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MUSE

A COMPUTER CODE FOR MULTIPLE SCATTERING CORRECTIONS

by

H. C. Honeck

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MUSE

A COMPUTER CODE FOR MULTIPLE SCATTERING CORRECTIONS

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The MUSE computer code is described herein as it existed on October 1, 1964. The code has been in continuous development for 1 year\$ and in its presented form has been applied successfully by General Atomic to the kind of problems discussed later in this report. However, the development and improvement of the code are being continued, so that duplication of results (or even close agreement) between problems run with the code as published and the code as it existed either before or after this time is not necessarily to be expected.

General Atomic has exercised due care in preparation, but does not warrant the merchantability, accuracy, and completeness of the code or of its description contained herein. The complexity of this kind of program precludes any guarantee to that effect. Therefore, any user must make his own determination of the suitability of the code for any specific use, and of the validity of the information produced by use of the code.

ABSTRACT

This report describes the theory and operation of the codes MUSE1 and MUSE2 designed to compute multiple scattering corrections for experiments measuring the angular cross section $\sigma_s(E, \theta)$ and the scattering kernel $\sigma_s(E' \rightarrow E, \theta)$, respectively.

CONTENTS

1. INTRODUCTION	1
2. THEORY	3
2.1. The Integral Transport Equation	3
2.2. The First Collision	5
2.3. Treatment of the Angular Variable	7
2.4. Treatment of the Spatial Variable	8
2.5. Treatment of the Energy Variable	9
2.6. The First Collision Source	9
2.7. Multiple Collisions	10
2.8. The Detector Response	11
3. NUMERICAL APPROXIMATIONS	13
3.1. The Spatial Variable	13
3.2. The Angular Variable	14
3.3. The Energy Variable	14
3.4. The Scattering Kernel	15
4. DESCRIPTION OF THE COMPUTER CODES	17
4.1. Problem Size and Storage Layout	17
4.2. Subroutines	23
4.3. Description of the Input Data	25
4.4. Description of the Output	29
4.5. The Auxiliary Code MUSIN	30
APPENDIX A - Spherical Harmonics	36
APPENDIX B - Listing of the Codes	39

1. INTRODUCTION

This report describes the theory and operating procedures for the code MUSE, which computes multiple scattering corrections for angular distribution measurements. Two versions of the code have been written. The first, MUSE1, computes corrections for a single differential experiment, for example, the angular cross section for thermal neutrons, $\sigma(E', \theta_0)$, where θ_0 is the scattering angle and E' is the incident energy. The second version, MUSE2, computes corrections for a double differential experiment, for example, the thermal neutron scattering kernel, $\sigma(E' \rightarrow E, \theta_0)$, where E is the final energy. Both versions allow a neutron to make up to four collisions in the sample.

The basic method in the MUSE code is a straightforward numerical solution of the integral transport equation with the following simplifications.

1. The sample is a homogeneous slab, two of whose dimensions are sufficiently large that the slab can be assumed to be infinite in extent.
2. The scattering kernel $\sigma(E' \rightarrow E, \theta_0)$ is represented by a matrix evaluated at an arbitrary energy mesh and a uniformly spaced mesh in $\cos \theta_0$ for the first collision. A Legendre expansion in $\cos \theta_0$ of the kernel is then used to compute higher order scattering terms. The present version of MUSE allows for P_3 scattering terms. Elastic and inelastic parts of the kernel are input separately to the program.

3. The sample is sufficiently thin so that the flux distribution across the sample can be approximated by a few step functions.
4. A theoretical scattering kernel is used. In a double differential experiment the scattering kernel is measured and if it is possible to construct a full kernel from the data, this kernel can be used in the code. A few iterations on the kernel should then produce a kernel free of multiple scattering corrections. In the single differential experiment there is not enough information measured to construct a kernel. Hence, a theoretical kernel must be used in the code.

The first and third approximations should be quite accurate since most samples are many mean free paths thick in two dimensions and only a few tenths of a mean free path thick. The truncation of the scattering kernel to terms in P_3 for the higher order collisions is adequate for most materials in the thermal range of neutron energies. However, at energies in the electron volt region both P_3 scattering and the multigroup approximation may be poor. Fortunately, in this energy region the cross section is near its minimum value and the multiple scattering correction is small. The last approximation is the most serious for the single differential experiment, but cannot be avoided. The multiple scattering corrections will probably be accurate if the measured and theoretical angular distributions are in good agreement.

2. THEORY

A schematic of a scattering sample is shown in Figure 1. A neutron enters the slab in a direction $\underline{\Omega}_1$, which lies in the X-Z plane and makes an angle θ_1 with the Z axis. After scattering into a new direction $\underline{\Omega}_2$, the neutron may make further collisions in the slab or escape the slab. Of the neutrons which escape the slab, only those lying in the X-Z plane ($\psi_2 = 0$ or π) are allowed to hit the detectors. Thus, the detectors are assumed to be in the plane formed by the incident neutron vector and the normal to the slab.

2.1. The Integral Transport Equation

The flux in a slab such as shown in Figure 1 is a solution of the transport equation which we write in the integral form.

$$\phi(Z, E, \underline{\Omega}) = \frac{1}{\mu} \int_0^Z dZ' H(Z', E, \underline{\Omega}) e^{-\Sigma(E) \left(\frac{Z-Z'}{\mu} \right)}, \quad \mu > 0, Z > Z' \quad (1)$$

$$= \frac{1}{-\mu} \int_Z^a dZ' H(Z', E, \underline{\Omega}) e^{-\Sigma(E) \left(\frac{Z-Z'}{\mu} \right)}, \quad \mu < 0, Z < Z'$$

$$H(Z', E, \underline{\Omega}) = \int d\underline{\Omega}' \int dE' \Sigma_s(E' \rightarrow E, \mu_0) \phi(Z', E', \underline{\Omega}') \quad (2)$$

$$\mu_0 = \underline{\Omega} \cdot \underline{\Omega}' \quad (3)$$

Here, $\mu = \cos \theta$, where θ is the angle between the neutron vector and the Z axis. $\phi(Z, E, \underline{\Omega})$ is the neutron flux, $\Sigma(E)$ is the total cross section, and $\Sigma_s(E' \rightarrow E, \mu_0)$ is the scattering kernel. The advantage of the integral formulation is that one obtains

Figure 1. Geometry.

accurate solutions with only a few spatial regions (mesh points) and that each iteration corresponds to one scattering order. Thus, the flux obtained by using the incident flux in Eq. (2) and inserting Eq. (2) into Eq. (1) is the first collision flux. Inserting this flux into Eq. (2), and Eq. (2) into Eq. (1), leads to the added flux due to second collisions. Thus, high order collisions are conveniently treated by iteration of Eq. (1) and (2).

We must now reduce these equations to a form suitable for solving on a digital computer. The procedure is as follows. We treat the first collision exactly and compute the detector response from the full angular scattering kernel. The first collision source is then resolved into spherical harmonic components which are used to compute second and higher order collision fluxes and detector responses.

2.2. The First Collision

The incident flux is assumed to be monoenergetic and in the direction $\underline{\Omega}_1$. The uncollided flux in the sample is then

$$\phi(Z, E, \underline{\Omega}) = e^{-\Sigma(E)Z/\mu} \delta(E - E_1) \delta(\underline{\Omega} - \underline{\Omega}_1) \quad (4)$$

Inserting this in Eq. (1) and (2) yields (for $\mu > 0$)

$$\phi(Z, E, \underline{\Omega}) = \frac{1}{\mu} \int_0^Z dz' e^{-\Sigma(E) \frac{(Z-z')}{\mu}} \Sigma_s(E_1 \rightarrow E, \mu_0) e^{-\Sigma(E_1) z' / \mu_1} \quad (5)$$

If we use subscript 2 to denote quantities seen by the detector, then the flux at the detector is

$$\begin{aligned}
\phi(a, E_2, \Omega_2) &= \Sigma_s(E_1 \rightarrow E_2, \mu_0) \frac{1}{\mu_2} \int_0^a dz' e^{-\Sigma(E_2) \left(\frac{z-z'}{\mu_2} \right) - \Sigma(E_1) \frac{z'}{\mu_1}} \\
&= \frac{a}{\mu_2} \left[\frac{e^{-x_1} - e^{-x_2}}{x_2 - x_1} \right] \Sigma_s(E_1 \rightarrow E_2, \mu_0)
\end{aligned} \tag{6}$$

where $\mu_0 = \Omega_1 \cdot \Omega_2$, $x_1 = a\Sigma(E_1)/\mu_1$, $x_2 = a\Sigma(E_2)/\mu_2$. For small values of x_1 and x_2 the quantity in brackets approaches $1 - \frac{1}{2}(x_2 + x_1)$ and is just the correction for sample attenuation. The term a/μ_2 is the effective sample thickness. We define the response as being

$$\begin{aligned}
R(E_2, \Omega_2) &= \frac{\mu_2}{a} \phi(a, E_2, \Omega_2) \\
&= \left[\frac{e^{-x_1} - e^{-x_2}}{x_2 - x_1} \right] \Sigma_s(E_1 \rightarrow E_2, \mu_0)
\end{aligned} \tag{7}$$

which is appropriate to the double differential experiment. The response for the single differential experiment is just the integral of Eq. (7) over all E_2 .

A similar derivation for $\mu_2 < 0$ leads to

$$R(E_2, \Omega_2) = \left[\frac{1 - e^{-(x_2 + x_1)}}{x_2 + x_1} \right] \Sigma_s(E_1 \rightarrow E_2, \mu_0) \tag{8}$$

where $x_1 = a\Sigma(E_1)/\mu_1$, and $x_2 = a\Sigma(E_2)/|\mu_2|$.

We next discuss the numerical methods for solving Eq. (1) for the higher order collisions.

2.3. Treatment of the Angular Variable

The quantities ϕ , H , and S are expanded in spherical harmonics (see Appendix A). For example,

$$\phi(Z, E, \underline{\Omega}) = \sum_{m=0}^{\infty} \sum_{\nu=-m}^m \phi_m^{\nu}(Z, E) Y_m^{\nu}(\underline{\Omega}) \quad (9)$$

The scattering kernel is expanded in a Legendre series and truncated after the fourth term.

$$\Sigma_S(E' \rightarrow E, \mu_0) = \frac{1}{2\pi} \sum_{k=0}^3 \left(\frac{2k+1}{2} \right) \Sigma_{S_k}(E' \rightarrow E) P_k(\mu_0) \quad (10)$$

Insert these expansions in Eq. (2) and use the addition theorem for the spherical harmonics (see Appendix A). There results

$$H_m^{\nu}(Z, E) = \int dE' \Sigma_{sm}(E' \rightarrow E) \phi_m^{\nu}(Z, E') \quad (11)$$

Next, insert expansions (11) and (9) into Eq. (1) and operate with $\int d\underline{\Omega} Y_m^{\nu}(\underline{\Omega})$.

$$\begin{aligned} \phi_n^{\nu}(Z, E) = \sum_{m=0}^3 \left\{ \int_0^Z dZ' H_m^{\nu}(Z', E) \frac{1}{2} \int_0^1 \frac{d\mu}{\mu} U_n^{\nu}(\mu) U_m^{\nu}(\mu) e^{-\Sigma(E) \left(\frac{Z-Z'}{\mu} \right)} \right. \\ \left. + \int_Z^a dZ' H_m^{\nu}(Z', E) \frac{1}{2} \int_{-1}^0 \frac{d\mu}{-\mu} U_n^{\nu}(\mu) U_m^{\nu}(\mu) e^{-\Sigma(E) \left(\frac{Z-Z'}{\mu} \right)} \right\} \end{aligned} \quad (12)$$

where the functions $U_n^{\nu}(\mu)$ are defined as

$$U_n^{\nu}(\mu) = \sqrt{4\pi} A_n^{\nu} P_n^{\nu}(\mu) \quad (13)$$

The sum on m in Eq. (12) has been truncated to $m=3$, because of the assumption that the scattering kernel can be represented by the first four terms in the Legendre expansion. Eq. (12) is valid for all n and $\nu \leq n$. It is necessary to consider only those terms for which $n \leq 3$ to obtain a solution to Eq. (12).

2.4. Treatment of the Spatial Variable

We divide the sample slab into J slabs of equal thickness, $\Delta = a/J$. Number the source slab with index j , and the receptor slab with index i . We average Eq. (12) over the i^{th} slab to obtain $\phi_{ni}^{\nu}(E)$.

$$\phi_{ni}^{\nu}(E) = \frac{1}{\Delta} \int_{z_i - \Delta}^{z_i} dz \phi_n^{\nu}(z, E) \quad (14)$$

Eq. (12) becomes

$$\phi_{ni}^{\nu}(E) = \sum_{m=0}^3 \sum_{j=1}^J H_{mj}^{\nu}(E) T_{nm\ell}^{\nu}(E) \quad (15)$$

where

$$T_{nm\ell}^{\nu}(E) = \frac{1}{2} \int_0^1 \frac{d\mu}{\mu} U_n^{\nu}(\mu) U_m^{\nu}(\mu) F_{\ell}(E, \mu), \quad \ell > 0$$

$$F_{\ell}(E, \mu) = \frac{\Delta}{x^2} e^{-\ell x} (e^x - 1) (1 - e^{-x}), \quad \ell > 0 \quad (16)$$

$$\ell = i - j, \quad x = \Delta \Sigma(E) / \mu$$

$$T_{nm\ell}^{\nu}(E) = (-1)^{n+m} T_{n,m,-\ell}^{\nu}(E), \quad \ell < 0$$

$$T_{nm0}^{\nu}(E) = \int_0^1 \frac{d\mu}{\mu} U_n^{\nu}(\mu) U_m^{\nu}(\mu) F_0(E, \mu), \quad n+m \text{ even}$$

$$= 0, \quad n+m \text{ odd}$$

$$F_0(E, \mu) = \frac{\Delta}{x^2} (x - 1 + e^{-x})$$

2.5. Treatment of the Energy Variable

The final step in the reduction of Eq. (1) to a matrix equation is the multigroup approximation in which all quantities are evaluated at a set of energies, E_k , $1 \leq k \leq K$. For the purpose of energy integration the set of weights ΔE_k are also defined. Eq. (11) and (15) become

$$\phi_{ni}^{\nu} = \sum_{m=0}^3 \sum_{j=1}^J H_{mj}^{\nu} T_{nm}^{\nu} \quad (17)$$

$$H_{mj}^{\nu} = \sum_{k=1}^K \Delta E_k \Sigma_{smk} \phi_{mj}^{\nu} \quad (18)$$

2.6. The First Collision Source

The incident flux is assumed to be monoenergetic and in the direction $\underline{\Omega}_1$. The uncollided flux in the slab is

$$\phi(Z, E, \underline{\Omega}) = e^{-\Sigma(E)Z/\mu} \delta(E - E_1) \delta(\underline{\Omega} - \underline{\Omega}_1) \quad (19)$$

which has spherical harmonic expansion coefficients

$$\phi_m^{\nu}(Z, E) = Y_m^{*\nu}(\underline{\Omega}_1) e^{-\Sigma(E)Z/\mu_1} \delta(E - E_1) \quad (20)$$

The first collision source is then obtained by folding this

with the scattering kernel.

$$H_m^v(Z, E) = \sum_{sm} (E_1 \rightarrow E) e^{-\Sigma(E_1)Z/\mu_1} A_{mm}^v P_m^v(\mu_1) \quad (21)$$

since it is assumed that $\psi_1 = 0$. Average Eq. (21) over the j^{th} slab and introduce the multigroup approximation.

$$H_{mj}^v = \sum_{sm} A_{ms}^v P_m^v(\mu_1) \frac{1}{x} (e^x - 1) e^{-jx} \quad (22)$$

$$x = \Sigma_{ms} \Delta/\mu_1$$

where μ_s is the energy group corresponding to E_1 .

2.7. Multiple Collisions

A complete solution of Eq. (17) and (18) by iteration would be quite time-consuming. Fortunately, if the sample is thin, we know that only a few collisions must be followed, and, since each collision corresponds to one iteration, only a few iterations are required. The procedure is then as follows. Compute a first collision source from the uncollided flux in the sample. Inserting this into Eq. (17) gives the first collision flux. Insert the first collision flux into Eq. (18), and compute H , the second collision source. Inserting this into Eq. (17) yields the second collision flux, and so forth. The individual terms can then be summed up to obtain the total source and flux from all collisions. Up to four collisions are treated in the present version of MUSE.

2.8. The Detector Response

We could obtain the flux leaving the sample from the flux computed above, but we would only have up to P_3 components. Additional components could be obtained from Eq. (17), since the sources have all been computed. We can avoid this lengthy procedure by returning to the original integral Eq. (1), where we now have computed the source function H to terms in P_3 . Insert the expansion for H into Eq. (1) and consider the case $\mu > 0$, $Z = a$, $\underline{\Omega} = \underline{\Omega}_2$.

$$\phi(a, E_2, \underline{\Omega}_2) = \frac{1}{\mu_2} \int_0^a dz' e^{-\Sigma(E_2) \left(\frac{Z-Z'}{\mu_2} \right)}$$

$$\sum_{n=0}^3 \sum_{\nu=-n}^n A_{n\nu}^{\nu P \nu}(\mu_2) e^{i\psi_2^{\nu}} H_n^{\nu}(Z', E_2) \quad (23)$$

This is then the flux of energy E_2 seen by a detector at $\underline{\Omega}_2$. Under ideal conditions of a very thin sample this flux is equal to $a\Sigma_s(E_1 \rightarrow E_2, \theta_0)/\mu_2$, so that the effective sample thickness is a/μ_2 . Define the response, $R(E_2, \theta_2)$, as

$$R(E_2, \theta_2) = \phi(a, E_2, \underline{\Omega}_2) \mu_2 / a$$

$$= \sum_{j=1}^J G_{\ell \ell}(\theta_2) \sum_{n=0}^3 \sum_{\nu=-n}^n A_{n\nu}^{\nu P \nu}(\mu_2) e^{i\psi_2^{\nu}} H_{nj}^{\nu} \quad (24)$$

where

$$G_{\ell \ell}(\theta_2) = \frac{\mu_2}{a\Sigma_{\ell}} e^{-(\ell-1)x} (1-e^{-x}) \quad (25)$$

$$x = \Delta \Sigma_{\#} / \mu_2$$

and $\#$ denotes the energy group corresponding to E_2 .

The following table gives the values of l and $e^{i\nu\psi_2}$ for various ranges of θ_2 . Note that ψ_2 can assume only the values 0 or π since the detector must be in the X-Z plane.

θ_2	l	$e^{i\nu\psi_2}$
$0 < \theta_2 < 90$	$J - j + 1$	1
$90 < \theta_2 < 180$	j	1
$180 < \theta_2 < 270$	j	$(-1)^\nu$
$270 < \theta_2 < 360$	$J - j + 1$	$(-1)^\nu$

The above equations are used in MUSE2.

If the detector is black, all neutrons are counted and the response is the energy integral of Eq. (23). Under ideal conditions of a very thin sample the flux is just $a\Sigma_s(E_1, \theta_0)/\mu_2$. Define the response, $R(\theta_2)$, as

$$R(\theta_2) = \frac{\mu_2}{a} \int dE_2 \phi(a, E_2, \Omega_2) \quad (26)$$

$$= \sum_{\# = 1}^K \Delta E_{\#} R(E_{\#}, \theta_2)$$

This equation is used in MUSE1.

3. NUMERICAL APPROXIMATIONS

The following sections describe the various numerical approximations used in the MUSE codes.

3.1. The Spatial Variable

Many of the formulas given in the preceding section must be evaluated by series expansions for small values of the arguments. The following expansions are used and are accurate to better than one part in 3×10^6 .

Equation (7), $y \leq 0.05$, $y = x_2 - x_1$

$$\left[\frac{e^{-x_1} - e^{-x_2}}{x_2 - x_1} \right] = e^{-x_2} \left[1 + \frac{1}{2} y + \frac{1}{6} y^2 + \frac{1}{24} y^3 \right] \quad (27)$$

Equation (8), $y \leq 0.05$, $y = x_2 + x_1$

$$\left[\frac{1 - e^{-y}}{y} \right] = 1 - \frac{1}{2} y + \frac{1}{6} y^2 - \frac{1}{24} y^3 \quad (28)$$

Equation (16), $x \leq 0.1$

$$\frac{\Delta}{x^2} e^{-lx} (e^x - 1) (1 - e^{-x}) = \Delta e^{-lx} \left(1 - \frac{1}{12} x^2 \right) \quad (29)$$

$$\frac{\Delta}{x^2} (x - 1 + e^{-x}) = \frac{\Delta}{2} \left(1 - \frac{1}{3} x + \frac{1}{12} x^2 - \frac{1}{60} x^3 \right) \quad (30)$$

Equation (22), $x \leq 0.05$

$$\frac{1}{x} e^{-jx} (e^x - 1) = e^{-jx} \left(1 + \frac{1}{2} x + \frac{1}{6} x^2 + \frac{1}{24} x^3 \right) \quad (31)$$

Equation (26), $x \leq 0.05$

$$1 - e^{-x} = x \left[1 - \frac{1}{2} x + \frac{1}{6} x^2 - \frac{1}{24} x^3 \right] \quad (32)$$

3.2. The Angular Variable

The integration over μ in Eq. (16) is done by Gauss quadrature.

$$\frac{1}{2} \int_0^1 \frac{d\mu}{\mu} U_n^\nu(\mu) U_m^\nu(\mu) F_\ell(E, \mu) =$$

$$\frac{1}{2} \sum_i \frac{W_i}{\mu_i} U_n^\nu(\mu_i) U_m^\nu(\mu_i) F_\ell(E, \mu_i) \quad (33)$$

The following table lists the angles and weights for an eight point quadrature integration.

<u>i</u>	<u>μ_i</u>	<u>W_i</u>
1	0.09501251	0.18945061
2	0.28160354	0.18260342
3	0.45801668	0.16915651
4	0.61787624	0.14959598
5	0.75540441	0.12462897
6	0.86563119	0.09515851
7	0.94457502	0.06225352
8	0.98940092	0.02715246

3.3. The Energy Variable

The energy scale is divided in an arbitrary set of points, E_k , and all quantities are evaluated at these points. The

integrations over the energy variable are defined by

$$\int dE f(E) = \sum_{k=1}^K \Delta E_k f(E_k) \quad (34)$$

where the following rules define the ΔE_k .

$$\Delta E_1 = \frac{1}{2}(E_1 + E_2)$$

$$\Delta E_k = \frac{1}{2}(E_{k+1} - E_{k-1}), \quad 1 < k < K \quad (35)$$

$$\Delta E_K = \frac{1}{2}(E_K - E_{K-1})$$

3.4. The Scattering Kernel

The input to the MUSE program is the scattering kernel

$$\frac{d^2 \sigma}{d\Omega d\epsilon} = \sigma(E_k \rightarrow E_{k'}, \mu_l) \quad (36)$$

where the E_k are the arbitrary energy points and the μ_l are a set of equally spaced points on the interval $(-1,1)$. The first collision response is determined directly from the full scattering kernel at integral values of k and k' , but arbitrary values of μ . Values at intermediate μ are obtained by linear interpolation.

If the kernel has a gas-like behavior for small energy transfer, the value of the kernel at $E_k = E_{k'}$, $\mu = 1$, is infinite. This value should be set to zero in the input, if the behavior is gas-like, and, hence, the linear interpolation will be

meaningless for μ near unity.

Higher order collisions are computed on the assumption that the kernel can be represented by a low order Legendre expansion.

$$\sigma(E' \rightarrow E, \mu) = \frac{1}{2\pi} \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \sigma_{\ell}(E' \rightarrow E) P_{\ell}(\mu) \quad (37)$$

$$\sigma_{\ell}(E' \rightarrow E) = 2\pi \int_{-1}^1 d\mu P_{\ell}(\mu) \sigma(E' \rightarrow E, \mu) \quad (38)$$

The integration over μ is done by Simpson's Rule. Let NA (an odd number) be the number of equally spaced values of μ in the interval $(-1,1)$, and let $\Delta\mu = 2/(NA-1)$. Then

$$\int_{-1}^1 f(\mu) d\mu = \frac{\Delta\mu}{3} \left[f_1 + f_{NA} + 4(f_2 + f_4 + \dots) + 2(f_3 + f_5 + \dots) \right] \quad (39)$$

The Legendre polynomials are computed using the recursion formula

$$P_{n+1}(\mu) = \frac{2n+1}{n+1} \mu P_n(\mu) - \frac{n}{n+1} P_{n-1}(\mu) \quad (40)$$

Again some difficulty arises if the near-elastic behavior is gas-like. In this region the kernel behaves like

$$\sigma(E \rightarrow E, \mu) = B(E) / \sqrt{1-\mu} \quad (41)$$

where $B(E)$ is some function of energy. We assume that $B(E)$ can be evaluated from the expression

$$B(E) = \sqrt{\Delta\mu} \sigma(E \rightarrow E, \mu_{NA-1}) \quad (42)$$

Then the integrals in the range $1-2\Delta\mu \leq \mu \leq 1$ can be evaluated analytically

$$\int_{1-2\Delta\mu}^1 d\mu \sigma(E-E, \mu) P_\ell(\mu) \approx B(E) \int_0^{2\Delta\mu} dx P_\ell(1-x)/\sqrt{x} \quad (43)$$

where $x = 1-\mu$. Since $\Delta\mu$ will be small, the $P_\ell(1-x)$ can be represented by

$$P_\ell(1-x) = 1 - a_\ell x \quad (44)$$

$$a_{\ell+1} = \frac{2\ell+1}{\ell+1} \left[1 + a_\ell - \frac{\ell}{2\ell+1} a_{\ell-1} \right] \quad (45)$$

$$a_0 = 0; a_1 = 1; a_2 = 3; a_3 = 6; \text{ etc.}$$

The final expression is then

$$\int_{1-2\Delta\mu}^1 d\mu \sigma(E-E, \mu) = \sigma(E-E, \mu_{NA-1}) 2\sqrt{2} \Delta\mu \left[1 - \frac{2}{3} \Delta\mu a_\ell \right] \quad (46)$$

The integral from $\mu = -1$ to $\mu = 1-2\Delta\mu$ is again done by Simpson's Rule.

4. DESCRIPTION OF THE COMPUTER CODES

The next few sections describe the MUSE codes written in Fortran to solve the equations given in the preceding sections. Except where noted, the comments apply to both MUSE1 and MUSE2.

4.1. Problem Size and Storage Layout

Nearly all important quantities are stored in COMMON

storage. The following table gives the layout of COMMON storage and a description of the quantities. The most frequently used arguments for arrays are also given. Below each argument list is a list of integers, L_1 to L_{11} , which, when assigned values, would appear in the DIMENSION statements. L_1 through L_{11} are defined as:

Symbol	Description	Typical Value
L_1	Maximum number of energy groups	30
L_2	Maximum number of spatial regions	5
L_3	Maximum number of incident energies	30
L_4	Maximum number of incident angles	4
L_5	Maximum number of quadrature angles	20
L_6	Maximum number of final angles	15
L_7	Maximum Legendre term number	3
L_8	$(L_7 + 1)^2$	16
L_9	$(L_7 + 1)(L_7 + 2)(2L_7 + 3)/6$	30
L_{10}	$L_7 + 1$	4
L_{11}	Number of angles used to represent the input kernel	31

COMMON Storage Layout

Name	Arguments, Size	Description	Eqn
FN	(I, NNU, K) (L_2, L_8, L_1)	ϕ_{ni}^v , $NNU = 1+n(n+1)+v$	17
H	(J, MNU, K) (L_2, L_8, L_1)	H_{mj}^v , $MNU = 1+m(m+1)+v$	18

COMMON Storage Layout (cont.)

Name	Arguments, Size	Description	Eqn
KX	-	Number of energy groups	-
JX	-	Number of spatial regions	-
THICK	-	Sample thickness (cm)	-
DEL	-	Region thickness = THICK/JX	-
N3	-	Maximum Legendre term considered	-
N3P	-	$N3 + 1$	-
LAX	-	Number of equally spaced cosines used to represent the angular distribution of the input scattering kernel	-
NAX	-	Number of quadrature integration angles	-
NA1X	-	Number of incident angles	-
NA2X	-	Number of final angles	-
KSX	-	Number of incident energies	-
DELA	-	$= 2/(LAX-1)$	-

COMMON Storage Layout (cont.)

Name	Arguments, Size	Description	Eqn
ANG	(NA) (L ₅)	Cosines of the quadrature angles, μ_i	33
ANGW	(NA) (L ₅)	Quadrature integration weights, w_i	33
TH1	(NA1) (L ₄)	Incident angles, θ_1 (degrees)	-
ANG1	(NA1) (L ₄)	Incident cosines, $\cos \theta_1$	-
TH2	(NA2) (L ₆)	Final angles, θ_2 (degrees)	-
ANG2	(NA2) (L ₆)	Final cosines, $\cos \theta_2$	-
KST	(KSP) (L ₁)	The energy group number, KS, for the KSP th incident energy	-
EN	(K) (L ₁)	The energy mesh, E_k	-
DE	(K) (L ₁)	Energy integration weights, ΔE_k	35

COMMON Storage Layout (cont.)

Name	Arguments, Size	Description	Eqn
XT	(K) (L ₁)	Total cross section, $\Sigma_{\ell} = \Sigma(E_{\ell})$	-
XSN	(K,N) (L ₁ ,L ₁₀)	Legendre moments of the scattering cross section, $\Sigma_{s\ell} = \Sigma_{s\ell}(E_{\ell})$, $\ell = N-1$	-
U	(NA, NNU) (L ₅ ,L ₈)	$U_n^v(\mu_{NA})$, $NNU = 1+n(n+1)+v$	13
F	(NA,L) (L ₅ ,L ₂)	$F_{\ell}(E, \mu_{NA})$ at a given E, $\ell = L-1$	16
PP	(L,K) (L ₁₁ ,L ₁)	$\Sigma_s(E \rightarrow E_K, \mu_L)$ at a given E This array is contained in FN	-
G	(L,K,NA2) (L ₂ ,L ₁ ,L ₆)	$G_{\ell\ell}(\theta_{NA2})$	25
T	(L,NMN,K) (L ₂ ,L ₉ ,L ₁)	$T_{nm\ell}^v(E_{\ell})$, $\ell = L-1$ $NMN = 1 + \frac{1}{6} n(n+1)(n+2)+m(m+1)+v$	16
P	(K,KP,N) (L ₁ ,L ₁ ,L ₁₀)	Legendre moments of the scattering kernel, $\Sigma_{s\ell}(E_{KP} \rightarrow E_K)$, $\ell = N-1$	38

COMMON Storage Layout (cont.)

Name	Arguments, Size	Description	Eqn
R	(NC, NA2) (3, L ₆)	Response at angle θ_{NA2} from collision NC-1 (MUSE1)	26
	or (NC, KF, NA2) (3, L ₁ , L ₆)	Response at angle θ_{NA2} and energy group KF from collision NC-1 (MUSE2)	25
DEAL	(NA2, NA1, KSP) (L ₆ , 4, L ₁)	Ideal response at angle θ_{NA2} from the KSP th incident energy and angle θ_{NA1} (MUSE1)	8
	or (KF, NA2, LKN) (L ₁ , L ₆ , 4)	Ideal response at angle θ_{NA2} and energy group KF from the KSP th incident energy and angle θ_{NA1} . LKN = NA1+NA1X*(KSP-1) (MUSE2)	7
RFC	(NA2, NA1, KSP) (L ₆ , 4, L ₁)	First collision response at angle θ_{NA2} from the KSP th incident energy and angle θ_{NA1} (MUSE1)	8
	or (KF, NA2, LKN) (L ₁ , L ₆ , 4)	First collision response at angle θ_{NA2} and energy group KF from the KSP th incident energy and angle θ_{NA1} . LKN = NA1+NA1X*(KSP-1) (MUSE2)	7

The COMMON storage requirements are then

$$L_1 \left[L_2 (L_6 + 2L_8 + L_9) + 8L_6 + L_{10} (L_1 + 1) + 4 \right] + 2L_4 \quad (\text{MUSE1})$$

$$+ L_5 (L_2 + L_8 + 2) + 5L_6 + 12$$

$$L_1 \left[L_2 (L_6 + 2L_8 + L_9) + 11L_6 + L_{10} (L_1 + 1) + 4 \right] + 2L_4 \quad (\text{MUSE2})$$

$$+ L_5 (L_2 + L_8 + 2) + 2L_6 + 12$$

which gives values of 19,545 (MUSE1) and 21,550 (MUSE2) for the typical values of L given previously.

The values of L_1 vs. L_2 for given L_7 are plotted in Figure 2 for 22,000 words of COMMON storage.

4.2. Subroutines

The MUSE codes are written as a collection of subroutines. A brief description of these subroutines is given in this section.

The main program is called MUSE. MUSE processes the card input data and, using subroutine KERN, processes the kernel input data from binary tape 11. MUSE then calls TNML to compute the transport kernel, SØURC to compute the source term for the second collision, ITER to compute the second collision flux and source, RESP to compute the second collision response, and so on for the third and fourth collisions. Finally, ØUT is called to print the results.

Subroutine KERN reads the energy mesh and scattering

Figure 2. Maximum number of spatial regions as a function of the number of groups (L_1) and the order of the P_n scattering (L_7) for 22,000 words of common storage

kernel from binary tape 11. As kernels for each incident energy group are processed, subroutine FCR is called to compute the first collision response (Eq. 7,8) and the ideal response.

Subroutine TNML computes the transport kernel (Eq. 16) and uses FLEU to compute $F(E, \mu)$ in Eq. 16 and YNNU to compute the Legendre polynomials $Y_n^l(\mu)$. Subroutine SØURC computes the first collision source term (Eq. 22).

One collision is followed by entering ITER, which evaluates Eq. 18 and 17, and then RESP, which computes the detector response from the current value of H_{mj}^v (Eq. 24 or 26).

Finally, subroutine ØUT collects the results and prints them.

4.3. Description of the Input Data

The input data consists of a deck of input cards and a binary tape containing the energy mesh and scattering kernel. The present code uses tape 11 for the kernel input tape. The following tables describe the input data.

Scattering Kernel Input (Binary Tape 11) for MUSE

Record	Symbol	Description
1	(HØLA(I), I=1,12), (HØLB(I), I=1,12), KX, LAX,	First title card Second title card Number of energy groups Number of equally spaced angular points

Scattering Kernel Input (Binary Tape 11) for MUSE (cont.)

Record	Symbol	Description
	N3P	Number of Legendre moments
2	(EN(K), K=1, KX), (DE(K), K=1, KX)	The energy mesh (ev) and increments
3	((XSN(J, N), J=1, KX), N=1, N3P)	Legendre moments of the cross section. $\sigma_{SL}(E_J)$, $L=N-1$
4	((P(K, J, N), K=1, KX), J=1, KX), N=1, N3P	Legendre moments of the kernel. $\sigma_{SL}(E_K \leftarrow E_J)$, $L=N-1$
J + 4	((PP(L, K), L=1, LAX), K=1, KX)	The kernel $\sigma_S(E_K \leftarrow E_J, \mu_L)$ at a fixed value of initial energy, E_J , final energy, E_K , and angles $\mu_L = -1 + \Delta\mu(L-1)$, $\Delta\mu = 2/(LAX-1)$

The tape will contain KX+4 records and an EOF mark.

Punched Card Input for MUSE

Item	Columns	Format	Symbol	Description
1	1 - 72	12A6	HØL	Title card, 72 characters
2	1 - 5	I5	KX	Number of energy groups (Note 1)

Punched Card Input for MUSE (cont.)

Item	Columns	Format	Symbol	Description
	6 - 10	I5	JX	Number of space mesh points (Note 1)
	11 - 15	I5	N3	Maximum Legendre term considered (Note 1)
	16 - 20	I5	NAX	Number of quadrature integration angles (≤ 20) (Note 2)
	21 - 25	I5	NALX	Number of incident angles (Note 3)
	26 - 30	I5	NA2X	Number of output angles ≤ 15
	31 - 35	I5	KSX	Number of incident energies (Note 3)
3	1 - 10	E10.5	THICK	Sample thickness (cm)
	11 - 20	E10.5	XA	Value of Σ_a ($\#T$) (cm^{-1})
	21 - 30	E10.5	SCALE	Scale factor (atoms/barn cm) for scattering kernel
4	-	7E10.5	ANG(N), N=1,NAX	The NAX values of the quadrature angles (Note 2)
5	-	7E10.5	ANGW(N), N=1,NAX	The NAX values of the quadrature weights (Note 2)
6	-	14I5	KST(K), K=1,KSX	Incident energy groups. If $KSX \geq KX$, omit this item. The program will set $KST(K)=K$, $K=1, KX$ (Note 3)

Punched Card Input for MUSE (cont.)

Item	Columns	Format	Symbol	Description
7	-	7E10.5	TH1(N), N=1,NA1X	Incident angles (degrees) measured from the normal to the sample
8	-	7E10.5	TH2(N), N=1,NA2X	Output angles (degrees) measured from the normal to the sample
9	1 - 5	I5	N3P	Number of Legendre components
10	-	7E10.5	XSN(1,N) N=1,N3P	Legendre compnents of the scattering cross section, σ_{SL} , L=N-1 (Note 4)
11	1 - 5	I5	LAX	Number of equally spaced angular points at which the cross section is evaluated ≤ 31 (Note 4)
12	-	7E10.5	PP(L,1), L=1,LAX	The angular cross section (Note 4)

Note 1. The maximum values of these quantities depends on the dimension statements. The current version allows for $KX \leq 30$, $JX \leq 5$, $N3 \leq 3$.

Punched Card Input for MUSE (cont.)

Note 2. An 8-point quadrature set is built in and is used if $NAX \leq 0$. If this set is to be used, omit items 4 and 5.

Note 3. In MUSE1, $NALX \leq 4$, and $KSX \leq KX$. In MUSE2 these quantities are further constrained to $NALX * KSX \leq 4$.

Note 4. Items 9 through 12 are used only in the special case of a one group problem, $KX = 1$. The quantity $PP(L,1)$ is the angular scattering cross section $\sigma_S(\mu_L)$ evaluated at

$$\mu_L = -1 + \Delta\mu(L-1)$$

$$\Delta\mu = 2/(LAX-1)$$

The kernel input tape is not used in this case.

Note 5. A second case may follow the first case. Two blank cards at the end of the deck will terminate the run.

4.4. Description of the Output

The printed output is accompanied by enough titles so that most quantities are well defined. The table headings require some further explanation.

"COL 1" is the first collision response of the detector given by Eq. (7) or (8) for MUSE2, and the energy integral of Eq. (7) or (8) for MUSE1.

"CØL 2," "CØL 3," and "CØL 4" are the second, third, and fourth collision responses of the detector given by Eq. (24) for MUSE2, and Eq. (26) for MUSE1.

"SUM" is the sum of the above four columns and is the total detector response.

"IDEAL" is the kernel $\Sigma_s(E' \rightarrow E, \mu)$ for MUSE2, and the cross section $\Sigma_s(E', \mu)$ for MUSE1, and is the value of "SUM" in the limit of the sample thickness approaching zero.

"CORRECTION" is the ratio "IDEAL"/"SUM" and is the number which multiplies the experimental value to obtain the corrected value.

In MUSE2 the energy integrals of "SUM" and "IDEAL" are printed at the bottom of the page and marked "INTEGRAL."

Sample output from MUSE1 and MUSE2 are given on the next two pages.

4.5. The Auxiliary Code MUSIN

The code MUSIN was written to facilitate preparation of the kernel input tape. MUSIN has four main features. First, most codes that compute scattering kernels utilize the detailed balance condition and compute $\sigma(E' \rightarrow E, \mu)$ at constant μ . This calculation is repeated for all μ and the order on the output tape is E, E', μ , where E is the most rapidly moving variable, and μ is the least rapidly moving variable. MUSIN rearranges this data so that the order is μ, E, E' as required by MUSE. A second feature is the ability to add a second kernel (in the

SLAB THICKNESS= 0.05486
 INCIDENT ENERGY= 0.0992
 INCIDENT ANGLE= 0.00

TH2	TH0	MU0	CORRECTION	COL 1	COL 2	COL 3	COL 4	SUM	IDEAL
0.	-0.	1.0000	4.993-01	3.535-02	3.453-02	5.871-03	1.325-03	7.707-02	3.848-02
10.00	10.00	0.9848	7.373-01	8.987-02	3.449-02	5.893-03	1.330-03	1.316-01	9.702-02
20.00	20.00	0.9397	9.254-01	2.511-01	3.435-02	5.954-03	1.342-03	2.927-01	2.709-01
30.00	30.00	0.8660	9.767-01	3.738-01	3.409-02	6.039-03	1.358-03	4.152-01	4.056-01
40.00	40.00	0.7660	9.742-01	3.239-01	3.360-02	6.121-03	1.373-03	3.550-01	3.556-01
50.00	50.00	0.6428	9.753-01	2.738-01	3.278-02	6.159-03	1.378-03	3.141-01	3.064-01
60.00	60.00	0.5000	9.861-01	2.242-01	3.139-02	6.089-03	1.357-03	2.630-01	2.594-01
70.00	70.00	0.3420	1.029+00	1.766-01	2.891-02	5.771-03	1.280-03	2.125-01	2.187-01
80.00	80.00	0.1736	1.215+00	1.218-01	2.338-02	4.741-03	1.038-03	1.510-01	1.835-01
90.00	90.00	-0.0000	4.534+06	2.633-08	6.280-09	1.195-09	2.456-10	3.405-08	1.544-01
100.00	100.00	-0.1736	1.168+00	8.613-02	2.055-02	4.617-03	1.036-03	1.123-01	1.313-01
110.00	110.00	-0.3420	9.291-01	8.996-02	2.455-02	5.667-03	1.282-03	1.215-01	1.128-01
120.00	120.00	-0.5000	8.444-01	8.391-02	2.530-02	5.982-03	1.363-03	1.165-01	9.841-02
130.00	130.00	-0.6428	7.988-01	7.720-02	2.503-02	6.041-03	1.385-03	1.097-01	8.760-02

Figure 3. Sample output page from MUSE 1

SLAB THICKNESS= 0.08128
 INCIDENT ENERGY= 0.0992
 INCIDENT ANGLE= 45.00
 FINAL ANGLE= 123.00
 SCATTERING ANGLE= 75.00
 COS OF SCAT. ANGLE= 0.2588

GROUP	E(FINAL)	CORRECTION	COL 1	COL 2	COL 3	COL 4	SUM	IDEAL
1	0.00025	1.261+00	1.178-02	9.539-03	5.218-03	2.383-03	2.892-02	3.646-02
2	0.00101	8.249-01	3.389-02	2.941-02	1.630-02	7.433-03	8.703-02	7.180-02
3	0.00228	7.075-01	5.881-02	5.296-02	2.963-02	1.346-02	1.549-01	1.096-01
4	0.00405	6.699-01	8.909-02	7.968-02	4.433-02	1.988-02	2.330-01	1.561-01
5	0.00632	6.768-01	1.316-01	1.104-01	5.986-02	2.621-02	3.281-01	2.220-01
6	0.00911	7.171-01	1.972-01	1.464-01	7.567-02	3.194-02	4.513-01	3.236-01
7	0.01240	7.794-01	3.002-01	1.892-01	9.122-02	3.567-02	6.173-01	4.812-01
8	0.01619	8.489-01	4.542-01	2.394-01	1.058-01	4.007-02	8.394-01	7.125-01
9	0.02049	9.106-01	6.623-01	2.958-01	1.193-01	4.191-02	1.118+00	1.018+00
10	0.02530	9.550-01	9.016-01	3.535-01	1.273-01	4.203-02	1.425+00	1.361+00
11	0.03061	9.791-01	1.115+00	4.009-01	1.309-01	4.040-02	1.687+00	1.652+00
12	0.03643	9.861-01	1.325+00	4.198-01	1.273-01	3.720-02	1.809+00	1.784+00
13	0.04275	9.840-01	1.181+00	3.920-01	1.161-01	3.286-02	1.722+00	1.694+00
14	0.04958	9.803-01	1.022+00	3.279-01	1.003-01	2.820-02	1.479+00	1.450+00
15	0.05692	9.838-01	8.892-01	2.728-01	8.526-02	2.409-02	1.272+00	1.252+00
16	0.06517	1.022+00	9.555-01	2.577-01	7.771-02	2.080-02	1.312+00	1.341+00
17	0.07484	1.070+00	1.314+00	3.110-01	7.717-02	1.828-02	1.720+00	1.841+00
18	0.08611	1.056+00	1.707+00	4.538-01	8.251-02	1.601-02	2.259+00	2.387+00
19	0.09917	9.684-01	1.596+00	6.046-01	7.946-02	1.276-02	2.293+00	2.221+00
20	0.11395	1.045+00	8.960-01	2.404-01	4.234-02	7.710-03	1.185+00	1.240+00
21	0.13122	1.074+00	3.640-01	8.060-02	1.898-02	4.055-03	4.676-01	5.021-01
22	0.15247	1.025+00	1.867-01	5.106-02	1.092-02	2.210-03	2.509-01	2.572-01
23	0.17899	1.030+00	7.126-02	1.856-02	4.325-03	9.100-04	9.505-02	9.791-02
24	0.21238	1.003+00	1.611-02	4.485-03	1.197-03	2.743-04	2.207-02	2.213-02
25	0.25461	9.933-01	2.690-03	8.038-04	2.375-04	5.819-05	3.991-03	3.964-03
26	0.30812	9.477-01	3.249-04	1.053-04	3.198-05	8.128-06	4.704-04	4.458-04
27	0.37594	9.307-01	2.197-05	6.695-06	2.047-06	5.228-07	3.123-05	2.907-05
28	0.46178	8.851-01	6.534-07	2.268-07	7.338-08	1.941-08	9.730-07	8.612-07
29	0.57016	8.424-01	8.212-09	3.201-09	1.085-09	2.969-10	1.079-08	1.078-08
30	0.70658	8.128-01	3.785-11	1.557-11	5.134-12	1.395-12	5.995-11	4.872-11
INTEGRAL							2.001-01	2.003-01

Figure 4. Sample output page from MUSE 2

form of Legendre components on decimal cards) to the main kernel. An elastic contribution case also can be read from decimal cards and added to the kernel. Finally, the Legendre moments of each of the above terms are computed and printed as a check on the data.

The input data for MUSIN consists of a binary kernel tape prepared by a code such as GAKER and a deck of decimal input cards. The output is a kernel tape suitable as input for MUSE, and printed output. The following tables describe the input data.

The program uses tape 11 for the kernel input, tape 12 for kernel output, tape 3 for scratch, tape 5 for decimal input, and tape 6 for printed output.

Scattering Kernel Input (Binary Tape 11) for MUSIN

Record	Symbol	Description
1	KX, LAX, (HØLA(I), I=1,12), (EN(K), K=1,KX)	Number of energy groups ≤ 40 Number of angular points (equally spaced) ≤ 31 Hollerith title Energy mesh (ev)
L+1	((PA(I, J), I=1,KX), J=1,KX)	The kernel $d^2\sigma/d\Omega dE$ evaluated at initial energy EN(J), final energy EN(I), and angle $\mu_L = -1 + (L-1)\Delta\mu$ $\Delta\mu = 2/(LAX-1)$

The tape will contain LAX + 1 records and an EOF mark.

Punched Card Input for MUSIN

Item	Columns	Format	Symbol	Description
1	1 - 72	12A6	HØLB	Title card, 72 characters
2	1 - 5	I5	NELT	If $NELT > 0$, an added elastic component will be read from cards (Item 4). If $NELT \leq 0$, the added elastic component will be zero.
	6 - 10	I5	NTK	Number of Legendre terms (≤ 4) of added kernel on decimal cards (Item 3) to be added to the kernel on tape 11.
	11 - 15	I5	N3	The Legendre components of the cross section to terms in N3 will be computed and printed.
	16 - 20	I5	NPT	If $NPT > 0$, the kernel output tape 12 will be printed.
	21 - 30	E10.5	SCALE	Scale factor applied to the added elastic component (Item 4).
	31 - 40	E10.5	SCALA	Scale factor applied to the added kernel (Item 3).
3	-	-	-	The added kernel decks (see format on page 35). NTK decks. Omit if $NTK \leq 0$.

Punched Card Input for MUSIN (cont.)

Item	Columns	Format	Symbol	Description
4	-	7E10.5	(X(L), L=1,LAX)	The added elastic component $\mu_L = -1 + (L-1)\Delta_L$, $\Delta_L = 2/(LAX-1)$ for each incident energy group. KX decks in all. Omit if NELT ≤ 0 .

Format of the Kernel Decks on Decimal Cards

Item	Columns	Format	Symbol	Description
1	1 - 54	9A6	HØL	54 character description of deck.
	55 - 60	I6	KX	Number of groups
	61 - 72	E12.5	T	Temperature (ev)
2	-	5E12.5	(B(KP,K), K=1,KP), KP=1,KX	The kernel $\sigma_{SL}(E' \rightarrow E)$, where index KP refers to E' , and K refers to E . Only the downscattering half is given. The remainder is constructed from detailed balance. See subroutine KERIN.

Except for the first card, this format is the standard GA format for kernel decks. A deck is punched for each Legendre component (denoted by l) of the kernel.

APPENDIX A.Spherical Harmonics

The spherical harmonic functions $Y_n^\nu(\underline{\Omega})$ are defined by the relationship

$$Y_n^\nu(\underline{\Omega}) = A_n^\nu P_n^\nu(\mu) e^{i\nu\psi}$$

where

$$A_n^\nu = \left[\frac{(2n+1)(n-\nu)!}{4\pi(n+\nu)!} \right]^{\frac{1}{2}}$$

and the $P_n^\nu(\mu)$ are the associated Legendre polynomials defined by

$$P_n^\nu(\mu) = (1-\mu^2)^{\frac{\nu}{2}} \frac{d^\nu}{d\mu^\nu} P_n(\mu)$$

and $P_n(\mu)$ are the ordinary Legendre polynomials. The $Y_n^\nu(\underline{\Omega})$ form an orthonormal set in the sense that

$$\int d\underline{\Omega} Y_n^\nu(\underline{\Omega}) Y_m^{*\beta}(\underline{\Omega}) = \delta_n^m \delta_\nu^\beta$$

where δ is the Kronecker delta, and the complex conjugate is defined by

$$Y_n^{*\nu}(\underline{\Omega}) = A_n^\nu P_n^\nu(\mu) e^{-i\nu\psi}$$

Several useful relations between the $P_n^\nu(\mu)$ are:

$$P_n^{-\nu}(\mu) = (-1)^\nu \frac{(n-\nu)!}{(n+\nu)!} P_n^\nu(\mu)$$

$$P_n^0(\mu) = P_n(\mu)$$

$$P_n^\nu(\mu) = 0 \text{ for } \nu > n$$

$$\mu P_n^\nu(\mu) = \frac{(n+\nu)}{(2n+1)} P_{n-1}^\nu(\mu) + \frac{(n+1-\nu)}{(2n+1)} P_{n+1}^\nu(\mu)$$

$$P_n^\nu(\mu) = \frac{1}{(2n+1)\sqrt{1-\mu^2}} \left[P_{n+1}^{\nu+1}(\mu) - P_{n-1}^{\nu+1}(\mu) \right]$$

$$P_n^\nu(\mu) = \frac{1}{(2n+1)\sqrt{1-\mu^2}} \left[(n+2-\nu)(\nu-1-n) P_{n+1}^{\nu-1}(\mu) + (n+\nu-1)(n+\nu) P_{n-1}^{\nu-1}(\mu) \right]$$

In the treatment of the scattering kernel it is necessary to relate the cosine of the scattering angle, μ_0 , with the directions $\underline{\Omega}$ and $\underline{\Omega}'$. Formally, $\mu_0 = \underline{\Omega} \cdot \underline{\Omega}'$. The addition theorem is useful here.

$$P_n(\mu_0) = \sum_{\nu=-n}^n \frac{(n-\nu)!}{(n+\nu)!} P_n^\nu(\mu) P_n^\nu(\mu') e^{i\nu(\Psi-\Psi')}$$

The following table lists the $P_n^\nu(\mu)$ for $n \leq 3$. The index k denotes the position in a one-dimensional array.

k	n	ν	$P_n^\nu(\mu)$	$4\pi (A_n^\nu)^2$
1	0	0	1	1
2	1	-1	$-\frac{1}{2}z$	6
3		0	μ	3
4		1	z	3/2
5	2	-2	$\frac{1}{8}z^2$	120
6		-1	$-\frac{1}{2}\mu z$	30
7		0	$\frac{1}{2}(3\mu^2-1)$	5
8		1	$3\mu z$	5/6
9		2	$3z^2$	5/24
10	3	-3	$-\frac{1}{48}z^3$	4340
11		-2	$\frac{1}{8}\mu z^2$	840
12		-1	$-\frac{1}{24}z(15\mu^2-3)$	84
13		0	$\frac{1}{2}\mu(5\mu^2-3)$	7
14		1	$\frac{1}{2}z(15\mu^2-3)$	7/12
15		2	$15\mu z^2$	7/120
16		3	$15z^3$	7/620

$$z = \sqrt{1-\mu^2}$$

$$k = 1+n(n+1)+\nu$$

APPENDIX B.Listing of the Codes

A listing of the Fortran cards for MUSIN, MUSE1, and MUSE2 is given on the following pages.

```

$IBFTG MUSIN LIST,DECK,REF
C=====MULTIPLE SCATTERING CODE, MUSIN=====
CMUSIN PREPARE INPUT TAPE FOR MUSE
  DIMENSION P(31,40,15),PA(40,40),PB(40,40,4),PC(31,40),
  1PD(40,40,4),PE(31,40),XS(40,10),XSA(40,10),XSE(40,10),XST(40,10),
  2XSL(10),X(40),EN(40),DE(40),HOLA(12),HOLB(12)
  COMMON P
  EQUIVALENCE (PB,P),(PC,P(6401)),(PD,P(7641)),(PE,P(14041)),
  1(XS,P(15281)),(XSA,P(15681)),(XSE,P(16081)),(XST,P(16481))
10 READ(5,20)HOLB,NELT,NTK,N3,NPT,SCALE,SCALA
20 FORMAT(12A6/4I5,2E10.5)
  REWIND 11
  READ(11)KX,LAX,HOLA,(EN(K),K=1,KX)
  WRITE(6,30)HOLA,HOLB,KX,LAX,NELT,SCALE,NTK,SCALA,N3
30 FORMAT(10H1MUSIN RUN//1H012A6/1H 12A6/15,9H ENERGIES/
  115,7H ANGLES/14H ELASTIC TEST=12,8H SCALE=1PE13.5/
  215,33H ADDED LEGENDRE TERMS WITH SCALE=1PE13.5/
  319H EDIT TO TERMS IN P12)
  DO 40 K=2,KX
40 DE(K)=0.5*(EN(K+1)-EN(K-1))
  DE(1)=0.5*(EN(1)+EN(2))
  DE(KX)=0.5*(EN(KX)-EN(KX-1))
  DEL=2./FLOAT(LAX-1)
  REWIND 3
  N3P=N3+1
  KDX=15
  KI=1
  KF=KDX
45 IF(KF-KX)60,60,50
50 KF=KX
60 KD=KF-KI+1
  DO 80 L=1,LAX
  READ(11)((PA(I,J),I=1,KX),J=1,KX)
  DO 70 J=1,KD
  JP=J+KI-1
  DO 70 I=1,KX
70 P(L,I,J)=PA(I,JP)
80 CONTINUE
  DO 90 J=1,KD
90 WRITE(3)((P(L,I,J),L=1,LAX),I=1,KX)
  REWIND 11
  IF(KF-KX)100,110,110
100 READ(11)
  KI=KI+KDX
  KF=KF+KDX
  GO TO 45
110 REWIND 3
  REWIND 12
  WRITE(12)HOLA,HOLB,KX,LAX,N3P
  WRITE(12)(EN(K),K=1,KX),(DE(K),K=1,KX)

```



```

      DO 120 N=1,N3P
      DO 120 J=1,KX
      XSA(J,N)=0.0
      DO 120 I=1,KX
120  PB(I,J,N)=0.0
      IF(NTK)160,160,130
130  WRITE(6,135)
135  FORMAT(19H0ADDED KERNEL IDENT)
      DO 150 N=1,NTK
      CALL KERIN(PA,EN,DE,X,SCALA)
      DO 140 J=1,KX
      XSA(J,N)=X(J)
      DO 140 I=1,KX
140  PB(I,J,N)=PA(I,J)
150  CONTINUE
160  IF(NELT)185,185,165
165  DO 175 J=1,KX
      READ(5,170)(X(L),L=1,LAX)
170  FORMAT(7E10.5)
      DO 175 L=1,LAX
      X(L)=X(L)*SCALE
175  PE(L,J)=X(L)
      CALL SIMP(X,XSL,LAX,N3,1)
      DO 180 N=1,N3P
180  XSE(J,N)=XSL(N)
      GO TO 200
185  DO 195 J=1,KX
      DO 190 L=1,LAX
190  PE(L,J)=0.0
      DO 195 N=1,N3P
195  XSE(J,N)=0.0
200  DO 220 J=1,KX
      READ(3)((PC(L,I),L=1,LAX),I=1,KX)
      DO 205 N=1,N3P
205  XS(J,N)=0.0
      DO 215 I=1,KX
      DO 210 L=1,LAX
210  X(L)=PC(L,I)
      CALL SIMP(X,XSL,LAX,N3,I=J)
      DO 215 N=1,N3P
      PD(I,J,N)=XSL(N)+PB(I,J,N)
215  XS(J,N)=XS(J,N)+XSL(N)*DE(I)
      DO 220 N=1,N3P
      XST(J,N)=XS(J,N)+XSA(J,N)+XSE(J,N)
220  PD(J,J,N)=PD(J,J,N)+XSE(J,N)/DE(J)
      REWIND 3
      WRITE(12)((XST(J,N),J=1,KX),N=1,N3P)
      WRITE(12)((PD(I,J,N),I=1,KX),J=1,KX),N=1,N3P)
      DO 255 J=1,KX
      READ(3)((PC(L,I),L=1,LAX),I=1,KX)

```

```

DO 245 I=1,KX
A=-1.0
DO 240 L=1,LAX
IF(L-LAX)235,225,235
225 IF(I-J)235,230,235
230 IF(PC(L,I))235,240,235
235 PC(L,I)=PC(L,I)+(PB(I,J,1)+3.0*A*PB(I,J,2)+2.5*(3.0*A*A-1.0)*
1PB(I,J,3)+3.5*A*(5.0*A*A-3.0)*PB(I,J,4))/12.5663708
240 A=A+DEL
245 CONTINUE
DO 250 L=1,LAX
250 PC(L,J)=PC(L,J)+PE(L,J)/DE(J)
255 WRITE(12)((PC(L,I),L=1,LAX),I=1,KX)
END FILE 12
REWIND 12
REWIND 3
REWIND 11
WRITE (6,310)
310 FORMAT(39HOLEGENDRE MOMENTS OF THE CROSS SECTIONS)
WRITE(6,320)
320 FORMAT(12HOMAIN KERNEL)
DO 330 N=1,N3P
M=N-1
330 WRITE(6,340)M,(XS(J,N),J=1,KX)
340 FORMAT(8HOSIGMA SI1/(1P5E15.5))
IF(NTK)380,380,350
350 WRITE(6,360)
360 FORMAT(13HODDED KERNEL)
DO 370 N=1,NIK
M=N-1
370 WRITE(6,340)M,(XSA(J,N),J=1,KX)
380 IF(NELT)420,420,390
390 WRITE(6,400)
400 FORMAT(14HODDED ELASTIC)
DO 410 N=1,N3P
M=N-1
410 WRITE(6,340)M,(XSE(J,N),J=1,KX)
420 IF(NPT)500,500,430
430 READ(12)HOLA,HOLB,KX,LAX,N3P
READ(12)(EN(K),K=1,KX),(DE(K),K=1,KX)
WRITE(6,432)HOLA,HOLB,KX,LAX,N3P,(EN(K),K=1,KX)
432 FORMAT(30H1FULL PRINT OF MUSE INPUT TAPE//1H 12A6/
11H 12A6/4H KX=I5/5H LAX=I5/5H N3P=I5//
212H ENERGY MESH/(1P10E11.4))
WRITE(6,434)(DE(K),K=1,KX)
434 FORMAT(18H0ENERGY INCREMENTS/(1P10E11.4))
READ(12)((XST(J,N),J=1,KX),N=1,N3P)
READ(12)((PD(I,J,N),I=1,KX),J=1,KX),N=1,N3P)
DO 436 N=1,N3P
M=N-1

```

```

436 WRITE(6,438)M,(XST(J,N),J=1,KX)
438 FORMAT(8H0SIGMA SI2/((1P10E11.4))
DO 442 N=1,N3P
M=N-1
WRITE(6,440)M
440 FORMAT(2H1PI2,20H COMPONENT OF KERNEL)
DO 442 J=1,KX
442 WRITE(6,444)J,(PD(I,J,N),I=1,KX)
444 FORMAT(15H0INCIDENT GROUP I4/((1P10E11.4))
DO 490 J=1,KX
READ(12)((PC(L,I),L=1,LAX),I=1,KX)
DO 445 L=1,LAX
PA(L,1)=X(L)
DO 445 I=1,KX
445 PA(L,1)=PA(L,1)+PC(L,I)*DE(I)
WRITE(6,450)J
450 FORMAT(22H1INCIDENT ENERGY GROUP I4)
DO 460 I=1,KX
460 WRITE(6,470)I,(PC(L,I),L=1,LAX)
470 FORMAT(38H0 ANGULAR DIST, FINAL ENERGY GROUP I4/((1P10E11.4))
WRITE(6,480)(PE(L,J),L=1,LAX)
480 FORMAT(26H0 ELASTIC ANGULAR DIST/((1P10E11.4))
WRITE(6,485)J,(PA(L,1),L=1,LAX)
485 FORMAT(39H0INTEGRATED ANGULAR DISTRIBUTION, GROUP I5/((1P10E11.4))
490 CONTINUE
REWIND 12
500 CALL EXIT
END

```

\$IBFTC SIMP LIST,REF

C=====MULTIPLE SCATTERING CODE, MUSEIN=====

C.SIMP SIMPSON RULE INTEGRATION

SUBROUTINE SIMP(XS,XSL,LAX,N3,KMKP)

DIMENSION XS(31),XSL(20),W(31),P(20)

10 LAXP=LAX

DELA=2./FLOAT(LAX-1)

N3P=N3+1

IF(KMKP)40,20,40

20 IF(XS(LAX))30,30,40

30 LAXP=LAX-2

40 DO50L=1,N3P

50 XSL(L)=0.0

DO60N=2,LAXP,2

W(N)=8.*3.1415927*DELA/3.

60 W(N-1)=0.5*W(N)

W(1)=W(2)/4.

W(LAXP)=W(1)

X=-1.0

DO90N=1,LAXP

P(1)=1.0

XSL(1)=XSL(1)+W(N)*XS(N)

```

      IF(N3)90,90,70
70 DO80L=1,N3
   M=L-1
   P(L+1)=(FLOAT(2*M+1)*X*P(L)-FLOAT(M)*P(L-1))/FLOAT(M+1)
80 XSL(L+1)=XSL(L+1)+W(N)*P(L+1)*XS(N)
90 X=X+DELA
   IF(KMKP)140,100,140
100 IF(XS(LAX))110,110,140
110 RTD=XS(LAX-1)*DELA*17.7715
   P(1)=0.0
   XSL(1)=XSL(1)+RTD
   IF(N3)140,140,120
120 DO130L=1,N3
   M=L-1
   P(L+1)=(FLOAT(2*M+1)/FLOAT(M+1))*(1.+P(L)-FLOAT(M)*P(L-1)/FLOAT(M+
11))
130 XSL(L+1)=XSL(L+1)+RTD*(1.-2.*P(L+1)*DELA/3.)
140 RETURN
   END

```

\$IBFIC KERIN

C=====MULTIPLE SCATTERING CODE MUSIN=====

CKERIN

```

      SUBROUTINE KERIN(P,EN,DE,XS,A)
      DIMENSION P(40,40),B(40,40),EN(40),DE(40),XS(40),HOL(9)
10 READ(5,20) HOL,KX,T
20 FORMAT(9A6,I6,E12.5)
   WRITE(6,30) HOL,KX,T
30 FORMAT(1H A6,8A6,6H KX=I3,5H T=F8.5)
   READ(5,35) ((B(KP,K),K=1,KP),KP=1,KX)
35 FORMAT(5E12.5)
   KXP=KX-1
   DO 40 KP=1,KXP
   KPP=KP-1
   DO 40 K=1,KPP
40 B(K,KP)=B(KP,K)*EN(KP)*EXP(-(EN(K)-EN(KP))/T)/EN(K)
   DO 50 KP=1,KX
   XS(KP)=0.0
   DO 50 K=1,KX
   XS(KP)=XS(KP)+A*B(KP,K)*DE(K)
50 P(K,KP)=A*B(KP,K)
   RETURN
   END

```

\$IBFTC MAIN LIST,REF

C=====MULTIPLE SCATTERING CODE, MUSE1=====

CMUSE1 MAIN PROGRAM

```

    DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
    1KST(30),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
    2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
    3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
    COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
    1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F, G,T,P,R,DEAL,RFC
    EQUIVALENCE (PP,FN),(X,FN(2000))
    DIMENSION HOL(12)
100 READ(5,110)HOL,KX,JX,N3,NAX,NA1X,NA2X,KSX,THICK,XA,SCALE
110 FORMAT (12A6/7I5/3E10.5)
    N3P=N3+1
    IF(NAX)113,113,115
113 NAX=8
    ANG(1)=.09501251
    ANG(2)=.28160354
    ANG(3)=.45801668
    ANG(4)=.61787624
    ANG(5)=.75540441
    ANG(6)=.86563119
    ANG(7)=.94457502
    ANG(8)=.98940092
    ANGW(1)=.18945061
    ANGW(2)=.18260342
    ANGW(3)=.16915651
    ANGW(4)=.14959598
    ANGW(5)=.12462897
    ANGW(6)=.09515851
    ANGW(7)=.06225352
    ANGW(8)=.02715246
    GO TO 125
115 READ(5,120)(ANG(N),N=1,NAX)
120 FORMAT(7E10.5)
    READ (5,120)(ANGW(N),N=1,NAX)
125 IF(KSX-KX)130,150,150
130 READ (5,140)(KST(K),K=1,KSX)
140 FORMAT(14I5)
    GO TO 170
150 KSX=KX
    DO 160K=1,KX
160 KST(K)=K
170 READ (5,120)(TH1(N),N=1,NA1X)
    READ (5,120)(TH2(N),N=1,NA2X)
    DEL=THICK/FLOAT(JX)
    WRITE (6,175)HOL
175 FORMAT('88H1MUSE1, MULTIPLE SCATTERING CORRECTIONS FOR A SINGLE DI
    FFERENTIAL SCATTERING EXPERIMENT. //1H 12A6)
    DO 180N=1,NA1X
180 ANG1(N)=COS(TH1(N)*3.1415927/180.0)

```

```

DO 190N=1,NA2X
190 ANG2(N)=ABS(COS(TH2(N)*3.1415927/180.0))
CALL KERN(SCALE,XA)
300 WRITE (6,310)JX,N3,THICK,XA
310 FORMAT(14H SPACE POINTS=I2/19H MAX LEGENDRE TERM=
1I2/18H SAMPLE THICKNESS=F10.5/4H XA=1PE12.4)
WRITE (6,320)NAX,(ANG(N),N=1,NAX)
320 FORMAT(19H0INTEGRATION ANGLES I10/(5F15.8))
WRITE (6,330)NAX,(ANGW(N),N=1,NAX)
330 FORMAT(20H0INTEGRATION WEIGHTSI10/(5F15.8))
WRITE (6,340)KX,(EN(K),K=1,KX)
340 FORMAT(9H0ENERGIES I10/(1P5E15.5))
WRITE(6,345)KX,(DE(K),K=1,KX)
345 FORMAT(15H0ENERGY WEIGHTSI10/(1P5E15.5))
WRITE (6,350)KXS,(KST(K),K=1,KXS)
350 FORMAT(23H0INCIDENT ENERGY GROUPSI10/(15I5))
WRITE (6,360)NA1X,(TH1(N),N=1,NA1X)
360 FORMAT(16H0INCIDENT ANGLES I10/(10F8.2))
WRITE (6,370)NA2X,(TH2(N),N=1,NA2X)
370 FORMAT(14H00OUTPUT ANGLES I10/(10F8.2))
M=0
DO 395N=1,N3P
380 WRITE (6,390)M,KX,(XSN(K,N),K=1,KX)
390 FORMAT(8H0SIGMA SI2,I10/(1P5E15.5))
395 M=M+1
500 CALL SLITE (1)
CALL TNML
DO 520NA1=1,NA1X
DO 520KSP=1,KXS
KS=KST(KSP)
CALL SOURC (KS,ANG1(NA1))
CALL ITER(-1)
CALL ITER(0)
CALL RESP(1)
CALL ITER(0)
CALL RESP(2)
CALL ITER(1)
CALL RESP(3)
520 CALL OUT(KSP,NA1)
GOTO100
END
$IBFTC KERN LIST,REF
C=====MULTIPLE SCATTERING CODE, MUSE1=====
CKERN1 PROCESS INPUT SCATTERING KERNEL
SUBROUTINE KERN(SCALE,XA)
DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
1KST(30),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KXS,DELA,ANG,
1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F, G,T,P,R,DEAL,RFC

```

```

EQUIVALENCE (PP,FN),(X,FN(2000))
DIMENSION HOLA(12),HOLB(12),XSL(20)
10 IF(KX-1)20,20,40
20 READ(5,30)N3PX,(XSN(1,N),N=1,N3PX)
30 FORMAT(15/(7E10.5))
   READ(5,30)LAX,(PP(L,1),L=1,LAX)
   DO 35 N=1,N3PX
35 P(1,1,N)=XSN(1,N)
   KX=1
   EN(1)=1.0
   DE(1)=1.0
   GO TO 90
40 READ(11)HOLA,HOLB,KXX,LAX,N3PX
   WRITE(6,50)HOLA,HOLB
50 FORMAT(1H012A6/1H 12A6)
   IF(KXX-KX)60,80,60
60 WRITE(6,70)
70 FORMAT(18HOWRONG KERNEL TAPE)
   CALL EXIT
80 READ(11)((EN(K),K=1,KX),(DE(K),K=1,KX)
   READ(11)((XSN(KP,N),KP=1,KX),N=1,N3PX)
   READ(11)((P(K,KP,N),K=1,KX),KP=1,KX),N=1,N3PX)
90 DO 100 N=1,N3PX
   DO 100 KP=1,KX
   XSN(KP,N)=SCALE*XSN(KP,N)
   DO 100 K=1,KX
100 P(K,KP,N)=SCALE*P(K,KP,N)
   DO 105 K=1,KX
105 XT(K)=XSN(K,1)+XA*SQRT(0.0253/EN(K))
   DELA=2.0/FLD(1,LAX-1)
   DO 170 KP=1,KX
   IF(KX-1)120,120,110
110 READ(11)((PP(L,K),L=1,LAX),K=1,KX)
120 DO 130 K=1,KX
   DO 130 L=1,LAX
130 PP(L,K)=SCALE*PP(L,K)
   DO150KS=1,KSX
   KSP=KS
   IF(KP-KST(KS))150,160,150
150 CONTINUE
   GOTO170
160 CALL FCR(KP,KSP)
170 CONTINUE
   IF(KX-1)190,190,180
180 REWIND 11
190 RETURN
END
$IBFTC FCR LIST,REF
C=====MULTIPLE SCATTERING CODE, MUSE1=====
CFCR 1 FIRST COLLISION RESPONSE
SUBROUTINE FCR(KP,KSP)

```

```

    DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
    1KST(30),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
    2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
    3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
    COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
    1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,  G,T,P,R,DEAL,RFC
    EQUIVALENCE (PP,FN),(X,FN(2000))
10 DO 110 NA1=1,NA1X
    X1=THICK*XT(KP)/ANG1(NA1)
    DO 110 NA2=1,NA2X
    THO=TH2(NA2)-TH1(NA1)
    CTHO=COS(THO*3.1415927/180.0)
    DEAL(NA2,NA1,KSP)=0.0
    RFC(NA2,NA1,KSP)=0.0
    L=1.+(CTHO+1.)/DELA
    IF(L-LAX)15,12,12
12 L=LAX-1
15 FRACT=(CTHO+1.-DELA*FLOAT(L-1))/DELA
    DO 100 KF=1,KX
    ANS=PP(L,KF)+FRACT*(PP(L+1,KF)-PP(L,KF))
    X2=THICK*XT(KF)/ANG2(NA2)
    IF(COS(TH2(NA2)*3.1415927/180.0))60,20,20
20 Y=X2-X1
    IF(ABS(Y)-.05)30,30,40
30 A=EXP(-X2)*(1.+Y*(0.5+Y*(0.16666667+Y*0.041666667)))
    GO TO 90
40 A=(EXP(-X1)-EXP(-X2))/Y
    GO TO 90
60 Y=X1+X2
    IF(Y-0.05)70,70,80
70 A=1.+Y*(-0.5+Y*(0.16666667-Y*0.041666667))
    GO TO 90
80 A=(1.-EXP(-Y))/Y
90 RFC(NA2,NA1,KSP)=RFC(NA2,NA1,KSP)+A*ANS*DE(KF)
    DEAL(NA2,NA1,KSP)=DEAL(NA2,NA1,KSP)+ANS*DE(KF)
100 CONTINUE
110 CONTINUE
    RETURN
    END
$IBFTC OUT LIST,REF
C=====MULTIPLE SCATTERING CODE, MUSE1=====
COUT 1 OUTPUT ROUTINE
    SUBROUTINE OUT(KSP,NA1)
    DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
    1KST(30),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
    2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
    3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
    COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
    1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,  G,T,P,R,DEAL,RFC
    EQUIVALENCE (PP,FN),(X,FN(2000))
100 KS=KST(KSP)

```



```

WRITE (6,140)THICK,EN(KS),ANG1(NA1)
WRITE (6,150)
WRITE (6,160)
DO 120 NA2=1,NA2X
TH0=TH2(NA2)-TH1(NA1)
CTHO=COS(TH0*3.1415927/180.0)
SUM=RFC(NA2,NA1,KSP)
DO 110 NC=1,3
110 SUM=SUM+R(NC,NA2)
CORR=DEAL(NA2,NA1,KSP)/SUM
WRITE (6,170)TH2(NA2),TH0,CTHO,CORR,RFC(NA2,NA1,KSP),(R(NC,NA2),
INC=1,3),SUM,DEAL(NA2,NA1,KSP)
120 CONTINUE
RETURN
140 FORMAT(16H1SLAB THICKNESS=F8.5/17H INCIDENT ENERGY=F8.4/16H INCIDE
INT ANGLE=F8.2)
150 FORMAT(101H0 TH2      TH0      MU0      CORRECTION      COL 1      COL
12      COL 3      COL 4      SUM      IDEAL)
160 FORMAT(103H -----
1-----
170 FORMAT(0PF7.2,F9.2,F9.4,1PE12.3,6E11.3)
END

$IBFTC YNNU LIST,REF
C=====MULTI-LE SCATTERING CODE, MUSE1=====
CYNNU1 ASSOCIATED LEGENDRE POLYNOMIALS
SUBROUTINE YNNU(X,Y,U)
DIMENSION P(16),U(16),A(16),B(16),Y(16)
10 CALL SLITET(1,K)
GOTO(20,40),K
20 B(1)=1.0
B(2)=6.0
B(3)=3.0
B(4)=1.5
B(5)=120.0
B(6)=30.0
B(7)=5.0
B(8)=5.0/6.0
B(9)=5.0/24.0
B(10)=4340.0
B(11)=840.0
B(12)=84.0
B(13)=7.0
B(14)=7.0/12.0
B(15)=7.0/120.0
B(16)=7.0/620.0
DO 30 N=1,16
B(N)=SQRT(B(N))
30 A(N)=B(N)*0.2820948
40 X2=X*X
Z=SQRT(1.0-X2)
Z2=1.0-X2

```

```

P(1)=1.0
P(2)=-0.5*Z
P(3)=X
P(4)=Z
P(5)=Z2/8.0
P(6)=-0.5*X*Z
P(7)=0.5*(3.0*X2-1.0)
P(8)=3.0*X*Z
P(9)=3.0*Z2
P(10)=-Z2*Z/48.0
P(11)=X*Z2/8.0
P(12)=-Z*(15.0*X2-3.0)/24.0
P(13)=0.5*(5.0*X2-3.0)*X
P(14)=0.5*Z*(15.0*X2-3.0)
P(15)=15.0*X*Z2
P(16)=15.0*Z*Z2
DO 50 N=1,16
Y(N)=A(N)*P(N)
50 U(N)=B(N)*P(N)
RETURN
END

```

\$IBFTC ITER LIST,REF

C=====MULTIPLE SCATTERING CODE, MUSE1=====

CITER1 ITERATION PROGRAM

```

SUBROUTINE ITER(NTAG)
  DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
  1KST(30),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
  2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
  3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
  COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
  1ANGW,1H1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F, G,T,P,R,DEAL,RFC
  EQUIVALENCE (PP,FN),(X,FN(2000))
  IF(NTAG)145,100,100
100 MNU=1
  M=0
110 NU=-M
  MP=M+1
115 DO 120J=1,JX
  DO 120K=1,KX
  H(J,MNU,K)=0.0
  DO 120KP=1,KX
120 H(J,MNU,K)=H(J,MNU,K)+P(K,KP,MP)*FN(J,MNU,KP)*DE(KP)
  NU=NU+1
  MNU=MNU+1
  IF(NU-M)115,115,130
130 M=M+1
  IF(M-N3)110,110,140
140 IF(NTAG)145,145,250
145 DO 240K=1,KX
  N=0
150 NU=-N

```

```

155 NNU=1+N*(N+1)+NU
    DO 158 I=1,JX
158 FN(I,NNU,K)=0.0
    M=0
160 CALL NMNU(NMN,N,M,NU)
    IF(NMN)210,210,170
170 MNU=1+M*(M+1)+NU
    SIGNA=1.0
    NPM=N+M
    IF(NPM-2*(NPM/2))180,180,175
175 SIGNA=-1.0
180 DO 200I=1,JX
    DO 200J=1,JX
    SGN=1.0
    IF(I-J)185,190,190
185 SGN =SIGNA
190 L=1+IABS(I-J)
200 FN(I,NNU,K)=FN(I,NNU,K)+H(J,MNU,K)*T(L,NMN,K)*SGN
210 M=M+1
    IF(M-N3)160,160,220
220 NU=NU+1
    IF(NU-N)155,155,230
230 N=N+1
    IF(N-N3)150,150,240
240 CONTINUE
250 CONTINUE
    RETURN
    END

```

\$IBFTC TNML LIST,REF

C=====MULTIPLE SCATTERING CODE, MUSE1=====

CTNML1 TRANSPORT AND OUTPUT KERNELS

SUBROUTINE TNML

```

    DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
    1KST(30),FN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
    2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
    3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
    COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
    1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F, G,I,P,R,DEAL,RFC
    EQUIVALENCE (PP,FN),(X,FN(2000))
    DIMENSION A(16),TEM(16)
100 DO 110NA=1,NAX
    CALL YNNU(ANG(NA),TEM,A)
    DO 110N=1,16
110 U(NA,N)=A(N)
    DO 200K=1,KX
    DO 120NA=1,NAX
    CALL FLEU(DEL,XT(K),ANG(NA),JX,A)
    DO 120L=1,JX
120 F(NA,L)=A(L)
    NMN=1
    N=0

```

```

130 M=0
150 NU=-M
160 NP=1+N*(N+1)+NU
    MP=1+M*(M+1)+NU
    DO 170L=1,JX
        T(L,NMN,K)=0.0
    DO 170NA=1,NAX
170 T(L,NMN,K)=T(L,NMN,K)+ANGW(NA)*U(NA,NP)*U(NA,MP)*F(NA,L)/ANG(NA)
    NPM=N+M
    IF(NPM-2*(NPM/2))177,177,173
173 T(1,NMN,K)=0.0
177 NMN=NMN+1
    NU=NU+1
    IF(NU-M)160,160,180
180 M=M+1
    IF(M-N)150,150,190
190 N=N+1
    IF(N-N3)130,130,200
200 CONTINUE
    DO 250NA=1,NA2X
    DO 250K=1,KX
        Z=DEL*XT(K)/ANG2(NA)
        E=EXP(-Z)
        IF(Z-0.05)210,210,220
210 AA=Z*(1.0+Z*(-0.5+Z*(0.16666667+Z*(-.41666667E-01))))
        GO TO 230
220 AA=1.0-E
230 BB=AA*ANG2(NA)/(THICK*XT(K))
    DO 240J=1,JX
        G(J,K,NA)=BB
240 BB=BB*E
250 CONTINUE
    RETURN
    END
$IBFIC SOURC LIST,REF
C=====MULTIPLE SCATTERING CODE, MUSE1=====
CSDUR1 FIRST COLLISION SOURCE
    SUBROUTINE SOURC(KS,ANGA)
        DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
        1KST(30),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
        2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
        3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
        COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
        1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F, G,T,P,R,DEAL,RFC
        EQUIVALENCE (PP,FN),(X,FN(2000))
        DIMENSION Y(16),B(16)
100 CALL YNNU(ANGA,Y,B)
        Z=XT(KS)*DEL/ANGA
        E=EXP(-Z)
        IF(Z-0.05)120,120,130
120 AA=1.0+Z*(0.5+Z*(0.16666667+Z*.41666667E-01))

```

```

      GO TO 140
130  AA=((1.0/E)-1.0)/Z
140  BB=E*AA
      DO 150 J=1,JX
      B(J)=BB
150  BB=BB*E
      DO 200 K=1,KX
      M=0
160  NU=-M
170  MNU=1+M*(M+1)+NU
      MP=M+1
      DO 180 J=1,JX
      H(J,MNU,K)=P(K,KS,MP)*B(J)*Y(MNU)
180  FN(J,MNU,K)=0.0
      NU=NU+1
      IF(NU-M)170,170,190
190  M=M+1
      IF(M-N3)160,160,200
200  CONTINUE
      RETURN
      END

$IBFTC FLEU      LIST,REF
C=====MULTIPLE SCATTERING CODE, MUSE1=====
CFLEU1  FL(E,MU) TRANSPORT KERNEL
      SUBROUTINE FLEU(DEL,SIG,ANG,J,F)
      DIMENSION F(5)
10  X=DEL*SIG/ABS(ANG)
      E=EXP(-X)
      IF(X-0.1)20,20,30
20  F(1)=C.5*DEL*(1.0+X*(-0.33333333+X*(.83333333E-01-.16666667E-01*X)
1))
      C=DEL*(1.0-.83333333E-01*X*X)
      GO TO 40
30  F(1)=(E-1.0+X)*DEL/(X*X)
      C=DEL*(1.0-E)*((1.0/E)-1.0)/(X*X)
40  B=C*E*0.5
      IF(J-1)60,60,45
45  DO 50 N=2,J
      F(N)=B
50  B=B*E
60  RETURN
      END

$IBFTC NMNU
C=====MULTIPLE SCATTERING CODE, MUSE1=====
CNMNU1  NMNU=1+N(N+1)(2N+1)/6+M(M+1)+NU
      SUBROUTINE NMNU(NM,N,M,NU)
10  NMN=0
20  NA=N
      MA=M
      IF(N-M)30,40,40
30  NA=M

```

```

      MA=N
40  IF(MA-IABS(NU))60,50,50
50  NMN=1+(NA*(NA+1)*(2*NA+1))/6+MA*(MA+1)+NU
60  RETURN
      END
$IBFTC RESP      LIST,REF
C=====MULTIPLE SCATTERING CODE, MUSE1=====
CRSP1  DETECTOR RESPONSE CALCULATION
      SUBROUTINE RESP(NC)
          DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
          1KST(30),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
          2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
          3R(3,15),DEAL(15, 4,30),RFC(15, 4,30)
          COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
          1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,      G,T,P,R,DEAL,RFC
          EQUIVALENCE (PP,FN),(X,FN(2000))
          DIMENSION A(16),B(16),C(16)
100  DO 320 NA2=1,NA2X
          THP=1H2(NA2)
          IF(THP-90.0)110,110,120
110  LAR=1
          GO TO 200
120  IF(THP-180.0)130,130,140
130  LAR=2
          GO TO 200
140  IF(THP-270.0)150,150,160
150  LAR=3
          GO TO 200
160  IF(THP-360.0)170,170,180
170  LAR=4
          GO TO 200
180  WRITE(6,190)THP
190  FORMAT(5HOTH2=F6.1,17H GREATER THAN 360)
          CALL EXIT
200  CALL YNNU(ANG2(NA2),A,C)
          CALL YNNU(-ANG2(NA2),B,C)
          DO 320 KF=1,KX
          AA=0.0
          DO 310 KF=1,KX
          N=0
210  NU=-N
220  NNU=1+N*(N+1)+NU
          SGN=1.
          IF(NU-2*(NU/2))230,240,230
230  SGN=-1.
240  DO 290 J=1,JX
          JP=JX J+1
          GO TO (250,260,270,280),LAR
250  AA=AA+A(NNU)*H(J,NNU,KF)*G(JP,KF,NA2)*DE(KF)
          GO TO 290
260  AA=AA+B(NNU)*H(J,NNU,KF)*G(J ,KF,NA2)*DE(KF)

```

```
GO TO 290
270 AA=AA+B(NNU)*H(J, NNU, KF)*G(J, KF, NA2)*DE(KF)*SGN
GO TO 290
280 AA=AA+A(NNU)*H(J, NNU, KF)*G(JP, KF, NA2)*DE(KF)*SGN
290 CONTINUE
    NU=NU+1
    IF(NU-N) 220, 220, 300
300 N=N+1
    IF(N-N3) 210, 210, 310
310 CONTINUE
320 R(NC, NA2)=AA
    RETURN
    END
```

```

$IBFTC MAIN LIST,REF
C=====MULTIPLE SCATTERING CODE, MUSE2=====
CMUSE2 MAIN PROGRAM
  DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
  1KST( 4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
  2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
  3R(3,30,15),DEAL(30,15,4),RFC(30,15,4)
  COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
  1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,G,T,P,R,DEAL,RFC
  EQUIVALENCE (PP,FN),(X,FN(2000))
  DIMENSION HOL(12)
100 READ(5,110)HOL,KX,JX,N3,NAX,NA1X,NA2X,KSX,THICK,XA,SCALE
110 FORMAT(12A6/7I5/3E10.5)
  N3P=N3+1
  IF(NAX)113,113,115
113 NAX=8
  ANG(1)=.09501251
  ANG(2)=.28160354
  ANG(3)=.45801668
  ANG(4)=.61787624
  ANG(5)=.75540441
  ANG(6)=.86563119
  ANG(7)=.94457502
  ANG(8)=.98940092
  ANGW(1)=.18945061
  ANGW(2)=.18260342
  ANGW(3)=.16915651
  ANGW(4)=.14959598
  ANGW(5)=.12462897
  ANGW(6)=.09515851
  ANGW(7)=.06225352
  ANGW(8)=.02715246
  GO TO 125
115 READ(5,120)(ANG(N),N=1,NAX)
120 FORMAT(7E10.5)
  READ (5,120)(ANGW(N),N=1,NAX)
125 IF(KSX-KX)130,150,150
130 READ (5,140)(KST(K),K=1,KSX)
140 FORMAT(14I5)
  GO TO 170
150 KSX=KX
  DO 160K=1,KX
160 KST(K)=K
170 READ (5,120)(TH1(N),N=1,NA1X)
  READ (5,120)(TH2(N),N=1,NA2X)
  DEL=THICK/FLOAT(JX)
  WRITE (6,175)HOL
175 FORMAT(18H1MUSE2, MULTIPLE SCATTERING CORRECTIONS FOR A DOUBLE DI-
  FFERENTIAL SCATTERING EXPERIMENT //1H 12A6)
  DO 180N=1,NA1X

```



```

180 ANG1(N)=COS(TH1(N)*3.1415927/180.0)
DO 190N=1,NA2X
190 ANG2(N)=ABS(COS(TH2(N)*3.1415927/180.0))
CALL KERN(SCALE,XA)
300 WRITE (6,310)JX,N3,THICK,XA
310 FORMAT(14H SPACE POINTS=I2/19H MAX LEGENDRE TERM=
1I2/18H SAMPLE THICKNESS=F10.5/4H XA=1PE12.4)
WRITE (6,320)NAX,(ANG(N),N=1,NAX)
320 FORMAT(19HOINTEGRATION ANGLES I10/(5F15.8))
WRITE (6,330)NAX,(ANGW(N),N=1,NAX)
330 FORMAT(20HOINTEGRATION WEIGHTS I10/(5F15.8))
WRITE (6,340)KX,(EN(K),K=1,KX)
340 FORMAT(9HOENERGIES I10/(1P5E15.5))
WRITE(6,345)KX,(DE(K),K=1,KX)
345 FORMAT(15HOENERGY WEIGHTS I10/(1P5E15.5))
WRITE (6,350)KSX,(KST(K),K=1,KSX)
350 FORMAT(23HOINCIDENT ENERGY GROUPS I10/(15I5))
WRITE (6,360)NA1X,(TH1(N),N=1,NA1X)
360 FORMAT(16HOINCIDENT ANGLES I10/(10F8.2))
WRITE (6,370)NA2X,(TH2(N),N=1,NA2X)
370 FORMAT(14HOUTPUT ANGLES I10/(10F8.2))
M=0
DO 395N=1,N3P
380 WRITE (6,390)M,KX,(XSN(K,N),K=1,KX)
390 FORMAT(8HOSIGMA SI2,I10/(1P5E15.5))
395 M=M+1
500 CALL SLITE (1)
CALL TNML
DO 520NA1=1,NA1X
DO 520KSP=1,KSX
KS=KST(KSP)
CALL SOURC (KS,ANG1(NA1))
CALL ITER(-1)
CALL ITER(0)
CALL RESP(1)
CALL ITER(0)
CALL RESP(2)
CALL ITER(1)
CALL RESP(3)
520 CALL OUT(KSP,NA1)
GOTO100
END

```

\$1BFTC KERN LIST,REF

C=====MULTIPLE SCATTERING CODE, MUSE2=====

CKERN2 PROCESS INPUT SCATTERING KERNEL

SUBROUTINE KERN(SCALE,XA)

```

DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
1KST( 4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
3R(3,30,15),DEAL(30,15,4),RFC(30,15,4)

```

```

COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,G,T,P,R,DEAL,REFC
EQUIVALENCE (PP,FN),(X,FN(2000))
DIMENSION HOLA(12),HOLB(12),XSL(20)
10 IF(KX-1)20,20,40
20 READ(5,30)N3PX,(XSN(1,N),N=1,N3PX)
30 FORMAT(15/(7E10.5))
READ(5,30)LAX,(PP(L,1),L=1,LAX)
DO 35 N=1,N3PX
35 P(1,1,N)=XSN(1,N)
KX=1
EN(1)=1.0
DE(1)=1.0
GO TO 90
40 READ(11)HOLA,HOLB,KXX,LAX,N3PX
WRITE(6,50)HOLA,HOLB
50 FORMAT(1H012A6/1H 12A6)
IF(KXX-KX)60,80,60
60 WRITE(6,70)
70 FORMAT(18HOWRONG KERNEL TAPE)
CALL EXIT
80 READ(11)(EN(K),K=1,KX),(DE(K),K=1,KX)
READ(11)((XSN(KP,N),KP=1,KX),N=1,N3PX)
READ(11)((P(K,KP,N),K=1,KX),KP=1,KX),N=1,N3PX)
90 DO 100 N=1,N3PX
DO 100 KP=1,KX
XSN(KP,N)=SCALE*XSN(KP,N)
DO 100 K=1,KX
100 P(K,KP,N)=SCALE*P(K,KP,N)
DO 105 K=1,KX
105 XT(K)=XSN(K,1)+XA*SQRT(0.0253/EN(K))
DELA=2.0/FLOAT(LAX-1)
DO 170 KP=1,KX
IF(KX-1)120,120,110
110 READ(11)((PP(L,K),L=1,LAX),K=1,KX)
120 DO 130 K=1,KX
DO 130 L=1,LAX
130 PP(L,K)=SCALE*PP(L,K)
DO 150 KS=1,KSX
KSP=KS
IF(KP-KST(KS))150,160,150
150 CONTINUE
GOTO 170
160 CALL FCR(KP,KSP)
170 CONTINUE
IF(KX-1)190,190,180
180 REWIND 11
190 RETURN
END
$IBFTC FCR LIST,REF

```

*=====MULTIPLE SCATTERING CODE, MUSE2=====

CFCR 2 FIRST COLLISION RESPONSE

SUBROUTINE FCR(KP,KSP)

DIMENSION ANG(20),ANGW(20),TH1(4),ANG1(4),TH2(15),ANG2(15),
1KST(4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
3R(3,30,15),DEAL(30,15,4),RFC(30,15,4)
COMMON EN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,G,T,P,R,DEAL,RFC
EQUIVALENCE (PP,FN),(X,FN(2000))

10 DO 110 NA1=1,NA1X

LKN=NA1+NA1X*(KSP-1)

X1=THICK*XT(KP)/ANG1(NA1)

DO 110 NA2=1,NA2X

THO=TH2(NA2)-TH1(NA1)

CTHO=COS(THO*3.1415927/180.0)

L=1.+(CTHO+1.)/DELA

IF(L-LAX)15,12,12

12 L=LAX-1

15 FRACT=(CTHO+1.-DELA*FLOAT(L-1))/DELA

DO 100 KF=1,KX

ANS=PP(L,KF)+FRACT*(PP(L+1,KF)-PP(L,KF))

DEAL(KF,NA2,LKN)=ANS

X2=THICK*XT(KF)/ANG2(NA2)

IF(COS(TH2(NA2)*3.1415927/180.0))60,20,20

20 Y=X2-X1

IF(ABS(Y)-.05)30,30,40

30 A=EXP(-X2)*(1.+Y*(0.5+Y*(0.16666667+Y*0.041666667)))

GO TO 90

40 A=(EXP(-X1)-EXP(-X2))/Y

GO TO 90

60 Y=X1+X2

IF(Y-0.05)70,70,80

70 A=1.+Y*(-0.5+Y*(0.16666667-Y*0.041666667))

GOTO 90

80 A=(1.-EXP(-Y))/Y

90 RFC(KF,NA2,LKN)=A*ANS

100 CONTINUE

110 CONTINUE

RETURN

END

\$IBFTC OUT LIST,REF

*=====MULTIPLE SCATTERING CODE, MUSE2=====

SUBROUTINE OUT(KSP,NA1)

COUT 2 OUTPUT ROUTINE

DIMENSION ANG(20),ANGW(20),TH1(4),ANG1(4),TH2(15),ANG2(15),
1KST(4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
3R(3,30,15),DEAL(30,15,4),RFC(30,15,4)
COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,

```

1 LANGW, TH1, ANG1, TH2, ANG2, KST, EN, DE, XT, XSN, U, F, G, T, P, R, DEAL, RFC
EQUIVALENCE (PP, FN), (X, FN(2000))
100 LKN=NA1+NA1X*(KSP-1)
    KS=KST(KSP)
    DO 200 NA2=1, NA2X
        THO=TH2(NA2)-TH1(NA1)
        CTHO=COS(THO*3.1415927/180.0)
        WRITE(6, 130) THICK, EN(KS), TH1(NA1), TH2(NA2), THO, CTHO
130 FORMAT(16H1SLAB THICKNESS=F8.5/17H INCIDENT ENERGY=
1F8.4/16H INCIDENT ANGLE=F8.2/13H FINAL ANGLE=F8.2/
218H SCATTERING ANGLE=F8.2/20H COS OF SCAT. ANGLE=
3F8.4)
    WRITE(6, 140)
140 FORMAT(109HOGROUP E(FINAL) CORRECTION COL 1 COL 2
1COL 3 COL 4 SUM IDEAL
    WRITE(6, 150)
150 FORMAT(111H -----)
1-----)
    SUM1=0.0
    SUM2=0.0
    DO180KF=1, KX
    SUM=RFC(KF, NA2, LKN)
    DO160NC=1, 3
160 SUM=SUM+R(NC, KF, NA2)
    CORR=DEAL(KF, NA2, LKN)/SUM
    WRITE(6, 170) KF, EN(KF), CORR, RFC(KF, NA2, LKN), (R(NC, KF, NA2),
1NC=1, 3), SUM, DEAL(KF, NA2, LKN)
170 FORMAT(14, F11.5, 1P7E12.3)
    SUM1=SUM1+SUM*DE(KF)
    SUM2=SUM2+DEAL(KF, NA2, LKN)*DE(KF)
180 CONTINUE
    WRITE(6, 190) SUM1, SUM2
190 FORMAT(79X21H-----/68X8HINTEGRAL1P2E12.3)
200 CONTINUE
    RETURN
    END
$IBFTC YNNU LIST, REF
*=====MULTIPLE SCATTERING CODE, MUSE2=====
CYNNU2 ASSOCIATED LEGENDRE POLYNOMIALS
    SUBROUTINE YNNU(X, Y, U)
    DIMENSION P(16), U(16), A(16), B(16), Y(16)
10 CALL SLITET(1, K)
    GOTO(20, 40), K
20 B(1)=1.0
    B(2)=6.0
    B(3)=3.0
    B(4)=1.5
    B(5)=120.0
    B(6)=30.0
    B(7)=5.0

```

```

B(8)=5.0/6.0
B(9)=5.0/24.0
B(10)=4340.0
B(11)=840.0
B(12)=84.0
B(13)=7.0
B(14)=7.0/12.0
B(15)=7.0/120.0
B(16)=7.0/620.0
DO 30 N=1,16
  B(N)=SQRT(B(N))
30 A(N)=B(N)*0.2820948
40 X2=X*X
  Z=SQRT(1.0-X2)
  Z2=1.0-X2
  P(1)=1.0
  P(2)=-0.5*Z
  P(3)=X
  P(4)=Z
  P(5)=Z2/8.0
  P(6)=-0.5*X*Z
  P(7)=0.5*(3.0*X2-1.0)
  P(8)=3.0*X*Z
  P(9)=3.0*Z2
  P(10)=-Z2*Z/48.0
  P(11)=X*Z2/8.0
  P(12)=-Z*(15.0*X2-3.0)/24.0
  P(13)=0.5*(5.0*X2-3.0)*X
  P(14)=0.5*Z*(15.0*X2-3.0)
  P(15)=15.0*X*Z2
  P(16)=15.0*Z*Z2
DO 50 N=1,16
  Y(N)=A(N)*P(N)
50 U(N)=B(N)*P(N)
  RETURN
  END

```

```

$IBFTC ITER LIST,REF

```

```

=====MULTIPLE SCATTERING CODE, MUSE2=====

```

```

CITER2 ITERATION PROGRAM

```

```

  SUBROUTINE ITER(NTAG)

```

```

    DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
    1KST( 4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
    2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
    3R(3,30,15),DEAL(30,15,4),RFC(30,15,4)
    COMMON EN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
    1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,G,T,P,R,DEAL,RFC
    EQUIVALENCE (PP,FN),(X,FN(2000))
    IF(NTAG)145,100,100

```

```

100 MNU=1

```

```

  M=0

```

```

110 NU=-M
    MP=M+1
115 DO 120J=1,JX
    DO 120K=1,KX
    H(J,MNU,K)=0.0
    DO 120KP=1,KX
120 H(J,MNU,K)=H(J,MNU,K)+P(K,KP,MP)*FN(J,MNU,KP)*DE(KP)
    NU=NU+1
    MNU=MNU+1
    IF(NU-M)115,115,130
130 M=M+1
    IF(M-N3)110,110,140
140 IF(NTAG)145,145,250
145 DO 240K=1,KX
    N=0
150 NU=-N
155 NNU=1+N*(N+1)+NU
    DO 158 I=1,JX
158 FN(I,NNU,K)=0.0
    M=0
160 CALL NMNU(NMN,N,M,NU)
    IF(NMN)210,210,170
170 MNU=1+M*(M+1)+NU
    SIGNA=1.0
    NPM=N+M
    IF(NPM-2*(NPM/2))180,180,175
175 SIGNA=-1.0
180 DO 200I=1,JX
    DO 200J=1,JX
    SGN=1.0
    IF(I-J)185,190,190
185 SGN=-SIGNA
190 L=1+IABS(I-J)
200 FN(I,NNU,K)=FN(I,NNU,K)+H(J,MNU,K)*T(L,NMN,K)*SGN
210 M=M+1
    IF(M-N3)160,160,220
220 NU=NU+1
    IF(NU-N)155,155,230
230 N=N+1
    IF(N-N3)150,150,240
240 CONTINUE
250 CONTINUE
    RETURN
    END
$IBFTC TNML LIST,REF
=====MULTIPLE SCATTERING CODE, MUSE2=====
CTNML2 TRANSPORT AND OUTPUT KERNELS
SUBROUTINE TNML
    DIMENSION ANG(20),ANGW(20),TH1( 4),ANG1( 4),TH2(15),ANG2(15),
    1KST( 4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),

```

```

2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
3R(3,30,15),DEAL(30,15,4),RFC(30,15,4)
COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,G,T,P,R,DEAL,RFC
EQUIVALENCE (PP,FN),(X,FN(2000))
DIMENSION A(16),TEM(16)
100 DO 110NA=1,NAX
    CALL YNNU(ANG(NA),TEM,A)
    DO 110N=1,16
110 U(NA,N)=A(N)
    DO 200K=1,KX
    DO 120NA=1,NAX
    CALL FLEU(DEL,XT(K),ANG(NA),JX,A)
    DO 120L=1,JX
120 F(NA,L)=A(L)
    NMN=1
    N=0
130 M=0
150 NU=-M
160 NP=1+N*(N+1)+NU
    MP=1+M*(M+1)+NU
    DO 170L=1,JX
    T(L,NMN,K)=0.0
    DO 170NA=1,NAX
170 T(L,NMN,K)=T(L,NMN,K)+ANGW(NA)*U(NA,NP)*U(NA,MP)*F(NA,L)/ANG(NA)
    NPM=N+M
    IF(NPM-2*(NPM/2))177,177,173
173 T(1,NMN,K)=0.0
177 NMN=NMN+1
    NU=NU+1
    IF(NU-M)160,160,180
180 M=M+1
    IF(M-N)150,150,190
190 N=N+1
    IF(N-N3)130,130,200
200 CONTINUE
    DO 250NA=1,NA2X
    DO 250K=1,KX
    Z=DEL*XT(K)/ANG2(NA)
    E=EXP(-Z)
    IF(Z-0.05)210,210,220
210 AA=Z*(1.0+Z*(-0.5+Z*(0.16666667+Z*(-.41666667E-01))))
    GO TO 230
220 AA=1.0-E
230 BB=AA*ANG2(NA)/(THICK*XT(K))
    DO 240J=1,JX
    G(J,K,NA)=BB
240 BB=BB*E
250 CONTINUE
    RETURN

```

END

\$IBFTC SOURC LIST,REF

*=====MULTIPLE SCATTERING CODE, MUSE2=====

CSOUR2 FIRST COLLISION SOURCE

SUBROUTINE SOURC(KS,ANGA)

DIMENSION ANG(20),ANGW(20),TH1(4),ANG1(4),TH2(15),ANG2(15),
 1KST(4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
 2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
 3R(3,30,15),DEAL(30,15,4),RFC(30,15,4)

COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
 1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,G,T,P,R,DEAL,RFC

EQUIVALENCE (PP,FN),(X,FN(2000))

DIMENSION Y(16),B(16)

100 CALL YNNU(ANGA,Y,B)

Z=XT(KS)*DEL/ANGA

E=EXP(-Z)

IF(Z-0.05)120,120,130

120 AA=1.0+Z*(0.5+Z*(0.16666667+Z*.41666667E-01))

GO TO 140

130 AA=((1.0/E)-1.0)/Z

140 BB=E*AA

DO 150 J=1,JX

B(J)=BB

150 BB=BB*E

DO 200 K=1,KX

M=0

160 NU=-M

170 MNU=1+M*(M+1)+NU

MP=M+1

DO 180 J=1,JX

H(J,MNU,K)=P(K,KS,MP)*B(J)*Y(MNU)

180 FN(J,MNU,K)=0.0

NU=NU+1

IF(NU-M)170,170,190

190 M=M+1

IF(M-N3)160,160,200

200 CONTINUE

RETURN

END

\$IBFTC FLEU LIST,REF

*=====MULTIPLE SCATTERING CODE, MUSE2=====

CFLEU2 FL(E,MU) TRANSPORT KERNEL

SUBROUTINE FLEU(DEL,SIG,ANG,J,F)

DIMENSION F(5)

10 X=DEL*SIG/ABS(ANG)

E=EXP(-X)

IF(X-0.1)20,20,30

20 F(1)=0.5*DEL*(1.0+X*(-0.33333333+X*(.83333333E-01-.16666667E-01*X)
 1))

C=DEL*(1.0-.83333333E-01*X*X)


```

      GO TO 40
30  F(1)=(E-1.0+X)*DEL/(X*X)
      C=DEL*(1.0-E)*((1.0/E)-1.0)/(X*X)
40  B=C*E*0.5
      IF(J-1)60,60,45
45  DO 50 N=2,J
      F(N)=B
50  B=B*E
60  RETURN
      END

```

\$IBFTC NMNU

=====MULTIPLE SCATTERING CODE, MUSE2=====

CNMNU2 NMNU=1+N(N+1)(2N+1)/6+M(M+1)+NU

SUBROUTINE NMNU(NMN,N,M,NU)

```

10  NMN=0
20  NA=N
      MA=M
      IF(N-M)30,40,40
30  NA=M
      MA=N
40  IF(MA-IABS(NU))60,50,50
50  NMN=1+(NA*(NA+1)*(2*NA+1))/6+MA*(MA+1)+NU
60  RETURN
      END

```

\$IBFTC RESP LIST,REF

=====MULTIPLE SCATTERING CODE, MUSE2=====

CRESP2 DETECTOR RESPONSE CALCULATION

SUBROUTINE RESP(NC)

```

      DIMENSION ANG(20),ANGW(20),TH1(4),ANG1(4),TH2(15),ANG2(15),
      IKST(4),EN(30),DE(30),XT(30),X(31),XSN(30,4),U(20,16),F(20,5),
      2PP(31,30),FN(5,16,30),H(5,16,30),G(5,30,15),T(5,30,30),P(30,30,4),
      3R(3,30,15),DEAL(30,15,4),KFC(30,15,4)
      COMMON FN,H,KX,JX,THICK,DEL,N3,N3P,LAX,NAX,NA1X,NA2X,KSX,DELA,ANG,
      1ANGW,TH1,ANG1,TH2,ANG2,KST,EN,DE,XT,XSN,U,F,G,T,P,R,DEAL,RFC
      EQUIVALENCE (PP,FN),(X,FN(2000))
      DIMENSION A(16),B(16),C(16)
100  DO 320 NA2=1,NA2X
      THP=TH2(NA2)
      IF(THP-90.0)110,110,120
110  LAR=1
      GO TO 200
120  IF(THP-180.)130,130,140
130  LAR=2
      GO TO 200
140  IF(THP-270.)150,150,160
150  LAR=3
      GO TO 200
160  IF(THP-360.)170,170,180
170  LAR=4
      GO TO 200

```

```

180 WRITE(6,190)THP
190 FORMAT(5H0TH2=F6.1,17H GREATER THAN 360)
    CALL EXIT
200 CALL YNNU(ANG2(NA2),A,C)
    CALL YNNU(-ANG2(NA2),B,C)
    DO 320 KF=1,KX
    R(NC,KF,NA2)=0.
    N=0
210 NU=-N
220 NNU=1+N*(N+1)*NU
    SGN=1.
    IF(NU-2*(NU/2))230,240,230
230 SGN=-1.
240 DO 290 J=1,JX
    JP=JX-J+1
    GO TO (250,260,270,280),LAR
250 R(NC,KF,NA2)=R(NC,KF,NA2)+A(NNU)*H(J,NNU,KF)*G(JP,KF,NA2)
    GO TO 290
260 R(NC,KF,NA2)=R(NC,KF,NA2)+B(NNU)*H(J,NNU,KF)*G(J,KF,NA2)
    GO TO 290
270 R(NC,KF,NA2)=R(NC,KF,NA2)+B(NNU)*H(J,NNU,KF)*G(J,KF,NA2)*SGN
    GO TO 290
280 R(NC,KF,NA2)=R(NC,KF,NA2)+A(NNU)*H(J,NNU,KF)*G(JP,KF,NA2)*SGN
290 CONTINUE
    NU=NU+1
    IF(NU-N)220,220,300
300 N=N+1
    IF(N=N3)210,210,310
310 CONTINUE
320 CONTINUE
    RETURN
    END

```