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MICROINSTABILITIES IN INHOMOGENEOUS PLASMA

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INTRODUCTION

Our effort to investigate microinstabilities in inhomogeneous magnetized plasma the past year has been particularly productive in providing the basis for analysing a number of problems of current interest to experimentalists in controlled thermonuclear research. Although a number of papers have recently dealt with these instabilities, a certain degree of confusion has arisen—at least in our mind—concerning the various results since they are generally obscured by many assumptions and approximations. A systematic derivation of the despersion equation for electromagnetic oscillations in anisotropic magnetized plasma incorporating density, temperature, and magnetic field gradients was in our judgment mandatory in order to fully assess other work and to effectively chart the path of our investigations. This derivation was carried out primarily by Mr. W. M. Farr in his doctoral dissertation where in—within the linear theory of plasma oscillations—he obtained from the Vlasov equation and the full set of Maxwell equations a general dispersion equation for electromagnetic disturbances propagating in an inhomogeneous anisotropic plasma situated in nonuniform magnetic field. This equation was found to properly reduce in certain limiting cases to equations employed by others in investigating some of these instabilities but with the added advantage of truly assessing the extent of validity of available results and their relation to CTR experiments. With such an equation as a basis, we have set out to examine a number of instabilities which could arise in inhomogeneous plasma with the hope of assessing the simaltaneous effect of gradients on oscillations near integral multiples of the ion cyclotron frequency. Emphasis is placed on these high frequency modes since they are continuously making their presence felt in mirror machines. In what follows a short description of the work underway along with some of the initial results is presented.

Incidentally since his graduation, Dr. Farr has joined the Sherwood Theoretical group at the Oak Ridge National Laboratory.

A. THE "LOCAL APPROXIMATION" IN INSTABILITY STUDIES

Invariably all investigations of universal, temperature gradient, and other instabilities in inhomogeneous magnetized plasma invoke the concept of "local approximation." Though widely used and accepted, the mathematical origin—and to some extent the concomitant physical interpretation and validity—has never been fully dileneated in the literature. For example, in the study of universal modes in a plasma with spatial variation in one dimension, say along x, the customary procedure is to write the equilibrium particle distribution function as

$$F_0(\vec{r},\vec{v}) = F_0(v_\perp^2, v_{11}^2) + \frac{v_y}{\Omega}\frac{\partial f_o}{\partial x} \qquad (1)$$

where it is implied that F_o is a function of the constants of motion: The perpendicular and parallel energies and the canonical angular momentum, $x + v_y/\Omega$. The x coordinate of the origin is chosen for convenience and the derivative is evaluated at that point. The justification for the above expansion is that v_y/Ω is of the order of the average Larmor radius and so long as it is much smaller than the characteristic distance of the density gradient it is an acceptable expansion. The point, however, arises that since v_y takes on values from zero to infinity the validity of the expansion could be in doubt. Moreover, suppose the plasma under consideration is situated in a curved magnetic field stimulated by a gravitational force g. In that case the equilibrium distribution function is a function of an additional constant of motion, namely $v_\perp^2 - 2gx$ where $v_\perp$ is particle velocity normal to the magnetic field. In that case how does one proceed to expand; what is the expansion parameter and is the "local approximation" still valid?

In order to resolve this "brushed aside" question and to establish a consistent mathematical formulation for attacking such problems we return to the basic equations (the Vlasov and Maxwell equations) and for convenience specialize to the case of electrostatic oscillations. In that case the linearized equations of the system are simply

$$\frac{\partial f_1}{\partial t} + \vec{v}_o\cdot\nabla f_1 + [g\,\vec{e}_x + \Omega\,(\vec{v}\times\vec{e}_z)]\cdot\frac{\partial f_1}{\partial\vec{v}} = -\frac{e}{m}\,\vec{E}_1\cdot\frac{\partial f_o}{\partial\vec{v}} \qquad (2)$$

$$\nabla\cdot\vec{E}_1 = -\nabla^2\varphi = 4\pi\sum_j e_j\int f_1\,d^3v \qquad (3)$$

where as usual neutral equilibrium plasma in uniform magnetic and gravitational fields is assumed. Since the spatial variations are along x one can choose the perturbations to be of the form $\varphi = \varphi(x)\exp i\,(k_y y + k_z z - wt)$ and a solution to equations (2) and (3) is obtained by integrating over the unperturbed particle orbits. The result is

$$\left[\frac{d^2}{dx^2} - k^2\right]\varphi(\vec{r},t=o) = -4\pi\sum_j\frac{e_i}{m_j}\int d^3v\left\{2\frac{\partial f_o}{\partial(v_\perp^2-2gx)}\varphi(\vec{r})\right.$$

$$+\,i\left[2w\frac{\partial f_o}{\partial(v_\perp^2-2gx)} + \frac{k_\perp}{\Omega_j}\frac{\partial f_o}{\partial(x+\frac{v_y}{\Omega})} + 2\,k_{11}\,v_{11}\left(\frac{\partial f_o}{\partial v_{11}^2}\right.\right.$$

$$\left.\left.\left. - \frac{\partial f_o}{\partial(v_\perp^2-2gx)}\right)\right]\int_{-\infty}^{o}\varphi(\vec{r}',t')dt'\right\} \qquad (4)$$

This equation is a very complex integro-differential equation and generally very difficult to solve. To make it accessible, we first expand f_o in Taylor series about $\eta = v_y/\Omega = o$ assuming that f_o is an entire function of η and keep all the terms in the expansion since v_y/Ω takes on all possible real values under the integral over velocity space, i.e., we let

$$f_o(x + \frac{v_y}{\Omega}, v_\perp^2 - 2gx, v_{11}^2) = \sum_{n=o}^{\infty} \frac{(\frac{\bar{v}_y}{\Omega})^n}{n!} \left\{\frac{\partial^n}{\partial\eta^n} f_o (x + \eta, v_\perp^2 - 2gx, v_{11}^2)\right\}_{\eta=o}$$

in which case Equation (4) becomes

$$\left[\frac{d^2}{dx^2} - k^2 \quad \varphi(\vec{r},o)\right] = -4\pi \sum_j \frac{e_j}{m_j} \int d^3v \sum_{n=o}^{\infty} \left(\frac{v_\perp}{\Omega}\right)^n (\sin\alpha)^n \frac{\partial^n}{\partial\eta^n}$$

$$\left\{2 \frac{\partial f_o(\eta)}{\partial(v_\perp^2 - 2gx)} \varphi(\vec{r}) + i \left[2w \frac{\partial f_o(\eta)}{\partial(v_\perp^2 - 2gx)} + \frac{k_\perp}{\Omega} \frac{\partial f_o}{\partial\eta}\right.\right.$$

$$\left.\left. + 2 k_{11} v_{11} \left(\frac{\partial f_o}{\partial v_{11}^2} - \frac{\partial f_o}{\partial(v_\perp^2 - 2gx)}\right)\right] \int_{-\infty}^{o} \varphi(\vec{r}', t')dt'\right\}_{\eta=o} \tag{5}$$

where α is the angle in velocity space. We note that the time integration has some maximum absolute value, N, and that the absolute value of, say, the kth term is less than

$$\frac{N}{k!} \frac{\partial^k}{\partial\eta^k}\left| <\psi> \right| = \frac{N}{k!} \frac{\partial^k}{\partial\eta^k}\left| \int\psi \, d^3v \right| \tag{6}$$

where the quantity in the angular bracket is some velocity moment which is a function of x and η. We assume that all such quantities are of the same order and set them equal to the largest $\rho_j^k G_k(x,\eta)$ where $\rho_j = (v_\perp/\Omega)_j$ is the average Larmor radius of the jth specie. The function G_k may then be written as

$$G_k (x,\eta) = G_k \left[n (x + \eta), T_\perp (x + \eta), T_{11} (x + \eta), x\right] \tag{7}$$

where the particle density n, and the temperatures are assumed to be slowly varying functions of their arguments. In fact we let them vary so slowly that

$$G_o (x,\eta) \gg \rho_j \frac{\partial}{\partial\eta} G_1 (x,\eta) \; \ldots . \gg \frac{\rho_j^k}{k!} \frac{\partial^k}{\partial\eta^k} G_k (x,\eta)$$

and keep only the first two terms. This then allows us to write

$$f_o\ (x + \frac{v_y}{\Omega},\ v_\perp^2 - 2gx, v_{11}^2) \simeq f_o\ (x, v_\perp^2 - 2gx, v_{11}^2)$$

$$+ \frac{v_y}{\Omega} \cdot \frac{\partial}{\partial\eta}\ f_o\ (x + \eta, v_\perp^2 - 2gx, v_{11}^2)\ _{\eta=o} \tag{8}$$

We are now in a position to justify the "local approximation." To do so we return to Equation (5) and expand the quantity $\varphi(x')$ that appears under the time integral about the value of x appearing in the rest of the equation, i.e.,

$$\varphi(x') = \sum_{\ell=o}^{\infty} \frac{1}{\ell!} \left[x(t') - x\right] \frac{\partial^\ell}{\partial x^\ell} \varphi(x) \tag{9}$$

We substitute (9) into (5) with the result that the latter is reduced from an integro-differential equation to a purely differential equation in x of infinite order when all the velocity integrations have been performed. The result may then be put in the form

$$\sum_{n=o}^{\infty} A_n\ (x)\ \frac{d^n\varphi(x)}{dx^n} = o \tag{10}$$

The x-dependence of the potential φ must be contained in the x-dependence of n, $T_\perp$, and T_{11}. Therefore if we write $\varphi(x)$ as power series in $(x-x_o)$, as in (9), the coefficients of $(x-x_o)^\ell$ must be proportional to

$$\frac{d^\ell}{dx^\ell}\ n(x),\ \frac{d^\ell T_\perp(x)}{dx^\ell},\ \text{etc.}$$

Moreover, if we restrict attention to a range in x of the order of few Larmor radii we can assume that only the first two terms in the expansion contribute significantly. This means that φ is linear in x consistent with the approximation made earlier. In view of this Equation (10) reduces to

$$A_1\ (x)\ \frac{d\varphi(x)}{dx} + A_o\ (x)\ \varphi\ (x) = o \tag{11}$$

The local approximation implies that we need only keep the second term in the above equation or simply

$$A_o(x)\ \varphi(x) = o \tag{12}$$

Since $d\varphi(x)/dx$ is first order in quantities like $(\rho_1\ 1/n\ dn/dx)$, the zero order term in $A_1(x)$ should vanish-identically and this can be demonstrated in a straight forward manner. Since Equation (12) is valid for all x, the origin x = o, can be conveniently chosen and Equation (8) becomes

$$f_o\left(x + \frac{v_y}{\Omega},\ v_\perp^2 - 2gx,\ v_{11}^2\right) \simeq f_o(o,\ v_\perp^2, v_{11}^2) + \frac{v_y}{\Omega}\frac{\partial}{\partial\eta} f_o(\eta, v_\perp^2, v_{11}^2)\Big|_{\eta=o} \tag{13}$$

which is what people normally write. It must be noted, however, that Equation (12) is not valid for sheared magnetic field and for such problems one must use Equation (4) in its entirety.

B. ELECTROSTATIC INSTABILITIES IN A PLASMA WITH DENSITY AND TEMPERATURE GRADIENTS IN A CURVED MAGNETIC FIELD

We have utilized the results of the first section to investigate the effects of temperature and density gradients along with field curvature on the ion cyclotron electrostatic instabilities. Although this study is still in progress, we have obtained the following preliminary results.

1. Type a Electron-Ion Instability

This mode which has come to be known as the "loss cone" instability has been investigated at $w \sim \Omega_i$ in uniform plasma by Hall, et al.,[1] and for $w >> \Omega_i$ by Post and Rosenbluth.[2] Upon inclusion of density and temperature gradients the stability condition for this mode at $w \sim \ell\Omega_i$ for a bi Maxwellian plasma becomes

$$\ell + \frac{1}{k_\perp}\frac{n_i'}{n_i}\ (\ell^2 - \lambda) - \frac{\lambda}{k_\perp}\frac{T_{\perp i}'}{T_{\perp i}}\left[(\ell^2 - \lambda)\left(1 - \frac{I_\ell'(\lambda)}{I_\ell(\lambda)}\right) + \frac{\ell^2}{\lambda}\right] > o \tag{14}$$

where $\lambda = 1/2\ k_\perp^2\rho_i^2$, and $I_\ell(\lambda)$ is the Bessel function of imaginary argument. We note that the density and temperature gradients have opposing effects. Moreover the temperature gradient effect is more strongly influenced by finite Larmor radius effects. In the absence of temperature gradients we observe that a negative density gradient (as is the case in experiments) is destabilizing. Furthermore, those wavelengths such that $\lambda = \ell^2$ are totally immune to density gradient effects. For $\ell = 1$ these correspond to wavelenths of the order of the ion Larmor radius. For a plasma whose ions are monoenergetic, i.e., having delta function distribution in perpendicular energy as presently exists in ALICE, the stability condition is exactly the opposite of (14), i.e., the left hand side must be less than zero. In this case it is readily seen that the (negative) density gradient has a stabilizing effect. This may explain—even for a modest density gradient—why certain modes, namely, those for which $\ell^2 << \lambda$ are hard to observe.

2. The Cyclotron Drift Instability in Curved Fields

The cyclotron drift instability of Mikhailovskii and Timofeev has been reexamined in the presence of a temperature gradient and field curvature as simulated by a gravitational force. Denoting by δ the ratio of the temperature gradient to the density gradient, i.e., $\delta = d \ln T/d \ln n$ we find that $\delta \geq 2$ stabilizes these modes. They are the modes whose frequency is shifted due to the ion gravitational drift and which propagate normal to the magnetic field. The mechanism responsible in this instability is the strong coupling of the wave to the ion gyrational motion. It is of interest to note that while $\delta \geq 2$ stabilizes these modes, it is destabilizing to low frequency modes, i.e., $w << \Omega_i$ as has been found by Galeev, et al.[4] For $\delta < 2$ the stability condition for the short wavelength ($\lambda >> 1$) cyclotron drift modes is given by

$$(1 - \frac{\delta}{2}) R \frac{n_i'}{n_i} < 2 \frac{T_{11_i} + T_{11_e}}{T_{\perp i}} \tag{15}$$

where the field radius of curvature R is limited by

$$\frac{R}{\rho_i} < \sqrt{\frac{m_e}{m_i}} \frac{T_{11_i} + T_{11_e}}{T_{\perp i}} \tag{16}$$

In a machine such as ALICE where $T_{11_i} \simeq T_{11_e}$ and $T_{\perp i} \simeq 10\, T_{11_i}$, and assuming that the temperature and density gradients are comparable so that $\delta=1$, a field with radius of curvature of 4 ion Larmor radii or more at the center of the machine is readily susceptible to this instability. This tolerable value of R becomes even smaller when the anisotropy is more accute—another reason why energy spreading in the ion perpendicular energy is so essential. The stability condition given by (15) is, however, somewhat stringent in that it assumes very large wavelengths along the machine. When this restriction is modified to accommodate finite-length devices (as is being done now) the above stability condition may very well be significantly altered.

3. Convective vs. Absolute Instabilities

From the stand point of the experimentalist it is not sufficient to ascertain whether a certain mode is unstable; rather, to what extent such instability could affect plasma confinement. In the absence of knowledge concerning the nonlenear effects of these instabilities it is quite useful to determine whether an instability is convective or absolute. If convective, the instability simply walks out beyond the limits of the machine and if it does so fast enough it could well be ignored. On the other hand if the instability is absolute it lingers on in the device and means of coping with it must then be found. To ascertain this aspect of these instabilities one must examine the roots of the dispersion equation in both the complex k and w planes. This is being carried out in connection with the various instabilities mentioned earlier. However, since a number of criteria have recently been advanced in the literature[5-7] we have thus far confined our effort to making a comparative study of these criteria and deciding which one is more readily amenable to the cases of interest.

In addition to the above investigations we are studying the effect of magnetic field shear on these instabilities and have initiated work on the stability of electromagnetic oscillations in inhomogeneous plasma. The latter is important in high-beta devices and in the ion cyclotron resonance heating experiments.

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