$\bar{\sigma}_1 = 7.8 \times 10^{-4}.$$ 

Since

$$E^* = 2.014 \times 10^6 \text{ psi},$$

$$\sigma_1 = 1574 \text{ psi},$$

it is found from Fig. 10 of Ref. 3 for

$$\bar{\sigma}_r / \bar{\sigma}_\theta = 1,$$

that the stress intensity factor $K = 1$. Therefore, the ligament stress intensity can be found as follows (see Ref. 3):

$$\sigma_{\text{eff}} = K \left( \frac{R}{H} \right) \sigma_1 = \frac{2.32}{0.375} \times 1574 = 9738 \text{ psi}.$$ 

Peak stress is defined in Ref. 3 as

$$\sigma_{\text{max}} = Y \sigma_1 + p,$$

where $p$ is the pressure acting on the surface and $Y$ the stress ratio, depending upon the ligament efficiency and the state of stresses.

The value $Y = 7.6$ is obtained from Fig. 12 of Ref. 3 for a ligament efficiency of 0.162. Therefore,

$$\sigma_{\text{max}} = 7.6 \times 1574 + 30 = 11992 \text{ psi}.$$
The stress intensities are calculated similarly for the middle and bottom plates. Results are shown in Table 7. For the same ligament efficiency, and depending on the allowable stresses $S_m$, it is seen that the thickness of the top plate could be reduced and the thicknesses of the remaining plates could be increased. Such an adjustment of the plate thickness would improve the design. Calculations based on the methods of this report would then have to be repeated with the new plate dimensions.

Table 7

<table>
<thead>
<tr>
<th>Plates</th>
<th>$\sigma_{\text{eff}}$</th>
<th>$\sigma_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>9700</td>
<td>12000</td>
</tr>
<tr>
<td>Middle</td>
<td>41600</td>
<td>51100</td>
</tr>
<tr>
<td>Bottom</td>
<td>48800</td>
<td>60000</td>
</tr>
</tbody>
</table>
IV. CONCLUDING REMARKS

A general solution for the bending of a core support plate subjected to mechanical and thermal loadings was obtained. The solution incorporates boundary conditions having varying degrees of rotational constraint. However, to evaluate the effectiveness of the constraint provided by the supporting structure, one must resort to experimental techniques for establishing the rotational effectiveness. The optimum values of $k$, the rotational restraint factor, can be obtained by invoking the theorem of minimum total potential energy.

Although the report considers only linear temperature variation through the thickness of the plate, other variations can be readily treated through Eq. (2). The solution obtained was applied to the FARET core support model.

The method of solution can be extended to ring plates, stepped plates, and plates with variable thickness subjected to mechanical and thermal loadings.

ACKNOWLEDGMENTS

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The numerical results were obtained with the IBM-704 at Argonne. Gratitude is owed to Mrs. D. Bingaman and her computing group who helped with the computation, and to Miss J. Wilson of the design and drafting group who prepared the figures.
REFERENCES


2. T. R. Bump, Argonne National Laboratory (Private communication).


