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CORE DESIGN DEVELOPMENT NEEDS IN RELATION TO FUEL FAILURE PROPAGATION, SODIUM BOILING AND CLAD/FUEL-SODIUM THERMAL INTERACTION

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U.S. ATOMIC ENERGY COMMISSION
CONTRACT AT(04-3)-189 PROJECT AGREEMENT 10

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ABSTRACT

A comprehensive assessment of safety related core design information needs was conducted. A basic hypothesis was that core performance under off-design and accident conditions will affect the choice of core design parameters. The economic consequences of fuel failure effects and random accidents were compared with fault tree results based on a "mean-annual-cost" concept.

It is concluded that fuel failure effects dominate the expected cost penalties. Principal development recommendations are for integral tests of: 1) fuel failure propagation and 2) loss-of-cooling consequences.

INTRODUCTION

Volume one of this report contained the fault tree results, conclusions and recommendations of this study. This second volume contains the details of the review and analysis of relevant phenomena.

This study was begun with an organization of the events and mechanisms describing possibly hazardous and costly malfunctions of an LMFBR. This organization took the form of a fault tree. A review of the state-of-the-art was used to determine the initial estimates of event frequency and transition probabilities. The resultant output of the fault tree in terms of mean-annual-cost showed which mechanisms and events were the most costly.

Specific questions were formulated from the dominant sequence of events identified from this preliminary exercise of the fault trees. This second volume reports the results of the studies aimed at answering these questions. Since this program was not continued into 1971, these analyses represent the interim status of the expected results. Therefore, the results presented are sometimes fragmentary and incomplete. These interim results were nevertheless used to formulate the final fault tree analysis and determination of priorities in order to present as complete an analysis as possible.

These key questions are arranged below in groups to show the generic dependence of the phenomena of interest. The measure of the significance of these phenomena is their effect on the core components. The damage threshold of the fuel pins and channels are thus needed. The first three questions identified relate to the damage threshold.

1. What degree of local overheating can be tolerated without clad failure, on a fuel pin operating at full power in the reactor? For how long? This question is addressed in Section 4.1.
2. What degree of local overheating can be tolerated without failure, on a channel wall of a fuel bundle operating at full power? This question is discussed in Section 5.1.
3. How much energy is required to cause failure of either a fuel pin or a channel wall? This question is discussed in Section 4.2 for fuel pins and 5.2 for channels.

Questions 4-6 relate to one mechanism of possible fuel pin failure propagation, fuel swelling. This is discussed in Section 6.1.

4. How much time is required for either fuel swelling or pellet ratcheting, or both, an initial hair-line crack failure in an operating fuel pin progress from the state where only fission gas is being released to the coolant to the stage where significant quantities of sodium are admitted to the pin?
5. What is the rate and extent of fuel pin swelling due to chemical interaction of fuel and sodium in a fuel pin which is operating at full power with a large area clad failure? To what extent will the reaction product extrude from the crack and reduce the coolant flow area locally?
6. What is the relationship between diametral swelling of one fuel pin and the degree of local overheating of its neighbors? To what extent does bowing of the pins contribute to the local overheating?

Questions 7-11 relate to the conditions and consequences of burst fuel failures involving fuel ejection into the coolant stream. These questions are discussed in Section 6.2.

7. Under what conditions can either fission gas pressure or sodium vapor pressure in the central void and fuel matrix suddenly expel fuel from a fuel pin which is operating with a large area clad failure? How much is expelled? What will be the size distribution of the particles?
8. How much fuel material will be rapidly expelled into the coolant, by fission gas or sodium vapor pressure after a burst failure occurs? What is the size distribution of the fuel particles?
9. What sizes and shapes of fuel and clad debris from either a large area failure or a burst failure are most likely to be trapped in the core region of an operating fuel bundle (both between adjacent pins and between pins and the channel wall)?
10. What is the effect of trapped fuel debris and clad debris on the local clad temperature of an operating fuel pin? To what extent does bowing of the pins contribute to the overheating?
11. What is the effect of trapped fuel debris and clad debris (either external or internal) on the local channel wall temperature? To what extent does bowing of the pins and of the channel walls contribute to the overheating?

The fuel-sodium thermal interaction possible when a pin bursts, question 12, is discussed in Section 7.3.

12. How much energy will be contained in the shock wave produced when several operating fuel pins burst and hot fuel contacts the coolant? What will be the effect of fission gas?

The last group of questions relates to the consequences of bundle loss-of-flow accidents (or complete deterioration of a fuel failure propagation situation). The mechanisms are discussed in Section 7. The energy absorption question (#18) is discussed in Section 5.3.

13. Under what conditions (fuel and sodium temperature, sodium velocity, channel pressure drop, degree of contact) will sodium boiling due to sudden contact with hot fuel in one fuel bundle void the initial channel? Under what conditions will it force sodium vapor or fission gas into the inlet plenum?
14. What stresses and strains are generated in the fuel pins, the spacers, and the channel walls by hydraulic forces when sodium re-enters a channel which is not blocked but was voided by either a large vapor bubble or a large fission gas bubble entering at the inlet?
15. How much energy will be contained in the shock wave produced by "sodium hammer" when sodium re-enters a blocked fuel bundle after channel voiding has occurred? What will be the effect of fission gas?
16. What stresses and strains are generated in the fuel pins and the channel wall by hydraulic forces when sodium re-enters a blocked channel (exclude the energy dissipated in shock waves)? What will be the effect of fission gas?
17. How much energy will be contained in the shock wave produced when the coolant contacts molten clad or molten fuel upon re-entry after channel voiding has occurred? What will be the effect of fission gas?
18. What fraction of the energy contained in a shock wave in the core coolant will be absorbed in the fuel pins and in the channel walls?

4.0 OPERATING SAFETY LIMITS FOR FUEL PINS

In evaluating the state of the art for safety research and development with reference to core design needs, a measure of the significance of potential disturbances is necessary. The logical yardstick is the damage threshold of core components. The preliminary fault tree analysis pinpointed those safety limits of highest priority. Fuel pin safety limits are discussed in this section and fuel channel safety limits in the following section. This section reviews and describes possible fuel pin damage thresholds, their mechanisms and possible consequences. The damage threshold of interest is loss of clad integrity. In this case fission gas, solid fission products and fuel may be lost from the fuel pin and sodium may enter the pin. This condition will be referred to as the operating safety limit.

The safety limit will be discussed in terms of three types of fuel pin perturbations:

1. External (to the clad) interruption of normal heat removal by flow reduction or flow blockage.
Only uniform clad overheating has been considered. Two and three dimensional effects of local cooling interruption should result in a damage threshold at a higher clad temperature and hence application of the uniform temperature results should be conservative.
2. Fuel melting within the clad
The principal perturbation leading to fuel melting is an overpower transient. In addition, consideration was given to the possible contribution of fuel melting to the clad damage threshold during fuel pin undercooling.
3. Mechanical perturbations in excess of design loads.
Three basic loading mechanisms considered were excess spacer contact loads, bending loads due to spacer restraint, and shock pulse loads.

4.1 CLADDING OVERHEATING (K. R. Absher)

4.1.1 Introduction

In the event of fuel pin undercooling that results from some flow perturbation, the temperature of the cladding rises. Consequently, the probability of fuel pin failure increases. An evaluation of the expected operating lifetime of the fuel pin in the overheated condition gives an indication of the likelihood of failure in a given time period at a given temperature.

The effects of uniform fuel pin overheating to be considered in this section are:

1. The change in mechanical properties of the irradiated cladding material with temperature;
2. The increase in corrosion of the cladding surface by sodium with increasing temperature; and
3. The increased attack of the cladding inside diameter by fuel and/or fission products with increasing temperature.

Consideration of these last two effects required gross extrapolation of existing data to high temperatures, significantly reducing the confidence of predicting their influence on the safety limit.

In this section, thermal stresses due to gradients across the pin are not considered. The overheating is assumed to be localized, so that the total fission gas pressure in the pin is not significantly affected. Effects of sudden fission gas release and fuel volume expansion due to fuel melting were not considered in this analysis. The analysis performed in this study uses the following model:

- a. Stress in the cladding is caused by internal pressures due to fission gas (this could be generalized to include fuel-clad mechanical interaction if a model could be developed to describe it);
- b. The clad is attacked by the fuel and/or fission products; in order to account for localized attack that creates notch-like damage, a stress concentration factor of two is used (based on observed reduction of burst pressures of irradiated cladding);
- c. The clad is corroded by flowing sodium at the elevated temperature;
- d. Three different fuel pin failure criteria were used:
 1. Conservative creep rupture criterion: the fuel pin fails at time, T , if the stress in the cladding at that time is sufficient to cause stress rupture of the clad in the time available from imposition of the overtemperature situation (this assumes that the stress in the clad at any point in time has existed over the duration of the overheating. This is a pessimistic assumption for sealed pins because the stress in the clad is monotonically increasing).

2. Optimistic criterion: the fuel pin fails when the stress in the cladding reaches the tensile strength of the cladding at that temperature or at twice the time-to-failure predicted by the conservative creep rupture criterion (the lower of these two times-to-failure is used);
3. Damage accumulation criterion: the fuel pin fails when a fractional time to failure damage index (based on creep-routine considerations) reaches unity.

The various cases considered are identified in Table 4.1. The results of the analysis appear in Table 4.2 (conservative criteria), Table 4.3 (damage accumulation criterion) and Table 4.4 (optimistic criterion). These tables show the expected lifetime (in hours) of an overheated fuel pin at the various temperatures considered. This data is also graphically shown in Figures 4-7 to 4-17.

Two areas of pressing need were encountered in this work. First, the development of an in-pile failure criterion for the cladding is needed. The failure models used in the above over-temperature transient were specifically for pressure induced primary stress rupture. A general failure criterion is needed which will include, but not be limited to, the primary stress damage. Of the proposed models for steady-state clad failure, the strain damage accumulative model is the most versatile. For general use, especially to include fuel failure effects, a transient strain damage accumulative model is desirable. A vital element of such a model would be the burst failure and creep-rupture effects for transient perturbations. This is particularly important since the FFTF and the first cores of the demonstration plant will utilize sealed and pressurized fuel pins. The ultimate clad failure criterion would probably utilize a strain energy damage accumulation model, since complex effects such as radiation damage could potentially be correlated.

Empirical data for a wide range of clad loading perturbations are necessary to correlate with the available models. In-pile creep-rupture data are, perhaps, most urgently needed for confirming this type of analysis.

There are creep-rupture data available for irradiated steel but the tests were done out-of-pile and thus do not account for radiation-enhanced creep, nor is it clear what the failure point is when creep is radiation-enhanced.

The second area in which information is needed is in the characterization of cladding attack. Work in Task E of PA-10 is not only directed at characterizing the attack but is also attempting to find ways to prevent or limit the attack. The operating life of an overheated fuel pin would be greatly increased if the cladding attack could be controlled (see results). Control of clad attack could ease the strength requirements of the cladding, thus allowing thinner cladding to be used for the same pressure requirements (if overtemperature is included in the design criteria).

4.1.2 Discussion

The study presented here analyzes a fuel pin that is instantaneously and uniformly overheated to some given cladding inside diameter temperature. In the normal, steady-state operation of the pin, it is assumed that cladding attack by fuel and/or fission products is negligible and outside diameter corrosion takes place at the rate of 0.7 mil/year corresponding to a coolant temperature of 1300°F.⁵

Releases of trapped fission gases due to the temperature increase are not expected to significantly stress the cladding. Melting of the fuel could be significant if high density, high power fuel is used. The worst overheating case considered could cause up to 20% volume melting of the fuel if the centerline is at melting under normal conditions. Assuming a 10% $\Delta V/V$ increase of fuel specific volume on melting, the maximum net expansion of the fuel is 2% $\Delta V/V$ locally. A large enough central void would accommodate this swelling. The size of the void depends largely on the initial fuel density and the pin power. Typical central void sizes for pins producing roughly 15 KW/FT with initial densities of 85-90% T.D. are in the range of 3-8% of the fuel volume. This size void should be adequate to accommodate the volume change. Thus, this effect may be neglected as an approximation.

4.1.2.1 Fuel Clad Interface Temperatures

The thermal perturbations due to given amounts of fuel swelling are discussed in Section 6.1. The amount of swelling to be expected, however, is unknown at the present time. Therefore, we must consider a wide range of flow restrictions that swelling could cause. The results of the thermal analysis indicate that the range of temperatures that should be considered is from normal temperatures up to 1800°F. At higher temperatures, a more complete analysis is necessary to include the effects of sodium boiling. If a limit can be put on the swelling of a failed pin, then a limiting temperature can be found for the adjacent pin. This limiting temperature will determine the minimum time-to-failure of

the adjacent pin due to the overheating. Other types of flow restriction may also cause overheating. Whatever the cause, this analysis should give the expected lifetime of a pin for a given degree of overheating.

Table 4-1. Case Identification

FIRST LETTER:	<p>"S" designates sealed pins</p> <p>"V" designates vented pins</p>
LAST LETTER:	<p>designates the time at which overheating first starts:</p> <p>"B": Beginning-of-life (0 hours)</p> <p>"M": Middle-of-life (8500 hours)</p> <p>"E": End-of-life (16,000 hours)</p> <p>(Residence time of pin in reactor = 17,000 hours)</p>
NUMBER:	<p>For sealed pins:</p> <p>1: 10 mil clad, EOL Pressure = 1000 psi</p> <p>2: 10 mil clad, EOL Pressure = 500 psi</p> <p>3: 15 mil clad, EOL Pressure = 1000 psi</p> <p>4: 15 mil clad, EOL Pressure = 500 psi</p> <p>For vented pins:</p> <p>1: 10 mil clad, short hold-up vent (50 psi)</p> <p>2: 10 mil clad, medium hold-up vent (100 psi)</p> <p>3: 10 mil clad, long hold-up vent (300 psi)</p> <p>4: 15 mil clad, short hold-up vent (50 psi)</p> <p>5: 15 mil clad, medium hold-up vent (100 psi)</p> <p>6: 15 mil clad, long hold-up vent (300 psi)</p>

Table 4-2. Conservative Failure Times
(In hours from the start of overheating)

Temp. °F =	1200	1300	1400	1500	1600	1700	1800
S1B	6300	3500	1500	410	175	95	60
S1M	480	80	11	0.3	0	0	0
S1E	12	0	0	0	0	0	0
S2B	11,500	6100	2600	500	180	100	60
S2M	3800	500	120	33	6	0	0
S2E	320	57	7	0	0	0	0
S3B	10,000	6000	3300	1500	400	170	100
S3M	2900	456	86	15	1	0	0
S3E	185	19	1	0	0	0	0
S4B	—	10,800	5790	2500	470	180	100
S4M	—	3730	520	124	36	8	1
S4E	—	370	71	11	1	0	0
V1B	—	—	7030	430	180	96	58
V1M	—	—	4000	240	100	51	25
V1E	—	—	—	200	90	44	22
V2B	—	—	1300	350	164	92	56
V2M	—	—	540	150	60	24	8
V2E	—	—	385	124	50	19	6
V3B	—	870	450	230	130	80	50
V3M	—	353	82	18	2	0	0
V3E	—	277	64	12	1	0	0
V4B	—	—	—	5300	410	174	98
V4M	—	—	—	3740	243	106	54
V4E	—	—	—	—	215	96	50
V5B	—	—	—	950	340	160	94
V5M	—	—	—	570	150	60	30
V5E	—	—	—	440	135	57	24
V6B	—	13,250	850	440	230	130	80
V6M	—	—	360	86	21	3	0
V6E	—	—	300	73	17	2	0

Note: Blanks indicate that failure does not occur during normal fuel pin lifetime.

Table 4-3. Failure Times Assuming Damage Accumulation
(In hours from the start of overheating)

Temp. °F =	1200	1300	1400	1500	1600	1700	1800
S1B	7650	4220	1865	461	176	94	56
S1M	567	95	66	3	0	0	0
S1E	3	0	0	0	0	0	0
S2B	14,000	7640	3275	540	185	95	60
S2M	4855	660	164	42	12	0	0
S2E	506	85	56	0	0	0	0
S3B	12,300	7560	4200	1850	443	181	100
S3M	3470	554	108	17	6	0	0
S3E	280	28	2	0	0	0	0
S4B	—	13,540	7330	3130	523	177	100
S4M	—	5030	673	150	37	9	2
S4E	—	570	107	17	6	0	0
V1B	—	—	9340	411	162	95	60
V1M	—	—	5450	280	120	60	32
V1E	—	—	2130	234	102	53	28
V2B	—	—	1500	352	153	88	55
V2M	—	—	686	178	75	31	10
V2E	—	—	473	153	62	25	8
V3B	—	1019	563	231	145	86	53
V3M	—	482	140	44	8	0	0
V3E	—	356	135	35	4	0	0
V4B	—	—	—	7040	460	175	100
V4M	—	—	—	4920	280	120	66
V4E	—	—	—	—	254	114	61
V5B	—	—	—	1422	380	173	98
V5M	—	—	—	802	185	81	36
V5E	—	—	—	590	165	73	32
V6B	—	16,700	1031	529	262	144	87
V6M	—	—	424	99	25	4	0
V6E	—	—	317	86	20	3	0

Note: Blanks indicate that failure does not occur during normal fuel pin lifetime.

Table 4-4. Optimistic Failure Times
(In hours from the start of overheating)

Temp. °F =	1200	1300	1400	1500	1600	1700	1800
S1B	12,600	7000	3000	520	185	100	60
S1M	960	160	22	1	0	0	0
S1E	24	0	0	0	0	0	0
S2B	—	12,200	5200	590	190	100	60
S2M	7600	1000	240	66	12	0.2	0
S2E	640	114	14	0	0	0	0
S3B	—	12,000	6600	3000	400	180	100
S3M	5800	910	172	30	2	0	0
S3E	370	38	2	0	0	0	0
S4B	—	—	12,160	5000	580	180	100
S4M	—	7460	1040	248	72	16	1
S4E	—	520	100	40	1	0	0
V1B	—	—	14,060	540	190	100	60
V1M	—	—	8000	340	135	70	38
V1E	—	—	—	275	120	60	33
V2B	—	—	2600	456	164	97	59
V2M	—	—	1080	260	105	50	16
V2E	—	—	770	214	50	38	6
V3B	—	1740	880	320	130	88	55
V3M	—	700	164	36	4	0	0
V3E	—	550	130	24	2	0	0
V4B	—	—	—	10,600	520	184	100
V4M	—	—	—	750	350	140	74
V4E	—	—	—	—	300	130	68
V5B	—	—	—	1900	440	180	100
V5M	—	—	—	1140	270	110	54
V5E	—	—	—	880	233	100	49
V6B	—	—	1700	880	320	155	90
V6M	—	—	720	175	42	6	0
V6E	—	—	600	73	34	4	0

Note: Blanks indicate that failure does not occur during normal fuel pin lifetime.

4.1.2.2 Cladding Mechanical Properties

Three different criteria are considered for the mechanical failure of the cladding; a conservative criterion, a damage accumulation criterion, and an optimistic criterion.

The conservative failure criterion uses creep rupture as the failure mode. At any time, T , in the life of the overheated fuel pin, a stress in the cladding is calculable. It is conservatively assumed that the calculated stress has existed in the cladding over the entire time period, T (this is conservative for sealed pins because the stress increases continuously in the cladding). Failure is assumed to occur if that stress corresponds to a time-to-rupture equal to " T " at the temperature under consideration.

The importance of creep rupture in irradiated steel is shown in Figure 4-1 where the failure strength curves for 6000 and 18000 hours fall well below the tensile strength curve plotted versus temperature. Data from BNWL are used in this study to provide information on creep rupture of irradiated cladding. Unfortunately, in-pile creep is largely an area of uncertainty with little applicable data available. The data used are for out-of-pile tests on irradiated stainless steel^{1,2,3,4}. Thus, radiation-enhanced creep is not accounted for. The data were obtained for temperatures of 1400°F and lower. Extrapolation of these data to 1800°F was necessary. A curve fit to these data yields the following formula:

$$U = \text{Exp} \left[9.65 - 1.79 \times 10^{-4} (T + 460) \left(\frac{\ln(H + 1)}{2.3036} + 20 \right) \right]$$

where:

U = Failure stress (ksi)

T = Clad temperature (°F), and,

H = Time-to-rupture (hours).

The curves in Figure 4-1 were obtained by setting $H = 0, 6000, 18000$ respectively.

Figure 4-2 shows the rupture stress for small times-to-rupture. It was based on the same information as was Figure 4-1. The plots in Figure 4-2 show that the applied stress to cause rupture decreases very rapidly initially as the time-at-stress increases.

The damage accumulation criterion is based on the assumption that the creep of the cladding under stress produces an increment of damage to the cladding over an increment of time. The amount of damage is accumulated over the time at stress until failure is reached. The damage index used here is the ratio of the time at stress to the time-to-rupture at that stress. For each time increment of length, T_i , the stress in the cladding is calculated, and the time to rupture, T_r , is calculated from equation (1). The damage index is T_i/T_r . This index is accumulated until the sum is equal to one at which time failure occurs.

The optimistic criterion is actually the lower of two separate criteria. One of these criteria assumes the tensile strength of the cladding at the temperature of interest to be the mechanical limit. As the cladding is thinned and the internal pressure increases (or remains roughly constant, as for vented pins), the hoop stress in the cladding increases. Failure is assumed to occur when this stress reaches the tensile strength of the cladding. The tensile strength of irradiated 304 and 316 stainless steel is plotted in Figure 4-1 (the tensile strength is the uppermost curve; time-to-rupture, $t_R = 0$). The other optimistic criterion takes creep rupture into account. The conservative failure criterion predicted failure at time, T , when the stress in the cladding is sufficient to cause creep rupture in a time period, T . If it is optimistically assumed that the cladding is stress-free up to time, T , and then the stress is applied to the cladding, failure would occur at time $2T$.

4.1.2.3 Cladding Corrosion

The corrosion and erosion of the cladding by flowing sodium at elevated temperatures will reduce the failure threshold of the fuel pin. Corrosion of the stainless steels by flowing sodium has been investigated at GE⁵, AI⁶ and the UKAEA⁵. The data are restricted to normal operating conditions, however. The maximum temperature reported in the literature is 1340°F (UK). This is approximately the cladding outside diameter temperature at the outlet of the hot pin at overpower (17.9 kw/ft) for the GE 1000 MWe LMFBR⁷. Gross extrapolation of the data is needed to examine the

Figure 4-1 Rupture Stress of Irradiated Type 304 and 316 Stainless Steel

Figure 4-2 Stress for Rupture as a Function of Time-To-Rupture
Type 304 & 316 Stainless Steel

temperature range of interest. Fig. 4-3 shows the corrosion rate versus temperature that was obtained from the formula, given by Wozadlo and Spalaris.⁵ The formula is:

$$\ln CR = 14.55 + 0.435 \ln (v) - 0.000958 \ln (L/D)_1 - 0.122 \ln (L/D)_2 \\ + 1.455 \left(\frac{1}{t+1} \right) + 1.156 \ln O_2 - 23827/(T + 460),$$

where:

CR = corrosion rate (mg/dm²-mo)

v = sodium velocity (ft/sec)

(L/D)₁ = isothermal downstream factor

(L/D)₂ = iso-velocity downstream factor

O₂ = oxygen content of sodium (ppm)

t = exposure time (months)

T = temperature (°F).

Values used in calculations:

V = 25 ft/sec

(L/D)₁ = 0

(L/D)₂ = 0

O₂ = 10 ppm, and

t = 24 months.

Since 1 mg/dm²-mo = 0.00615 mil/year, the equation used for corrosion rate in mils per hour is:

$$C = \exp [18.67 - 23827/(T + 460)] \times 0.00615/8760.$$

A ΔT across the cladding of 50°F was assumed.

4.1.2.4 Fuel-Clad Interaction

Out-of-Pile Experience

Out-of-pile tests of fuel-clad compatibility at 1400°F for 1000 hours have indicated excellent compatibility of stoichiometric and hypostoichiometric fuel with the austenitic alloys tested (SS 304, 316, 321, 347 and Incoloy 800). Hyperstoichiometric fuel (O:M = 2.04) caused reaction zones of up to 1.4 mils on the cladding.⁸ According to thermodynamic studies reported in Reference 9, cladding is oxidized by all hyperstoichiometric compositions of mixed-oxide fuel. Although chromium has the highest affinity of the cladding constituents for oxygen, other oxides (FeO, Fe₂O₃ and NiO) may form due to the exhaustion of chromium at the cladding surface. The availability of chromium for oxidation is dependent on its diffusion out of the metallic solution of the cladding. This diffusion is relatively slow and the other cladding constituents will consequently be oxidized.

Figure 4-4 shows the free energy of formation for the oxides of stainless steel constituents and fuel.⁹ A smaller (more negative) free energy of formation indicates a greater affinity for oxygen. Thus, the mixed-oxide fuel may oxidize other materials that lie below it on the free energy plot. For example, (UPu)O_{2.04} can oxidize any of the pure cladding constituents at temperatures below 1450°C, while (UPu)O_{1.96} can oxidize only the silicon below 1600°C (and that reaction is marginal).

The conclusion from this out-of-pile work is that mixed oxide fuel with O:M ≤ 2.00 is fully compatible with the primary cladding candidates at 1400°F for at least 1000 hours. However, our interest is for high burnup fuel, longer

periods of time and higher temperatures. High fuel burnup releases free oxygen capable of oxidizing the clad and creates corrosive fission products that can attack the cladding.

Figure 4-3 Corrosion Rate vs Temperature for Stainless Steel Type 304 & 316

In-Pile Clad Attack Experience

There have been many in-pile instances of attack of the cladding Inside Diameter by fuel and/or fission products. The depth of attack of thirty-two fast and thermal flux-irradiated pins and their cladding Inside Diameter temperatures appear in Reference 10. Failed pins were not included. An empirical curve-fit was imposed on the points yielding the following conservative equation:

$$P = -1.0650 \times 10^{-1} + 2.64833 \times 10^{-4} T - 2.24999 \times 10^{-7} T^2 + 6.66666 \times 10^{-11} T^3$$

where:

P = penetration depth in inches and

T = temperature of cladding inside diameter in degrees Fahrenheit.

This curve-fit is shown in the following Figure 4-5.

Figure 4-4 Free Energy of Formation for Oxides of Stainless Steel Constituents and Mixed-Oxide Fuel

Figure 4-5 Fuel-Clad Interaction

This formula conservatively accounts for the uncertainty in the data. Irradiation times for the pins varied from 5000 to 12,000 hours. A clear time dependence was not observed. The equation is based on data for temperatures under 1400°F. Use of this equation for temperatures up to 1800°F requires gross extrapolation of the data, but such a representation is needed for this study.

Other examples of cladding attack were compared to the above formula. Failed pin B3C¹¹ exhibited localized attack, the depth of which agreed very well with the empirical curve. Five points at different temperatures all matched the curve very well. The estimated time of the B3C failure is roughly 1000 hours (5500 MWd/Te). Pin B3B¹¹ also had localized cladding attack and the depth of attack versus the temperatures corresponded well with the curve. B3B failed at roughly 3000 hours. Two pins were found not to fit on the curve. Pin E1D^{12,13} with five hours irradiation and pin E5A¹⁴ with 122 hours irradiation. Both pins lie below the curve.

The pins with 1000 to 12,000 hours irradiation time did not exhibit an increase in the depth of attack with increasing time for a given temperature. This may indicate a saturation of attack at 1000 hours or less. For this study the depth of attack is assumed to increase exponentially for the first 1000 hours and to saturate at that time. The

formula used to define the exponential is somewhat arbitrary in that there are only two short-time points to fit it to. The formula used is:

$$A = (F + .05) \left[1 - \left(\frac{.05}{F + .05} \right)^{.001H} \right]$$

where:

A = depth of attack (mils) at time H (hours) and

F = final saturation depth of attack (from cladding attack formula).

The plot of depth-of-attack versus time for the various temperatures appears in Figure 4-6.

Presently, there does not appear to be an acceptable explanation of the difference between the localized attack in failed pins B3B and B3C and the uniform attack seen in E5B-1 and E5B-2. Since the data from both cases fit the curve, this difference will not be considered further in this study. All attack will be considered as a clad-thinning process with a stress concentration factor of two included to account for effects of localized attack. Burst tests of EBR-II test pins at PNL resulted in the irradiated pins failing at roughly half the hoop stress as the unirradiated control pins.

Effect of Thermal Gradient on Attack

The thermal gradient across the cladding may have an important effect on cladding attack behavior. Lauritzen⁸ suggests that the commonly observed precipitation of complex carbides in grain boundaries on the cladding inside diameter is probably due to migration of the carbon up the thermal gradient. Since the heat flux through the cladding determines the thermal gradient across it, a higher power level may increase the amount of grain boundary precipitation, even if the clad inside diameter temperature is unchanged. When the carbon migrates to the grain boundaries, it may precipitate as a chromium carbide (e.g., Cr_{23}C_6). This depletes the grain boundary area of chromium and removes its passivating protection locally. This area is then more open to attack by corrosive agents. The conclusion is that higher power pins may be more subject to cladding attack than lower power pins, even at the same inside diameter temperatures. There is presently no quantitative way to factor this into the analysis here, however.

Effect of Pin Radial Power Profile on Clad Attack

A thermal flux may aggravate the attack because, for the same linear power of a pin, more fissions take place adjacent to the cladding in a thermal flux than in a fast flux. Thus, more corrosive fission products and more oxygen will be available at the cladding surface. Again, there is no clear way to treat this analytically, so data from fast and thermal flux irradiations are used together.

4.1.3 Analysis

Two simple time-share codes were written to combine the effects of cladding strength change, outside diameter corrosion and inside diameter attack. The codes take time-steps through the life of the fuel pin. At each step, the stress in the cladding and the allowable stress are calculated and compared. When the allowable stress is exceeded or the damage index exceeds unity, the code linearly interpolates to find the failure time.

The stress in the cladding is taken to be the simple hoop stress with a stress concentration factor of two that accounts for effects of localized attack.

$S = 2PR/T$, where

P = internal pressure

R = pin radius

T = Clad thickness after corrosion and attack

2 = Stress concentration factor.

The residence time of a fuel bundle in the core is assumed to be two years (~ 18,000 hours).

Several cases were considered in this study. Parameters include:

Temperature (1200, 1300, 1400, 1500, 1600, 1700, 1800°F)

Cladding Thickness (0.010, 0.015 inch)

Figure 4-6 Assumed Cladding Attack Rate

Fission Gas Accommodation

Sealed: 1000 psi final pressure

500 psi final pressure

Vented: short hold-up (50 psi)

long hold-up (100, 300 psi)

Onset of overheating: (beginning, middle, and near end of life).

The cases are identified in Table 4-1.

For sealed pins, the pressure is assumed to increase linearly from the beginning of life. For vented pins, the constant pressure is arbitrarily assumed to be established at 1000 hours after the beginning of life.

4.1.4 Results

Tables 4-2, 4-3 and 4-4 present the time to failure from the initiation of overheating for the cases described in Table 4-1. Blank spaces indicate that failure does not occur during the two year operating lifetime of the fuel pin. Sample results are plotted in Figures 4-7 through 4-10 for a cladding inside diameter temperature of 1400°F. Beginning-of-life and middle-of-life overheating cases are considered. The stress in the cladding is plotted as a function of time. Also appearing on the figures are two curves representing the conservative failure criterion and the optimistic (tensile strength) failure criterion. The curve that decreases with time represents the stress versus time-to-rupture for SS 316 at 1400°F. The horizontal line is the tensile strength at 1400°F. The intersection of the stress curve and the strength curve indicates failure at that time.

The results show large differences between the optimistic and conservative failure criteria for failures involving long time periods. This is to be expected because of the importance of creep-rupture in these cases. In Figures 4-11 and 4-12, the time to failure is plotted versus the cladding inside diameter temperature based on the conservative and optimistic criteria for three beginning-of-life and middle-of-life cases.

Figure 4-7 Stress Histories for Sealed Pins

Figure 4-8 Stress Histories of Sealed Pins

The relative importance of the effects considered in this study is shown by example in Figures 4-13 and 4-14. In Figure 4-13, case S3B (15 mil clad, 1000 psi final pressure) was run with the cladding attack varied, with the corrosion varied and with the failure criterion varied. Figure 4-14 shows the same information for case V2B (10 mil clad, 100 psi constant pressure). Clearly, the corrosion is relatively unimportant compared to attack and choice of failure criterion. The corrosion becomes somewhat more important, however, when the overheating is initiated at middle or end of life. The degree of attack is extremely important in the higher temperatures ($>1350^{\circ}$) while the choice of failure criterion is of major importance for the lower temperatures ($<1600^{\circ}$ F).

Figure 4-15 shows the effect of internal pressure on the time to failure of a fuel pin. The overheating is assumed to be started at the middle-of-life and the clad thickness is 15 mils. The lower pressure (vented) pins appear to have a distinct advantage over the high pressure (sealed) pins in terms of expected lifetime at overtemperature conditions. The cumulative damage criterion was used in defining the failure point for Figure 4-15.

Figure 4-16 compares the three criteria for failure. The results from the damage accumulation criterion lie within the uncertainty band defined by the optimistic and conservative failure criterion.

The results are sensitive to the stress concentration factor used. The time to failure for pins that survive over 1000 hours is roughly inversely proportional to this factor. (If the stress concentration factor is doubled, the pin is expected to survive the over-temperature situation only half as long.) However, if the time to failure is short (<1000 hours), the stress concentration factor has little effect on the failure time. The reason for this is that the failure time is dominated by the attack rate during the first 1000 hours.

Figure 4-17 shows the instantaneous failure limits for each of six cases. BOL, MOL and EOL designate beginning, middle and end of life, respectively. Consider a pin with 10 mil clad near the end of life. Corrosion of the outside diameter has decreased its clad thickness by about 1.4 mil and may have introduced stress concentrating locations. If this pin is suddenly overheated to a clad temperature of 1500° F, an internal pressure of about 480 psi will instantaneously burst the pin.

Figure 4-9 Stress Histories for Vented Pins

4.1.5 Recommendations

The depth and rate of attack on the cladding inside diameter appears to be dominant among the factors considered here in determining the operating life at a given temperature of an overheated fuel pin. It is recommended, therefore, that the PA-10 Task E work on cladding attack be maintained at a high priority in order to understand the phenomena involved and to develop means of controlling attack. If it is a requirement that fuel pins be able to withstand temperature transients as discussed here, then the solution to the problem of cladding attack is a very important goal for Task E. As shown in Figure 4-13, the control of cladding attack may be the most important consideration in prolonging the lives of overheated fuel pins.

The mechanical properties of in-pile cladding are also important in the determination of the operating life of an overheated fuel pin. The most important information that this study needs on cladding mechanical properties is the in-pile creep-rupture behavior. A cladding failure criterion should be developed and checked experimentally. PA-10 Task L is working in the area of biaxial and uniaxial creep of irradiated materials and is testing the cumulative strain damage theory for Type 316 stainless steel at 1300°F¹⁶. If it is found that this theory is valid, it could then be applied to the analysis done here to yield realistic failure times. The development of a realistic failure criterion is the item of highest priority in the area of cladding mechanical properties.

The problem discussed here appeared to be less sensitive to outside diameter corrosion rate than to inside diameter attack and cladding strength loss. The work, however, assumed that the formula derived for temperatures below 1300°F is applicable to much higher temperatures. Some high-temperature corrosion tests under PA-15 would be valuable as a check of this assumption but this is not a high priority need.

In conclusion, it is strongly recommended that PA-10 Task E continue its investigation of cladding attack with special emphasis on its control, and that PA-10 Task L develop a realistic in-pile failure model for cladding.

Figure 4-10 Stress Histories of Vented Pins

Figure 4-11 Time-To-Failure vs Temperature

Figure 4-12 Time-To-Failure vs Temperature

Figure 4-13 Operating Life for Case S3B

Figure 4-14. Operating Life for Case V2B

Figure 4-15 Sealed-Vented Comparison (15 Mil Clad)

Figure 4-16 Comparison of Optimistic, Conservative and Damage Accumulation Failure Criteria for Case S3B

Figure 4-17 Burst Pressure vs Clad Temperature

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4.2 OVERPOWER AND FUEL MELTING PERTURBATIONS (G. R. Thomas, L. M. McWethy)

4.2.1 Introduction

The primary concern of this section is the failure threshold (clad breach) of fuel pins subjected to overpower transients or undercooling involving fuel melting. Fuel melting contributes additional loading mechanisms to the clad and in addition, imposes added considerations to post failure core behavior.

In the past, the predominant accident considered to lead to fuel melting has been the overpower transient. The key questions in this regard have been the failure threshold and the post failure work-energy release. The failure threshold variation with fuel design parameters can potentially influence design optimization.

The second perturbation which could involve fuel melting is undercooling, either locally, whole bundle, or whole core. For the highest rated fuel pins operating near center-melt, undercooling would cause local fuel melting at the peak power location. This would potentially influence the failure threshold and post-failure consequences.

4.2.2 General Review, Overpower Transients

Dickerman¹ presented a general review of safety areas, including overpower transients, noting the extensive research and documentation of power transient tests. This discussion will be limited to results considered relevant to the question of the failure threshold under overpower conditions.

In a general discussion of test results, Thomas and Field^{2,5} noted phenomena which bear on the failure threshold of overpower fuel pins.

One of the more consistent experimentally observed phenomena involves the transient movement of molten oxide fuel, both within the fuel pin prior to failure, and in the form of expulsion of this fuel from the pin once failure has occurred.

During post-irradiation examination of several of Task C specimens, evidence has been found of direct contact between the molten oxide fuel and the inner surface of the stainless steel cladding. There was no evidence of incipient clad melting, or even a significant degree of grain growth in the cladding, despite the extremely high temperatures of the molten oxide fuel ($>5000^{\circ}\text{F}$). From this data and confirming heat transfer analysis, it was concluded that clad melting per se would not be a primary failure mechanism in overpower transients, unless the external cooling capability were also reduced.

In several experiments involving cladding failures, evidence of significant fuel displacement interior to the cladding has been noted both above and below the region of the failure^{2,3}. In one unfailed case³, molten fuel was forced upward a distance of some 10 inches through a small diameter hole (0.070 inch diameter) in an inert UO_2 blanket located above the fuel column, without causing failure of the fuel pin cladding. It is believed that such movement is caused by a combination of forces arising from the large volume expansion of the oxide fuel, as it passes through its melting point (approximately 10%), plus the expansion of entrapped gases within the molten fuel as a result of the rapid transient heating during the experiment.

It is this mode of pressurization of the cladding which can cause loss of clad integrity when the burst strength is exceeded. Design and operating parameters which affect the transient pressure attained and the clad burst strength will then influence the failure threshold under overpower conditions.

4.2.3 Model Formulation

These considerations led to formulation of a predictive model of clad failure under overpower conditions⁴, which is summarized here.

Primary factors which affect the failure threshold are:

1. Radially-dependent fission gas production and release prior to and during the transient.
2. Cladding temperature during the transient.
3. Thermal volume of the fuel and cladding, including the volume change associated with the fuel phase changes.
4. Cladding strength and ductility as a function of temperature and irradiation history.
5. Fuel and cladding swelling.
6. Rate of energy input during the transient.

The effect of the above parameters on the cladding failure threshold was evaluated assuming complete axial restraint. This model assumes that if sufficient energy is generated in the fuel rod during a short period of time, the central region of the fuel begins to melt and expand due to the phase change volume expansion of approximately 9.6%. The outer, cooler parts of the fuel also thermally expand and are cracked. As the molten portion of the fuel increases in temperature and spreads radially towards the cladding, fission gas bubbles, initially entrapped in the solid fuel matrix, are released within the molten fuel. This enables the bubbles to act as a freely compressible medium that hydrostatically pressurizes the molten core of the fuel rod. The gas occupies a diminishing volume as a result of the significant increase in the fuel volume during melting, plus additional thermal expansion above the melting temperature. The decrease in gas volume causes a rise in pressure which is transmitted through the outer solid fuel to the cladding. Inherent in this model is the assumption that the fission gas released during melting does not have time to reach the plenum during the transient.

An idealized stress strain curve was used. Since the strain at which a burst will occur will presumably vary with burnup, a burst strain of 1% was assumed at high fluences while at zero burnup, the ultimate value was chosen to be 4%. The failure strain at intermediate burnups were assumed to vary linearly with burnup.

The single most significant parameter which affects the amount of melted fuel volume required to burst the

cladding was found to be fuel burnup. Burnup affects four parameters required in the analysis: fission gas production, fuel swelling, cladding swelling, and cladding failure strain. Of these four, the two with the greatest effect on failure are fission gas production and cladding failure strain.

As burnup increases, the total fission gas inventory increases, and therefore, the fission gas remaining in the fuel after "steady-state" release increases. Consequently, the volume of fission gas released in the melted fuel during an overpower transient also increases. Combined with decreasing cladding ductility due to irradiation, the increase in burnup causes a sharp decrease in the amount of fuel melting required to fail the cladding.

Inspection of Figure 4-18 indicates that lower fuel density requires a greater molten fuel volume and hence a greater energy input to the fuel in order to fail the cladding. In this particular case, to emphasize the effect of density and burnup, cladding temperature was assumed constant. At 100,000 MWd/Te, the fission gas inventory is large so that increasing the fuel porosity only increases the volume the fission gas can occupy by a small amount. Conversely, increasing the fuel porosity at low burnup or low fission gas inventory, results in a significant increase in the required fuel melting volume to cause cladding failure.

Figure 4-19 illustrates typical pressure versus time curves as a function of burnup as calculated by this model. Note that the present model assumes that burnup affects clad ductility but not yield strength. The rapid buildup of pressure, once fuel melting is initiated, is quite obvious from the slope of the pressure curves.

In both Figures 4-18 and 4-19 the energy input rate used was 97 kw/ft-sec. This corresponds to an LMFBR reactivity insertion rate of about \$15/sec, which is felt to be quite severe. It also closely simulates the conditions which test pin C4B was subjected to in the TREAT facility. This test is reviewed in Section 4.2.4.

4.2.4 Experimental Confirmation

A review of the PA-10, Task C-1 transient irradiation test program was performed in an attempt to answer the question of whether any fuel performance could be used as an indicator that a portion of the core had exceeded an established overpower criterion.

The results of the review are certainly far from a statistical evaluation — There are too many important parameters, and too few tests; but the results do lend credence to several tentative conclusions based on comparative results and analysis. To date, the Task C-1 irradiations have involved 27 fuel specimens, some of which have only seen steady-state irradiations as comparative controls. The specimens that have undergone TREAT irradiation represent a large share of all transient irradiations performed on oxide fuel in sodium.

The majority of these Task C-1 experiments would be classified in the range from the high side of credible accidents (fuel molten or just below melting) to severe hypothetical accidents (fuel volume melt fractions up to 80%).

Analyses of the completed tests indicate the possibility of establishing a maximum allowable fuel-volume-melt criterion for determining transient overpower failure thresholds (such as transient cladding deformation, or actual cladding breach).

At present, the wide variety of test and fabrication parameters, and the small total number of tests eliminates the possibility of establishing a meaningful statistical analysis for predicting failure thresholds; but the tests do indicate that a clad-breach threshold for TREAT tests of sodium-cooled oxide pins is a function of length-to-diameter ratio (L/D), smeared fuel density (ρ_s), fuel volume melt (V_m), and burnup (B.U.) and probably a function of the linear power as a function of time, and for the cladding, the neutron flux spectrum and total nvt, plus others. The trends produced by the first four variables above are indicated by the partial summary of Task C transient irradiations presented in Table 4-5.

An intuitive correlation of the clad-breach probability, for the Task C fuel specimens irradiated in the TREAT reactor, indicates a possible functional relationship between four variables, L/D , ρ_s , V_m , and B.U. of the form

$$P = A (L/D)^a (\rho_s)^b (V_m)^c + E (B.U.)^f$$

where:

A, a, b, c, E, and f are all positive constants.

Figure 4-18 Fuel Melt Fraction Required to Fail Clad

Figure 4-19

**Table 4-5. Partial Summary of Task C Experimental Results
Indicating Functional Relationships Involved in a
Fuel Volume Melt-Clad Breach Failure Criterion**

Functional Relationship	Fuel Specimen	L/D	Smeared Density	Burnup MWD/Te	Smeared Melt Volume (%)	Failed
Fuel Column Length to Diameter Ratio	C1D	$\sim 11.7^{(a)}$	92.3	$\sim 0^{(b)}$	~ 93	No
	C2C	27.8	91.8	$\sim 0^{(b)}$	~ 80	Yes
	C4A	66.0	90.4	$\sim 0^{(b)}$	56-63	Yes
Fuel Column Smeared Density	C4C	65.5	80.1	$\sim 0^{(b)}$	67-70	No
	C4A	66.0	90.4	$\sim 0^{(b)}$	56-63	Yes
	C2B	27.9	90.6	$\sim 0^{(b)}$	~ 63	No
Effect of Prior Irradiation	C3C	27.7	92.6	$\sim 64,000^{(c)}$	~ 67	Yes ^(d)
	C3E	28.0	92.9	$\sim 63,000^{(c)}$	16-20	No
	C5A	112.3	90.0	$\sim 18,000^{(c)}$	~ 35	Yes ^(e)

- (a) SEFOR-sized fuel specimen with UO_2 fuel; all others are 0.250 inch outside diameter clad with mixed oxide fuel (except UO_2 fuel in C5A specimen).
- (b) TREAT reactor irradiation only, burnup < 0.1 MWd/Te.
- (c) Preirradiated at the GETR Test Facility.
- (d) A breach in the cladding did not occur, but molten fuel penetrated into and melted through the combined fuel pin lower end plug-pressure transducer housing, resulting in expulsion of 30% of the total fuel volume from the fuel pin.
- (e) Also note apparent effect of large L/D .

An initial correlation of the Task C-1 tests involving gas-bonded fuel specimens was obtained using this basic relationship (18 specimens correlated), and it shown in Figure 4-20. This correlation, which is applicable only to Task C-1 tests to date, gives some indication of validity to the above relationship, while at the same time pointing out limitations inherent in an incomplete matrix of the possible design and testing parameters. The correlation uses the following values: $A = a = 1$; $b = c = E = 2$; $P = 14$. With these values, however, the correlation shows failure for burnups in excess 70,000 MWD/Te even at zero fuel melt volume (the highest burnup specimen tested to date by Task C had a burnup of $\sim 64,000$ MWD/Te). There is no physical indication of such a low transient failure threshold limit. In fact, the model discussed previously indicates that fuel specimens can undergo at least 10-20% transient molten fuel volume with no significant associated damage, even at target burnups. But, with the demonstrated range of fuel failure thresholds shown in Table 4-5 and the lack of a sufficient number of tests to establish a statistical basis for predictions, it is difficult at present to establish empirical criteria for allowable fuel volume melt in overpower conditions. The inadequacy of the specific experimental correlation presented in Figure 4-20 probably stems from failure to include other potential variables; e.g., linear power as a function of time (with associated fission gas release history) nvt., etc.

Since it is believed that fuel specimens can survive some transient fuel melting (as indicated by the test results and trends presented in Table 4-5), and the indications are that their failure threshold is a complex function of many design

Figure 4-20 Failure (Breach of Primary Containment) Correlation

and test variables, it would seem possible to experimentally establish a transient fuel volume melt threshold with operationally acceptable error bands (e.g., $\pm 10\%$) for a specific fuel pin design and associated specific steady-state and transient irradiation history (thereby eliminating many significant variables affecting such a volume-melt-criterion, e.g., L/D , ρ_s , and time dependent linear power). In addition, application of the general fuel failure model discussed previously, to the actual conditions of specific TREAT experiments, is useful in establishing confidence in the model, and in providing a basis for detailed comparison of the tests to hypothetical reactor conditions. One such application is shown in Figure 4-21, illustrating the pressure versus time function calculated for TREAT test C4B. Comparison of this figure with the hypothetical curves in Figure 4-19 indicates that the TREAT case is probably more severe, and that failure occurred near the predicted conditions.

The major conclusions of this study are:

1. The most reliable indicated fuel failure threshold criterion, based on the Task C-1 transient irradiation experiments, is the transient fuel volume melt. However, at present, the total number of transient overpower tests performed on quarter-inch diameter oxide pins in sodium (the Task C-1 program represents virtually all such tests) is insufficient to statistically determine a transient fuel-volume-melt criterion which would not have unduly restrictive error bands.
2. Indications obtained from a study of the completed Task C-1 experiments indicate that the failure threshold, (breach of the cladding) is a complex function of at least the following: the fuel column length to diameter ratio (possibly only significant in TREAT tests [initiated from approximately zero power] and start-up accidents), fuel smeared density, fuel transient volume-melt function, prior total fuel irradiation history (burnup), specific irradiation history (linear power as a function of time), associated steady-state fission gas release, neutron energy spectrum, and cladding fluence (nvt).
3. The experimental correlation is in basic agreement with the theoretical model formulation presented in Section 4.2.3. It appears that the understanding of the overpower failure threshold has progressed to the stage of showing quantitative correlations and predictions, for transients on the time scale of the TREAT tests. The confidence level for these results are not yet determined, emphasizing the need for more data in this area.

Figure 4-21

4.2.5 Over-enriched Fuel Pin

A special case of an overpower transient is a zone two fuel pin in a zone one location, i.e., locally over-enriched. This is different from the power transient principally due to the rate of melting. An over-enriched pin would be subjected to a startup transient, a slow (on the order of hours) increase of power starting from the as fabricated (zero burnup) condition. Such a transient has not been experimentally tested to date.

Considerations which make this different are:

- Essentially steady state temperature profiles in the fuel will result from the slow startup rate.
- Time scales are long enough to allow entrained gas (e.g., porosity gas) to diffuse from the potential molten region without inducing high pressures.
- The fuel and clad would not be in intimate (and possibly bonded) contact, perhaps permitting unrestrained axial expansion, raising the failure threshold.
- If failure does not occur initially, cyclic operation may induce failure by a fuel ratcheting expansion mechanism each time melting occurs.

The available models suggest a melt fraction of greater than 80% would be necessary to induce clad rupture on a previously unirradiated pin (<90% smeared density); a greater melt fraction than would be anticipated from a zone two fuel pin in the center of zone one.

4.2.6 Applicability to Undercooling Transients

No specific analysis or data on undercooling failure modes involving fuel melting have been found. However, some of the understanding gained from the above data on overpower fuel melting does contribute to understanding this potentially important phenomenon.

Potential initiating mechanisms for undercooling include core flow reductions, bundle blockages and local flow undercooling due to fuel swelling and/or debris. Like the startup transient, these perturbations probably develop slowly compared to power transients.

The principal differences between undercooling and overpower fuel melting accidents are:

- Clad expansion is faster than fuel (at least prior to attaining fuel-melt temperatures), relieving any fuel-clad contact loads and opening a fuel-clad gap.
- The quantity fuel melted before clad rupture is probably lower due to the higher clad temperatures.
- Fuel melting is localized for the cases of local undercooling, with potential axial redistribution of fuel melt away from the undercooling perturbation.

For the undercooling perturbations of most interest, those due to trapped debris or fuel swelling, the last point suggests that in highly rated pins operating near center melt, a local undercooling will tend to be somewhat self-correcting. Fuel melt should fall into the central void space below the undercooling elevation, reducing the power generation rate locally and limiting the clad temperature perturbation. This presumes the slumping fuel does not bridge the central void immediately below the hotspot by rapid solidification on colder fuel.

The failure model developed for overpower transients is based on a clad stress due to released fission gas on fuel melting. If a fuel column central void is assumed to be gas tight, then fuel melting and fission gas release could likewise affect the failure threshold of undercooled pins by increasing clad stress. More likely, however, the opening of the fuel-clad gap on undercooling will permit any generated pressure in the central void to crack the brittle fuel region and vent the pressure to the plenum or fission gas vent. The available central void volume should accommodate any fuel volume change on melting and permit axial redistribution of molten fuel away from the hot spot, reducing the severity of the over-temperature condition.

Thus, the tentative conclusion drawn is that due to the slower time scale of undercooling transients, the observed mobility of molten fuel^{2,3,5}, and the faster clad expansion on undercooling, it is anticipated that the clad failure threshold on undercooling of a pin normally operating near central melt conditions will be controlled by the steady state fission gas pressure and the clad rupture properties at the excess temperatures similar to a non-molten case. Fuel redistribution may in fact increase the tolerance for local overheating perturbations.

4.2.7 Design Significance of Fuel Melting Accidents

Design parameters which can potentially influence fuel melt considerations are:

- Fuel Density — affects available volume expansion space
- Axial Restraint — similarly affects available volume expansion space (note that proposed axial fuel separators such as SEFOR or PFR are not being seriously considered for LMFBR use). High burnup pins suffer some degree of axial restraint due to fuel-clad bonding or wedging. Also, rapid transient heating can induce an effective axial restraint even in zero-burnup fuel².
- Annular Pellets — demonstrated capability of accommodating volume expansion makes this a viable choice *if* design for severe overpower (>20% melt fraction) is considered necessary.
- Fission Gas Accommodation — the pin pressure affects the starting stress on the clad as well as the quantity of gas being compressed by fuel expansion on melting.
- Fuel Burnup — lowered failure threshold with burnup might influence peak design burnup *if* design for severe overpower is considered necessary.

A study of alternative designs under fuel melt conditions has not been made. In particular, the design philosophy of requiring survival after defined overpower of fuel melting accidents has not been delineated by any reactor design to date (FFTF, Demo Plant or LMFBR). More applicable to the fault tree analysis would be a comparison of changes in fuel cycle costs for changes in the above parameters versus the change in mean annual cost of accident accommodation. This comparison and the choice of a logical design can be accomplished when the fuel failure threshold and failure consequences are predictable within acceptable confidence levels.

4.2.8 Conclusion

- The state of the art for failure threshold under fuel melt conditions is entering the early quantitative stage but confidence levels have not yet been defined.
- Continuing modeling and correlation holds promise in the near future for a reliable predictive failure model.
- Economic comparisons of possible design choices have not been made, prohibiting design recommendations.
- The economic impact of power transient accidents is considered to be negligible.
- The occurrence of fuel melting does not appear to significantly influence the failure threshold of undercooling accidents.
- On the basis of mean-annual-cost, research into fuel melting failure thresholds is considered low priority. The following recommendations have not been reiterated in Section 3, Conclusions and Recommendations for this reason.

4.2.9 Recommendations

- Testing of failure thresholds under fuel melt conditions is generally ahead of analysis and modeling. Every effort should be made to extract by analysis the available information on failure threshold and post-failure behavior from transient irradiation tests.
- On failure threshold experiments, fuel pin pressure transducers should be incorporated — where possible — to indicate failure times, failure pressures and post-failure expansion characteristics.
- Clad strain gages should be used if feasible to provide data on material ductility and to correlate pin pressure measurements.
- Predictive failure modeling efforts should be emphasized to utilize the growing experimental data.
- Out-of-pile tests should be performed on very-short-term (0.1-5 seconds) strain properties of design representative cladding materials. These tests should vary pressure and/or temperature as a function of time.

SECTION 4.2 REFERENCES

1. Dickerman, C. E., "Studies of Fast Reactor Core Behavior Under accident Conditions", *Nuclear Safety*, 11 (3):195 (may-June 1970).
2. Thomas, G. R., and Field, J. H., "Transient Overpower Irradiation of Axially-Restrained Zero-Burnup Fast Reactor Fuel Specimens", GEAP-13562, Sept., 1969.
3. Hikido, T., and Field, J. H., "Molten Fuel Movement in Transient Overpower Tests of Irradiated Oxide Fuel", GEAP-13543, Sept., 1969.
4. GEAP-10028-34, "Sodium-Cooled Reactors, Fast Ceramic Reactor Development Program, Thirty-Fourth Quarterly Report", Feb-April, 1970.
5. Field, J. H., and Thomas, G. R., "Transient Overpower Tests and Thermal Analysis for Sodium-Cooled Reactor Fuel", ASME 11th National Heat Transfer Conference, Aug. 3-6, 1969., Minn., Minn.

4.3 MECHANICAL PERTURBATIONS IN EXCESS OF DESIGN LOADS (W. R. Gee, Jr., J. M. Cole)

4.3.1 Introduction

Over the lifetime of a fuel pin, it may be subjected to mechanical loadings that exceed the design limits. During normal operation there are the forces due to fission gas inside the pin and due to fuel swelling and thermal ratcheting of the pellets. The latter could lead to high local stresses in the cladding. Under abnormal conditions (e.g., failure of an adjacent fuel pin), there could be external pressure pulses due to fuel-sodium thermal interaction as discussed in Section 7.3. If the failed pin swells or if failure debris is trapped, a significant thermal gradient could be established across a sound fuel pin. This gradient could cause bowing and create bending stresses in the pin. This gradient could cause bowing and create bending stresses in the pin. The support of the fuel pins by wire wrap or grid spacers leads to high local stresses at the contact points.

4.3.2 Spacer Contact Force

The detailed analysis of a fuel pin under a thermal gradient is presented in Section 6. According to this analysis, the safety limits are determined by the spacer contact forces.

The fuel pin cladding is treated as a thin-walled hollow cylinder. From Reference (1) it is found that the local stress due to a force distributed over a small area of the surface of a hollow cylinder varies as follows:

$$\sigma_{\max} = k_1 \frac{P_2}{t^2}$$

where:

P = total force (normal to the surface only)

t = cylinder wall thickness.

The coefficient k_1 is tabulated in Reference (1) and depends strongly on the parameters:

$$\frac{A}{R^2} = \text{Ratio of loaded area to the square of the mean radius of the cylinder.}$$

$$\frac{R}{t} = \text{Ratio of the mean radius of the cylinder to the wall thickness.}$$

As A/R^2 becomes very small, k_1 approaches 2.40. For the case of a 50 mil wire at right angles to 0.250 inch diameter cylinder the contact area may be found from Roark to vary as follows:

$$A = 6.86 \times 10^{-6} p^{2/3}, \text{ in}^2$$

Thus for our reference pin design (see Table 4-6) k_1 can be estimated only by extrapolating the data from Reference (1). This process yields a value of approximately 2.0 for k_1 . This value is probably appropriate, also, for grid spacers which use "dimples" at the spacer-pin contact point. Consequently, for the reference pins:

$$\sigma_{\max} = 20,000 P, \text{ psi (since } t = 0.010 \text{ inch).}$$

Assuming that the yield strength of irradiated 316 stainless steel is approximately 60,000 psi, a force of 3 lbs. causes local yielding of the clad. While it is recognized that this does not represent the failure load, nor does it consider the consequences of failure, it does represent a conservative estimate of a fuel pin safety limit.

4.3.3 Fuel Pin Response to Pressure Pulses

The fuel pin diameter appears to make it insensitive to all but the very short duration pulses that, hypothetically, can originate from fuel-sodium thermal interaction (see Section 7.3, Figure 7-56). These pulses could impose a lateral load on the fuel pin that would deflect it sufficiently to impact the adjacent pin. A comparison has been made of the ability of fuel pins and channels to resist the loads due to pressure pulses. The analytical method appears in Section 5.3 and Appendix A. Graphs of fuel pin response to loads are also shown in Appendix A. The analysis considers a fuel pin with an outside diameter of 0.23 inch, a cladding thickness of 0.014 inch, a support spacing of 6 inches, and a yield strength of 13,000 psi (the clad temperature is higher than the channel temperature).

The irradiated fuel pin cladding will most likely be quite brittle and be capable of accepting only small amounts of strain. The analysis conducted here indicated that relatively small shock pulses may be sufficient to fail the pins. For example, a pulse that deposits an energy density of only 0.095 in-lb/in² will deflect the pin a distance of 0.1 inch. This deflection would occur at the center of the span between spacer contact points. The maximum strain in this case was computed to be 0.029. Brittle cladding subjected to such conditions would certainly be very near its failure point.

A deposited energy density of 0.025 in-lb/in² causes yielding of the clad in bending. The calculations also indicated that the pin cross-section would not be plastically deformed (flattened) until the energy density deposited on the pin was about 0.0078 in-lb/in.² The computation of the support loads indicates that the fuel pin cladding would be excessively deformed at the spacer contact points by the pressure equivalent to a deposited energy density of .00016 in-lb/in.² If the local damage is unacceptable, then flexible spacers would be necessary to reduce the cladding deformation.

From this comparison, it appears that the spacer contact strains are the first threat to clad integrity followed by collapse of the fuel pin, and lastly, pin bending strains. These conclusions are based largely on elastic stress limits. It is recognized that the plastic strain that would exceed the clad ductility may not occur at the same energy ratios as calculated for the yield strain and that the mode of failure may be different, emphasizing the need for dynamic clad failure tests.

The calculations made for the channel wall indicated that the wall would deform 0.1 inch for an energy density of 10 in-lb/in² with a maximum strain of 0.09. Comparing this with the energy necessary to deform the pins by the same amount it appears that the fuel pins are much more susceptible to damage from shock loading than the channels are in the event of very short duration pulses. For pulses of greater duration, the channels might become the more susceptible.

4.3.4 Conclusions and Recommendations

The work done here serves to emphasize that contact loads due to the presence of pin spacers may be quite important as a failure mode for fuel pins. The local strains caused by such loads may be quite large. Whether or not the clad can tolerate this deformation without failing depends largely on its ductility at any given irradiation level. In any case the capability of the pins to withstand other types of loading will be reduced by these local yielding effects.

More information is required in order that such influences can be properly evaluated. Development of a method to predict clad failure due to spacer contact loads under both short and long term loading is recommended.

The resistance to deformation of the fuel pin cladding at a spacer support point for a dynamic loading has not been thoroughly investigated due to the lack of an analytical tool. The development of such a tool would be valuable in performing this type of analysis. The assessment of the relative flexibility of the fuel pin as a beam and the spacers is necessary to determine how the total absorbed energy is divided between the pin and spacer. Such an assessment is recommended.

SECTION 4.3 REFERENCES

1. Roark, R. J., "Formulas for Stress and Strain," 4th Edition, Table XII, Item 7. McGraw-Hill Book Company, N.Y., 1965

5.0 OPERATING SAFETY LIMITS FOR FUEL CHANNELS

It is generally considered that the propagation of a local fault from one fuel assembly to its neighbors represents a serious safety problem. The ability of the channel to contain local faults is discussed in this chapter. The categories of channel perturbation considered are:

1. Uniform overheating due to loss of coolant flow
2. Internal pressurization, either in the acoustic (shock pulse) or inertial regime.
3. Asymmetric external loads from the deformation of an adjacent channel.

5.1 UNIFORM OVERHEATING OF THE CHANNEL

Both the strength and ductility of the core structural material decrease with increasing temperature. Strength and elongation variations for steady state conditions are illustrated in Figure 5-1. These trends apply to irradiated material.¹ Yield strength is shown to decrease linearly with temperature.

As the channel wall becomes overheated it becomes less capable of resisting normal loadings such as those imposed by internal pressures. Figure 5-2 shows the limiting internal pressure which causes yielding of the channel walls as a function of the wall temperature. The pressure retention capabilities fall off rather steeply as the wall goes from 1000°F to 1400°F or higher. For irradiated materials a condition which causes yielding may also cause failure since the available ductility is very low. Thus the limiting pressures shown in Figure 5-2 may also be conservatively assumed to define failure conditions for such a channel. Temperature increases are thus seen to be quite important in defining the capabilities of the channel. The FFTF configuration was utilized for this example calculation.

Uniform overheating of the channel also reduces its ability to resist the effects of shock and impulsive loadings. Quantitative effects are illustrated in Figure 5-3 for the FFTF channel configuration.² Strain and deflection of the wall are shown as a function of the structure temperature for several levels of absorbed energy. This type of loading is discussed further in Section 5.3.

For a given level of energy absorption both the channel wall strain and deflection increase as the temperature increases. If the wall material has a low ductility because of its irradiated condition it may not be capable of accepting the increased strains predicted. Overheating effects, therefore, may well mean the difference between failure and survival of the channel during such an incident. In addition, the larger deflections which accrue from the higher material temperatures increase the likelihood of damage to and interference with adjacent core structures.

Figures 5-2 and 5-3 are only examples of the deleterious effects of channel overheating and the resultant loss of strength. They do indicate the manner in which the operating safety limits for the channel are reduced.

Figure 5-1(a) Recommended Tensile Properties Above Total Fluences of 10^{22} nvt As A Function of Test Temperature – Type 304 and 316 Stainless Steel

Figure 5-1(b) Recommended Tensile Properties as a Function of Total Fluence At 600° and 700°C – Type 304 and 316 Stainless Steel

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Figure 5-2 Channel Internal Pressure Which Causes Yielding as a Function of Channel Wall Temperature

Figure 5-3 Variation of Channel Response to Internal Loads as a Function of Channel Temperature

SECTION 5.1 REFERENCES

1. GEAP-10062, "Mechanical Properties of Irradiated Type-304 and Type-316 Stainless Steel in Fast Reactors", Comprelli, F. A., Busboom, H. J. and Gebhardt, M. F., June, 1969.
2. BNWL-955, "FFTF Reference Concept Summary Descriptions", January 1969.

5.2 RESPONSE TO DYNAMIC LOADING

In prior work (GEAP-10059⁵), a method of solution for dynamic problems by static simulation was illustrated in the solution of a hexagonal duct for a shock pulse originating internally. Utilizing this method the energy required to deform the structural material is computed in a succession of static solutions of increasing load and/or deformation. This is then compared with the energy available from dynamic loading in order to arrive at the equivalent static load solution. It is necessary to carry such a solution into the plastic range in order to secure significant energy absorption. The prior effort employed a finite element solution and plastic material properties, but was limited to small deflection theory and consequent restricted deformation. This limitation has been removed by the present method of analysis. This method is applied to both internal and external pressure loads on the hexagonal channel walls.

The prior effort also assessed the deposition of energy on a barrier by a combination of acoustic theory and impact approximation for impulsive load. A limited indication of the opacity of the barrier as a function of the wave length and natural period of vibration was obtained.

The assessment of energy deposition on a barrier immersed in a fluid medium has not really been improved in this study. Some simplification of the acoustic laws is shown in a formula for transmission of energy which indicates the opacity of the barrier and the conditions controlling its opacity. The transmission of acoustic energy is discussed next.

5.2.1 TRANSMISSION, REFLECTION AND ABSORPTION OF SHOCK IMPULSE:

There appear to be at least three regimes in the sonic transmission of plane pressure pulses that require differing techniques of analysis for assessing the reflection, transmission and absorption of energy by an obstacle immersed in the fluid traversed by the pulse. The three regimes for normal incidence are indicated as:

1. For a thin solid (where the thickness in the direction of travel of the pulse is much less than a wave length of the pulse), the wall appears to be nearly transparent to the pulse. In this case, the pulse seems to carry the solid along with the particle motion of the fluid, with energy consistent with the particle displacement absorbed by the wall and the remainder transmitted. An approximation of the energy deposited can be obtained by consideration of the amplitude of the particle motion of the fluid forcing a deflection of the flexible solid, with the energy associated with plastic deformation of the solid absorbed. It would appear necessary that the intensity of such a pulse be great enough to deform the solid plastically and that relatively rigid portions of the solid might not fit into this regime. It would also appear necessary that the dimensions of the wall perpendicular to the direction of travel be equal to, or greater than, a wave length for this response. This regime has not been investigated sufficiently by either analytical or experimental methods to instill confidence in such speculation and only limited evidence is available to support the indicated procedure.
2. For a thick solid (where the thickness in the direction of travel of the pulse is very much greater than the wave length of the pulse), the acoustic laws of transmission, reflection and absorption apply at each interface in the elastic range.
3. For intermediate thickness of solid (where the thickness in the direction of the travel of the pulse is a wave length or more) the phenomena are less well known. It would seem prudent for energy absorption in the solid to consider both acoustic absorption and impulsive loading as additive, just as in the thick solid. The present state-of-the-art does not seem to permit a correction for the relative velocities of the wall and the pressure pulse.

The acoustic principles that are of interest in the propagation of plane waves of dilation normal to the plane of an obstacle immersed in a fluid can be discussed more briefly if a set of symbols is described. The symbols to be employed are:

SYMBOL	DESCRIPTION
P	Pressure (psig)
λ	Wave length of pulse (including fictitious negative image) (inch)
h	Object dimension in direction of travel of pulse (inch)
ρ	Mass density of material $\left(\frac{W}{gV} \cdot \frac{\text{lb. Sec}^2}{\text{in.}^4} \right)$
C	Velocity of sound (pulse velocity) (inch per second)
v	Velocity of material (inch per second)
n	Any integer
1,2,3	Subscripts to indicate region (see figure); 1 and 3 are the liquid, 2 is the solid.
A_r	Reflected amplitude of pulse (inch)
A_i	Amplitude of pulse in subscript region (inch)
E_r	Reflected energy of pulse (inch lb)
E_i	Energy of pulse in subscript region (inch lb)

It is not intended that development of equations, formulas and proofs be furnished here. Texts on acoustics as well as some of the references must suffice for those interested in the subject. Only a limited exposition on the pertinent parameters is intended for the reader interested in the specific phenomena. The parameters which appear to control the acoustic transmission of dilational waves are ρ , c , λ , and h . For transmission at normal incidence from one semi-infinite medium to another, the pressure amplitude and the energy of the reflected and transmitted waves are a function of the characteristic impedance, ρc , of the media, such that:

$$\frac{A_r}{A_1} = \frac{\rho_2 C_2 - \rho_1 C_1}{\rho_2 C_2 + \rho_1 C_1}$$

$$\frac{A_2}{A_1} = \frac{2\rho_2 C_2}{\rho_2 C_2 + \rho_1 C_1}$$

$$\frac{E_r}{E_1} = \left(\frac{\rho_2 C_2 - \rho_1 C_1}{\rho_2 C_2 + \rho_1 C_1} \right)^2$$

$$\frac{E_2}{E_1} = \frac{4\rho_2 C_2 \rho_1 C_1}{(\rho_2 C_2 + \rho_1 C_1)^2}$$

The transmission in an infinite medium through an immersed obstacle is a function of the thickness of the obstacle, h , and the wave length of the pulse, λ , in addition to the impedances, $\rho_i C_i$.

$$\frac{E_3}{E_1} = \frac{4}{4 \cos^2 \left(\frac{2\pi h}{\lambda} \right) + \left(\frac{\rho_2 C_2}{\rho_1 C_1} + \frac{\rho_1 C_1}{\rho_2 C_2} \right)^2 \sin^2 \left(\frac{2\pi h}{\lambda} \right)} \quad (1)$$

$$\text{if } h \ll \lambda: \cos \left(2\pi \frac{h}{\lambda} \right) \approx 1, \sin \left(2\pi \frac{h}{\lambda} \right) \approx 0$$

$$\text{then: } \frac{E_3}{E_1} = 1$$

$$\text{if } h = \frac{n\lambda}{2}: \cos \left(\frac{2\pi h}{\lambda} \right) = 1, \sin \left(2\pi \frac{h}{\lambda} \right) = 0$$

$$\text{then: } \frac{E_3}{E_1} = 1$$

$$\text{if } h = \frac{(2n-1)\lambda}{4}: \cos \left(\frac{2\pi h}{\lambda} \right) = 0, \sin \left(2\pi \frac{h}{\lambda} \right) = 1$$

$$\text{then: } \frac{E_3}{E_1} = \frac{4}{\left(\frac{\rho_2 C_2}{\rho_1 C_1} + \frac{\rho_1 C_1}{\rho_2 C_2} \right)^2}$$

It can be seen that for an obstacle thickness very short, when compared to the wave length of the pulse, or a multiple of a half-wave length in thickness, the obstacle is relatively transparent. Where the thickness is one-quarter wave length in extent, the obstacle is relatively opaque, if the impedances are much different. It becomes clear that the wave length of the pulse is very significant in the propagation of the energy. The difficulty of precisely predicting the wave form for the multiplicity of possible initiating conditions appears formidable. An examination of the effect of the

duration of the pulse in relation to the period of natural vibration of the obstacle in the failure mode indicates an additional dependence on the wave length of the pulse. If the pulse duration is less than the natural period, an impulsive loading will be imposed on the obstacle, resulting in a velocity and inertial energy deposition. A pulse duration greater than the natural period acts as a quasi-static pressure.

It would appear that a prediction of damage from a pressure pulse originating from contact of liquid sodium and overheated fuel rods that a worst case assumption would be necessary for either deposition or transmission of energy. A combination of acoustic energy absorption and impulsive energy absorbed from the reflected wave (as illustrated in Reference 5) without accounting for thickness of obstacle, is probably a worst case for energy absorbed. For reflected energy, an acoustic reflection is probably the maximum. For transmitted energy, it would appear possible to transmit the wave nearly undiminished. The prospect for properly portioning the energy of the pulse between absorbed, transmitted and reflected appears dismal unless the wave length or period of the pulse can be estimated with fair accuracy and confidence.

An effort has been made to correlate the test reported in Reference 4, where an explosive charge detonated in a fuel pin of an EBR-II subassembly in water. An approximate pressure vs time is reported, as is the deformation of the duct. The presence of a cluster of fuel pins surrounding the central one containing the charge is reported to have negligible effect on the deformation of the duct when compared with the single pin exploding. The explosive charge of 125 mg of Bullseye Pistol Powder is reported to release less than 0.15 Btu. The pressure-time pulse at the duct wall was measured for twenty explosions with a maximum peak pressure of 3150 psi and an average peak pressure of 2600 psi. The rise time was very short, less than 20 μ sec, but the pulse endured for about 200 μ sec. The decay pressure is reported to be defined by the equation

$$P(t) = P_0 e^{-t/\theta}$$

where

$P(t)$ = Pressure at time, t (psi)

t = Time from peak pressure (second)

θ = Time constant of $58 (10)^{-6}$ seconds

P_0 = Peak pressure (psi)

e = Base of Naperian logarithms

The maximum value of deformation of each flat is reported as .0206 inches. From other sources, it was determined that the duct was a hexagonal cylinder of 1.3025 inches flat dimension, with 0.040 inch thickness of 304 stainless steel. The fuel cladding tubes of the same material were 0.174 inch outside diameter and .009 inch wall thickness. Using a linear rise to a peak pressure of 3000 psi and the indicated decay curve, the impulse (momentum) of the pulse was computed as approximately 0.15 lb.sec/inch². On the same basis, the energy of the pulse computed as about 31 in.lb/inch². The energy absorbed by the duct when the deformation of the flat is 0.0206 inch is computed as about 0.5 in.lb/inch². The wave length of the pulse in water is $\lambda = C \times T = .0004 \times (65,400) = 26.16$ inches and in steel is $.0004 \times (220,000) = 88$ inches. The wave length when compared with the wall thickness of the duct of 0.04 inch indicates that the duct wall is very thin for this pulse and should transmit the pulse undiminished, by acoustic theory. The minimal energy absorbed by the duct would tend to confirm this theory. The natural period of vibration for the duct is about 0.00014 second as compared to 0.0002 second for the duration of the pulse.

The best correlation obtained was the particle travel of the fluid during the pulse compared to the observed deformation of the wall. By integrating $\delta = \int P/k \, dx$ over the length of the pulse, one obtains a travel of 0.024 inch for a particle of the water. In this integral, P is a function of position in the pulse and k is the bulk modulus of elasticity of the fluid. This compares very well with the observed plastic deformation of 0.0206 inch at the center of the duct wall. This evidence would indicate that the flexible span of the thin wall was simply forced to follow the motion of the water as the pulse passed.

The clad tubes used in the experiment were also assessed. The empty clad tubes had a bending frequency of about 93 cps with a period of .011 second, while water filling changed the frequency to 82 and period to .012 second. Sand

fill resulted in frequency of 74 cps and period of .0135 second. The outside diameter of the tubes equal to 0.17 inch when compared to the wave length of the pulse in water of 26 inches would indicate little, if any, interruption of the pulse by the tubes. For a circular shell the frequency is about 37,000 cps with a period of $.000027 < .0002$ (duration of pulse) so that a quasi-static pressure is indicated for collapse of the tubes. The elastic buckling strength of the tubes for external pressure is about 10,000 psi < 3000 psi peak pressure. Assuming 36,000 psi as yield strength of the material, yielding of the tube (plastic instability) might occur at about 3700 psi < 3000 psi peak pressure. Reference 4 reports little energy absorbed by the tubes and this report is consistent with the pulse duration and dimension of tube.

5.2.2 Analysis by Static Simulation of Shock Pulse Loading

In the analysis of core components for accidents in a liquid-metal-cooled fast breeder reactor, there have been postulated dynamic loading from a shock pulse originating from vapor ejection of the coolant followed by re-entry of the liquid coolant. The shock pulse may originate from thermal expansion incident to the violent heating of the liquid, to the vapor explosion, to the liquid hammer following the abrupt halt of the liquid column, or to some combination of these factors. The concern here will be confined to the pulse wave traveling at, or faster than, sonic speed that emanates from the site, rather than the longer lived pressure front that may follow close behind. It may be noted that the solution may be applied to the wave or to the subsequent quasi-static pressure.

The shock pulse is a dynamic loading of high pressure and brief duration that results in an impulse load on the barrier that may be categorized by the energy deposited. This energy must be absorbed by the internal energy of the deformed barrier and the problem is one of relating the internal energy and accompanying deformation to the energy deposited. If the deformation is consistent with the loading, then the internal energy will also be consistent, regardless of the manner of imposing the deformation. It is proposed that a static loading resulting in a similar deformation mode can be employed to relate deformation, energy absorbed, stresses and strains for the barrier.

Analyses employing this simulation have been performed for hexagonal fuel channels with pulses originating both internal and external to the channel. In addition, analysis has been performed for fuel rods supported at intervals by spacers. The method of analysis hinges upon the choice of a static loading that will simulate the deformation pattern imposed by an inertial loading resulting from the shock pulse. An exact simulation is difficult as the true loading is a function of the resistance to deformation of each point of the structure. A judicious choice can frequently be made that is conservative, i.e., that results in a less efficient energy absorption than the exact loading. Following the choice of load, the structure is analyzed for successive increasing loads and deformations extending well into the plastic range and approaching the factor limiting the ultimate usefulness of the structure. Equating the energy deposited by the pulse to the internal energy indicates the appropriate load equivalence and corresponding stress, deformation and strain. A limit load solution employing perfectly elastic-plastic material properties is utilized in combination with a membrane solution where appropriate.

For the channel loaded from the inside, the loading has been chosen as uniform for the cross section and a computer solution utilized for the successive analyses required. This loading will result in more bending and less membrane deformation in the hexagonal cross section than would the inertial loading so that it is a less efficient (and therefore conservative) energy absorber. The channel is successively analyzed in the elastic range for bending deformation of the walls, for elasto-plastic bending, and finally for plastic membrane resistance with adjacent panels furnishing the abutment reactions necessary. Graphs in Appendix A are examples of the resultant relations between energy, maximum deformation, simulated load and strain for the channel. Appendix A also gives the method of solution that has been programmed for the computer.

The externally loaded channel has one flat loaded uniformly and is supported at the corners of the opposite flat. In this case, the deformation of the entire cross section contributes to energy absorption from the one loaded flat and the ability to resist load by membrane action is absent. Graphs in Appendix A are examples of the resultant relations between energy, maximum deformation, simulated load and strain for this loading condition. The method of solution is also contained in Appendix A.

The fuel rod is loaded as a beam with fixed supports (approximates continuous beam over multiple supports). Reactions to support membrane forces are not available so beam action is the only load resistance available. Graphs in Appendix A are examples of the resultant relations of energy, maximum deformation, simulated load and strain. The solution is outlined in Appendix A.

5.2.3 Discussion of Results

The factors limiting the resistance of the structural elements of the core to dynamic loading are currently considered to be either excessive deformation or strain in excess of the ductility of the material. Excessive deformation will impact adjacent channels or the fuel pins with consequent damage and danger of propagation. Excessive strain may rupture the channel with similar results. The rate of strain may also be a factor in limiting the structural resistance but the available evidence is insufficient to properly assess this factor. The rate of strain is not considered in this analysis.

A parameter study has been made to study the effect of yield strength, hexagonal channel dimension, and wall thickness on the two factors which evidently limit the ability of the channel to absorb energy (resist dynamic loading). The studies have been limited to three variations in each of the three parameters and the conclusions must be limited by the extent of the study. The size and thickness parameters have been centered around the dimensions of the FFTF channel (4.615 inches across flat outside diameter, 0.14 inch wall thickness and inside corner radius of 0.16 inch). The yield strengths have been chosen in an attempt to bracket the beginning of life and the end of life for type 304 SS at 1000°F but may also be considered for temperature or material variation. The modulus of elasticity is a slowly varying property with temperature and has been used as a constant value of 22.6×10^6 for the study. A computation for maximum strain rate has been included in the analysis of the channel for internal load, but not for the external load solution. If sufficient information is accumulated to justify it, the computation of velocity of strain can be easily added for the external load solution.

In the investigation of the fuel pin, the reference pin of type 304 stainless steel with an outside diameter of 0.23 inch and a wall thickness of 0.014 inch has been chosen as sufficiently representative for study. The critical factors limiting the ability of the pin to absorb energy are the same as those for the channel (deformation and strain) and the parameters varied are the yield strength and span between spacers. The range of yield strengths is slightly lower than that for the channels to conform to the somewhat higher temperature of clad versus channel.

The maximum deflection of the channel wall appears to be a function of inverse thickness, span, and inverse of yield strength

$$\left(\delta = f\left(\frac{1}{h}, F, \frac{1}{\sigma_y}\right) \right)$$

The maximum strain in the wall is a function of thickness, inverse span and inverse of yield strength

$$\left(\epsilon = f\left(h, \frac{1}{F}, \frac{1}{\sigma_y}\right) \right)$$

The thickness appears to have a stronger influence on deflection than on strain for constant energy absorption, for the ranges examined. The span dimension appears to be a weaker factor in strain than in deflection. The yield strength appears a strong factor in both deflection and strain response to energy absorption.

The fuel pin response to incident energy appears strongly influenced by yield strength. The deflection is greatly affected by the span but the strain appears independent of span for specified incident energy.

The shock loadings postulated to date are incompletely characterized and cover a large range. A severe example of the first shock wave due to fuel/clad-sodium thermal interaction on sodium re-entry⁵ appears to be an energy density of 18.8 in. lb per square inch. The basis for this conservative estimate of energy is presented in Section 7.3.4.3. If we assume the wall to be opaque to this pulse about 20% will be absorbed and the remainder reflected.⁵ For an internal loading, the reference channel would have a maximum deformation of 0.055 inch and a strain of 0.048 in early life with a yield strength of 14,000 psi. It would appear that this would offer little threat to the integrity of the channel. Near end of life the yield strength might be increased to 34000 psi with a consequent degradation of ductility. In this case, the deflection would be 0.028 inch and the strain 0.02. This case is marginal, but the channel would still have a good chance for survival. For external loading of the channel in early life, the deflection would compute as 0.119 inch and the strain as 0.00377. The strain is negligible but the deflection may be excessive. Late in life the deflection would compute as 0.056 and the strain as 0.0024. The channel should be adequate to survive this loading at design

temperatures. As the accident progresses, the channel will become overheated and likely fail due to repeated loads and reduced strength and ductility. The fuel pin would not be competent to survive such loading, but the duration of the pulse would determine how much of the load was imposed on the pin.

No other dynamic loads which might be imposed on channels have been detailed, preventing further comparison and correlation of this model of channel energy accommodation.

5.2.4 Conclusions and Recommendations

- Experimental results of channel deformation by static and dynamic loads are needed to confirm the analysis of channel deflection and failure loads. Dynamic loading by pneumatics, hydraulics, or explosives should be considered to simulate the range of pressure pulse types to be expected. Prototypic integral tests of channel containment of violent boiling, sodium hammer or pin bursts (such as EBR-II Mark E tests) should follow. The environment of the channel should be simulated.
- Analysis and, where necessary, testing of local thermal and mechanical perturbations of channels must be conducted to complete the understanding of channel response to possible reactor situations.
- Extension of the static simulation model of structural response to include dynamic effects is recommended. Provision for thermal loads and for axial and circumferential variation of normal loads is recommended.
- Both analysis and experiments appear necessary to permit prediction of channel response to accident conditions. A key uncertainty is the effect of irradiation on material properties. Material property tests of cladding are expected to provide the required data, permitting analytical prediction of their influence on channel operating safety limits.

SECTION 5.2 REFERENCES

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6.0 LONG TERM FUEL FAILURE PROPAGATION EFFECTS

LMFBR core performance beyond the design condition is reviewed in the next two sections with particular regard to the priority questions identified by the initial fault tree analysis. Those phenomena which seem to influence the progression of a fault and contribute to failure propagation are emphasized. An important consideration in evaluating failure propagation is the relationship of the possible safety system response to the propagation mechanisms. This section considers propagation mechanisms which probably are slower than available instrumentation response times. The principal mechanisms are fuel-sodium reaction product swelling and fuel-clad debris blockage. The following Section 7.0 considers propagation mechanisms which may occur faster than the expected instrumentation response times.

Three of the long term fuel propagation mechanisms will be discussed in this section. The first problem area is that caused by fuel-sodium reaction product swelling. The second deals with fuel failure propagation resulting from debris trapped within the fuel pin matrix. Both of these topics treat fuel pin failures propagating from one pin to another. The third problem area deals with the possible evolution from one type of failure to another more dangerous type within a single pin.

6.1 FUEL-SODIUM REACTION PRODUCT SWELLING (W.R. Gee, Jr.)

Mixed oxide fuels have been observed to swell considerably when exposed to sodium.^{1,2} * A chemical reaction is believed to take place. The reaction product is less dense than the fuel, and thus its formation results in the swelling of the material. Fuel pins with cladding failures which allow sodium to contact the fuel are potential candidates for this swelling. The growth could restrict the flow to adjacent pins, possibly leading to their overheating and eventual failure. Thus the rate and extent of swelling that can be expected from a pin operating with a large area failure is important to the probability of failure propagation.

Several of the task B and E tests of PA-10 have been directed toward determining the characteristics and dependencies of the fuel-sodium reaction. Capsule B9A tested the effect of oxygen content of mixed oxide fuel on the operation of a defected fuel pin in flowing sodium.³ The capsule contained two purposely defected pins; one with O:M of 1.95 and the other with O:M of 1.99. The pin with the larger O:M ratio experienced severe expansion of the fuel in the defect region. The fuel-sodium reaction product extended 80 mils into the flowing coolant stream.⁴ Diametral expansion of the cladding due to the swelling was a maximum of 70 mils for the pin with O:M = 1.99 and 20 mils for the pin with O:M = 1.95.⁵

The large uncertainty in the B9A experiment was the level of sodium purity. Ideally, the oxygen level should have been maintained at 5-10 ppm to typify LMFBR conditions. It was probably much higher than this during the experiment, however.⁶ Capsule B9C should provide information on the effect of coolant purity on fuel specimens failed during full power operation.⁶

There have been several failures of Task F encapsulated pins. Diametral increases of the cladding have typically been in the range of 50 to 60 mils, maximum. The large diametral increases have hypothetically been attributed to fuel-sodium reaction product swelling. In encapsulated specimens, the limited amount of sodium could possibly lead to more swelling than in a typical LMFBR system with a reservoir of relatively pure sodium that is essentially infinite.⁷ The pure, flowing sodium could continuously remove excess oxygen from the fuel and thus retard the swelling due to reaction product formation.

The rate of swelling attributed to the reaction product formation was investigated with the purposely defected pin, E1J-B2.⁶ This pin was failed at a fabricated notch in the cladding by taking advantage of the differential thermal expansion between the fuel and the cladding. After failure, the average ΔD of the fuel pin was determined from neutron radiography after 2, 30, 120, and 1440 minutes of irradiation. The data of ΔD versus irradiation time plotted on log paper as a straight line. The equation of the line is: $\Delta D \text{ (mils)} = 3.88 T^{-2.5}$, where "T" is the irradiation time in minutes. E1J-B2 contained stoichiometric fuel and operated at a power level of 19-22 kw/ft. A single point at T = 24 days from

*Analysis (P.A. 53) completed since this work was done indicates that fuel-sodium reaction product swelling is limited by oxygen availability. A small clad defect apparently will admit sufficient sodium to form the reaction product and cause swelling. As soon as a significant size crack is formed (~ 15 mils), however, oxygen diffusion from the fuel to the sodium is sufficient to deplete the oxygen supply and to stop the reaction. Fuel swelling is thus limited and does not appear to be a serious problem.

the B9A pin with O:M = 1.99 fell on the same line.⁶ The applicability of these results to LMFBR cores is not clear because of the uncertainty in oxygen level in the capsule test.

Related work is being performed under Task E, where the density, thermal conductivity and x-ray parameters of the reaction product are under investigation. Aitken and Evans⁸ are performing thermodynamic and experimental studies of sodium-fuel interactions under PA-53. Analysis of the sodium for oxygen content is emphasized in the reporting. The work is being done out-of-pile, however, and may not be truly representative of the in-pile case with irradiated fuel.

Information from the European programs indicates that there have been many fuel pin failures in DFR, Rapsodie and BR-5. There have been no reports, however, of swelling of failed fuel pins that caused flow blockage and propagation of failures to other pins. This is possibly an indication that fuel-sodium reaction product swelling is not as serious in large volume flowing sodium (or NaK) systems as it is in encapsulated tests.

Analysis of Fuel Pin Swelling Effects

The chemical reaction between the fuel and the coolant can cause fuel swelling. As the fuel pin swells in the region of a clad failure it restricts flow in the surrounding coolant passages. This causes an asymmetrical overheating of the adjacent pins. The adjacent pins will tend to bow toward the swollen pin, further reducing the coolant flow area between them and producing stresses in their clad due to restraint of the bowing by the fuel pin spacers. These stresses, together with the local overheating could cause failure of the adjacent pin.

The effect, on the temperatures and stresses in the adjacent pins, of swelling of one initial pin is discussed below.

SECTION 6.1 REFERENCES

1. Sodium-Cooled Reactors, Fast Ceramic Reactor Development Program, Twenty-Seventh Quarterly Report, May-July 1969, GE-BRDO, August 1968 (GEAP-5677).
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8. GEAP-12032, "A Thermodynamic Data Program Involving Plutonia and Urania at High Temperatures", 8th Quarterly Report, PA-53, E. A. Aitken, S. K. Evans, September 1969.

6.1.1 Structural Analysis

The structural problem is illustrated, schematically, in Figure 6-1. A fuel pin is exposed to a linear transverse temperature gradient over part of its length, 2ℓ . As the pin bows in the direction of increasing temperature, it is partially restrained by reaction forces at the spacer contact points. However, the spacers are also elastic and are represented here as springs with fixed spring constants.

The solution of this problem is complex, and it was not possible to produce an analytical solution in the limited time available. Consequently, the problem was studied using an existing structural analysis computer code (MASS).

The following characteristics and properties were held constant during the study:

Fuel Pin Pitch,	0.30 inches
Fuel Pin Outside Diameter	0.25 inches
Fuel Pin Clad Thickness,	0.01 inches
Modulus of Elasticity,	22×10^6 lb/in ²
Coefficient of Thermal Expansion	1×10^{-5} in/in ^{°F}
Temperature Difference (linear gradient),	100°F
Spring Constant for Spacer Force:	
Spiral Wire,	38.70×10^{-6} in/lb
Grid,	$\left\{ \begin{array}{l} 8.50 \times 10^{-6} \text{ in/lb} \\ 45.50 \times 10^{-6} \text{ in/lb} \end{array} \right.$

The fuel pin design values (pitch, diameter, wall thickness) are based on the GE 1000 MW(e) Design Study.¹ The modulus of elasticity and the coefficient of thermal expansion are typical of Type 316 stainless steel at 1000°F². The transverse temperature gradient was held constant at 400°F/in, since the deflection and stress vary directly with the magnitude of the gradient. For very large gradients non-linearities will occur, due to local variation of the material properties. These effects were not studied.

Several cases were studied in which the overheated length and/or the distance between spacer contact points were varied. The parameters of concern are given in Table 6-1 for each case. The heated length refers to the length of the pin over which the ΔT exists.

In estimating the restraint due to spiral wire spacers, the model shown on Figure 6-2 was used. Consider pin "B" bowing toward the space between pins "D" and "E". Pin "B" is restrained in that direction by four contact points in each turn of the spiral wire. The plan view on Figure 6-2 shows that the forces at the contact points are radially directed and that adjacent points are 60° apart circumferentially. The elevation view shows that the axial spacing between contact points alternates from one-sixth to one-third of the spiral wire lead distance. Referring back to Figure 6-1 S_1 equals one-third and S_2 equals one-sixth of the spiral wire lead distance. In using the two-dimensional model of Figure 6-1 it must be remembered that the calculated contact forces represent only the component of contact force in the direction of movement of the rod.

The spring constant for the spiral wire spaced bundle was calculated by assuming that the restraining force (component in the direction of movement) is eventually transmitted to the middle of the channel wall, and that the channel wall deflects as a beam with built in ends. The width of the beam is assumed to equal the spiral wire lead distance.

In estimating the restraint due to grid spacers, the model shown on Figure 6-3 was used. The grids are assumed to be equally spaced in the axial direction ($S_1 = S_2$). However, it is also assumed that each grid is rotated by 60° relative to its neighbor above and below it. Thus if pin "B" is bowing towards the space between pins "D" and "E" the effective spring constant alternates between two values. One spring constant is characteristic of the orientation shown on Figures 6-3 for bowing towards D and E and, the other (which is stiffer) is characteristic of bowing towards "A" and "D". For the chosen pin pitch and diameter, the thickness of the grid material is only ten mils. The height of each grid member is arbitrarily taken as one-half inch.

Figure 6-1 Schematic Diagram of the Structural Problem

Table 6-1. Case Characteristics

Case No.	Spacer Type	Lead or Spacing, in.	Heated Length, 2ℓ , in.
1	Spiral Wire	6	2
2	Spiral Wire	6	8
3	Spiral Wire	6	16
4	Spiral Wire	9	3
5	Spiral Wire	12	4
6	Grid	2	2
7	Grid	2	6
8	Grid	2	10
9	Grid	3	3
10	Grid	4	4

Figure 6-2 Spiral Wire Spacer Contact Points

Figure 6-3 Grid Spacer Contact Points

The results of the structural analysis are presented on Figures 6-4 through 6-11. From Figures 6-4 and 6-6 it can be seen that the largest deflection occurs when the thermal gradient extends over the span between two spacer contact points, only. Whereas, Figures 6-8 and 6-10 show that the largest bending stress results when the thermal gradient extends over three or more spans.

Figures 6-5 and 6-7 show the effect on maximum deflection of increasing the distance between spacer contact points. It is significant to note how small the deflections are, compared to the fuel pin spacing. Considering only the bending stresses, it appears that temperature differences in excess of 500°F can be tolerated for the cases shown. However, it appears that the local stresses due to spacer contact forces are high.

The magnitudes of these forces which can result from a thermal gradient across the pin are shown in Table 6-2. A ΔT of 100°F was assumed throughout. The spacer number in this table refers to the successive axial spacer contact points starting from the one nearest the center of the overheated length.

Assuming that the yield strength of irradiated 316 stainless steel is approximately 60,000 psi, the force causing yielding is 3 pounds (see Section 4.3.2). The corresponding ΔT above which plastic strain will occur can be defined for each case as shown in Table 6-3.

These results indicate that cladding ΔT 's on the order of 80°F to 150°F can cause stress at the spacer contact point in excess of the yield point. It is uncertain at this time whether or not such local yielding of the clad would eventually result in failure of the pin.

Consequently, these ΔT 's do not necessarily represent failure points, but simply provide an index for evaluating the ΔT 's which are calculated in the thermal analysis.

6.1.2 Thermal Analysis

The thermal analysis was done using an interchannel flow mixing computer code. The calculational model was a 37-pin bundle with an active core height of 30 inches and with the axial power distribution typical of the peak power bundle of Reference 1. The code printed out the coolant temperature in each interpin flow channel of the 37-pin bundle for each of ten axial nodes.

The central pin of the 37-pin array was allowed to swell with every cross-section remaining circular, and three parameters were studied. The three were: the amount of diametral increase; the axial length of the swollen region; and, the effectiveness of interchannel flow mixing. In all cases the swollen region begins at the core mid-plane and extends toward the outlet.

The results are presented on Figures 6-12 through 6-15. Figure 6-12 shows how the temperature difference across an adjacent pin is affected by the amount of swelling of the central pin. In this figure the swollen region extends over one calculational node (three inches); and no interchannel flow mixing is assumed. It appears that, for these conditions, swelling of the central pin up to 35% $\Delta D/D$ is structurally acceptable. This conclusion ignores the accelerated degradation of clad properties which will result due to the increased temperature. (Note: At 40% $\Delta D/D$, the swollen pin will touch the adjacent pins.)

It should be noted that the calculational model overestimates the ΔT downstream of the swollen region. The flow channels adjacent to the swollen pin have increased flow area downstream of the swollen region. Therefore, some coolant from other channels must be diverted to these channels to provide the extra flow. This diverted coolant has a lower temperature than the coolant already in the channel. Thus, a decrease in the average temperature should result. However, this effect is not included in the current calculations.

Figure 6-13 shows how the temperature difference across an adjacent pin is affected by the amount of interchannel flow mixing. This effect is characterized by an interchannel mixing factor, F , which is the equivalent fraction of the flow from one channel which is interchanged with an adjacent channel, over one calculational node. Experimental measurements show that " F " is between 1/3 and 2/3 for pins with spiral wire spacers.^{4,5,6} For grid spacers the value is somewhat less.⁶ Figure 6-13 shows that the effect of interchannel mixing becomes increasingly important as the amount of swelling increases.

Figures 6-14 and 6-15 show how the temperature difference across an adjacent pin is affected by the axial extent of the swollen zone on the central pin. Figure 6-14 shows that if the interchannel mixing factor is at the lower extreme of the data for wire wrapped pins and if the axial extent of swelling exceeds six inches, for 30% swelling, or three inches, for 40% swelling, then failure of the adjacent pin is likely. However, Figure 6-15 shows that if the interchannel

Figure 6-4 Effect on Fuel Pin Deflection of the Axial Extent of the Thermal Gradient, (2v), Spiral Wire Spacers.

Figure 6-5 Effect on Fuel Pin Deflection of the Spiral Wire Lead Distance

Figure 6-6 Effect on Fuel Pin Deflection of the Axial Extent of the Thermal Gradient ($2l$), Grid Spacers

Figure 6-7 Effect on Fuel Pin Deflection of the Distance Between Grid Spacers

Figure 6-8 Effect on Fuel Pin Bending Stress of the Axial Extent (2ψ) of the Thermal Gradient, Spiral Wire Spacers

Figure 6-9 Effect On Fuel Pin Bending Stress Of The Spiral Wire Lead Distance

Figure 6-10 Effect on Fuel Pin Bending Stress of the Axial Extent (2 ψ) of the Thermal Gradient, Grid Spacers

Figure 6-11 Effect on Fuel Pin Bending Stress of the Distance Between Grid Spacers

Figure 6-12 Effect on Adjacent Pin ΔT of Initial Pin Swelling ($\Delta L = 3''$, $F = 0$)

Figure 6-13 Effect on Adjacent Pin ΔT of the Interchannel Flow Sweeping Factor ($\Delta L = 3''$)

Figure 6-14 Effect on Adjacent Pin ΔT of the Axial Extent of Initial Pin Swelling (Flow Sweeping = 1/3)

Figure 6-15 Effect on Adjacent Pin ΔT of the Axial Extent of Initial Pin Swelling (Flow Sweeping = 2/3)

mixing factor is at the upper end of the experimental range and if swelling can be limited to less than 30% $\Delta D/D$, then the swollen zone can be allowed to extend over the entire upper half of the fuel pin.

The reference design¹ employed a pin P/D of 1.2 and a spiral wire lead of 6 inches. Recent experimental evidence indicates that the interchannel flow mixing for this configuration should be very near 1/3.⁶ Thus the plots in Figure 6-14 are most applicable for this design.

The analysis performed here indicated that even moderate thermal gradients across a fuel pin can lead to substantial forces at the spacer contact points. In Section 4.3.2 it was pointed out that forces on this order may be sufficient to cause local yielding of the cladding, and possibly even failure of the pin. The overheated pins will also be threatened by the potential for accelerated attack of the clad inside diameter if the affected pins continue to operate at high temperatures. It is important, therefore, to minimize both the temperature rise and the temperature gradient experienced by fuel pins adjacent to a flow disturbance. This may be accomplished to a certain extent by controlling the degree of interchannel coolant mixing. Investigations should be carried out to define the dependence of this mixing on fuel assembly design parameters such as pin spacing, spacer lead, and spacer configuration. Research in this area is recommended.

In analyzing the effects of fuel pin swelling, only two major areas of uncertainty were found. One uncertainty is in predicting the amount of interchannel flow mixing within a fuel bundle. Although some data is available, more precision is required. In addition the extent of the flow mixing must be defined in terms of the fuel assembly design parameters. The allowable axial extent of fuel swelling in the failed pin is strongly affected by varying the assumed amount of mixing within the range of existing data. This parameter can be well characterized by ex-reactor tests.

The second major uncertainty is in evaluating the damage to fuel pin cladding due to excessive forces at the spacer contact points. The extent to which the clad can relieve these forces through either thermal or irradiation induced creep remains to be evaluated. However, more data is required on the extent of irradiation induced creep and on the total strain to failure of cladding material operating in a fast neutron flux. These two properties can best be determined by in-reactor tests.

Table 6-2. Pin to Spacer Contact Force in the Direction
of the Deflection (pounds), $\Delta T = 100^\circ F$

Case No. →	1	2	3	4	5	6	7	8	9	10
Spacer No. ↓										
1	2.97	0.39	0	2.62	2.09	1.83	1.40	0.35	1.26	0.95
2	2.81	1.67	0.01	2.79	2.29	2.27	0.07	1.84	1.58	1.20
3	0.27	1.32	0.08	0.24	0.30	0.52	1.79	0	0.41	0.32
4	0.10	2.53	0.17	0.07	0.11	0.11	0.39	1.85	0.10	0.08
5	0.02	0.17	2.53	0	0.01	0.02	0.07	0.44	0.02	0.02
6		0.08	1.32					0.09		
7		0.01	1.65					0.02		
8			0.29							
9			0.10							

Table 6-3. Safety Limits Based on Mechanical Loading

Case No.	Allowable ΔT, °F*
1	87.5
2	103
3	103
4	93.2
5	114
6	114
7	145
8	141
9	164
10	216

*Temperature gradient across a pin which produces forces at spacer contact points sufficient to cause local yielding in the clad.

SUBSECTIONS 6.1.1 AND 6.1.2 REFERENCES

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6.1.3 Conclusions and Recommendations

Fuel-Sodium Reaction Product Swelling*

The most important conclusion from the work in the area of fuel-sodium reaction product swelling is that a fuel pin failure in an LMFBR may have the potential for serious swelling. The failed fuel pins that have exhibited large amounts of swelling have been encapsulated pins with relatively small amounts of sodium available. This includes the B9A test in the forced-convection capsule that apparently operated with high levels of oxygen in the sodium. The B9C test should yield valuable information on failed fuel pin swelling in sodium of the purity expected for LMFBR's, provided that the purity in the capsule can be maintained.

It is concluded that there is not enough evidence available to date to conservatively predict anything less for LMFBR pins than the swelling observed in encapsulated pins.

The recommendations that result from this review are directed toward the ongoing work in Tasks B and E of PA-10. The success of simulating an LMFBR environment in the Series B9 forced-convection capsules is very dependent on the control of the sodium purity. The importance of this parameter makes it highly desirable to have a reliable monitoring and clean-up system for the sodium loop. Even if the oxygen content cannot be maintained at the 5-10 ppm level, it is important to know what it is during the experiment.

Another consideration that could warrant investigation is the initial failure geometry. It appears that any fuel pin defect at power produces a large cladding breach relative to that necessary to allow sodium entry. The large breach would allow sodium to circulate into and out of the pin and thus, continuously remove the excess oxygen from the fuel. If, however, the failure starts out very small, the sodium may leak slowly into the pin and, instead of carrying the excess oxygen away, may form a region of fuel-sodium reaction product. It is conceivable that this reaction product could form a protective layer preventing the pure sodium from removing the oxygen when the failure enlarges. Instead, the sodium could diffuse through the reaction-product layer and form new reaction product when it reaches a region in the fuel pin where sufficient oxygen is available.

Investigation of this hypothetical effect would require a more refined defect mechanism. For the near-term experimental effort, it is recognized that the reliability of the mechanism is more important than the defect size. In later work, however, the influence of defect size should be considered.

Structural and Thermal Analysis

The following are the conclusions of the study of the effects of fuel-sodium reaction product swelling:

1. The deflection of a fuel pin is greatest when the temperature gradient is imposed over a single free span between two spacer contact points.
2. The largest bending stress occurs when the temperature gradient is imposed over three or more spans.
3. For spiral wire leads less than twelve inches and for grid spacer axial spacings of less than four inches, the allowable temperature gradient across a fuel pin is more likely to be limited by the strains due to the spacer contact forces than to either the magnitude of pin bowing or of the bending stresses.
4. If the swollen region is short (less than three inches) then the ΔT across the adjacent pin does not produce either excessive stresses or excessive bowing, for a uniform swelling up to 35% $\Delta D/D$.
5. If the swollen region is long (greater than six inches) then the acceptable amount of swelling is very sensitive to the amount of interchannel mixing which can be assured.

*Analysis (P.A. 53) completed since this work was done indicates that fuel-sodium reaction product swelling is limited by oxygen availability. A small clad defect apparently will admit sufficient sodium to form the reaction product and cause swelling. As soon as a significant size crack is available (~ 15 mils) however, oxygen diffusion from the fuel to the sodium is sufficient to deplete the oxygen supply and to stop the reaction.

6.2 FUEL-CLAD DEBRIS BLOCKAGE

Fuel pin failures may be caused and propagated by the presence of debris which somehow becomes trapped within the fuel pin array. Such debris would most likely originate from the fuel pins themselves since it is unlikely that particulate matter of significant size will be carried in by the coolant flow. Any debris would most likely consist of pieces of fuel, cladding, or spacers which originated from a failed fuel pin.

No evidence has been found whereby debris trapping is known to have caused fuel failures in any liquid metal cooled reactors. This does not preclude the potential importance of this failure mechanism in LMFBR's, however. On the contrary, the use of the fault trees has indicated that this could be a very significant problem area.

Several investigations were directed at removing some of the uncertainty attached to this failure mechanism. First an attempt was made to characterize the size and type of debris originating from failed fuel pins. Next the geometry of a typical LMFBR fuel pin array was studied to determine the potential trapping sites and to compare these site sizes with the debris sizes found from the initial effort. Finally a thermal analysis was undertaken to assess the effect of trapped debris on the nearby pins and channel wall.

6.2.1 Assessment of Failed Fuel Pin Debris and Geometry (K.R. Absher)

Characterization of the debris and the changes in pin geometry resulting from a failed fuel pin is necessary in order to properly assess the potential for fuel pin failure propagation. This area must be considered because the safety and economic consequences of a fuel pin failure are sensitive to the amount and size distribution of failure debris and to the extent of pin geometry changes. Three potential effects of interest are: 1) fuel-sodium thermal interaction when hot fuel is expelled into the sodium, 2) local flow blockage due to failure debris or diametral increases of the pin, and 3) contamination of primary system.

6.2.1.1 Fuel Debris

The amount of fuel material that can be expelled into the coolant does not appear to be inherently limited by any work-energy consideration if there exists a large fission gas pressure in the central void (several thousand psi). The only identified case of fuel expulsion is from TREAT tests with fuel melting. This is not applicable, because of the high temperatures and the very fluid state of the fuel during expulsion.

Fuel pin failures have occurred in the PA-10 Task F fast flux irradiations and fuel loss has been observed. These results have been studied to determine the mechanisms of fuel loss from a failed pin. It is not known when the failures occurred and for how long they were irradiated in the failed condition. Fuel loss could have been due to weakening of the fuel from sodium logging and fallout in-pile or due to losses during examination. Rough estimates of fuel loss at various axial positions of pins F2F, F4F and F20 were made. All estimates were below 15%. Clad restraint was lost for up to 90° in F4F, 130° in F20, and was essentially completely lost at one axial position of F2F. These were encapsulated pins so there was no flowing coolant to wash fuel out. Even in the most severely affected location of F2F (no clad restraint), most of the fuel seemed to hold together while a small ring of apparently brittle fuel was lost. This behavior does not suggest fuel expulsion due to fission gas pressure in the central void, but rather passive fallout of fuel chunks.

Pins B3B and B3C suffered severe failures.² Pin B3B had negligible fuel loss. Pin B3C, however, suffered nearly 100% fuel loss in the failure zone. About 8% of the total fuel was lost. This pin failed at about 5,500 MWd/Te but remained in-pile until it had 53,000 MWd/Te. Thus, the fuel had a very long time in which to become weakened and to fall out. It is also possible that all of this fuel debris was expelled at the time of failure.

There does not appear to be an analytical means of predicting the size distribution of expelled fuel particles. The approach used here is to inspect photographs of the cross-section of previous failures and to measure the size of fuel particles that appear to be unrestrained. It is assumed that these particles are the most likely to be expelled. Implicit in this approach is the assumption that the fuel cracking due to fuel expulsion is similar to the cracking when the pin power is reduced and it is removed from the core.

Measurements were made of fuel particles in failed pins from Task F photographs of F2F, F4F and F20. Illustrations from PA-10 Quarterlies^{3,4} were also used but to a lesser degree. Task C pins could not be used because the once-molten condition of the fuel makes them atypical. Roughly one hundred measurements of the dimensions of individually identifiable fuel particles were made. The average dimension was about 0.030 inch. The maximum was

about 0.088 inch. The distribution is shown in Figure 6-16. If a severe failure occurs (without much fuel melting) a conservative estimate of debris is 15% of the fuel in the failure zone in pieces with maximum dimensions up to 0.1 inch.

6.2.1.2 Cladding and Wire Wrap Debris

Pieces of cladding can cause flow blockages if they move into the coolant subchannel. A piece of cladding can move into the coolant subchannel if it is removed from the fuel element due to cladding fracture. Task F photographs of failed pins F2F, F20 and F4F show longitudinal fractures through the cladding at the same axial elevation. The potential exists for a portion of the cladding between these longitudinal cracks to be dislodged and become "debris" or to be preferentially distorted and obstruct flow locally.

The circumferential distance between the parallel longitudinal fractures varied between 0.02 and 0.24 inch. We have little basis for estimating the expected length of a cladding fragment resulting from these longitudinal fractures. Pin F4F exhibited two pieces roughly 0.12 inch long but there was clad melting during the failure.

The spacers are potential sources of failure debris. If the wire wrap or grid is in the failure zone, it may break and cause flow blockage. This type of occurrence has not been observed.

6.2.1.3 Fuel Pin Geometry Changes

A change in pin geometry can cause flow reduction locally. In the following examples, the cladding suffered significant diametral increases.

Pin	ΔD (mils)	O/M	Clad Material	Fuel Type	% T.D.	B.U. (MWd/Te)
F2F	53	1.98	Incoloy 800	Pellet	94.6	58,000
F20	56	2.03	347 SS	Pellet	94.0	71,000
F4F	31	2.00	Incoloy 800	Pellet	86.3	34,000
B3B	50	2.00	347 SS	Pellet	86	18,000
B3C	85	2.00	347 SS	Pellet	88	53,000*
B9A-1	65	1.99	316 SS	Pellet	89	9,300
B9A-2	20	1.95	316 SS	Pellet	89	9,300

* Failure occurred at 5,500 MWd/Te.

The cause of the cladding distortion is not clear in most cases. It could have been a rapid distortion due to forces generated at the time of failure as postulated for B3C² or it could have been a long-term distortion due to fuel-sodium reaction product swelling as postulated in B9A-1,2⁵.

T. Lauritzen performed some burst tests of .250 inch outside diameter, .015 inch wall thickness stainless steel tubing.¹ In the region of tube failure, gross distortion was evident. This distortion typically extended over an axial length of one half inch. The more severe failures produced large area, somewhat flat, pieces of tubing debris that could cause local flow blockage in a coolant channel. The largest piece of this description would be 0.5 inch x 0.785 inch (the latter dimension corresponds to the cladding circumference). This type of failure is not expected in FCR cores because the presence of sodium around the tube during a burst failure would prevent such severe deformation. This was illustrated in some ANL tests⁶ in which an explosive charge was detonated inside tubing. The manner in which the tubing failed differed with the surrounding medium. If the pin was immersed in water, the wall ruptured in a single longitudinal tear. If the test was in air, the wall rupture progressed at 45° angles and several fragments broke loose. This indicates that liquid sodium in the channel will greatly decrease the probability of clad fragments being dislodged as a result of a failure.

Figure 6-16 Size Distribution

6.2.1.4 Recommendations

In order to increase our ability to characterize the debris and geometry resulting from fuel pin failures and, thus, to estimate the probable consequences of failures, there must be fuel pin tests run to failure. These tests must be adequately instrumented in order to give a rapid indication of failure. The test may then be removed from the core in order to isolate the failure effects from post-failure behavior. It is not necessary to design fuel pin tests specifically to study this topic. The surveillance of pin failures in other tests should provide adequate information on the type of debris to be expected under various conditions. This presumes, of course, that there will be many fuel pin tests run to failure in the future and that they will be adequately instrumented.

The subject discussed here would be best served by careful inspection of the failed pins. Physical measurement of actual debris or potential debris is a simple means to arrive at an expected size distribution of material that could be forced into the coolant stream. This information can be used to estimate the likelihood of local flow blockages.

It is recommended that future burst tests of clad materials be run in a more typical medium such as water or sodium. This will give a better representation of in-pile conditions. Thus, the cladding geometry after failure should be more representative. High temperatures of the tests, such as described in Reference 1 (1100-1300°F), will require use of sodium or NaK bath. Out-of-pile burst tests of irradiated fuel pins would be a logical extension to show directly the possibilities of debris ejection, even though it simulates only bursts at zero power. There may be a limiting fuel particle size due to the thermal shock when fuel comes in contact with sodium. Burst failures may be simulated by dropping various sized pieces of hot fuel into sodium and measuring the resultant particle sizes. Thermal shock to a fuel pin at the initiation of a large area failure could be studied by fabricating an annular fuel pin with a defect mechanism in the cladding and a heater in the annulus. When the fuel pin is at temperature in sodium, the pin is purposely failed. The particle size of the potential debris could be observed in the pin. An alternative, more difficult method would be to have the defect extend over a sufficient length to permit the cladding to open up and allow the free particles to fall out and be collected at the bottom of the sodium vessel.

SUBSECTION 6.2.1 REFERENCES

1. GEAP-13550, "The Influence of Surface Defects on Short-Term Mechanical Properties of Austenitic Stainless Steel Tubing", T. Lauritzen, A. Montes, and A. Conti, December 1969.
2. GEAP-13620, "Behavior of Failed Mixed Oxide Fuel in Flowing Liquid Metal", K. Gregoire, et al., to be published.
3. GEAP-5584, "FCR 25th Quarterly Report", March 1968.
4. GEAP-10028-32, "FCR 32nd Quarterly Report", November 1969.
5. GEAP-10028-34, "Thirty-Fourth Quarterly Report PA-10", June 1970.
6. ANL-7445, "Reactor Development Program Progress Report", April 1968.

6.2.2 Trapping of Debris Within a Fuel Assembly (F.A. Sciacca)

The purpose of this effort was to identify the areas within a fuel assembly where solid particles might be trapped. This was done as a function of the subassembly design parameters, namely the pin diameter, pitch-to-diameter ratio, spacer type, and the pin-to-channel wall spacing. These are the major influencing parameters. In addition an attempt was made to estimate whether fuel particles which enter the coolant streams will be carried along with the flow or will fall downward due to gravitational forces. The present effort limited itself to a study of trapping sites within the tube bundle of the fuel assembly. Thus possible blockages at the channel inlet were excluded.

Debris which may collect in a fuel assembly will most likely originate from failed fuel pins. The debris may consist of fuel particles or pieces of cladding or spacers. Any loose material will be trapped within the fuel pin matrix if it is larger than the coolant flow paths available to it.

The foregoing discussion gives an indication of the sizes of fuel particles and other debris which may originate from failed fuel pins. It pointed out that most of the loose fuel particles will have a maximum dimension of less than 0.10 inches. The average size noted was on the order of 0.030 inches. The measurements of fuel debris which were made utilized photographs of the cross-sections of irradiated fuel pins which had failed. The particles measured

remained in the pin. No data is available at this time of the sizes of particles which have actually been lost from a pin due to a failure.

6.2.2.1 Wire Wrap Spacers

The configuration used to define the trapping sites for wire wrapped fuel pin arrays is shown in Figure 6-17. A typical arrangement is shown, and there are spacers on all pins. At the channel wall the spacing between the pins and the wall may either be the full diameter of the wire spacer or a smaller spacing may be used. The latter has been proposed as a means for obtaining a more uniform temperature distribution within the assembly. Both types of pin-to-wall spacing designs were studied.

The shaded area in Figure 6-17 indicates the largest passage available to a particle of debris which somehow enters the fuel pin matrix. For the purposes of the present study it was assumed that all debris had essentially spherical or cylindrical shape. The characteristic dimension assumed for such particles was their diameter. Thus if a particle has a diameter larger than that of a circle which can be inscribed inside the shaded area of Figure 6-17, the particle is assumed to have been trapped. Particles which are smaller in size at least have the potential for escaping. Similarly any particle which can pass through the minimum spacing between pins has a high probability of passing through the array.

Within a typical LMFBR fuel assembly the flow is expected to be highly turbulent. The flow will undoubtedly have some influence on particle trapping. It may serve to wedge small particles into unlikely spaces where they would normally not be trapped. This may be especially true of the swirling action set up by wire wrap spacers. On the other hand the flow may act as a driving force on the particles which will prevent them from being trapped as long as their dimensions are not excessive. Since the flow effect is totally uncertain, it could not be directly factored into the study at this time. Potential trapping of debris, therefore, is defined solely in terms of geometrical considerations.

Figure 6-18 illustrates the minimum particle diameter which will be trapped by the shaded area in Figure 6-17. All particles larger than this will definitely be trapped within the confines of the space defined by three adjacent pins and their wire spacers. It may be noted that any such particle would have to originate from one of these pins or their spacers. This may tend to localize the failures, at least for a period of time. The possibility exists, of course, of several small particles entering a restriction simultaneously and becoming trapped by an action whereby they wedge themselves together against the trapping site. In this case the particles may originate from non-local sources.

The particle trapping sizes shown in Figure 6-18 are given as a function of pin diameter and pitch-to-diameter ratio. It has been assumed that the physical dimensions of fuel debris will be essentially independent of pin diameter and pin spacing. It naturally follows that particles have a greater probability of being trapped whenever smaller pins and/or smaller pitch-to-diameter ratios are employed in the design. For typical values of a pin diameter of 0.25 inches and a P/D of 1.20, Figure 6-18 shows that particles with diameters of 0.067 inches or greater will be trapped.

The above discussion of particle sizes indicated that most of the particles studied had dimensions less than this. Thus most fuel particles originating from failed fuel pins should not be trapped between fuel pins.

Along the channel wall there appears to be even less likelihood of particle entrapment. The flow passages here are larger than in the bundle interior. This is the case even when a "flattened wire" type spacing is used at the channel wall. When a flattened wire was used it was assumed that the pin-to-wall spacing was held constant at 0.030 inches. Using this assumption it appears that the size of particles which will be trapped next to the channel wall is insensitive to the pitch-to-diameter ratio employed, at least over the range studied. This is not the case when the pin-to-wall spacing is defined by the normal wire wrap thickness. These observations are shown in Figure 6-19.

The foregoing discussion was concerned with particle trapping sites defined by the shaded area of Figure 6-17 or similar sites adjacent to the channel wall. Wire wrap spacers will cause some swirling of the coolant flow. This type of coolant movement could serve to wedge particles into the gap between two neighboring fuel pins. The size of this gap defines the lower limit on particle sizes which one might expect to be trapped in such locations. Figure 6-20 is a plot of this particle size and its relationship to pin diameter and pin P/D. This figure is independent of the type of spacer employed.

6.2.2.2 Grid Spacers

Figure 6-21 shows a typical grid type spacer layout. The beams composing this spacer do not lie in the same horizontal plane. It was assumed that each of the beams had a projected area which was of sufficient width to fill the

Figure 6-17 Spiral Wire Spacer

Figure 6-18 Minimum Particle Size Which Will Be Trapped In A Wire Wrap Fuel Assembly

Figure 6-19 Minimum Particle Size Which May Be Trapped Between Fuel Pins and the Channel Wall

Figure 6-20 Minimum Particle Size Which May Be Trapped in the Gap Between Adjacent Fuel Pins

Figure 6-21 Grid Spacer

space between the rows of pins. The numbered areas in Figure 6-21 indicate the various flow paths which must be considered as potential trapping sites. Areas No. 3 and No. 4 are assumed to lie adjacent to the channel wall.

The minimum particle sizes which will be trapped in any of the grid spacer flow passages are shown in Figures 6-22 and 6-23. The former figure shows particle trapping sizes corresponding to the interior trapping sites as well as a No. 4 type site. The latter figure shows the trapping size requirements associated with the type No. 3 areas adjacent to the channel wall.

Fairly small debris particles may be trapped by a site such as area No. 2 of Figure 6-22. It need only be on the order of .020 to 0.030 inches. If the particles are much larger than this, however, they will likely be deflected away into a larger passageway. This is the case since the spacer beams do not lie in the same plane. Because the No. 2 sites will not trap particles which are either too large or too small, the probability of trapping at this location is reduced. Also, particles will not pile up behind one another at this site since they would more likely be deflected off into an adjacent No. 1 type flow passage.

Along the channel wall Figure 6-23 indicates that increasing the pin P/D results in smaller particles being trapped. This apparent anomaly results when the spacer beams grow in their projected area as the P/D increases. This increased spacer dimension reduces the passage flow areas along the wall. This result is also dependent on the assumption of a constant pin-to-wall spacing. Both 0.050 and 0.030 inch spacings were employed in Figure 6-23.

Mixed oxide fuels have been known to swell considerably when exposed to sodium.^{2,3} A chemical reaction is believed to take place. The resulting compound formed by the fuel and coolant is less dense than the original fuel material. Fuel particles which enter the coolant stream may undergo such a reaction. If they are trapped in the fuel assembly the swelling action may serve to wedge them in even more tightly. Such effects are uncertain at this time.

6.2.2.3 Coolant Drag Forces

In order to estimate whether or not fuel particles entering the coolant stream would be carried along with the stream or fall downward due to gravitational forces, the coolant forces on the debris were calculated. The drag force relationship used was applicable to a smooth sphere.⁴ Since the debris of interest is far from being smooth and spherical, the calculated drag force was arbitrarily doubled. If the drag force is greater than the weight of the particle, the object will be carried upward with the coolant stream. If the opposite is true, the particle may fall downward.

Figure 6-24 shows the particle sizes at which the drag forces balance the gravitational forces for fuel particles with a density of 10 gm/cc. The sizes are plotted as a function of coolant velocity. Normal LMFBR coolant velocities are in the range of 20 to 30 ft/sec. Classical drag forces, therefore, will be adequate to carry along only those particles which are on the order of 0.037 inches or smaller.

The small particles which can escape being trapped will be carried upward, at least to the regions where the coolant velocities decrease. There many of the small particles would likely remain suspended until the coolant flow decreases. When the velocities drop, even the small particles will likely fall back into the tube bundle of the fuel assembly. If they are not trapped in this region they may fall out of the lower portion of the fuel bundle. In this way fuel particles may eventually end up at the bottom of the reactor vessel or on the reactor grid plate. This may pose a contamination problem. The fuel assemblies can be designed such that this type of problem can be avoided if necessary.

6.2.2.4 Conclusions and Recommendations

The trapping sites available to debris which may be present in a fuel assembly are largely dependent on the type of fuel pin spacers employed. In the present effort only wire wrap and grid type spacers were studied. It was also limited to one example of each type.

A comparison of the trapping site sizes of typical LMFBR fuel assembly designs with the fuel debris dimensions reported in Section 6.2.1 indicate that some of the particles may be too large to pass through the fuel pin array and would be trapped. The bulk of the fuel particles are small, however, and should be able to pass through without being trapped. In general, it is only the larger particles, which are believed to be present in fewer numbers, that will be caught.

The results of this study indicate that simple straight beam-type grid spacers will trap smaller particles than will wire wrap spacers. The grid spacer studied had the characteristic that it could possibly trap relatively small particles. These small particle trapping sites, however, would not support any significant axial pile-up of particles behind the initial entrapment. Thus these sites could lead to small flow blockages, but would not allow debris to accumulate and

Figure 6-22 Minimum Particle Sizes Which May Be Trapped in a Grid Spacer Fuel Assembly

Figure 6-23 Minimum Particle Size Which May Be Trapped Between Fuel Pins and the Channel Wall

Figure 6-24 Size of Fuel Particles at Which Coolant Drag Forces Balance Gravitational Forces

cause a more severe blockage. Wire wrap spacers are more restrictive than grid spacers when comparing trapping sites which would support axial pile-up of fuel debris. That is, wire spacer fuel pin arrays will trap smaller particles at this type of site than will a grid spacer array such as that studied. Visual and radioactive trace tests of debris blockage are recommended to demonstrate the size of the particles which will be trapped in typical fuel bundles.

The work done here serves to define the trapping sites present in an LMFBR fuel assembly. The relationships found, however, relate only to the trapping potential of given sites based on their geometrical characteristics and those assumed for the debris. Particle trapping may also occur by virtue of a wedging action or because of flow influences. Whether or not debris will be caught due to these latter mechanisms can best be determined by performing tests.

Test facilities presently exist where particle trapping could be investigated. The General Electric ESADA fuel assembly water test loop is one such facility. Lead shot could be used to simulate fuel particles. If necessary the lead could be made radioactive to facilitate tracing its path through the fuel assembly. The ANL transparent test section¹ could also be used to visually follow the progress of debris injected into the fuel assemblies. The tests should be aimed at defining the probability of debris trapping as a function of the fuel assembly design parameters, the particle size and size distribution, and the coolant flow rates. This type of test, together with the information gained from the investigations recommended in Section 6.2.1 would serve to reduce much of the uncertainty presently associated with the problem of debris trapping.

6.2.3 Thermal Effects of Trapped Fuel Debris on Fuel Pins (F.A. Sciacca)

Fuel debris which becomes trapped between fuel pins or between the fuel pins and the channel wall will undoubtedly cause some local overheating. The overheating may lead to failures in the fuel pins affected, and it may even cause damage to the channel wall. To study this problem an analysis was made of a fuel pin adjacent to a channel wall between which fuel debris was assumed to have collected.

The work sought to explore the effects of several parameters on the resulting channel wall and cladding temperature. These parameters included the pin-to-wall spacing, channel wall thickness, channel-to-channel gap, and the extent of the blockage. The heat transfer coefficient between the trapped fuel particle and the solid surfaces in contact with it was also varied.

6.2.3.1 Analytical Model

The thermal analysis of the consequences of trapped debris was carried out using the THTD computer program.⁵ This program allows for the thermal analysis of two and three dimensional systems in both steady state and transient modes.

The present thermal analysis limited itself to a detailed examination of a small portion of a typical LMFBR channel wall and a fuel pin next to it. The geometry and nodal structure used are shown in Figure 6-25.

The following conditions and assumptions were used in the analysis:

1. The geometry and pin performance were patterned after the GE design described in GEAP-5678.⁶ The pin had a peak power generation rate of 16.23 kw/ft.
2. The debris was assumed to consist of fuel particles which were generating heat at the same volumetric rate as the adjacent fuel pin.
3. The axial length of the blockage was varied from a minimum of one inch to a maximum of six inches. The blockage cross-section was held constant over the length. The blockage was assumed to start at the core axial midplane.
4. The coolant flow nodes had their flow rates adjusted to reflect the velocity profiles which might be expected for the geometries studied. Local flow conditions were predicted using an interchannel flow mixing calculational model. Heat transfer between coolant nodes was assumed to occur by conduction only.
5. The outside boundaries of the model shown in Figure 6-25 were assumed to be adiabatic. This implies that the blockage conditions are repeated at similar sites along the channel wall.
6. Coolant was assumed to be flowing at a velocity of 1.6 ft/sec in the gap between adjacent fuel assemblies. This velocity was based on an assumed leakage flow of two percent of the total core flow.

Figure 6-25. Computer Model

6.2.3.2 Fuel Pin Cladding Temperatures

The overheating effects on a fuel pin due to trapped fuel debris are much more severe than those experienced by the channel wall. This is to be expected since heat removal from the pin is stopped or seriously restricted wherever the cladding is in contact with the debris.

Figure 6-26 shows the magnitude of the cladding hot spot temperature increase caused by fuel debris. In general, the values are a factor of three higher than those experienced by the channel wall.

For the cases of moderate blockages, a 10° arc along the cladding outer surface was assumed to be in contact with the fuel debris. The resultant overheating was on the order of 400°F . A severe blockage assumed that a 40° arc was in contact with the trapped debris. This case indicated that the peak cladding temperature would rise to about 1300°F higher than what would be expected without the blockage. This is a very severe amount of overheating.

The local cladding temperature perturbations caused by fuel debris are shown in Figures 6-27 and 6-28. Both figures show the cladding mid-wall temperatures around 180° of its circumference. Several axial locations are shown, each of which is assumed to be in contact with the fuel debris. The temperature rises very sharply in the region of the fuel debris due to the fact that heat transfer from the cladding hot spots is occurring only by conduction to the cooler regions of the clad. This mode of heat transfer is relatively ineffective for the clad thickness assumed in the analysis (10 mils).

The cladding hot spot temperatures for a moderate blockage are at least 1500°F . This over-temperature exists only over a small segment of the cladding in contact with the fuel debris. At the present time it is very doubtful that the cladding integrity could be maintained for a substantial period of time once it is subjected to such high temperatures. The chemical attack on both the inside diameter and outside diameter would be accelerated. At 1500°F some data suggest that the penetration depth of the chemical attack would approach 10 mils. See Section 4.1. In addition the mechanical strength of the cladding would deteriorate. Thus it would be less capable of withstanding any mechanical loads imposed upon it (see Section 4.1).

All of the present calculations assumed the blockage started near the core axial midplane. Slightly higher cladding temperatures might be predicted if the blockage was assumed to occur near the core outlet, even though the power generation rate is less at that location. Thus the temperature values predicted here may be somewhat optimistic.

The severe blockage resulted in extremely high cladding temperatures. The peak values calculated, in fact, approach the melting point of stainless steels. The cladding would most likely fail under such conditions.

The results of this analysis are consistent with those obtained from other studies wherein fuel was assumed to blanket a portion of a fuel pin's outer surface.^{7,8}

The presence of fuel debris adjacent to a fuel pin may also lead to some fuel melting within the pin. The moderate blockage case calculations indicated that only a small amount of fuel would melt. The severe blockage case wherein a 40° arc of the pin clad was in contact with the debris resulted in significantly more fuel melting. The calculation showed that fuel may melt out to almost half the radius of the fuel. With a central void present, the molten fuel would run down and solidify at a lower axial position. The power generation and resultant heat flux and temperature perturbation would then be less than predicted by the above model, making the above results somewhat conservative. The lack of a sufficient void to accommodate the molten fuel would lead to an additional pressure buildup in the heated region because of the volume increase experienced when the fuel melts. Since the cladding integrity is seriously jeopardized under the assumed blockage conditions, pressurization due to molten fuel could lead to a rapid expulsion of the molten material from the pin. The probability of this occurring and its possible consequences were not further analyzed due to lack of time and resources.

The calculations made during this study employed a minimum axial length of one inch and a maximum length of six inches. If the blockage was short relative to the length covered in this analysis, then it could be expected that the resulting thermal perturbation on both cladding and channel wall would be reduced.

The present calculations do give an indication of the effect of the axial length of the blockage. Figures 6-27 and 6-28 showed the temperature variation along the fuel pin circumference. The temperatures went from maximum to minimum in a distance of about 0.06 inches. Clad positions .06 inches away from the block center were only slightly affected by the presence of the debris. This information suggests that the temperature profiles would be essentially fully established at a distance on the order of 0.06 inches from the beginning of the blockage. If the blockage extended

Figure 6-26 Local Clad Temperature Increase Due to Trapped Fuel Debris

for an axial distance on this same order or was shorter, then the resulting peak temperature would most likely be less than those calculated by the present analysis.

6.2.3.3 Local Channel Wall Temperatures

Local thermal perturbations of the channel will most likely be caused by the effects of trapped fuel debris or distorted fuel pins which contact the channel wall. The consequences of such disturbances are very dependent on the degree of overheating which results. At a minimum, the hot spots in the channel wall will create local thermal stresses and possibly some local yielding. Internal pressure acting on such an area may be sufficient to cause local distortions or even to rupture the channel at that location. At the upper extreme molten fuel could contact the channel wall and cause it to melt.

Local overheating effects would be of the greatest consequence if they led to failure of the channel in some way. Such a failure would enhance the probability of further failure propagation. Coolant flow would likely be diverted out of the channel through the failure area and into the channel-to-channel gap spacing. The flow to the fuel pin regions

Figure 6-27 Thermal Gradients in Fuel Pin Cladding Due to Fuel Debris

downstream of the failure would be decreased, causing overheating. If these pins were sufficiently undercooled they too would fail.

Local overheating conditions will be more difficult to detect than those associated with uniform overheating. Local effects, therefore, will likely remain uncorrected for longer periods of time. This time element increases the probability of the local conditions leading to further failure propagation.

The effects of trapped fuel debris or other such flow blockage events are intuitively felt to be more consequential than the effects of bowed pins on the channel wall. Therefore, the debris trapping problem was chosen for study in an attempt to define the channel operating safety limits as they are affected by local thermal perturbations.

The extent of the local channel overheating due to trapped fuel debris is shown in Figure 6-29 for the section of the channel wall nearest the centerline of the blockage debris. This figure can also be interpreted as the ΔT between the hottest spot along the channel wall and the balance of the wall which is unaffected by the presence of the debris.

The results indicate that the particle size (extent of blockage) and the channel-to-channel gap spacing have the strongest influence on the channel temperatures. Parameters such as channel wall thickness, pin to wall spacing, and contact coefficient have a rather small effect. Over the range explored for these latter parameters, the temperature increase varied by less than 50°F.

The model uses one inch axial nodes, which does not show the effect of axial heat transport. Figure 6-29 assumes that the temperature perturbation is fairly well established after the blockage has progressed for only one-half inch. Short lengths were not studied, however, so the effects of short blockages are somewhat uncertain. Extending the blockage axially beyond one inch raised the peak temperatures, but only marginally above the initial ΔT incurred at the first node.

Figure 6-28 Thermal Gradients in Fuel Pin Cladding Due To Fuel Debris

The cross-sectional areas of the fuel debris assumed in the analysis are noted below, along with the equivalent particle diameters if they are assumed to be cylindrical in shape. The particle sizes vary, depending on the spacing available between the pin and the channel wall:

	Pin-to-Wall Spacing, in.	Debris Cross- Section Area, inch ²	Equivalent Particle Diameter, inch
Moderate Blockage	0.050	0.0064	0.064
	0.030	0.0051	0.057
Severe Blockage	0.050	0.0099	0.079

With the severe blockage the channel wall width in intimate contact with the fuel particle was more than twice the width assumed for the moderate blockage case. This greater coverage results in a channel wall ΔT which is nearly three times that occurring from the moderate case.

The cases investigated in which the channel-to-channel gap distance was varied indicated that heat transfer through

Figure 6-29 Local Channel Wall Temperature Increase Due To Trapped Fuel Debris

the channel wall to the gap coolant plays a significant roll in minimizing the channel wall temperature. When the gap spacing was assumed to be negligible the peak channel wall ΔT was about 250°F . The outer wall surface was assumed to be adiabatic in this case. Such an assumption is very pessimistic and tends to maximize the resultant overheating. In reality heat transfer would occur to the adjacent channel which is in contact with the overheated one. The opposite extreme to this case utilized a channel-to-channel gap of 0.25 inches. In this case the wall overheating amounted to only about 100°F .

Examples of the computed local temperature variations along the channel wall caused by the fuel debris are shown in Figures 6-30 and 6-31. Temperature profiles of both the inside wall surface and the outside surface are shown for a single elevation. The distance covered extends from A to B of Figure 6-25.

For the moderate blockage Figure 6-30 shows that the inside surface temperatures vary by about 100°F over this distance. The outside surface temperature variation is about 30°F .

The severe blockage case, Figure 6-31, shows a temperature variation along the wall of 420°F along the inside wall and 110°F on the outer wall. The cross-over of temperatures is due to the fact that relatively stagnant ($<2\text{fps}$) coolant is in contact with the outside surfaces and conduction heat transfer tends to equalize the outer surface temperatures.

6.2.3.4 Channel Deflection Analysis

The present thermal analysis has pointed out that trapped fuel particles can cause significant local cladding and channel wall hot spots. These hot spots will result in thermal stresses which, in combination with the mechanical loads imposed on the pins and channel, may cause some local yielding. Only a small amount of yielding is needed, however, in order to relieve the thermal stresses created by the hot spots. The resulting strains on the channel wall will be small, and it is not expected that the wall will fail because of this type of deformation.

Figure 6-30 Predicted Local Temperature Variation Along Channel Wall Caused By Fuel Debris

The presence of fuel debris next to the channel wall will lead to higher temperatures for the inside surface than for the outside surface of the wall. This type of ΔT would tend to bow the heated wall inward. If the deflections were significant stresses could be set up in the fuel pins at the spacer contact points. In Section 4.3 it was shown that stresses in excess of the yield strength can be generated by relatively small loads of this type (3 pounds). A deflection analysis was made to establish whether or not the thermal condition in the channel wall could cause such problems.

The calculations assumed the channel wall to have a typical hexagonal cross section. One flat of the hex was overheated. The overtemperature profiles used in the analysis are sketched below. All gradients were assumed to be linear.

An example case was studied with the temperature conditions indicated in the above sketch. The hot wall temperature was on the order of 150°F above the balance of the channel cross-section. The thermal gradient through the hot side wall thickness was assumed to be about 100°F. The analysis of these conditions indicated that the hot side of the channel would bow inward a maximum amount of slightly more than 1 mil.

The configuration used in this analysis assumed a channel size of five inches across flats with a 0.100 inch wall thickness. The walls of such a channel would deflect outward about 0.8 mils for each 10 psi of net internal pressure. Superimposing the movement caused by the thermal conditions on those created by the pressurization would simply reduce the outward deflection to some extent. If the fuel pin array were tightly packed at the axial location where the

overheating occurred the thermally induced channel movement could impose additional loads on the pins. This possibility appears to be somewhat remote since most fuel assemblies will be fabricated with clearances of at least 20 mils.

6.2.3.5 Transient Analysis of a Molten Fuel Blockage

Studies have been made in the past of the response of the channel wall to contact with molten fuel.^{9,9} In most cases, however, the channel wall outer surface was assumed to be in contact with the wall of the adjacent channel. Such a condition maximizes the possibility of failure propagation from one fuel assembly to the next.

Most of the cases of the present study employed a substantial coolant gap between fuel assemblies. Such a gap may be necessary to accommodate steel swelling. This gap may also serve to retard failure propagation from one channel to another.

A brief transient analysis has been made in which molten fuel was assumed to contact the inner surface of the channel wall. The model used is shown in Figure 6-25, with the severe blockage debris configuration applied. The fuel was assumed to be molten at the instant of contact with the wall. A large coolant gap between channels was assumed to exist. The presence of molten fuel within the fuel assembly would most likely be accompanied by a flow decrease. To simulate this effect, the coolant velocity inside the channel was assumed to be sharply reduced. The coolant, however, was assumed to be present.

The transient response of both the fuel pin cladding and channel wall to contact with molten fuel is shown in Figure 6-32. The channel wall node nearest the molten fuel shows a sharp temperature rise, but it does not become hot enough to melt. The highest wall temperature calculated was slightly less than 1900°F. It reaches this value after only about a half second from the onset of contact with the fuel. This hot node cools rapidly from this point. After ten seconds have elapsed its temperature begins to approach the value calculated for the case of contact with non-molten fuel debris.

The transient analysis calculations have shown that the channel wall would not be seriously affected by contact with a small amount of molten fuel. The channel wall does not melt, but it does lose mechanical strength simply because its temperature increases. In this condition it would be less capable of withstanding any mechanical loads imposed upon it, such as a shock wave loading which may be generated by a fuel-coolant interaction.

The response of the fuel pin cladding indicates that the cladding would reach its melting point after only about a tenth of a second from the onset of contact with the molten fuel. This pin failure might also release additional molten fuel into the immediate area.

Figure 6-31 Local Temperature Variation Along Channel Wall Caused By Fuel Debris

The transient analysis of molten fuel debris is thus qualitatively similar to the steady state case. In both cases, the affected fuel pin fails and the channel wall is probably not seriously affected. The presence of molten fuel and clad does add the possibility of violent boiling. This consequence of the mode of failure has not been further analyzed. Its importance in future testing and analysis is well recognized.

6.2.3.6 Conclusions and Recommendations

The analysis indicated that a sufficiently large fuel debris blockage can cause significant overheating of the fuel pin cladding. These high temperatures might cause failure due to reduction of clad strength and ductility and would promote chemical attack of the clad, making it less capable of resisting any mechanical loading. More severe blockages can raise the cladding temperature to very near its melting point. Pin failure is anticipated under such conditions. The pins affected can potentially fail in such a way as to add to the blockage and further extend the failure propagation.

For the type of blockages analyzed it is not expected that the overheating of the channel wall will cause it to fail. The design parameter which has the greatest influence on the channel overheating is the coolant gap between channels. Parameters such as the wall thickness and the pin-to-wall spacing are much less sensitive in their influence on the peak temperatures resulting from the blockage. The channel hot spots created by the blockage will induce thermal stresses and possibly some deflections. The stresses may cause some local yielding of the channel wall. The deflections will be minor, and should not pose a further hazard to the fuel pins.

It is recommended that additional analytical effort be directed toward more adequately defining the consequences of fuel debris being trapped. Such an effort should attempt to follow the sequence of events initiated by the blockage.

Figure 6-32 Thermal Response of Channel Wall and Fuel Cladding to Contact with Molten Fuel

That is, for a given assumed collection of fuel debris the analysis should attempt to establish the time to failure of the affected fuel pins, the manner in which they fail, whether or not the failed pins will release additional fuel, etc. Such an analysis is necessary in order to more adequately establish the probability of failure propagation to nearby fuel pins and possibly even to adjacent fuel assemblies. The parameters which should be studied are:

- the size and shape of particles necessary to cause a given amount of overtemperature in the fuel pins
- clad thickness
- axial location of blockage
- pin linear power
- channel wall thickness
- channel to channel gap

The analytical studies should be supported and verified by in-pile tests. Tests are needed whereby a fuel debris blockage occurs adjacent to an operating fuel pin. The tests should seek to define the manner in which the pin fails, and the possible effects of that failure on adjacent fuel pins

The design criteria for fuel channels should be reviewed with the objective of defining necessary criteria concerning channel integrity.

If channel criteria requiring limited deflection, containment of flow and/or pressure disturbances are found necessary, a program of analysis and experiments should be undertaken for expected operating loads as well as abnormal loads due to accidents or fuel failures.

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6.3 DEGRADATION OF FUEL FAILURES WITHIN A SINGLE PIN (K. R. Absher)

Many fuel pin failures will likely originate as hairline cracks or pinholes in the cladding. If the reactor continues to operate with pins possessing these defects, the failures will have an opportunity to progress to more consequential states, such as large area failures or burst failures.

These latter states, in turn, are more likely to cause propagation of the damage to adjacent pins than was the initial small failure. It is important to identify the mechanisms which cause small failures to degrade to larger ones. Design features of the reactor and fuel may then be incorporated to minimize this degradation and its consequences.

This section discusses first the progression of a hairline crack or pinhole failure in the clad to a large area type failure. The mechanisms which further degrade this failure to a burst type failure are then considered.

6.3.1 The Progression of Crack-Type Fuel Pin Failures to Large Area Failures

The question to be addressed here deals with the progression of a hairline crack, or pinhole failure. The most likely way in which this may occur is for internal pressures due to fuel swelling and/or thermal ratcheting to stress the cladding and cause the crack to open up. Fission gas pressure in the central void may be a consideration if it stresses the cladding. This mechanism is discussed in Section 6.3.2.

6.3.1.1 Sodium Leakage

The crack size that is just large enough to admit significant amounts of sodium is quite important to this study because of possible fuel-sodium reaction product swelling. From work done at LASL¹ on sodium leakage through stainless steel tubes, it appears that sodium will penetrate extremely small breaches of the cladding. The leakage is also sensitive to the temperature. At a temperature of 400°C (752°F), a helium leak rate of 10^{-4} atm cm³/sec* does not appear to permit significant sodium leakage. At 650°C (1202°F), however, a leak rate of only 10^{-6} atm cm³/sec will allow significant sodium leakage according to the LASL work. This indicates that sodium leakage into a small failure may differ significantly between the core inlet (cool) and outlet (hot).

A leak rate of 10^{-6} atm cm³/sec is very small. Fission gas in a 15 kw/ft, 30 inch long pin is produced at the rate of 40×10^{-6} atm cm³/sec. If it is very conservatively assumed that fission gas leaks at the same rate as helium does, the fission gas loss is still only 2.5% of the fission gas production in the pin. If the temperature is 1200°F, this minute leak will admit sodium. From this example, it may be qualitatively concluded that any failure that allows "significant" release of fission gas will also admit "significant" quantities of sodium to the pin. This argument is presented only to establish the potential for sodium ingress. It is realized that if the leak rate is smaller than the production rate, a continuously increasing positive pressure difference will exist between the inside of the cladding and the lower pressure of the coolant. It is conceivable that sodium could leak in against the pressure gradient. Sodium wets stainless steel extremely well at operating temperatures and could "creep" in along the edges of the crack. This is a hypothetical mechanism for in-leakage of sodium.

The significance of the sodium in the pin presumably lies in the potential fuel-sodium reaction product swelling of the pin as discussed in Section 6.1. It also includes the possibility of fuel and/or solid fission products being leached or swept out of the pin into the primary system. From the above discussion it appears highly likely that crack type failures will allow some sodium to come in contact with the fuel. The definition of this type failure, however, still precludes the loss of any solid fission products or fuel from the pin. Thus, there is no "sweeping" of the sodium into and out of the failure as might be the case with the large area and burst type failures.

6.3.1.2 Mechanisms for Crack Progression

If the cracked cladding is loaded by fuel swelling, the crack may create very large stress concentrations. If this problem is treated elastically, it would indicate that the crack would progress for moderate internal loads and the cladding would be easily deflected to give a large area failure. The fact that the dimensions of the crack are small compared to the dimensions of the cladding, however, allows local stress relief (at the ends of the crack) to occur through very small amounts of plastic deformation. The result of this stress relief mechanism is that the very small

* atm cm³/sec = cubic centimeters at atmospheric pressure per second.

crack and pinhole failures do not seriously impair the strength of the cladding. This is not to say that these small failures will not weaken the cladding, just that the stress concentrations will be much smaller than would be predicted by elastic theory.

If the small amount of sodium that enters the pin reacts with the fuel and causes swelling, higher cladding stresses could result at the failure location. Although the fuel-sodium reaction is not well understood, there is some indication that a large volume of sodium is necessary to cause very much swelling. In the Task B sodium bonded pins of PA-10, the reaction product was formed but there was no unusual swelling of the pins due to internal pressure. Pin B9A, however, was purposely defected and exposed to flowing sodium.² One pin in this test with O:M = 1.99 suffered a 23% diametral increase while the other pin with O:M = 1.95 had 7% $\Delta d/d$ swelling. It is probable, however, that there was a relatively high concentration of oxide in the sodium. This degree of swelling is sufficient to severely restrict flow in its coolant channels with the attendant risk of propagation.

High local stresses caused by non-linear fuel swelling effects (ratcheting, fuel chips, pellet interface irregularities) could open up the initial crack failure. The initial failure would be the weakest point in the cladding and would provide a point of stress concentration, even after it has undergone stress relief through small deformations. The likelihood that this type of failure progression will occur cannot now be assessed because the anomalous fuel behavior cannot be adequately described.

Another mechanism for failure progression concerns stress-induced corrosion as a means of clad degradation in an area of stress concentration. If stress corrosion occurs, the region around the failure will be preferentially attacked because of the stress concentration. This will tend to worsen the stresses in that region. Again, this is not well enough understood to base analysis on it.

6.3.1.3 Fuel Pin Failure Experience

Most failed fuel pins exhibit large area failures as opposed to "crack" failures. Crack failures may have been observed in two different situations. In EBR-II, test pin F4D suffered a small failure that released some fission gas into its capsule. No failure could be located at room temperature in the subsequent examination.³ Later, metallographic examination revealed no evidence that sodium had entered the pin.⁴ It is possible, however, that the failure occurred above the capsule sodium level. The sodium level is about 13 inches below the top of the test pin in the GE capsules at room temperature.⁵ Encapsulated pins from EBR-II that exhibit large area failures include F2F, F20, F4F, F0M, F2D and E1H-33 (an F3B pin). In a DFR test bundle, twenty-three pins failed. Three of these pins had released fission gas but had no visible flaws. They were pressurized at room temperature to 80 psi with argon but did not leak.⁶ It was not reported whether sodium entered the fuel pins or not. It is possible that it did not because of the relatively short time between failure initiation and temperature reduction (probably less than a day).

These two cases of possible "crack" failures, compared to the much larger number of large area failures observed indicate that one of two things happens. Either initial crack failures readily progress to large area failures or most failures are initiated as large area failures. If the former conclusion is correct, then the rate at which the progression occurs is of interest. Although there is no way of predicting the rate of progression at the present time, it appears clear that it must occur in a time that is short compared to the life of the pin. This is because most failures are large area failures by the time the failed pins are examined.

6.3.1.4 Conclusions and Recommendations

Consideration of the progression of crack type fuel pin failures to large area failures resulted in the conclusion that essentially all fuel pin failures will admit sodium into the fuel matrix. The question that remains concerns the initial failure configuration; whether the initial failure is a pinhole or large area failure. This question is not critical so no experimental work is recommended for its investigation.

As a future consideration, the B9 series of PA-10 should include a fuel pin with a very small fabricated defect to simulate a crack failure. If a sufficiently small failure mechanism can be developed, an in-pile test should yield information on the opening up of the small failure. This type of test is also desirable for the investigation of the dynamics of fuel-sodium reaction product swelling for small defects.

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6.3.2 Progression of a Large Area Fuel Pin Failure to a Burst Type Failure

The conditions under which either fission gas pressure or sodium vapor pressure in the central void and fuel matrix can suddenly expel fuel from a fuel pin which is operating with a large area clad failure are important and must be studied. The mechanisms that relate to these conditions are important because they appear to be the most probable ways for a large area failure in one pin to progress to a burst failure in the same pin. This transition appears on the dominant paths in the fault trees.

6.3.2.1 Introduction

Internal pressure in a fuel pin with a large area failure will, if it exists, provide the driving force necessary to expel fuel into the coolant stream. This expulsion defines a "burst" failure and can lead to more serious consequences than the original large area failure. For this reason, it is important that we understand the behavior of a fuel pin with a large area failure with respect to its generation, retention and release of pressure. Consideration must be given to the sources of pressure and to the magnitude of pressure that can build up for a given failed fuel pin configuration. If small pressures crack the fuel and are released, then no fuel expulsion is expected. If the fuel is very strong, however, large pressures may be produced and when the failure point is reached, the pressure release may be violent and fuel may be swept out of the pin. At the present time the pressure necessary to cause fuel expulsion is not known.

Under normal or near-normal operating conditions, internal pressures may be generated in a fuel pin with a large area failure by two mechanisms. First, the fission gas atoms can migrate up the thermal gradient of the fuel pin and accumulate in the central void, increasing the pressure in the void. Second, liquid sodium that enters the fuel pin during shutdown will vaporize during the startup transient and may cause significant pressures. A calculation assuming 100% gas release to a 40 mil diameter central void by a pin operating at 15 kw/ft shows that the pressure buildup due to fission gas accumulation is 244 psi per day. Central void pressure is released to the plenum, vent or to the failure at each significant power change (e.g., shutdown). Therefore, long continuous runs could produce high pressures. An analysis of the second source of pressure indicates that for a ten hour startup period, the sodium vapor can escape in a time period that is small compared to the available time. Therefore, this second mechanism is not as important as the fission gas buildup.

A model of a fuel pin was used in analyzing the potential pressures that could be contained in the fuel. The model consists of a central void, a plastic region of fuel that transmits pressures hydrostatically and an outer, brittle region that must provide the restraint to internal pressures when cladding restraint is lost through failure. The thickness of the brittle region increases as the power produced by the pin decreases. Structural analyses were performed on the brittle region in order to determine if it can restrain large pressures.

In order for the analyses to be meaningful the strength of the fuel must be established. Out-of-pile data suggest that the strength in compression of UO_2 is in the range of 100,000 psi and strength in tension may be around 5000 psi.⁴ Confidence in these values is very low because the physical condition of fuel in-pile is much different than the unirradiated material.

As a first case, the brittle region of fuel is assumed to behave as a thick-walled cylinder (see Figure 6-33A). Total loss of cladding restraint is assumed.

If a 5000 psi tensile strength for the fuel is assumed, a maximum pressure of 1000 psi would build up in a pin operating at 18 kw/ft and a maximum pressure of 2000 psi could build-up in a pin operating at 11 kw/ft. Higher pressures would fracture the fuel.

When the original large area failure occurs, the contact of the sodium will lower the fuel surface temperature and cause a thermal shock to the pin. A thermal stress analysis was done showing that the tensile strength of the fuel in the outer regions would be exceeded when the fuel surface was lowered by 500°F. The thermal fractures would extend through most, if not all, of the brittle region. Thus, it is not likely that fuel will be able to act like a thick-walled cylinder as was suggested for the first case.

The other cases consider more realistic conditions. Partial cladding restraint is assumed (this is verified in most photographs of pin failures). If a section of the brittle fuel region spans the cladding failure, it could offer support as a curved beam supported at its ends by the cladding (Figure 6-33B). The beam itself could restrain large pressures for small failures. The limiting factor, however, is the strength of the cladding in bending (Figure 6-33C). This limits the

Figure 6-33 Models

maximum pressure in the fuel pin to 300 psi for 10 mil cladding on a 13 kw/ft pin. If 15 mil cladding is used, the maximum pressure is 650 psi for the 13 kw/ft pin.

If the fuel in the brittle region is severely fractured, it may not be able to act as a cylinder or as a beam. In this case, the fuel would not be able to sustain any tensile stresses. The configuration that could restrain internal pressures with compressive stresses only is a circular arch (Figure 6-33D). The arch spans the cladding breach and the ends of the arch are supported by the remaining cladding. Since the fuel is very strong in compression, the internal pressure is limited by the cladding. The structure may fail in two ways: the strength of the cladding may be exceeded or the cladding may deflect and destroy the arch geometry. The maximum pressure for any failure size is found to be 300 psi for 10 mil cladding and 720 psi for 15 mil cladding.

Thus, moderate pressures (less than 1000 psi) can be expected in a failed pin based on these conservative analyses.

6.3.2.2 Pressure Sources

6.3.2.2.1 Fission Gas Pressure

It is postulated that fission gas atoms migrate up the thermal gradient in a fuel pin and, thus, collect in the central void. The steadily increasing pressures in the central void due to fission gas can provide a driving force for the expulsion of fuel debris. It is also postulated that the fuel pin central void is a sealed chamber while the pin is operating at constant full power. A simple model was developed for these considerations. The model of a fuel pin at full power consists of a central void surrounded by a hot, plastic annulus. This plastic fuel seals the central void and is restrained by an outer, brittle layer of fuel and by the cladding.

Consider a fuel pin with a 0.040 inch diameter central void, operating at 15 kw/ft with a central temperature of 4500°F.

The fission gas generation rate is:

$$\begin{aligned}
 & 15 \frac{\text{kw}}{\text{ft}} \times \frac{6.24 \times 10^{15} \text{ MeV/sec}}{\text{kw}} \times \frac{1 \text{ fission}}{200 \text{ MeV}} = 4.68 \times 10^{14} \frac{\text{fissions}}{\text{sec-ft}} \\
 & 4.68 \times 10^{14} \frac{\text{fissions}}{\text{sec-ft}} \times 0.27 \frac{\text{gas atoms}}{\text{fission}} = 1.26 \times 10^{14} \frac{\text{gas atoms}}{\text{sec-ft}} \\
 & 1.26 \times 10^{14} \frac{\text{gas atoms}}{\text{sec-ft}} \times \frac{22.4 \text{ l}}{6.02 \times 10^{23} \text{ gas atoms}} \times 3.53 \times 10^{-2} \frac{\text{ft}^3}{\text{l}} = 1.67 \times 10^{-10} \frac{\text{ft}^3 \text{ at STP}}{\text{ft-sec}}
 \end{aligned}$$

For a 0.040 inch void diameter

$$\begin{aligned}
 \text{void volume} &= 8.76 \times 10^{-6} \frac{\text{ft}^3}{\text{ft (of length)}} \\
 1.67 \times 10^{-10} \frac{\text{ft}^3 \text{ at STP}}{\text{ft-sec}} \times \frac{4960^\circ \text{R}}{492^\circ \text{R}} &= 1.68 \times 10^{-9} \frac{\text{ft}^3 \text{ at } 4500^\circ \text{F}}{\text{ft-sec}}
 \end{aligned}$$

The time necessary to build up one atmosphere of pressure at 4500°F in the void is:

$$\begin{aligned}
 & \frac{8.76 \times 10^{-6} \text{ ft}^3/\text{ft}}{1.68 \times 10^{-9} \text{ ft}^3/\text{ft-sec}} - 5.21 \times 10^3 \text{ sec} = 0.0603 \text{ day} \\
 & \frac{1 \text{ atmosphere}}{0.0603 \text{ day}} \times \frac{14.7 \text{ psi}}{\text{atmosphere}} = 244 \frac{\text{psi}}{\text{day}}
 \end{aligned}$$

Thus if 100% of the fission gas migrated to the central void, the pressure in the void would increase at the rate of 244 psi for every day of continuous operation. If at any time the reactor changes power significantly (e.g., shutdown), the fuel pin will crack and the central void pressure will be released to the fuel pin plenum or vent. If, however, a full power run of two weeks or a month is possible, high pressures (3000 to 7000 psi) are conceivable for the central void.

The fuel and cladding together are capable of restraining these pressures. If the cladding fails, however, it is unlikely that the fuel would have the mechanical strength and integrity to restrain the internal pressures. This aspect will be discussed in more detail in later sections.

There are two documents^{1,2} that report the pressure histories of operating oxide fuel pins. A detailed comparison of these reports appears in Appendix B-I. Briefly, Reference 1 reports on twelve fuel pins that exhibited step fission gas releases at reactor shutdowns, indicating that fission gas was retained in the fuel matrix or central void during continuous operation. Reference 2 reports on one pin that exhibited a continuous fission gas release during operation. This continuous release was due to the higher temperatures of that pin and to the fact that most of its gas was generated in the outer regions of the pin where its release to the fuel-clad gap was more likely. The fission density for this pin increased greatly near the fuel surface because the pin was highly enriched and irradiated in a thermal flux. The twelve pins that exhibited step releases were only slightly enriched and, therefore, are more representative of LMFBR fuel pins in their power and temperature profiles.

6.3.2.2.2 Sodium Vapor Pressure

Large pressures could be produced in a failed fuel pin if sodium enters the pin when the reactor is shut down and then vaporizes during startup. This problem of sodium logging has been investigated analytically and experimentally.³ Pessimistic analyses indicate that sodium vapor pressure at fuel pin center temperatures could fail the cladding of a sound pin. Experimentally, sodium was purposely admitted to fuel pins and they were subjected to start up transients 200 times the normal rate. No failures resulted.

There is an important difference between those experiments and the problem under consideration here. The experimental pins either had cladding that was sound or that had a single, 0.005 inch diameter hole drilled in them. In either case, substantial restraint to internal pressures was available from the cladding. In a pin with a large area failure, however, the restraint must be provided by the fuel itself. It is not clear that the fuel has the strength and integrity to restrain the pressures that can be generated.

If it is assumed that sodium fills the cracks and central void in the fuel pin, it then becomes important to know how the sodium will escape when the temperatures start rising. Appendix B-II contains some simple calculations of the rate at which sodium could escape from the fuel pin through cracks in the fuel, and of the peak pressure which could develop in the central void.

For the assumed conditions it appears that the sodium vapor pressure will not be excessive in the fuel matrix during the startup period.

6.3.2.3 Stress Considerations

6.3.2.3.1 Fuel Structural Model

The model used in the stress considerations is consistent with the model used in the fission gas buildup considerations. The model consists of a central void, surrounded by an annular region of plastic fuel, surrounded by an annular region of brittle fuel. The plastic-brittle interface is assumed to occur at about 2200°F. It is also assumed that the plastic fuel behaves as a fluid, transmitting contained pressures hydrostatically to the brittle region. The brittle region provides the structural member that bridges the cladding failure.

Calculations based on this model have been made in order to determine the pressures that can be developed in the fuel pin before the fuel matrix fails and releases the pressure. The larger the pressures developed, the more serious the consequence that pressure release may have.

6.3.2.3.2 Fuel Strength

There is some information on the strength of uranium dioxide. There is a problem in that brittle materials do not have unique strengths. It is known that fracture strength decreases rapidly with increased porosity. It has also been found that the fracture strength of UO_2 is somewhat higher at elevated temperatures (1830°F) than at room temperature.⁴ Average moduli of rupture reported in Reference 4 ranged from 7000 to 20,000 psi. The modulus of rupture always exceeds the true stress that has been reached in the material at the time of failure, however, the difference is small for brittle materials such as uranium dioxide.⁵

In two reports in the *Journal of Nuclear Materials*,^{6,7} beams of UO_2 were supported at the ends and subjected to point loads at the center of the beam. In tests above 1440°F , maximum fiber stress of 7000 psi was sustained without failure. At room temperature, the bending strength of $\text{UO}_{2.00}$ (density = 10.4 gm/cc) was reported as 8800 psi. At temperatures of 2550°F , UO_2 beams sustained stresses of 5000, 12,000 and 15,000 psi without fracture. At these temperatures, however, there was considerable creep (strain rates up to $5 \times 10^{-7} \text{ sec}^{-1}$). UO_2 at elevated temperatures has been stressed in compression up to 25,500 psi⁸ and 15,000 psi.⁹ Reference 4 gives the compressive strength as 60,000 to 140,000 psi at room temperature.

These data vary widely but yield rough indications of the strength that can be exhibited by fuel in a reactor. The increased porosity due to fission events or due to fabricated voids may decrease the strength.

The strength of the fuel is important because it limits the pressures that can build up in a failed fuel pin. If the fuel is very weak or cracked, pressures will continuously bleed off from the pin. If the fuel can form a strong pressure containment, large pressures can be generated, providing a potential for "burst" failures.

6.3.2.3.3 Hoop Stresses Due to Internal Pressures

If a pressure is developed in the central void due to fission gas or sodium vapor, the pressure is transmitted hydrostatically by the plastic region to the brittle region. If the brittle region is not cracked, it acts like a thick-walled cylinder under internal pressure loading. Equations for radial and tangential stresses in a thick-walled cylinder with internal pressure were obtained from Reference 5.

$$S_r = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} - \frac{(P_i - P_o) r_i^2 r_o^2}{(r_o^2 - r_i^2) r^2}$$

$$S_t = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} + \frac{(P_i - P_o) r_i^2 r_o^2}{(r_o^2 - r_i^2) r^2}$$

For a typical 0.250 inch outside diameter fuel pin operating at 12 kw/ft:

$$\begin{aligned} r_o &= 0.111 \text{ inch} \\ r_i &= 0.089 \text{ inch} \\ P_o &= 50 \text{ psia} \\ P_i &= \text{Variable (1000, 3000, 7000 psia)} \end{aligned}$$

Plots of the tangential and radial stresses appear in Figure 6-34. A similar case was run but with a cooler pin (larger brittle region). The effect is a thicker-walled cylinder containing the same pressures, leading to lower stresses in the fuel. Results for the cooler pin are shown in Figure 6-35.

If a tensile strength of 5000 psi is assumed for the fuel, the maximum pressure that can be restrained is about 1000 psi for the hotter pin. The cooler pin could probably restrain pressures of up to 2000 psi. These calculations are for a sound layer of brittle fuel acting as a thick-walled cylinder. As will be shown later, it is probable that the thermal shock following a large area failure will crack the brittle layer. In this case, very little internal pressure could be restrained. However, if there is some type of fission induced "healing" of the cracks, the brittle fuel could again restrain pressures.

The hoop stresses calculated here are for the extreme case of zero cladding restraint. Thus the cladding must be totally removed from the fuel or opened up so severely that it offers no support to the fuel.

Figure 6-34 Hoop Stresses for Hot Pin (18 kw/ft)

Figure 6-35 Hoop Stresses for Cool Pin (11 kw/ft)

6.3.2.3.4 Thermal Stresses Due to Sodium Ingress

If a fuel pin fails at power and sodium is admitted to the fuel-clad gap, the thermal conductivity across the gap will increase greatly. In the considerations here, it is assumed that the sound fuel pin has a 500°F temperature drop across the gap (corresponding to a gap coefficient of 1560 Btu/hr-ft²-°F for a 15 kw/ft pin).

As a first estimate, it was assumed that the operating fuel pin failed and was instantaneously sodium bonded around its circumference. This sodium bonding was assumed to lower the fuel surface temperature by 500°F. The temperature changes across the pin at various times after failure appear in Figure 6-36 (computed from curves given in Reference 10). Since it is assumed that the fuel has no thermal stresses before the failure, the stresses after the failure are due solely to the temperature gradient from sodium bonding. Thus the "ΔT" on Figure 6-36 represents the difference between the instantaneous and the initial steady state temperatures.

In order to find the stress profiles across the pin as time progresses, a small computer program was written to solve the equation for tangential stress in a thick-walled cylinder with a temperature profile T(r):

$$S(r) = \frac{\alpha E}{1-\nu} \cdot \frac{1}{r^2} \left[\frac{r^2 + a^2}{b^2 - a^2} \int_a^b rT(r)dr + \int_a^r rT(r)dr - T(r)r^2 \right] \quad (\text{Ref. 11})$$

where a = inner radius

b = outer radius

α = coefficient of thermal expansion (5 x 10⁻⁶/°F used⁴)

ν = Poisson's ratio (0.302 used⁴)

E = Young's modulus (24 x 10⁶ psi used⁴).

The results of the equation are shown in Figures 6-37A and 6-37B. The outer ring is put in tension by the cooling because it attempts to contract around an unyielding cylinder. Thus, the cracking would start at the outside edge and progress inward. If a maximum tensile stress of 5000 psi is allowed, it appears that the brittle region will suffer extensive fracture due to the thermal shock. The actual extent of thermal cracking is not amenable to analysis because the formation of the initial surface cracks would tend to alleviate the stresses through the rest of the structure. It is clear, however, that cracking will occur and a weaker structure will result. This fracturing may do the following: 1) permit fall-out from the failed cladding, 2) provide volume between the cracks to accommodate expansion due to internal pressures, or 3) provide a direct path for pressure release from the fuel. On the other hand, if a section of sound fuel bridges the cladding failure, it may still provide restraint as a simple beam.

6.3.2.3.5 Failed Pin with Partial Clad Restraint

Most cladding failures only remove restraint from a portion of the circumference of the fuel pin. The size of the failure may be defined by the angle over which clad restraint is lost.

If the failure is spanned by a piece of brittle fuel, internal pressure may be restrained by simple beam support. The beam is curved but it is thin enough to treat it with standard beam formulas. The resultant formula is:

$$\text{Maximum Stress} = \frac{Mc}{I} = \frac{3PR^2 A^2}{2h^2} = 117 PA^2$$

P = Internal pressure in psi

R = Pin radius, inches (0.111 inch)

A = Failure size, radians

h = Thickness of brittle fuel region (~ 0.02 inch)

Maximum stress = 117 PA²

Figure 6-36 Pin Temperature Changes After Bonding

If the fuel is assumed to fracture at 5000 psi, Figure 6-38 shows the maximum internal pressures expected as a function of failure size. The hotter pin has less strength because the brittle region is thinner than in the lower power pin.

Also plotted on Figure 6-38 is the limit due to the strength of the cladding. The method of finding these limits is shown at the end of Appendix B-III. The worst case is for the cooler pin with 15 mil cladding. Internal pressures of up to 660 psi can be developed.

As shown in the section on thermal stresses, it is likely that the brittle fuel in the failure region will be severely fractured and, therefore, unable to display tensile strength. If the failure is not too large, the brittle fuel can restrain pressures in the central regions by acting as an arch (Figure 6-39). If it is assumed that the arch is loaded hydrostatically, the arch should be circular. Reference 12 treats symmetrical two-hinged arches. The application of this theory to the present problem is presented in detail in Appendix B-III. It appears that this mode of restraint would be most effective for failures of 25° or less. The strength of the arch is sensitive to the brittle region thickness because this limits the thickness of the arch. This is particularly important for large failures where the circular arch may come close to the plastic region of the fuel. As shown in Appendix B-III, however, it is not the fuel strength that limits the amount of restraint available. At pressures of 300 psi for 10 mil clad and 720 psi for 15 mil clad, the ultimate strength of cladding is exceeded for the worst-case cladding failure size.

Figure 6-37A Thermal Stress In Fuel

Figure 6-37B Thermal Stress In Fuel

6.3.2.4 Recommendations

The buildup of fission gas pressure in the central void appears to be the most likely mechanism that has a potential for producing large internal pressures. The irradiation of fuel pins instrumented with fission gas pressure transducers would give valuable information on the holdup and release of the gas. The pressure transducer could be located in the gas plenum as in previous tests.^{1,2} The EBR-II Instrumented Subassembly allows for pressure instrumentation of this type. A more useful but more difficult test would measure the pressures in the central void directly. Such a test is described in Appendix B-IV.

In addition to the above in-pile testing, out-of-pile tests are recommended in order to study the behavior of a fuel pin under internal pressure. One recommended out-of-pile test is to fabricate a fuel structure to mockup the brittle region of an in-pile fuel pin and to pressurize that structure to failure. The parameters that should be varied are:

1. thickness of the fuel (each thickness corresponds to a specific fuel pin temperature profile);
2. temperature (even though the gradient cannot be mocked up, the magnitude should be); and
3. clad failure size.

Figure 6-38 Internal Pressure of Failed Fuel

Figure 6-39 Arch-Restraint Model

Figure 6-40 schematically shows the experimental configuration. The fuel should fit tightly in the cladding because initial gaps are filled during operation. The fuel may be fabricated as one column (should be sintered), inserted into the cladding, and then an axial hole may be drilled through the column to remove the fuel that would be plastic during normal operation since plastic fuel does not add significantly to the structural strength. This hole should be filled with a fluid to mockup the plastic fuel. Various defect configurations and sizes can then be machined in the cladding. The test specimen is then brought to temperature and slowly pressurized. The important results of this experiment are the pressure at failure and the pressure necessary to cause expulsion of fuel material.

Figure 6-40 Out-Of-Pile Burst Test

This test is conservative in that it uses unirradiated fuel. Irradiation increases the defects and porosity in the fuel. Porosity greatly reduces the fracture strength of a brittle material. There is also no thermal stress cracking in the out-of-pile fuel specimen. This leads to pessimistic results because larger pressures could be produced, leading to potentially more severe failures.

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7.0 RAPID FUEL FAILURE PROPAGATION EFFECTS

The principal propagation phenomena which may occur rapidly with respect to instrumentation response times involve sodium boiling and fuel-clad/sodium thermal interaction (violent boiling). The interest in rapid propagation is not, however, limited to those cases where the reactor continues to operate after the fault begins. Even with reactor scram, a local fault involving sodium boiling and/or violent boiling may propagate damage sufficiently to increase the economic penalties substantially.

Specific application of the current state-of-the-art of knowledge on sodium boiling and/or violent boiling to reactor accident conditions was not undertaken in this study. Rather, a qualitative review of propagation mechanisms was conducted, and in some cases, additional analysis performed to gain preliminary understanding. This review formed the basis for judgments as to priority development needs.

Section 7.1 discusses the general characteristics of sodium boiling, including the threshold for boiling, boiling stability and thermal hydraulics.

Section 7.2 discusses the potential damage mechanisms associated with sodium boiling, vapor bubble collapse and sodium hammer.

Section 7.3 discusses the potential damage mechanism of fuel-clad/sodium thermal interaction (violent boiling).

7.1 GENERAL CHARACTERISTICS OF SODIUM BOILING (L.M. McWethy, W.A. Sangster)

Introduction

Sodium boiling phenomena affect the progression and consequences of every accident which exceeds the boiling threshold. This section discusses in general terms three areas of general interest. These are:

- Boiling Threshold
- Sodium Superheat
- Boiling Hydrodynamics

7.1.1 Probability of Boiling and Its Detection

An accident situation, whether local or affecting the entire core, is generally considered to be more serious if sodium boiling occurs. It is common in accident analysis to assume that fuel cooling ceases on boiling initiation. This leads to the earliest predictions of clad and fuel meltdown. Reactivity changes due to voiding are instituted at the highest credible superheat and the worst voiding pattern. The models for sodium boiling under rapid transient situations are in the early stages of development and only model specific phenomena of current interest, not the expected behavior of the accident chain. Typical of these areas are the conditions controlling the inception of boiling.

Flow reduction accidents have been categorized for arbitrary flow reductions as well as for specific malfunctions¹. With current technology for instrumentation response only one postulated whole core accident required a design change to avoid sodium boiling, the guillotine inlet pipe break. The conservative assumption was made that zero superheat was required to initiate boiling, requiring the fastest instrumentation response and scram action.

For a single bundle flow reduction of undefined source, a reduction of about 50% was considered tolerable from the standpoint of not sustaining rapid fuel damage. Up to 80% reduction can be sustained if accompanied by a reactor scram. A rapid flow reduction to a single fuel assembly seems credible only if due to an external object or objects. Since the Fermi incident, inlet nozzles have been carefully designed to preclude serious blockage by flat plates^{2,3}. Similar design precautions should reduce significantly the probabilities of finding foreign objects in the inlet coolant stream and their causing significant flow reductions. Analysis of the instrumentation requirements for local flow reductions¹ led to the conclusion that flow meters are required to detect and shutdown a reactor subjected to rapid ($\sim < 1$ second) and severe flow reductions (50-80%) to prevent fuel damage and sodium boiling. Thermocouples are thermally buffered from the core by axial blankets and vents in the case of rapid flow coast downs and noise detectors probably only indicate sodium boiling, i.e., a post-incident detector as far as the initial flow reduction is concerned.

For blockages caused by fuel pin debris, from within a bundle itself, the blockage rate (burst fuel propagation rate) is intuitively slower than one second per row in the radial direction. Detection of the local blockages by flowmeters or thermocouples is unlikely until the blockage size is significant. Keaton⁴ indicated the sensitivity limits of

thermal noise analysis to be ten subchannels (the flow area formed by three adjacent fuel pins) blocked for a length of 12 inches. This is also the size blockage at which the sodium boiling threshold might be reached⁵ for LMFBR conditions. This indicates that local blockages could involve the sodium boiling phenomenon before fuel bundle instrumentation is capable of detecting the situation. Such a blockage would necessarily involve failure of several fuel pins which would provide another indication of failure propagation, i.e., the continuing build up of fission gas and fission products in the coolant, but fission product detectors tend to be quite slow (~ 20 seconds). (Note the exception, the diving bell fuel pin which might not release fission gas and fission products on failure.)

The operating philosophy of the reactor for shutdown on indication of multiple failures may then control the probability of a local blockage degenerating into a local boiling situation.

The conclusions to be drawn from this study are:

- Whole core flow reductions will not involve sodium boiling unless accompanied by loss of safety system action. The probability of this occurring is likely to be very low; for example, less than 10^{-9} by design.¹
- Every initiating mechanism for bundle inlet blockage which can be imagined is (or will be) guarded against by design. This leaves unknown the probability of bundle blockage due to unspecified mechanisms.
- Small local blockages within the tube bundle have a relatively low probability of detection with current state-of-the-art; the probability of a boiling incident in the reactor is thus apparently controlled by the probability of experiencing a gross (> 10 sub-channels) local blockage.

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7.1.2 SODIUM SUPERHEAT

The additional unknown of the boiling threshold is the superheat to initiate boiling. This has been extensively investigated in recent years. Holtz¹⁰ formulated the model which appears to best explain the principal relationships influencing sodium superheat for pool boiling. This model has been expanded and discussed extensively^{1,2}. For a flow blockage condition, a superheat in the range of 200 to 250°F is predicted by this model.^{1,7} For most real accident situations (partial blockages and flow coastdowns) the sodium velocity is significant and this is considered a key variable. A generally sharp decrease in superheat has been observed with increasing velocity^{1,6,8}, however, recent tests^{3,9} indicate the effect may not eliminate superheat considerations as was thought earlier. Under sodium blowdown conditions at 20 fps, Fauske³ reports 79°F superheat. Under conditions similar to reactor conditions except pressure level (5 psia instead of ~ 25 psia), Logan⁹ reports superheats to 100°F at 6 fps. This is not much lower than reported for similar low velocity tests,⁶ (generally less than 150°F). The effect of the higher reactor pressure is also expected to lower the superheat of interest compared to current tests. High pressure tests resulted in superheats of 5 to 100°F in an unpredictable manner,¹⁸ typical of other data scatter.

Singer⁴ recently reported a test which seemed to eliminate the heat flux as a parameter affecting superheat. Deitrich⁵ analyzed the possible influence of radiation on superheat, concluding that fission fragments could establish an upper limit on superheat. For reactor conditions this is in the range of 300 to 400°F, far above the available predictions and data. Presumably this would apply only to a failed fuel element since the presence of fission fragments is required.

One such incident possibly involving radiation effects has been reported,¹¹ where a superheat of about 1260°F, could be inferred from simultaneous measurement of the pressure and temperature. This involved the violent meltdown of a fuel pin in the ANL piston autoclave. Fission fragments were certainly present, plus presumably fission gas bubbles and other nucleation sites. The observed flashing of superheated liquid sodium from 1700°C to saturation

Figure 7-1 Significance of Sodium Superheat in Loss-of-Flow Accidents

at $1000^\circ C$ certainly is in contradiction to all sodium superheat data taken in pin geometries and to Deitrich's hypothesis concerning fission fragments. A possible explanation is that two-phase conditions inside the autoclave effectively decoupled the pressure transducer from the thermocouple until a shock wave could provide relief to a local boiling situation, i.e., boiling was acoustically constrained. This depends on the relative location of thermocouple and pressure transducer and the plenum expansion column. Not enough is known of this case for further speculation and it cannot be compared to sodium boiling inception in a pin bundle in any case.

The conclusion one is forced to draw from the studies of sodium superheat is that superheat is probably not high and will likely exhibit a good deal of scatter. This scatter has been considered as a random variable⁷ or it could be due to unknown influences of the testing procedures. It appears significant that no researcher in the field has chosen to predict the superheat to be expected in LMFBR accident situations. Generally, superheat values less than $150^\circ F$ appear typical¹⁴ and are supported by analysis and experiment. In reviewing European experience, Schlechtendahl¹⁹ concludes that there is insufficient data to predict superheat at the present time.

The reason for the extensive study of sodium superheat lies with the positive void coefficient of LMFBR's and the concern with the DBA. Superheat also will influence to some degree the progression of any incident involving sodium boiling. In both cases, superheat is likely to dictate the flow regime following the inception of boiling. Slug flow is reported to occur above $20^\circ C$ superheat and dispersed two phase flow below.¹³ In addition, flashing may not occur at all for certain conditions below a threshold of superheat. For superheated Freon, Grolmes¹² reports a stable liquid-vapor interface without flashing with the threshold superheat a function of container diameter.

The Freon flashing may be due to development of a thermal boundary layer but this seems unlikely in sodium. These cases show the possible influence of superheat on the sodium behavior after boiling inception.

Superheat for Loss of Flow Accidents

Analysis of the significance of sodium superheat has suggested a criterion for defining the conditions under which the sodium superheat is important. "Low" superheat can be considered the regime where the value of superheat to initiate boiling does *not* have a commanding influence on the subsequent rate of reactivity insertion due to voiding. For

significant superheats, perhaps greater than 40-50°F, inertia limited slug flow will occur on boiling initiation. The rate of voiding and hence rate of reactivity change will be a function of the integrated average pressure acting on the slug. This average pressure is a function of both the initiating superheat and the temperature transient experienced by the vapor bubble.

"Low" superheat could be considered the regime when the temperature increase of the sodium remaining in the core (driving the vapor bubble temperature and pressure up) during the initial voiding process is larger than the initial superheat. A simple example is the FFTF flow blockage case,¹⁷ Figure 7-1, illustrating this criterion. The voiding time shown was calculated for a constant driving pressure equal to saturation at the initiating superheat, i.e., the slowest time to void. The temperature increase causes a higher average pressure and resultant shorter voiding time.

If the superheat to initiate boiling were 65°F (point A), the time to void at the corresponding saturation pressure would be about 0.2 second (point B). However, during this time interval, the temperature of the sodium (either trapped in the core or remaining as a film on pins and spacers), and consequently the vapor bubble temperature, would increase about 200°F (point C). Assuming inertia limited voiding, the peak temperature and corresponding pressure in the core then corresponds to 265°F superheat at the time the vapor bubble passes the core boundary. The average driving pressure then corresponds to some saturation temperature between 65 and 265°F superheat.

This indicates that a minimum effective voiding pressure exists because of the transient temperature increase for low superheats, or that the voiding rate for "low" superheat is not strongly dependent on the value of superheat but rather on the temperature transient. In contrast, the voiding rate with "high" superheat (point D) is clearly a strong function of the value of superheat and resultant vapor pressure, and the temperature transient would have minor influence on the voiding rate.

For the complete loss of flow in FFTF,⁵ the value of superheat dividing the arbitrary "high" and "low" regimes (where the ΔT during expulsion is equal to the initial superheat) is about 130°F. For comparison, superheat tests which closely simulate LMFBR conditions⁶ show the superheat to be generally less than 100°F, with the majority of runs less than 60°F, i.e., in the "low" superheat category.

The major discrepancies in these tests were:

1. Pressures were sub-atmospheric (typically 5 psia).
2. No spacers were present.

The effects of spacers and higher pressures in reactor are expected to severely reduce the probability of achieving significant superheat. The effect of sodium corrosion is also important, as "Little or no superheating is obtained when the heater surface has been attacked by the sodium, producing well-defined surface prominences and pits."¹⁵ This characterization of surface conditions is confirmed by GE work,¹⁶ which also indicates the surface character may be strongly influenced by heat flux.

Except for the new fuel, corrosion effects will thus apparently reduce sodium superheat. These effects, the corrosion, pressure, spacers and velocity, suggest caution in using atypical experimental results (particularly pool boiling tests) to predict reactor accident sodium superheat.

The conclusions from this review of sodium boiling inception are:

- a. Superheat is likely to be "low" for most reactor accidents.
- b. The initial boiling flow regime is as likely to be dispersed as to be slug flow. Flow regimes are discussed in Section 7.1.3.1.
- c. Voiding will likely develop slowly, dictated by the thermal transient, rather than violently due to high superheat.

Recommendations

- Since the question of superheat appears basically important only for whole core situations with simultaneous loss of protective system action, only specific superheat tests to support licensing arguments should be run (and then under prototypical conditions of pressure and temperature history, heat flux, sodium oxide control, etc.) and only after the incentive (lower DBA energy release) has been calculated.
- For the FFTF reactor, superheat tests are not important in this respect due to the low sodium voiding reactivity (<\$1).
- Superheat should be *noted* in general sodium boiling tests, but there appears no identifiable logic for continuing separate superheat tests that are not in support of licensing specific designs.

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7.1.3 Sodium Boiling Hydrodynamics and Heat Transfer

The key questions identified in the fault tree analysis which pertain to sodium boiling hydrodynamics and heat transfer are:

1. Is sodium boiling stable?
2. What conditions define the rate and coherence of voiding?
3. What conditions define the threshold for clad and fuel meltdown?

These areas will be reviewed briefly in an attempt to highlight the most relevant research. Fauske¹ reviewed the majority of the relevant sodium boiling hydrodynamics research. Of principal interest were the models for sodium voiding. In general, the available models each treat a single flow regime and operationally suffer frequent numerical instabilities. With the exception of the BLOW-II code,² none of these models has been used extensively to correlate experimental data or to predict reactor accident conditions. None of the models appear capable of realistically predicting the sodium boiling hydrodynamics and heat transfer much beyond the point of initial bubble growth.

7.1.3.1 Boiling Stability for a Fuel Bundle

A discussion of sodium boiling instability by Fauske¹ summarizes the conditions for pool boiling. Entirely different behavior is observed with high power density tests simulating fuel rods. In the only relevant tests which might be used directly, sodium boiling was found to be extremely unstable for pressures (to 47 psia) and heat fluxes

(450,000 Btu/hr-ft²) in the range of interest.³ Unfortunately work was discontinued on this high temperature facility in mid-1968 with only a few data runs taken.

Schlechtandahl's BLOW-II results² certainly demonstrate the violent chugging predicted with high superheat and a slug flow model. Mention is made of a study by D. Smidt showing that two phase sodium boiling is unstable under reactor conditions.

Logan's tests have shown stable boiling in testing for sodium superheat,⁴ but the test is run in a constant flow loop and hence might be forced to be stable.

Different stability conditions might be expected depending on the manner of flow reduction. Loss of flow from pump failure or pipe breaks would result in a depressurization of the core inlet, giving the least pressure difference to sustain two-phase flow. In contrast, a partial flow blockage to a single bundle which initiates boiling permits the full pump head to sustain two-phase flow if the single phase pressure drop becomes insignificant. This latter condition would have a greater tendency to be stable.

In order to study further this question of boiling hydrodynamic stability, a detailed review of sodium two-phase flow was conducted.

Two-Phase Flow Regimes, Pressure Drop, and Void Fraction for Sodium

An initial phase of the review was the investigation of several mechanistic areas related to the understanding of sodium boiling phenomena. One such area is the hydrodynamics of two-phase flow. The available data are for steady state conditions and the applicability to rapid transients in an LMFBR is currently uncertain.

1. Flow Regimes

This important area is in the most preliminary form. No actual data for sodium have been found and the curves of Figures 7-2 thru 7-5 are derived from a Baker⁵ plot which in turn was derived from air-water data. The Baker plot is meant to be applicable to any two-phase mixture and has been converted to sodium at two different pressures. Figures 7-2 and 7-3 are for sodium at 10 psia. Most of the two-phase sodium test programs at AI and ANL (reviewed below) have been conducted at sub-atmospheric pressures due to temperature limitations in the loops (e.g., saturation temperature = 1540°F for sodium at 10 psia). The second pair of curves, Figures 7-4 and 7-5, are for sodium at 50 psia which is more typical of reactor design conditions. The effect of higher pressure is to permit bubble flow out to higher qualities.

All four figures show the flow regime expected for any combination of quality and flow rate. Most of the sodium test data for void fractions and pressure drop have been in the region of less than 14 percent quality and mass flux from 10⁵ to 10⁶ lb/hr-ft².

Probably both bubble and dispersed/annular flow regimes were experienced. (Note that dispersed flow is similar to annular flow except that liquid droplets are encountered in the vapor core.)

Although the curves in the figures indicate an abrupt transition from one regime to another, these are more likely to be gradual changes as evidenced by the continuous trends observed in the data on pressure drop, slip ratio, and void fraction as a function of quality. Thus, for example, at a given flow rate the transition from bubble flow to annular flow might be expected to be characterized by a regime of bubble concentration near the central core of the final annular flow pattern.

An unanswered question at this time is whether these tube test results are generally applicable to rod bundles. The effect of the rods and especially the spacers may be to alter these trends considerably. Spiral wires may inhibit annular flow; grid spacers may break up large bubbles into smaller one.

2. Pressure Drop

Two-phase friction pressure drop multipliers for sodium were generated based on actual sodium⁶ data which are

Figure 7-2 Two Phase Flow Patterns for Sodium at 10 PSIA, Thru 14% Quality

Figure 7-3 Two Phase Flow Patterns for Sodium at 10 PSIA, Thru 90% Quality

Figure 7-4. Two Phase Flow Patterns for Sodium at 50 PSIA, Thru 15% Quality

Figure 7-5 Two Phase Flow Patterns for Sodium at 50 PSIA, Thru 90% Quality

found to be in good agreement with non-metallic fluids.⁷ Figure 7-6 shows the two phase* multiplier as a function of quality and pressure.

$$\frac{\Delta P}{\Delta L} = f \left(\frac{W}{A} \right)^2 \frac{\Phi_{fo}^2}{2g\rho D}$$

where:

- W is the total mass flow (both vapor and liquid),
- ρ is the liquid density,
- D is the usual equivalent hydraulic diameter,
- A is the total cross sectional area.
- Φ_{fo}^2 is the two phase multiplier

The multiplier of Figure 7-6 is the more common type which assumes that the total mass flow is flowing as a liquid and the effect of the vapor is included in the multiplier.

The curves of Figure 7-6 have been adjusted such that the 100 percent quality points agree with an all vapor pressure drop.

3. Void Fraction

Sodium void fraction data of AI⁶ and ANL⁸ indicate some disagreement as shown in Figure 7-7 (void fraction as a function of quality for sodium at low pressure ~10 psia, near where the experiments were performed). For a given quality the ANL data (which agree with the Lockhart-Martinelli curves of Reference 7) lie considerably above AI void fraction data. The preponderance of data follows the Lockhart-Martinelli correlation, recommending its use for steady state sodium hydrodynamics.

The general conclusion from reviewing the sodium boiling hydrodynamics is that the well established two-phase flow relationships for non-metallic fluids are adequate for steady state conditions. Since the thermal conductivity of sodium is very much higher than say water, it is anticipated that *transient* hydrodynamics will be influenced strongly by the thermal environment of the boiling regime and hence caution seems warranted in directly applying two-phase hydrodynamics to rapid sodium boiling transients.

The results of this study were applied to full power reactor conditions, with the results in Figure 7-8 for a uniform power bundle or a single isolated fuel pin showing the pressure drop and temperature versus flow relationships for steady state conditions. The "check" curve is noted beginning at about 1/3 normal flow. For the case of a loss of pump head situation below ~1/3 flow, the flow suffers a transient as it seeks the new equilibrium level at about 3,000 lb/hr. For flow blockage, the higher available head (here 60 psi) results in about 7,000 lb/hr. flow. In both cases, however, the exit flow is superheated sodium vapor, indicating that sodium boiling under even full pressure is not able to cool an LMFBR fuel bundle. The flow transient in crossing the "check" curve must then be classified as unstable since a new equilibrium situation is not achieved.

A complicating factor for most fuel bundles is the effect of the power gradient and edge flow channels. Power/flow mismatch has been shown to affect the fuel bundle "check" curve,⁹ indicating that the bundle pressure drop may increase only slightly above the single phase value for a narrow range of bundle flow below boiling initiation. Provided this relationship is "flat" enough, it is credible that a two-phase flow situation caused by an inlet blockage might be "stable" in the sense that the bundle flow would not seek a new minimum on the low end of the "check" curve. Because this phenomenon is due to flow redistribution from the hot to the cold side of the bundle, it may mean flow starvation and clad melting even faster (than for uniform power and flow) for the hot side of a skewed power bundle or the center of a bundle without edge flow control.

7.1.3.2 Boiling Stability for Local Blockages

The boiling stability around a local blockage within a tube bundle is essentially unknown at this time. The parallel channel "check" curves effect discussed above⁹ certainly applies here and indicates that hydrodynamically stable local

Figure 7-6 Sodium Two-Phase Friction Pressure Drop Multiplier vs. Quality At Five Pressures

Figure 7-7 Void Fraction vs. Quality for Sodium at 10 PSIA

Figure 7-8 Steady State Bundle Hydrodynamics

two-phase flow might exist and will cause the local single phase velocity to increase adjacent to the blockage. Local boiling stability is discussed further in Section 7.2.1, *vapor bubble collapse*. It must be conservatively assumed at this time that any incident of local boiling within a pin bundle is unstable. Boiling is assumed to propagate to the entire fuel assembly.

7.1.3.3 The Rate and Coherence of Voiding

Voiding is of interest for whole core accidents in reactors with a positive void coefficient. Fauske¹ reviews the background on voiding models. If the superheat is sufficiently high, the initial voiding rate is rather easily calculated for individual channels unaffected by system hydraulics. Behavior beyond initial voiding is uncertain and no models or tests are directly applicable. Schlechtendahl² discusses the most serious limitation of current models, the restriction of a single flow regime. Until a continuous calculation can be accomplished with several flow regimes, the hydrodynamics after initial voiding will be subject to large uncertainties. Even for low superheat, equilibrium models may not be adequate because the flow may be unstable. The ability to calculate the hydrodynamics beyond initial voiding becomes critical for the situation of non-coherent voiding of parallel channels. Without this tool, the void states of different channels can be assumed to be random² beyond the initial voiding or some non-random pattern,¹⁰ both subject to argument for licensing purposes.

An effect which may eliminate the need for sophisticated voiding models is the sensitivity of reactivity to void fraction and void location. It may be possible to show that in all instances involving sodium voiding at power, the void fraction is very high (>0.9) for all conditions following the initial voiding, and that the void location is predictable within acceptable bounds.

Since the reactivity effect of a single channel voiding is relatively minor ($\sim 10\%$), the incentive for better models does not exist for single channel accidents. If propagation of a single channel accident occurred, the reactivity effects might be important, dependent on propagation time scale and assuming no control system action.

7.1.3.4 Clad and Fuel Meltdown

In reviewing the literature, no data or analysis directly applicable to sodium heat transfer beyond initial voiding was found. Of principal interest is the cooling rate of the clad and fuel from boiling initiation onward. Since it appears that sodium boiling is unstable, the key question concerns the timing and location of clad failure (if it has not failed at temperatures below boiling), clad melting and fuel melting.

Sodium boiling is important in this sense because it leads to possible fuel/clad-sodium thermal interaction (violent boiling) and reactivity changes caused by fuel slumping. For local blockages, involving sodium boiling and clad and/or fuel melting, the subsequent concern is generation of pressures capable of causing pin failure propagation and possibly channel failure. For whole bundle accidents, the concern is of generation of bundle-to-bundle propagation pressures.

The available evidence for sodium boiling heat transfer was reviewed. Atomics International studied steady state boiling heat transfer extensively several years ago. Measurements of critical heat flux showed values generally above 10^6 Btu/hr-ft² for low qualities ($<5\%$) and low pressure (8 psia).¹² This data might extrapolate to qualities below 10% for reactor pressures (>20 psia). Burnout then would not be expected for low quality sodium boiling for steady state peak heat flux conditions, conditions which would be apt to occur only over a short length at the beginning of a boiling section.

This data suggests that fuel pin cooling is adequate if a two-phase forced convection flow can be maintained at reasonable qualities, which seems unlikely at full power. This excellent heat transfer, however, contributes to the instability of sodium hydrodynamics. The high rates of heat transfer in moderate temperature gradients provides the driving pressure from vaporization to make pressure drop and possibly critical flow the controlling phenomenon.

Transient heat transfer is known to affect significantly the hydrodynamics of voiding. Ford, et al¹³ discussed the influence of a thermal boundary layer formed during Freon evaporation and condensation on the subsequent voiding rate. Because of the low conductivity of Freon, this thermal boundary is very thin and dramatically limits the voiding rate, i.e., a thermally limited voiding rate.

In contrast, sodium conductivity is such that a thermal boundary layer develops in the adjoining structural materials in both evaporation and condensation.¹⁴ Sodium boiling hydrodynamics then couples the heat sources and sinks in a fuel bundle during the transient. The fact that the heat sources and sinks in a fuel bundle undergoing a rapid

transient are extremely complex leads to caution in using steady state sodium hydrodynamic correlations.

The effect of non-uniform heating on transient voiding was noted by Singer, et al¹⁵ and the steady state effect (Baroczy)⁹ was noted earlier. Schlechtendahl² attributed discrepancies between tests and a slug flow model to axial vapor and two-phase flow within a slug flow vapor bubble. Radial heat transport during boiling has been noted to dramatically affect the voiding of a simulated loss of flow test run by Pfefferlen.¹⁶

This coupling of heat transfer and hydrodynamics during rapid overheating could significantly affect the timing and location of pin failures, clad and/or fuel melting and even channel failure. There is currently no comprehensive model even proposed to account for these effects. If an accurate modeling of violent boiling is to be attained, the events leading up to meltdown must similarly be accurately modeled. The alternative to modeling is integral testing in which intermediate events like sodium boiling and clad/fuel meltdown are ignored.

7.1.3.5 Conclusions and Recommendations

1. The superheat to initiate sodium boiling is likely to be low and therefore relatively unimportant for local flow reductions and for whole core accidents.
2. The most likely way to initiate boiling is local boiling due to a flow blockage inside a fuel bundle, which is likely to be unstable, leading to whole bundle boiling.
3. Review of the literature suggests that whole bundle sodium boiling under full power conditions is unstable, i.e., divergent to a very low flow condition.
4. Eventual clad and fuel meltdown is probable but no data or models are presently available to accurately predict the timing and pattern of meltdown.
5. Sodium condensation effects are seemingly neglected in LMFBR studies. Vague mention is made of reentry (a condensation problem) as it influences molten fuel-sodium interaction. Whereas sodium boiling, after initiation, may not be strongly influenced by the presence of fission gas or other contaminants, condensation rates will surely be strongly affected. The problem of inert gas cell pressurization following the DBA may be related to this problem. The course of all accidents involving sodium boiling is determined as much by condensation as by boiling. Flow choking, as predicted in the FFTF studies, is due to the condensation on cold surfaces. This area requires further study.
6. A series of integral, prototypical tests is necessary as soon as possible to provide guidance to the phenomenological tests, prototypical out-of-pile tests and further in-pile tests.

Recommended Sodium Boiling Experimental Program

1. Sodium boiling tests (out-of-pile) are recommended, with the following general criteria:
 - continuation of PA-10, Task C-2 mockup 1A type tests
 - multi-pin test section
 - simulating expected reactor conditions
 - pressure and temperature
 - axial flow restrictions
 - channel mockup
 - run power/flow mismatches, initially with shutdown, later to destruction.

A tungsten heater, clad with steel, might be melted in a sodium loop to correlate with the more expensive in-pile tests. This could yield useful data concerning clad fragmentation and violent boiling before heater destruction and power cutoff occurred. The severely complicated geometry problem of a disrupted bundle would be simulated.

Potential development of a UO₂ electric heater by Atomics International¹⁷ holds promise of clad and fuel meltdown tests into the regime of gross geometry distortion, simulating a loss of flow situation well into the violent boiling regime.

2. In-pile integral experiments are recommended.
 - PA-10, Task C-2 tests appear adequate
 - It is recommended to emphasize some initial prototypical tests for guidance of further tests. Specifically, the final state of a loss-of-flow incident should be determined to provide a measure of the significance of the accident.

Serious structural damage to a simulated channel, for instance, would mean the damage mechanisms should be examined further. These implications are further discussed in Section 7.3.2

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7.2 SODIUM HAMMER AND VAPOR BUBBLE COLLAPSE

Introduction

The study of sodium boiling effects has identified several priority questions deserving further attention. One such question is the mechanical consequences of sodium boiling. It is generally agreed that the pressurization due to initial vaporization of superheated sodium will impose negligible additional loads on the pins and channels, see Section 7.1.2 on Sodium Superheat. Sodium vapor condensation, however, can cause a significant conversion of thermal energy differences to compressive strain energy due to local pressurization. The two categories of conversion are sodium hammer, taken here to mean axial collapse of a cylindrical bubble (normally thought of as filling a subassembly cross section) and collapse of a spherical bubble (normally small in size).

7.2.1 Vapor Bubble Collapse

7.2.1.1 Accident Description

A sufficiently large blockage in an LMFBR fuel bundle will result in sub-cooled boiling (with respect to the overall bundle flow) of the sodium when conduction and convection are sufficiently impeded. Subsequently, cooling of the hottest region of the blockage will be assisted by the boiling mechanism. Immediately following boiling initiation (and perhaps for a significant length of time afterward) a sub-cooled boiling regime will describe the coolant hydrodynamics.

Sodium vapor bubbles will be generated in and/or behind the blockage and condensed as they are transported by natural and forced convection to adjacent colder sodium streams.

The additional flow perturbation due to the two-phase pressure drop tends to further reduce the local as well as the bundle flow. This question of local boiling stability has been recognized in LMFBR literature but no analysis, testing or conjecture, has been published which contributes to its answer. Intuitively, local sodium boiling is very unstable because of the high specific volume of sodium vapor at operating pressures, the high specific powers typical of LMFBRs and, the small hydraulic diameter and resulting high pressure drops. If the local boiling is sufficiently unstable, vapor bubble collapse may be of no concern, but this conclusion cannot be drawn with certainty at this time. For sealed pin designs the collapse of sodium vapor bubbles adds an additional mechanism of damage propagation and possibly the additional flow degradation mechanism of fission gas release upon failure of fuel pins not yet adversely affected by the thermal perturbation of the blockage.

The mechanism of conversion of a thermal difference between the bubble and surrounding fluid to fluid compressive strain and kinetic energy involves the following:

- a. Transport of a vapor bubble into subcooled sodium
- b. Rapid condensation of the vapor onto the bubble wall lowering the bubble pressure and forming a *potential energy* source.
- c. Acceleration inward of the surrounding liquid by the pressure difference between ambient conditions and the remaining vapor; i.e. conversion of the *potential energy* to kinetic energy.
- d. Deceleration of the liquid on complete collapse (conversion of kinetic energy to fluid strain energy at high pressure) or if vapor remains (is not condensed completely) conversion of kinetic energy to fluid strain energy *and* vapor strain energy.
- e. Generation and propagation outward at sonic velocity of a pressure pulse (fluid strain energy and kinetic energy) by the expansion of the compressed vapor and fluid.
- f. Several expansions and compressions of the vapor may occur before condensation is complete.

This pressure pulse may damage adjacent fuel pins and channels.

The pins may already be overheated and subject to thermal stresses due to skewed cooling and high spacer contact stresses as well as normal fission gas pressure stresses. The added aggravation of bubble collapse pressures could cause pin failure propagation.

Normally, collapse would occur in spaces between pins due to the temperature gradient from heated wall to bulk coolant, but in the blanket or plenum region, with low or zero heat flux, collapse could occur on the pin surface. This may be similar to cavitation in its effects.

The need is to characterize the possible pressure pulse energy generated by local subcooled boiling with vapor bubble collapse and to compare with the damage thresholds of irradiated, possibly overheated, and possibly stressed fuel pins and channels.

7.2.1.2 State-of-the-Art Review

Peppler¹ noted the possible damage mechanism due to sodium bubble collapse. Assuming the vapor behaves as an ideal gas, the momentum controlled collapse (Rayleigh equation) at 200°C subcooling results in pressures as great as 50 atm lasting for a few microseconds. Since the assumed gas does not condense as would vapor, the bubble rebounds on a period of about 0.3 milliseconds. Peppler concludes that a damage potential analysis requires accounting of the dynamic response of the core structure.

Judd^{2,3,4,5} has extensively analyzed sodium vapor bubble collapse. Using the momentum equations of Hickling and Plesset, Judd³ calculates that in 1 atm. sodium and 280°C subcooling, the initial bubble collapse is inertia controlled but followed by a condensation limit as the remaining vapor is compressed. This leads to rebound and several cycles to effect complete condensation. At each rebound, very high pressures are generated (~8000 atm.) which steepen into a shock wave whose intensity decays as 1/r. The peak pressure in the liquid is 200 atm. at the original vapor bubble radius of 10 cm.

Uncertainties of this analysis include the sodium equation of state, since the peak pressure is above the critical pressure, and the question of bubble stability. Collapsing bubbles are unstable but the shock waves are formed even if

collapse occurs as many small bubbles rather than one symmetric bubble and experiments indicate comparable pressures occur.

A more refined calculation⁴ apparently confirmed the initial inertia dominated collapse, followed by a damped oscillation limited by heat transfer.

A later numerical bubble collapse model⁵ coupled the vapor thermodynamics with Hickling and Plesset's bubble collapse equations. This model calculates the initial collapse and rebound of a sodium vapor bubble, but not subsequent secondary collapse cycles. The results are expressed as pressure history and pulse energy as a function of bubble size with subcooling, thermal boundary layer, and vapor state equation as parameters. The later two parameters alter the results significantly (orders of magnitude) but no indication is given by the author of what a reasonable assumption might be. A wide range of pulse energy (about half the peak pressure) is calculated, from 1.5 to 185 joules for the initial collapse depending on the assumptions used. This energy level is noted to be less than the initial potential energy of the cavity ($P_{\infty}V_0$), which is intuitively the upper limit on available energy. The absorption of latent heat of the vapor acts to retard the bubble collapse and reduce the available work. Because condensation phenomena are not included in Judd's model, a realistic estimate of the rate of bubble collapse cannot be found. The conclusion drawn by Judd is that the available energy could be as large as 4.4% of the potential energy, assuming adiabatic compression of the vapor (no condensation). This value increases to 44% with zero vapor phase resistance to condensation up to the critical temperature; but with condensation limited by the rate of heat transfer into the liquid (a thermal boundary layer effect). With zero liquid film resistance, presumably the energy of the bubble collapse pressure wave would equal the potential energy.

At 44% of the potential energy level, it is interesting to compare the energy intensity with the channel wall absorption capability. The pressure pulse will deform the wall about 5 mils, producing ~1% strain and absorbing about 40% of the calculated pulse energy. Since the pressure pulse is spherical, and the channel analysis assumed a plane wave, the comparison is not expected to be accurate, but does indicate that collapse of a single large bubble may produce significant damage in the channel wall. Judd's model also shows the pressure pulse slowly developing into a shock wave over a distance of 2 feet, which is very large compared to the free distances in a fuel bundle. Reflections of pressure wave from pins, channels and area changes will influence the development of the pressure pulse, suggesting that a simultaneous solution of bubble collapse equations and structural response is necessary to analytically determine the significance of bubble collapse.

An experiment to simulate bubble collapse in water by P. G. Kosky and G. A. Henwood⁶ yielded similar pressure (~10,000 psi for a bubble radius of 2 inches). The method used a column of water terminating in a cone. A pressure transducer at the apex recorded the bubble collapse pressures resulting from sudden application of pressure at the opposite end of the column. This technique combines a linear pressure pulse in the tube with a simulated spherical pressure pulse in the cone. The peak pressure recorded does not represent the maximum since the transducer results in a flat ended cone, preventing complete concentration of the spherical pressure pulse. In addition, the measured pressure decay from cone transducer to the cone edge behaves as $1/r^2$ rather than $1/r$ as expected, indicating complications of the pressure response due to the solid wall and/or the flat ended cone. This test does not seem to be representative of sodium vapor bubble collapse. The authors noted a decrease in peak pressure with increasing gas content of the initial bubble.

A cavitation study by I. R. Jones and D. H. Edwards⁷ uses a spark discharge to generate a water vapor bubble on the end of a pressure bar. A transducer measures the force applied to the bar due to the vapor bubble collapse. Knowing the bubble size from photographs, the peak pressures were estimated to be between 10^4 and 10^7 atm, with 10^5 atm the most likely. These fall in the same range as the predictions of Judd. This study measured directly the force exerted by bubble collapse, which is a direct measure of the damage potential. Figure 7-9 shows the peak force and an estimate of the wave impulse taken from the data of Jones and Edwards.

This shows a peak force and impulse approximately proportional to the square of the initial bubble radius, in agreement with the pulse energy calculations of Judd.⁵

For bubbles contained between fuel pins, the peak force is in the range of 1 to 2 lbs., which by themselves would not appear capable of causing pin damage.

An overall review of the damage potential due to bubble collapse is given by F. G. Hammitt.⁸ Damage is attributed to either the high pressures generated by bubble collapse or by the formation of a high velocity liquid jet which strikes

Figure 7-9 Bubble Collapse Force, Cavitation
Water at 1 atm
(I. R. Jones & D. H. Edwards)⁽⁷⁾

the surface. Damage potential is a function of the subcooling, the gas content, material strength and hardness, bubble diameter, distances of collapse from the wall, and symmetry of collapse.

Damage occurs by mechanical attack as well as combined cavitation attack and corrosion. Mechanical damage is characterized by cratering analogous to liquid impact tests. The pressure and velocity fields generated by bubble collapse theoretically go to infinity at zero radius according to the classical Rayleigh collapse calculations. The impulse magnitudes are proportional to bubble size and pressure difference. In real situations, a spectrum of bubble sizes might result in a few bubbles collapsing on the surface and causing cratering but many more collapsing with insufficient strength to cause permanent deformation. These repeated blows can contribute to a fatigue failure of the surface and to material coldworking. The material properties are thus affected, with increased strength and hardness tending to increase damage resistance but increased brittleness reducing damage resistance. Only small numbers of craters are formed in relation to the number of bubbles collapsing, indicating that bubble diameter, wall distance and bubble orientation (if collapse is not symmetric) contribute to reducing cavitation damage.

Unsymmetric collapse of bubbles can be approximately toroidal and this can produce a small microjet of liquid penetrating the bubble after a volume reduction of about a factor of 10-100. This microjet is one explanation of the mechanical damage effects and the analogy with water impact tests.

Collapsing bubbles tend to move toward a solid surface and this may show a significant effect upon the pressure exerted on the wall. If the wall is not rigid (e.g., a fuel pin) the collapsing bubble draws the wall toward the bubble, followed by the high pressure impulse pushing the wall away.

For ideal collapse the potential energy of the cavity $(P_{\infty} - P_v)\Delta V$ can be converted to kinetic energy. This may give rise to the very high local velocities leading to microjet damage, and to the high local pressures as the collapse is restrained in liquid compression.

For cavitation caused by flowing fluids, the damage rate is proportional to the 6th power of the stream velocity. This is a result of a rapidly changing local pressure field in a streamline. This same effect could bear on sodium bubble collapse with cross flow in tube bundles and spacers causing local changes in velocity and pressure as well as temperature. The effect of entrained or dissolved gas is noted to substantially reduce cavitation damage.

The "thermodynamic" effect of fluid temperature changes the vapor pressure, as well as restraining bubble growth and collapse if liquid and vapor heat transfer inhibit phase change. Damage is not affected by thermal restraints as long as a modified "thermodynamic parameter" is typical of low vapor pressure fluids.⁹ The transition value between heat transfer and inertia controlled collapse is assumed to be at a thermodynamic parameter of 1000 (apparently at 1360°F for sodium). Bubble collapse damage potential is thus not restrained by thermal effects below $\approx 260^\circ\text{F}$ subcooling.

The best correlation of a single mechanical property for metals to measure cavitation resistance is the ultimate resilience $(1/2 \sigma_t^2/E)$ representing the energy per unit volume necessary to cause brittle failure. The third principal variable affecting cavitation damage is the pressure causing bubble collapse.

The recommended correlation of cavitation damage is shown in Figure 7-10 for a typical fast reactor pressure of 23 psia. The damage rate is not sensitive to the pressure except near the boiling point, and there the damage is negligible. The bubble thermodynamics dominate the correlation of cavitation damage, significant damage occurring only in the inertia dominated regime where vapor pressure does not retard collapse.

Local boiling behind a blockage may lead to bubble collapse in very subcooled sodium (temperature near normal operating conditions) and adjacent pins may experience cavitation damage. The energy potential for cavitation damage is also the mechanism for rupture of the clad acting as a thin wall vessel. The principal difference may be the bubble size. The cavitation correlation assumes a constant but unknown cavity size. With subcooled boiling the bubble size is also unknown and represents the most important variable affecting the damage potential of bubble collapse.

A rigorous solution of bubble collapse equations is reported by Theofanous, et al.,¹⁰ Example calculations are included which are qualitatively similar to the results of Judd.⁵ The peak pressure is always near the critical point (450 atm) see Figure 7-9, for the subcoolings (100 to 300°C) and initial bubble sizes (0.3 and 3 cm) assumed. No energy calculations are included but note is made of the need to include the fuel pin dynamic response in order to evaluate damage potential.

7.2.1.3 Analysis of Bubble Collapse

With the exception of Judd,⁵ there has been no reference in the literature to the energy content of pressure waves resulting from bubble collapse. Judd's results showed a two order of magnitude variation depending on the assumptions, with no recommended correlation. In an attempt to understand the energy sources due to bubble collapse, the compressive strain energy in the liquid and vapor has been compared for both Judd's and Theofanous' sample cases.

The liquid strain energy is taken as:

$$U = 1/2 \cdot m \left(c \frac{\delta p}{\rho_{\infty}} \right)^2$$

$$= 1/2 \int_{r_2}^{r_1} (4\pi r^2) \rho \frac{c^2}{E'} (\delta P)^2 dr$$

Figure 7-10 316 SS Cavitation Damage (R. Garcia and F.G. Hammitt)⁹

where: δP is the pressure of the fluid at radius r above ambient pressure

c is sonic velocity

ρ is fluid density

E' is bulk modulus of the fluid.

The vapor energy is taken as the adiabatic work of compression from saturation to the peak pressure, and for rebound, the energy is the adiabatic expansion work down to ambient pressure. The results of these comparisons are shown in the following table.

Energy Comparisons, Bubble Collapse

Reference	Judd ⁵	Judd ⁵	Theofanous ¹⁰
Bubble diameter, cm	10	3	3
Minimum bubble diameter, cm	0.68	0.093	0.21
Subcooling, °C	280	280	100
Saturation pressure, atm	0.036	0.036	0.36
Ambient pressure, atm	1	1	1
Peak pressure in bubble, atm	380	4300	400
Approximate pressure distribution at rebound, atm	$P = 240r^{-1.19}$	—	$P = 77.6r^{-1.05}$
Fluid strain energy at rebound, in-lb _f	~200*	14**	~35
Vapor compressive work at rebound, in-lb _f	2400	71.4	58.6
Vapor expansion work from rebound to ambient pressure, in-lb _f	1720	60	53.4
Potential energy of bubble, in-lb ($P_{\infty} - P_0$)V ₀	3800	101	99

The results are interpreted as follows:

1. A large fraction of the potential energy of the bubble results in vapor compression (~60%).
2. This stored energy is available to expand against the surroundings *if* condensation is not effective. Isentropic compression of the vapor (assuming ideal gas behavior) results in conditions near the critical point indicating little condensation will take place initially.

The energy available from vapor expansion is roughly half the initial potential energy.

3. The fluid strain energy at rebound is significant in Theofanous' calculation (~35%), but relatively minor in Judd's calculation (~5%).

This appears to be due to the effect of bubble size and the assumption of condensation overriding the higher subcooling of Judd's case. Judd's calculation of fluid strain energy shows 4.4% at 10 cm initial radius and 10.3% at 3 cm initial radius. The additional factor of 3 of Theofanous' case is presumed due to Theofanous' assumption of continuous condensation (with a unity condensation coefficient) compared to Judd's adiabatic vapor compression. Judd indicates there is negligible increase in pressure wave energy after rebound, and does not discuss the disposition of the stored vapor energy. Theofanous' does not extend his analysis beyond the minimum radius (rebound).

4. As a minimum, the fluid strain energy is available in the pressure wave to interact with the surroundings. As a maximum, the vapor expansion work must be included. But this total is nearly equal to the potential energy. In fact, probably most of the potential energy appears as pressure pulse energy, with the majority occurring on the first collapse. Figure 7-11 shows the potential energy of the bubble as a function of temperature and radius. Saturation is slightly above 1700°F at which point the potential energy is zero. The vapor pressure decrease with increasing subcooling shows that 50% of the maximum potential energy is achieved with only 100°F subcooling. It is noted that the total potential energy of a collapsing 0.1 inch radius bubble is about equal to the plastic limit load of a fuel pin bending between 6 inch spacer supports (see Figure 7-11). A bubble larger than the coolant flow passages is thus necessary to cause pin damage in the bending mode.

*Compares to 161 in-lb_f Judd calculates for energy above half the peak pressure.

**Judd's calculation.

$$PE = (P_{\infty} - P_{sat}) \frac{4}{3} \pi r^3$$

$$P_{\infty} = 23 \text{ PSIA}$$

Figure 7-11 Vapor Bubble Potential Energy

Since there is evidence that bubbles growing in a tube bundle remain spherical beyond the dimension of a fuel pin, it may be possible for bubbles larger than pins to collapse in a tube bundle, possibly damaging pins by the impulse imparted to the pin. The potential energy in the bubbles is proportional to bubble volume. This collapse could occur even in the core region because of the short life of the vapor bubble (typically less than 10 msec) compared to the fuel time constant (~ 1 sec). Nothing in the literature herein was found to limit the size range of bubbles considered.

No data or methods appear available to predict the efficiency of energy deposition onto the fuel pins or channels. Because the wave length of a bubble collapse pressure wave is longer^{5, 10, 7} than the pin diameter, pin energy absorption is probably not significant¹¹ unless collapse occurs in close proximity to the fuel pin, analogous to cavitation.

Similarly, the cavitation impulse data⁷ for a 1/2-inch bubble, if exerted on the mass of a fuel pin between 6-inch supports, results in an energy deposition less than 0.1% of the bubble potential energy and a factor of 10 less than the pin elastic limit in bending. This indicates that the energy deposition averaged over a beam length of the full pin might be negligible. The available data shows large potential energies but very low energy absorptions by the fuel pins in the bending mode, such that bending damage should be negligible.

7.2.1.4 Conclusions & Recommendations

The potential for propagation of fuel pin defects due to vapor bubble collapse behind or adjacent to a local boiling incident cannot be assessed with the current state-of-the-art. Propagation appears more likely due to bubbles collapsing on or very near a fuel pin rather than due to the pressure wave acting on pins farther away. The potential energy of large bubbles is large enough to be capable of damaging channel walls and pins, but the transmission of the energy in the form of long pressure waves should not result in enough energy disposition to cause significant damage except perhaps at the source.

Three damage mechanisms to fuel pins were considered. With the fuel pin acting as a beam between spacer contacts, the impulse of the bubble collapse pressure wave will cause a bending load and resultant stresses and strains. Observed impulses from water vapor bubble collapse are negligible when acting on a fuel pin with 6-inch supports. The same conclusion should be valid for sodium vapor bubbles.

A second mechanism is deformation of the clad locally by the high collapse pressures and/or impact of liquid microjets. This damage mode is analogous to spacer contact damage, being a thin membrane bending deformation. No analysis or experimental data was found during this brief study to be applicable to this damage mode, prohibiting the comparison of energy potential of bubble collapse to clad damage thresholds. The third damage mode is removal of material at the surface due to cavitation damage. The peak damage rate is less than one mil/hour, indicating this damage mode is significant only for sodium boiling incidences of long duration. This is because detection and shutdown are possible from acoustic noise detection after several seconds or minutes and because other consequences of local blockage may be more significant (i.e. boiling instability or molten fuel/clad sodium thermal interaction).

In summary, the state-of-the-art prevents completion of this assessment of damage potential due to vapor bubble collapse. The uncertainties of this study will be weighed against other possible mechanisms in finally establishing a priority of items for future research.

Recommendations

Analysis must be conducted in the areas of local clad damage thresholds and local energy deposition from collapsing vapor bubbles to provide an estimate of the consequences. This will permit a relative evaluation of this mechanism in comparison to many others.

Experimentally, evidence of vapor bubble collapse damage might be obtained from any test involving sodium boiling. These include the C-2 series, and the ANL, PNL, and ORNL boiling tests. One specific test at ANL, the fission gas blanketing test, could perhaps be modified to inject sodium vapor rather than fission gas into a pin array. This would permit a direct measurement of vapor bubble collapse damage potential and subsequent design oriented studies if damage were found to be significant. Design features such as spacers, fission gas pressure, clad ductility and clad strength could be varied.

ORNL's fuel element failure propagation loop facility will be operated in the boiling mode at the end of each single phase sodium test of the temperature perturbations due to local blockages. These boiling tests might provide answers to the question of local boiling stability and vapor bubble collapse damage potential.

To derive data on bubble collapse phenomenon, the instrumentation should include sufficient thermocouples to monitor the radial and axial temperature distribution as well as clad and channel strain gages, accelerometers and pressure transducers, and post operation examination of heater damage.

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7.2.2 Sodium Hammer

Introduction

A key question identified in the fault tree analysis of failure propagation was the energy content of a shock wave produced by sodium hammer.¹ of chief concern are the mechanical effects of sodium hammer as they affect rapid failure propagation.

If failed fuel or system debris causes a local blockage sufficient to cause boiling, the collapse of vapor bubbles in surrounding sub-cooled sodium may possibly generate pressure pulses capable of damaging adjacent fuel pins and channels. The most direct analogy is to cavitation phenomenon.

If the above blockage degenerates to the point of boiling in the entire channel and a slug flow chugging regime ensues, sodium hammer may occur on vapor condensation and slug reentry into the fuel bundles. The energy potential is sufficiently large to cause concern for the integrity of the adjacent bundles.

7.2.2.1 Discussion of Sodium Hammer

If condensation rates are sufficiently rapid, sodium hammer can potentially generate shock waves capable of damaging channel walls and fuel pins. This is a condition of sudden linear restraint of a column of sodium, analogous to water hammer on sudden valve closure. Like vapor bubble collapse, this phenomenon occurs due to vapor condensation but is associated with the entire fuel bundle. Sodium hammer requires two columns of liquid contacting after the intervening vapor bubble has condensed.

The usual initiating event considered is a loss of coolant flow or partial bundle blockage. Overheating of the entire fuel bundle occurs, followed by sodium boiling. If voiding and subsequent reentry occur in the *slug* flow regions, the *worst* conditions for sodium hammer occur. Sodium hammer probably also occurs if rapid condensation of any high void fraction flow regime occurs. The pressures generated in sodium hammer are a function of the flow regime, void distribution in the channel, geometry of the bundle, and thermal-hydraulic conditions which dictate the evaporation and condensation of sodium. Another complicating factor is the presence of non-condensable gases from fuel pin failures which tend to damp sodium hammer pressures.

The need is to characterize possible pressure pulses generated by sodium hammer and to compare with the damage thresholds of irradiated, possibly overheated and possibly stressed fuel pins and channels.

7.2.2.2 State-of-the-Art Review

The damage potential of sodium hammer phenomenon was pointed out by Peppler, et al.,² noting that reentry velocities of 10-20 m/sec could be attained. The kinetic energy is capable of causing "significant local deformations". Caution was urged that the surroundings must be accounted for in predicting the consequences of sodium hammer. Reference was made to an experimental program aimed at observing these effects.

Grass, et al.³ measured pressures in excess of 650 psi in a single tube sodium boiling test, presumably due to sodium hammer.

Smidt, et al.⁴ notes that due to continued vaporization of sodium, complete bubble collapse and sodium hammer is highly unlikely. Because it represents the fastest failure propagation mechanism, it is given deserved attention.

Peppler and Schultheiss⁵ report pressure pulses to 450 psi lasting for 1 msec on single tube sodium boiling tests. Such pressure pulses were noted both under heated conditions and after heater shutdown when the final vapor bubble condensed. Also noted was the condensation of a vapor bubble between an ejected sodium slug and a reentering slug, indicating sodium hammer by two slugs moving toward one another. This suggests that damaging pressure pulses could be generated without complete collapse of all voids in a boiling channel.

Schlechtendahl⁶ describes the results of a loss of pump accident as predicted with the BLOW-II code. The first two vapor bubbles collapse completely, giving rise to sodium hammer. Although the impact velocity or pressure is not given, the graphical results indicate the velocity is about 18 fps at impact. Schlechtendahl notes that a real system is damped by axial vapor transport which might prohibit complete recondensation as predicted by BLOW-II.

On a complete flow blockage case, the BLOW-II model predicts only one bubble collapse.

The BLOW-II model is a single bubble slug flow model and therefore would not predict multiple slug ejection as observed by Peppler and Schultheiss.⁵

The available reactor safety literature points out the possible danger of sodium hammer, notes factors which reduce the probability of it occurring but offers no analytical or experimental evidence that it won't occur. One is forced to conservatively conclude that sodium hammer is a problem deserving of further attention.

7.2.2.3 Analysis of Sodium Hammer

Sodium hammer is caused by the condensation of sodium vapor in sub-cooled surroundings, which might be a sodium plenum or an unheated part to the fuel assembly. In rapid loss of flow accidents, the blankets, plenum and channel represent heat sinks which lag the temperature transient of the fueled region, permitting rapid vapor condensation.

With a low pressure formed in the bubble, the over-pressure causes liquid acceleration until the pressure of the vapor (or gas) in the bubble is higher than ambient. With small amounts of vapor present, essentially complete collapse can occur, leading to the maximum impact velocities of the on-rushing liquid slugs. The deceleration of the slug occurs as a shock wave, the magnitude of which is dictated by the structural response as well as the liquid compressibility.

The driving pressure causing liquid accelerating into a low pressure bubble is dependent on

1. the reactor system design
2. core pressure drop
3. accident sequence

In the LMFBF, the inlet and exit plenum pressures will generally dictate these driving pressures. The inlet plenum pressure is about 90 psi at full flow. The exit plenum pressure varies from 23 psia to 28 psia between zero and full flow.

Since the accident of principal concern is a single bundle flow blockage at full power, the appropriate pressures are 90 psia at the inlet and 28 psia at the outlet. For small vapor slugs collapsing, the initial liquid slug acceleration is controlled by inertia. For zero bubble pressure the slug acceleration is 2.9 P/L times gravity. For a four foot slug under 28 psi, the acceleration is 20 times gravity. In four inches of travel, a velocity of 21 fps is achieved. At these velocities in a tube bundle, frictional resistance becomes limiting.

Because of this frictional resistance, the top slug of sodium, under ~28 psia driving head, will decelerate as it enters the tube bundles with velocities between 60 and 20 fps being typical.⁷ The velocity on impact of two columns will then depend on the location of bubble collapse. This is most likely to occur near the upper core interface near the original nucleation site. Velocities could be in the neighborhood of 20-30 fps. The relief of the sodium hammer pressure occurs with the time of the pulse to find a free surface or expansions and return. The available expansions are the instrument section and the two plenums. From the upper core interface the instrument expansion is ~4 ft and the plenum is ~6 ft.

If multiple slugs occur, the nearest bubble may also act as a free surface.

The model investigated⁸ for calculating the sodium hammer shock wave velocity and pressure assumes an instantaneous elastic response of the structure to the imposed pressure. The dynamic description of the radial accommodation to sodium hammer pressures is not included. Provided that the duration of the pressure pulse is much longer than the response time of the radial accommodation, the dynamic response should have negligible effect on the resulting pressure. It is seen later that for long slugs of sodium (greater than say 2 ft) this condition is valid, permitting straight forward calculation of the sodium hammer.

The sodium hammer pressure will deform the containing structure until an equilibrium pressure is achieved. For the hexagonal channel, the response to a hydrodynamic pressure as calculated in Section 5.3 provides the needed data.

For this simplified calculation of sodium hammer, the following nomenclature is used:

Symbol	Definition	Example Value
A_p	Cross sectional area of channel	16.6 in ²
D	Pipe diameter with equal area as the channel	4.6 in
d	Pin diameter	.23 in
L	Slug length to a free surface	4 ft
E	Modulus of elasticity, steel	20×10^6 psi
E'	Modulus of elasticity, sodium	6.3×10^5 psi
T	Channel or pipe wall thickness	.140 in.
P	Sodium hammer pressure	psi
t	Clad thickness	.015 in.
V_w	Sodium hammer propagation velocity	
V	Sodium impact velocity	
ΔV_p	Channel volume change per unit length under pressure P	
ΔV_f	Sodium volume change per unit length under pressure P	
ρ	Sodium density	50 lbm/ft ³
λ	Period of channel wall	0.21 msec
τ	Time for wave propagation to free surface and return	
δ	Maximum channel deflection	
ϵ	Maximum channel strain	

The basic equation of sodium hammer for propagation velocity is:

$$V_w = \frac{\sqrt{\frac{E'}{\rho}}}{\sqrt{1 + \frac{\Delta V_p}{\Delta V_f}}}$$

where the numerator is sonic velocity in undisturbed sodium, and the denominator is the reduction due to radial accommodation of the strain.

The resulting sodium hammer pressure is:

$$P = \frac{p V_w V}{144 g_c}$$

The pressure pulse duration is:

$$\tau = \frac{2L}{V_w}$$

The sodium volume change is:

$$\Delta V_f = A_p \frac{P}{E'} \quad \text{per unit length}$$

The elastic response of a circular pipe of the same area as the channel is:

$$\Delta V_p = \frac{\pi}{4} \frac{D^3 P}{TE}$$

Combined with the propagation velocity, the last two result in the classical water hammer equation

$$V_w = \frac{\sqrt{\frac{E'}{p}}}{\sqrt{1 + \frac{E' D}{ET}}}$$

The results of the calculations for sodium hammer are shown in Figures 7-12, 7-13, and 7-14 and described below. The FFTF driver fuel bundle geometry is assumed for these calculations.

Case Descriptions		
Case #	Description	τ , msec (L = 4 ft.)
1)	Rigid boundary	1.05
2)	Empty pipe	1.49
3)	Pipe with tube bundle	2.14
4)	Empty hex channel	<8.7
5)	Hex channel w Tube bundle	<13.8
6)	Hex channel w Tube bundle and adjacent wall effect	<9.8

1. A rigid boundary results in a propagation velocity equal to the velocity of sound in sodium or ~7600 fps. The resulting pressure is:

$$P = 82.6 V \quad [\text{psi}]$$

2. For a circular pipe of the same flow area and wall thickness as the channel, the propagation velocity is 5360 fps and resulting pressure is:

$$P = 57.8 V \quad [\text{psi}]$$

The pipe volume change is 1.035 times the fluid compression in this case.

3. For a pipe with a tube bundle, the flow area reduction due to the presence of the tubes reduces the energy of the sodium hammer proportionately. In addition, the tubes help accommodate the pressure with a compressive strain. The volume change of the tubes is only 71% of that of the fluid itself.

The resulting propagation velocity is 3740 fps and pressure is:

$$P = 40.3 V \quad [\text{psi}]$$

4. The response of a hexagonal channel to sodium hammer pressures is extremely non-linear, but the results of the channel deformation analysis, Section 5.3, permit evaluation of the channel response. The channel volume change due to an imposed internal pressure permits calculation of the impact velocity, propagation velocity and pressure relationship. This requires the assumption of an instantaneous deformation of the channel to an equilibrium position at which the channel can sustain the required pressure loading. This assumption should be reasonable if the characteristic deformation period of the channel is much less than the period of the imposed pressure pulse. Figure 7-13 shows the duration of the pulse (for a four foot column) to be in the range of 5-15 msec. This is sufficiently long compared to the channel period of 0.21 msec to give confidence in the assumption as a first approximation.

The channel deformation shows a plastic region where continued deformation occurs at constant pressure. This causes the "flat" pressure vs impact velocity response of Figure 7-12. The sodium hammer results for hexagonal channel in air are shown in Figures 7-12, 7-13, and 7-14.

Figure 7-12 Sodium Hammer Pressures

Figure 7-13 Pulse Propagation Velocity and Duration

Figure 7-14 Channel Deflection and Strain

5. Within the tube bundle the fuel pins reduce the flow area and the resultant sodium hammer. In addition, the frictional resistance to flow prohibits a high impact velocity.
6. As a first estimate to include the influence of the surroundings on the channel response, it was assumed that two equal channel walls must deflect to accommodate a sodium hammer pulse. The deformation is thus characteristic of one-half the imposed pressure. This gives a much stiffer containment to the sodium hammer, with higher pressures but less strain than the single channel wall response. First results of the deformation of a channel due to an external load indicate a much more flexible or soft response than to an internal load. This indicates the correct response to sodium hammer in a multi-channel array is probably between cases 5 and 6. For the expected sodium hammer velocities up to 30 fps, the cross-hatched area in Figures 7-12 and 7-14 indicate the range of probable response.

Energy Balance Comparison

The preceding analysis was based on the volume accommodation of sodium hammer compression. Figure 7-15 shows the comparison of the specific kinetic energy of the sodium column compared to the calculated strain energy absorbed by the channel. For cases 5 and 6, the strain energy is between 60 and 80% higher than the imposed fluid kinetic energy. An energy balance is thus not satisfied for this calculation of sodium hammer response.

The energy transferred to fluid strain and to the fuel pins are each less than 1% of the imposed kinetic energy and thus are of no consequence.

This energy comparison shows the results of Figures 7-12 to 7-14 to be very conservative in the estimates of channel strain and deflection. This also suggests that a two dimensional dynamic calculation might be necessary to gain a more accurate estimate of sodium hammer effects. It appears likely that the response is not axially uniform, with the largest channel deflections at the point of impact and decreasing as the wave progresses away. These same considerations suggest that the classical pattern of a pressure wave traveling to the inlet and reflecting as a rarefaction wave may be modified by the "plastic" response of the channel.

The energy of the sodium hammer is almost entirely dissipated in plastic strain energy in the channel. The returning rarefaction wave causes an acceleration of the sodium towards the inlet but at much reduced velocity. The rarefaction wave velocity is thus likely to be near acoustic velocity. For example, case 5 at an impact velocity of 26 fps produces a pressure of 191 psi and a pulse propagation velocity of 670 fps. On rebound at the channel inlet, the rarefaction wave, if traveling at sonic velocity (7640 fps) produces a reverse sodium velocity of 2.3 fps, more than a factor of ten reduction from a purely elastic case. This probable response also means the pressure pulse duration is less than calculated, again indicating the conservatism of the calculated results.

Considerations of the Channel Surroundings

A basic assumption of the preceding was that the channels are in air, eliminating consideration of the channel surroundings. The response in a sodium bath may be dynamically different than in air, further complicating analysis of sodium hammer response. As the pressure due to sodium hammer is imposed inside a channel, the channel wall accelerates outward and is restrained by the compressibility of the surrounding sodium. A pressure wave is thus generated in the sodium, and propagated radially outward.

Assuming this pulse to propagate at the same velocity is the axial sodium hammer pulse, the following table shows the time for the sodium compression to accommodate the required volume expansion of the channel.

Case 5:			
Impact Velocity	Pressure	Channel Deflection	Characteristic Time
fps	psi	mils	msec
16.1	191	.01	.05
26.4	191	.05	.16
45.2	284	.10	.18

Figure 7-15 Comparison of Sodium Column Kinetic Energy and Channel Strain Energy

These times for accommodation are less than the channel period ($\sim .21$ msec) and much less than the duration of the pressure pulse. This comparison indicates the calculated one dimensional response in air may be tolerable as a first estimate.

A rigorous prediction of the propagation and reflection characteristics of the sodium hammer pressure pulse in an array of hexagonal channels appears beyond the state of the art.

7.2.2.4 Sodium Hammer – Radial Loads on Pins

Assuming slug flow of two contacting columns of sodium in a tube bundle, the water hammer analogy showed that the channel wall could not sustain a high pressure but would yield until an equilibrium pressure was achieved. This requires almost all the energy of the column of sodium to be transferred to the channel wall. Deformations are on the order of 50 mils at mid-flat of the wall. This necessarily presumes rapid radial flow of sodium to the available volume, i.e., the energy of compression in the sodium is relieved by wall movement. Sodium in the tube bundle center must move laterally toward the edge of the bundle and toward the center of the flat. This necessarily requires the fuel pins to move or for sodium to flow around tubes. In idealized form, a shock wave moves up the channel. Above the shock is undisturbed geometry. Below is a channel wall which has been deformed outward ~ 50 mils at maximum. This requires a shearing action of the fuel pins at the shock wave, unless the sodium flows around the pins.

The possibility exists for this shearing action to deposit energy in the pins, and possibly fail them in some manner. Severe radial pressure gradients could result within the tube bundle and not really affect the channel wall deformation.

Also, if radial impedance is high, the channel corners may be subjected to higher pressures. There could then be extremely complex radial compression waves behind the axial shock wave. The shape of the axial shock could conceivably be non-planar, perhaps like a Mach cone, with an associated pressure profile, causing the radial movement of sodium to the walls.

Analysis of Radial Effect of Sodium Hammer

The radial effect of sodium hammer involves the pressure gradient due to sodium flowing across the fuel pins. The key uncertainty is the length of the interaction zone (shock pulse), L_s . It is reasoned that this length will be controlled by the channel wall deflection period.

Description of Model

Assume uniform velocity and deflection toward wall.

The time for radial movement is $t_s = \frac{L_s}{V_w}$. Also, t_s is the time for deflection of wall (characteristic period of wall)–

Area for movement – $S \times (1) \text{ in.}^2/\text{in.}$

Volume accommodation – $\Delta V_p \text{ in.}^3/\text{in.}$

$$\begin{aligned}\Delta V_p &= 6A V_c t_s \\ &= 6S \times (1) V_c \frac{L_s}{V_w}\end{aligned}$$

Therefore the radial velocity is:

$$V_c = \frac{\Delta V_p V_w}{6S L_s}$$

The radial velocity at the channel wall is then

$$V_c = \frac{\Delta V_p}{6St_s} \frac{(1000)}{(12)} \text{ [fps]}$$

Note that the peak radial velocity could be imagined to be twice as high at center of the flats because of the two dimensional flow patterns.

For the case 5 above, a hex channel tube bundle with a flat dimension of $S = 2.35$ inches, Table 7-1 shows the results of sodium impact on the average radial velocity at the channel wall.

Table 7-1. Radial Velocity Due to Sodium Hammer

Sodium Hammer Impact Velocity, V_{impact} tps	Sodium Hammer Pressure P psi	Channel Volume Accommodation $\text{in.}^3/\text{in.}$	Channel Natural Period t_s msec	Shock Width L_s inches	Radial Velocity At Wall V_c fps
3.93	90	.005	.052	1.32	0.57
16.1	191	.065	.080	1.05	4.83
26.4	191	.252	.202	1.62	7.4
45.2	284	.504	.235	1.64	12.7

This indicates that the velocities are reasonable for the assumed shock width based on the channel natural period. The pressure drop across the pins can be evaluated by standard cross-flow correlations as follows:

Pressure drop — cross flow over pins from Kreith⁹.

$$\Delta P = \frac{f' G_{\max}^2 N}{\rho (2.09 \times 10^8)} \left(\frac{\mu_s}{\mu_b} \right)^{0.14}$$

$$f' = \left[0.25 + \frac{0.118}{\left(\frac{S_T - 1}{D_o} \right)^{1.08}} \right] \left(\frac{G_{\max} D_o}{\mu_b} \right)^{-0.16}$$

Where:

S_T = half pitch

$G_{\max}$ = mass flow at minimum area

N = number of transverse rows

For Na @ 1300°F, μ = .05 lbm/hr ft
 ρ = 50 lbm/ft³

and assuming: D_O = 0.23 inch

S_T = 0.1425 inch

Assuming the fuel pins are stationary, Table 7-2 shows the resulting pressure drop across the outer row of fuel pins (the highest gradient).

Table 7-2. Pressure Drop in Cross Flow

Na Hammer Impact Velocity V_{impact} fps	Maximum Velocity Between Pins $V_{\max}$ fps	Mass Flow Rate lbm/sec-ft ²	Reynolds Number Re	Friction Factor f'	Pressure Drop, ΔP psi	Force on Pin $F = \Delta P L_s D$ lbf
3.93	2.95	147	203,000	.0312	—	—
16.1	25	1250	1,730,000	.0222	0.21	0.08
26.4	38.4	1920	2,660,000	.0218	0.35	0.13
45.2	66	3300	4,560,000	.020	0.95	0.36

The cumulative pressure drop radially is shown in Figure 7-16 for a large maximum flow rate to illustrate the pressure profile necessary to cause the radial flow of sodium.

The pressure gradient across the outer row of pins is noted to be very low, less than 1 psi. The conclusion is reached that the radial pressure profiles are negligible compared to the axial pressure change across the shock front, supporting the conclusion that the one dimensional analysis is probably adequate.

The low radial velocities and particularly the pressure gradients suggest that radial impedance will not significantly modify the previous channel deformation results. The fuel pins should not deflect significantly under these loads and will be relatively undisturbed.

Of greater significance perhaps is the effect of the radial kinetic energy contained in the fluid adjacent to the wall. In conjunction with the sodium outside the wall, the additional momentum may force the channel beyond its statically stable point, with a resulting lowering of the pressure and an elastic (or plastic) rebound. This could be a mechanism whereby the gross radial deformation results in higher conversion of axial energy sources into radial channel strain than a simplified water hammer analysis might suggest. The average radial velocities within the shock wave are the same order of magnitude as the sodium impact velocities. This effect has not been analyzed and no experimental data are known to be applicable to this area.

Figure 7-16 Radial Pressure Gradient Within Shock Pulse

7.2.2.5 Hydraulic Forces on Reentry

Another postulated damage mechanism is due to the hydraulic forces of sodium reentering a voided fuel bundle. Voiding might be due to sodium boiling at high superheat or a large gas bubble. Damage mechanisms are: the impact of sodium column on either end of the tube bundle; and vibration and pressure gradients due to excessively high sodium velocities.

Consideration of the *maximum* velocities attainable in a fuel bundle lead to the following:

- for:
- inlet plenum pressure of 100 psia
 - exit plenum pressure of 23 psia
 - inlet and exit loss coefficients of 1.0 (fueled region velocity heads)
 - core sodium fraction of 0.5

The maximum velocities are ~70 fps in the inlet region (corresponding to ~140 fps in the tube bundle) and 30 fps in the exit region above the tube bundle (corresponding to ~60 fps in the tube bundle).

It appears intuitively unlikely that any real fuel bundle will have an orifice loss coefficient of less than 1.0.

The pressure on the end of the tube bundle when the column impacts will be somewhat less than the steady state acceleration pressure drop because of compressibility effects but the error is less than 6% at 60 fps entrance velocity and will be ignored (a conservative assumption).

The resulting pressure on the tube bundle end is ~20 psi @ 30 fps and 108 psi @ 70 fps. This produces a force on a fuel pin of from 1 to 5.4 lbf, and a compressive stress of from 90 to 480 psi. This stress level should be negligible. The pin could elastically buckle under this sudden load, but it requires a length of 55 inches at a force of 5.4 lbf. With an average unsupported length of 6 inches, buckling seems unlikely.

The second damage mode, due to vibration or adverse pressure gradients, has not been evaluated but intuitively will not be significant since the liquid column decelerates rapidly on entering the tube bundle and vibration typically causes a fretting at spacer supports which requires sustained operation for substantial damage.

7.2.2.6 Conclusions & Recommendations

1. The maximum channel strain imposed due to a calculation of slug flow sodium hammer at 30 fps should lie between 2.5 and 5.5%. With the large uncertainties in irradiated steel ductility, particularly during an overtemperature transient and at high strain rates, it must be concluded that strains in this range might induce channel failures and possibly fuel pin failures in adjacent channels. Future analysis must consider the simultaneous response of the surrounding channels and their tube bundles to sodium hammer in a fuel assembly.

Dynamic solutions, particularly in the radial direction, are necessary, not only to determine the magnitude of pressures due to sodium hammer and fuel, clad, sodium thermal interaction but to simultaneously determine the response of adjacent channels and fuel pins.

2. For gross core accidents, the radial accommodation should be negligible (assuming radial *coherence* of pressure pulses), perhaps permitting calculation with axial effects only.
3. The thermal-hydraulics of sodium boiling must be understood to determine if and when a sodium hammer or violent boiling situation might develop. It appears intuitively that the thermal-hydraulics will prevent any gross sodium hammer from developing, but this has not been demonstrated analytically or experimentally.

7.2.2.7 Test Recommendations

1. Sodium boiling test programs must make every effort to simulate the reactor environment and geometry to establish credibility in predicting sodium boiling consequences like sodium hammer. Pressure instrumentation must be capable of detecting the magnitude and duration of pressure pulses produced by bubble collapse and sodium hammer. Since intervening two-phase conditions may muffle these pressures, many transducers at various elevations appear necessary.

For multi-pin boiling tests, the channel should be simulated, allowing a soft response to sodium hammer.

2. An out-of-pile test of channel containment capabilities for sodium hammer is recommended, using water hammer techniques. Reentry sodium hammer can be simulated with an evacuated channel and a rupture diaphragm in a water test, see figure 7-17. The fuel pins and surrounding fuel bundles can be simulated in a water tank to represent the core. Variables to consider include:

Design Variables:	Channel wall thickness Hexagon corner radius Material ductility (cold worked or annealed)
Test Variables:	Pressure differences across slug Gas content (pressure) of evacuated space to simulate vapor or fission gas Length for accelerating the slug Axial location of lower liquid level (either in tube bundle or above) Channel surroundings <ul style="list-style-type: none"> — air — water — empty channels in water — fuel bundles in water
Measurements:	Pressure taps on primary and surrounding bundles Accelerometers and strain gages on channels and pins Post test measurements for residual strains.

This test may indicate the containment capabilities for molten fuel sodium interaction pressures as well.

Figure 7-17 Test Layout for Water Hammer Effect on Core

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7.3 MOLTEN FUEL/CLAD-SODIUM THERMAL INTERACTIONS

7.3.1 Introduction

The fault tree analysis suggests several ways in which hot particles of fuel and cladding might be brought into contact with sodium in the event of a failure in one assembly. Similar mechanisms can be postulated for hypothetical whole-core (DBA) incidents. The temperature difference between hot particles and coolant would be extreme. Molten UO_2 may be in the 6000°F to 7000°F range, and coolant would initially be below its boiling point (<1600°F).

Since expansion of the coolant would initially be constrained by its acoustic properties (compressibility), its inertia, and the rigidity of the containing structure (e.g., channel walls), much of the energy thus transferred would generate within the coolant a very high pressure. Eventually, the pressure would overcome the constraining forces, and expansion of the coolant would transmit energy to the containing structures. This expansion can, under some conditions, occur in a dramatic, explosive way. Since it may also be accompanied by vaporization of some or all of the affected coolant, the term "violent boiling" is also often used to describe the thermal interaction process.

Violent boiling is of concern as a potential mechanism for damage propagation and for its contribution to the total mechanical work of a DBA. In the former (single assembly failure) context, it may be initiated by:

1. Expulsion of fuel and/or clad from a failed pin into the coolant stream, or
2. re-entry of sodium into a previously voided flow channel where pin failure has since occurred.

These initiating events lead to different geometric concepts of the fuel/clad-coolant interaction process. The first case suggests a "volume mixing" concept — hot particles are dispersed throughout a given coolant volume. The initial contact may be followed by fragmentation of particles caused by internal thermal stresses or by the shock of bubble collapse.

Several alternatives must be considered in the case of coolant reentry. Overheating in the voided channel could cause pin failure and, similar to the first case, mixing of fuel/clad particles with reentering sodium. If severe pin failure causes molten fuel and metal to collect at the bottom of the voided channel, then a horizontal contact surface would exist — at least momentarily — between reentering sodium and molten fuel/clad material. Still a third concept fits the case of reentering sodium contacting overheated, but still intact pins. In this instance, a vertical contact surface is established, and the fluid response is characterized by a plane pressure wave which moves toward the channel wall. Since no fuel is involved, this is more appropriately termed a clad-coolant interaction.

The initiating events described involve just two basic concepts of the fuel/clad-coolant interaction process in a single-assembly failure:

1. Mixing of overheated fuel/clad particles in a given coolant volume,
2. Surface contact between a body of overheated fuel/clad material and another body of coolant.

Innumerable specific geometric arrangements can be postulated, but the simplified view of the process covered in these two concepts is reasonably representative of all of them.

The same concepts apply in DBA analysis. In fact, one hypothetical event leading to a DBA is the rapid propagation of failure starting in one or more assemblies. In this sense, there is no real distinction between the "damage propagation" situation and the DBA situation. In any case, the contribution of fuel/clad-sodium thermal interactions to the total work energy of the DBA is of interest. Two cases must be considered:

1. The expansion and vaporization of core materials in the absence of sodium (the "sodium out" condition). The usual approach to DBA analysis is to estimate the work energy of isentropic expansion of the core to a threshold pressure (ambient or higher). However, substantial residual thermal energy remains in the expanded core. Eventually, core fragments may contact sodium, and the resulting interaction may contribute significantly to the total work energy.
2. Combined core expansion and core-sodium thermal interaction ("sodium in" condition).

Analysis of a typical hypothetical DBA would probably necessitate consideration of both conditions, since both could exist during disassembly and expansion of the core.

The focus of this study is on two specific questions derived from the fault tree analysis. These are:

1. How much energy is contained in the shock wave produced when several pins burst and the hot fuel contacts the coolant?

2. How much energy will be contained in the shock wave produced when the coolant contacts molten clad or molten fuel upon reentry after channel voiding has occurred?

These questions define immediate and specific information needs, and have guided the evaluation of experimental and analytical programs related to the thermal interaction phenomenon. The attempt to answer them leads very quickly to consideration of the more basic concepts discussed above (i.e., interaction of fuel/clad particles mixed with sodium and interaction of fuel or clad material in plane surface contact with sodium).

In this section, the state-of-the-art in experimental and analytical work on fuel-coolant interactions is reviewed. Recommendations are made for developing the information needed, and the relationships of existing programs to these recommendations are discussed. The experimental work reviewed includes that of GE, ANL and TRW. Analytical approaches covered are the Hicks-Menzies concept, analytical studies made by GE, and the analytical modeling work of ANL.

Among the earliest attempts to predict analytically the damage potential associated with thermal interaction is the work of Hicks and Menzies reported in ANL-7120.¹ The idea was originally introduced there as a result of the then current approach to DBA analysis which involved estimating the work potential due to expansion of heated core materials to ambient conditions. The latter was considered to be a conservative estimate of the damage potential in a DBA situation. However, it left unanswered the question of what happens to residual energy in the core materials after their expansion to ambient pressure. Conceivably the force of the first burst could disperse these hot materials into the coolant and lead to further, and perhaps more violent, interactions. This concern led to the development by Hicks and Menzies of the "classical" thermodynamic equilibrium approach to thermal interaction modeling. This is discussed further in Section 7.3.4.

A less conservative and more realistic model of the interaction process was developed by GE.² In this case, account was taken of the finite rate of heat transfer from molten fuel or clad materials to sodium and also credit was taken for the fact that the exchange of energy may terminate before the fuel and sodium reach thermal equilibrium. This led to estimates of the damage potential which were considerably lower than those of Hicks-Menzies but were at the same time highly conservative.

The slab geometry model published in GEAP-10059² and subsequent refinements of it both in the conversion to spherical geometry and in the degree of conservatism inherent in the thermodynamic modeling are described below in Section 7.3.4.

More recent modeling efforts have been based on hydrodynamic models as opposed to thermodynamic limit approaches. These models comprise essentially solutions of the linearized Navier-Stokes equations (i.e., the wave equation). A relatively simple wave propagation model in slab geometry coupled with a transient heat conduction model was developed at GE and used to verify some of the assumptions involved in the earlier thermodynamic modeling. In addition, ANL is now working on a hydrodynamic model based on bulk heating of a specified volume of coolant into which fuel particles are dispersed and which is constrained in its expansion by the surrounding volume of unheated coolant.

Table 7-3 provides a qualitative comparison of the work energy levels predicted by these various modeling schemes and experiments and the associated "efficiencies" predicted for the conversion of thermal to mechanical energy. These estimates cover a range of five decades in magnitude.

The table represents the present state of knowledge regarding the damage potential of fuel-sodium thermal interactions. It is well known that the Hicks-Menzies thermodynamic equilibrium model gives a highly conservative work energy estimate. At the same time, it is possible that the GE spherical geometry thermodynamic analysis (at the other end of the range) gives results that are on the low side because it does not treat interactions between fuel particles. With increasing knowledge gained from experimental work and more recent analytical modelling efforts, the trend has been toward acceptance of work energy levels on the low end of this scale. The ANL S-series tests provide strong evidence that the "efficiency" of thermal to mechanical energy conversion in a prototypical fuel assembly would be on the order of 10^{-5} (10^{-3} %).

A systematic evaluation of each of the data points of Table 7-3 leads to a probability distribution on the thermal to mechanical energy conversion efficiency of fuel sodium thermal interaction. Taken in conjunction with the structural response studies described in Section 5.0, this in turn provides probability estimates for the occurrence of

specific undesired events referred to in the fault tree analysis. These include: fuel rod failure due to shock loading; channel failure due to internal shock loading; and channel failure due to external shock loading. Estimates of probabilities of occurrences of these events are given in Table 7-4. The procedure leading to these estimates is described below.

The procedure used to develop a probability distribution curve from the data summarized in Table 7-3 is to consider individually the various estimates of thermal to mechanical energy conversion efficiency and arrive at an estimate of the probability that the specific value of conversion efficiency might be exceeded in actuality. The aggregate of these probability estimates then forms a complimentary cumulative probability distribution on conversion efficiency. That is, the probability that conversion efficiency will exceed a given value is plotted as a function of that value. The assignment of a probability to each of the calculated energy conversion values of Table 7-3 is made from a consideration of the analytical model used in the calculation, including the fundamental assumptions and the level of sophistication of the model.

The *relative* degree of certainty associated with each of these analytical predictions is more readily established than are absolute probability levels. However, two of the tabulated values serve to establish extreme ends of the distribution curve. These are discussed below.

1. Hicks-Menzies Analysis

The Hicks-Menzies model provides a thermodynamic limit on energy conversion based on ideal fluid expansion and thermal equilibrium heat transfer. With allowances for the uncertainties of initial conditions, material properties and numerical approximations (e.g., roundoff errors), the possibility of exceeding this ideal limit (18%) is not zero, but is nearly so. A 1% probability is given to that possibility.

2. G.E. Analysis – Spherical Geometry, More Realistic Thermodynamic Model

No attempt was made in this analysis to derive a conservative result. The work energy calculated includes only that of liquid expansion. In addition, heat transfer is assumed to be terminated in a very short time (on the order of microseconds) due to coolant vaporization. When the interactions of many fuel particles are considered, the

Table 7-3. Summary of Work-Energy and Thermal-Mechanical Conversion Estimates

Source of Data	Work in-lb/in ²	Energy Joules/gm UO ₂	Conversion ⁽¹⁾ %
Hicks-Menzies Analysis ¹	—	306	18
GE Analysis — GEAP-10059 ²	13	35 ⁽²⁾	2.1
GE Analysis — Spherical Geometry Conservative Thermodynamic Limit ⁽³⁾			
(a) Particle Radius = 0.0016 in.	4.7	6.3	0.38
(b) Particle Radius = 0.010 in.	8.1	1.8	0.11
GE Analysis — Spherical Geometry More Realistic Thermodynamic Model ⁽³⁾			
(a) Particle Radius = 0.0016 in.	0.0037	0.0051	0.00031
(b) Particle Radius = 0.010 in.	0.0065	0.0014	0.000086
GE Analysis — Plane Geometry Hydrodynamic Solution			
(a) Particle Radius = 0.0016 in.	0.13	0.18 ⁽⁴⁾	0.010
(b) Particle Radius = 0.010 in.	0.13	0.028 ⁽⁴⁾	0.0016
ANL Analysis — Linear Constraint Hydrodynamic Solution ⁽⁵⁾	—	19	1.2
ANL Experiment, S-Series TREAT Piston Autoclave ⁴			
(a) S2	—	0.012	0.0011
(b) S3	—	0.014	0.00070
(c) S4	—	0.042	0.0014

Notes:

(1) Conversion factor is defined as (work energy) ÷ (total energy in fuel), where total energy in fuel = $(C_p)_{\text{fuel}} \cdot T_{\text{fuel}}$.

(2) Based on fuel particle radius of 0.0016 in., and 13 in-lb/in² of particle surface.

(3) "Conservative limit is based on the thermodynamic model used in GEAP-10059. "More realistic" model uses constant volume heating followed by liquid expansion, so work is only $\int P dv$.

(4) Based on the fuel particle radius shown, and 0.13 in-lb/in² of particle surface.

(5) Calculated from data published in reference 4.

Table 7-4. Summary of Damage Thresholds and Their Probabilities of Occurrence

Event	Shock Energy Required for Event to Occur*	Probability that Shock Energy Might Exceed Thresholds**
1. Pin Failure Due to Shock Load	0.25 in-lb/in ²	.80
2. Channel Failure Due to <i>Internal</i> Shock Load	50 in-lb/in ²	.04
3. Channel Failure Due to <i>External</i> Shock Load	25 in-lb/in ²	.05

duration of heat transfer may be considerably longer. It is, therefore, believed that the energy conversion efficiencies of $3.1 \times 10^{-4}\%$ and $8.6 \times 10^{-5}\%$ resulting from this analysis will be exceeded with probability 99%.

The only experimental data included in Table 7-3 are the results of the S-2, S-3, and S-4 tests of ANL. The resultant energy conversion efficiencies are approximately 10^{-5} . Evaluation of an uncertainty for this figure encompasses consideration of the degree of prototypicality associated with the experimental design and accuracy of experimental measurements and of the data reduction analysis. These factors are discussed in some detail in Section 7.3.4. Consideration of them yields no clear reason to believe that the efficiency result of 10^{-5} is either too high or too low. Therefore, a probability of 50% is assigned to this single experimental data point. This establishes a third point on the probability distribution curve.

The three points just determined define the general shape of the curve. Probability assignments are made for each of the remaining analytically determined values of Table 7-3 by considering the credibility of the analysis relative to the first three points. The resulting probability values are discussed below, and the probability distribution is plotted in Figure 7-18.

G.E. Analysis – GEAP-10059

The analysis published in GEAP-10059² describes a conservative bound on the work energy of thermal interaction. However, a somewhat arbitrary limitation is imposed on the duration of heat transfer from fuel to sodium. Thus, there is some possibility that this "upper limit" could be exceeded. This occurrence is assigned a probability of 2%.

G.E. Analysis – Spherical Geometry Conservative Thermodynamic Limit

In this extension of the work published in GEAP-10059² the conservative limit estimate was made for the case of spherical geometry, considering a single fuel particle. The calculated work energy includes only that of liquid expansion. The likelihood that the calculated value might be exceeded is a little higher than the probability associated with the previous slab geometry analysis. Probabilities of 5% and 10%, respectively, were assigned to the two values of .38% and .11% associated with two different particle radii. This reflects the fact that the smaller sized particle (0.0016 inches) is less likely to occur.^{1 6}

G.E. Analysis – Plane Geometry Hydrodynamic Solution

While this analysis involves more realistic modeling of the fluid response, it is also subject to the same assumptions and conditions as the thermodynamic analyses referred to above (liquid expansion only, heat transfer cutoff in $< 1 \mu\text{-sec}$). The "work energy" estimate includes both compression energy and fluid kinetic energy and may be considered relatively more conservative in that sense. Probabilities of 20% and 35% are assigned to the efficiencies of .010% and .0016% (for .0016 and .010 inches, respectively).

*These estimates are based on the analytical results given in Section 5.0.

**Based on propagation of a *Single Pin Failure* (see text).

Figure 7-18 Thermal Interaction Energy Conversion Efficiency Probability Distribution

ANL Analysis – Linear Constraint Hydrodynamic Solution

The ANL model concept is considered to be realistic. However, the data published were developed with a version of the model which assumes a constant heat transfer rate. Since the latter would actually tend to decrease with time (as the thermal potential decreases), the assumption may be conservative. The probability that conversion efficiency of 1.2% would be exceeded is, therefore, taken as 4%.

It has been estimated (see Section 5.0) that significant pin damage might result from the absorption of 0.05 inch-lb of shock energy per in² of pin surface. Other significant damage thresholds are: ~ 5 in-lb/in² for channel failure due to external loading and ~ 10 in-lb/in² for channel failure due to internal loading. These studies also show that the absorbed energy amounts to about 20% of the incident energy, thus the minimum shock wave energy for each type of damage can be estimated. The likelihood of achieving these levels is related to energy conversion efficiency, energy density of the fuel material and the amount of fuel interacting with sodium. The energy contained in molten fuel at 6000°F is about 1700 joules/gm. Instantaneous failure of an entire fuel pin in a typical fuel assembly would release about 0.4 gms of fuel per square inch of channel surface.* Thus, the total *thermal* energy would be 6000 in-lb/in² of channel surface. The failure level (50 in-lb/in², from Table 7-4 for internal shock loading of the channel would be reached provided the energy conversion efficiency is 0.85%. Thus, it is concluded (from Figure 7-18) that channel failure due to *internal* shock loading from fuel-sodium thermal interaction might occur with 4% probability if *one pin fails*.

*Based on the following parameters:

Hexagonal channel width: 4.3" across flats, inside dim.
Pin outside diameter: 0.25"
Pellet outside diameter: 0.22"
Smear Density: 85% of t.d.

The probability of failure propagation from a single pin to adjacent pins is determined in a similar manner. With the assumption that the shock load due to single pin failure is equally distributed over the adjacent pins, the probability is 65%. If all of the load appears on *one* pin, the probability for failure is 95%. The latter assumption is very conservative, and even the former assumption (distribution of the load over adjacent pins) may be on the conservative side. It seems reasonable, in the light of these considerations, to establish the probability for pin-to-pin damage propagation at 80%.

The analysis of Section 5.0 suggests a failure threshold for external loading of a channel at 25 in-lb/in². Utilizing the previously described analysis, the resulting probability is 5%. In order for this event to occur, the shock must be transmitted from an adjacent assembly. It is theoretically possible for this to occur even without failure of the first assembly. In any case, no account has been taken here of the efficiency of transmission from one assembly to another. This would diminish the likelihood of failure due to external loading.

The conclusion to be drawn from the foregoing analysis is that the shock pulse due to failure of a single pin is unlikely to cause failure of the channel, but is likely to damage adjacent pins. This may lead to the generation of further shocks, and their cumulative effect on the channel is not known. Should very many secondary pin failures occur at the same time, then channel failure would become more probable. Better understanding of the mechanisms of pin-to-pin damage propagation (e.g., the rate of fuel fragment mixing with liquid coolant) is needed in order to establish with reasonable confidence the probability of assembly-to-assembly damage propagation under all conditions.

Another level of uncertainty is added by the estimates of failure thresholds. These are based on impulse loading by very narrow (~ 1 μ -sec width) pressure pulses (see Section 5.0). Without further knowledge of the actual character of shock pulses created by fuel-sodium interaction, it is impossible to know the degree of conservatism behind the failure values.

7.3.2 Conclusions and Recommendations

7.3.2.1 Information Needs

The broad objectives of fuel-coolant interaction studies are:

1. To establish realistic levels of thermal to work energy conversion efficiency in prototypical configurations.
2. To provide such analytical tools as are necessary in order to design for the minimization of failure propagation due to the fuel-coolant interaction mechanism.
3. To provide analytical tools needed to accomplish DBA analysis as a basis for plant licensing.

The aggregate of currently available experimental and analytical data leaves some uncertainty as to whether or not the thermal interaction mechanism is significant in either the damage propagation or DBA context. Analytical estimates range from the admittedly conservative values of 10% associated with Hicks-Menzies analysis to efficiencies of 10^{-5} or less associated with other thermodynamic models based on liquid expansion. A similar range is suggested by experimental results. Laboratory tests such as those of TRW employing shock tubes show that large energy releases result from thermal interaction. More prototypical tests, most notably the S-Series tests of ANL, give stronger evidence of much lower efficiencies on the order of 10^{-5} . If the latter figure is typical then it is not likely that fuel-coolant thermal interactions are of significance in assembly to assembly damage propagation or in DBA situations.

The primary need at the present time is to establish under prototypical conditions the range of pressure generation and work energy which can be expected in an operating LMFBR. In order to provide data which can be extrapolated by the reactor designer, the influences of the following must be determined:

- effects of burnup (this includes the effect of burnup on clad and channel wall ductility, the effect of fission gases in mitigating the pressure pulses generated during thermal interaction, and others),
- the effect of continued coolant flow during the failure of pins and subsequent expulsion of fuel — particularly the effect of channel voiding with expulsion of sodium into the plenum,
- the effect of fuel pin length,
- the effect of the number of pins,
- the effect of axial restraint of fuel expansion.

There is also a need to develop better understanding of the basic mechanisms involved in the thermal interaction

process. However, this should not be interpreted as a primary information goal. The need for this type of information is largely dependent on the outcome of an initial testing and analytical program aimed at determining realistic pressure generation and work energy data in prototypical situations. The important information needs in this area are:

- fragmentation data (including the rate of fragmentation of over-heated fuel and clad particles and the extent of fragmentation, i.e., ultimate particle size distributions),
- heat transfer rates,
- rate of mixing of particles with sodium and extent of mixing,
- understanding of phenomena which limit the duration of the interaction (e.g., blanketing of heat transfer surfaces by non-condensable gas or sodium vapor),
- the effect of non-condensable gas in cushioning pressure generation.

The need for sophisticated modeling techniques and extensive laboratory experimentation to develop data of and understanding of basic mechanisms is directly related to the magnitude of the thermal interaction problem. Clearly if the results of the ANL S-Series tests are prototypical, the only need is to establish enough basic information to correlate further integral test results and to allow extrapolation of them for the purpose of reactor design and DBA analysis. This does not imply a need for full understanding of such things as fragmentation and heat transfer rate. The pursuit of such detailed information should be carried out only after a realistic perspective has been gained by the acquisition of information based on integral type tests.

7.3.2.2 Recommended Programs

The immediate emphasis should be placed on a program involving a series of increasingly prototypical in-pile loss-of-flow tests. This program should be accompanied by an analytical program which includes sufficient model development and analysis to support the conclusions of the test program. It is important to realize that an analytical program of this type does not require thorough understanding of basic mechanisms. It is possible to utilize models having uncertainties about basic mechanisms or about the magnitudes of inputs which involve basic mechanisms. By normalizing the analytical results to the test results, the model can then be applied in order to compare one test to another by varying the parameter of significance. This type of model is not required to produce accurately absolute results, but only to substantiate hypotheses regarding the experimental results. The tests should progress very soon to systems which include multiple pins, flowing sodium with space for expansion above the shield zone and realistic pin lengths. The test series should include both clean and irradiated fuel. The primary measured variable of these tests should be the pressure transient developed in the assembly.

In keeping with this recommendation the schedule of the GE C-2 series of forced convection capsule tests should be accelerated, and the scope of the program should be broadened to cover a wider range of experimental parameters (burnup, flow, temperature, pin length, etc.).

The C-1 series of overpower transient capsules should eventually be modified to include flowing sodium and more typical geometry (multiple pins, with capsule containment designed for prototypical radial heat transfer conditions). This is not essential in the context of failure propagation, but will provide necessary information of thermal interaction damage for DBA analysis. Strong consideration should be given to conducting these tests in SEFOR, which offers conditions much more prototypical of fast reactors than does TREAT.

With respect to analytical modeling the present emphasis should be on correlation of the data obtained from the test series. For this purpose the following recommendations are made:

1. Present analytical efforts should be directed principally toward the correlation of experimental results.
2. Further development should be based on the hydrodynamic model approach rather than on purely thermodynamic model concepts. The latter models are inherently based on conservative assumptions and do not accurately describe the dynamics of fuel coolant interaction. The rate effects determine the extent of pressure generation. The use of modeling concepts which are based entirely on equilibrium thermodynamics leaves too many unanswered questions.
3. The first model refinement which should be accomplished is to incorporate the effects of non-condensable gases. These effects include limiting the heat transfer rate by blanketing or partly blanketing the heat transfer surfaces and effectively increasing the compressibility of the medium during the early part of the transient.
4. Examination of the S-Series test data suggests that multiple shock wave reflections are occurring within the test

capsule. This is an important consideration in estimating the thermal to mechanical conversion efficiency since it is necessary to distinguish between reflected pulses and original pulses generated by successive pin failures. None of the currently used modeling techniques are able to account for wave reflections and the further sophistication of the models to accomplish this should be considered in the near future.

5. The structural accommodation of violent boiling pressures should be included, either explicitly or with appropriate empirical correlations of channel dynamic response.

The need for other laboratory tests directed toward the understanding and establishment of basic parameters is largely contingent on the knowledge gained from the prototypical test series. While it may not be practical to delay entirely this type of testing until the completion of the prototypical test series, the program should place a very low priority on this type of testing until pressure generation and thermal to mechanical conversion efficiencies have been established with greater certainty. At that time further heat transfer tests and fragmentation tests and related tests may or may not be warranted. If they are it should be realized that one of the most significant parameters necessary to predict both damage propagation and the response to DBA conditions is the extent of mixing of fuel and coolant, including the geometric detail. This particular piece of information will never be available and the best that can be hoped for is to treat it somehow in a parametric fashion both in design and in DBA analysis. The undesirability of this approach is clear because it appears at this time to result in unnecessarily conservative work energy estimates.

If early experimental programs lead to the conclusion that LMFBR design and licensing will depend in a significant way on the analytical modeling of fuel-coolant thermal interaction effects, then the further sophistication of analytical modeling techniques is indicated. A primary need will be to incorporate the interactions of the expanding fuel and coolant with the containing structure. Such interactions include the expansion of the containment under stress, which significantly relieves the pressure load, and the multiple reflections of shock waves due to various material interfaces and free surfaces. The need for this level of sophistication is not established yet. Should it become so, consideration should be given to the use of two-dimensional or even three-dimensional stress propagation codes such as the hydrodynamic and plastic/elastic codes under development at ANL.^{14, 15} In order to apply these tools in the context of the fuel-coolant thermal interaction, it will be necessary to couple them with a heat transfer model providing the energy input source. Given this source the codes as presently conceived are capable of developing the initial shock wave and following it through the coolant and also internally in the containing structure (the latter leading to prediction of internal stresses). Additional refinements which may be needed are the inclusion of the effects of heat conduction and real fluid and metal viscosities. Both of these dissipating mechanisms may significantly reduce the conservatism of the model.

7.3.2.3 Relationship to Existing Programs

GE Experimental Programs

The most valuable existing test program with respect to damage propagation is the GE C-2 series. These tests have the necessary potential for prototypical modeling, including realistic pin lengths, flowing sodium, the ability to establish typical initial conditions, and to minimize edge effects by the use of multiple-pin arrays. This program should be accelerated, and expanded to cover a wider range of initial conditions. The Task C-2 loss of flow tests (under PA-10) will be run in-pile (GETR) in forced convection capsules. Pin sizes in C10A through C10E will be nearly prototypical (2-3 ft. active length, 15 in. upper blanket, 0.5 in. lower blanket, 15 in. plenum, UO₂ fuel). The initial capsules will contain one pin, but will provide information on the effects of burnup, and the rate of flow decay. Test C10E will use a 7 pin array in a hardened flux. Subsequent tests should be consistent with the objective of increasingly prototypical design, and with the information needs indicated in the previous section.

The potential for damage propagation will be directly measured in C-2 capsules by post-test examination of the capsule, including measurement of distortion in the simulated channel wall. A transient pressure history will also be recorded, but the transducer is located at the end of the capsule. The pressure response at this point may not represent loading on the channel wall.

C10X is the first in-pile test of this series. (It follows Mockup 1A which was run in November of 1969 and was an out-of-pile simulation of the C-2 series of tests.) The C10X capsule will have static sodium, and is designed primarily to determine the level of damage resulting from fuel/coolant thermal interaction and to assure the adequacy of containment of the capsule design.

ANL Experimental Programs

The ANL S series tests utilize a piston autoclave in TREAT to evaluate work energy release from fuel/clad-sodium thermal interactions. The test series, as planned, does lead to successively more prototypical condition. The S-6 test will be of a 7 pin FFTF assembly. Both power excursion and flow blockage tests are planned. The latter will be simulated in stagnant sodium. A flowing sodium environment is necessary to establish representative steady state condition prior to the flow blockage.

Work energy will be measured in terms of piston motion, thus enabling the determination of energy conversion efficiency. The comparison of efficiencies for flow blockage and power transient conditions will be useful in DBA analysis.

Some problems arise in trying to apply the S series data. By changing the experiment design, it may be possible to resolve some of these difficulties.

- a. The initial sodium temperature (160 to 200°C) in these tests is too low. Future tests (after S6) may be run at 400°C if the heater design modification works properly. While 400°C is an improvement, 500°C or higher is needed to simulate reactor conditions.
- b. The present piston design is inadequate for measuring the mechanical energy of the system. The piston "hangs up" early in the transient with the result that pressure generation after that time is atypical. The design should be modified to minimize the effect of the piston itself on experimental results.
- c. The gas plenum attached to each fuel rod appears to be a holdover from an earlier design. If it is there to mock up a real plenum, it should be internally connected to the fuelled portion of the rod. The extreme axial restriction to fuel movement that the present design provides does not reflect the LMFBR design philosophy.
- d. The very short fuel specimens used in the S series tests, combined with their axial positioning, result in nearly flat axial power profiles over their lengths. If longer fuel pins were used, and/or the test autoclave moved axially, so that the power profile is less flat, the fuel failure would be more representative of what occurs in a reactor, which would not have a flat axial power distribution.
- e. The current radial restraint (gunbarrel) is atypical, and should be replaced with a channel mockup.

The ANL Impact (I series) of TREAT tests resemble the TRW out-of-pile shock tube tests and are more related to the understanding of basic phenomena than to prototypical conditions. It is quite possible that relatively high energy-conversion efficiencies will be obtained if the program is carried out in the presently conceived fashion, as the experiments have been designed in a fashion which should maximize the energy conversion. The design is relatively simple — a mass of sodium will impact as a homogeneous slug (of fluid) on a mass of molten UO_2 . This geometry lends itself to analysis, but does not in any way represent a configuration obtainable in a credible accident situation. The possibility exists that these results would then be misapplied in the analysis of sodium voiding and reentry accidents, which the test does not mock up; it is more unlikely that large, homogeneous slugs of either sodium or fuel would exist inside the reactor. These tests are considered to be low in priority.

Other phenomenological experiments which have been conducted at ANL are the fragmentation and heat transfer studies. The former include molten UO_2 -Sodium interactions, and also tin-water and NaCl-water combinations. These studies are continuing. The heat transfer studies are presently considered by ANL to be of low priority.

Power excursion tests are not of prime concern in the present context of damage propagation, because the overpower condition would occur almost simultaneously in a large number of fuel assemblies. Thus it is a condition to be considered in a DBA situation, rather than in relation to failure propagation. However, the fuel/clad-sodium thermal interaction process is common to both situations, and it is appropriate to give some consideration to overpower transient testing in this discussion.

The GE C-1 series is a stagnant sodium capsule with capability for multiple pin and arrays. This series could be expanded to provide data of thermal interaction damage by making the capsule design similar to that of C-2, and introducing flowing sodium. The use of the ANL Mk II loop for this purpose is currently being studied. The requirements to produce a power transient limits the reactor facilities available for this purpose. The present tests are designed for TREAT.

The adequacy of TREAT for all of these tests is questionable, and is most questionable for the purpose of loss of flow testing. This is primarily due to:

1. The flux depression in mixed oxide specimens resulting from the fact that TREAT is a thermal reactor,

2. The TREAT design, which has required initiation of transients from zero power.

An analytical comparison of the response of a specimen in TREAT, and its response to a ramp insertion accident shows that TREAT provides an acceptable simulation of this overpower transient, provided that zero burnup fuel is used.^{2,3} The simulation may not be so good with irradiated fuel. The radial temperature skewing due to TREAT characteristics tends to locate the peak temperature in the region of higher fission gas retention, thus possibly leading to unrealistically early failure.

In loss of flow tests, this effect is compounded by the inherent limitation on integrated power, and by the necessity of starting the transient from zero power rather than from a normal steady operating condition. Modification of the control system may allow the approximation of a square pulse, but it is very doubtful that it can be maintained long enough to simulate a flow coastdown after allowing the coolant temperature to rise to a level typical of LMFBR operation. Thus, it would seem exceedingly difficult, if not impossible, to overcome the inappropriate thermal balance between fuel and coolant for flow coastdown testing.

These tests could be conducted in SEFOR after completion (in early 1972) of the current SEFOR experimental program. SEFOR would provide the right fast reactor spectrum and, in addition, is designed to allow power transients from an initial steady-state power condition. The use of SEFOR for transient overpower tests should be given strong consideration. While the present C-2 program on GETR appears adequate for the flow coastdown series, other flow coastdown tests in SEFOR might also be considered to take advantage of the hard spectrum and potential for following a loss of flow transient for a much longer time than is permitted in a water reactor.

Analytical Model Studies

With regard to analytical modeling, the concepts which appear to be consistent with requirements previously outlined are the bulk heating model presently under development by ANL and the hydrodynamic model which has been applied in slab geometry by GE.

The ANL concept has also been applied by PNL for the purpose of DBA analysis. Since the results indicated are only roughly a factor of 3 lower in efficiency than Hicks-Menzies results, it is not likely that this approach would be helpful in the DBA analysis of a 1000 MWe LMFBR without further refinement of either the modeling approach or the basic inputs or both. The ANL bulk heating model is a clean, easily understandable concept and is readily adaptable to many modeling situations. Most of the unknowns associated with the model are in the input parameters, for example, the model incorporates a time dependent heat transfer rate. This parameter can reflect many of the presently unknown phenomena, including rate and extent of fuel-coolant mixing, rate and extent of fuel fragmentation, the effect of vapor blanketing, etc.

The ANL model assumes constant pressure and temperature in the heated zone. The GE hydrodynamic model does not make this assumption but utilizes instead a solution of the transient heat conduction equation in linear geometry. The coupling of these two concepts would be quite useful. The effect of thermal conduction across the boundary between the heated and unheated zones is of particular interest since the ANL model makes the conservative assumption that all of the thermal energy is contained in the heated zone.

7.3.3 Review of Experimental Results (M.B. Goldstein)

7.3.3.1 Introduction

Current experimental programs at both GE (PA-10) and ANL relate directly to information needs described in Section 7.3.2. In addition, the out-of-pile shock tube tests conducted several years ago by TRW provided some basic information. These test results are reviewed in this section, and related test programs proposed by GE and ANL are discussed.

The tests are divided into two categories:

1. Integral tests, in which prototypicality is a major criterion of the test design and the resultant effect of all parameters of the thermal interaction process is measured, and,
2. Phenomenological tests, in which the objective is understanding of basic phenomena, and the test design allows measurement of one parameter (e.g., rate of heat transfer from UO_2 particles to sodium).

In the first category, tests already run or proposed include simulation of both overpower transient and loss of coolant flow conditions. In the present context, the main emphasis is on the latter. The subject of failure propagation implies initiating events which affect few assemblies. This is not the case with overpower transients, which are more of concern with respect to contribution of violent boiling to DBA work energy.

In the next section (7.3.3.2), the review is summarized and conclusions are presented. Further details of existing and proposed test programs are found in subsequent discussion.

7.3.3.2 Summary and Conclusions

Of the pertinent test programs presently proposed, the one with the greatest potential for achieving prototypical conditions is the GE C-2 series.²⁰ These flow coastdown tests will be run in GETR with flowing sodium. The initial tests will have one pin, but later ones will have seven pins. Pin lengths will be realistic, and irradiated pins will be used.

Piston Autoclave Tests (ANL)

The ANL piston autoclave(s)¹⁸ series of tests has shown that the energy conversion efficiency in large power transients may be on the order of 0.001 to 0.05%.¹¹ The first three significant tests (S_2 , S_3 , and S_4) indicate that the efficiency increases with the fuel melt fraction, and with the rate of heat addition due to the transient's period. The S_5 test* indicates that the absence of gas in the fuel pin results in much higher conversion efficiencies in a particular transient; the efficiency of the S_5 appears to be about 20 times that of the S_3 , which has a similar energy deposition rate and level.

Some problems arise in trying to apply the S series data to the analysis of large power transients. By changing the experiment design, it may be possible to resolve some of these difficulties.

- a. The initial sodium temperature (160 to 200°C) in these tests is too low. Future tests (after S_6) may be run at 400°C if a heater design modification works properly. The temperature should be increased to 500°C, which is more prototypical, because cladding properties change significantly in the 400°C to 500°C range.
- b. The piston design is inadequate for measuring the mechanical energy of the system. Once the piston starts to lock up, the pressure pulses that follow are not necessarily similar to those that would have occurred if there was more "compressible" space in the system. The piston design should be modified or replaced with a more accommodating design, which would be more representative of a reactor loop (with a large amount of compressive gas above it). In examining the output of the transient, all pressure pulses following the "locking up" of the piston must be examined to make sure that they are not reflections of a previous pressure wave. If some pulses are reflections, then the previous pulses did not transfer all of their energy to the piston and/or shock absorber, and the summed impulses of the waves yield an overestimate of the change of momentum of the idealized piston.
- c. The gas plenum attached to each fuel rod appears to be a holdover from an earlier design. If it is there to mock up a real plenum, it should be internally connected to the fuelled portion of the rod. The extreme axial restriction of fuel movement that the present design provides does not reflect the LMFB design philosophy, and could result in untypically early failure of the pin.
- d. The very short fuel specimens used in the S series tests, combined with their axial positioning, result in nearly flat axial power profiles over their lengths. If longer fuel pins were used, and/or the test autoclave moved axially, so that the power profile is less flat, the fuel failure would be more representative of what occurs in a reactor, which will not have a flat axial power distribution.
- e. The current radial constraint (gun barrel) is atypical and should be replaced with a channel mockup.

Testing of Irradiated Pins

The influence of irradiation on the work energy of fuel/clad-sodium thermal interaction must be determined. Ideally, this should be done by testing pins which have been pre-irradiated in a fast flux to high (50,000-100,000 MWD/Te) burnup levels. Since this cannot be accomplished within a reasonable time with prototypical fuel, the current practice is to accelerate burnup by using fully enriched fuel. The cladding is then under-irradiated relative to the fuel.

*Preliminary output data supplied by R.W. Wright (ANL) at information exchange meeting - Sunnyvale - July 13, 14 - 1970.

The loss of cladding ductility with irradiation directly affects the pin's failure point, thus it could be a significant factor in determining the over-all effect of thermal interactions.

It may be feasible to mock up the expected loss of ductility in irradiated cladding by using highly cold-worked cladding in test specimens. If these specimens are also pre-irradiated to achieve the fuel burnup desired, the possibility of undesired effects on cold-worked clad would have to be investigated. Such effects could be avoided by using un-irradiated fuel. In this case, some of the properties of irradiated fuel pellets could be simulated. For example, annular pellets could be used, and fission gas could be simulated by water vapor, if the pellets were allowed to absorb water before the test.

While these tests would not eliminate the need for testing of irradiated pins, they would provide a relatively inexpensive means of determining the effects of cladding irradiation on thermal interaction. The feasibility of these tests has not been established, but it should be evaluated.

Impact Tests

Many of the theories and models related to the fuel-clad-sodium thermal interaction problems are based on information obtained by studying fuel/clad-water interactions during the course of thermal reactor safety programs. Out-of-pile shock tube work by TRW showed that the conversion of thermal to mechanical energy is very inefficient when hot, solid metal interacts with water, and that the efficiency (and the pressure pulse size) increases dramatically when the metal is molten.¹ These tests also showed that the efficiency and pressure pulse of the interaction are sharply reduced by the presence of a small amount of non-compressible gas which cushions the initial expansion of heated sodium.^{1,2} ANL is extending this work to the sodium situation by employing an impact test in TREAT, in which a column of sodium will impact on a molten column of UO_2 .

The proposed ANL Impact (I) Series in TREAT closely resembles the TRW out-of-pile shock tube tests. It is quite possible that relatively high energy conversion efficiencies will be obtained in the I series, as the experiment has been designed with a geometry that lends itself to basic understanding rather than prototypicality. The possibility then exists that the test results could be incorrectly applied to predictive analysis of sodium voiding and reentry accidents. These tests are considered to be of low priority.

Fragmentation and Heat Transfer Tests

Fragmentation of fuel and/or cladding during a transient increases the amount of heat transfer area available, and may thus increase the heat transferred, resulting in higher pressure pulses and higher energy conversion efficiency. There has been a large number of experiments to examine the means of producing fragmentation of metals and fuel upon entry into sodium.^{7,9,10} In general, solid metals remain intact, and molten metals sometimes fragment. Molten metals whose melting points are above the critical temperature for water did not fragment when dropped into cold water. Solid UO_2 pellets were found to crack (or craze) but did not fragment when dropped into sodium, while liquid UO_2 does fragment when dropped into sodium. Fragmentation work is continuing at ANL, with molten UO_2 -sodium interactions, tin-water, and molten salt (NaCl)-water interactions all being investigated.

The early work with water led to the hypothesis that liquid metal will fragment if its melting point is within the region of violent (nucleate or transition) boiling for the coolant. Bubble collapse causes large pressure pulses which cause droplet fragmentation. This temperature level extends up to about the coolant's critical temperature. Solid metals remain intact when dropped into either water or sodium.

The mean particle diameter of the fragments resulting when molten stainless steel is dropped into sodium was found to decrease as the sodium temperature was raised from 250 to 800°C.^{11,21} However, later measurements of the debris showed that the reduction was not as large as originally reported.²² Work with solid UO_2 and sodium showed the pellet cracking and crazing, but not fragmenting. When liquid, UO_2 dropped into sodium has a violent interaction with much fragmentation.

The work with sodium does not always seem to agree with the work done with water. Part of the difficulty arises from the use of the droplet temperature rather than the interface temperature (between the coolant and the molten metal). If the *interface* temperature between the drop and the coolant is calculated (using a zero film resistance), it is found to be very nearly the same as the drop temperature when water is the coolant. In the case of a liquid metal

coolant, the interface temperature is closer to the average of the drop and the coolant temperatures.* It is obvious that the bulk of the drop may be well above the sodium's boiling point while the interface is well below it, so that boiling does not occur and the drop does not fragment. Similarly, the bulk of a very hot drop may be above the coolant's critical temperature, while the interface temperature is well below it, so that nucleate (violent) rather than film (quiet) boiling occurs, and the particle fragments.

In the case of UO_2 , the calculated interface temperature when molten UO_2 is dropped into relatively cold (250°C) sodium is below the sodium's boiling point. However, fragmentation occurs. It is quite possible that the fragmentation process is different in the case of basically brittle materials, like UO_2 . If the "skin" of the UO_2 drop becomes solid, there is a large contraction in the skin, due to the shrinkage (9%) in the change of state plus the shrinkage due to the large temperature drop (relative to the UO_2) within the drop times the coefficient of expansion. The brittle UO_2 "skin" cannot take very much tension, and fractures. Sodium bubble growth and collapse may not contribute to the fracturing at all. Pressure generation due to sudden heating of the liquid sodium could also contribute to fragmentation forces (see Section 7.3.3.3).

By elevating the sodium temperature to $350\text{--}400^\circ\text{C}$, the interface between molten UO_2 and the sodium should be raised above the boiling point. The resultant particle size distribution may be quite different that the sizes that have been seen in the tests with cooler sodium.

The work on fragmentation should be completely reviewed by ANL, and an attempt should be made to develop the fragmentation models that are required to describe the phenomenon reported to date.

7.3.3.3 The TRW Work^{1,2,3,4,5,6} Reviewed

The TRW reactor safety research work which is relevant to the fuel-sodium interaction problem falls into two general categories. The first of these is the generation of destructive pressures by thermal methods under reactor conditions. The second is transient steam void growth and collapse. The TRW research program was concluded at the end of 1966.

7.3.3.3.1 The Generation of Destructive Pressures

Several types of experiments were used to determine the method of creating high pressure levels during a reactor transient. The earliest tests used a flat plate fuel element and the KEWB reactor as a pulsed heat source. Another group of tests employed a stainless steel electrical heater which could achieve more rapid thermal transients than those obtainable with the reactor heat source. These tests indicated that the pressures obtained due to the heat flux from solid (plate) sources are relatively low, and are further reduced if incondensable gases are present in the system. Later tests involved the use of a shock tube, in which various heat sources could be evaluated. The pressures obtained by the interaction of molten metal and water were very large in comparison to the pressures obtained with the solid sources. In several cases, pressures exceeding the critical pressures of water were recorded.

$$\begin{aligned}
 * \quad T_{\text{interface}} &= T_{\text{sink}} + \phi \times (T_{\text{source}} - T_{\text{sink}}) \\
 \phi &= \frac{(K/\sqrt{a})_{\text{source}}}{(K/\sqrt{a})_{\text{source}} + (K/\sqrt{a})_{\text{sink}}} \\
 K &= \text{thermal conductivity (Btu/hr-ft-}^\circ\text{F)} \\
 a &= \text{thermal diffusivity (ft}^2\text{/hr)}
 \end{aligned}$$

For 250°C sodium, $K/\sqrt{a} = 28.33$. No data have been found for the thermal conductivity or thermal diffusivity of molten steel. The density and specific heat are fairly well determined. The thermal conductivity below the melting point of 304 stainless steel may be as low as 14 or as high as 18 Btu/hr-ft- $^\circ\text{F}$. The thermal conductivity of molten metals are usually lower than their conductivities in solid state. If K of molten 304 stainless = 14, its temperature must be greater than 1475°C if the interface temperature with 250°C sodium is to be at or above 882°C (the boiling point of sodium). If $K = 12$, the steel temperature must be above 1530°C . The data presented in Table 7-7 indicates that the 1425°C stainless steel drop did not fragment, while the 1550°C drop did. If the hypothesis (that fragmentations will not occur until the interface temperature exceeds the coolant's boiling point) is correct, the thermal conductivity of the molten steel is between 12 and 14. This hypothesis should be relatively easy to prove (or disprove). If it is true, it presents a relatively easy way to determine the thermal conductivity of molten metals.

Experiments with Flat Plate Heat Sources

In one group of these experiments, an aluminum clad, enriched uranium — 10% Mo disc immersed in water was fission heated by neutron bursts with reactor periods as short as 3 milliseconds. The apparatus, shown in Figure 7-19, was located in the reflector of the KEWB reactor. Results of two typical transients are shown in Figure 7-20 and 7-21. The results are quite similar, although the transient shown in Figure 7-20 had an initial water pressure of 15 psia (the latter was at 120 psia) and had a smaller heat input than the transient shown in Figure 7-21. The magnitude of the first pressure pulse increases with loading (on the piston) and decreases with reactor period. It also increases with initial pressure to a maximum at about 380 psia (initial pressure) and then appears to decrease slowly. The difference between the first pressure peak and the initial pressure level never became more than 140 psi, which should not be large enough to cause appreciable damage. However, the water hammer pressure pulses approached 1000 psi, and may be large enough to cause damage. The magnitude of these "aftershocks" appears to be governed by the system restraint (piston loading), which is comparable to a hydrostatic head in the system. The results of the group of experiments are listed in Table 7-5.

In order to determine the maximum possible pressures which could be produced by a solid slab source, the equipment shown in Figure 7-19 was modified so that the piston could not move. In these fixed volume experiments, the only expansion space available was from the compression of the water, and the flexibility of the channel walls. Pressures of about 1800 psi (slightly more than one half of the critical pressure) were achieved during these tests, although measured disc (heat source) temperatures approached 500°C, well above the critical temperature for water. The pressure-time history appears to depend on the peak temperature, but the peak pressure is insensitive to peak temperature. Figure 7-22 shows two typical pressure-time histories of fixed volume runs. Apparently, all runs with disc temperatures higher than 335°C looked like the higher temperature run (two peak pressure pulse), while the lower temperature runs had single peaks. In each case, boiling appears to start at about 200°C, and vapor blanketing starts at about 310°C.

Figure 7-23 shows the initial peak pressure obtained as a function of the piston loading for a 5 ms reactor period, and a 350°C maximum (disc) temperature. A locked piston run is included to show the result of those tests when the same temperature rise was employed. The peak pressures obtained varied with the cube root of the piston loading.

Figure 7-19 Sketch of Rolling Diaphragm Piston Seal System

Figure 7-20 Sketch of Temperature, Pressure, Dilation, and Reactor Power Histories for a 5 ms Transient with 30 gm/cm² Piston Loading and Atmospheric Initial Pressure

Figure 7-21 Sketch of Temperature, Pressure, Dilation and Reactor Power Histories for a 3 ms Transient with 36 gram/cm² Piston Loading and 120 PSIA Initial Pressure

Figure 7-19 Sketch of Rolling Diaphragm Piston Seal System

Figure 7-20 Sketch of Temperature, Pressure, Dilation, and Reactor Power Histories for a 5 ms Transient with 30 gm/cm² Piston Loading and Atmospheric Initial Pressure

Figure 7-21 Sketch of Temperature, Pressure, Dilation and Reactor Power Histories for a 3 ms Transient with 36 gram/cm² Piston Loading and 120 PSIA Initial Pressure

Table 7-5. Pressure And Void Data For Runs At Various
Initial Pressures And Ambient Initial Temperature

Initial Pressure	Reactor Period	Piston Loading	1st Peak Pressure	1st Pressure Minimum	Magnitude of Water Hammer Pressure				1st Void Maximum	Max. Void Growth Rate	Adiabatic Peak Temperature
					Pulses (psi)						
					1st	2nd	3rd	4th			
psia	ms	gm/cm ²	psia	psia					cm	cm/sec	°C
	O-Ring Seal		Disc A								
15	5.0	30	45	5	100	100	100	100	.008	2.1	500
15	4.9	120	65	5	180	190	190	180	.013	1.7	500
15	4.9	230	75	5	150	160	120	70	>.03	2.8	500
110	5.0	30	170	50	170	160	160	160	.007	3.4	500
100	5.0	120	190	20	370	370	370	360	.012	3.2	500
100	5.0	230	200	30	940	750	590	460	>.03	4.7	500
	Diaphragm Seal		Disc B								
100	4.9	36	150	30	180	230	140	90	.007	4.4	500
120	4.0	36	190	50	340	370	380	340	.008	6.1	570
120	3.0	36	210	50	220	310	310	370	.006	5.4	660
190	5.2	36	250	100	380	560	490	450	.002	1.8	490
310	5.0	36	380	190	330	420	340	280	.008	4.1	500
380	5.2	36	450	200	630	590	620	400	-----	-----	490
380	4.2	36	480	170	720	650	510	400	.004	7.5	550
380	3.0	36	520	130	740	560	450	400	.024	8.3	660
1000	4.2	36	1080	690	670	560	490	410	.011	6.4	550
1000	3.0	36	1130	700	570	490	420	390	.008	8.3	660
1000	2.9	38	1120	670	600	500	520	500	.009	6.4	670
1500	3.0	36	1580	1340	280	220	210	200	.004	4.7	660

Figure 7-22 Pressure-Time Histories for Fixed Volume Runs with Reactor Periods and Maximum Temperatures as Indicated

Figure 7-23 Initial Peak Pressures as a Function of Piston Loading for Runs With 5ms Reactor Period and 350°C Maximum Temperature

Cylindrical Heater

A thin tubular stainless steel heater was used to produce temperature transients in another experiment. Transients of 700°C in 120 seconds were achieved. The heat sink (water) was contained in a rigid closed capsule. In one configuration, the capsule was completely filled with water, giving 0.58 cm^3 of water per cm^2 of heater surface. In the other configuration, the capsule contained a large air bubble, equivalent to 0.012 cm^3 per cm^2 of heater surface (about 2% of the water's volume). The pressure as a function of time is shown in Figure 7-24, for the two cases. The lower trace, for the full capsule, shows the mean pressure rising slowly to about 1150 psig, with a 3500 cps, 500 psig amplitude oscillation that is the characteristic "organ-pipe" frequency of the capsule cavity. In the upper trace, with the air bubble present, the pressure rise is truncated after 150 msec when the initial pressure wave returned from the bubble surface as a rarefaction, and produced vapor blanketing of the heated surface. After several milliseconds, vapor collapse occurs and a water hammer pulse is seen. This looks quite similar to the KEWB capsule test, with the unlocked piston (space available for pressure accommodation).

These experiments illustrate the difficulty in producing high pressures by heating solid surfaces. Although temperatures well in excess of the 347°C critical temperature were produced with a wide range of heating periods, the maximum measured pressures were only about one half the critical pressure (of water).

The initial pressure pulse appears to be brought about by the water expanding as it is heated. The following pulses are due to vapor condensation and water returning to the heated surface and suddenly stopped — in other words, water hammer.

The Shock Tube Tests

The relatively low pressures obtained with the (solid) plate type heat sources led to the development of the shock tube test, in which either solid or liquid heat sources could be employed to try to produce destructive pressures by thermal energy transfer methods. Figure 7-25 is a drawing of the apparatus used in the later experiments.^{1,2,4} It differs considerably from the apparatus used earlier.⁵ The water column is 3 feet high. The diaphragm is ruptured by opening the solenoid valve (above the water column) to the atmosphere. A shock wave is produced, which breaks the diaphragm. The volume above the crucible, shown in the diagram to be connected to a vacuum line, is also connected to an evacuated water-filled flask at room temperature. As a result, the evacuated volume actually contains a slight amount of water vapor (about 1 cm of Hg) which condenses as the advancing water column approaches the heat source.

Many of the early modifications of the apparatus were made in an attempt to get more consistent water impacts on the heat source. The diaphragm was first ruptured by piercing it with a needle, which caused a progressive rupture, rather than an "instantaneous" one. The small quantity of water which reached the heat source before the bulk of the water column started moving produced water vapor which was believed to cushion the impact of the main body of water. The rupture method used later (the shockwave) gave more complete rupture, and resulted in consistent impact pressured on the heat source. It may be that gas contained in the small amount of water that impacted on the heat source was the real culprit, as it appears that a) the water used was not extensively out-gassed before the experiment and b) the water vapor should have been condensed on the advancing water column's front face as it does in the later apparatus.

Solid and molten aluminum, molten silver, and UO_2 powder were all used as heat sources. The results of the experiments using the metallic heat sources are shown in Figure 7-26. The pressure levels achieved by the solid heat sources were quite low, but the molten metal sources gave very high pressures. The molten silver case, at 1130°C (170°C above its melting point) was run to make sure that the high pressures seen with the molten $\text{Al-H}_2\text{O}$ interaction were not due to a chemical interaction; silver does not react chemically with water. All of the runs with molten metal heat sources at 1000°C or higher resulted in crucible failure.

The early results of the test were all for temperatures of 950°C or less, and tentative hypothesis were reached that a) the high pressure steam is produced only by the sensible heat of the molten metal (aluminum), and b) the fraction of the sensible heat converted to high pressure steam (and mechanical energy) increases linearly with elevation in temperatures above the melting point. These tentative conclusions were not understood, as the linear increase in the fraction of the molten aluminum's sensible heat that goes into high pressure steam cannot continue to increase indefinitely as the aluminum temperature increases. The complete results shown in Figure 7-26 have more spread than the earlier data, and do not extend to high enough temperatures to give real support to the hypothesis.

Figure 7-24 Pressure history Sketches for Electrically Heated Fixed Volume Experiments with 2% Air Bubble Above and Capsule Full of Water Below

Figure 7-25 Drawing of the Shock Tube Experiment

Figure 7-26 Temperature Dependence of the Peak Pressure from Water Impact upon Aluminum or Silver. Arrows Indicate Shots Where Pressure Rise Was Terminated by Crucible Failure.

The thermal conversion efficiencies observed in this experiment were orders of magnitude greater than those observed in the tests using plate type, solid heat sources described previously. For example, a 450°C temperature rise in 15 milliseconds in the KEWB capsule test produced an efficiency of only 5×10^{-4} for the production of high pressure steam, and 2×10^{-5} for conversion of thermal energy into kinetic energy. Figure 7-27 shows the pressure pulse following water impact upon 950°C aluminum. The depression in the pressure level at about 1.5 msec is the arrival of the unloading wave from the top of the water column. The energy* in the high pressure steam 1 msec after the start of the pressure pulse was about 40% of the sensible heat of the 950°C molten aluminum, and the kinetic energy of the water column was about 5% of the sensible heat at the same time.

Several runs were made with hot powdered UO_2 as the heat source. The water impact (180 psia) dispersed the UO_2 powder into the water, but peak pressures of only 240 psia and 320 psia were produced at UO_2 temperatures of 800°C and 1200°C respectively. The powder was only compacted (initially) to about one-third of theoretical density. Following the initial contact, a period of several milliseconds was spent compacting the UO_2 powder, during which the pressure stayed at a few psia, before the real impact pressure and dispersal began. Due to the poor thermal conductivity of the UO_2 powder, and the use of a single thermocouple, the quoted temperatures of the UO_2 cannot be assigned labels like "bulk" or "average" — they are really local temperatures in a poorly conducting, discontinuous material. The possibility also exists that compliant open spaces exist in the compacted powder, which would reduce the pressure level. The conclusion was reached that the transient heat flux from the powdered UO_2 was about one-fourth that obtainable from aluminum at the same temperature. The UO_2 powder was much finer than the dispersed aluminum or silver residue in the shock tube tests. It was *not* concluded that molten, or near molten UO_2 could not produce the very high pressure that the (relatively) cool, molten aluminum produced.

7.3.3.3.2 Transient Steam Void Growth and Collapse

The apparatus used for this work is shown in Figure 7-19. The steam-void volume (dilation), and power histories which are typical of those observed during a 5 millisecond period transient with ambient initial temperature and atmospheric initial pressure and with a 30 gpm/cm² piston loading are shown in Figure 7-20. As the disc temperature rises during the reactor power excursion, the pressure rises, first due to thermal expansion and then due to nucleate boiling (Pressure 1).** When nucleate boiling progresses into film boiling (Pressure 2) and a vapor blanket forms, the pressure drops to a constant partial vacuum (Pressure 3) (about 4 psia). The void volume, after the initial pressure pulse, follows a nearly parabolic history (Dilation 1, 2) due to the constant deceleration pressure upon the inertial load. When the void collapses (Dilation 3) a water-hammer impact pressure pulse occurs (Pressure 4). The water then rebounds, a vapor blanket forms, the pressure drops again to 4 psia and the parabolic history repeats through several impacts until the energy is dissipated. The void history following the initial pressure pulse is determined by the piston loading, the impulse it has received, and the pressure difference across the piston during the period of vapor blanketing. Thus, the impulse, peak pressure, and rate of void growth immediately following the initial pressure pulse (which are not independent) appear to be the important phenomena.

The results of these tests are shown in Table 7-5. The collapse of the bubble, and the maximum bubble size, appear to be a function of the amount of initial subcooling. Some of the results of this investigation are shown in Table 7-6, which indicated that with only 15°C subcooling, the pressure within the bubble stayed above one atmosphere. If an open tube had been used, the bubble could have completely expelled the water without collapsing.

The pressure impulse given to the inertial load was about the same over this range of subcooling. The maximum rate of void growth was therefore also about the same at all subcooling; about 2 cm³/sec per cm² of heated surface. The pressure within the bubble increases with decreasing subcooling, from about 4 psia at 80°C subcooling to about 19 psia with 15°C subcooling. The deceleration provided by the low pressure (vacuum) bubble decreases until it appears that if the experiment were not confined, net water expulsion would have been achieved.²

*Pg. 69-70, Ref. 2, the assumption is made that the pressure pulse was due to the formation of high pressure steam. The necessary amount of steam was then calculated.

Pressure 1) refers to the number 1 on the pressure history in Figure 7.3.3 and indicates the time of occurrence of the phenomenon which is being described.

Figure 7-27 Pressure History from Water Impact Upon Molten Aluminum At 950°C

The time period between bubble growth, collapse, and reappearance increases with reduced subcooling. The effect on the magnitude of the second pressure pulse (the start of the water-hammering) does not appear to have been investigated, although a few figures² indicate that the second pressure pulse is increased in magnitude with reduced subcooling.

7.3.3.4 The ANL Work Reviewed

The ANL experimental effort may be divided into three work categories; laboratory simulations (primarily molten material – coolant contacting experiments), heat transfer experiments, and in-pile simulations involving transient tests in TREAT. In each area, there have usually been both experiments involving sodium as the coolant, and water as the coolant. In several cases, other coolants were employed to check out the hypothesis that had been derived. In several

Table 7-6. 124 Gram Loading 5 ms Period

Run	Initial Subcooling	Initial Pressure	Max. Temp.	Initial Peak Pressure	Max. Void Growth Rate	Initial Impulse	Deceleration Pressure	Maximum Void
	°C	psig	°C	psig	cm/sec	psi-sec	psig	cm
208	80	0	350	34	1.7	.11	- 10	.007
209	45	0	420	24	1.9	.09	- 8	.012
210	45	0	420	25	1.9	.09	- 9	.013
211	20	0	470	29	2.9	.11	- 1.5	Stop at .03
212	45	35	440	86	3.5	.16	-28	.015
213	80	0	320	26	1.8	.10	-10	.009
215	80	0	330	27	1.9	.12	-10	.012
217	15	0	475	41	5.1	.17	+ 4	Stop at .03

cases, the results obtained using sodium (and in one case, mercury⁷) did not appear to agree with those obtained with water. In general, these difficulties appeared due to the improper comparison of the heat source's temperature, in relation to the coolant's critical point, or boiling point. When water is the coolant, the heat source's temperature is very nearly the same as the interface (between the source and the sink) temperature. If the *interface** temperature is related to the coolant's critical point and/or boiling point, the results of the studies employing different coolants appear fairly consistent.

7.3.3.4.1 Laboratory Simulations

Pressure and Bubble Formation Work

A series of experiments were made using an electrically heated cylindrical heat source and an annular heat sink — consisting of either water, methanol, ethylene bromide, or acetone.⁸ The wall was heated at adiabatic rates of up to 14,000°C/sec; when losses are considered, the true rate is about 60% of this value. The liquids were subcooled from 0 to 110°C, the initial liquid velocity varied from 0 to 9 meters/second, and the system was always operated at an initial pressure of one atmosphere. Although the temperature of the heated surface exceeded 400°C in several experiments (over 400 individual experiments were run) the maximum transient pressure was only about 16 atmospheres.

In many respects this work was similar to the TRW work reported in Section 7.3.3.3 (Transient Steam Void Growth and Bubble Collapse). The highest pressures are achieved with highly subcooled coolant, and the bubble size increases with decreasing subcooling. With small amounts of subcooling, the coolant would be completely expelled from the test section. The highest initial pressure pulses were achieved (in stagnant water) with relatively small amounts of subcooling, while the later pulses peaked with greater amounts of subcooling. The initial small pressure pulse corresponds to the sudden growth of vapor bubbles. For highly subcooled liquid, the vapor recondenses, causing a partial vacuum, and pulls the liquid back into contact with the hot test section. Upon contact, the liquid vaporizes, and the cycle is repeated until the heated surface has sufficiently cooled so as not to vaporize additional liquid.

As the initial temperature of the liquid is increased, a larger fraction of the input energy goes into latent heat as opposed to sensible heat, resulting in more vapor formation and a larger initial pressure pulse. When the liquid is close to saturation, a still larger and broader initial pressure pulse is created, and the liquid may be completely expelled from the heated section.

Two of the important conclusions drawn by Singer as a result of this work were a) the essential mechanisms that occurred during the transient process were not qualitatively affected by the type of fluid used, and b) initial coolant velocities up to about 9 m/sec had little effect on the boiling dynamics.

*Based on a conduction model.

Molten Material – Coolant Interaction Studies

A large number of experiments fall into this category. In one group of experiments, inductively heated small samples (~0.3 cc) of various metals at temperatures of 1500 to 2400°C were dropped into liquid sodium to determine the amount of fragmentation.^{7,9,10} In general, there was no evidence of chemical interaction, solid metals remained intact, and some of the molten metal drops fragmented. A series of experiments were made in which the sodium temperature was elevated for a fixed (2300-2400°C) stainless steel molten drop temperature. The mean particle diameter was found to decrease slightly with increasing sodium temperature (Figure 7-28).

Measurements were also made of the effect of the steel's temperature on the fragment size when the sodium temperature was held at 250°C (Figure 7-29). Some solid UO₂ pellets were heated (radiative transfer) to temperature of 900°C and 2100°C. The pellets showed some degree of cracking (or crazing) when dropped into 250°C sodium, but did not fragment.

Table 7-7 gives the results of some of the high temperature drop tests into sodium.¹¹

Other work used water as the heat sink.^{7,12} Molten metals whose melting points were above the critical temperature for water (373°C) did not fragment when dropped into cold (0°C) water. Upon elevation of the water temperature to 60°C, the low melting point metals remained intact upon impact. The results of this work¹¹ are shown in Table 7-8. The Weber* number was found¹² to be a useful indicator of the dynamic effects on the fragmentation process. Figure 7-30 shows some of the results of this work; zinc did not fragment. The results with gallium are shown in Table 7-9 and indicate that fragmentation is independent of particle temperature up to a certain point (between 300-400°C with gallium-water), and that after going through a region (~400°C) of more violent interacting, increased temperature does not appear to increase the fragmentation process.

The work with water led to the hypothesis that molten metal dropped into water will fragment if its melting point is within the region of violent (nucleate or transition) boiling. Nucleate boiling in highly subcooled water results in high pressure water hammer pulses, caused by bubble collapse. As discussed earlier, when the amount of subcooling is reduced, the bubble collapse rate decreases, and the water hammer pressure pulses are small. This explains the non-fragmentation of molten tin, etc. in less subcooled (60°C) water. Thus the fragmentation appears to be a result of the asymmetric pressure (force) loadings on the drop's surface brought about by the rapid vapor (bubble) collapse rate in the subcooled water.

If the metal has solidified while still at a temperature level that supports stable (film) boiling, fragmentation should not occur. Note that the molten metals with freezing points above water's critical point in Table 7-8 did not fragment.

This hypothesis led to many difficulties when it was tested with data employing a liquid metal coolant (heat sink). Examination of Table 7-7 shows that several molten metals do not fragment in sodium (b.p. 882°C) until their temperatures are several hundreds of degrees above sodium's boiling point. Zirconium does not fragment at 2100°C but does at 2350°C, which is in the vicinity of the critical temperature of sodium. A platinum sample at 1900°C (m.p. 1710°C) was dropped into mercury (b.p. 357°C, critical temperature 1450°C) and fragmented, which is in definite contradiction to the part of the hypothesis covering the coolant's critical temperature and the heat source's melting point.

The "drop tests" are still in progress. Some work, with tin and water, shows that the maximum fragmentation occurs with tin temperatures in the 400-500°C temperature range. The projected area, which increases inversely with fragment size, increases with the Weber number. Fragmentation is found to be relatively insensitive to tin temperature when the water temperature is 50°C or more (less than 50°C subcooled). Other drop tests with water and molten metals are being made to measure the pressure transients generated by the bubble growth, the time delay (till nucleate boiling) incurred when the metal temperature is above the critical temperature, and the effect of subcooling level on the pressure level.

Work employing molten UO₂ and sodium is still in an early phase of the project. Some data have been presented.^{13,19} One drop consisting of 8.8 grams of UO₂ at 2840-2940°C poured into a pot of 200°C sodium resulted in the formation of at least 10 fragments, and a peak pressure pulse of 81 psi with a half-width of 1 millisecond.

*Weber number - dimensionless, $= 2rpv^2/\sigma$. r = radius of drop, p = density of coolant, v = contact velocity of drop, and σ = molten metal-coolant interfacial tension. The last item is not found in many handbooks or easily measured.

Figure 7-28 Particle Size Distribution of Type 304 Stainless Steel Samples
At ~ 2200 To 2300° C Dropped Into Sodium²²

In the second test, about 7.4 grams of UO_2 at 2900°C dropped into a tank containing 310 cm³ of sodium at 200°C. Two pressure pulses were recorded. The larger pulse exceeded the instrument range, but the extrapolated value of 350 psi is four times greater than the pressure recorded in the first experiment.

Fifty grams of sodium, and about 20% of the UO_2 , was expelled from the tank. The calculated work done on the sodium corresponds to 0.05 joules (~0.012 calories)/gm of UO_2 . If only the heat of fusion plus the energy above the melting point is considered available for conversion of thermal energy to mechanical work, the conversion efficiency is about 0.02%.

The UO_2 particles expelled from the tank have a mean diameter of 500 μ , and have smooth, spherical surfaces. Fragments inside the tank have a mean diameter of 200 μ , and are angular and rough-surfaced.

A third test, in which molten UO_2 was dropped into sodium at 400°C has been performed. The data are being analyzed.

Figure 7-29 Particle Size Distribution of Type 301 Stainless Steel Samples Dropped Into 250°C Sodium

Table 7-7. Results of Dropping Materials Into 250°C Sodium
(Sample Volume, ~0.3 cc; Height of Drop, 2 ft)

Material	Melting Point (°C)	Drop Temperatures At Which the Samples Remained	
		Intact (°C)	Fragmented (°C)
Tantalum	2980	1950, 2200, 2400	—
UO ₂	~2700	900, 2100	—
Molybdenum	2600	2200	—
Niobium	2470	2400	—
Niobium-1 w/o Zirconium	~2450	2200	—
Zirconium	1852	1700, 1860, 1900 2000, 2100	2350
Titanium	1812	—	2050
Vanadium	1730	1800, 1950	2200
Vanadium-20 w/o Titanium	~1700	1800	2100
Nickel	1453	1400, 1600, 1800	2100
Mild Steel	~1450	1500, 1600	2000, 2400
Type 304 Stainless Steel*	~1400	1425	1550, 1600, 1800 2000, 2100, 2200 2300, 2400
Uranium	1133	1200	1700, 2000, 2450
Silver	961	—	1900
Aluminum	660	900	1200, 1550, 1950

*Type 304 SS samples were also dropped at a temperature of 2200°C into sodium maintained at 450, 600, 700, and 820°C; fragmentation occurred in all cases.

Table 7-8. Results on Dropping Various Materials Into Water
(Sample Volume, ~0.3 cc; Height of Drop, 2 ft)

Material	Melting Point (°C)	Sample Behavior for Indicated Drop Temperature (°C)			
		Water at 0°C		Water at 60°C	
		Intact	Fragmented	Intact	Fragmented
Gold	1063	2600	—	—*	—*
Silver	961	1900	—	—*	—*
Aluminum	660	825	—	825	—
Zinc	420	825	—	825	—
Lead	327	—	825	825	—
Bismuth	271	—	825	825	—
Tin	232	—	825	825	—
Rose's Metal	96	—	600	—*	—*
Wood's Metal	70	—	600	—*	—*

*No experiments were done in 60°C water.

**Table 7-9 Fragmentation of Liquid Gallium
Dropped Into 30°C Water**
(Average of Five 0.37-cm Dia. Drops)

Drop Height (in.)	Weber Number	Number of Fragments as Indicated Initial Gallium Temperature				
		50°C	100°C	300°C	400°C	600°C
2	5.3	—	—	—	20.4	10.2
4	11.5	—	—	—	1.8	9.2
8	23	1.0	1.1	1.0	9.3	1.6
16	46	3.4	3.5	2.8	6.7	2.5
32	92	10.5	6.5	6.6	7.4	—

**Figure 7-30. Plot of Weber Number vs Number of Fragments
for Various Molten Metals Dropped into Room
Temperature Water.**

7.3.3.4.2 Heat Transfer Experiments

This work has consisted primarily of a group of transient heat transfer experiments, employing both water and liquid sodium as the heat sinks, and a metal sphere on a swinging arm apparatus as the heat source. The basic experimental apparatus consists of a swinging arm with a knife blade attached to the arm. A small metal sphere is attached to a thermocouple at the end of the blade. While the arm is in the raised position, the metal sphere (usually 0.5 inches in diameter) is heated in an electric furnace. The arm is then released and drops the sphere through a tank containing either sodium or water.

The work included the effect of the heat source's velocity, the sphere temperature, and the sink's temperature on the rate of heat transfer. The results using water are fairly predictable. Nucleate boiling starts when the sphere temperature is about 125°C, and the rate of heat transfer increases with increasing sphere temperature until the transition to film boiling occurs. For relatively cold water (20-25°C) this occurs at a (silver) sphere temperature of about 350°C.^{11,14} The effect of elevated water temperature is to reduce the transition temperature, or in other words to lower the "burnout", or DNB — departure from nucleate boiling — point. The reduction in the DNB heat flux with reduced subcooling is quite dramatic, and is shown in Figure 7-31.¹¹ This also shows that gas inclusion in the water reduces the DNB temperature, with the most dramatic differential appearing in the "medium" subcooling regime; note the difference in the heat transfer rate at the DNB with 60°C boiled water (degassed) and the 60°C "normal" water.

The work with sodium is more difficult due to the problems of maintaining the liquid sodium at elevated temperatures. There appear to be three different heat transfer regimes when a liquid metal is the heat sink. The first of these regimes consists of heat sources below the coolant boiling point, and with the coolant relatively cold. Work in this regime¹⁴ employing sodium between 200-450°C and 330-800°C wall temperature for the Ta (tantalum) heat source yield results which were best correlated by the following expression:

$$Nu = 2 + 0.39 (Re Pr)^{0.5}$$

Theoretically, the Nusselt number due to pure conduction is 2 when the Reynolds number is zero. The 0.5 exponential power corresponds to that given by the results of potential (frictionless) theory.

When the sodium temperature was kept low, but the sphere temperature was elevated beyond the boiling point (of sodium), the following equation¹⁵ was found to yield the correct shape to the heat transfer versus ΔT^* results:

$$Q/A = K \left(\frac{U_{\infty}}{D} C_p K_L P_L \right)^{.5} (T_L - T_B)$$

The experimental results have considerable scatter; it is not unusual to find 4 or 5 measurements taken at one "fixed" condition (T , U , T_B etc.) in which the measurements of Q/A vary by a factor of 2. The correlations appear to be the best when a value of 0.675 is employed for K . K theoretically is a function of the angle at which flow separation from the sphere occurs, and is 0 at 0°, 0.80 at 90°, and 1.13 at 180° (no separation).

A simpler correlation, given in the same reference,¹⁴ is:

$$Q/A \text{ (cal/cm}^2\text{-sec)} = C(T_L - T_B)^{0.88} \text{ where } C \text{ depends on } T_B \text{ and } V.$$

$$C = 3.76 \quad V = 10 \text{ feet/sec, } T_B = 300^\circ\text{C}$$

$$C = 2.82 \quad V = 6 \text{ feet/sec, } T_B = 300^\circ\text{C}$$

$$C = 2.96 \quad V = 10 \text{ feet/sec, } T_B = 450^\circ\text{C}$$

* $\Delta T = T_L - T_B$ where T_L = temperature of the liquid at the sphere surface (T interface?) and T_B = bulk liquid temperature. Temperatures are usually in °C.

Figure 7-31. Heat Flux From Swinging-Arm Experiment With $\frac{1}{4}$ -in. dia Silver Sphere Moving Through Water at 13.9 ft/sec

The third heat transfer regime appears when the sodium temperature is elevated to 500°C or more. Figure 7-32 was presented at a recent conference¹¹ by ANL personnel. It looks quite similar to what occurs at the DNB point with water. This work has not been found in the ANL reports. A cursory examination of the "break points" indicates that these are where the "interface" temperature reaches the sodium boiling point. Apparently, the heat transfer coefficient in the nucleate boiling regime is lower than that in the pure conduction regime. The heat transfer experiments are being phased out at ANL.

Figure 7-32. Heat-Transfer Rate for 1/2-in.-dia Tantalum Sphere in Sodium
(Sphere speed corrected to 10 ft/sec)

7.3.3.4.3 In-Pile Experiments

Charts A and B were presented at a recent meeting.¹¹ Chart A shows the proposed ANL TREAT (transient test reactor) time-table. Chart B is a more comprehensive summary of test program, including proposed GE (PA-10) tests and others. To date, most of the work has been done with clean (unirradiated) fuel. Pins have been assembled and irradiated in other reactors — e.g., a 19 pin assembly in EBR-II¹⁶ — for use in other tests.

Examination of the test program shows two groups of tests that are currently being conducted, and that apply to the fuel-coolant interaction program. These are the S series (the piston autoclave), which has had 4 successful experiment runs, and the I series, which still appears to be in the checkout phase.

Impact — Series I

The apparatus used in the I series of tests is shown in Figure 7-33.¹⁷ As shown, the apparatus will evaluate the consequences of the impact of water on molten UO_2 in preparation for a similar test with sodium. A nuclear transient of energy equivalent to about 400 cal/g UO_2 will melt about 250 g of UO_2 . The water column will be released just before the end of the transient. The (single?) pressure transducer atop the chamber will measure the pressures generated.

As shown in Chart A, 6 tests will be run in this series with sodium, after the initial calibration with water.

Piston-Autoclave — Series S

Figures 7-34 and 7-35 show the piston-autoclave apparatus used in the S series tests. This series of tests gives an indication of the efficiency of the conversion of nuclear to mechanical energy. However, the sodium is relatively cold at the start of the transient (200°C)¹⁸ and if the pressure pulses are caused by the interaction of molten UO_2 and sodium, the reaction could be more violent than the reaction seen with less subcooling (hotter sodium) as would be expected during normal reactor operation. Table 7-10 gives some of the results of tests S_2 , S_3 , and S_4 . Figure 7-36 has traces of the piston displacement, reactor power, pressure pulses, and sodium temperature for run S_3 . The pressure data and some piston velocity data for run S_4 is shown in Figure 7-37.

The pressure pulses may be due to the expansion of the bonding gas after pin failure.¹¹ The use of non-irradiated pins also leads to questions as to the usefulness of this information, as pins with a small amount of irradiation will have smaller clad-oxide gaps, and possibly, gas filled central "voids" which could provide a mechanism for the dispersal of fuel out of the clad into the sodium.

Other problems arise because the piston "locks up" and is unable to absorb the mechanical energy of the material within the autoclave. Therefore, the method of converting the pressure-impulse data to momentum changes and eventually to energy conversion efficiency is not exact; the method probably gives an upper limit.

Preliminary data from the S_5 test (no gas) indicates that with an S_3 (size) transient, about 20 times as much energy is converted to work as when there is gas present. The efficiencies shown on Table 7-10 are very rough, and reworking of the data indicates that the S_3 may be as high as 0.03% efficient, and the S_4 still higher.

Chart A
Proposed Experiments In Treat

	FY 1970			FY 1971			FY 1972		
Fuel Movement Power Excursion	E-1 Checkout-1 MK II, 1 Pin UO ₂ Completed	E-2 Checkout-2 MK II, 1 Pin UO ₂ Completed	E-3 MK II, 3 Pin UO ₂ IRR 6%	E-4 MK II 1 Pin FFTF Or UO ₂		E-5 MK II 7 Pin FFTF Or UO ₂	E-6 MK II 7 Pin FFTF Or XO40 IRR 5%	E-7 MK II 7 Pin FFTF Or XO40 IRR 5%	
Fuel Movement Loss of Flow and Assembly Blockage			L-1 MK II 1 Pin* FFTF or UO ₂	L-2 MK II 1 Pin* PFR	L-3 MK II 7 Pin PFR IRR 5%	L-4 MK II 7 Pin FFTF Or XO40 IRR 5%	L-5 MK II 7 Pin PFR IRR 5%		
Fuel Movement Loss of Flow (Dry Slumping)					F-1 Transp. 7 Pin UO ₂ in Argon	F-2 Transp. 7 Pin UO ₂ in Argon		F-3 Transp. 7 Pin UO ₂ in Argon	
Behavior of Small Amounts of Released Molten Fuel				D-1 MK II 7 Pin UO ₂ Enr. Pellet	D-2 MK II 7 Pin UO ₂ Enr. Pellet		D-3 MK II 7 Pin UO ₂ Fuel Block	D-4 MK II 7 Pin UO ₂ Fuel Block	D-5 MK II 7 Pin UO ₂ Enr. Pellet
Actual Work Energy Release From Molten Fuel Coolant Interaction	S-1,2,3,4 Piston 5 Pin, UO ₂ P.E. Completed	S-5 Piston 5 Pin UO ₂ (Evac) P.E.	S-6 Piston 7 Pin UO ₂ FFTF P.E.	S-7 Column 7 Pin FFTF Flow Block	S-8 Column 7 Pin FFTF P.E.	S-9 Column FP Simu Flow Block	S-10 Column FP Simu P.E.	S-11 Column FP Simu Flow Block	
Actual Work Energy Release From Molten Fuel Coolant Interaction		I-1 Impact Column Calibration	I-2, 3 Impact Column Molten UO ₂	I-4 Impact Molten UO ₂ -SS	I-5 Impact Molten UO ₂ -SS	I-6 Impact Molten PuO ₂ -UO ₂ -SS	I-7 Impact Molten PuO ₂ -UO ₂ -SS		
Characterization of Sodium Expulsion and Potential Re-Entry Processes								R-1, 2, 3, 4, 5, 6. Modified Autoclave: (Open Inlet Simulation) Or MOD MK-II Loop: (Restricted Inlet Simulation) UO ₂ 7 Pins - Accurate FFTF Simulation	
Ability of Fuel Assembly Cans to Resist Melt- Through							G-1 MK II 7 Pin UO ₂ Melt - Through	G-2 MK II 7 Pin UO ₂ Melt - Through	
Transient Fuel Element Failure Thresholds		H-1 MK II PFR Completed	H-2 MK II FFTF		H-3 MK II 7 Pin PFR IRR 5%				

* 7 Pins Will Be Used If Possible

Chart B

In-Pile Testing Sequence of Experiments

TESTS		FY70	FY71	FY72	FY73
FEFPL	Checkout			(C)	
	Fuel Element Fail Prop (Clad Rupt)			(P1)	(P4)
	Rate of Prop: Melting			(P2)	(P5)
	Blockage			(P3)	(P6) (P8)
	Failure Propagation				(P7) (P9) (P10)
Treat MK II	Fuel Move - Power Excursion	(E1) (E2)	(E3) (E4)	(E5) (E6) (E7)	
	Fuel Move - Loss of Flow		(L1) (L2) (L3) (L4) (L5)		Continued FFTF
	Fuel Move - Slump		(F1) (F2)	(F3)	Experiments or
	Release of Small Amts of Molten Fuel		(D1) (D2) (D3) (D4) (D5)		Demo Plant
	Assembly Melt Thru			(G1) (G2)	Simulation Tests
	Trans Fail Threshold		(H1) (H2) (H3)		
	Power Excursion Molten Fuel Piston Ass'y Blockage Coolant Interaction Impact Column-Re-Entry	(1,2) (3,4)	(S5) (S6) (S7) (S8,9) (S10) (S11)		
Treat Static	Expulsion and Re-Entry	(I1)	(I2) (I3) (I4) (I5) (I6) (I7)	(R1) (R2,3) (R4) (R5,6)	
	Treat - Axially Res Fuel C4	(A,B,C) (I,J)	(C4 X)	(C4-G, C4-H)	(C4-E, F, K, L)
GE PA10	GETR Mobility of Molten Fuel C5	(C5 A) (C5-B)			
	Treat - Defected Na Bonded Fuel Perf. C6		(C6-A(1,2,3))	(C6-B(1,2,3))	
	GETR FCC Loss of Flow Re-Entry C10		(C10-X)	(C10-A, C10-B)	
	Treat - MK II Failure Threshold		Work Plan Being Developed Schedule Not Available		
PNL Treat Tests	Test Fuel Pins To Safety Limit Conditions		FY 71, 72 Schedules Unknown		
	GETR -59				
	EBR II -1, 2				
EBR II Project	EBR II Fuel Failure Tests		Series Proposed to Test FFTF Fuel to Failure		
	Treat Tests: Gas Release I		(I)		
	Fuel Motion II			(II)

Figure 7-33. Apparatus for Treat Facility Will Evaluate Consequences of Water Column Impacting on Molten UO₂

Figure 7-34. Piston Autoclave Apparatus

Figure 7-35 Top View of 9-Pin Fuel Cluster For High Energy Meltdown Experiments In Treat In A Sodium Environment

Table 7-10. Summary of Results of Piston Autoclave Experiments

Experiment Number	S2	S3	S4
Reactor Energy, MW-sec	495	669	1050
Fission Energy in Test Fuel, cal/gUO ₂	340	460	720
Reactor Period, msec	63	53	34
Peak Power, MW	2310	2664	6650
Amplitude of Largest Pressure Pulse, psi	180	540	1800
Impulse from Largest Pressure Pulse, dyne-sec	2×10^4	8.5×10^4	18.4×10^4
Average Pulse Duration, msec	~3	2.3	0.9
Maximum Piston Velocity, cm/sec	193	472	807
Maximum Piston Impulse, dyne-sec	2×10^4	7.7×10^4	9.5×10^4
Fission Energy at First Pressure Pulse, cal/gUO ₂	234	444	480
Maximum Work Done on Piston, ergs.	3.6×10^6	17.1×10^6	35.8×10^6
Maximum Event Temperature, °C	710	1100	>3000
Conversion of Nuclear to Mechanical Energy	$1.1 \times 10^{-5}\alpha$	0.7×10^{-5}	1.4×10^{-5}
Nature of Specimens After Test	One fuel rod failed, others intact	All fuel rods destroyed, dummyrods failed by holes in clad	Complete destruction of fuel and dummyrods

α Based on the nuclear energy input to the single failed pin only.

Figure 7-36 Treat S-3 Results

Figure 7-37 Treat S-4 Results

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7.3.4 Analytical Modeling

7.3.4.1 Introduction

The principal effect of fuel/clad-coolant thermal interaction is the dynamic pressure loading (i.e.; pressure-time history) it imposes on fuel pins, channel walls or core containment structure. A useful, but less informative parameter is the mechanical work of sodium expansion due to the thermal interaction process. The latter is obtainable from the pressure history if the volume history is also known, since the work is:

$$\text{Work} = \int P(t) \frac{dv(t)}{dt} dt$$

The basic elements of the process, which must be modeled are:

1. The rate of heat transfer from fuel or clad to sodium; and
2. The constraints which prevent thermal expansion of the heated sodium, and thereby cause pressurization of the heated coolant.

In an exact model, the second element would include the elastic-plastic response of channel walls or other containment which, by expanding under the pressure loading, tend to reduce the pressure.

Current analytical models of thermal interaction fall into two categories 1) those which use equilibrium thermodynamics to estimate the total work (i.e., the value of the integral in equation 1); and 2) those which use coolant hydrodynamics to estimate the pressure history. The equilibrium thermodynamics approach is limited by the fact that it is not a dynamic model. An estimate of the total energy *potential* for deforming structure is determined, but no information is given of time dependence. Hydrodynamic models are more realistic, but so far have excluded interactions with containing structure.

This section will review and discuss specific analytical models in each of these categories. The review covers the work of Hicks and Menzies on their thermodynamic equilibrium model, several different models by GE, including:

1. Thermodynamic equilibrium models in both linear and spherical geometry, which apply respectively, to surface contact and mixing of fuel/clad and coolant.
2. A hydrodynamic model in linear geometry, including space and time dependent estimates of pressure, density, temperature, and particle velocity,

and the hydrodynamic model of ANL.

A useful relative measure of the effect of fuel/clad-coolant interaction is the "efficiency" of conversion from thermal to mechanical energy – defined as the ratio of expansion work to initial internal energy of the fuel. The calculated efficiency, when compared with that measured, also serves to indicate the degree of conservatism of the model. The models reviewed give efficiency estimates ranging from 18% to .002% for similar conditions. Experimental efficiencies on the order of .001% have been obtained. At the latter efficiency, the interaction of the entire core of a large (1000 MWe) LMFBR with its coolant would only accomplish 1 MW-sec of work (based on fuel specific energy of 1000 joules/gm, which is about that required for meltdown, and 100,000 Kg of fuel). This amount of work in a DBA is insignificant. On the other hand, if we were to believe that 18% efficiency is more realistic, the thermal interaction problem would create an unacceptable blast containment requirement on the plant design.

7.3.4.2 Summary and Conclusions

Analytical models based on the application of equilibrium thermodynamics include the thermodynamic equilibrium model of Hicks and Menzies and two versions (slab and spherical geometry) of a model concept introduced by GE in a study of a hypothetical loss of flow to a FFTF fuel assembly.² (2)

The Hicks and Menzies model assumes that fuel and sodium instantaneously mix and reach a thermal equilibrium. The heated sodium then expands to do work on its surroundings. Two extremes of the resulting work are given by:

1. "Equilibrium Expansion" in which the fuel and sodium remain in thermal equilibrium, thus heat transfer to the sodium continues.

- 2 "Adiabatic Expansion" in which heat transfer from fuel to sodium stops when the initial equilibrium condition is reached.

The model is conceptually simple and conservative.

The GE thermodynamic model is less conservative. Thermodynamic equilibrium is not assumed. The initial contact between fuel and sodium results in a high pressure generation in the sodium at the heated interface. As the pressure wave moves away from the interface, vapor is allowed to form and heat transfer is interrupted before equilibrium can be established.

The further development of equilibrium thermodynamic models such as those just described is not recommended for the correlation of thermal interaction experiments or for design or DBA analysis. Such models can be applied with confidence only when very conservative modeling assumptions are made.

Further analytical modeling efforts should be based on hydrodynamic principles which will allow more realistic treatment of this dynamics problem. The current modeling concept of ANL, and the plane geometry hydrodynamic model of GE fall into this category and should be further exploited.

The GE model is based on an analytic solution in plane geometry of a describing system of equations including:

1. The wave equation which describes hydrodynamic response of the coolant.
2. The Fourier heat conduction equation, which approximates an energy equation
3. A linear equation of state for sodium.

The present version of the model employs some simplifying assumptions which should be explored (e.g., items 2 and 3 above) but is very simple and useful in application to situations where a plane interface between overheated fuel or (more likely) cladding can be postulated. The model should be further developed. In addition to exploration of the effects of the simplifying assumptions, inclusion of reflected wave effects should be considered.

The ANL model concept is based on the instantaneous mixing of heated particles in a finite volume of coolant. The free thermal expansion of the heated coolant is prevented by surrounding fluid. Either acoustic or inertial constraint can be assumed. A time dependent model of heat transfer from fuel to coolant is used. The fundamental assumptions are:

1. There is no heat transfer across the interface between heated and unheated liquid.
2. The heated liquid is in a state of uniform thermal equilibrium.

Within these simplifying assumptions it has wide application and its continued use and development is encouraged. Suggested refinements include:

1. Accounting for the effects of non-condensable gases in reducing the effect of thermal interaction
2. Inclusion of acoustic wave reflections
3. Accounting for the effects of structural expansion under pressure loading.

7.3.4.3 Thermodynamic Limit Calculations

7.3.4.3.1 Introduction

The application of basic equilibrium thermodynamic principles, including the first and second laws of thermodynamics as well as equations of state, enables us to predict an upper limit of the efficiency for conversion of the thermal energy contained in heated fuel and clad materials into mechanical work without considering the rate of occurrence of the conversion (dynamic effects). In reality the actual amount of mechanical work performed is determined by the coupled response of the structure and the pressure generation due to the thermal interaction. The thermodynamic models of course do not take account of this. The work performed is simply based on the integral, $\int P dv$ due to expansion of the heated coolant during the thermodynamic process. Such predictions are by their very nature quite conservative. They have been used to gain intuition and to define the need for more precise analytical modeling. The thermodynamic analysis described here will include that of Hicks and Menzies and the later analysis reported in GEAP-10059² which was originally made in conjunction with damage propagation estimates on the FFTF

7.3.4.3.2 Hicks and Menzies Thermodynamic Equilibrium Model

The concept of the Hicks-Menzies model is based on the following description of the thermal interaction process: hot fuel and sodium are brought instantaneously into intimate contact and undergo an exchange of energy such that they reach thermal equilibrium (i.e., they reach the same temperature). It is assumed that no expansion of the sodium occurs during this interval so that the process is a constant volume one. In some applications of the concept the constant volume path is approximated by a path following the saturated liquid line, since in reality constant volume lines are very close to the saturation line. The process is illustrated in Figure 7-38, which shows the continuance of this constant volume process above the critical point to that point at which the fuel and sodium are in equilibrium (state point 2 of the T-S diagram). The mechanical work is performed in the subsequent process. Two kinds of assumptions are used regarding the thermodynamic process here. In the first one, it is assumed that no further exchange of heat takes place between fuel and sodium. In this case the sodium simply expands isentropically on the path 2-3' to some threshold pressure or to ambient pressure. In the second, more conservative approximation, fuel and sodium are assumed to remain in thermal equilibrium so that heat continues to flow from the fuel into the sodium until the threshold or final pressure is reached. This case is illustrated by the path 2-3-4-5.

The energy conversion efficiency from this model is roughly 10%. Various modifications of the model including improvements in the equation of state⁹ and the inclusion of heat of fusion of the fuel material¹⁰ lead to variations from this figure. The original analysis by Hicks and Menzies did not account for the variation of sodium and fuel physical properties with temperature or for phase changes in the fuel or for the extension of the sodium temperature above the critical point. A later analysis by PNL¹⁰ did make these refinements. A comparison of results is indicated in Table 7-11 which also shows the comparative results of the equilibrium and adiabatic assumption for the sodium expansion process.

Table 7-11. Comparison of Thermodynamic Equilibrium Calculations¹⁰

	Isentropic Expansion (PNL)	Equilibrium Expansion (PNL)	Hicks Menzies
Equilibrium Temperature (°K)	3428	3428	3940
Equilibrium Pressure (ATM)	616	616	1008
Work of Expansion to 1 ATM (MW-sec)	541	1228	1513

While the thermal equilibrium model has the advantage of being conservative and conceptually simple, situations in which the conservatism of this approach to DBA analysis can be tolerated are extremely limited. Further development of it for DBA analysis or for design analysis with respect to damage propagation is not recommended.

7.3.4.3.3 Slab Geometry Thermodynamic Model

This section describes a model of fuel/clad-sodium thermal interaction which is less conservative than the thermodynamic equilibrium concept, but still employs equilibrium thermodynamic principles to estimate an upper bound on work energy. The model was originally developed and used by GE in a study of damage propagation following loss of flow in an FFTF fuel assembly².

Compared to the results of Hicks and Menzies this is a much lower estimate because credit is taken for a mechanism which prevents thermal equilibrium from occurring – vapor blanketing of the heat transfer surface between

Figure 7-38 Hicks-Menzies Thermodynamic Equilibrium Model

fuel and coolant. This interruption of heat transfer may be only temporary. Condensation of the vapor film may allow the interaction to resume, thus the process may be viewed as a cyclic one which in the limit, approaches a thermal equilibrium condition. However, it is not this limit which is of concern with regard to damage propagation, but rather the first "cycle" or impulse since this imposes the greatest load on channel walls.

The original model, as described in Reference 2, attempted to define the partitioning of work energy into "acoustic" and "inertial" components. The former is defined in Reference 2 as the work done during the very early stage of the process, when the constraint on expansion of the liquid sodium is essentially acoustic (i.e., determined by the compressibility of the fluid). The definition given of "inertial" work is that which is done against the inertial constraint of surrounding fluid (i.e., the response is characterized by acceleration of an incompressible fluid).

The (arbitrary) criterion used is that "inertial work" is done by the expansion associated with coolant vaporization because a time lag must be associated with such vapor formation. The analysis, which served the purpose of establishing conservative upper bounds on both work components, determined "acoustic work" by expansion of the heated liquid to ambient pressure. The "inertial work" was then determined from the volume change due to vaporization at ambient pressure.

A more realistic view would be that all of the work is done during what was termed the "acoustic work" part of the process, and that some coolant vaporization might occur during this time. No attempt has been made to add this refinement to the model. The present discussion covers only the so-called "acoustic work" calculation. A detailed derivation is given to show how a substantially lower but still conservative upper limit of "acoustic work" can be derived.

The concept upon which this thermodynamic model is developed is that of a plane interface between hot fuel or clad material and liquid sodium coolant. This concept is descriptive of two situations:

1. The reentry of a slug of condensed coolant into a previously voided channel, where continued heating has resulted in fuel pin failure. In this case, the slab geometry solution simulates energy propagation in the axial direction.
2. Contact of reentering liquid sodium with an overheated fuel pin which still has essentially its original shape. In this case, the slab geometry solution represents energy propagated in the form of a plane wave from the heated interface to the channel wall. The clad-coolant interaction is the important consideration in this instance.

This modeling concept has limited application. It was useful in the original context as a means of establishing a conservative limit on the potential for damage propagation beyond the initially affected assembly following loss of flow. The establishment of surface contact between overheated core materials and coolant is conceivable in this situation. However, pin failure may result in the ejection of hot fuel and clad particles into the coolant stream, and subsequent particle fragmentation. Thus, an intimate mixture of hot particles in coolant may soon be created instead of a plane interface.

The model equations are developed in the following paragraphs.

Summary of Events

The conditions which lead to and which accompany the generation of the shock pulse are:

1. Fuel is heated to a high temperature in the absence of coolant, then the hot fuel and sodium are abruptly brought into intimate contact.
2. Heat flows from the fuel to the sodium at a very high rate, thus creating a high temperature gradient in a thin layer of coolant near the fuel.
3. The rate of heat addition is initially much higher than the rate at which the thin coolant layer can dissipate the energy by expansion against adjacent coolant. Therefore, a high pressure buildup occurs.
4. Energy contained in the highly compressed coolant layer is transferred to surrounding coolant in the form of a steep-fronted pressure wave, or "shock" pulse, leaving the originally affected layer in an energy and pressure state near its initial state.
5. As the pressure at the fuel-coolant interface is reduced (by propagation of the shock wave away from the interface) below the saturation pressure (at the interface temperature), vaporization occurs. The rate of heat

transfer drops to essentially zero due to the resistance imposed by the vapor film. Thus the total heat energy transferred from hot fuel to coolant is limited to that which is transferred before the interface pressure drops to the saturation condition.

The vapor film separating hot core materials from liquid coolant may condense and allow the process to repeat itself. The model described here represents only the first cycle of this repetitive sequence of events. The subsequent pulses should be less severe so the maximum potential for rupture of the channel wall is indicated by the energy content of the first pulse.

Heat Transfer Model

The model for the heat transfer process utilizes the following basic assumptions:

1. The fuel-coolant system can be treated in semi-infinite slab geometry.
2. The contact resistance between fuel and liquid coolant is zero.

The heat transfer is thus governed by conduction in the two materials, and the temperature profile at time $t > 0$ has the shape given in Figure 7-39.

Figure 7-39 Temperature Profile at Time $t > 0$

The coolant temperature equation is:

$$T_c(x, t) = \phi (T_{F0} - T_{C0}) \left[\operatorname{erfc} \left(\frac{x}{2\sqrt{a_c t}} \right) \right] + T_{C0} \quad (1)$$

where

$$\phi \equiv \frac{K_F / \sqrt{a_F}}{K_F / \sqrt{a_F} + K_C / \sqrt{a_C}} \quad (2)$$

K $\equiv$ Thermal conductivity

a $\equiv$ Thermal diffusivity

T_{F0} $\equiv$ Initial fuel temperature

T_{C0} $\equiv$ Initial coolant temperature

The average of $T_c(x, t)$ over any distance, X , is:

$$\bar{T}(X, t) = \frac{\phi (T_{F0} - T_{C0}) \int_0^{X/2\sqrt{a_c t}} [\operatorname{erfc}(x)] dx}{\left(X/2\sqrt{a_c t} \right)} + T_{C0} \quad (3)$$

Since

$$\operatorname{erfc}(z) \rightarrow 0 \text{ as } z \rightarrow \infty,$$

we can describe the position of the "thermal front" at any time by,

$$X_{\text{thermal}} = 8\sqrt{a_c t} \quad (4)$$

and the velocity of propagation of the thermal front by

$$\frac{dX_{\text{thermal}}}{dt} = 4\sqrt{a_c/t} \quad (5)$$

Thermodynamic Model

Before it is possible to study the thermodynamics of a system, it is necessary to establish its boundaries. Having done this, thermodynamic principles may be applied to predict the amount and form of energy which crosses the system boundaries. Define the thermodynamic system here to be that mass of coolant which is originally within a distance, X_S , of the fuel-coolant interface, where X_S describes the position of the shock front at the time that heat transfer from the fuel terminates

This definition is better understood with reference to Figure 7-40 which shows temperature and pressure profiles in the system at four different stages following initial contact of fuel with sodium. The first graph depicts initial temperature and pressure conditions ($t = 0$). Figure 7-40(b), for time t_1 , shows the temperature profile, $T_c(x, t_1)$, as well as the average temperature of the thermodynamic system, $T_{AVG}(t_1)$. In addition, the pressure profile, $P(x, t_1)$, the system pressure, $P_{AVG}(t_1)$, and the average pressure behind the shock front $\bar{P}_{shock}(t_1)$ are shown. At this time (t_1), the pressure disturbance has not propagated beyond the "thermal front" at $X_{thermal}$, but the thermal front velocity, $dX_{thermal}/dt$, has decreased so that it is just equal to the acoustic velocity. The thermal front velocity continues to decrease; therefore at a later time, such as t_2 in Figure 7-40(c) the pressure front is ahead of the thermal front. The final graph, Figure 7-40(d), depicts temperature and pressure at $t = t_3$, the precise time at which the average pressure behind the shock is equal to the saturation pressure corresponding to interface temperature, $T_{x=0}$. This condition serves to define the system boundary, X_S . As a consequence of this definition, the system pressure, $P_{AVG}(t)$ increases during the transient, while the pressure behind the shock front decreases.

In reality, the local pressure at $x=0$ would decrease to the saturation value some time before t_3 . The present model therefore overestimates the amount of heat transferred prior to the occurrence of local boiling at the fuel-coolant interface, and is thus conservative in this regard.

The model implies that the system undergoes a series of state changes, as shown in the p-v diagram of Figure 7-41.

State 1 is the initial condition of the system, as in Figure 7-40(a) for time $t=0$. Heat is transferred to the system, bringing it to state 2, at which time heat transfer stops. Thus, state point 2 describes the system as shown in Figure 7-40(d), where the acoustic wave front has just reached (but not crossed) the system boundary. The system then expands back to its initial pressure, P_∞ , along the constant entropy path from 2 to 3.

A key assumption of the analysis is that the system volume is the same at point 2 as at point 1. (This is *not* equivalent to the assumption of a *constant volume* process.) Since P_2 is also known, the state of the system at point 2 is thus fully described. An argument based on the changes in entropy of the system and the heat reservoir provides an upper limit on the amount of heat which could be transferred to the system in any process between states 1 and 2 (see equations 8 through 11, below). Thus, by application of the first law of thermodynamics,

$$d'w = d'Q - du \quad (6)$$

a corresponding upper limit on the work done between these two points is obtained.

This assumption leads to a very conservative estimate of the mechanical work developed by the process. A reasonable assumption is that the system goes from state 1 to state 2 in a constant volume process.

Since the work of expansion is

$$d'w = p dv \quad (7)$$

this would immediately lead to the conclusion that no work is done between points 1 and 2.

Figure 7-40 Pressure and Temperature Profiles

Figure 7-41 Thermodynamic Processes of the Model

Further discussion of the point is found later (see "Discussion of the Model"). In the process between points 1 and 2, the system receives heat from a reservoir (the fuel) which remains at a constant temperature, T_0 . No other heat energy crosses the system boundaries. According to the principle of increase of entropy,⁷ the total entropy change of the system and its surroundings must be equal to or greater than zero, or

$$ds + ds_0 \geq 0, \quad (8)$$

where ds is the change in system entropy and ds_0 is that of the surroundings.

The heat added to the system is $d'Q$, therefore the heat added to the surroundings is given by

$$d'Q_0 = -d'Q \quad (9)$$

The derivation is based on the equation,

$$ds_0 = \frac{d'Q_0}{T_0} \quad (10)$$

It then follows, from equations 6, 8, 9 and 10 that

$$d'W \leq T_0 ds - du. \quad (11)$$

The entropy change of a system is equal to the heat-temperature quotient only in a reversible process.⁸ In an irreversible process, the entropy change is greater.⁸

$$ds_0 > \frac{d'Q_0}{T_0} \quad (12)$$

Inequality¹¹ can be derived from another argument which also leads to a substantially lower upper limit as will be shown later

Since both entropy and internal energy are point functions, this leads to:

$$W_{1 \rightarrow 2} \leq T_0 (s_2 - s_1) - (u_2 - u_1) \quad (13)$$

The changes in entropy and internal energy are the same for any process which leads from state 1 to state 2. Therefore, without postulating that the change actually takes place along a constant volume line, we can correctly find Δs and Δu by using a constant volume line. Thus,

$$ds = C_v \frac{dT}{T} \quad (14)$$

and

$$du = C_v dT \quad (15)$$

Consequently

$$W_{1 \rightarrow 2} \approx T_{C0} \ln \left(T_{AVG}(t_1) / T_{C0} \right) - C_v \left(T_{AVG}(t_1) - T_{C0} \right) \quad (16)$$

where $T_{AVG}(t_1)$ refers to the system temperature at state point 2, as in Figure 7-40(d). This is, in turn, calculated according to a constant volume pressure increase from P_∞ to P_{SAT}

$$T_{AVG}(t_1) - T_{C0} = \frac{\beta_T}{\alpha} \left(P_{SAT} - P_\infty \right) \quad (17)$$

where

β_T = the isothermal compressibility,

α = the thermal expansion coefficient.

The work done in isentropic expansion of the system between points 2 and 3 is

$$W_{2 \rightarrow 3} = \int_{P_{SAT}}^{P_\infty} p dv \quad (18)$$

But in an isentropic process

$$dv = -v \beta_S dP \quad (19)$$

where β_S is the adiabatic compressibility. Thus,

$$W_{2 \rightarrow 3} = - \int_{P_{SAT}}^{P_\infty} \beta_S P v dP \quad (20)$$

Since the volume change is small, this is adequately approximated by,

$$W_{2 \rightarrow 3} \approx \beta_S v \int_{P_\infty}^{P_{SAT}} P dP = \frac{\beta_S v}{2} \left(P_{SAT} - P_\infty \right) \left(P_{SAT} + P_\infty \right) \quad (21)$$

Numerical Evaluation

The details of the calculation of *total acoustic work* based on the model² described above are given below, and should be helpful in understanding both the reported work and the comparison with the revised model which results in a lower limit of acoustic work.

$$W = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} \leq \left\{ T_0 C_v \ln \left[T_{AVG}(t_3) / T_{C0} \right] - C_v \left[T_{AVG}(t_3) - T_{C0} \right] \right\} + \frac{\beta_S v}{2} (P_{SAT} - P_\infty) (P_{SAT} + P_\infty) \quad (22)$$

The initial conditions assumed are

$$P_\infty = 1 \text{ atm,}$$

$$T_{C0} = 1400^\circ \text{F,}$$

$$T_{F0} = 6000^\circ \text{F}$$

In addition, the following properties are used:

$$K_F \quad (\text{Fuel thermal conductivity}) = 0.00486 \text{ cal/sec-cm-}^\circ\text{C,}$$

$$a_F \quad (\text{Fuel thermal diffusivity}) = 0.00389 \text{ cm}^2/\text{sec,}$$

$$K_c \quad (\text{Coolant thermal conductivity}) = 0.1321 \text{ cal/sec-cm-}^\circ\text{C}$$

$$a_c \quad (\text{Coolant thermal diffusivity}) = 0.570 \text{ cm}^2/\text{sec,}$$

$$\left(\frac{\alpha}{\beta_T} \right) \quad (\text{Thermal pressure coefficient of Na}) = 5.69 \text{ atmos/}^\circ\text{F,}$$

$$C \quad (\text{Sonic velocity in Na}) = 2.28 \times 10^5 \text{ cm/sec,}$$

$$v \quad (\text{Specific volume of Na}) = 2.08 \times 10^{-2} \text{ ft}^3/\text{lbm}$$

$$\beta_S \quad (\text{Isentropic compressibility of Na}) = 2.78 \times 10^{-5} \text{ atmos}^{-1}$$

$$C_v \quad (\text{Specific heat @ const volume}) = 0.305 \text{ Btu/lbm-}^\circ\text{F}$$

To calculate the fuel-sodium interface temperature, $T_{\text{interface}}$, set $x = 0$ in equation 1,

$$T_{\text{interface}} = \phi \left(T_{F0} - T_{C0} \right) + T_{C0} \quad (23)$$

From equation 2

$$\phi = \frac{1}{1 + \frac{k_c}{k_F} \sqrt{a_F/a_c}} = \frac{1}{1 + \frac{0.1321}{0.00486} \sqrt{\frac{0.00389}{0.570}}} = 0.308$$

$$T_{\text{interface}} = 0.308 \times (6000 - 1400) + 1400 = 2818^\circ\text{F}.$$

The system pressure at state point 2 is $P_{\text{AVG}}(t_3)$, which is the interface saturation pressure:

$$P_{\text{AVG}}(t_3) - P_{\text{SAT}}(2818^\circ\text{F}) = 46 \text{ atm}.$$

Then equation 12 gives

$$T_{\text{AVG}}(t_3) = 1400 + \frac{1}{5.69} (46 - 1) = 1408^\circ\text{F}$$

Now, take

$$T_0 = T_{\text{interface}} = 2818^\circ\text{F} = 3278^\circ\text{R}.$$

The acoustic work is determined as follows.

$$W_{1 \rightarrow 2} \leq 3278 \times 0.305 \times \ln \left(\frac{1408 + 460}{1400 + 460} \right) - 0.305 (1408 - 1400)$$

$$= 1.75 \text{ Btu/lbm}$$

$$= 16.300 \text{ in-lb/lbm}$$

$$W_{2 \rightarrow 3} = \frac{2.78 \times 10^{-5} \times 2.08 \times 10^{-2}}{2} \times 45 \times 47$$

$$= 6.13 \times 10^{-4} \frac{\text{atmos-ft}^3}{\text{lbm}}$$

$$= 15.6 \text{ in-lb/lbm}$$

$$W = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} \leq 1.63 \times 10^4 \text{ in-lb/lbm}$$

This is the highly conservative upper limit on total acoustic work for the chosen parameters using the model of Reference 2.

In order to express this result in the more convenient units of work per unit area, it is necessary to determine the extent, X_S of the thermodynamic system. This requires that we determine the location of the pressure front at the time that the average temperature behind the pressure front is $T_{\text{AVG}}(t_3) = 1408^\circ\text{F}$. Equations 3 through 5 are involved in

this determination. The pressure and thermal fronts are coincident up to the time, t_1 , that the thermal front velocity decreases to the acoustic velocity. That is:

$$4\sqrt{a_c/t_1} = C, \quad (24)$$

$$t_1 = \frac{16 a_c}{C^2} = \frac{16 \times 0.570}{(2.28 \times 10^5)^2} = 1.92 \times 10^{-10} \text{ sec.}$$

The thermal/pressure front at that time is at:

$$\begin{aligned} X_1 &= 8\sqrt{a_c t_1} = 8\sqrt{0.570 \times 1.92 \times 10^{-10}} \\ &= 8.35 \times 10^{-5} \text{ cm.} \end{aligned}$$

It is assumed that the acoustic wave front propagates at a constant velocity, C , as it moves away from the point, X_1 . Therefore, the system boundary X_S is given by:

$$X_S = X_1 + (t_3 - t_1) C,$$

or

$$X_S = 8.35 \times 10^{-5} + (t_3 - 1.92 \times 10^{-10}) \times 2.28 \times 10^5 \quad (25)$$

In addition, equation (3) gives:

$$T_{AVG} - T_{C0} = 7.9 = \frac{\left(\phi T_{F0} - T_{C0} \right) \int_0^{X_S/2\sqrt{a_c t_3}} \text{erfc}(x) dx}{\left(X_S/2\sqrt{a_c t_3} \right)} \quad (26)$$

The simultaneous solution of equations 25 and 26 provides values of t_3 and X_S . The solution is simplified if we consider that,

$$\text{erfc}(x) \approx 0 \text{ for } x \geq 4,$$

and

$$\int_0^4 \operatorname{erfc}(x) dx \approx \frac{1}{\sqrt{\pi}}.$$

Then, from equation 26.

$$X_s = \frac{\phi(T_{F0} - T_{C0}) \cdot 2\sqrt{a_c t_3}}{7.9 \times \sqrt{\pi}} = 148\sqrt{t_3} \quad (27)$$

From equations 25 and 27, it is found that

$$X_s = 0.0963 \text{ cm} = 0.0379 \text{ in.}$$

and

$$t_3 = 0.422 \times 10^{-6} \text{ seconds}$$

The coolant density is

$$\rho = \frac{1}{v} = 48 \frac{\text{lbm}}{\text{ft}^3} = 0.0278 \frac{\text{lbm}}{\text{in}^3}.$$

and the mass of the thermodynamic system per unit area is

$$\frac{M}{A} = \rho \times X = 0.0278 \times 0.0379 = 0.001052 \frac{\text{lbm}}{\text{in}^2}$$

Thus,

$$W_{1 \rightarrow 2} \leq 16,300 \times 0.001052 = 18.8 \text{ in-lb/in}^2$$

$$W_{2 \rightarrow 3} = 15.6 \times 0.001052 = 0.0156 \text{ in-lb/in}^2$$

$$W \leq 18.8 \text{ in-lb/in}^2$$

This is the form in which the conservative estimates of total acoustic work were presented in Reference 2.

Discussion of the Model

In the description of the thermodynamic process which takes place between state points 1 and 2, the model includes no explicit assumption of the nature of this process, assuming only that the initial and final specific volumes are the same. It is easy to accept on a physical intuition basis the assumption that the process in question is a constant volume process. The "realistic" estimate, based on this assumption, would include only the work of isentropic expansion from point 2 to point 3.

Aside from this, inequality 11 can be derived from another argument, which does not include an explicit description of the heating process between state 1 and 2. Consider the generally applicable statement,⁸

$$ds \geq \frac{d'Q}{T} \quad (28)$$

which refers in this instance to the entropy of, heat addition to, and temperature of the thermodynamic system as previously defined. This, together with the first law as stated in equation 6 leads to the inequality,

$$d'w \leq Tds - du. \quad (29)$$

Since $d'Q$, the heat added to the system, is positive, ds is also positive (from inequality 28). Also, according to the heat conduction model of Section 3.6.2, temperature, T , is a monotonically increasing function during the heat transfer process. It follows that

$$Tds \leq T_{AVG}(t_3) ds, \quad (30)$$

where $T_{AVG}(t_3)$, it will be recalled, is the system temperature at the end of the process. Therefore, it can be said from 29 and 30 that,

$$d'W \leq T_{AVG}(t_3) ds - du. \quad (31)$$

Inequality 11 follows directly, provided that the reservoir temperature, T_0 , is greater than $T_{AVG}(t_3)$. In view of the heat conduction model postulated, it is logical to assume that this is the case.

It is concluded that both inequalities 11 and 31 give valid upper limits on work performed by the thermodynamic system during the heat transfer process. In fact, inequality 11 is valid for any "reservoir" temperature, T_0 , which is at least as great as $T_{AVG}(t_3)$. If in the numerical evaluation of Section 3.6.4, we substitute $T_{AVG}(t_3) = 1408^\circ\text{F}$ for the value originally used (2818°F), the calculated work ($W_{1 \rightarrow 2}$) reduces from 16,300 in-lb/lbm to 47 in-lb/lbm. The value of $W_{2 \rightarrow 3}$ (15.6 in-lb/lbm) is unchanged. The total acoustic work estimate has been reduced from 16,316 to 63 in-lb/lbm and is an upper limit in the same sense as the original value.

Inertial Work

The model² also defines an upper limit on the "inertial work" - the work energy associated with sodium vapor expansion following emission of the initial shock wave. The derivation of this value is given in Reference 2 and is not repeated here. In the original analysis, the inertial component comprises about 10% of the total work energy release. As indicated in the previous section, that model highly overestimates the acoustic work component, and the ratio should be reversed ("acoustic work" comprises less than 10% of the total).

Further development of this concept as a tool for design or DBA related analysis is not recommended. In order to achieve the degree of realism necessary for correlation of experimental results, design or DBA analysis a dynamic model must be employed. While this attempts to define the true scale on which the violent boiling interaction takes place, it does so in a somewhat arbitrary way. This is acceptable in deliberately conservative scoping studies, such as that of Reference 2, but would not be acceptable in the aforementioned applications.

7.3.4.3.4 GE Spherical Geometry Thermodynamic Model

The slab geometry model described in the previous section has application to situations in which plane surface contact between overheated fuel/clad and coolant can be postulated. The spherical version is based on the concept of a single fuel (or clad) particle instantaneously immersed in a large body of liquid sodium, and is therefore applicable where such surface contact does not exist. Except for geometric effects, the spherical model relies on the same development as does the slab geometry version. A single spherical particle is considered, and the analysis covers just the mechanical energy release up to the time of initial film blanketing of the particle.

Interactions between particles are not covered in this model. A simplistic view of such interaction might be that the spherical waves generated by a large number of particles in a single plane add up to a plane wave of the same intensity (in-lb/in²) as the single particle wave. In reality the situation is considerably more complicated. The particles

are not arranged in a single plane and they do not generate spherical waves in such a coherent manner that the result is a simple plane wave. In addition, the cut-off of heat transfer by vapor blanketing may be significantly delayed by such interference effects. A simple study based on a system comprised of several particles would provide useful insight to the significance of wave interference effects.

The single-particle spherical model is described below. Results of some parameter studies utilizing the model are also given.

Spherical Geometry Model

The basic difference between this and the slab geometry model of Section 7.3.4.3.3 is in the heat conduction equation. The thermodynamic analysis itself is not changed. In the spherical case the thermodynamic system is bounded internally by the spherical fuel particle surface. The external boundary is determined by the location of the shock front at the time that vapor blanketing occurs around the fuel particle.

The coolant temperature profile is given by,

$$T(r,t) = (T_{\text{interface}} - T_{\text{co}}) \frac{a}{r} \cdot \text{erfc} \left(\frac{r-a}{2\sqrt{a_c t}} \right) + T_{\text{co}}, \quad r \geq a, \quad (1)$$

where

- $T_{\text{interface}}$ = fuel-coolant interface temperature,
- a = fuel particle radius,
- r = radius,
- a_c = thermal diffusivity of the coolant,
- t = time
- T_{co} = initial coolant temperature.

This solution follows from the boundary condition that $T_{\text{interface}}$ is constant with time. The interface temperature is calculated in accordance with the slab geometry model, that is,

$$T_{\text{interface}} = \frac{T_{\text{fo}} - T_{\text{co}}}{1 + \frac{K_c}{K_f} \sqrt{a_f/a_c}} + T_{\text{co}}, \quad (2)$$

where,

- T_{fo} = initial fuel temperature,
- K_c, K_f = thermal conductivities of coolant, fuel, respectively,
- a_f, a_c = thermal diffusivities of fuel, coolant, respectively.

The temperature rise $(T - T_{\text{co}})$ at any time approaches zero with increasing radius. We define for the purpose of our analysis a "thermal front", which is a finite distance at which the temperature rise is essentially zero. It is assumed here that the error function becomes zero when its argument is equal to 4. Thus the thermal front is given by,

$$r_{\text{th}}(t) = a + 8 \sqrt{a_c t}. \quad (3)$$

and its propagation velocity is,

$$U_{th} = \frac{dr_{th}}{dt} = 4 \sqrt{a_c t}. \quad (4)$$

The results obtained are quite insensitive to the choice of constants in determining the "thermal front". Tests in which the value varied from 3.0 to 8.0 showed no difference.

On the basis of the temperature equation and the definition of the thermal front as expressed in equation 2, the average temperature of *heated* coolant is then given as follows:

$$\bar{T}_{th}(t) = \frac{3a \sqrt{a_c t} (T_{interface} - T_{co})}{(a + 8 \sqrt{a_c t})^3 - a^3} \left[\frac{2a}{\sqrt{\pi}} + \sqrt{a_c t} \right] + T_{co} \quad (5)$$

As shown in equation 4, the thermal front velocity decreases with time. The shock front is assumed to propagate with a constant velocity, U , equal to the sonic velocity. Hence, the shock front rapidly moves ahead of the thermal front, and the average temperature of *pressurized* coolant, given in terms of the volume ratio of heated to pressurized coolant, is,

$$\bar{T}_{press}(t) = \frac{V_{therm}(t)}{V_{press}(t)} \left[T_{therm}(t) - T_{co} \right] + T_{co} \quad (6)$$

The thermodynamic system is heated at constant volume to the point at which its average pressure is equal to the saturation pressure at the fuel-coolant interface. At this point it is assumed that vapor blanketing prevents further transfer of energy from the fuel particle to the coolant. The system then expands isentropically to its initial pressure state, P_o , and the work of this expansion is considered to be the shock damage potential. The temperature change in a system heated at constant volume is related to its pressure change by

$$T - T_{co} = \frac{\beta_T}{\alpha} (P - P_o), \quad (7)$$

where

β_T = the isothermal compressibility, and
 α = the temperature coefficient of expansion.

Therefore, the average temperature of the system at the beginning of the expansion process is defined as in equation 8:

$$\bar{T}_{press} = T_{co} + \frac{\beta_T}{\alpha} \left[P_{sat}(T_{interface}) - P_o \right] \quad (8)$$

The work potential is then:

$$W = \int_{P_{sat}}^{P_o} P dv \approx \frac{1}{2} \beta_s v (P_{sat} - P_o) (P_{sat} + P_o), \quad (9)$$

with

β_s = the adiabatic compressibility, and
 v = specific volume.

This expresses the work in terms of energy per unit mass of fluid contained in the thermodynamic system. The remainder of the analysis is devoted to determining the extent of the thermodynamic system based on the previously mentioned definition (i.e., the position of the shock front when vapor blanketing occurs). We need to determine two points on the time scale. The first of these is the time at which the shock front separates from the thermal front. This is considered to be the point in time at which the thermal velocity is equal to the acoustic velocity. The time and position associated with this event are given in equations 10 and 11.

$$t_{sep} = 16 \alpha_c / U^2 \quad (10)$$

$$V_{sep} = a + 8 \sqrt{\alpha_c t_{sep}} \quad (11)$$

The second event occurs at "saturation" time (i.e., when vapor blanketing occurs at the fuel-coolant interface). At this time, the *average* pressure behind the shock front is equal to saturation pressure at the interface. In accordance with our definition of the thermodynamic system, the average pressure behind the shock front at this time is also the average pressure of the system at the beginning of the isentropic expansion. The system temperature at this point is defined in both equations 6 and 8. Therefore,

$$\frac{\beta_T}{\alpha} \left[P_{sat}(T_{interface}) - P_o \right] = \frac{V_{therm}(t_{sat})}{V_{press}(t_{sat})} \left[T_{therm}(t_{sat}) - T_{co} \right] \quad (12)$$

where t_{sat} is the time to saturation. In addition, the system boundary is

$$r_{sat} = r_{sep} + U(t_{sat} - t_{sep}) \quad (13)$$

The combination of equations 5, 12, and 13 yields:

$$\frac{3a \sqrt{\alpha_c t_{sat}} (T_{interface} - T_{co})}{\left[r_{sep} + U(t_{sat} - t_{sep}) \right]^3 - a^3} \left[\frac{2a}{\sqrt{\pi}} + \sqrt{\alpha_c t_{sat}} \right] = \frac{\beta_T}{\alpha} \left[P_{sat} - P_o \right] \quad (14)$$

An iterative procedure leads to a solution of (14) for the "saturation" time t_{sat} . Then equation 13 gives the boundary of the thermodynamic system, and we are able to normalize the work energy result in terms of energy per unit surface area of fuel, or energy per unit mass of fuel.

The model attempts to account for the effect of liquid superheat at the fuel/clad-coolant interface by a modification of the saturation pressure calculation. Thus, P_{sat} is given by,¹¹

$$\log(P_{sat}) = 4.521 - 9396/(T_{interface} - T_{sh} + 460) \quad (15)$$

where: T_{sh} = the amount of superheat, °F, and P_{sat} is given in atmospheres. Equation 15 implies that the liquid, which vaporizes at a temperature of $T_{interface}$ °F, is in a superheated state at P_{sat} atmos.

The effect of superheat, according to this model, is then to prolong the period of heat transfer from fuel (or cladding) to sodium, and to reduce the maximum pressure of the thermodynamic system.

Discussion of Results

Figure 7-42 shows the effect of particle size. Work energy is given in both in-lb/sq in. of particle surface and in Joules/gm of particle. In the case shown, the initial fuel temperature was 6000°F, the initial coolant temperature was 1400°F and there was no superheat at the fuel-coolant interface. The work in units of in-lb/sq in. approaches asymptotically a value which is equal to that obtained in the slab geometry analysis. For all practical purposes the utilization of a very large particle radius in the spherical geometry analysis gives the same results as the slab geometry analysis.

Figures 7-43 and 7-44 indicate the variation of work energy with initial fuel and coolant temperatures.

Fragmentation studies show that a large fraction of the fuel particles will have diameters in the range, 10 mils to 100 mils. Thus, the results of these calculations suggest work energy levels on the order of 0.01 in-lb/in².

Figure 7-45 shows the calculated work energy of a clad-sodium thermal interaction. This is determined from the same model, by replacing the UO₂ physical properties with those of cladding material. Comparison of Figures 7-42 and 7-45 shows that clad material at 3000°F generates about 1/3 as much work energy as UO₂ at 6000°F. This is not a significantly different level, and the result suggests that clad-sodium interaction could generate a damaging pressure pulse even before UO₂ is expelled from a failed fuel pin.

Figure 7-42 Work Potential vs. Fuel Particle Size

Figure 7-43 Work Potential vs Initial Fuel Temperature

Figure 7-44 Work Potential vs Initial Coolant Temperature

Figure 1-45 Work Potential of Clad Sodium Interaction vs Particle Size

Some parametric studies of the variation of work energy with superheat were attempted with this model, but the results were contrary to logical expectation. The model predicted that work energy decreases, which is an unreal result. Superheat at the fuel/clad-sodium interface prolongs the heat transfer period, which increases the thermal energy transferred to coolant and should also increase work energy. Figure 7-46 shows the effect on transferred energy as predicted by the model. The unreal behavior of the model in this regard is associated with the fact that the thermodynamic system changes with superheat. Examination of the model equations will show that, for expansion to one atmosphere, the work (in-lb/in²) is very nearly proportional to the product $P_{sat}^2 \cdot V$,

$$W \propto P_{sat}^2 \cdot V, \quad (16)$$

when all parameters except the superheat are held constant. In equation (16) P_{sat} refers to coolant saturation pressure at $(T_{interface} - T_{superheat})$, and V is the volume of the thermodynamic system, as implicitly described by V_{sat} of equation (13). Figure 7-47 shows the variation of the factors in equation 16 with superheat for a particle radius of 0.100-in., fuel temperature of 6000°F, and coolant temperature of 1400°F.

The volume of the thermodynamic system (V) is seen in Figure 7-47 to increase as T_{sh} increases. This trend is predictable from the basic concept, in which the thermodynamic system is defined to be the body of coolant behind the shock front at the time that vaporization occurs at the interface. At the same time, P_{sat} — the pressure of the thermodynamic system — decreases with superheat (see equation 15). In the particular example shown the net result, which is given in equation 16, is a decrease in work energy, W , as Figure 7-47 shows.

Modified Spherical Geometry Model

A logical model change to avoid this atypical behavior is to define a thermodynamic system which does not change with superheat. Since the original definition was somewhat arbitrarily chosen, this is equally acceptable.

Figure 7-46 Effect of Superheat on Transferred Energy

Figure 7-47 Unrealistic Calculation of Effect of Superheat on Work Energy as Predicted by Original Spherical Geometry Thermodynamic Model.

The newer version utilizes a definition of the thermodynamic system which is dependent on the coolant/fuel mass ratio. The latter quantity is treated as an input parameter. The concept is illustrated in Figure 7-48. The thermodynamic system includes the coolant bounded internally by the particle surface (radius, a) and externally by the surface of radius, r_c . The radius, r_c , is dependent only on the mass ratio of coolant to fuel, thus the thermodynamic system does not change as other parameters (e.g., superheat) are varied.

Except for this difference, the constant mass model is like the original model. The saturation pressure is determined as before, and energy transfer continues until the average pressure behind the wave front (i.e., in the zone between $r = a$ and $r = r_{sat}$) is equal to the saturation pressure. At this point, the average pressure in the thermodynamic system, P_c , is determined as

$$P_c = \frac{r_{sat}^3 - a^3}{r_c^3 - a^3} (P_{sat} - P_o) + P_o \quad (17)$$

The work of isentropic expansion of the system from P_c to P_o is then taken as the work energy of the process.

Figures 7-49 through 7-55 give the results of studies using the constant mass model. It will be seen from comparison of Figures 7-42 and 7-55 that the constant mass model gives lower work energy estimates by a factor of ~10. This difference is due to the coolant/fuel mass ratio of 50.0 chosen for this study. The work energy decreases as the mass ratio is increased from its minimum possible value (this is defined below). Figure 7-50 illustrates this trend for a particular set of initial conditions.

Figure 7-48 "Constant Mass" Thermodynamic Model

Figure 7-49 Work Potential vs Particle Size (Constant Mass Model)

Figure 7-50 Work vs. Coolant/Fuel Mass Ratio

Figure 7-51 Work Potential vs. Initial Fuel Temperature (Constant Mass Model)

Figure 7-52 Work Potential vs. Initial Coolant Temperature (Constant Mass Model)

Figure 7-53 Effect of Superheat on Work Potential (Constant Mass Model)

Figure 7-54 Effect of Superheat At Variable Particle Sizes

Figure 7-55 Work Potential of Clad Sodium Interaction vs. Particle Size (Constant Mass Model)

It is clear from Figure 7-48 that the model applies only when $r_c \geq r_{sat}$, and this criterion establishes the minimum possible mass ratio. A ratio of 50.0 was chosen for the parameter studies because this is the smallest value which meets the "minimum ratio" criterion for the entire range of study. A realistic mass ratio would be much lower (the normal average over the core is ~ 0.1). The trends indicated in these studies would not be significantly altered by using a more realistic mass ratio.

In Figure 7-51 work energy, predicted by the constant mass model, is seen to increase with increasing fuel temperature. The same trend is found with the original model (see Figure 7-43) but the shape of the curve is quite different. An even greater contrast appears when Figure 7-44, which shows work versus coolant temperature as calculated by the original model, is compared to Figure 7-52, which shows the work versus coolant temperature as calculated by the modified model. The original model shows that work increases with coolant temperature. The modified model shows that work decreases with coolant temperature.

Both variations of the model concept have a logical basis. Aside from the obvious behavior with respect to variations of superheat, no attempt has been made to determine which model is "right." The comparison really serves to illustrate the precision (or lack thereof) implied in the model concept, and should be taken in that light. Further attempts to justify one or the other of the two results on the basis of real behavior would only obscure the fact that the thermal interaction process is not very accurately described by this modeling approach.

The effect of superheat is described in Figures 7-53 and 7-54 and the trend exhibited is intuitively acceptable. Figure 7-55 shows work energy for a clad-coolant interaction to complete the comparison of the original and the constant mass versions of the model.

The "minimum possible mass ratio" discussed above, can be interpreted to define the minimum particle spacing for which there is no interaction between individual particles. Since mass ratios well below these minimum values are probable in many cases, it is clear that particle interactions must be treated in order to achieve more realistic predictions. Some insight as to the effects of interactions could be developed by superimposing the results of two or three single particle calculations from this model. However, this would be of limited value.

From the need for a multiple particle modeling approach, and the difficulties mentioned in reconciling the behavior of this thermodynamic model with reality, it is concluded that the equilibrium thermodynamic model concept should be abandoned at this point. Efforts to develop more precise models, which will correlate experimental results should use concepts which more realistically describe the fluid response.

7.3.4.4 Hydrodynamic Models

7.3.4.4.1 Introduction

The thermodynamic limit modeling approach yields ultra-conservative estimates of the work available for damage to adjacent fuel pins or channel walls. With the incentive for more realistic modeling, recent work has been based on the generation and propagation of pressure pulses in a fluid. The geometric arrangement of fuel and sodium in a realistic situation is far from clean. The models can only crudely approximate the geometry. Some models have been based on surface contact between a large body of fuel and another large body of sodium which leads to a slab geometry approach. Other models assume that fuel particles are finely dispersed in a body of coolant so that the response is that of a bulk heated fluid. It is possible that both of these concepts are necessary in the analysis of a DBA or single assembly accident. One can visualize the generation of a plane surface contact in the case of reentry of condensed coolant into a channel which has overheated and melted. On the other hand, single or multiple fuel pin failure may result in the injection of fuel particles into the coolant stream and later fragmentation of the particles so that the bulk heated fluid concept is more applicable. The modeling concepts described here are all based on solutions of the Wave equation, which implies the assumption that the waves generated are of infinitesimal amplitude. The coupling of hydrodynamic response, based on conservation of mass and momentum, with energy considerations either in the form of heat conduction or in the form of the first law of thermodynamics leads to the solutions described below. The first of these calculates the pressure wave generation and propagation in slab geometry and the second set of equations is based on the current modeling concept presently in use at ANL. The latter is based on bulk heating of the fluid which is subjected to either a linear or spherical constraint by the surrounding unheated fluid.

7.3.4.4.2 Plane Geometry Wave Solution

The slab geometry model described in Section 7.3.4.3.3 attempts to determine a thermodynamic limit on work energy generated in a body of liquid coolant which has instantaneous plane surface contact with heated fuel or clad material. The model shown in this section was developed by GE and describes in a more precise manner the same situation. The simultaneous solution in plane geometry of hydrodynamic, temperature, and state equations approximates the pressure wave generated and moving away from the heated interface.

The describing equations are:

1. Mass Continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0 \quad (1)$$

where

ρ = fluid density
 u = fluid velocity

2. Conservation of Momentum

$$g_c \frac{\partial p}{\partial x} + \rho u \frac{\partial u}{\partial x} + \rho \frac{\partial u}{\partial t} = 0 \quad (2)$$

3. Conservation of Energy

$$T = T_{co} + (T_{interface} - T_{co}) \operatorname{erfc} \frac{x}{2\sqrt{\alpha t}} \quad (3)$$

4. Equation of State

$$\rho = \rho_0 + b_p (p - p_0) + b_T (T - T_0) \quad (4)$$

The hydrodynamics equations (1) and (2) are precise. Equation 3 expresses in only an approximate way the first law of thermodynamics, since it shows only heat conduction and ignores the conversion of thermal to mechanical energy. The equation of state is linearized (the coefficients b_p and b_T are treated as constants). In order to obtain an analytical solution of this system of equations, (1) and (2) are linearized in the manner leading to the wave equation. Specifically, the terms $\mu \frac{\partial \rho}{\partial x}$ of equation (1) and $\rho u \frac{\partial u}{\partial x}$ of equation (2) are dropped.

The boundary condition $\left. \frac{\partial p}{\partial x} \right|_{x=0} = 0$

= 0, is used to obtain a unique solution. In addition, the solution is required to exist for all $x > 0$.

A typical result is shown in Figure 7-56 where the wave profile is given for two different points in time. The greater than peak pressure is determined entirely by the interface temperature, and is not attenuated with time because dissipative mechanisms are not included in the model. The "energy" of the wave includes:

$$\text{potential energy} = \frac{1}{2} \beta_s \int (p(x)^2 - p_0^2) dx, \text{ in-lb/in}^2$$

and

$$\text{kinetic energy} = \frac{1}{2} \int \rho u^2 dx, \text{ in-lb/in}^2$$

Table 7-12 shows the wave energy, and total heat transferred from fuel to coolant as functions of time.

For the initial conditions (sodium temperature, 1400°F and UO_2 temperature 6000°F) of this case, the thermodynamic model (Section 7.3.4.3.3) predicts that the *average* pressure behind the wave front is reduced to the saturation pressure at the interface after 0.575 μ -sec. The wave profile at that instant is shown in Figure 7-56. Table 7-12 shows that the wave energy changes very slowly, even if heat transfer persists for a much longer time.

A detailed derivation of the model equations, determined by the simultaneous solutions of equations 1 through 4, is found in Appendix C. The linearization of equations 1 and 2 makes it possible to obtain an analytic solution, thus avoiding the stability and truncation error problems usually encountered in numerical solutions.

The hydrodynamic model concept appears at this time to be the most suitable one for carrying forward the technology of thermal interaction modeling. The one just described is, in its present form, applicable to situations in

Figure 7-56 Pressure Wave Shape

Table 7-12. Wave Energy and Total Energy Versus Time

UO₂ Initial Temp. = 6000°F, Na Initial Temp. = 1400°F

Time (seconds)	Total Thermal Energy Transferred (in-lb/in ²)	Wave Energy (Kinetic + Potential) (in-lb/in ²)	Wave/Total (%)
1.0×10^{-8}	3.71	.0738	2.0
3.0×10^{-8}	6.43	.0872	1.4
1.0×10^{-7}	11.7	.103	.88
3.0×10^{-7}	20.3	.120	.59
575×10^{-6}	28.1	.129	.46
1.0×10^{-6}	37.1	.138	.37
3.0×10^{-6}	64.3	.156	.24
1.0×10^{-5}	117.3	.179	.15

which a plane interface between overheated fuel or cladding and coolant can be postulated (e.g., coolant re-entry to a voided channel where pins are overheated but still intact) Refinements which should be considered are

- 1 The replacement of the heat conduction equation with an energy balance equation which accounts for heat conduction and the conversion of thermal energy to mechanical energy (and vice versa)
- 2 Inclusion of reflected wave effects.
- 3 The use of a more exact equation of state.

In addition, a simple modification of the model applies to situations in which fuel or cladding particles are dispersed in a finite volume of coolant. This modification could be accomplished by replacement of the heat conduction equation (3) with a two-region conduction equation in which one region has a volumetric heat source. This concept—a bulk heated zone with acoustic constraint—is employed by ANL in the hydrodynamic model described in Section 7.3.4.4.3. However, the ANL model does not take account of heat conduction into the restraining fluid, which would tend to reduce the pressure generation. This recommended modification of the GE plane geometry model could be accomplished analytically (numerical solution would not be required) and would provide useful insight to the degree of conservatism inherent in the ANL model.

The continued development of both hydrodynamic models (the one described here, and the ANL model to be discussed in the next section) is recommended.

7.3.4.4.3 ANL Analytical Model (3,4,12,13)

The concept presently in use at ANL for modeling of the fuel-sodium thermal interaction is based on the system illustrated in Figure 7-57. A specified volume of sodium coolant is said to contain a uniform, volume-distributed heat source. This represents the presence of fuel or clad particles, and implies that they have been uniformly and instantaneously dispersed in the coolant. Without any external constraint the heated volume would expand, and no pressure buildup would occur. In reality, the heated fuel is surrounded by unheated coolant, which acts to restrain the expansion, thus generating pressure. The balance of these competing forces (thermal expansion versus constraint) determines the pressure versus time history in the system.

Figure 7-57 ANL Bulk Heating Model

The entire solution is given by the following system of equations:

1. Heat transfer from individual particles to coolant:

$$q'' = f(T_{\text{fuel}}, T) \quad (1)$$

2. Conservation of Energy (1st Law of Thermodynamics):

$$\frac{dH}{dt} = \frac{dQ}{dt} + \frac{Vdp}{dt} \quad (2)$$

3. Equation of State:

$$H = H(p, V) \quad (3)$$

4. Constraint.

$$p = f\left(V, \frac{dV}{dt}, \frac{d^2V}{dt^2}\right) \quad (4)$$

The rate of heat generation, dQ/dt , is related to the particle surface heat flux, q'' , by the particle size and distribution (e.g., mass ratio and mixing zone length).

The constraint is related to the compressibility and inertia of the surrounding medium. In order to simplify the solution, the system is solved in two different regimes: 1) "acoustic" constraint, and 2) "inertial" constraint. In the first regime, the fluid response is assumed to be described by the wave equation, so the pressure is a linear function of the rate of expansion,

$$\frac{dZ}{dt} = \frac{p(t) - p_0}{\rho_0 C_0} \quad (5)$$

This solution applies only up to the time that wave reflections might appear at the heated zone interface, or for

$$t < \frac{2L}{C_0} \quad (6)$$

Waves reflected from a free surface at the end of the channel would act to "unload" the pressure, thus possibly allowing vapor formation to blanket the heat transfer surface and stop the interaction. Reflections from rigid surfaces would have the opposite effect – a pressure increase. Real channel geometry is sufficiently complicated that the effect of reflections is difficult to predict.

The inertial approximation assumes the surrounding fluid to be incompressible so that

$$\frac{d^2 Z}{dt^2} = \frac{p(t) - p_0}{\rho_0 L} \quad (7)$$

This is a valid assumption when t is larger than $2L/c$, and the fluid velocity is small compared with acoustic velocity

In addition to the linear constraint model just described, the concept has been applied in spherical geometry to simulate a three dimensional constraint.

The important basic assumptions of this model are:

1. There is no heat transfer across the interface between heated and unheated zones.
2. The heated coolant is in a state of uniform thermodynamic equilibrium.

In addition to the assumptions, the arbitrary parameters of the model are:

1. Initial conditions (mixing length, particle size, fuel/coolant mass ratio).
2. Heat transfer rate (fragmentation rate, heat transfer mechanisms, etc.).
3. Limiting conditions (unloading wave or other mechanisms which terminate the interaction).

The model is theoretically well founded. If proper choices are made of the parameters, it should predict very well the pressure generation for most conceivable situations of interest.

A description of the model, and some results of its application were published in ANL 7561.³ These data were based on the properties of water and served primarily to illustrate the method. A later presentation⁴ provided some data on sodium. Figures 7-58 through 7-61 show some of these results. The parameter, a , is the ratio of channel flow area to coolant mass in the heated zone, as illustrated in Figure 7-62. Figures 7-62 and 7-63 relate the model parameters, such as a and q , to primary quantities such as particle surface heat flux and coolant fuel mass ratio. Figure 7-64 shows the "acoustic period" for both water and sodium as a function of distance.

The original model was based on an assumed constant heat transfer rate. It has since been revised to include a time-dependent heat transfer model and to incorporate some corrections in application of the basic concept. The revised model is described in Reference 12 but no results have been published.

The same model concept was used by PNL for analysis of work energy release in a DBA.⁶

The continued development of this model concept is recommended. Refinements which should be incorporated in the model include

- 1 The effects of non-condensable gases. In any failure which allows fuel and sodium to interact either fill gas or fission gas will be expelled along with fuel particles. This gas will act to reduce the effect of thermal interaction by blanketing or partially blanketing heat transfer surfaces and by cushioning the initial expansion of heated sodium.
- 2 The inclusion of acoustic wave reflections to correlate capsule test data, which must be affected by such reflections
- 3 Accounting for the effects of structural expansion under pressure loading. This may, at least in the acoustic constraint regime, be incorporated into Item 2

In addition, the effects of the two basic modeling assumptions listed above (1 No heat transfer from the heated zone; 2. Uniform equilibrium in the heated zone) should be investigated. The application of the GE plane geometry hydrodynamic model for this purpose is discussed in the previous section (7.3.4.4.2)

Figure 7-58 Pressure Generated by Addition of Heat to Sodium under Spherical Acoustic Constraint

Figure 7-59 Pressure Generated by Addition of Heat to Sodium under Spherical Acoustic Constraint

Figure 7-60 Pressure Generated by Addition of Heat to Sodium under Spherical Acoustic Constraint

Figure 7-61 Pressure Generated by Addition of Heat to Sodium under Spherical Acoustic Constraint

Figure 7-62 Coolant Mass Model

Figure 7-63 Relationship of Heat Generation in Coolant and Fuel Particle Size

Figure 7-64 Acoustic Periods for Water and Sodium

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APPENDIX A (J. M. Cole)**INTRODUCTION**

Appendix A contains the derivations for the analysis employed in the computer programs for investigating structural response to dynamic loading. It also includes the listing of the computer program and graphs of the results from limited parameter studies of the channels and fuel pins. Table A-1 indicates the parameters and range used in these studies

Table A-1.**Parameter Studies for Hexagonal Channel Duct**

Parameter	Values			Units
Span	2.1	2.35	2.6	in.
Thickness	0.1	0.14	0.18	in
Yield Strength	14,000	24,000	34,000	psi

Parameter Studies for Fuel Pin

Parameter	Values			Units
Span	4.6	8.10	12.14	in.
Yield Strength	13,000	23,000	33,000	psi

1.0 CHANNEL CONTAINMENT CAPABILITIES FOR INTERNAL SHOCK LOADS

For absorption of energy from an internal load where the walls of a hexagonal duct are given an instantaneous velocity the load distribution may be approximated as a uniform loading. This loading will not be uniform due to the larger resistance to deformation at the corners relative to the resistance at the center of the flats. The larger deceleration at the corners will tend to peak the load at the corners, resulting in an increase in the proportion of the load carried as a membrane load in the walls and decrease the proportion carried by bending of the walls. The actual loading should result in smaller values of radial deflection at the center of the flat, smaller total strain at the corners and greater energy absorption than the equivalent uniform loading. An equivalent uniform loading will be employed in this model for study of a plane symmetrical, impulsive load to the duct wall where the axial extent of the loading is of such length and sufficiently uniform to justify the use of a plane strain solution.

The derivations utilized for the computer solution of an internal shock pulse acting on a hexagonal channel duct are provided initially. The computer program is labeled 'HEXD', for identification only. Illustrative results from this program are shown on Figures A-1, A-2 and A-3. The listing of the computer and graphs of the solutions obtained follow the derivations.

Figure A-1 Channel Response to Internal Load - Varying Thickness

Figure A-2 Channel Response to Internal Load - Increased Yield Strength

HEX-INTERNAL
PRESSURE

SS 304
F = 2.6 IN.

T = 1,000°F
E = 22.6(10)⁶ PSI

$\delta y = 14,000$ PSI

" = 24,000 PSI

Figure A-3 Channel Response to Internal Loads - Varying Yield Strength and Wall Thickness

Discussion of Derivation

By reason of its symmetry, the hexagonal duct may be solved as a slab with fixed supports and a plane stress solution initially employed can be transformed to a plane strain solution by substitution of suitable elastic constants. The principal regime of interest is concerned with the solution of the slab while portions are stressed beyond the elastic limit but the elastic solution serves as an initial point. A rigorous solution is probably not justified in the existing status, and certainly will not be attempted. A perfectly elastic-plastic material will be assumed (strain hardening neglected) but membrane action will not be neglected for large deformations.

1.1 ELASTIC CHANNEL RESPONSE

The elastic solution is well known and illustrated by Figure A-4 and the following equation for elastic channel response.

Legend:

- σ = Stress – psi
- M = Moment – in. lb.
- b = Width – in. – (1)
- h = thickness – in.
- p = pressure – psi
- S = Span – in
- U = Energy – in. lb./in.²
- δ = Deflection – in.
- θ = Deflection – radians
- R = Radius – in.
- ϵ = strain
- x = locus – in., abscissa
- E = Adjusted Youngs Modulus of Elasticity – psi
- ν = Poisson's Ratio
- N = axial force – lb.
- I = Moment of Inertia – (in.)⁴

Subscripts:

- e = elastic
- p = plastic
- y = yield limit

Equation's for elastic channel response:

$$\begin{aligned}
 u &= \frac{Mh}{2I} + \frac{N}{h} = \frac{6M}{h^2} + \frac{\sqrt{3pS}}{2h} \\
 \sigma_y &= \frac{6M}{h^2} + \frac{\sqrt{3pS}}{2h} = \frac{pS^2}{2h^2} + \frac{\sqrt{3pS}}{2h} \\
 p_y &= \frac{2h^2 \sigma_y}{S(S + \sqrt{3h})} \quad (1.1.1) \\
 y &= \frac{P}{2EI} \left(\frac{Sx^3}{6} - \frac{x^4}{12} - \frac{S^2 x^2}{12} \right)
 \end{aligned}$$

Figure A-4 Geometry, Loading and Moment Diagram of Elastic Regime of Channel Response to Internal Loads.

$$\begin{aligned}
 U &= \frac{1}{2} \int_0^S p y dx \\
 &= \frac{p^2}{2Eh^3} \int_0^S \left(Sx^3 - \frac{x^4}{2} - \frac{S^2 x^2}{2} \right) dx \\
 &= \frac{p^2}{2Eh^3} \left[\frac{Sx^4}{4} - \frac{x^5}{10} - \frac{S^2 x^3}{6} \right]_0^S \\
 &= - \frac{p^2 S^5}{120Eh^3} \text{ per side or } = - \frac{p^2 S^4}{120Eh^3} \text{ in. lb./in.}^2 \quad (1.1.2)
 \end{aligned}$$

$$\delta_y = - \frac{p^2 S^4}{32Eh^3} \quad (1.1.3)$$

$$\epsilon = \frac{-\sigma_y}{E} \quad (1.1.4)$$

Note: The equations utilized for the computer solution are identified by number.

1.2 CHANNEL RESPONSE WHEN TWO PLASTIC HINGES FORM

For loading beyond the elastic limit when a plastic hinge gradually develops at the support (corner) there is a stress re-distribution at the cross sections that are yielding plastically and yielding extends outward from the support. An approximate solution for the load that causes a fully plastic hinge at the support will be attempted as an interim solution, before attempting the limit load. At the plastic hinge, the stress-distribution will be approximated as shown on Figure A-5.

The extent of the plastic zone(s) can be approximated by geometrical model as illustrated on the accompanying figures and equations for two plastic hinges.

The moment at the support, $PS^2/12$, is equated to the resistance of the plastic stress distribution, $\sigma_y \left(\frac{1-r^2}{4} \right) h^2$

to obtain the loading for this intermediate condition. The interface between the elastic and plastic zones is determined by equating the elastic resistance of the cross section to the moment equation.

The deflection and the energy absorbed can be approximated by treating the elastic portion of the beam as partially restrained between the plastic hinges, for the load greater than the elastically supported load.

Two -- Plastic hinges at supports.

Moment at hinge (Resisting)

$$M_p = \frac{h^2 (1-r^2) \sigma_y}{4}$$

$$\frac{\sqrt{3pS}}{2} = \sigma_y r h \quad (\text{Axial load})$$

$$p = \frac{2\sigma_y r h}{3S} \quad (1.2.1)$$

Figure A-5 Loading and Moment Diagrams for the Plastic Regime of Channel Response to Internal Loads

$$M_h = \frac{pS^2}{12} = \frac{\sigma_y r h S}{6\sqrt{3}}$$

$$\frac{h^2 (1-r^2) \sigma_y}{4} = \frac{\sigma_y r h S}{6\sqrt{3}}$$

$$r^2 + \frac{2rS}{3\sqrt{3}h} = 1$$

$$\left(r + \frac{S}{3\sqrt{3}h}\right)^2 = 1 + \left(\frac{S}{3\sqrt{3}h}\right)^2$$

$$r = \sqrt{1 + \left(\frac{S}{3\sqrt{3}h}\right)^2} - \frac{S}{3\sqrt{3}h}$$

(1.2.2)

$$M_e = \frac{\left(\sigma_y - \frac{\sqrt{3}pS}{2}\right)h^2}{6} = \frac{pSx}{2} - \frac{px^2}{2} - \frac{pS^2}{12}$$

$$x^2 - Sx + \left[\frac{S^2}{6} - \frac{\left(\sigma_y - \frac{\sqrt{3}pS}{2}\right)h^2}{3p} \right] = 0$$

$$x = \frac{S \pm \sqrt{S^2 - 4 \left[\frac{S^2}{6} - \frac{\left(\sigma_y - \frac{\sqrt{3}pS}{2}\right)h^2}{3p} \right]}}{2}$$

(1.2.3)

Treating the elastic part of the beam as partially restrained results in the following:

$$\theta_x = \frac{p(S-2x)^3}{2Eh^3} - \frac{6M_e(S-2x)}{Eh^3} \quad (1.2.4)$$

$$\epsilon_2 = \epsilon_1 + \frac{h\theta}{2x} \quad (1.2.5)$$

$$EI \frac{d^2 y}{dx^2} = \frac{pSx}{2} - \frac{px^2}{2} - M_0$$

$$EI \frac{dy}{dx} = \frac{pSx^2}{4} - \frac{px^3}{6} - M_0 x + C$$

$$@ x = S/2; \frac{dy}{dx} = 0 \quad \therefore$$

$$C = \frac{M_0 S}{2} - \frac{pS^3}{24}$$

$$EI \frac{dy}{dx} = \frac{p S x^2}{4} - \frac{p x^3}{6} - M_o x + \frac{M_o S}{2} - \frac{p S^3}{24}$$

$$EI y = \frac{p S x^3}{12} - \frac{p x^4}{24} - \frac{M_o x^2}{2} + \frac{M_o S x}{2} - \frac{p S^3 x}{24} + C \quad (1.2.6)$$

$$@ x = 0, y = 0, \therefore C = 0$$

$$EI y = \frac{p S x^3}{12} - \frac{p x^4}{24} - \frac{M_o x^2}{2} + \frac{M_o S x}{2} - \frac{p S^3 x}{24}$$

$$\text{if } M_o = M_e \approx \frac{2}{3} \frac{p S^2}{12} = \frac{p S^2}{18}$$

$$EI y = - \frac{p x^4}{24} + \frac{p S x^3}{12} - \frac{p S^3 x}{24} - \frac{p S^2 x^2}{36} + \frac{p S^3 x}{36}$$

$$= - \frac{p x^4}{24} + \frac{p S x^3}{12} - \frac{p S^2 x^2}{36} - \frac{p S^3 x}{72}$$

$$\Delta U = \frac{2P_1 + \Delta p}{2EI} \int \Delta y dx$$

$$= - \frac{[2P_1 + (P_2 - P_1)] (P_2 - P_1)}{2EI} \int_0^S \left(\frac{x^4}{24} - \frac{S x^3}{12} + \frac{S^2 x^2}{36} + \frac{S^3 x}{72} \right)$$

$$= - \frac{(P_2^2 - P_1^2)}{2EI} \left[\frac{S^5}{120} - \frac{S^5}{48} + \frac{S^5}{108} + \frac{S^5}{144} \right]$$

$$= - \frac{(P_2^2 - P_1^2) S^5}{540EI} \text{ or } - \frac{(P_2^2 - P_1^2) S^4}{540EI} \text{ in. lb./in.}^2 \quad (1.2.7)$$

$$U_2 = U + \Delta U$$

$$EI \delta = \frac{P_2 S^4}{384} + \frac{P_2 S^4}{96} - \frac{P_2 S^4}{144} - \frac{P_2 S^4}{144}$$

$$\delta = \frac{7 P_2 S^4}{1152EI} \quad (1.2.8)$$

1.3 ELASTIC LIMIT AT CENTER OF SPAN

If an additional increment of load is placed on the beam, there will eventually be yielding at the center of the span progressing to a third plastic hinge with increasing load. This limit load is the largest that the flat can carry by beam action. The load required to initiate yielding at the center is obtained by equating the elastic resistance to the center moment. The limit load for two plastic hinges is reached when yielding at the center begins. The derivation follows:

Limit Load for Two-Plastic Hinges – yield stress at center of span.

$$M_p = \frac{h^2 (1-r^2) \sigma_y}{4}$$

$$p = \frac{2\sigma_y r h}{\sqrt{3} S} \quad (1.3.1)$$

$$M_e = \frac{\left(\sigma_y - \frac{\sqrt{3} p S}{2h}\right) h^2}{6} = M_\xi$$

$$M_\xi = \frac{p S^2}{8} - \frac{h^2 (1-r^2) \sigma_y}{4}$$

$$\frac{\left(\sigma_y - \frac{\sqrt{3} p S}{2h}\right) h^2}{6} = \frac{\sigma_y r h S}{4\sqrt{3}} - \frac{h^2 (1-r^2) \sigma_y}{4}$$

$$\frac{2(1-r)}{3} = \frac{r S}{\sqrt{3} h} + r^2 - 1$$

$$r^2 + r \left(\frac{S}{3h} + \frac{2}{3} \right) = 1 + \frac{2}{3}$$

$$\left[r + \left(\frac{S}{2\sqrt{3}h} + \frac{1}{3} \right) \right]^2 = \frac{5}{3} + \left(\frac{S}{2\sqrt{3}h} + \frac{1}{3} \right)^2$$

$$r = \sqrt{\frac{5}{3} + \left(\frac{S}{2\sqrt{3}h} + \frac{1}{3} \right)^2} - \frac{S}{2\sqrt{3}h} - \frac{1}{3}$$

(1.3.2)

$$p = \frac{2\sigma_y r h}{\sqrt{3} S}$$

$$M_e = M_p + \frac{p S x}{2} - \frac{p x^2}{2} = 0$$

$$x^2 - S x + \frac{2}{p} (M_p - M_e) = 0$$

$$x = \frac{S \pm \sqrt{S^2 - 8 \frac{(M_p - M_e)}{p}}}{2}$$

(1.3.3)

$$\theta_x = \frac{p (S - 2x)^3}{2Eh^3} - \frac{6 M_e (S - 2x)}{Eh^3}$$

(1.3.4)

$$\bar{\epsilon}_3 = \frac{h\theta}{2x} + \epsilon_1$$

(1.3.5)

The energy and deflection can be approximated as before.

$$U_3 = U_1 - \frac{(p_3^2 - p_1^2) S^4}{540 EI} \quad (1.3.6)$$

$$\delta = \frac{-7 \cdot p_3 S^4}{1152 EI} \quad (1.3.7)$$

1.4 THREE PLASTIC HINGES, MAXIMUM LOAD FOR BENDING

When fully plastic hinges develop at both the supports and the center of the span, the beam cannot resist greater load and membrane forces must be employed to secure greater resistance to load. The limit load for the beam is obtained by equating the resistance of the plastic stress distribution to the moment of the center of the span. The derivation is:

Three-Hinge – Limit Load for Bending Resistance

$$M_p \quad \frac{\sigma_y h^2 (1-r^2)}{4} = \frac{p S^2}{16}$$

$$p = \frac{2\sigma_y r h}{\sqrt{3} S} \quad (1.4.1)$$

$$\frac{h(1-r^2)}{4} = \frac{rS}{8\sqrt{3}}$$

$$r = \frac{rS}{2\sqrt{3}h} = 1$$

$$\left(r + \frac{S}{4\sqrt{3}h}\right)^2 = 1 + \left(\frac{S}{4\sqrt{3}h}\right)^2$$

$$r = \sqrt{1 + \left(\frac{S}{4\sqrt{3}h}\right)^2} - \frac{S}{4\sqrt{3}h} \quad (1.4.2)$$

$$M_e = \frac{\left(\sigma_y \frac{\sqrt{3} p S}{2}\right) h^2}{6}$$

$$\pm M_e - \frac{p S^2}{16} + \frac{p S x}{2} - \frac{p x^2}{2} = 0$$

$$x^2 - Sx + \frac{p S^2}{8} + \frac{2M_e}{p} = 0$$

$$x = \frac{S - \sqrt{S^2 - 4\left(\frac{S^2}{8} + \frac{2M_e}{p}\right)}}{2} \quad (1.4.3)$$

Adjusting E for plasticity:

$$E^* = 2E \left[\frac{S - x_1 + x_2}{S} \right]$$

$$\theta_x = \frac{p(S-2x_1)^3}{2E^* h^3} - \frac{6 M_e (S-2x_1)}{E^* h^3} \quad (1.4.4)$$

$$\bar{\epsilon}_4 = \epsilon_1 + \frac{h\theta}{2x_1} \quad (1.4.5)$$

$$U_4 = U_1 - \frac{p_4^2 - p_1^2}{540E^* I} \quad (1.4.6)$$

$$\delta = \frac{7 p_4 S^4}{1152E^* I} \quad (1.4.7)$$

Note: To transform the foregoing plane stress solution into a plane strain solution, suitable for a long cylinder, it is necessary to substitute an alternate value for the normal modulus of elasticity:

$$E = \frac{YM}{1-\nu^2} \quad (1.1)$$

For corner radii, it has been observed that a correction of the span for the reduction in length of flat is proper. Comparison with more rigorous methods of analysis indicates that in the elastic range, the effective span of the beam approximates the dimension of the flat between radius centers. See Figure A-6. In this study, it has been assumed that this correction can be extended into the plastic range.

Figure A-6

1.5 MEMBRANE LOADING OF HEX FLAT

For loading beyond the ability of the section resistance by bending, it can be assumed that yielding sufficient to allow increased resistance by membrane action will be necessary. The necessary abutment reactions will be provided by adjacent spans, for uniform loading. The yielding must supply the necessary deformation to sustain the load with yield stress in the member. The differential equation is a simple derivation and can be integrated for a solution of the relation between deflection and load for yield stress in the member.

Membrane Loading — Loads in excess of the beam resistance — see Figure A-7.

$$\frac{d^2 y}{dx^2} = - \frac{p}{T}$$

(Arc of circle for $p = C$)

$$\frac{dy}{dx} = - \frac{px}{T} + C$$

$$@ x = S/2, \frac{dy}{dx} = 0 \quad C = \frac{pS}{2T}$$

$$\frac{dy}{dx} = - \frac{px}{T} + \frac{pS}{2T} = \frac{p}{T} \left(\frac{S}{2} - x \right)$$

$$y = \frac{px^2}{2T} + \frac{pSx}{2T} + C$$

$$@ x = 0, y = 0; x = S, y = 0 \quad C = 0$$

$$y = \frac{px}{2T} (S - x)$$

$$T = \frac{px}{2y} (S - x)$$

$$@ x = S/2, y = \delta$$

$$T = \frac{pS^2}{8\delta}$$

(Uniform pressure)

(1.5.1)

$$y = \frac{4\delta x}{S^2} (S - x)$$

$$y' = \frac{8\delta}{S^2} \left(\frac{S}{2} - x \right)$$

$$\begin{aligned} \Delta S &= \frac{1}{2} \int_0^S y'^2 dx \\ &= \frac{32 \delta^2}{S^4} \int_0^S (S/2 - x)^2 dx \end{aligned}$$

$$= \frac{32 \delta^2}{S^4} \int_0^S \left[\frac{S^2 x}{4} - \frac{S x^2}{2} + \frac{x^3}{3} \right] dx$$

$$= \frac{8 \delta^2}{3 S}$$

$$\epsilon_m = \frac{\Delta S}{S} = \frac{8 \delta^2}{3 S^2} \quad (1.5.2)$$

Either:

$$\sigma_m = E \epsilon = \frac{8 E \delta^2}{3 S^2}, \quad T = \frac{8 E \delta^2 h}{3 S^2}$$

or

$$\sigma_m = \sigma_y, \quad T = \sigma_y h$$

$$p = \frac{8 \delta T}{S^2}$$

$$u = \frac{p}{2} \int_0^S y dx$$

$$= \frac{2 p \delta}{S^2} \int_0^S x (S - x) dx$$

$$= \frac{2 p \delta}{S^2} \left(\frac{S^3}{2} - \frac{S^3}{3} \right)$$

$$= \frac{p \delta S}{3} \quad \text{or} \quad \frac{p \delta}{3} \text{ in. lb./in.}^2 \quad (1.5.3)$$

$$\theta_1 = \frac{dy}{dx} = \frac{4 \delta}{S} \quad (\text{rotation @ support}) \quad (1.5.4)$$

$$\epsilon_1 = \frac{h \theta_1}{2 x_4} = \frac{2 h \delta}{S x_4}$$

$$\epsilon = \frac{2 h \delta}{S x_4} + \frac{8 \delta^2}{3 S^2} \quad (1.5.5)$$

For loads exceeding the limit load, the incremental loads will be resisted only by membrane forces.
 For loading less than the limit load, it does not seem that the membrane forces are significant.

Figure A-7 Membrane Resistance of Hex Flat

1.6 VELOCITY OF STRAIN

For computing the velocity of strain, it is assumed that a uniform deceleration from an initial velocity is attained and that the energy absorbed can be equated to the initial kinetic energy. The initial velocity is a function of the energy absorbed. ($E = \frac{Mv^2}{2}$)

Velocity of strain:

$$u = \frac{wv^2}{2g} = \frac{h\rho v^2}{2g}$$

$$v^2 = \frac{2gu}{h\rho}$$

$$\delta = \frac{v}{z} (t)$$

$$\epsilon = \frac{\dot{\epsilon}}{2} (t)$$

$$\frac{\epsilon}{\delta} = \frac{\dot{\epsilon}}{2} \div \frac{v}{2}$$

$$\epsilon = \frac{\dot{\epsilon} v}{\delta}$$

w = weight - lb.
 ρ = unit weight - lb./in.³
 v = velocity - in./sec.
 t = time
 g = gravitation constant

(1.6.1)

1.7 HEXD COMPUTER PROGRAM LISTING AND GRAPHS OF RESULTS

```

S      IDENT  AP49,JMC-SV200,NONPRO
S      FORTRAN NDECK
S      INCODE 19MF
C      HEXD
C      THIS PROGRAM SOLVES A HEXAGONAL DUCT FOR AN INTERNAL IMPULSE LOAD
C      INPUT=YIELD STRESS+RSI, MODULUS+PSI, THICK+IN, WIDTH OF FLAT+IN,
C      DEFLECTION STEP+IN, NO OF STEPS, POISSON, DENSITY+PCI
C      OUTPUT= DEFLECTION OF FLAT, EQUIV PRESS LOAD, MAX STRAIN, ENERGY
C      ABSORBED = LB-IN/IN**2, STRAIN RATE
C      DIMENSION F(20),P(20),D(20),U(20),V(20)
C      DIMENSION X(20),Y(20),XIN(20),LRSXY1(6),LRSXY2(6),LRSXY3(6)
C      DIMENSION LBH(7),LRS1(7,2),LRS2(3,3),LRS3(2,3),KEY1(3,1)
C      DIMENSION KEY2(3,1),KEY3(3,1)
C      EQUIVALENCE (LRS1,LRS2,LRS3 )
C      WRITE(6,10)
10  FORMAT(7H) HEXD )
C      READ(5,20) E1,D1,AN,R,S,L1
20  FORMAT(4F15.8/1F15.8,1F5)
C      WRITE(6,41)E1,D1,AN,R,S,L1
41  FORMAT(5H(E1=F10.3,SH ,D1=F10.3,SH ,AN=F10.3,4H ,R=F10.3,
14H ,S=F10.3,5H ,L1=F10.2H ))
C      READ(5,5) (LRS1(J,3),J=1,3)
5  FORMAT(3A6)
60  READ(5,6)H, (LRS1(J,2),J=1,2)
6  FORMAT(F15.8,3A6)
80  WRITE (6,50)
80  FORMAT (10HRELASTIC-1)
C      READ(5,30) F, (LRS1(J,1),J=1,3)
30  FORMAT(5X,F15.8,10X,3A6)
C      WRITE(6,55)F,H
55  FORMAT(4HMF= ,F6.2,14,4X,7H H=,F6.2,1H.)
C      E2 = E1/(1.-AN**2)
C      E(1) = S/F2
C      P(1) = S**2/(F*(F/2.+0.86607*H))
C      AM1 = (S-.86607*P(1)*F/H)**2/6.
C      D(1) = P(1)*F**4/(72.*F2*H**3)
C      U(1) = (P(1)**2*F**4)/(120.*H**3*F2)
C      R3 = SQRT( 1.+F**2/(27.*H**2))-F/(3.464*H)
C      P(2) = R3*H*S/(.866*F)
C      AM2 = (S-.866*P(2)*F/H)**2/6.
C      D(2) = P(2)*F**2/12.
C      R4 = R4.*F**4/(1152.*F2*H**3)
C      D(2) = P(2)*R1
C      R2 = R1*1152./3780.
C      U(2) = U(1) + ((P(2)**2-P(1)**2)*R2)
C      X1 = (F-SQRT(F**2-4.*(F**2/6.-2.*AM1/P(2))))/2.
C      R2 = ((F-2.*X1)/(E2*H**3))*(P(2)*(F-2.*X1)**2/2.-6.*AM1)
C      E(2) = H*A2/(2.*X1) +E(1)
C      R3 = SQRT(S./3.+(F/(3.464*H)+1./3.))**2 - (F/(3.464*H)+1./3.)
C      P(3) = R3*H*S/(.866*F)
C      D(3) = P(3)*R1
C      U(3) = U(1) + (P(3)**2-P(1)**2)*R2
C      AM3 = (S-.866*P(3)*F/H)**2/6.
C      AM2 = H**2*(1.-R2**2)*S/4.
C      X1 = (F-SQRT(E**2- R4*(AM2-AM1)/F(3)))/2.
C      R2 = ((F-2.*X1)/(F2*H**3))*(P(3)*(F-2.*X1)**2/2.-6.*AM1)
C      F(3) = H*A3/(2.*X1) +E(1)
C      P4 = SQRT( 1.+F**2/(120.*H**2))-F/(4.*H**4)
C      D(4) = (P4*S**4)/(.866*H)
C      AM4 = (S-.866*P(4)*F/H)**2/6.
C      X1 = (F-SQRT(F**2-4.*(F**2/6.-2.*AM1/P(4))))/2.
C      X2 = (F-SQRT(F**2-4.*(F**2/6.-2.*AM1/P(4))))/2.
C      F4 = (X2-Y1)*F2*2./((7.*F)
C      D(4) = P(4)*R1*F2/F4
C      V4=X1
C      Z = R.*F2*D(4)**2/(7.*F**2.)
C      IF (7.GT.S) GO TO 120
C      P4 = (64.*H*F2*U(4)**3)/(3.*F**4)
C      A4 = ((F-2.*X1)/(R4*H**3))*((-14)*F-2.*X1)**2 /7.-6.*AM1)
C      D(4) = AMAX1(P(4),P4)
C      U(4) = U(1)+(P(4)**2-P(1)**2)*R2*F2/F4
C      GO TO 135

```

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```

120 P(4) = A.*S*H*D(4)/F**2
    A4 = 4.*D(4)/F
    U(4) = P(4)*D(4)/3.
135 F(4) = H*A4/(2.*X1) + A.*D(4)**2/(3.*F**2) + F(1)
    DO 170 J = 5,L1
    VV = J-4
    D(J) = VV*D1
    Z = A.*F2*D(J)**2/(3.*F**2)
    IF (Z.GT. S) GO TO 160
    P4 = 7*H*P.*D(J)/F**2
    P(J) = AMAX1(P(4),P4)
    GO TO 170
160 P(J) = (A.*S*H*D(J))/F**2
    IF (P(J).GT.P(4)) GO TO 170
    P(J) = P(4)
170 A = 4.*D(J)/F
    U(J) = P(J)*D(J)/3.
    E(J) = H*A/(2.*X1) + A.*D(J)**2/(3.*F**2) + E(1)
190 CONTINUE
    WRITE(6,200)
200 FORMAT(70H0 J= T= VF= V=
    1 VM= )
    DO 220 J = 1,L1
    VM = U(J)/P(J)
    VD = (772.*U(J)/(H*D))**0.5
    T = 2.*D(J)/VD
    V(J) = F(J)*VD/D(J)
    WRITE(6,210) J,T,V(J),VD,VM
210 FORMAT(15,4F15.8)
220 CONTINUE
    WRITE(6,230)
230 FORMAT(72H0 I= D(1) P(1) E(1)
    1 U(1) )
    DO 250 J = 1,L1
    WRITE(6,240) J,D(J),P(J),F(J),U(J)
240 FORMAT(15,F13.5,F13.5,F13.5,F13.5)
    X(J) = U(J)
250 CONTINUE
    W = 3.59*(F2*H**2*736./((17.*D**4))**0.5
    T = 1./W
    WRITE(6,260) W,T
260 FORMAT(4H0W= .F10.5,4Y.3H T= . F13.5)

```

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```

XIN(1)=1
XIN(2)=1
XIN(3)=0
XIN(4)=0
XIN(5)=0
XIN(6)=0
XIN(7)=0
XIN(8)=0
XIN(9)=3
XIN(10)=3
XIN(11)=4
XIN(12)=3
ND(1,1)=1.1
ND(2,1)=1
ND(3,1)=1
ND(4,1)=1
ND(5,1)=3
DATA LBXY1(1)/18H ENERGY- IN.LB./SI/
DATA LBXY2(1)/18H ENERGY- IN.LB./SI/
DATA LBXY3(1)/18H ENERGY- IN.LB./SI/
DATA LB(1)/30H WORK-INTERVAL CAD.F=23,000,000/
DATA LBXY1(4)/18H DEFLECTION- IN. /
DATA LBXY2(4)/18H PRESSURE- PSI /
DATA LBXY3(4)/18H STRAIN /
DATA KXY1(1,1)/18H DEFLECTION/ENERGY /
DATA KXY2(1,1)/18H PRESSURE/ENERGY /
DATA KXY3(1,1)/18H STRAIN/ENERGY /
DO 35 JU=1,7
DO 25 K=1,11
GO TO(36,37,38),JU

```

```

36 Y(K)=D(K)
GO TO 25
37 Y(K)=P(K)
GO TO 25
38 Y(K)=F(K)
25 CONTINUE
GO TO(45,46,47),IJ
45 CALL DRAW(X,Y,XIN,ND,LXV1,LH,LBSH1,KEY1)
GO TO 35
46 CALL DRAW(X,Y,XIN,ND,LXV2,LH,LBSH2,KEY2)
GO TO 35
47 CALL DRAW(X,Y,XIN,ND,LXV3,LH,LBSH3,KEY3)

```

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```

35 CONTINUE
270 CONTINUE
CALL EXIT
END

```

```

$ OPTION FORTRAN
$ EXECUTE
$ LIMITS F,20K,1000,
$ TAPE 30,X10,...,DRAW
$ COMMENT SAVE TAPE FC 39 FOR PLOT CHG A249 SNJMP 14279,JMC.
$ INCODE IRMF
27.056 0.02 .30 .20
14000. 10
Y.S.=14.000 PSI.
.04 .04 IN. THICK
1.3 FLAT DIM=1.3 IN.
$ ENDJOB

```

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Figure A-8 Deflection vs Energy

Figure A-9 Strain vs Energy

Figure A-10 Pressure vs Energy

Figure A-11 Deflection vs Energy

Figure A-12 Strain vs Energy

Figure A-13 Pressure vs Energy

Figure A-14 Deflection versus Energy

Figure A-15 Strain versus Energy

Figure A-16 Pressure versus Energy

2.0 CHANNEL RESISTANCE TO SHOCK LOADS

For external loading which may occur from transmission of impulse from adjacent ducts, the loading of the duct will be unsymmetrical, i.e., one side only, with resistance to translation from inertia and bearing at the corners of adjacent ducts. A severe case is loading on one face only of the hexagonal duct with bending resistance the only contributor preventing collapse of the duct. This case can be examined by a slight alteration of the methods developed and probably represents the most severe loading.

The initial step in the solution is a plane strain solution of the hexagonal cross section under pressure loading of one flat reacted on the two opposite corners (see Figure A-17). It is assumed that these reactions may be supplied either by interference from adjacent channels or by the beam resistance of the loaded channel from axially remote supports. The method employing virtual work is selected for determining the redundant internal loads.

The derivations utilized for the computer solution of an external shock pulse acting on a hexagonal channel duct are provided in the following figures and computation steps. The computer program is labeled "HEXDOUT". Illustrative results from this program are shown on Figures A-18 and A-19. The listing of the computer program and graphs of the solutions obtained follow all the derivations (see section 2.5).

Item	Location				
	0	1	2	3	4
M:	0	$\frac{pSx}{2}$	$\frac{pSx}{2}$	$\frac{px}{2}(s-x)$	
ds:	dx	2dx	2dx	dx	
m_H :	0	$-\sqrt{3}x$	$-\sqrt{3}(s-x)$	$-\sqrt{3}S$	
m_M	1	1	1	1	

$$\delta_{H_p} = \int \frac{M m ds}{EI}$$

$$EI \delta_{H_p} = -\sqrt{3} p \int_0^{-s/2} Sx^2 dx - \sqrt{3} p S \int_{-s/2}^0 x(s-x) dx - \frac{\sqrt{3} p S}{2} \int_0^{s/2} x(s-x) dx$$

$$= -\sqrt{3} p \left\{ \int_0^{-s/2} \left[\frac{Sx^3}{3} \right] + \int_{-s/2}^0 \left[\frac{S^2 x^2}{2} - \frac{Sx^3}{3} \right] + \frac{1}{2} \int_0^{s/2} \left[\frac{S^2 x^2}{2} - \frac{Sx^3}{3} \right] \right\}$$

$$= \frac{\sqrt{3} p}{24} \left[S^4 + 2S^4 - S^4 \right]$$

$$= \frac{\sqrt{3} p S^4}{12}$$

$$= -\frac{5pS^3}{24}$$

EXTERNAL LOADING OF DUCT

Figure A-17 Loading and Geometry For External Loads

SS-304 T 1000° F
 F = 2.6 IN., E = 22.6(10)⁶ PSI
 h = 0.1 IN. ① ν = 0.3
 " = 0.14 IN. ② σ_y = 14,000 PSI

SS-304: $T = 1000^{\circ}\text{F}$
 $F = 2.6 \text{ IN.}$, $E = 22.6(10)^6 \text{ PSI}$
 $h = 0.1 \text{ IN.}$, $\nu = 0.3$
 $\sigma_Y = 24,000 \text{ PSI}$

Figure A-19 Channel Response to External Loads - Increased Yield Strength

$$\delta_{H_h} = H \int \frac{m^2 ds}{EI}$$

$$\begin{aligned} EI \delta_{H_h} &= 6H \int_0^{-s/2} x^2 dx + 6H \int_{-s/2}^0 (s-x)^2 dx + 3HS^2 \int_0^{s/2} dx \\ &= 6H \left[\frac{x^3}{3} \right]_0^{-s/2} + 6H \left[S^2 x - Sx^2 + \frac{x^3}{3} \right]_{-s/2}^0 + 3HS^2 \left[x \right]_0^{s/2} \\ &= \frac{HS^3}{4} + \frac{7HS^3}{4} + \frac{6HS^3}{4} \\ &= \frac{7HS^3}{2} \end{aligned}$$

$$\delta_{H_m} = M \int \frac{m^2 ds}{EI} \quad \delta_{\theta_h} = M \int \frac{m^2 ds}{EI}$$

$$\begin{aligned} EI \delta_{H_m} &= -2\sqrt{3}M \int_0^{-s/2} x dx - 2\sqrt{3}M \int_{-s/2}^0 (s-x) dx - \sqrt{3}MS \int_0^{s/2} dx \\ &= -2\sqrt{3}M \left\{ \left[\frac{x^2}{2} \right]_0^{-s/2} + \left[Sx - \frac{x^2}{2} \right]_{-s/2}^0 + \frac{1}{2} \left[sx \right]_0^{s/2} \right\} \\ &= -\frac{\sqrt{3}M}{4} (S^2 + 3S^2 + 2S^2) = -\frac{\sqrt{3}mS^2}{2} \end{aligned}$$

$$\delta_{\theta_m} = M \int \frac{m^2 ds}{EI}$$

$$\begin{aligned} EI \delta_{\theta_m} &= M \left\{ \int_0^{s/2} dx + 2 \int_0^{s/2} dx + 2 \int_0^{s/2} dx + \int_0^{s/2} dx \right\} \\ &= M \left[x + 2x + 2x + x \right]_0^{s/2} \\ &= 3MS \end{aligned}$$

$$\delta_{\theta_p} = \int \frac{Mm ds}{EI}$$

$$\begin{aligned} EI \delta_{\theta_p} &= -pS \int_0^{-s/2} x dx - pS \int_{-s/2}^0 x dx + \frac{p}{2} \int_0^{s/2} x(s-x) dx \\ &= -p \left[\frac{Sx^2}{2} \right]_0^{-s/2} - pS \left[\frac{x^2}{2} \right]_{-s/2}^0 + \frac{p}{2} \left[\frac{Sx^2}{2} - \frac{x^3}{3} \right]_0^{s/2} \\ &= \frac{pS^3}{24} (3 + 3 - 1) \end{aligned}$$

Deflection equations:

$$\textcircled{1} \delta_H = 0 = \frac{\sqrt{3} P S^4}{12 EI} + \frac{7 H S^3}{2 EI} - \frac{3 \sqrt{3} M S^2}{2 EI}$$

$$\textcircled{2} \delta_\theta = 0 = -\frac{5 p S^3}{24 EI} - \frac{3 \sqrt{3} H S^3}{2 EI} + \frac{3 M S}{EI}$$

$$\textcircled{1} \frac{p S^2}{18} + \frac{7 H S}{3 \sqrt{3}} - M = 0$$

$$\textcircled{2} -\frac{5 p S^2}{72} - \frac{\sqrt{3} H S}{2} + M = 0$$

$$\textcircled{1} \leftarrow \textcircled{2} -\frac{p S^2}{72} + .48111 H S = 0$$

$$H = \frac{p S}{34.64} = .02886 p S$$

$$\textcircled{2} M_0 - .06944 p S^2 - .025 p S^2 = 0$$

$$M_0 = 0.09444 p S^2$$

$$M_2 = -0.09444 p S^2 + 0.02886 p S \left(\frac{\sqrt{3}}{2} S \right) + \frac{p S^2}{4}$$

$$= 0.18055 p S^2$$

$$M_3 = 0.09444 p S^2 - \frac{p S}{34.64} (\sqrt{3} S) = 0.04444 p S^2$$

$$M_4 + 0.09444 p S^2 + \frac{p S^2}{4} - \frac{p S^2}{8} - 0.02886 p S (\sqrt{3} S) = 0$$

$$M_4 = -0.16946 p S^2$$

Using the solution for the distributed load and the solution for the corner load in an additional application of virtual work yields the deflection of the corners.

Deflection of corners of duct:

	Location (see Figure A-17)				
Item	0	1	2	3	4
M		$-\frac{pSx}{2}$	$-\frac{pSx}{2}$		
m		$-x$	$-x$		
ds		$2\,dx$	$2\,dx$		

$$\delta_{1-3p} = \int \frac{M m d s}{EI}$$

$$\begin{aligned} EI \delta_{1-3p} &= p S \int_0^{s/2} x^2 dx + p S \int_0^{s/2} x^2 dx \\ &= 2 p S \left[\frac{x^3}{3} \right]_0^{s/2} \\ &= \frac{p S^4}{12} \end{aligned}$$

$$EI \delta_{1-3H} = H \int \frac{m m d s}{EI}$$

$$\begin{aligned} &= 2 (.02886 p S) \int_0^{s/2} \sqrt{3} x^2 dx + 2\sqrt{3} (.02886 p S) \int_0^{s/2} (S - 2x) dx \\ &= .09997 p S \left[\frac{x^3}{3} + \frac{Sx^2}{2} - \frac{x^3}{3} \right]_0^{s/2} \\ &= \frac{.09997 p S^4}{8} \end{aligned}$$

$$EI \delta_{1-3M} = M \int m m d s$$

$$\begin{aligned} &= -.09444 p S^2 \int_0^{s/2} 2x dx - .09444 p S^2 \int_0^{s/2} 2x dx \\ &= - 18888 p S^2 \int_0^{s/2} x^2 dx \\ &= - \frac{18888 p S^4}{4} \end{aligned}$$

$$\begin{aligned} EI \delta_{1-3\Sigma} &= \frac{p S^4}{12} + \frac{.29991 p S^4}{24} - \frac{18888 p S^4}{4} \\ &= \frac{1.16663 p S^4}{24} = 0.0486 p S^4 \end{aligned}$$

2.1 ELASTIC RESPONSE

By regarding the loaded flat as a partially restrained beam with deflection of the support, the deflection under the load and thus the energy may be computed.

Deflection of Flat plus corners (see Equation 1.2.6).

$$EI_y = -\frac{p x^4}{24} + \frac{p S x^3}{12} + \frac{M_o x^2}{2} - \frac{M_o S x}{2} - \frac{p S^3 x}{24} - .0486 p S$$

Note, reversal of sign of M_o and addition of corner deflection.

@ $x = s/2$:

$$\begin{aligned} EI\delta &= p S^4 \left[-\frac{1}{384} + \frac{1}{96} + \frac{.04444}{8} - \frac{.04444}{4} - \frac{1}{48} - .0486 \right] \\ &= -p S^4 \left[\frac{5}{384} + \frac{.04444}{8} + .0486 \right] \\ &= -0.06717 p S^4 \end{aligned} \quad (2.1.1)$$

$$\begin{aligned} EI_y &= -\frac{p x^4}{24} + \frac{p S x^3}{12} + 0.02222 p S^2 x^2 - 0.02222 p S^3 x \\ &\quad - \frac{p S^3 x}{24} - 0.0486 p S^4 \\ &= -\frac{p x^4}{24} + \frac{p S x^3}{12} + 0.02222 p S^2 x^2 - 0.06389 S^3 x \\ &\quad - 0.0486 S^4 \end{aligned}$$

$$\begin{aligned} EISU &= \frac{1}{2} \int_0^s p y \, ds \\ &= \frac{p^2}{2} \int_0^s \left(-\frac{x^4}{24} + \frac{S x^3}{12} + 0.02222 S^2 x^2 - .06389 S^3 x - .0486 S^4 \right) dx \\ &= \frac{p^2}{2} \left[-\frac{x^5}{120} + \frac{S x^4}{48} + \frac{.02222 S^2 x^3}{3} - \frac{.06387 S^3 x^2}{2} - .0486 S^4 x \right] \\ &= -.03032 p^2 S^5 \end{aligned} \quad (2.1.2)$$

For the limit of elastic loading, the maximum moment is equated to the yield strength of the section, as before, to find the yield load. The deflection and energy are then computed for this load.

$$\sigma_y = \frac{6M}{h^2} + \frac{.44744 p S}{h}$$

$$M_y = \frac{(h \sigma_y - .44744 p S) h}{6}$$

$$M_2 = 0.1806 p S^2 \quad (\text{Critical moment @ corner})$$

$$0.1806 p S^2 = \frac{h (h \sigma_y - .44744 p S)}{6}$$

$$p(.1806 S^2 + \frac{.44744 S h}{6}) = \frac{h^2 \sigma_y}{6}$$

$$p = \frac{h^2 \sigma_y}{1.0836 S^2 + .44744 h S} \quad (2.1.3)$$

$$\epsilon = \frac{-\sigma_y}{E} \quad (2.1.4)$$

2.2 TWO PLASTIC HINGES REGIME

The resistance of the plastic stress distribution is equated to the maximum moment and the load corresponding to two fully developed hinges is computed. The deflection and energy may then be ascertained. To determine an approximate value for the strain, it is first necessary to determine the limit of the plastic zone by equating the elastic moment resistance to the bending moment equation. Following this, a virtual work solution of the rotation at the hinge and assuming that the curvature is uniformly distributed in the plastic zone results in an approximation of the strain.

Two-Plastic Hinges:

$$M_p = 0.1806 p S^2 = \frac{h^2 (1-r^2) \sigma_y}{4}$$

$$.44744 p S = \sigma_y r h$$

$$M_p = \frac{.1806 \sigma_y r h S}{.44744}$$

$$\frac{h^2 (1-r^2) \sigma_y}{4} = \frac{.1806 \sigma_y r h S}{.44744}$$

$$r^2 + 1.6145 \frac{r S}{h} = 1$$

$$r + \frac{1.6145 S}{2 h} = 1 + .80725 \frac{S}{h}^2$$

$$r = \sqrt{1 + .80725 \frac{S}{h}^2} - .80725 \frac{S}{h} \quad (2.2.1)$$

$$p = \frac{r h \sigma_y}{.44744 S} \quad (2.2.2)$$

$$M_e = \left(\frac{\sigma_y - \frac{.44744 p S^2}{h} h^2}{6} \right) = .1806 p S^2 - p \frac{Sx}{2} + H (1.73206 x)$$

$$x (.5 p S - (.02886 (1.73206 x) p S) = .1806 p S^2 - M_e$$

$$x = \frac{M_p - M_e}{p S (.5 - .05)} \quad (2.2.3)$$

Using reduced modulus for plasticity:

$$E^x = 0.9 E \quad (2.2.4)$$

$$\delta = - \frac{25.795 p S^4}{32 E^x h^3} \quad (2.2.5)$$

$$U = - \frac{.36384 p^2 S^4}{E^x h^3} \quad (2.2.6)$$

Location (see Figure A-20)

Item	1	2	3
M	$-.1806 p S^2$	$-.1806 p S^2$	
M _H	$.025 p S^2$	$.05 (s-x) p S$	
M _p	$\frac{p S^2}{4}$	$\frac{p S}{2} (s-x)$	
m	1	1	
d s	d x	2 d x	

$$\begin{aligned} EI \delta_{\theta_3} &= \int M m ds \\ &= p S^2 \int_0^{s/2} (.025 - .1806 + .25) dx + p S \int_{s/2}^s (.1806 S + .55 (s-x) (2 dx) \\ &= p S^2 \int_0^{s/2} (.0944 dx) + 2 p S \int_{s/2}^s (.3694 S - .55 x) dx \\ &= p S^2 \left(\frac{.0944 S}{2} \right) + 2 p S \left(\frac{.3694 S^2}{2} - \frac{.55 (3 S^2)}{8} \right) \end{aligned}$$

$$= .0041 p S^3 \quad (2.2.7)$$

$$\epsilon = \frac{h \theta_3}{2x} + \epsilon_1 \quad (2.2.8)$$

ROTATION AT HINGE

Figure A-20 Geometry for Rotation at Hinge

2.3 THREE PLASTIC HINGES – MAXIMUM LOAD

For the ultimate load (three hinges), the resistance of the plastic stress distribution is equated to the moment at the center of the loaded flat when it reaches the value at the prior formed plastic hinges. The load thus computed is employed with a reduced value for the modulus in plasticity to estimate the deflection and energy at this limit. The extent of the plastic zone and the average plastic strain are computed as before.

Following the formation of three plastic hinges the section offers no further resistance to load and additional deformation results in an increase in rotation of the hinges but not an increase in the plastic zones or the loading. The energy and strain are then functions of deformation only, and are easily computed from the deflection

Three-Plastic Hinges Collapse Load:

$$\begin{aligned} M_p &= \frac{h^2 (1 - r^2) \sigma_y}{4} = \frac{M_2 + M_d}{2} \\ &= \frac{(.18055 + .16745) p S^2}{2} = 175 p S^2 \end{aligned} \quad (2.3.1)$$

$$.02887 p S = \sigma_y r h$$

$$M_p = \frac{.175 \sigma_y r h S}{.02887}$$

$$\frac{h^2 (1 - r^2) \sigma_y}{4} = \frac{.175 \sigma_y r h S}{.02887}$$

$$h^2 r^2 \sigma_y + 23.476 \sigma_y r h S = h^2 \sigma_y$$

$$r^2 + 23.476 \frac{r S}{h} = 1$$

$$\left(r + 11.738 \frac{S}{h}\right)^2 = 1 + \left(11.738 \frac{S}{h}\right)^2$$

$$r = \sqrt{1 + \left(11.738 \frac{S}{h}\right)^2} - 11.738 \frac{S}{h} \quad (2.3.2)$$

$$p = \frac{r h \sigma_y}{.02887 S} \quad (2.3.3)$$

Using reduced modulus for plasticity:

$$E^* = \frac{2}{3} E \quad (2.3.4)$$

$$\theta = \frac{.0492 p S^3}{E^* h^3} \quad (2.3.5)$$

$$M_e = \frac{\left(\sigma_y - \frac{.02887 p S}{h}\right) h^2}{6} \quad (2.3.6)$$

$$x = \sqrt{\frac{2 (M_p - M_e)}{p}} \quad (2.3.7)$$

$$\epsilon_3 = \frac{h \theta}{2x} + \epsilon_1 \quad (2.3.8)$$

$$\delta = \frac{25.795 p S^4}{32 E^* h^3} \quad (2.3.9)$$

$$U = - \frac{.36384 p^2 S^4}{E^* h^3} \quad (2.3.10)$$

2.4 PLASTIC REGIME

For loading greater than required to develop the three hinges, the section would be unstable and collapse for static loading. However, additional energy will be absorbed with additional deflection, and this energy can be computed as a function of the deflection.

The bending collapse load is the maximum load the section can withstand as membrane action is impossible.

Fully Plastic – Following development of third hinge:

$$p = p_3 \quad (2.4.1)$$

$$\theta = \frac{\delta}{16.384 S} \quad (2.4.2)$$

$$\epsilon = \frac{h \theta}{2 x_3} + \epsilon_1 \quad (2.4.3)$$

$$U = \frac{p \delta}{2.2155} \quad (2.4.4)$$

2.5 HEXDOUT COMPUTER PROGRAM LISTING AND GRAPHS OF RESULTS

```

C      SNIMR      14001
C      JPRINT     A220,JMC-SV200,XY0000
C      FORTTRAN
C      INCODE     1RMF
C
C      HEXDOUT    D ON FLAT OF HEX DUCT REACTED BY OPPOSITE CORNER LOADS
C      PLANE STRAIN SOLUTION OF HEX DUCT WITH LOADED FLAT
C      INPUT(N,NU,F),EN,DE(NU),H(1),H(N),SY(1),SY(N),F(1),F(N)
C      FORMAT(214,4F16.5/(4G16.5))
      DIMENSION EP(20),P(20),DE(20),H(20),V(20)
      DIMENSION H(10),SY(10),F(10)
      DIMENSION XX(20),YY(20),XIN(20),NR(5,1),L5XY(6),LRYX2(6),LRYX3(6)
      DIMENSION LPM(7),LSSH1(3,3),LSSH2(3,3),LSSH3(3,3),KXY1(3,1)
      DIMENSION KXY2(3,1),KXY3(3,1)
      EQUIVALENCE(LSSH1,LSSH2,LSSH3)
      WRITE(6,10)
10  FORMAT(10H1 HEXDOUT )
      READ(5,20)N,NU,F1,EN,FD,SY(1)
20  FORMAT(214,4F16.5)
      WRITE(6,11)N,NU,F1,EN,FD,SY(1)
11  FORMAT(4H0 N=,I3,F4 NU=,I3,F1 EN=,I3,F4 FD=,I3,F4
      LPM      FD=.512,5,TH SY(1)=.112,5,51
      K=1
      DOA(5,5)(LSSH1(J,1),J=1,3)
      FORMAT(34F)
      DO 40 J=1,N
40  F12(1,4)=J*(LPM(1,5),I=1,3)
      FORMAT(F10,F,3A6)
      DO 40 L=1,N
      F12(5,20)=F(L)*(LSSH1(1,1),I=1,3)
30  FORMAT(4Y,5F10,F,10K,3A6)
      F= F1/(1.-FN**2)
      P(1)= H(J)**2*SY(K)/( F(L)*(1.0024*F(L)+.44744*H(J)))
      DE(1)= .00607*P(1)*F(L)**4/(F*H(J)**3)
      U(1)= P(1)/(1/2.2155)
      F1(1)= SY(K)/F
      F= F1/(1.-FN**2)
      P= SQRT(1.+(1.30725*F(L)/H(J)**2)-.00125*F(L)/H(J))
      P(2)= F*H(J)*SY(K)/(1.44744*F(L))
      PM= .18044*P(1)*F(L)**2
      FM= (SY(K)-.44744*P(2)*F(L)/H(J))*H(J)**2/6.
      X= (PM-FM)/(P(2)*F(L)*(1.45))
      DE(2)= .00607*P(2)*F(L)**4/(F*H(J)**3)
      U(2)= P(2)*DE(2)/2.2155
                                     04-10-71 -1 5 000
      TH= DE(2)/(2.220 *F(L))
      F1(2)=H(J)*TH/(2.*X)+F1(1)
      F=(2./3.)* F1/(1.-FN**2)
      P= SQRT(1.+(12.*F(L)/H(J)**2)-12.177*F(L)/H(J))
      P(3)= P*H(J)*SY(K)/(1.02007*F(L))
      PM= .16044*P(1)*F(L)**2
      FM= (SY(K)-.02007*P(2)*F(L)/H(J))*H(J)**2/6.
      DE(3)= .00607*P(3)*F(L)**4/(F*H(J)**3)
      U(3)= P(3)*DE(3)/2.2155
      TH= DE(3)/(2.220 *F(L))
      F1(3)=12*H(J)/(2.*X)+F1(1)
      V=NU
      D= FD/Y
      IF(D-DE(3)) 70,90,90
70  IF(D**2-DE(3))=0,90,90
90  IF(D**2-DE(3))102,103,100
100 DE(4)= 0
      T= 105
      T1(1)= 3.*D
      T2= 105
100 DE(4)= 3.*D
      T2= 105

```

```

103 DE(4) = 4.00
105 NN = NU + 5
DO 120 I = 1, NN
  DE(I) = DE(I) + FLOAT(I-4)*D
  P(I) = P(3)
  U(I) = P(I)*DE(I)/2.2185
  TH = DE(I)/(3.239 *P(I))
  EP(I) = H(J)*TH / (2.*X1+EP(I))
120 CONTINUE
  WRITE(6,15) J,X,L,H(J),SV(K),F(L)
16 FORMAT(3HJ=,I3,3H K=,I3,3H L=,I3,6H H(J)=,G12.5,7H SV(K)=,G12.5,
1 6H F(L)=,G12.5)
  WRITE(6,17)
17 FORMAT(7CHO      1      P(I)      DE(I)      EP(I)
1      U(I) )
  DO 140 I = 1, NN
  WRITE(6,19) I,P(I),DE(I),EP(I),U(I)
19 FORMAT(6H      ,I4,4(4X,G12.5))
  XX(I) = U(I)
140 CONTINUE

```

04-19-71 41 5 000

```

XIN(1)=1
XIN(2)=1
XIN(3)=0
XIN(4)=0
XIN(5)=0
XIN(6)=0
XIN(7)=0
XIN(8)=0
XIN(9)=3
XIN(10)=3
XIN(11)=4
XIN(12)=3
NP(1,1)=NN
NP(2,1)=1
NP(3,1)=1
NP(4,1)=1
NP(5,1)=1
DATA LBXY1(1)/18H ENERGY- IN.LB./SI/
DATA LBXY2(1)/18H ENERGY- IN.LB./SI/
DATA LBXY3(1)/18H ENERGY- IN.LB./SI/
DATA LBH(1)/30H HEXNOUT-FLATLOAD,6=22.656
DATA LBXY1(4)/18H DEFLECTION- IN. /
DATA LBXY2(4)/18H PRESSURE- PSI /
DATA LBXY3(4)/18H STRAIN /
DATA KEY1(1,1)/18HDEFLECTION/ENERGY /
DATA KEY2(1,1)/18HPRESSURE/ENERGY /
DATA KEY3(1,1)/18HSTRAIN/ENERGY /
DO 35 IJ=1,3
DO 25 I=1,NN
GO TO(36,37,38 ),IJ
36 YY(I)=DE(I)
GO TO 25
37 YY(I)=P(I)
GO TO 25
38 YY(I)=EP(I)
39 CONTINUE
DO 40 IJ=45,46,47),IJ
40 CALL DRAW(XX,YY,XIN,NP,LBXY1,LBH,LBSH1,KEY1)
GO TO 35
41 CALL DRAW(XX,YY,XIN,NP,LBXY2,LBH,LBSH2,KEY2)
GO TO 35
42 CALL DRAW(XX,YY,XIN,NP,LBXY3,LBH,LBSH3,KEY3)

```

04-19-71 -1 5 000

```

35 CONTINUE
40 CONTINUE
  CALL EXIT
  END
#      OPTION FORTRAN
$      EXECUTE
$      TAPE      39,X1D,...,DRAW
$      INCODE    IBMF
      3  10  22500000.
Y.S.=34,000 PSI.
      .1      .1 IN. THICK
          2.1      FLAT DIM=2.1 IN.
          2.35     FLAT DIM=2.35 IN.
          2.6      FLAT DIM=2.6 IN.
      .14     .14 IN. THICK
          2.1      FLAT DIM=2.1 IN.
          2.35     FLAT DIM=2.35 IN.
          2.6      FLAT DIM=2.6 IN.
      .18     .18 IN. THICK
          2.1      FLAT DIM=2.1 IN.
          2.35     FLAT DIM=2.35 IN.
          2.6      FLAT DIM=2.6 IN.
#      ENDJOB

```

Figure A-21 Deflection vs Energy

Figure A-22 Strain vs Energy

Figure A-23 Pressure vs Energy

Figure A-24 Deflection vs Energy

Figure A-25 Strain vs Energy

Figure A-26 Pressure vs Energy

Figure A-27 Deflection vs Energy

Figure A-28 Strain vs Energy

Figure A-29 Pressure vs Energy

3.0 FUEL TUBE RESPONSE TO EXTERNAL LOADS

The loading of the fuel tubes may be simulated in a similar manner by a uniform loading over multiple supports approximated by a fixed end beam with uniform load. The elastic solution is well known and the equations require no explanation.

The derivations utilized for the computer solution of an external shock loading acting on a continuous fuel tube over multiple supports spaced at equal intervals are provided on the following pages. The computer program is labeled "CONTUBE". Illustrative results from this program are shown on Figures A-30 and A-31. The listing of the computer program and graphs of the solutions obtained follow the derivations. The elastic load limit is illustrated on Figure A-32.

3.1 ELASTIC RESPONSE

$$\sigma_y = \frac{MD}{2I} = \frac{pD^2 L^2}{24I} = \frac{pDL^2}{12\pi r^2 t}$$

$$pD = \frac{12 \sigma_y I}{r L^2} = \frac{6 \pi r^2 t}{L^2} \quad (3.1.1)$$

$$y = -\frac{pD}{2EI} \left(\frac{x^4}{12} - \frac{Lx^3}{6} + \frac{L^2 x^2}{12} \right)$$

$$\begin{aligned} EILU &= \frac{1}{2} \int_0^L pD y dx \\ &= \frac{pD^2}{4} \int_0^L \left(\frac{x^4}{12} - \frac{2Lx^3}{12} + \frac{L^2 x^2}{12} \right) dx \\ &= \frac{pD^2}{4} \left[\frac{x^5}{60} - \frac{Lx^4}{24} + \frac{L^2 x^3}{36} \right]_0^L \\ &= \frac{pD^2 L^5}{1440} \end{aligned}$$

$$U = \frac{pD^2 L^4}{1440 EI} \quad \text{in. lb./in.}^2 \quad (3.1.2)$$

$$\delta = \frac{pD L^4}{384 EI} \quad (3.1.3)$$

$$\epsilon = -\frac{\sigma_y}{E} \quad (3.1.4)$$

Figure A-30 Continuous Fuel Rod Tube

* LOAD ON TUBE = $P \cdot (O.D.)$

SS-304, $T = 1100^{\circ}F$
 O.D. = 0.23 IN.
 $t = 0.014$ IN.
 $S = 8.0$ IN.
 $E = 22.2(10)^6$ PSI, $\nu = 0.3$
 $\sigma_y = 23,000$ PSI

Figure A-31 Continuous Fuel Tube

Figure A-32 Fuel Tube Loading Diagram

3.2 TWO PLASTIC HINGES

In the plastic range, the development of the two hinges at the supports follows the pattern established for the flat of the hexagonal channel, but the plastic resistance of the tubular section varies slightly from the rectangular section and requires computing. The deflection, energy and rotation at the support are obtained, as before, for a partially restrained beam. The load is obtained by equating the plastic resistance of the section to the moment at the support for the yielded condition where the center of the span moment has reached the elastic resistance of the section. The average strain is computed as a function of the rotation at the support and the extent of the plastic zone. See Figure A-33 for stress distribution and Figure A-34 for two hinge load.

Two-Plastic Hinges:

Plastic

$$M_p = 4 \sigma_y \int_0^{\pi/2} r \sin \Theta \cdot r d\Theta \cdot t$$

$$= 4 \sigma_y r^2 t \int_0^{\pi/2} \sin \Theta d\Theta$$

$$= 4 \sigma_y r^2 t \left[-\cos \Theta \right]_0^{\pi/2}$$

$$= 4 \sigma_y r^2 t = \frac{4 \sigma_y I}{\pi r}$$

(3.2.1)

Elastic

$$M_e = 4 \sigma_y \int_0^{\pi/2} r (\sin \Theta)^2 r d\Theta \cdot t$$

$$= 4 \sigma_y r^2 t \int_0^{\pi/2} \sin^2 \Theta \cdot d\Theta$$

$$= 4 \sigma_y r^2 t \left[\frac{\Theta}{2} - \frac{1}{4} \sin 2\Theta \right]_0^{\pi/2}$$

$$= \pi \sigma_y r^2 t = \sigma_y \frac{I}{r}$$

(3.2.2)

$$M_p = 4 \sigma_y r^2 t = .07 p D L^2$$

$$p D = \frac{400 \sigma_y r^2 t}{7 L^2}$$

(3.2.3)

$$M_e = \frac{p D L x}{2} - \frac{p D x^2}{2} - .07 p D L^2 = - .055 p D L^2$$

(3.2.4)

$$.03 L^2 - L x + x^2 = 0$$

$$x = \frac{L - \sqrt{L^2 - .12 L^2}}{2}$$

$$= \frac{L}{2} \left(1 - \sqrt{1 - .12} \right)$$

$$= 0.03096 L \quad (3.2.5)$$

$$E^* = 0.9 E \quad (\text{reduction for plasticity}) \quad (3.2.6)$$

$$E^* I_y = - \frac{p D x^4}{24} + \frac{p D L x^3}{12} - \frac{p D L^3 x}{24} - \frac{M_o x^2}{2} + \frac{M_o L x}{2}$$

$$E^* I \delta = - \frac{p D L^4}{384} + \frac{p D L^4}{96} - \frac{p D L^4}{48} - \frac{M_o L^2}{8} + \frac{M_o L^2}{4}$$

$$= - \frac{5 p D L^4}{384} + \frac{M_o L^2}{8}$$

$$= - \frac{5 p D L^4}{384} + .006875 p D L^4$$

$$= - .0061458 p D L^4 \quad (3.2.7)$$

$$E^* I U = \frac{1}{2} p D \int y dx$$

$$= \frac{(p D)^2}{2} \int_0^L \left(- \frac{x^4}{24} + \frac{L x^3}{12} - \frac{L^3 x}{24} - \frac{.005 L^2 x^2}{2} + \frac{.055 L^3 x}{2} \right) dx$$

$$= \frac{(p D)^2}{2} \left(- \frac{L^5}{120} + \frac{L^5}{40} - \frac{L^5}{48} - \frac{.055 L^5}{6} + \frac{.055 L^5}{4} \right)$$

$$= - \frac{0.45 (p D)^2 L^5}{240}$$

$$U = - \frac{0.45 (p D)^2 L^4}{240 E^* I}$$

in. lb./in.²

(3.2.8)

$$\Theta = \frac{dy}{dx} = \frac{1}{E^* I} \left(\frac{p D L x^2}{4} - \frac{p D x^3}{6} - M_o x + \frac{M_o L}{2} - \frac{p D L^3}{24} \right)$$

@ x = 0:

$$E^* I \Theta = \left(\frac{M_o L}{2} - \frac{p D L^3}{24} \right)$$

$$= \frac{.055 L}{2} - \frac{p D L^3}{24}$$

$$= -.01417 p D L^3$$

(3.2.4)

$$\epsilon = \epsilon_{3.1} + \frac{h \Theta}{2x}$$

(3.2.5)

ELASTIC

PLASTIC

STRESS DISTRIBUTION

$$M_e = 4 \int_0^{\pi/2} \sigma_y \sin \theta \cdot \gamma \sin \theta \cdot r d\theta \cdot t$$

$$M_p = 4 \int_0^{\pi/2} \sigma_y \cdot \gamma \sin \theta \cdot r d\theta \cdot t$$

Figure A-33 Fuel Tube Stress Distribution

ELASTIC:

$$M_x = M_{\frac{L}{2}} = \frac{M_e}{M_e + M_p} \left(\frac{pDL^2}{8} \right)$$

$$= \frac{1}{1 + \frac{4}{\pi}} \left(\frac{pDL^2}{8} \right)$$

$$= .055 pDL^2$$

Figure A-34 Fuel Tube Loading Diagram

3.3 THREE PLASTIC HINGES – COLLAPSE LOAD

The limit load for three hinges is determined by the plastic resistance of the tube at the center of the span. At this time, the moment at the center will equal the moment at the support and both will equal one-half of the moment at the center of a simply supported beam of equal length. The limit of the plastic zone is determined as before by equating the elastic resistance of the section to the bending moment equation to locate the point at which elastic response is possible. The value of the modulus of elasticity is arbitrarily reduced to account for the plasticity and the deflection is computed for a partially restrained beam. The energy is computed as the integral of the load multiplied by the deflection (external work = internal) across the span. The average strain is computed from the rotation at the support distributed through the plastic zone plus the yield strain. See Figure A-35 for load diagram.

Three-Plastic Hinge – Maximum Load

$$M_o = 4 \sigma_y r^2 t = \frac{p D L^2}{16}$$

$$p D = \frac{64 \sigma_y r^2 t}{L^2} \quad (3.3.1)$$

$$M_x = \frac{p D L x}{2} - \frac{p D x^2}{2} - \frac{p D L^2}{16} = - \frac{\pi p D L^2}{64}$$

$$x^2 - L x + \frac{L^2}{8} - \frac{\pi L^2}{32} = 0$$

$$x = \frac{L - \sqrt{L^2 - .10732 L^2}}{2} \quad (3.3.2)$$

$$E^* = \frac{2 E}{3} \quad (\text{reduction for plasticity}) \quad (3.3.3)$$

$$E^* I \delta = - \frac{5 p D L^4}{384} + \frac{M_o L^2}{8}$$

$$= - \frac{5 p D L^4}{384} + \frac{p D L^4}{128}$$

$$= - \frac{2 p D L^4}{384} \quad (3.3.4)$$

$$E^* I L U = \frac{(p D)^2}{2} \int_0^L \left(-\frac{x^4}{24} + \frac{L x^3}{12} - \frac{L^3 x}{24} - \frac{.0625 L^2 x^2}{2} + \frac{.0625 L^3 x}{2} \right)$$

$$= \frac{(p D)^2}{2} \left[-\frac{L^5}{120} + \frac{L^5}{48} - \frac{L^5}{48} - \frac{.0625 L^5}{6} + \frac{.0625 L^5}{4} \right]$$

$$= U_{3.3} - \frac{p D (\delta_i - \delta_3)}{4} \quad \text{in. lb./in.}^2 \quad (3.4.4)$$

ELASTIC:

$$M_x = \frac{M_e}{2Mp} \left(\frac{pDL^2}{8} \right)$$

$$= \frac{1}{8} \left(\frac{pDL^2}{8} \right)$$

$$\pi$$

$$\frac{\pi pDL^2}{64}$$

LIMIT FOR 3-HINGE LOAD

Figure A-35 Load Diagram for Three Hinge Regime

$$= -\frac{(pD)^2 L^5}{240} + \frac{(pD)^2 L^5}{384}$$

$$U = -\frac{(pD)^2 L^4}{640 E^* I} \quad \text{in. lb./in.}^2 \quad (3.3.5)$$

$$E^* I \Theta = \frac{M_e L}{2} - \frac{p D L^3}{24}$$

$$= \frac{.04909 p D L^3}{2} - \frac{p D L^3}{24}$$

$$= -.017125 p D L^3 \quad (3.3.6)$$

$$\epsilon = \epsilon_{3.1} + \frac{h \Theta}{2 x} \quad (3.3.7)$$

3.4 PLASTIC REGIME

There does not appear to be a mechanism for furnishing the tensile abutment restraints necessary for membrane action of the tube and the bending resistance is the limiting load. For additional plastic deformation, the load is assumed constant and the energy and strain are a function of deflection. See Figure A-36 for this regime.

Plastic Range:

$$\alpha = -\frac{2(\delta_i - \delta_3)}{L} \quad (3.4.1)$$

$$p_i D = p_3 D \quad (3.4.2)$$

$$\epsilon_i = \epsilon_{3.1} + \frac{(\Theta_3 + \alpha)}{2 x_3} \quad (3.4.3)$$

$$U_i = U_{3.3} - \frac{p D}{2 L} \int y dx$$

$$= U_{3.3} - \frac{p D}{L} \int_0^{L/2} \frac{2(\delta_i - \delta_3)}{L} x dx$$

$$= U_{3.3} - \frac{2 p D (\delta_i - \delta_3)}{L^2} \left[\frac{x^2}{2} \right]_0^{L/2}$$

Figure A-36 Plastic Deformation Regime – After Developing Third Hinge

3.5 CONTUBE COMPUTER PROGRAM LISTING AND GRAPHS OF RESULTS

```

* IDENT 4220,JMC-SV200,NONE
* FOOTRAN
* INCODE 1,TIME

CCONTUBE= CONTINUE
C FUEL PIN= ENERGY APP
C INPUT RI,H,S(1),DS, DY,E,SY,PNU,RO,LN,LS
C FORMAT (6E12,5/3F12,5,2/4)
  DIMENSION XIN(20), NP(2,1), LRYV1(6), LRYH(7), LRYH(7,2), KE(1,3,1)
  DIMENSION EP(20),P(20),S(10),Y(20),LRYV2(6),LRYV3(6)
  DIMENSION KEY2(2,1),KEY3(2,1)
  WRITE(6,10)
10 FORMAT(10H) CONTUBE=
  READ(5,15) RI,H,S(1),DS, DY,E,SY,PNU,RO,LN,LS
15 FORMAT(6F12, 5/3F12,5,2/4)

  WRITE(6,20) RI,H,S(1),DS, DY,E,SY,PNU,RO,LN,LS
20 FORMAT(4HRI=,G12,3,2H H=,G12,3,5H S(1)=,G12,3,4H DS=,G12,3,
  1 4H DY=,G12,3,2H E=,G12,3,7/4H SY=,G12,3,5H PNU=,
  2G12,5,4H RO=,G12,3,4H LN=,14,4H LS=,14)
30 FORMAT(14H (1) PLASTIC,S=,G12,5)
  DO 45 I= 1,LS
40 FORMAT(8V,3A6)
  READ(5,40) (LRSH(J,3),J=1,3)
  S(I)=S(1)+FLOAT(I-1)*DS
  WRITE(6,30) S(I)
  EYF=3.14159*((R1+H)**4-R1**4)/4.
  PP = 12.*SY*EYF/((R1+H)*S(I)**2)
  P(1)= PP/(2.*(R1+H))
  Y(1)= PP*S(I)**4/(384.*E*EYF)
  U(1)= PP*Y(1)/7.75
  ED(1)= SV/
  PP= 19.197*SY*EYF/(R1+H)*S(I)**2)
  P(2)= PP/(2.*(R1+H))
  Y(2)= .00614*PP*S(I)**4/(384.*E*EYF)
  U(2)= .20566*PP*Y(1)
  TH= 2.20566*Y(2)/S(I)
  ED(2)=ED(1)+(R1+H)*TH/(.03066*S(I))
  PP= 64.*SV*EYF/(3.14159*S(I)**2*(R1+H))
  EF= 2.*E/7.
  P(3)= PP/(2.*(R1+H))
  Y(3)= PP*S(I)**4/(192.*E*EYF)
  U(3)= .3*PP*Y(3)
  TH= 2.20566*Y(3)/S(I)
  ED(3)=ED(1)+(R1+H)*TH/(.03066*S(I))
  Y(4)= (CONTI(Y(2)/DY**1.5)*DY
  DO 120 J= 4,LN
  Y(J)= Y(4)*T**FLAT(J-4)
  P(J)= P(3)
  U(J)=U(3)+ PP*(Y(J)-Y(3))/4.
  AL= 2.*(Y(J)-Y(3))/S(I)
  ED(J)=ED(1)+(R1+H)*(AL+TH)/(.03066*S(I))
120 CONTINUE
30 FORMAT( 72H0 I J P(J) Y(J) EP(J)
  1 U(J)
  WRITE(6,80)
  DO 100 J=1,LN
  WRITE(6,110) I,J,P(J),Y(J),EP(J),U(J)

```

```

110 FORMAT(2H0 ,214.4(4X,312.5))
100 CONTINUE
  XIN(1)=1
  XIN(2)=1
  XIN(3)=0.0
  XIN(4)=0.0
  XIN(5)=0.0
  XIN(6)=0.0
  XIN(7)=0
  XIN(8)=0
  XIN(9)=3
  XIN(10)=3
  XIN(11)=4
  XIN(12)=3
  ND(1,1)=LN
  ND(2,1)=1
  ND(3,1)=1
  ND(4,1)=1
  ND(5,1)=3
  DATA LXXY(1)/1PHENERGY (IN LB/ST) /
  DATA LXXY2(1)/1PHENERGY (IN LB/ST) /
  DATA LXXY3(1)/1PHENERGY (IN LB/ST) /
  DATA LXXY(4)/1PHDEFLECTION (IN.) /
  DATA LXXY2(4)/1PH PRESSURE (PSI) /
  DATA LXXY3(4)/1PH STRAIN (IN/IN) /
  DATA LBH(1)/42HPA-10 TASK G6.3 FUFL DIN.F=22200000 PSI
  DATA LBSH(1,1)/18HDE=.23 IN. /
  DATA LBSH(1,2)/18HDE=.014.YS=33000PSI /
  WRITE(6,60)(LBSH(J,3),J=1,3)
107 DATA KEY1(1,1) /1PHDEFLECTION /
  DATA KEY2(1,1) /1PHPRESSURE /
  DATA KEY3(1,1) /1PHEPSILON /
  CALL DRAW ( U,Y,XIN,NP,LXXY1,LBH,LBSH , KEY1)
  CALL DRAW ( U,Z,XIN,NP,LXXY2,LBH,LBSH , KEY2)
  CALL DRAW ( U,FP,XIN,NP,LXXY3,LBH,LBSH , KEY3)
45 CONTINUE
  CALL EXIT
  END

```

```

$ OPTION FORTRAN
$ EXECUTE

```

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```

$ TAPF 30.X10....DRAW
$ INCODE 18MF

```

```

.101 .014 0. 2. .05 22.2 E+06
33000. .3 .1 6
  SPAN= 4 IN.
  SPAN= 6 IN.
  SPAN= 8 IN.
  SPAN= 10 IN.
  SPAN= 12 IN.
  SPAN= 14 IN.
$ ENDJOB

```

Figure A-37 Fuel Tube Response - Deflection vs Energy - $S_Y = 13,000$

Figure A-30 Fuel Tube Response - Strain vs Energy

Figure A-39 Fuel Tube Response - Pressure vs Energy

Figure A-40 Fuel Tube Response - Deflection vs Energy

Figure A-41 Fuel Tube Response – Strain vs Energy

Figure A-42 Fuel Tube Response - Pressure vs Energy

Figure A-43¹ Fuel Tube Response - Deflection vs Energy

Figure A-44 Fuel Tube Response - Strain vs Energy

Figure A-45 Fuel Tube Response — Pressure vs Energy

APPENDIX B (K.R. Absher)

Potential Mechanisms for Fuel Expulsion

This appendix contains details of the investigation into the mechanisms that could result in the expulsion of fuel from a failed fuel pin.

B-I Comparison of the Fission Gas Release Characteristics of Two Fuel Pins, B3B and LFT

B-II Escape of Sodium from a Failed Pin during Startup

B-III Failed Clad-Fuel Arch Support of Internal Pressure

B-IV Recommended Experiment to Investigate Internal Pressure Behavior

APPENDIX B-I COMPARISON OF THE FISSION GAS RELEASE CHARACTERISTICS OF TWO FUEL PINS, B3B AND LFT

A group of twelve pins have been irradiated in the NRX Reactor in Chalk River.¹ The pins were fitted with pressure transducers to provide data on the pressures developed in the pins. Pin LFT will be discussed here as a representative of that group. The recorded pressures in pin LFT are shown in Figure B-1 as a function of time. The reactor power is plotted against the same abscissa. Notice that the pressure remains constant during periods of reactor full power runs and that the pressure goes through a step increase when the reactor is shut down and brought up to power again.

Pins B3B and B3C were irradiated in GETR and were fitted with pressure transducers.² The transducer on B3C failed shortly after irradiation began. The pressures of pin B3B are shown in Figure B-2 as a function of time. The reactor power level may be inferred from the clad thermocouple plot. Notice that the pressure in the pin shows a continuous increase during a single-power reactor run. The step pressure increases that characterized the Canadian pins were not observed in B3B. The following considerations are an attempt to explain this discrepancy.

Table B-1 presents data of the two pins as reported in References 4 and 5. The first consideration is the radial distribution of fissions and therefore fission gas atom births. The macroscopic cross sections of the two pins for thermal neutrons is approximately 27 in.^{-1} (B3B) and 1.0 in.^{-1} (LFT). The difference is due to the higher concentration of fissile atoms in B3B. The corresponding mean free paths for thermal neutrons are 0.027 in. (B3B) and 1.0 in. (LFT). These figures indicate that thermal neutrons have a low probability of penetrating very far into B3B but have about a fifty-fifty chance of passing through LFT completely. The distributions of flux and power density are shown in Figure B-3.

Another consideration is the pin temperature. The higher the local temperature, the more mobile are the fission gas atoms. The calculations that follow were used to estimate the temperature difference from the central void to the surface of the fuel. The results are shown in Figure B-4. Clearly, fission gas has greater mobility in pin B3B, especially around the periphery of the fuel.

The fuel density of pin B3B is slightly less than that of LFT (90% T.D. versus 93.5% T.P.). Notley observed that a decrease in fuel density results in an increase in gas release. Thus, B3B would release gas more easily than LFT.

Calculation of Temperature Difference Across the Fuel

R_o = Fuel outer radius

R_i = Central void radius

Σ_a = Macroscopic absorption cross section in fuel

Consider a thin cylindrical shell concentric with the fuel pin.

$$\text{Thickness} = dr$$

$$\text{Radius} = r$$

$$\text{Length} = L$$

$$\text{Radial heat flux} = q_r(r)$$

$$\text{Rate of heat flowing into shell} = 2\pi r L q_r(r)$$

$$\text{Rate of heat flowing out of shell} = 2\pi(r+\Delta r) L q_r(r+\Delta r)$$

$$\text{Rate of heat generation in shell} = 2\pi r \Delta r L S(r)$$

$$S(r) = \text{radially dependent volumetric heat generation rate.}$$

$$2\pi r L q_r(r) - 2\pi(r+\Delta r) L q_r(r+\Delta r) + 2\pi r \Delta r L S(r) = 0$$

Divide by $2\pi \Delta r L$:

$$\frac{r q_r(r)}{\Delta r} - \frac{(r+\Delta r) q_r(r+\Delta r)}{\Delta r} = -rs(r)$$

$$\frac{r q_r(r) \Big|_{r=r+\Delta r} - r q_r(r) \Big|_{r=r}}{\Delta r} = rs(r)$$

Taking the limit as $\Delta r \rightarrow 0$ gives:

$$\frac{d}{dr} [r q_r(r)] = rs(r)$$

Assume that $s(r)$ is proportional to the thermal flux at any point:

$$s(r) = C_1 \phi(r).$$

With this geometry,

$$\phi(r) = C_2 \exp[(r-R) \Sigma_a]$$

where C_2 is the flux at the fuel surface. Let $C_1 C_2 \equiv A$

$$s(r) = A \exp[(r-R) \Sigma_a]$$

$$\frac{d}{dr} [r q_r(r)] = r A \exp[(r-R) \Sigma_a]$$

$$r q_r(r) = A \int r \cdot \exp[(r-R) \Sigma_a] dr = A e^{-R \Sigma_a} \left[\frac{e^{r \Sigma_a}}{\Sigma_a^2} (r \Sigma_a - 1) + C \right]$$

$$q_r(r) = A e^{-R\Sigma_a} \left[\frac{e^{r\Sigma_a}}{\Sigma_a^2} \left(\Sigma_a - \frac{1}{r} \right) + \frac{c}{r} \right]$$

To evaluate "c", let $q_r(r = R_i) = 0$

$$\frac{c}{R_i} + \frac{e^{R_i\Sigma_a}}{\Sigma_a^2} \left(\Sigma_a - \frac{1}{R_i} \right) = 0$$

$$c = \frac{e^{R_i\Sigma_a}}{\Sigma_a^2} (1 - \Sigma_a R_i)$$

$$q_r(r) = -K \frac{dT}{dr}$$

$$-dT = \frac{Ae^{-R\Sigma_a}}{k} \left[\frac{e^{r\Sigma_a}}{\Sigma_a^2} \left(\Sigma_a - \frac{1}{r} \right) + \frac{c}{r} \right] dr$$

$$\begin{aligned} T_i - T_o &= \frac{Ae^{-R\Sigma_a}}{k} \left[\int_{R_i}^{R_o} \frac{e^{r\Sigma_a}}{\Sigma_a} dr - \int_{R_i}^{R_o} \frac{Rr\Sigma_a}{r\Sigma_a^2} dr + c \int_{R_i}^{R_o} \frac{1}{r} dr \right] \\ &= \frac{Ae^{-R\Sigma_a}}{k} \left\{ \frac{1}{\Sigma_a^2} (e^{R\Sigma_a} - e^{R_i\Sigma_a}) - \frac{1}{\Sigma_a^2} \left[(\ln R_o + \Sigma_a R_o + \frac{\Sigma_a^2 R_o^2}{2.21} + \frac{\Sigma_a^3 R_o^3}{3.31} + \dots) - (\ln R_i + \Sigma_a R_i + \frac{\Sigma_a^2 R_i^2}{2.21} + \dots) \right] \right. \\ &\quad \left. + c (\ln R_o - \ln R_i) \right\} \end{aligned}$$

"A" is still unknown. It can be found from the linear heat generation rate, Q.

$$\int_{R_i}^{R_o} 2\pi r s(r) dr = Q$$

$$\begin{aligned}
 \int_{R_i}^{R_o} 2\pi r A e^{(r-R)\Sigma_a} dr &= 2\pi A e^{-R\Sigma_a} \int_{R_i}^{R_o} r e^{r\Sigma_a} dr \\
 &= 2\pi A e^{-R\Sigma_a} \left[\frac{e^{r\Sigma_a}}{\Sigma_a^2} (r\Sigma_a - 1) \right]_{R_i}^{R_o} \\
 &= \frac{2\pi A e^{-R\Sigma_a}}{\Sigma_a} \left[R_o e^{R_o\Sigma_a} - \frac{e^{R_o\Sigma_a}}{\Sigma_a} - R_i e^{R_i\Sigma_a} + \frac{e^{R_i\Sigma_a}}{\Sigma_a} \right]
 \end{aligned}$$

$$A = \frac{Q \Sigma_a e^{R_o\Sigma_a}}{2\pi \left[R_o e^{R_o\Sigma_a} - \frac{e^{R_o\Sigma_a}}{\Sigma_a} - R_i e^{R_i\Sigma_a} + \frac{e^{R_i\Sigma_a}}{\Sigma_a} \right]}$$

Thus, all parameters are known and $T_i - T_o$ can be computed.

	Pin B3B	Pin LFT
R_i	= 0.020 inch	0.030 inch
R_o	= 0.1085 inch	0.3825 inch
Σ_a	= 27 inch ⁻¹ = 324 ft ⁻¹	1.0 in. ⁻¹ = 12 ft. ⁻¹
$e^{R_o\Sigma_a}$	= 18.7	1.465
$e^{R_i\Sigma_a}$	= 1.715	1.04
Q	= 25 kw/ft = 8.54 x 10 ⁴ Btu/hr-ft	24 kw/ft = 8.19 x 10 ⁴ Btu/hr-ft
k	= 1.24 Btu/hr-ft-°F	2.5 Btu/hr-ft-°F
A	= 7.66 x 10 ⁸ Btu/hr-ft ³	3.95 x 10 ⁷ Btu/hr-ft
$T_i - T_o$	= 3060°F	3940°F

The three differences between B3B and LFT presented above may explain why the gas release from B3B was continuous while that from LFT was not. Fission gas was generated near the outer perimeter of B3B while it was fairly evenly distributed over LFT. The B3B temperatures were about 1000°F higher than LFT for similar radial points in the outer regions of fuel. Thus, gas in B3B was better able to move by thermal diffusion than in LFT. The lower density of B3B fuel also enhanced gas release.

Assuming that the discrepancy between the gas release patterns has been explained, it is still necessary to determine which pin was more representative of an LMFBR pin. The radial flux shape across LFT was definitely more representative than the B3B shape. The pin size (0.250 inch outside diameter), materials (mixed oxide), temperatures and density of B3B, however, were more representative. Variables that were not representative for either pin included neutron energy spectrum and the length of continuous runs. LFT was not run for longer than 11 days without a shutdown while B3B was limited to about 5 day runs in the latter half of its irradiation (it had one run of about 18 days early in the test). These periods of constant operation are much shorter than the potential month-long runs of LMFBR's. Overall, stepwise gas release exhibited by LFT is more representative of an LMFBR fuel pin because of its flat power profile, indicating that the fuel may retain the gas in the central void, and the gas will be available to cause fuel ejection on failure.

Figure B-1 The Measured Gas Pressure Versus Time of Operation. Element LFT (from Reference 1)

Figure B-2 Pin B3B Pressure And Temperature History

Table B-1

	B3B	LFT
Fuel	(0.75 U - 0.25 Pu)O ₂ .00	UO ₂ .005
Clad Outside Diameter Temperature	1250°F	410°F
Linear Heat Generation	21-26 kw/ft	22-24 kw/ft
Fuel Diameter	0.217	0.765
Pin Diameter	0.250	0.803
Clad Thickness	0.015	0.018
Gap	0.003	0.004
Enrichment	40% U-235	2.4% U-235
Density	90% Pellet	93.7% Pellet
Fuel Surface Temperature	1800°F	860°F
Burnup	18 MWd/kg	3.3 MWd/kg
Gas Volume Generated	33 cc stp	28 cc stp

Figure B-3 Radial Power Density Profiles

Figure B-4 Temperature Profiles

APPENDIX B-II

ESCAPE OF SODIUM FROM A FAILED PIN DURING STARTUP

If there is a large area failure, the fuel is exposed to sodium. Thermal stress cracking at shutdown will give the sodium a path to the central region of the fuel. This can be seen in photographs of irradiated fuel pin cross-sections. In the isothermal state of shutdown, the fuel pin central void could fill with liquid sodium, as could the cracks. When the reactor starts back up, the sodium will expand and be forced out of the pin. At first, just the thermal expansion will be effective. When the fuel temperature reaches the local sodium boiling point, the vapor pressure will drive the sodium out. The flow rate out the crack will depend on whether the sodium is in the liquid or vapor state.

Another consideration is that, during startup, the fuel is expanding and the cracks are closing. If the flow path is sufficiently restricted, large pressures may build up in the pin.

The calculations are based on a linear rise to power. The nominal case is a ten-hour startup. Fuel centerline temperature is given as a function of time:

$$T(t) = (400t + 400)^{\circ}\text{F} = (222T + 478)^{\circ}\text{K},$$

where "t" is the time in hours from the beginning of startup. The sodium vapor pressure in the central void is assumed to correspond to the fuel centerline temperature. These vapor pressures can be found from Figure B-5.

The dimensions of the crack through which the sodium flows are somewhat arbitrary. Photographs of fuel pin cross-sections indicate that a conservative crack width is 0.001 inch. The length of the crack is similar to the fuel radius, 0.100 inch. The crack height is very arbitrary. It could be set at the height of a pellet but a more conservative value of 0.100 inch is used.

Assume a constant coefficient of linear expansion for the fuel of $10^{-5}/^{\circ}\text{F}$. The circumference of the pin at mid-radius is 0.315 inch. From studying many photographs of fuel pins it appears that a fuel pin typically has six or more radial cracks dividing the fuel into six or more segments. The length of fuel associated with the specific crack under consideration is then $0.315/6 = 0.0525$ inch. The temperature change necessary to close the crack will then be: $\Delta T = 0.001 \text{ inch} / (0.0525 \text{ inch}) (10^{-5} / ^{\circ}\text{K}) = 1900^{\circ}\text{K} = 3430^{\circ}\text{F}$.

The temperature change will be 3430°F at $3430^{\circ}\text{F} / 400^{\circ}\text{F/hr} = 8.6$ hours after the startup begins. Conservatively assume that the crack closes up completely at $T = 8$ hours. The width of the crack can then be given as a function of time into the startup:

$$W(t) = 0.001 - 1.25 \times 10^{-4} t$$

It is assumed that the length and height of the crack are constant.

Now calculate the mass flow rate of liquid sodium through the crack as a function of time. Use the formula:

$$P_i(t) - P_o = \frac{\rho V^2}{2} \left(1 + \frac{fL}{R} \right)$$

$$\rho = 45 \text{ lbm/ft}^3$$

$$f \approx 0.005$$

$$P_o = 50 \text{ psi}$$

$$R = \text{Area/Perimeter} = \frac{10^{-4} - 1.25 \times 10^{-5} t}{0.202 - 2.5 \times 10^{-4} t}$$

Figure B-5 Sodium Vapor-Vapor Pressure (ATMS) Versus Temperature

$$V(t)^2 = \frac{2(p_i - p_o)}{\rho \left(1 + \frac{fL}{R}\right)}$$

$$\text{Mass flow rate} = M = A(t) \cdot V(t) \cdot \rho$$

A simple calculation shows that the liquid sodium would be expelled in a very short time.

Since the case of liquid sodium escape is no problem, consider the case of sodium vapor escape. For the flow of a compressible fluid through a thin passage the following formula is applicable:

$$Q = \frac{bh^3}{692\mu LP_2} (P_1^2 - P_2^2) \text{ (Ref. 3)}$$

where:

- b = crack height = 0.1 in.
- h = crack width = 0.001 in. (initially)
- L = crack length = 0.1 in.
- μ = viscosity — see Table B-2
- P_1 = internal pressure — see Table B-2
- P_2 = external pressure = 50 psi

The results of the calculations are shown in Table B-2. The pressure is too slight to cause much flow during the first three hours. Most of the vapor is expelled during the fourth and fifth hours of startup.

Figure B-6 shows the pressure that builds up in the pin while the sodium vapor attempts to escape. If the sodium could not escape, the pressure would follow the sodium vapor pressure curve. The sodium is completely expelled at $t = 5.3$ hours, according to Figure B-7. The maximum pressure is 170 psi. The pressure then decays rapidly to the outside pressure. At $t = 8$ hours, the crack closes up and the pressure increases in direct proportion to the temperature. This increase is insignificant. The other plots on Figure B-6 are the results of using faster startups. If, instead of a ten hour startup, a ten minute startup is assumed, the sodium still escapes but the maximum pressure gets up to over 1000 psi. According to the plots, a seven minute startup can be tolerated. A faster startup causes the crack to close before the sodium has escaped. The pressure then follows the vapor pressure curve.

The maximum pressure developed during the seven-minute startup is in excess of 2000 psi. For a normal startup, however, the pressures are much lower. The calculations are conservative because credit has not been taken for the increased conductivity of the fuel pin due to the presence of sodium. This higher conductivity would result in lower fuel temperatures for the same power generation rate. Thus, the thermal transient would be less severe than assumed here.

Table B-2. Sodium Vapor Flow From Fuel Pin

t (hr)	P vapor (psi)	T Q _L (°F)	$\mu \frac{\text{lb}_f\text{-sec}}{\text{in.}^2}$	Crack Width h(in.)	Flow Rate ft ³ /min	Specific Volume ft ³ /lbm	Mass Flow Rate lbm/min	Mass Remaining (lbm)
0	.0000027	400	5.0×10^{-9}	.001	1.4×10^{-10}	1.3×10^8	1.1×10^{-18}	97×10^{-5}
1	.01	800	7.0	.000875	2.8×10^{-7}	1.1×10^5	2.5×10^{-12}	97
2	1.3	1200	9.0	.00075	1.8×10^{-5}	900	2×10^{-8}	97
3	.3	1600	10.9	.000625	9.5×10^{-5}	80	1.2×10^{-6}	90
4	79	2000	12.8	.0005	4×10^{-4}	11	3.6×10^{-5}	0
5	190	2400	14.8	.000375	5.7×10^{-4}	4	1.4×10^{-4}	0
6	613	2800	16.8	.000250	1.17×10^{-3}	1.4	8.4×10^{-4}	0
7	1220	3200	18.7	.000125	4.86×10^{-4}	1.1	4.4×10^{-4}	0
8	2125	3600	20.7	0	0	—	0	0
9	3370	4000	22.6	0	0	—	0	0
10	5000	4400	24.6	0	0	—	0	0

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Figure B-6 Sodium Logged Pin Pressure During Startup

Figure B-7 Sodium Vapor Flow From a Sodium Logged Pin During Startup

APPENDIX B-III ARCH SUPPORT OF INTERNAL PRESSURES

The purpose of this analysis is to determine if the fuel can restrain internal pressures with its compressive strength by forming an arch over the cladding failure. In this configuration, even if the fuel was fractured due to thermal shock, it could still have structural strength.

The treatment here is based on a section in Timoshenko and Young's *Theory of Structures* entitled "Symmetrical Two-Hinged Arches." In the representation of Figure 7:

$$H = \frac{\int_0^s N \cos \phi \, ds}{\int_0^s \cos^2 \phi \, ds}$$

where

s = arc length

H = horizontal force on cladding

N = force normal to cladding

Since it is a circular arch, $dx = ds \cos \phi$

$$H = \frac{2 \int_0^{\ell/2} N \, dx}{\int_0^{\ell/2} \cos \phi \, dx} = \frac{\int_0^{\ell/2} \frac{Px^2}{r} \, dx}{\int_0^{\ell/2} \frac{(r^2 - x^2)^{1/2}}{r} \, dx}$$

where

P = hydrostatic pressure in plastic fuel.

Evaluating the definite integrals:

$$H = - \frac{P\ell^3}{6\ell (r^2 - \ell^2/4)^{1/2} + 12 r^2 \sin^{-1} (\ell/2r)}$$

$$N = H/\sin \theta/2$$

where θ defines the size of the cladding failure.

$$N = \frac{P\ell^3}{6 \sin \frac{\theta}{2} \left[\ell \left(r^2 - \ell \frac{2}{4} \right)^{1/2} + 2r^2 \left(\frac{\pi}{2} - \frac{\theta}{2} \right) \right]}$$

$$\sin \frac{\theta}{2} = \frac{\ell/2}{R}$$

$$N = \frac{8PR^3 \sin^2 \frac{\theta}{2}}{6R \sin \frac{\theta}{2} \left(r^2 - R^2 \sin^2 \frac{\theta}{2} \right)^{1/2} + 6r^2 \left(\frac{\pi}{2} - \frac{\theta}{2} \right)}$$

$$\text{But } r = R \tan \frac{\theta}{2}$$

$$N = \frac{2PR \sin^2 \frac{\theta}{2}}{3 \sin \frac{\theta}{2} \left(\tan^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \right)^{1/2} + 3 \tan^2 \frac{\theta}{2} \left(\frac{\pi}{2} - \frac{\theta}{2} \right)}$$

$$(\tan^2 \theta - \sin^2 \theta)^{1/2} = \frac{\sin^2 \theta}{\cos \theta}$$

$$N = \frac{2PR \sin^2 \frac{\theta}{2}}{3 \sin \frac{\theta}{2} \cdot \frac{\sin^2 \frac{\theta}{2}}{\cos \frac{\theta}{2}} + 3 \frac{\sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2}} \left(\frac{\pi}{2} - \frac{\theta}{2} \right)}$$

$$= \frac{2PR}{3 \tan \frac{\theta}{2} + 3 \left(\frac{\pi}{2} - \frac{\theta}{2} \right) / \cos^2 \frac{\theta}{2}}$$

For the stress in the arch, divide by the arch thickness.

The results of time-share computer codes run for this case are shown in Figure B-8. Figure B-8 shows that, as the size of the failure increases, the stress in the arch for a given internal pressure increases rapidly. Because the fuel is strong in compression, however, it is possible for very large pressures to be contained. Since fuel strength does not limit the pressures to small values, we must consider the strength of the cladding. The cladding must bear the load from the arch.

The maximum stress in the cladding occurs at the point of maximum moment. Simple beam analysis can be used for thin curved members such as the cladding (see Figure B-9). The bending moment is calculated from the following formula:

$$M = F_H (R \cos A/2 - \sqrt{R^2 - X^2}) + F_V (X - R \sin A/2)$$

$$F_H = P \sin A/2$$

$$F_V = P \cos A/2$$

Find the maximum moment:

$$\frac{dM}{dX} = 0 = F_H (-1/2) (R^2 - X^2)^{-1/2} (-2X) + F_V$$

$$\frac{X}{\sqrt{R^2 - X^2}} = -\frac{F_V}{F_H} = -\cot A/2$$

A plot of X versus $X/\sqrt{R^2 - X^2}$ appears in Figure B-10. To find the position of maximum moment of angle A , locate $\cot A/2$ on the ordinate and read the value of " X " on the abscissa. The maximum moments are then calculated from the above formula.

A thin curved beam may be treated quite accurately with the simple beam formula:

$$\sigma = MC/I$$

For a 0.250 inch outside diameter clad with 0.010 inch wall thickness, and considering a one inch length of clad,

$$\frac{C}{I} = \frac{h/2}{bh^3/12} = \frac{6}{bh^2} = 6 \times 10^4$$

If 0.015 inch wall thickness is used, $C/I = 2.67 \times 10^4$. The following table gives values of maximum stress for a 100 pound loading:

Figure B-8 Defected Fuel Arch Stress

Figure B-9 Cladding Model For Stress Calculations

Figure B-10

A	cot A/2	$X_{\max}$	$M_{\max}$ (P=100)	$\sigma_{\max}$ (P=100)
1.15°	95.5	0.1250 inch	12.5 in-lbf	750,000 psi
12.6°	9.06	0.1242	12.1	726,000
24.1°	4.69	0.1222	11.5	690,000
35.5°	3.29	0.1198	10.5	630,000
47.0°	2.51	0.1142	8.5	510,000

Clearly, these stresses are far beyond the ultimate strength of the cladding ($\sim 70,000$ psi). Figure B-11 shows that the ultimate strength of the cladding is exceeded by a 17 pound force in the worst case. A seventeen pound force corresponds to an internal pressure of about 375 psi. If 0.015 inch thick cladding is used, an internal pressure of about 850 psi could be restrained.

Another limitation on the internal pressure arises from the deflection of the cladding under the loading. If it is assumed that the arch buckles and fails when the deflection of the cladding is equal to the thickness of the arch, a limit can be calculated for internal pressure.

First, the thickness of the arch for each failure size (angle) is found from Figure B-9. The force on the cladding necessary to cause a deflection equal to the arch thickness is found from Figure B-12 for each failure size. The internal

Figure B-11 Stress in Failed Cladding

Figure B-12 Deflection of Failed Clad

pressure necessary to cause that force is found from Figure B-13. The internal pressure necessary to deform the clad enough to fail the arch is now known.

The strength limits and the deflection limits have been plotted in Figure B-14. Ten-mil cladding can restrain internal pressures of up to 300 psi. Fifteen mil cladding can restrain up to 720 psi. These pressures put an upper limit on pressures that can be developed in a failed fuel pin. The thinner cladding has a safety advantage in this respect because it deforms at a lower internal pressure, than with the thicker cladding. A lower driving force is then available to expel fuel. The pressure necessary to expel this fuel is not presently known.

Figure B-13 Force on Clad Due Internal Pressure

Figure B-14 Strength and Deflection Limits of Failed Fuel

APPENDIX B-IV

RECOMMENDED EXPERIMENT TO INVESTIGATE INTERNAL PRESSURE BEHAVIOR

An experiment to measure the buildup of fission gas in the central void of an operating fuel pin could answer many questions concerning the behavior of a fuel pin and the potential consequences of pin failure. It is felt that such a test is possible with present facilities.

The test pin configuration is shown in Figure B-15. The central void in the active fuel may be allowed to form naturally. The ceramic plug may be depleted uranium. Its main purpose is to provide a gas path from the hot fuel up into cooler temperature regions where the pressure transducers are located. The active fuel should not be highly enriched because a large flux depression in the pin will alter the fission gas distribution. The fuel column does not have to be long. There should be several pellets, however, in order to include the pellet-interface effects.

The output from the two pressure transducers should give a good picture of the gas behavior during operation and power changes. The model presented in this memo would predict that the transducer that measures central void pressure would indicate continuously increasing pressures during steady operation. The other transducer measures the pressure in the gas plenum. It should remain constant or rise very slowly due to gas release from the periphery of the fuel pin during steady operation. At the time of a significant power change (e.g., shutdown), the void transducer should drop and the plenum transducer should jump. They should equilibrate so they indicate the same pressure.

A valuable addition to this experiment would be the use of a remotely initiated defect mechanism to provide a large area failure in the cladding. Pressure measurements would then provide evidence that the unrestrained fuel can or cannot contain pressures in the central void.

Figure B-15 Test Pin Configuration

REFERENCES – APPENDIX B

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2. GEAP-5158, "Sodium-Cooled Reactors Fast Ceramic Reactor Development Program, 18th Quarterly Report, February – April, 1966," Aug. 1966.
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APPENDIX C

DERIVATION OF THE "WAVE" EQUATION (P.M. Tschamper)

1.0 INTRODUCTION

This appendix describes the derivation of the "WAVE" model of fuel/clad thermal interactions originally developed by D. Bender at GE. The model applies to prediction of the hydrodynamic response (in slab geometry) of coolant in extended (plane) surface contact with hot fuel or clad material. Simultaneous solution of the WAVE equation with a simplified equation of state and a transient heat conduction equation yields an estimate of the pressure profile in the coolant as a function of time.

A one dimensional geometry is presumed and a plane through which heat is transferred into the semi-infinite system represents the boundary between fuel and coolant.

The following nomenclature is used throughout the derivation;

x	=	linear distance away from the fuel coolant interface
t	=	time
$p(x,t)$	=	pressure at distance x , time t
$\rho(x,t)$	=	density at distance x , time t
$u(x,t)$	=	velocity at distance x , time t
α	=	thermal diffusivity of coolant
$T(x,t)$	=	temperature at distance x , time t
g_c	=	$32.2 \text{ (lbm-ft)/(lbf-sec}^2\text{)}$

Figure C-1 identifies the representation of the model.

The basic equations that have to be solved simultaneously are:

1. Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) = 0 \quad (1)$$

2. Momentum Equation:

$$g_c \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} (\rho u^2) + \frac{\partial}{\partial t} (\rho u) = 0 \quad (2)$$

3. Energy (rather conduction) Equation:

$$T = T_o + (T_{\text{interface}} - T_o) \operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha t}} \right) \quad (3)$$

4. Equation of State:

$$\rho = \rho_o + b_p (p - p_o) + b_T (T - T_o) \quad (4)$$

Figure C-1. The WAVE Fuel/Sodium Interaction Model

2.0 NONDIMENSIONALIZED PARAMETERS

It is convenient, though not necessary, to define nondimensionalized parameters which will be used during the manipulations of the equations.

$$\gamma^2 = b_p / g_c, \quad \text{sec}^2 / \text{ft}^2 \quad (5)$$

$$x^* = \frac{x}{\alpha \gamma}, \quad \text{no units} \quad (6)$$

$$t^* = \frac{t}{\alpha \gamma^2}, \quad \text{no units} \quad (7)$$

$$T^* = (T - T_0) / (T_{\text{interface}} - T_0), \quad \text{no units} \quad (8)$$

$$p^* = (p - p_0) / \hat{p}, \quad \text{no units} \quad (9)$$

$$\hat{p} = \rho_0 / (g_c \gamma^2), \quad \text{lbf} / \text{ft}^2 \quad (10)$$

$$\rho^* = \rho / \rho_0, \quad \text{no units} \quad (11)$$

$$u^* = u \gamma, \quad \text{no units} \quad (12)$$

$$b_p = \left(\frac{\partial \rho}{\partial p} \right)_{T=\text{const.}}, \quad b_T = \left(\frac{\partial \rho}{\partial T} \right)_{p=\text{const.}} \quad (13)$$

Note that the expression for γ^{-1} represents a propagation velocity which is equal to isothermal acoustic velocity.

$$v_p = \gamma^{-1} = \sqrt{g_c / b_p} = \sqrt{g_c \left(\frac{\partial p}{\partial \rho} \right)_{T=\text{const.}}}$$

3.0 THE CONTINUITY EQUATION

The continuity equation (1) is expanded to:

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} = 0 \quad (14)$$

By equations (12), (11), (7) and (6)

$$u = u^*/\gamma \text{ and } x = x^* \cdot \alpha \cdot \gamma, \rho = \rho_0 \cdot \rho^* \text{ and } t = t^* \cdot \alpha \cdot \gamma^2.$$

and equation (1) becomes

$$\frac{1}{\alpha \cdot \gamma^2} \rho_0 \frac{\partial \rho^*}{\partial t^*} + \rho_0 \cdot \rho^* \frac{1}{\gamma^2 \cdot \alpha} \frac{\partial u^*}{\partial x^*} + \rho_0 \frac{1}{\gamma^2 \alpha} u^* \frac{\partial \rho^*}{\partial x^*} = 0$$

or

$$\frac{\partial \rho^*}{\partial t^*} + \rho^* \frac{\partial u^*}{\partial x^*} + u^* \frac{\partial \rho^*}{\partial x^*} = 0 \quad (15)$$

assuming that the thermal diffusivity α and the propagation velocity included in γ^2 are constants. ρ_0 is the density of the coolant before the effect of the fuel-sodium interaction.

4.0 THE MOMENTUM EQUATION

The momentum equation (2) can be operated on to yield:

$$g_c \frac{\partial p}{\partial x} + 2\rho u \frac{\partial u}{\partial x} + u^2 \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial t} + u \frac{\partial \rho}{\partial t} = 0 \quad (16)$$

But with the help of the continuity relation of equation (14)

$$u \frac{\partial \rho}{\partial t} = -\rho u \frac{\partial u}{\partial x} - u^2 \frac{\partial \rho}{\partial x} \quad (17)$$

Substitution of (17) in (16) yields:

$$g_c \frac{\partial p}{\partial x} + \rho u \frac{\partial u}{\partial x} + \rho \frac{\partial u}{\partial t} = 0 \quad (18)$$

By equations (9), (10), (6), (11), (12) and (7)

$$p = p^* \cdot \hat{p} + p_o = p^* \cdot \rho_o / (g_c \cdot \gamma^2) + p_o$$

$$x = x^* \cdot \alpha \cdot \gamma$$

$$\rho = \rho_o \cdot \rho^*$$

$$u = u^* / \gamma$$

$$t = t^* \cdot \alpha \cdot \gamma^2$$

and (20) turns into

$$\frac{\partial p^*}{\partial x^*} + u \frac{\partial u^*}{\partial x^*} + \frac{\partial u^*}{\partial t^*} = 0 \quad (19)$$

if ρ^* is assumed to be 1.

Again it has been assumed that the thermal diffusivity α and the propagation velocity included in γ are constant.

5.0 THE HEAT CONDUCTION EQUATION

The heat conduction solution (equation 3) follows from the Fourier conduction equation:

$$\frac{dT(x,t)}{dt} = \alpha \frac{d^2 T}{dx^2}, \quad (20)$$

which is satisfied, in the half spaces, $X < 0$, and $X > 0$, by:¹

$$T_f = A_f + B_f \operatorname{erf} \frac{|x|}{2\sqrt{\alpha_f t}}, \quad X < 0, \quad (21)$$

$$T_c = A_c + B_c \operatorname{erf} \frac{x}{2\sqrt{\alpha_c t}}, \quad X > 0 \quad (22)$$

Here, we have used the subscripts, f and c, to denote the fuel and coolant properties, respectively. The four arbitrary constants, A_f , B_f , A_c and B_c are determined from the initial and boundary conditions. The initial conditions,

$$T_f(x, 0) = T_{fo} \quad (23)$$

and

$$T_c(x, 0) = T_{co} \quad (24)$$

lead to:

$$A_f + B_f = T_{fo} \quad (25)$$

and

$$A_c + B_c = T_{co} \quad (26)$$

At the boundary, $x = 0$, we have:

$$K_f \left. \frac{\partial T_f}{\partial x} \right|_{x=0} = K_c \left. \frac{\partial T_c}{\partial x} \right|_{x=0}, \quad t > 0, \quad (27)$$

and,

$$T_f(0, t) = T_c(0, t) \quad (28)$$

Thus,

$$A_f = A_c, \quad (29)$$

and,

$$\frac{K_f}{\sqrt{\alpha_f}} \cdot B_f = -\frac{K_c}{2\sqrt{\alpha_c}} \cdot B_c \quad (30)$$

In the above, K_f and K_c are the thermal conductivities of fuel and coolant, respectively.

After solving equations 25, 26, 29 and 30 for the coefficients, the solution in the space, $x > 0$, which is the one of interest, is

$$T_c = T_{co} + \frac{K_f/\sqrt{\alpha_f}}{K_c/\sqrt{\alpha_c} + K_f/\sqrt{\alpha_f}} (T_{fo} - T_{co}) \operatorname{erfc} \frac{x}{2\sqrt{\alpha_c t}} \quad (31)$$

Now, if equation 31 is applied at the boundary, $x=0$, we find that the interface temperature is constant with time:

$$T_{\text{interface}} = T(o,t) = T_o + \frac{K_f/\sqrt{\alpha_f}}{K_c/\sqrt{\alpha_c} + K_f/\sqrt{\alpha_f}} (T_{fo} - T_o) \quad (32)$$

where we have returned to the previously defined nomenclature,

$$T_c \equiv T(x, t), \text{ and,}$$

$$T_{co} \equiv T_o.$$

We can also, as seen from equation 32, express equation 31 in the form,

$$T(x, t) = T_o + (T_{\text{interface}} - T_o) \operatorname{erfc} \frac{x}{2\sqrt{\alpha_c t}} \quad (33)$$

which is used in equation 3.

Nondimensionalization

Equation (33) is operated on with the relationships (8), (6), and (7); hence:

$$T = T^* (T_{\text{interface}} - T_o) + T_o$$

$$x = x^* \cdot \alpha \cdot \gamma$$

$$t = t^* \cdot \alpha \cdot \gamma^2$$

The following nondimensionalized form results

$$T^* = \operatorname{erfc} \left(\frac{x^*}{2\sqrt{t^*}} \right) \quad (34)$$

6.0 EQUATION OF STATE

The equation of state can be nondimensionalized as follows. Equations (11), (9), and (8) are used together with the new definition.

$$\mu_T^2 = \frac{b_T}{\rho_0} \cdot (T_{\text{interface}} - T_0), \text{ no units} \quad (35)$$

Hence, equation (4) is transformed into

$$\rho^* = 1 + p^* \mu_T^2 \cdot T^* \quad (36)$$

7.0 SIMULTANEOUS SOLUTION OF THE SET OF THE DIFFERENTIAL EQUATIONS

The following four equations have to be solved simultaneously

$$\frac{\partial \rho^*}{\partial t^*} + \rho^* \frac{\partial u^*}{\partial x^*} + u^* \frac{\partial \rho^*}{\partial x^*} = 0 \quad (15 \text{ repeated})$$

$$\frac{\partial p^*}{\partial x^*} + u^* \frac{\partial u^*}{\partial x^*} + \frac{\partial u^*}{\partial t^*} = 0 \quad (19 \text{ repeated})$$

$$T^* - \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}}\right) = 0 \quad (34 \text{ repeated})$$

$$\rho^* - 1 - (p^* + \mu_T^2 T^*) = 0 \quad (36 \text{ repeated})$$

The following assumptions are made to represent a first order approximation to the problem:

- In equation (15) it is assumed that $\rho^* \approx 1$ and that consequently $\partial \rho^* / \partial x^* \approx 0$. Note that a similar approximation has already been executed on the equation (19) when it was nondimensionalized in paragraph 4.
- In equation (19) the term $\mu^* \partial u^* / \partial x^*$ has been neglected.
- Equation (36) has been used unaltered.

The equations that will be solved simultaneously then take on the form:

$$\text{Continuity: } \frac{\partial u^*}{\partial x^*} + \frac{\partial p^*}{\partial t^*} = 0 \quad (37)$$

$$\text{Momentum: } \frac{\partial p^*}{\partial x^*} + \frac{\partial u^*}{\partial t^*} = 0 \quad (38)$$

$$\text{Conduction: } T^* - \operatorname{erfc} \frac{x^*}{2\sqrt{t^*}} = 0 \quad (39)$$

$$\text{State: } \rho^* - 1 - (p^* + \mu_T^2 T^*) = 0 \quad (40)$$

The independent variables are x^* and t^* and the dependent ones u^* , ρ^* , p^* and T^* .

Using equation (40) and differentiating with respect to time yields:

$$\frac{\partial \rho^*}{\partial t^*} = \frac{\partial p^*}{\partial t^*} + \mu_T^2 \frac{\partial T^*}{\partial t^*} \quad (41)$$

Substituting this expression into the continuity equation furnishes the relationship with velocity:

$$\frac{\partial u^*}{\partial x^*} = - \frac{\partial p^*}{\partial t^*} - \mu_T^2 \frac{\partial T^*}{\partial t^*} \quad (42)$$

The partial derivative of the temperature T^* with respect to time is,²

$$\frac{\partial T^*}{\partial t^*} = \frac{x^*}{2\sqrt{\pi t^{*3}}} \cdot e^{-x^{*2}/4t^*} \quad (43)$$

We may now eliminate the temperature in equation (42) with (43) and obtain

$$\frac{\partial u^*}{\partial x^*} = -\mu_T^2 \cdot \frac{x^*}{2\sqrt{\pi t^{*3}}} \cdot e^{-x^{*2}/4t^*} - \frac{\partial p^*}{\partial t^*} \quad (44)$$

If equation (58) is transposed into the Laplace domain we have,³

$$\frac{\partial u^*}{\partial x^*} = -\mu_T^2 \cdot e^{-x^*} \cdot \sqrt{s} - s \cdot p^* \quad (45)$$

Now we use equation (38) and transpose it into the Laplace domain:

$$\frac{\partial p^*}{\partial x^*} = -s \cdot u^* \quad (46)$$

Solution of this equation for u^* and differentiation with respect to x^* yields:

$$\frac{\partial u^*}{\partial x^*} = -\frac{1}{s} \frac{\partial^2 p^*}{\partial x^{*2}} \quad (47)$$

Equating (47) with (44) and thus eliminating the velocity gradient term, $\partial u^*/\partial x^*$:

$$\frac{1}{s} \frac{\partial^2 p^*}{\partial x^{*2}} = \mu_T^2 e^{-x^*} \sqrt{s} + s \cdot p^* \quad (48)$$

or in the familiar form of a partial differential equation with a non-homogeneity term:

$$\frac{\partial^2 p^*}{\partial x^{*2}} - s^2 p^* = s \cdot \mu_T^2 e^{-x^*} \sqrt{s} \quad (49)$$

The solution of the homogeneous part is

$$p^* = C_1 \cdot e^{-s} \cdot x^* + C_2 \cdot e^s \cdot x^* \quad (50)$$

Since the solution of this problem cannot be unbounded, the coefficient C_2 must equal 0.

The particular solution will have to contain an exponential and is thus written in the form:

$$p^* = C_3 \cdot e^{-x^*} \sqrt{s} \quad (51)$$

Its first and second derivatives with respect to x^* are:

$$\frac{\partial p^*}{\partial x^*} = -C_3 \cdot \sqrt{s} \cdot e^{-x^*} \sqrt{s} \text{ and } \frac{\partial^2 p^*}{\partial x^{*2}} = tC_3 \cdot s \cdot e^{-x^*} \sqrt{s} \quad (52)$$

Introduction of (52) into (49) permits us to calculate the value of C_3 :

$$C_3 s \cdot e^{-x^*} \sqrt{s} - s^2 C_3 \cdot e^{-x^*} \sqrt{s} = s \mu_T^2 e^{-x^*} \sqrt{s}$$

or

$$C_3 = \frac{\mu_T^2}{(1-s)} \quad (53)$$

Therefore the complete solution of (62) is:

$$p^* = C_1 \cdot e^{-sx^*} + \frac{\mu_T^2}{(1-s)} e^{-x^*} \sqrt{s} \quad (54)$$

We need a boundary condition to calculate the value of C_1 . Assume:

$$\left. \frac{\partial p^*}{\partial x^*} \right]_{x^*=0} = 0 \quad (55)$$

This of course makes the value of C_1 :

$$C_1 = \mu_T^2 \cdot \frac{1}{\sqrt{s}(s-1)} \quad (56)$$

If equation (56) is substituted into equation (54) we find the equation for p^* which has to be back-transformed into the real domain:

$$p^*(s) = \mu_T^2 \cdot \frac{e^{-s \cdot x^*}}{\sqrt{s}(s-1)} - \mu_T^2 \frac{e^{-x^*} \sqrt{s}}{(s-1)} \quad (57)$$

Each quotient will be treated separately. First, transform the first one on the RHS, separating it from the constant μ_T^2 .

$$\mathcal{L}^{-1} \left[e^{-sx^*} \cdot \frac{1}{\sqrt{s}(s-1)} \right] = F(t^* - x^*) \text{ for } F(t^*) = 0 \text{ when } t^* < 0 \quad (58)$$

$$= e^{-(t^* - x^*)} \cdot \text{erf}(\sqrt{t^* - x^*}) \quad (59)$$

Note that a special case of the Faltung principle⁴ was used.

The complete transform of the first expression on the RHS of equation (57) then is:

$$R_1 = (\mu_T^2) \cdot e^{-(t^* - x^*)} \cdot \text{erf}(\sqrt{t^* - x^*}) \quad (60)$$

The second expression in equation (57) has to be transformed into partial fractions before the transform may be performed. Thus:

$$\mu_T^{-2} \cdot \frac{e^{-x^* \sqrt{s}}}{(s-1)} = \mu_T^{-2} \cdot \frac{1}{2} \cdot \left[\frac{e^{-x^* \sqrt{s}}}{\sqrt{s}(\sqrt{s}+1)} + \frac{e^{-x^* \sqrt{s}}}{\sqrt{s}(\sqrt{s}-1)} \right] \quad (61)$$

Both quotients in the square bracket can be transformed at once:⁵

$$\mathcal{L}^{-1} \left[\frac{e^{-x^* \sqrt{s}}}{\sqrt{s}(\sqrt{s}+1)} \right] = e^{x^*} \cdot e^{t^*} \cdot \operatorname{erfc} \left(\sqrt{t^*} + \frac{x^*}{2\sqrt{t^*}} \right) \quad (62)$$

and

$$\mathcal{L}^{-1} \left[\frac{e^{-x^* \sqrt{s}}}{\sqrt{s}(\sqrt{s}-1)} \right] = e^{-x^*} \cdot e^{t^*} \cdot \operatorname{erfc} \left(-\sqrt{t^*} + \frac{x^*}{2\sqrt{t^*}} \right) \quad (63)$$

Hence the complete transform of expression (61), the second term in equation (57) is:

$$R_2 = \mu_T^{-2} \cdot \frac{1}{2} \cdot e^{t^*} \left\{ e^{x^*} \operatorname{erfc} \left(\frac{x^*}{2\sqrt{t^*}} + \sqrt{t^*} \right) + e^{-x^*} \operatorname{erfc} \left(\frac{x^*}{2\sqrt{t^*}} - \sqrt{t^*} \right) \right\} \quad (64)$$

The complete solution for the parameter $p^*(x^*, t^*)$ is therefore:

$$p^*(x^*, t^*) = \mu_T^{-2} \cdot e^{-(t^* - x^*)} \cdot \operatorname{erf}(\sqrt{t^* - x^*}) - \mu_T^{-2} \cdot \frac{e^{t^*}}{2} \left\{ e^{x^*} \operatorname{erfc} \left(\frac{x^*}{2\sqrt{t^*}} + \sqrt{t^*} \right) + e^{-x^*} \operatorname{erfc} \left(\frac{x^*}{2\sqrt{t^*}} - \sqrt{t^*} \right) \right\} \quad (65)$$

This equation represents the solution for the nondimensionalized pressure p^* .

Let us now solve for the nondimensionalized velocity u^* . In equation (60) we have already presented the momentum equation in the Laplace domain:

$$\frac{\partial p^*}{\partial x^*} = -s \cdot u^* \quad (46 \text{ repeated})$$

But in equation (57) we have presented the dimensionless pressure parameter p^* as a function of the Laplace variable s . Therefore we only have to differentiate this equation with respect to x^* and will then be able to equate the result with the RHS of the above equation. Thus:

$$\frac{\partial p^*}{\partial x^*} = -\mu_T^{-2} \cdot s \cdot \frac{e^{-s \cdot x^*}}{\sqrt{s}(s-1)} + \mu_T^{-2} \cdot \frac{s}{\sqrt{s}} \cdot \frac{e^{-x^* \sqrt{s}}}{(s-1)} \quad (66)$$

and with (46) we obtain

$$u^*(s) = \mu_T^2 \left[\frac{e^{-s} x^*}{\sqrt{s}(s-1)} - \frac{e^{-x^*} \cdot \sqrt{s}}{(s-1)} \right] \quad (67)$$

As may easily be seen, the inverse Laplace transform of equation (67) involves exactly the same Laplace cases as used in the treatment of equation (57), though the signs of the partial fractions are different. We have therefore for the nondimensionalized velocity in the real domain of t^* the following expression:

$$u^*(x^*, t^*) = \mu_T^2 \left\{ e^{(t^* - x^*)} \cdot \operatorname{erf}(\sqrt{t^* - x^*}) + \frac{1}{2} e^{t^*} \left[e^{x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} + \sqrt{t^*}\right) - e^{-x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} - \sqrt{t^*}\right) \right] \right\} \quad (68)$$

The purpose of this calculation was to obtain the dependent variables p^* , u^* , T^* and ρ^* in terms of the independent variables x^* , t^* . The results are summarized in Table 1.

Table 1. Solutions of the Fuel/Sodium Interaction Problem

(in non-dimensionalized form)

$$p^*(x^*, t^*) = \mu_T^2 \cdot e^{-(t^* - x^*)} \cdot \operatorname{erf}(\sqrt{t^* - x^*}) - \mu_T^2 \cdot \frac{e^{t^*}}{2} \left\{ e^{x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} + \sqrt{t^*}\right) + e^{-x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} - \sqrt{t^*}\right) \right\}$$

$$u^*(x^*, t^*) = \mu_T^2 \left\{ e^{(t^* - x^*)} \cdot \operatorname{erf}(\sqrt{t^* - x^*}) + \frac{1}{2} e^{t^*} \left[e^{x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} + \sqrt{t^*}\right) - e^{-x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} - \sqrt{t^*}\right) \right] \right\}$$

$$T^*(x^*, t^*) = \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}}\right)$$

$$p^*(x^*, t^*) = 1 + \mu_T^2 \left\{ e^{-(t^* - x^*)} \cdot \operatorname{erf}(\sqrt{t^* - x^*}) - \frac{e^{t^*}}{2} \left[e^{x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} + \sqrt{t^*}\right) + e^{-x^*} \cdot \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}} - \sqrt{t^*}\right) \right] + \operatorname{erfc}\left(\frac{x^*}{2\sqrt{t^*}}\right) \right\}$$

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