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**GENERATION AND BLOCKING
OF EXPERIMENTAL POINTS
FOR FRACTIONAL FACTORIALS**

J. A. MAHAFFEY
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GENERATION AND BLOCKING OF EXPERIMENTAL
POINTS FOR FRACTIONAL FACTORIALS

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TABLE OF CONTENTS

INTRODUCTION	1
SUMMARY	2
DISCUSSION	2
Input-Output	2
2 ^m Series Program	4
3 ⁿ Series Program	6
Program Operation	8
Additional Applications	9
ACKNOWLEDGEMENT	11
REFERENCES	12
APPENDIX A	A-1
APPENDIX B	B-1
APPENDIX C	C-1
APPENDIX D	D-1
APPENDIX E	E-1
DISTRIBUTION	F-1

GENERATION AND BLOCKING OF EXPERIMENTAL POINTS FOR FRACTIONAL FACTORIALS

J. A. Mahaffey and J. E. Schlosser

INTRODUCTION

In a factorial experiment with t factors each at k levels, there are k^t possible combinations of levels of the factors. Each of these combinations will be called an experimental point, or simply a point. Various experimental designs are available for arranging these points into blocks. The purpose of this paper is to explain the use of computer programs to generate and block the points for many experimental designs using a factorial arrangement of treatments.

When the number of factors becomes large, sampling the points in a total factorial frequently becomes prohibitive in terms of cost or time. Three ways to decrease the number of points are:

- To decrease the number of factors
- To decrease the number of levels of some factors
- To sample a fractional replication of the total factorial.

It will be assumed that the first two ways have been considered in the experimental planning, and that the problem still exists. Thus, the remaining alternative becomes important.

In a single replication of a k^t factorial, each main effect is an average over the k^{t-1} combinations of levels of the other factors, and in effect has a k^{t-1} -fold replication, where a smaller replication will often suffice. Similarly, each two-way interaction is an average over the other k^{t-2} combinations of levels of the other factors, and so forth. If it is reasonably assumed that certain interactions are negligible, then main effects and other interactions can be estimated even though their effective replications are reduced by using a fractional factorial. This alternative has limitations; foremost, the chances of misinterpretation are increased.

For values of m through 16 certain $1/2^p$ ($p = 1, 2, \dots, 8$) fractional factorials of the 2^m series have been enumerated⁽¹⁾. For three level designs with values of n through 10, the experimental points for particular $1/3^q$ replicates ($q = 1, 2, 3, 4, 5$) of the 3^n series are listed⁽²⁾. For values of m through 9, irregular fractional replicates have been specified for the 2^m series⁽³⁾. Other designs, which permit estimation of main effects but no interactions, are also available^(4,5), as are plans that allow estimation of main effects and some two factor interactions⁽⁶⁾. The programs of interest here will generate the experimental points for any of these, and for other plans subject to the limitations due to computer size and time.

SUMMARY

The use of computer programs in designing experiments having factorial treatment arrangements for two, three, and four level designs is explained. The input and output are discussed and illustrated by examples, and a print-out of the programs and subroutines is included. Further possibilities for applications of these programs are suggested.

DISCUSSION

INPUT-OUTPUT

The programs are organized in the following way:

1. The input quantities are read.
2. The restrictions (resulting from the experimenter's assumption) are packed into binary words.
3. A subroutine initializes the machine language routines.
4. The admissible points are determined.
5. The experimental points are unpacked and blocked, and the output is printed.

The input consists of the restrictions and blocking restrictions. The restrictions specify which interactions are assumed to be negligible. This means that the sum of squares explained

by the specified combination cannot be partitioned from the residual (or error) sum of squares. As a consequence, certain information that could be used in estimation and testing is not available. The blocking restrictions specify what is to be confounded with blocks.

For a prime value of k , there are k numbers $(0, 1, \dots, k-1)$ which form a field modulo k . For values of k that are not prime, a Galois field can be defined with elements $0, 1, \dots, k-1$. These are the characters that can be used in specifying the restrictions and blocking restrictions. These characters in a k^t factorial, also represent the possible levels of each factor. Denote these factors as A, B, C , etc. Thus, A will have the values $0, 1, \dots, k-1$, as will each of the other factors.

Also part of a restriction is a parity indicator. This is an element of the field or Galois field to which a specified (by the restriction) linear combination of the levels of the factors must sum to be acceptable in the design. That is, those points will be included in the design satisfying all restrictions. The restrictions are specified by punching the proper characters in the first k columns of an IBM card, and the parity indicator in column 50.

Each blocking restriction is punched in the first k columns of a card. For example, in a $1/9$ replicate of a 3^4 experiment with restrictions $A + 2C = 0$ and $2B + C + D = 1$ and with the ABCD interaction confounded with blocks, the restrictions would be entered as

Column	1	2	3	4	.	.	.	50
	1	0	2	0	.	.	.	0
	0	2	1	1	.	.	.	1,

and the blocking restriction as

Column	1	2	3	4
	1	1	1	1.

Let the experimental points be represented as

$$x_1 \quad x_2 \quad x_3 \quad x_4,$$

where

$$x_i = 0, 1, 2 \text{ for } i = 1, 2, 3, 4.$$

Then the points acceptable to the design simultaneously satisfy

$$(1 \times x_1) + (0 \times x_2) + (2 \times x_3) + (0 \times x_4) = 0$$

and

$$(0 \times x_1) + (2 \times x_2) + (1 \times x_3) + (1 \times x_4) = 1.$$

Those 27 points that satisfy the first restriction are of the form

$$x_1 \quad x_2 \quad x_1 \quad x_4.$$

Of these, the nine which satisfy the second are of the form

$$x_1 \quad x_2 \quad x_1 \quad 2x_1 + x_2 + 1.$$

Then the blocking restriction divides these nine points into three blocks of three points each by determining whether the quantity $x_1 + 2x_2 + 1$ equals zero, one, or two, modulo three.

The output will consist of the restrictions, the blocking restrictions, the number of experimental points, and the blocked experimental points. In the output the parity indicator is the left hand side of the restriction. The alphabetic characters on the right hand side denoting the factors are not separated by addition signs. The blocking restrictions are printed similarly, with those points which are in the specific block beneath them. For each block, the entire list of experimental points is scanned, and those points satisfying all blocking restrictions are printed under the appropriate headings. The last blocking card entered is printed as the first hierarchical level in the output.

2^m SERIES PROGRAM

The essence of this program is its use of the binary computer logic. The program operates by testing the 2^m points

against the restrictions using the logical "and" instruction. To be acceptable, a point must satisfy all restrictions. An experimental point is masked with the first restriction. If the sum of the resulting digits modulo two matches the zero or one in column 50 of the first restriction card, the point is tested against the next restriction, etc. If not, the point is destroyed. Each time a point is accepted, it is stored for further use. The program begins with the point of all zeros; rejects or accepts this point; adds one to the least significant bit; rejects or accepts, etc.; until the point of all ones is reached and tested. Thus, in a 2^3 with the restriction $A + B + C = 1 \pmod{2}$, the four output points would be 001, 010, 100, and 111.

The program will operate with redundant restrictions. For example, the three restrictions $A + B + C + D = 1 \pmod{2}$, $B + C + D + E = 1 \pmod{2}$, and $A + E = 0 \pmod{2}$ are redundant, since any two add to the third. As a consequence, the number of restrictions may be greater than m . However, nonredundant restrictions minimize the running time of the program. If there are p nonredundant restrictions in a 2^m factorial, each restriction eliminates one-half of the remaining points, and 2^{m-p} experimental points are generated. It is interesting to note that if the restrictions are nonredundant, then the mean number of restrictions that a point is tested against is less than two. This is calculated in Appendix A. The restrictions should be consistent; that is, the simultaneous restrictions $A + B + C = 1 \pmod{2}$ and $A + B + C = 0 \pmod{2}$ should not be used. If such a combination is used, no output points will be generated because there are no points satisfying both restrictions. After the points have been selected, a second pass in the program arranges them into 2^s blocks ($s = 0, 1, 2, 3, 4, 5$; $2^s = 1, 2, 4, 8, 16, 32$). This can be extended if more than 32 blocks are desired. The confounded effects should be consistent with the restrictions under which the points were generated. Thus,

if the restrictions are $A + B + C + D + E = 1 \pmod{2}$ and $B + C + D = 1 \pmod{2}$, confounding the AE interaction with blocks is meaningless because it is not being investigated ($A + E = 0 \pmod{2}$). In this case, instead of getting output in two blocks as expected, the output would be in a single block.

3ⁿ SERIES PROGRAM

This program again tests the 3ⁿ points against the restrictions and accepts only those points satisfying all restrictions. In this case, each level of each factor is represented by two bits of a single word; the three levels are represented as 00, 01, and 10 in the experimental points and restrictions for 0, 1, and 2, respectively. The other possibility, 11, is eliminated by "anding" the point with 1010...10, shifting the result one place to the right, "anding" this to the original point, and testing if the result is zero or nonzero. The net effect is to "and" together the pairs of bits representing a factor, and to test if this is zero or nonzero. If the result is zero, the original point was admissible in terms of containing no 11 combinations. If the result is nonzero, then the point contains the inadmissible character, 11. To decrease the execution time of the program, which as a consequence avoids having more than one inadmissible combination in a point at any time, the addition of unity is not always to the least significant bit. The next point is determined in the following way:

1. If a current point is admissible, then the next point to be tested for admissibility is obtained by adding unity to the least significant bit.
2. If a current point is not admissible, then the next point to be tested for admissibility is obtained by adding the nonzero word, resulting from testing the admissibility of the current point, to the current point.

An illustration of this method is given in Appendix B. It should be noted that for any point containing an 11 combination,

all the characters to the right of this combination will be zero. This is a consequence of the method of generation. Thus, only inadmissible points will be skipped, making this procedure more efficient than repetitively adding unity to the least significant bit.

After a point is determined to be admissible, it is tested against the restrictions. To do this, it is desirable to add the bits of some resultant word modulo three, and compare the result to the 0, 1, or 2 of the restriction; that is, it is desirable to multiply coefficients of proper terms in the restriction by the corresponding bits of an experimental point and have the following table summarize the possible results.

	00	01	10
00	00	00	00
01	00	01 or 10	11
10	00	11	01 or 10

If an "anding" is executed between the restriction and the current point, the following table would summarize the possible operations.

"and"	00	01	10
00	00	00	00
01	00	01	00
10	00	00	10

Thus, the only combinations that do not give the desired results are 01 "and" 10 and 10 "and" 01. This can be corrected as follows:

1. "And" pairs of bits of the point and the restriction diagonally in both directions.
2. "Or" the two results.
3. Expand each single bit to two like bits.
4. "Or" this result with the result of "anding" the point and the restriction.

This procedure is illustrated in Appendix B.

The sum of the bits then corresponds to the sum of the proper products. If this sum modulo three matches the 0, 1, or 2 desired, the point is acceptable. Thus, in a 3^2 with the restriction $A + 2B = 0 \pmod{3}$, the output points would be determined to be 0000, 0101, and 1010, and would be printed as 00, 11, and 22 (assuming no blocking).

If the q restrictions are nonredundant, then the mean number of restrictions that a point is tested against is less than $3/2$. This is calculated in Appendix A. The blocking routine arranges the points into 3^s blocks ($s = 0, 1, 2, 3, 4, 5$; $3^s = 1, 3, 9, 27, 81, 243$).

PROGRAM OPERATION

The programs were written in FORTRAN IV and machine language for the IBM 7090 and the UNIVAC 1107 computers. For increased efficiency, binary ones and zeros, which represent restrictions and experimental points, are stored as individual bits or pairs of bits in the words of the computer. Thus, the word length of these computers establishes the basic limit of the routine at 35 variables for the 2^m series and at 17 variables for the 3^n series. However, because of the time required to run the program, these are not serious limitations.

When the number of variables becomes large, the time required to compute the acceptable experimental points for the 2^m series is the dominant time, and is $9 \times 2^{m-14}$ sec for the IBM 7090 and $1.5 \times 2^{m-14}$ sec for the UNIVAC 1107. Thus, 35 variables would require about 34 days on the 1107 and 200 days on the 7090. Presently, in this program, a limit of 25 variables was arbitrarily chosen; however, this limit could easily be extended. The great difference in running times is caused by the need to determine parity of masked words (i.e., the number of one bits in the word modulo two). On the 7090, this requires working with each individual bit in a loop requiring six machine cycles. The 1107 contains an instruction for this

purpose which performs the masking and determines the parity in one operation. It also leaves the word unmodified, thereby eliminating the need to restore it for further testing.

For the 3^n series program, a limit of 15 variables, coded for the 7090 only, was arbitrarily chosen. The 1107 would not yield the same large increase in speed on the 3^n series as it did in the 2^n case because of the more complex selection technique. For a large number of variables, the time for the 3^n series is approximately $4 \times 3^{n-8}$ sec.

A printout of the programs is given in Appendix E.

ADDITIONAL APPLICATIONS

There are several further problems that can be solved using these programs. Three of interest are:

- The generation of irregular fractions
- The generation of $2^m 3^n$ designs
- The generation of 4^r designs.

Irregular fractions can be generated by repeated execution of the programs. Use of irregular fraction replicates allows partial confounding of certain main effects and interactions. For example, to generate a 3/4 replicate of a 2^3 factorial, the program could be executed twice--once with the restriction $A + B = 1 \pmod{2}$, and once with the restrictions $A + B = 0 \pmod{2}$ and $A + C = 1 \pmod{2}$. The 1/4 fraction, for which $A + B = 0 \pmod{2}$ and $A + C = 0 \pmod{2}$, has been eliminated. In this case, only the three-way interaction cannot be estimated; whereas in any 1/2 replicate there are at least two effects or interactions that are not capable of being estimated. The generation of such an experiment is illustrated by Example (1) of Appendix D.

The fractions of the $2^m 3^n$ series can be found by combining the output of the two programs. These can be combined as suggested

in Reference 7, or may be combined by putting each two-level with each three-level combination obtained. To obtain designs satisfying a restriction such as $A + B + 2C = 0 \pmod{3}$, where A has two levels and B and C are three level factors, is not possible by using these programs. It could be programmed, but would be very time-consuming. Asymmetrical plans can be generated by use of the 3^n program as suggested in Reference 4, but these permit only the estimation of main effects in a $2^m 3^n$ experiment.

The generation of experiments of the 4^r series is possible by use of the two level program. These can be thought of as 2^{2r} designs, and each of the r four level factors can be represented by the combination of two two level factors. Here $0 = 00$, $1 = 01$, $2 = 10$, and $3 = 11$. The use of modular arithmetic is not valid since the integers modulo four are not a field (divisors of zero are present); whereas, the integers modulo two and three do form fields. However, by use of the theory of Galois Fields, the problem can be solved.

Consider the GF (2^2) with four elements and primitive mark $x + 1$. The addition and multiplication tables for this field are

Addition					Multiplication				
	0	1	x	x + 1		0	1	x	x + 1
0	0	1	x	x + 1	0	0	0	0	0
1		0	x + 1	x	1		1	x	x + 1
x			0	1	x			x + 1	1
x + 1				0	x + 1				x

where the four elements correspond to the four levels in increasing arithmetic order. These tables are found by performing the indicated operations modulo two and modulo $x^2 + x + 1$.

Designate the four level factors as $u_1, u_2, \dots$, and the two level factors as $V_{11}, V_{12}; V_{21}, V_{22}; \dots$ where V_{1i} and V_{2i}

correspond to the two characters representing u_i . A restriction in the 4^r case will be represented by two restrictions in the 2^{2r} situation, since one restriction with r variables at four levels each gives $4^{r-1} = 2^{2r-2}$ experimental points. To find the two corresponding restrictions, it is necessary to write the 4^r restriction in terms of the elements of the Galois Field. Then transform this restriction to the 2^m field by use of identities of the form

$$u_i = xV_{1i} + V_{2i}$$

and set the coefficients of equal powers of x on both sides of the equation equal. An example of this procedure is given in Appendix C. For analysis of 4^r experiments, a brief discussion is given in Reference 8. Example (3) in Appendix D illustrates the generation of points for 4^r plans.

For many experiments where the number of levels of the factors can be written as $2^i 3^j$, $i \geq 0$, $j \geq 0$, i and j integral, the experimental points can be generated by proper use of these programs. The experimental points for the designs given in References 4-6 can be easily generated by determining the independent aliases and writing these in the language of the programs. Points for designs not enumerated in these articles can thus be readily determined by use of the theory of these papers. Example (3) of Appendix D was taken from Reference 6.

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APPENDIX A

APPENDIX A

TO SHOW: The mean number of restrictions that a point is tested against is less than two for the 2^m program, and is less than $3/2$ for the 3^n program.

Let

p = fraction of the remaining points that is rejected by each restriction,

X = number of times a point is accepted preceeding its rejection.

Then

$$P(X = k) = p(1 - p)^k \quad k = 0, 1, 2, \dots$$

But this is the p.d.f.* of the geometric distribution and since k has no upper bound, it is the limiting case for the present problem. The mean of this distribution is $1/p$. But it is known that

$p = 1/2$ for the 2^m program

$p = 2/3$ for the 3^n program.

Thus, the desired numbers, $1/p$, are 2 and $3/2$, respectively.

* *Probability Density Function*

APPENDIX B

APPENDIX B

TO SHOW: The operation of the 3^n program to determine if a point contains any 11 combinations and (if so) to determine the next point to be tested for admissibility.

Let the current point be

10 00 01 11 00.

The following steps occur:

- (1) The current point is "anded" with 10 10...10.

Thus,

10 00 01 11 00

"and"

10 10 10 10 10

gives

10 00 00 10 00.

- (2) This is shifted one place to the right, thus giving

01 00 00 01 00.

- (3) This is "anded" to the original point. Thus,

10 00 01 11 00

"and"

01 00 00 01 00

gives

00 00 00 01 00.

- (4) This result is nonzero. The current point is inadmissible.

- (5) The new current point is found by adding the final result and the current point. Thus,

10 00 01 11 00

plus

00 00 00 01 00

gives

10 00 10 00 00.

This is now the current point, and can be shown to be admissible. The three inadmissible points,

10 00 01 11 01
 10 00 01 11 10
 10 00 01 11 11,

were skipped. No admissible points were skipped.

TO SHOW: The procedure which allows addition of bits, modulo three, to be compared to the 0,1, or 2 of a restriction in determining if the point satisfies the restriction.

If a point is 1 2 1 2 0 or

01 10 01 10 00

and a restriction is

10 10 01 01 10

(2A + 2B + C + D + 2E), then the following steps occur:

- (1) The point and the restriction are "anded". Thus,

01 10 01 10 00

"and"

10 10 01 01 10

gives

00 10 01 00 00.

- (2) The two are "anded" diagonally both directions.
 Thus,

01	10	01	10	00	"and"
↓	↓	↓	↓	↓	
10	10	01	01	10	

gives

0	0	0	1	0.
---	---	---	---	----

01	10	01	10	00	"and"
↓	↓	↓	↓	↓	
10	10	01	01	10	

gives

1	0	0	0	0.
---	---	---	---	----

(3) These two results are "ored". Thus,

0	0	0	1	0
---	---	---	---	---

"or"

1	0	0	0	0
---	---	---	---	---

gives

1	0	0	1	0.
---	---	---	---	----

(4) Expanding, the result is

11	00	00	11	00.
----	----	----	----	-----

(5) This result and the result of the original "anding" in (1) are "ored" together. Thus,

00	10	01	00	00
----	----	----	----	----

"or"

11	00	00	11	00
----	----	----	----	----

gives

11	10	01	11	00
----	----	----	----	----

(6) The sum of these bits is $6 \pmod{3} = 0$. The desired result is

$$\begin{aligned}
 &(2 \times 1) + (2 \times 2) + (1 \times 1) + (1 \times 2) \\
 &\quad + (2 \times 0) = 9 \pmod{3} \\
 &\quad = 0
 \end{aligned}$$

APPENDIX C

APPENDIX C

TO SHOW: Conversion of a four level restriction to two 2-level restrictions.

If the desired four level restriction is

$$u_1 + xu_2 = x + 1,$$

then, by substituting,

$$(xV_{11} + V_{12}) + x(xV_{21} + V_{22}) = x + 1.$$

Setting coefficients of powers of x equal, the restrictions become

$$V_{11} + V_{21} + V_{22} = 1,$$

and

$$V_{12} + V_{21} = 1,$$

or, in terms of the two-level programs,

$$A + C + D = 1 \pmod{2}$$

and

$$B + C = 1 \pmod{2}.$$

In a $4^2 = 2^4$ experiment, these restrictions result in the generation of the four points

A	B	C	D
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	0

or

u_1	u_2
0	x
1	1
x	$x + 1$
$x + 1$	0

These can be thought of as the points 02, 11, 23, and 30.

APPENDIX D

APPENDIX DEXAMPLE (1)

Reference: (3), Table 3A, p. 492

Experiment: 3/4 replicate of a 2^6

Number of experimental points: 48

Computer: UNIVAC 1107

Time required: 0.87 sec

Restrictions:

Run 1: $A + B + C + D = 0$, $A + B + E + F = 1$

Run 2: $A + B + C + D = 1$

Blocking restrictions: None

Comments: The same points could have been generated by the following sets of restrictions:

(1)

Run 1: $A + B + E + F = 0$, $A + B + C + D = 1$

Run 2: $A + B + E + F = 1$

(2)

Run 1: $A + B + C + D = 0$, $A + B + E + F = 1$

Run 2: $A + B + C + D = 1$, $A + B + E + F = 1$

Run 3: $A + B + C + D = 1$, $A + B + E + F = 0$

Further, rather than using $A + B + C + D$ and $A + B + E + F$, $C + D + E + F$ could have been used in appropriate combination with either of the other two, or in addition to both if $C + D + E + F$ were equal to their sum.

Computer output:

6 VARIABLES

2 RESTRICTIONS

	A	B	C	D	E	F
0	1	1	1	1	0	0
1	1	1	0	0	1	1

16 EXPERIMENTAL POINTS

0 =

A	B	C	D	E	F
1	0	1	0	0	0
0	1	1	0	0	0
1	0	0	1	0	0
0	1	0	1	0	0
0	0	0	0	1	0
1	1	0	0	1	0
0	0	1	1	1	0
1	1	1	1	1	0
0	0	0	0	0	1
1	1	0	0	0	1
0	0	1	1	0	1
1	1	1	1	0	1
1	0	1	0	1	1
0	1	1	0	1	1
1	0	0	1	1	1
0	1	0	1	1	1

1 =

A B C D E F

TIME REQUIRED WAS .404 SECONDS.

6 VARIABLES

1 RESTRICTIONS

	A	B	C	D	E	F
1	1	1	1	1	0	0

32 EXPERIMENTAL POINTS

0 =

A	B	C	D	E	F
1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
1	1	1	0	0	0
0	0	0	1	0	0
1	1	0	1	0	0
1	0	1	1	0	0
0	1	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0
1	1	1	0	1	0
0	0	0	1	1	0
1	1	0	1	1	0
1	0	1	1	1	0
0	1	1	1	1	0
1	0	0	0	0	1
0	1	0	0	0	1
0	0	1	0	0	1
1	1	1	0	0	1
0	0	0	1	0	1
1	1	0	1	0	1
1	0	1	1	0	1
0	1	1	1	0	1
1	0	0	0	1	1
0	1	0	0	1	1
0	0	1	0	1	1
1	1	1	0	1	1
0	0	0	1	1	1
1	1	0	1	1	1
1	0	1	1	1	1
0	1	1	1	1	1

1 =

A B C D E F

TIME REQUIRED WAS .466 SECONDS.

EXAMPLE (2)

Reference: (2), Plan 243.9.9, p. 35

Experiment: $\frac{1}{243}$ replicate of a 3^9 in 9 blocks of 9 units each.

Number of Experimental Points: 81

Computer: IBM 7090

Time Required: Approximately 12 seconds.

Restrictions: $B + C + D + E + F + G = 0$,
 $A + C + D + 2E + 2F + H = 0$,
 $A + B + 2D + 2E + F + I = 0$,
 $A + B + 2C + E + 2F = 0$,
 $A + 2B + 2C + D + F = 0$.

Blocking Restrictions: $A + B + 2C$, $B + C$

Comments: The restrictions and blocking restrictions in all of the 3^n experimental plans in Reference (2) are in bold print in the enumeration. In Reference (1), the 2^m series experimental plans, the independent aliases can be picked from the list, since they are numbers $2^0 = 1$, $2^1 = 2$, $2^2 = 4, \dots$.

Computer Output:

9 VARIABLES

5 RESTRICTIONS

	A	B	C	D	E	F	G	H	I
0	0	1	1	1	1	1	1	0	0
0	1	0	1	1	2	2	0	1	0
0	1	1	0	2	2	1	0	0	1
0	1	1	2	0	1	2	0	0	0
0	1	2	2	1	0	1	0	0	0

81 EXPERIMENTAL POINTS

$$0 = B \quad C$$

$$0 = A \quad B \quad 2C$$

A	B	C	D	E	F	G	H	I
0	0	0	0	0	0	0	0	0
1	1	2	2	0	0	1	1	0
2	2	1	1	0	0	2	2	0
0	0	0	1	2	2	1	0	1
1	1	2	0	2	2	2	1	1
2	2	1	2	2	2	0	2	1
0	0	0	2	1	1	2	0	2
1	1	2	1	1	1	0	1	2
2	2	1	0	1	1	1	2	2

1 = A B 2C

A	B	C	D	E	F	G	H	I
2	1	2	1	2	0	0	0	0
0	2	1	0	2	0	1	1	0
1	0	0	2	2	0	2	2	0
2	1	2	2	1	2	1	0	1
0	2	1	1	1	2	2	1	1
1	0	0	0	1	2	0	2	1
2	1	2	0	0	1	2	0	2
0	2	1	2	0	1	0	1	2
1	0	0	1	0	1	1	2	2

2 = A B 2C

A	B	C	D	E	F	G	H	I
1	2	1	2	1	0	0	0	0
2	0	0	1	1	0	1	1	0
0	1	2	0	1	0	2	2	0
1	2	1	0	0	2	1	0	1
2	0	0	2	0	2	2	1	1
0	1	2	1	0	2	0	2	1
1	2	1	1	2	1	2	0	2
2	0	0	0	2	1	0	1	2
0	1	2	2	2	1	1	2	2

1 = B C

0 = A B 2C

A	B	C	D	E	F	G	H	I
0	2	2	2	2	2	2	0	0
1	0	1	1	2	2	0	1	0
2	1	0	0	2	2	1	2	0
0	2	2	0	1	1	0	0	1
1	0	1	2	1	1	1	1	1
2	1	0	1	1	1	2	2	1
0	2	2	1	0	0	1	0	2
1	0	1	0	0	0	2	1	2
2	1	0	2	0	0	0	2	2

1 = A B 2C

A	B	C	D	E	F	G	H	I
2	0	1	0	1	2	2	0	0
0	1	0	2	1	2	0	1	0
1	2	2	1	1	2	1	2	0
2	0	1	1	0	1	0	0	1
0	1	0	0	0	1	1	1	1
1	2	2	2	0	1	2	2	1
2	0	1	2	2	0	1	0	2
0	1	0	1	2	0	2	1	2
1	2	2	0	2	0	0	2	2

2 = A B 2C

A	B	C	D	E	F	G	H	I
1	1	0	1	0	2	2	0	0
2	2	2	0	0	2	0	1	0
0	0	1	2	0	2	1	2	0
1	1	0	2	2	1	0	0	1
2	2	2	1	2	1	1	1	1
0	0	1	0	2	1	2	2	1
1	1	0	0	1	0	1	0	2
2	2	2	2	1	0	2	1	2
0	0	1	1	1	0	0	2	2

2 = B C

0 = A B 2C

A	B	C	D	E	F	G	H	I
0	1	1	1	1	1	1	0	0
1	2	0	0	1	1	2	1	0
2	0	2	2	1	1	0	2	0
0	1	1	2	0	0	2	0	1
1	2	0	1	0	0	0	1	1
2	0	2	0	0	0	1	2	1
0	1	1	0	2	2	0	0	2
1	2	0	2	2	2	1	1	2
2	0	2	1	2	2	2	2	2

1 = A B 2C

A	B	C	D	E	F	G	H	I
2	2	0	2	0	1	1	0	0
0	0	2	1	0	1	2	1	0
1	1	1	0	0	1	0	2	0
2	2	0	0	2	0	2	0	1
0	0	2	2	2	0	0	1	1
1	1	1	1	2	0	1	2	1
2	2	0	1	1	2	0	0	2
0	0	2	0	1	2	1	1	2
1	1	1	2	1	2	2	2	2

2 = A B 2C

A	B	C	D	E	F	G	H	I
1	0	2	0	2	1	1	0	0
2	1	1	2	2	1	2	1	0
0	2	0	1	2	1	0	2	0
1	0	2	1	1	0	2	0	1
2	1	1	0	1	0	0	1	1
0	2	0	2	1	0	1	2	1
1	0	2	2	0	2	0	0	2
2	1	1	1	0	2	1	1	2
0	2	0	0	0	2	2	2	2

EXAMPLE (3)

Reference: (6), Table 1, p. 58

Experiment: Plan which permits estimation of all main-effects and all two-factor interactions containing A for a 4^6 .

Number of Experimental Points: 64

Computer: UNIVAC 1107

Time Required: 1.90 seconds - essentially the time required for a 2^{12} .

Restrictions: $B + C = D$, $B + "2"C = E$, $B + "3"C = F$ or
 $B + C + D = 0$, $B + xC + E = 0$,
 $B + (x + 1) C + F = 0$

Four Level	Two Level
$u_2 + u_3 + u_4 = 0$	$C + E + G = 0$
	$D + F + H = 0$
$u_2 + xu_3 + u_5 = 0$	$C + E + F + I = 0$
	$D + E + J = 0$
$u_2 + (x + 1) u_3 + u_6 = 0$	$C + F + K = 0$
	$D + E + F + L = 0$

Comments: The arithmetic operations of the Galois Field (2^2) must be used in all manipulations. There is a temptation to write the second restriction as $(B + C) + C = D + C = E$. However, under the permissible operations $1 \cdot C + 1 \cdot C = (1 + 1)C = 0 \cdot C = 0$, and the net result is that the main-effects B and E are totally confounded if the above is done.

Computer Output:

	12 VARIABLES												6 RESTRICTIONS											
	A	B	C	D	E	F	G	H	I	J	K	L												
0	0	0	1	0	1	0	1	0	0	0	0	0												
0	0	0	0	1	0	1	0	1	0	0	0	0												
0	0	0	1	0	1	1	0	0	1	0	0	0												
0	0	0	0	1	1	0	0	0	0	1	0	0												
0	0	0	1	0	0	1	0	0	0	0	1	0												
0	0	0	0	1	1	1	0	0	0	0	0	1												

64 EXPERIMENTAL POINTS

APPENDIX E

APPENDIX E

The following is a listing of the Fortran IV main program for the 2^m series calculation, which is nearly the same for the IBM 7090 and UNIVAC 1107 computers (the 1107 requires the CALL SETIO statements found near the bottom of this page):

```

C
C
C      INPUT      FIRST CARD
C      COLUMNS   CONTENTS
C      1-5        NV=NUMBER OF VARIABLES          NOT OVER 25
C      6-10       NR=NUMBER OF RESTRICTIONS        NOT OVER 25
C      11-15      NB=NUMBER OF BLOCKING RESTRICTIONS NOT OVER 5
C
C
C      THEN NR CARDS CONTAINING 1 OR 0 IN EACH OF FIRST 35 COLUMNS
C      COLUMN 50 CONTAINS 0 IF EVEN PARITY SHOULD CAUSE SELECTION
C      1 IF OLD PARITY SHOULD CAUSE SELECTION
C
C
C      THEN NB CARDS CONTAINING 1 OR 0 IN EACH OF FIRST 35 COLUMNS
C
C      INTEGER BLANK,DIGIT(2),ALF
C      DIMENSION JUDY(40),L(39),MAHAF(35),LEMON(40)
C      DIMENSION MCOM(513),LOCK(35,5),ALF(26),MBLOCK(5),LIME(5)
C      DATA (ALF(I),I=1,26) /1HA,1HB,1HC,1HD,1HE,1HF,1HG,1HH,1HI,1HJ,
1      1HK,1HL,1HM,1HN,1HO,1HP,1HQ,1HR,1HS,1HT,
2      1HU,1HV,1HW,1HX,1HY,1HZ /, BLANK/1H /,
3      DIGIT(1)/0/,DIGIT(2)/1/
900 FORMAT(3I5)
910 FORMAT(35I1,14X11)
920 FORMAT( 1H1135,12H VARIABLES      110,15H RESTRICTIONS  ///)
929 FORMAT(41X 25(1XA1) /1H )
930 FORMAT(31X15,5X5512)
950 FORMAT( 1H0135,21H EXPERIMENTAL POINTS /1H )
951 FORMAT(35H0MORE THAN 512 EXPERIMENTAL POINTS /1H )
960 FORMAT(1X 39I3)
970 FORMAT(1H020X16HTIME REQUIRED WAS F7.3, 9H SECONDS.)
981 FORMAT(1H0          12,3H = 25(1XA1))
982 FORMAT(1H0 10X      12,3H = 25(1XA1))
983 FORMAT(1H0 20X      12,3H = 25(1XA1))
984 FORMAT(1H0 30X      12,3H = 25(1XA1))
985 FORMAT(1H0 40X      12,3H = 25(1XA1))
990 FORMAT(1H0 59X 25(1XA1) /1H )
991 FORMAT(      60X 25I2 )
      CALL SETIO (3,2)
      CALL SETIO (2,1)
10 CONTINUE
      CALL ETIME
      READ (2,900) NV,NR,NB

```

```

NB=MINO(05,MAXO(1,NB))
NR=MINO(35,MAXO(0,NR))
NV=MINO(25,MAXO(1,NV))
WRITE (3,920) NV,NR
WRITE (3,929) (ALF(J),J=1,NV)
DO 30 I=1,NR
  READ (2,910) MAHAF,LEMON(I)
  WRITE (3,930) LEMON(I),(MAHAF(J),J=1,NV)
  JUDY(I)=0
  DO 20 J=1,35
    JUDY(I)=JUDY(I)+ 2** (J-1)*MAHAF(J)
20 CONTINUE
30 CONTINUE
  DO 50 I=1,NB
    READ (2,910) (MAHAF(J) ,J=1,NV)
    MBLOCK(I)=0
    NZ=0
    DO 40 J=1,NV
      LOCK(J,I)=BLANK
      MBLOCK(I)=MBLOCK(I) + 2** (J-1)* MAHAF(J)
      IF(MAHAF(J).NE.1) GO TO 40
      NZ=NZ+1
    LOCK(NZ,I)=ALF(J)
40 CONTINUE
50 CONTINUE
  NUMBER=2** NV-1
  CALL SET UP(JUDY,NR,L,NV,NUMBER,LEMON)

```

C	VARIABLE	MEANING
C	JUDY	LIST OF BINARY RESTRICTIONS
C	NR	NUMBER OF RESTRICTIONS
C	L	OUTPUT ARRAY WHICH WILL BE SET TO 1 OR 0 FOR
C		EACH VARIABLE
C	NV	NUMBER OF VARIABLES
C	NUMBER	NUMBER OF INTEGERS TO TRY
C	LEMON	LIST OF PARITY INDICATIONS FOR RESTRICTIONS
C		

```

KA=1
KB=1
KC=1
KD=1
KE=1
GO TO(100,90,80,70,60) ,NB
60 KA=2
70 KB=2
80 KC=2
90 KD=2
100 KE=2

```



```

      MIKE=0
110  CALL GET NUM (MONEY)
C
C      MONEY WILL BE ZERO AT THE END. THAT IS IF THERE ARE NO MORE
C      POSSIBLE COMBINATIONS.
C
      IF(MONEY.EQ.0) GO TO 120
      MIKE=MIN(513,MIKE+1)
      MCOM(MIKE)=L(1)
      GO TO 110
120  CONTINUE
      IF(MIKE.GT.512) WRITE (3,951)
      WRITE (3,950) MIKE
      DO 240 LA=1,KA
      IF(KA.GT.1) WRITE (3,981) DIGIT(LA),(LOCK(J,5),J=1,NV)
      LIME(5)=LA-1
      DO 230 LB=1,KB
      IF(KB.GT.1) WRITE (3,982) DIGIT(LB),(LOCK(J,4),J=1,NV)
      LIME(4)=LB-1
      DO 220 LC=1,KC
      IF(KC.GT.1) WRITE (3,983) DIGIT(LC),(LOCK(J,3),J=1,NV)
      LIME(3)=LC-1
      DO 210 LD=1,KD
      IF(KD.GT.1) WRITE (3,984) DIGIT(LD),(LOCK(J,2),J=1,NV)
      LIME(2)=LD-1
      DO 200 LE=1,KE
      IF(KE.GT.1) WRITE (3,985) DIGIT(LE),(LOCK(J,1),J=1,NV)
      LIME(1)=LE-1
      WRITE (3,990) (ALF(J),J=1,NV)
      DO 140 LQ=1,MIKE
      KORE=1
      DO 130 LX=1,NB
      KORE=KORE* MASK(MCOM(LQ),MBLOCK(LX),LIME(LX))
130  CONTINUE
      IF(KORE.EQ.0) GO TO 140
      CALL UNPACK (MCOM(LQ),L)
      WRITE (3,991) (L(J),J=1,NV)
140  CONTINUE
200  CONTINUE
210  CONTINUE
220  CONTINUE
230  CONTINUE
240  CONTINUE
      CALL ETIMEF(TIME)
      WRITE (3,970) TIME
      GO TO 10
      END

```

The following subroutine was written in MAP language for the IBM 7090 for the 2^m series:

\$IBMAP	SELECT	100	
SETUP	SAVE		
	CLA*	4,4	
	ADD	3,4	
	STA	JUDY	
	CLA*	4,4	
	STA	NR1	
	ADD	8,4	
	STA	LEMON	
	CLA*	6,4	
	STA	NV1	
	STA	NV2	
	STA	NV3	
	ADD	5,4	
	STA	L	
	CLA	=-1	
	STO	CURNUM	
	CLA*	7,4	
	STO	MAX	
	RETURN	SETUP	
GETNUM	SAVE	1,2,4	GET NUM ENTRY
	STZ*	3,4	SET END FLAG TO ZERO
NEW	CLA	CURNUM	INCREMENT CURRENT NUMBER
	ADD	=1	
	CAS	MAX	COMPARE WITH END OF LIST
	RETURN	GETNUM	RETURN
	TRA	*+1	
	STO	CURNUM	STORE NEW CURRENT NUMBER
NR1	AXT	** ,1	XR1 IS RESTRICTION INDEX
	TXL	UNP,1,0	
START	CAL	CURNUM	
JUDY	ANA	** ,1	AND TOGETHER RESTRICTION AND CURRENT NUM
*			COMPUTE PARITY
NV1	AXT	** ,2	XR2 IS VARIABLE INDEX
	LDI	=0	ZERO INDICATORS
	LGR	1	SHIFT RIGHT ONE BIT
	TQP	*+2	
	IIR	1	IF BIT IS 1, INVERT PARITY INDICATOR
	TIX	*-3,2,1	CONTINUE LOOPING
	PIA		PUT INDICATORS INTO AC
LEMON	SUB	** ,1	SUBTRACT PARITY REQUEST
	TNZ	NEW	IF NOT EQUAL TRY NEW NUMBER
	TIX	START,1,1	
UNP	CAL	CURNUM	STORE RESULT
	STL*	3,4	SET FLAG
NV2	AXT	** ,2	
L	SLW	** ,2	

	RETURN	GETNUM	
CURNUM	PZE		
MAX	PZE		
UNPACK	SAVE	2	UNPACK NUMBER
	CLA	4,4	
	ADD	=25	
	STA	LL	
	CAL*	3,4	
	AXT	25,2	
	LDQ	=0	
	LGR	1	
	RQL	1	
LL	STQ	**,2	
	TIX	*-4,2,1	
	RETURN	UNPACK	
MASK	SAVE	2	AND TOGETHER FIRST TWO ARGUMENTS -
	CAL*	3,4	COMPUTE RESULTING PARITY.
	ANA*	4,4	IF PARITY IS EQUAL TO THIRD ARGUMENT
NV3	AXT	**,2	RETURN 1. OTHERWISE RETURN ZERO.
	LDI	=0	
	LGR	1	
	TQP	**+2	
	IIR	1	
	TIX	*-3,2,1	
	PIA		
	SUB*	5,4	
	COM		
	ANA	=1	
	RETURN	MASK	
	END		

The following subroutine was written in Sleuth II language for the UNIVAC 1107 for the 2^m series:

REGNAM	SA,H2	A0,LEMON
JFACTR	LA	A0,*1,B11
\$ (1)	SA,H2	A0,NR
SETUP* NOP	LA	A0,*3,B11
LA	ANA,XM	A0,1
ANA,XM	SA,H2	A0,NV
SA,H2	LA	A0,*4,B11
SA,H2	AA,XM	A0,1
LA	SA	A0,MAX
SA,H2	LA	A0,(01)
LA	SNA	A0,CURNUM
ANA,XM	J	7,B11

```

GETNUM* SR      R4,NTEMP
SX      B1,NTEMP+1
SZ      *0,B11
NEW     LA,XM    A0,1
AA      A0,CURNUM
TG      A0,MAX
J      RETURN
SA      A0,CURNUM
LX,XM   B1,1
NR      LK,XM    R4,0
JGD     R4,$+2
J      UNP
START  LA      A0,CURNUM
LEMON  TZ      0,B1
J      ODD
JUDY   TEP     A0,0,B1
J      NEW
J      LOOP
ODD    TOP     A0,0,B1
J      NEW
LOOP   AX,XM   B1,1
JGD    R4,LEMON
UNP    LA,XM   A0,1
SA     AC,*0,B11
LA     A0,CURNUM
L      SA      A0,0
RETURN LR      R4,NTEMP
LX     B1,NTEMP+1
J      2,B11
UNPACK* SX     B1,NTEMP+1
SR     R4,NTEMP
LA     A0,1,B11

```

```

ANA,XM  A0,1
SA,H2   A0,LL
LA      A0,*0,B11
LX,XM   B1,1
LR,XM   R4,0
NV      AND    A0,(01)
ULOOP   SA     A1,0,B1
LL      SSL    A0,1
AX,XM   B1,1
JGD     R4,ULOOP
LR      R4,NTEMP
LX      B1,NTEMP+1
J      3,B11
MASK*   LA     A0,*0,B11
TZ      *2,B11
J      ODDU
J      BAD
J      TEP     A0,*1,B11
J      BAD
J      GOOD
ODDU    TOP     A0,*1,B11
J      BAD
GOOD    LA,XM   A0,1
J      4,B11
BAD     LA,XM   A0,0
J      4,B11
$(2)
NTEMP  +      0
+      0
MAX    +      0
CURNUM +      0
END

```

The following is a listing of the Fortran IV main program for the 3ⁿ series calculation for the IBM 7090 computer:

```

$IBFTC JAM.      LIST,REF
C
C
C      INPUT      FIRST CARD
C      COLUMNS   CONTENTS
C      1-5        NV=NUMBER OF VARIABLES
C      6-10       NR=NUMBER OF RESTRICTIONS
C      11-15      NB=NUMBER OF BLOCKING RESTRICTIONS
C
C
C      THEN NR CARDS (AT LEAST 1 CARD) CONTAINING RESTRICTIONS
C      STARTING IN COLUMN 1 WITH PARITY INDICATOR IN COLUMN 50
C      THEN NB CARDS CONTAINING BLOCKING RESTRICTIONS STARTING IN
C      COLUMN 1
C
      INTEGER BLANK,DIGIT(3),ALF,ALF2(26)
      DIMENSION JUDY(40),L(39),MAHAF(35),LEMON(40)
      DIMENSION MCOM(513),LOCK(35,5),ALF(26),MBLOCK(5),LIME(5)
      DATA (ALF(I),I=1,26) /1HA,1HB,1HC,1HD,1HE,1HF,1HG,1HH,1HI,1HJ,
1      1HK,1HL,1HM,1HN,1HO,1HP,1HQ,1HR,1HS,1HT,
2      1HU,1HV,1HW,1HX,1HY,1HZ /, BLANK/1H /,
3      DIGIT(1)/0/,DIGIT(2)/1/,DIGIT(3)/2/
4, (ALF2(I),I=1,26) /2H2A,2H2B,2H2C,2H2D,2H2E,2H2F,2H2G,2H2H,
4      2H2I,2H2J,2H2K,2H2L,2H2M,2H2N,2H2O,2H2P,2H2Q,
4      2H2R,2H2S,2H2T,2H2U,2H2V,2H2W,2H2X,2H2Y,2H2Z/
900 FORMAT(3I5)
910 FORMAT(35I1,14XI1)
920 FORMAT( 1H1I25,12H VARIABLES      I10,15H RESTRICTIONS  ///)
929 FORMAT(31X 25(1XA1) /1H )
930 FORMAT(21XI5,5X35I2)
950 FORMAT( 1H0I25,21H EXPERIMENTAL POINTS /1H )
951 FORMAT(35H0MORE THAN 512 EXPERIMENTAL POINTS /1H )
960 FORMAT(1X 39I3)
981 FORMAT(1H0      I2,3H = 15(1XA2))
982 FORMAT(1H0 10X      I2,3H = 15(1XA2))
983 FORMAT(1H0 20X      I2,3H = 15(1XA2))
984 FORMAT(1H0 30X      I2,3H = 15(1XA2))
985 FORMAT(1H0 40X      I2,3H = 15(1XA2))
990 FORMAT(1H0 59X 25(1XA1) /1H )
991 FORMAT(      60X 25I2 )
10 CONTINUE
      READ (2,900) NV,NR,NB
      NR=MIN0(35,MAX0(0,NR))
      NV=MIN0(15,MAX0(1,NV))
      NB=MIN0(05,MAX0(1,NB))
      WRITE (3,920) NV,NR

```

```

WRITE (3,929) (ALF(J),J=1,NV)
DO 30 I=1,NR
READ (2,910) MAHAF,LEMON(I)
WRITE (3,930) LEMON(I),(MAHAF(J),J=1,NV)
JUDY(I)=0
DO 20 J=1,15
JUDY(I)=JUDY(I)+ MAHAF(J)*(2**(2*J-2))
20 CONTINUE
30 CONTINUE
DO 50 I=1,NB
READ (2,910)(MAHAF(J) ,J=1,NV)
MBLOCK(I)=0
NZ=0
DO 40 J=1,NV
LOCK(J,I)=BLANK
MBLOCK(I)=MBLOCK(I)+ MAHAF(J)*(2**(2*J-2))
IF(MAHAF(J).LT.1) GO TO 40
NZ=NZ+1
LOCK(NZ,I)=ALF(J)
IF(MAHAF(J).EQ.2) LOCK(NZ,I)=ALF2(J)
40 CONTINUE
50 CONTINUE
NUMBER=2**(2*NV) -1
CALL SET UP(JUDY,NR,L,NV,NUMBER,LEMON)
C
C VARIABLE MEANING
C JUDY LIST OF BINARY RESTRICTIONS
C NR NUMBER OF RESTRICTIONS
C L(1) ACCEPTABLE EXPERIMENTAL POINT
C NV NUMBER OF VARIABLES
C NUMBER NUMBER OF INTEGERS TO TRY
C LEMON LIST OF PARITY INDICATIONS FOR RESTRICTIONS
C
KA=1
KB=1
KC=1
KD=1
KE=1
GO TO(100,90,80,70,60) ,NB
60 KA=3
70 KB=3
80 KC=3
90 KD=3
100 KE=3
MIKE=0
110 CALL GET NUM (MONEY)
IF(MONEY.EQ.0) GO TO 120

```

```
C
C
C
C
MONEY WILL BE ZERO AT THE END. THAT IS IF THERE ARE NO MORE
    POSSIBLE COMBINATIONS.

MIKE=MIN0(513,MIKE+1)
MCOM(MIKE)=L(1)
GO TO 110
120 CONTINUE
    IF(MIKE.GT.512) WRITE (3,951)
    WRITE (3,950) MIKE
    DO 240 LA=1,KA
    IF(KA.GT.1) WRITE (3,981) DIGIT(LA),(LOCK(J,5),J=1,NV)
    LIME(5)=LA-1
    DO 230 LB=1,KB
    IF(KB.GT.1) WRITE (3,982) DIGIT(LB),(LOCK(J,4),J=1,NV)
    LIME(4)=LB-1
    DO 220 LC=1,KC
    IF(KC.GT.1) WRITE (3,983) DIGIT(LC),(LOCK(J,3),J=1,NV)
    LIME(3)=LC-1
    DO 210 LD=1,KD
    IF(KD.GT.1) WRITE (3,984) DIGIT(LD),(LOCK(J,2),J=1,NV)
    LIME(2)=LD-1
    DO 200 LE=1,KE
    IF(KE.GT.1) WRITE (3,985) DIGIT(LE),(LOCK(J,1),J=1,NV)
    LIME(1)=LE-1
    WRITE (3,990) (ALF(J),J=1,NV)
    DO 140 LQ=1,MIKE
    KORE=1
    DO 130 LX=1,NB
    KORE=KORE* MASK(MCOM(LQ),MBLOCK(LX),LIME(LX))
130 CONTINUE
    IF(KORE.EQ.0) GO TO 140
    CALL UNPACK (MCOM(LQ),L)
    WRITE (3,991) (L(J),J=1,NV)
140 CONTINUE
200 CONTINUE
210 CONTINUE
220 CONTINUE
230 CONTINUE
240 CONTINUE
GO TO 10
END
```

The following subroutine was written in MAP language for the IBM 7090 for the 3ⁿ series:

\$IBMAP	SELECT	100	
SETUP	SAVE		
	CLA*	4,4	
	ADD	3,4	
	STA	JUDY	
	STA	JUDY1	
	STA	JUDY2	
	CLA*	4,4	
	STA	NR1	
	ADD	8,4	
	STA	LEMON	
	CLA*	6,4	
	STA	NV2	
	ADD	5,4	
	STA	L	
	CLA*	6,4	
	ALS	1	
	STO	NV	
	CLA	=-1	
	STO	CURNUM	
	CLA*	7,4	
	STO	MAX	
	RETURN	SETUP	
GETNUM	SAVE	1,2,4	GET NUM ENTRY
	STZ*	3,4	SET END FLAG TO ZERO
	SXA	XR4,4	
	CLA	=1	
NEW	ADD	CURNUM	INCREMENT CURRENT NUMBER
	CAS	MAX	COMPARE WITH MAXIMUM NUMBER
	RETURN	GETNUM	AND RETURN IF FINISHED.
	TRA	*+1	
	STO	CURNUM	SAVE CURRENT NUMBER
	ANA	=0525252525252	TEST FOR ILLEGAL NUMBERS BY ANDING
	ARS	1	PAIRS OF BITS. IF THE NUMBER
	ANA	CURNUM	IS ILLEGAL THE AC WILL CONTAIN THE
	TNZ	NEW	INCREMENT TO GET NEXT POSSIBLE GOOD NUM.
NR1	AXT	**,1	XR1 IS RESTRICTION INDEX
	TXL	UNP,1,0	ARE THERE ANY RESTRICTIONS-
START	CAL	CURNUM	YES.
JUDY	ANA	**,1	AND TOGETHER RESTRICTION AND CURRENT NUM
	SLW	ANDED	

	CAL	CURNUM	AND TOGETHER THE LOW ORDER BIT OF THE
	ALS	1	CURRENT NUMBER AND THE HIGH ORDER
JUDY1	ANA	** , 1	BIT OF THE RESTRICTION.
	ANA	=0525252525252	AND SAVE
	SLW	FIRST	
JUDY2	CAL	** , 1	AND TOGETHER THE HIGH ORDER BIT OF THE
	ALS	1	CURRENT NUMBER AND THE LOW ORDER BIT
	ANA	CURNUM	OF THE RESTRICTION.
	ANA	=0525252525252	
	ORA	FIRST	OR THESE TWO RESULTS.
	SLW	FIRST	CHANGE EACH ONE BIT INTO TWO ONE BITS.
	ARS	1	
	ORA	FIRST	OR WITH ORIGINAL ANDED WORD TO FORM
	ORA	ANDED	DESIRED RESULT IN THE FORM OF THE NUMBER
	LXA	NV , 2	OF BITS. NOW COUNT THE
	AXT	0 , 4	NUMBER OF BITS
	LGR	1	
	TQP	*+2	
	TXI	*+1 , 4 , 1	
	TIX	*-3 , 2 , 1	
	PXA	, 4	TAKE SUM MODULO 3
	LRS	35	
	DVP	=3	
LEMON	SUB	** , 1	SUBTRACT PARITY REQUEST
	TNZ	NEW-1	
	TIX	START , 1 , 1	
UNP	CAL	CURNUM	STORE RESULT
XR4	AXT	** , 4	
	STL*	3 , 4	SET FLAG
NV2	AXT	** , 2	
L	SLW	** , 2	RETURN NUMBER
	RETURN	GETNUM	
CURNUM	PZE		
MAX	PZE		
CN	PZE		
NV	PZE		
ANDED	PZE		
FIRST	PZE		
UNPACK	SAVE	2	UNPACK NUMBER IN FIRST
	CLA	4 , 4	ARGUMENT AND STORE IN ARRAY
	ADD	=15	OF SECOND ARGUMENT.
	STA	LL	
	CAL*	3 , 4	
	AXT	15 , 2	
	LDQ	=0	
	LGR	2	
	RQL	2	

LL	STQ	** , 2	
	TIX	*-4 , 2 , 1	
	RETURN	UNPACK	
MASK	SAVE	2 , 1	SEE IF THE POINT (FIRST ARGUMENT)
	CAL*	3 , 4	SATISFIES BLOCKING RESTRICTIONS GIVEN
	ANA*	4 , 4	BY SECOND AND THIRD ARGUMENTS
	SLW	ANDED	
	CAL*	3 , 4	
	ALS	1	
	ANA*	4 , 4	
	ANA	=0525252525252	
	SLW	FIRST	
	CAL*	4 , 4	
	ALS	1	COMPUTE PRODUCTS AS ABOVE
	ANA*	3 , 4	
	ANA	=0525252525252	
	ORA	FIRST	
	SLW	FIRST	
	ARS	1	
	ORA	FIRST	
	ORA	ANDED	
	LXA	NV , 2	
	AXT	0 , 1	
	LGR	1	
	TQP	*+2	
	TXI	*+1 , 1 , 1	
	TIX	*-3 , 2 , 1	
	PXA	, 1	TAKE RESULT MODULO 3
	LRS	35	
	DVP	=3	
	SUB*	5 , 4	SUBTRACT DESIRED PARITY.
	LDQ	=1	
	TZE	*+2	IF EQUAL RETURN A 1.
	LDQ	=0	
	XCA		
	RETURN	MASK	
	END		

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