BNL 17218
EP&S 72- 3
CRISP 72 - 69
ISABELLE PROJECT

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

ACCELERATOR DEPARTMENT
Informal Report

INTERSECTING STORAGE ACCELERATOR NOTES

Empirical Formula for Pion Production In
Proton Proton Collision Up to 1500 GeV

C. L. Wang
Sept. 6, 1972

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EMPIRICAL FORMULA FOR PION PRODUCTION IN
PROTON PROTON COLLISION UP TO 1500 GeV

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Aside from general interest, the pion production cross section up to 200 GeV may be of special use in the consideration of $\pi p, \mu p, \bar{p}p$ or $\bar{p}e^+$ options¹ for the ISABELLE project. The attainable pion, muon and anti-proton fluxes, the optimum targeting energies, the interaction rates, etc., are strongly dependent on the relevant cross sections. The attached paper presents a fairly reliable estimate of the pion production in pp collision up to 1500 GeV based on the ISR data. The $\bar{p}$ production can be estimated from the published $\bar{p}/\pi^-$ ratios².

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EMPIRICAL FORMULA FOR PION PRODUCTION IN
PROTON PROTON COLLISION UP TO 1500 GeV*

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ABSTRACT

A semiempirical formula which well represents the double differential cross section for pion production in high energy pp collisions is presented. The formula agrees with all data between 20 and 1500 GeV to within a factor of ~ 2 and is consistent with scaling and limiting fragmentation. Also presented is a set of momentum spectra at some interesting energies.

I. Introduction

To add to the numerous models¹ of particle production in high energy collisions, such as the statistical model, fireball model, jet model, multi-peripheral model, isobar model, thermodynamic model, diffraction-dissociation model, emission model etc., countless papers on inclusive reactions has been initiated by the appearance of the hypotheses of limiting fragmentation² and scaling³. However, in spite of all these efforts, because of the tremendous complexity of high energy collisions, it remains impractical if not impossible to calculate the momentum and angular distribution of produced particles for 'daily use' by means of these models.

A simple analytical expression which can represent particle production cross section is of great value for many purposes, such as planning experiments, designing high energy secondary beams (pion, kaon, antiproton, muon and even neutrino), radiation shield calculations, Monte Carlo calculation of total absorption nuclear cascade detectors, to mention a few. Semiempirical formulae of CKP⁴ and of Trilling⁵ for pion production proved to be quite useful during the design studies of multihundred GeV proton synchrotrons, but they are generally considered rather qualitative. Eight-parameter formulae⁶ give fairly quantitative representation of pion, kaon and antiproton production between 10 and 35 GeV and have been widely in use. A five-parameter formula⁷ represents the pion production in proton nuclei collisions between 10 and 70 GeV well and may be extrapolated to 500 GeV⁸.

*Work performed under the auspices of the U.S. Atomic Energy Commission

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In this paper we present a four-parameter formula which fits the pion production data in pp collision between 20 and 1500 GeV. The formula is consistent with scaling and limiting fragmentation and may therefore be extrapolated to much higher energies. Constant improvement of the formula, as new data become available, will of course increase the reliability at all energies. Nevertheless even with the existing fragmentary data above a hundred GeV, it is believed that the formula presented here should be reliable to within a factor of 2 over a large incident energy region up to 1500 GeV. At 500 GeV the highest energy of proton synchrotrons for years to come, the formula fits all data points to within a factor of 1.5.

II. Data On $p + p \rightarrow \pi^\pm + \text{anything}$

The data utilized in the present analyses are from Allaby et al⁹ at 19.2 GeV/c, Anderson et al¹⁰ at 30 GeV/c, and Ratner et al¹¹ and Bertin et al¹² at the CERN ISR corresponding to laboratory momenta up to 1500 GeV/c.

To obtain the laboratory differential cross section from the ISR data, the center of mass (c.m.) differential cross section is obtained from the published invariant cross section $I (= E^* d^3 \sigma / dp^{*3})$ by

$$\frac{d^2 \sigma}{dp^* d\Omega^*} = \frac{p^{*2}}{E^*} I \quad (1)$$

where p^* and E^* are c.m. momentum and energy of the pions produced. The corresponding laboratory quantities are then obtained by the Lorentz transformation. Thus,

$$\frac{d^2 \sigma}{dp d\Omega} = \frac{E^* p^2}{E p^{*2}} \frac{d^2 \sigma}{dp^* d\Omega^*} \quad (2)$$

which is simply $I p^2 / E$.

III. Scaling Law and Empirical Formula

That the invariant cross section I is independent of the incident energy but depends only on $X (= p_\ell / p_m)$, where p_ℓ is the longitudinal momentum of the pion produced and p_m is the maximum kinematically allowed value of p_ℓ and on p_t , transverse momentum of the pion, ($= p \sin \theta$, where θ is the production angle) suggests a linear scaling law for the momentum spectrum. Namely,

$$\frac{d^2 \sigma}{dp d\Omega} = \frac{p^2}{E} I(X, p_t) \quad (3)$$

and for high energy secondaries $p \approx E$, thus

$$\frac{d^2\sigma}{dp d\Omega} = p_m \times I(X, p_t). \quad (\text{Linear scaling law}) \quad (4)$$

Therefore, for given X and p_t the differential cross section scales linearly with p_m , or essentially the momentum of the incident proton. Eq. (4) holds for any Lorentz frame of reference, and is identical with the scaling law obtained by Vander Velde¹³ utilizing the hypothesis of limiting fragmentation.

Now the kinematics require that the momentum spectrum vanish at $X = 1$. Detailed data between 10 and 70 GeV show that the longitudinal momentum distribution is well approximated by $\exp(-BX^C)$ with $C > 1$,^{6,7,8} and the transverse momentum distribution follows closely $\exp(-Dp_t)$.^{4,6,7} Thus eq. (4) can be well represented

$$Y = \frac{d^2\sigma}{dp d\Omega} = A p_m X (1 - X) \exp(-BX^C - Dp_t) \quad (5)$$

where A is the normalization constant, B and C dictate the shape of the longitudinal momentum spectrum and D is related to the average transverse momentum.

IV. Least Squares Analyses and Results

Least Squares analyses were carried out to determine the four parameters A , B , C , and D by minimizing the quantity

$$Q = \frac{1}{F} \sum_i \left[\frac{\log Y_i^e - \log Y_i^c}{\Delta Y_i^e / Y_i^e} \right]^2 \quad (2)$$

where the superscripts e and c refer to the experimental and calculated values respectively, ΔY_i^e is the error pertaining to i -th experimental datum and F is the number of degrees of freedom. The result is shown in Table I. There Q 's indicate that good fits were obtained.

Table I Values of Parameters

<u>Pion</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>Data</u>	<u>Q</u>
Negative	51.403	5.732	1.333	4.247	274	0.91
Positive	77.793	3.558	1.333	4.727	210	1.23

The goodness of fit can be visualized in Fig. 1 through 25. Fig. 1 shows the invariant cross section of π^- production plotted against X . The broken lines, calculated by Eq. 5 for 500 and 1500 GeV, are independent of the incident energy as expected by scaling and limiting distribution. In Fig. 2 through 25 are plotted the differential cross sections of π^- and π^+ production at various incident momenta

in a way convenient for 'daily use'. To obtain the number of pions produced per GeV/c per steradian, one simply divides the differential cross section by the pp absorption cross section.

V. Discussion

As can be seen from the figures, eq. 5 fits within a factor ~ 2 all data points varying six orders of magnitude in differential cross sections and two orders in incident energy. The empirical formula can thus be considered as a reasonably reliable representation of the differential cross sections.

At lower secondary momenta, the momentum spectra for different production angle are believed to cross over⁸ as evidenced by the pion production data in proton-nuclei collision. Such characteristics can be taken care of by replacing the term Dp_t by $D\theta (p-E \cos^F \theta)$ when data in the low momentum region become available.

For pion production in pp collision the data do support scaling and limiting fragmentation, thus leading to the linear scaling law. (eq. 4). However, there is evidence¹² that for antiprotons this is not the case, indicating that limiting distribution may not have been reached even at 1500 GeV. It is worthwhile to point out that for pion production in p-nuclei collision between 10 and 70 GeV, the data are more consistent with a square root rather than a linear scaling law.^{7,8} It is not clear whether p-nuclei collisions behave differently from pp col-¹⁴lisions or the normalization of those p-nuclei experiments are really unreliable. A simple measurement of zero degree π^- production from p-nuclei collisions as a function of incident energy should suffice to clarify the situation.

Acknowledgement - It is a pleasure to thank Dr. D. Berley for valuable discussions and Miss F. Munkenbeck and Mr. J. Zyzkind for their assistance in programing and analyses.

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Figure Captions

Fig. 1 Invariant cross section for $p+p \rightarrow \pi^- + \text{anything}$ vs X , showing the scaling behavior. The broken lines are calculated by Eq. 5. (See text). Data are from Ref. 12.

Fig. 2 through 13.

Double differential cross section for $p+p \rightarrow \pi^- + \text{anything}$ at 19.2, 30, 70, 100, 200, 225, 300, 500, 1100 and 1500 GeV/c calculated by Eq. 5. Data are from Ref. 9, 10 and 12.

Fig. 14 through 25.

Double differential cross section for $p+p \rightarrow \pi^+ + \text{anything}$ at 19.2, 30, 70, 100, 200, 300, 500, 1100 and 1500 GeV/c calculated by Eq. 5. Data are from Ref. 9, 10 11 and 12.

FIGURE 1

19.2 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 2

19.2 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 3

19.2 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 4

30 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 5

70 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 6

100 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 7

200 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 8

225 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 9

300 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 10

500 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 11

1100 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 12

500 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 11

1100 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 12

1500 GEV/C P-P. NEGATIVE PION PRODUCTION

FIGURE 13

19.2 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 14

19.2 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 15

19.2 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 16

30 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 17

70 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 18

100 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 19

200 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 20

300 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 21

500 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 22

1100 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 23

1100 GEV/C P-P. POSITIVE PION PRODUCTION

PION MOMENTUM(GEV/C)

FIGURE 24

1500 GEV/C P-P. POSITIVE PION PRODUCTION

FIGURE 25