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AN INFORMAL REPORT

The Low-Density Limit of the Theta Pinch

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UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG-36

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Printed in the United States of America. Available from
National Technical Information Service
U. S. Department of Commerce
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Springfield, Virginia 22151
Price: Printed Copy \$3.00, Microfiche \$0.95

LA-5019-MS

An Informal Report
UC 20

ISSUED: August 1972

The Low-Density Limit of the Theta Pinch

by

Michael Kaufmann*

*Max-Planck-Institut Fur Plasmaphysik, 8046 Garching, Bei Munchen, Germany.

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THE LOW-DENSITY LIMIT OF THE THETA PINCH

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ABSTRACT

In the following, the scaling of the onset of sheath broadening in θ -pinch implosions is derived on the basis of a number of assumptions. The most realistic is that the instability threshold is that of the electron-cyclotron-drift instability and that the electrons are heated by resistive or diffusive field mixing during the implosion. This scaling is used to order various existing high-voltage θ -pinch experiments and to predict the conditions for excessive broadening of the sheath in future θ pinches and other shock-heating experiments.

I. INTRODUCTION

The theta pinch is a successful device for producing a high ion-temperature plasma.¹⁻⁵ The ion heating is usually well described by the snowplow or similar models with additional adiabatic heating.⁵ According to these models and experimental results the ion temperature rises roughly proportional to the inverse filling density. But with decreasing filling density a limit of this dependence was found experimentally.³⁻⁵ Below this limit the ion temperature is lower than the scaling law predicts, and at the same time diffusion during the first compression weakens the effect of the driving field. The compressed plasma has β -values below 1 which become smaller with decreasing filling density.⁶ It turned out that the sheath broadening in this region cannot be explained by Spitzer's resistivity.^{5,7}

The enhanced diffusion during the first compression of a theta pinch has hitherto not been satisfactorily explained. Simple estimates show drift velocities in the sheath much less than the electron thermal velocity, and two-stream instabilities are therefore excluded.⁸ Ion acoustic waves have a much lower instability limit at the ion sound velocity and were regarded as responsible for the diffusion.⁷ But the essential condition for unstable ion acoustic waves that T_e be much larger than T_i is violated

in a high energy theta pinch, even during the first compression.⁸ Unstable Bernstein waves have a critical velocity⁹ which is low enough to be reached by the drift velocity in the sheath of a theta pinch, and so in all probability these instabilities are excited in the linear range.

Recent results of computer simulation studies presented a new situation.^{15,16,23,24} The mechanism in the nonlinear range seems to be different from the mechanism of the linear theory. In this completely new situation on the theoretical side the instability limit is unknown. Computer experiments hitherto cover only a parameter range with a drift velocity near the thermal velocity of the electrons. Besides this, in the computer experiments the starting point of the nonlinear instability is usually not treated in an appropriate way. A lower number of particles in a Debye cell in the computer studies compared with experiments produces too much thermal noise.

Future simulation studies should yield a limit of fast growing nonlinear instabilities, depending on the drift velocity in relation to other characteristic velocities of the plasma. The object of this paper is to get some idea of this critical drift velocity by analyzing existing experimental results. Such an empirical result should meanwhile

help us in different ways. The information may serve as a comparison with future simulation studies. At the moment it can be used for planning experiments and predicting the limit of good compression in a theta pinch. The empirical derivation of the dependence of such a limit can also initiate new experiments to prove a special dependence.

The possibility of empirically deriving a scaling law for the limit of a good compression is based on relatively new experiments. Older experiments^{1,5,19} were carried out in a narrow range of parameters. Compression experiments with increased tube radius and much higher voltages¹⁰⁻¹² extended this parameter range sufficiently to derive new scaling laws. In the new experiments the density limit for good theta pinch compression was reduced by three orders of magnitude compared with the older experiments and correspondingly high ion temperatures were reached.¹²

For the proposed derivation of scaling laws different assumptions about an instability limit of the drift velocity in the sheath will be made in the following. Together with other simplifying assumptions scaling laws showing the dependence of the critical filling density from other initial parameters of the pinch will be derived and compared with experimental results.

A. Derivation of the Critical Density Under Different Assumptions About the Diffusion Mechanism

1. Fundamental Assumptions. Different instability limits were assumed to be the major cause of the diffusion of the magnetic field during the first compression of a theta pinch. In order to derive simple scaling laws, it was assumed that instabilities in the unstable range always grow fast enough. This means that the instabilities should increase the electron temperature and decrease the drift velocity so fast that the parameters are always near the instability limit. For simplification it was therefore assumed that inside the sheath the parameters just satisfy the marginal limit of the instability which is considered. The way in which the instabilities grow in the unstable range, especially the nonlinear behavior, is not considered here.

Assuming such an instability limit to be just fulfilled inside the sheath gives a relation between the magnetic field jump, density, and electron temperature inside the sheath. To get another relation

between these quantities, an assumption about the ratio between the magnetic field energy and the dissipated energy in the sheath is made. The derivation of this ratio under not very stringent conditions is given in the Appendix.

It is assumed that the energy dissipated is gained by the electrons. It is further assumed that the magnetic field outside the plasma rises linearly with time. To determine an averaged magnetic field during compression, the time of the snowplow model was taken to give the time for the first compression. Even, if one assumes a model in which the ions are elastically reflected at the magnetic field jump, the compression time will not be essentially changed. The critical initial density $n_{c,e}$ is defined as the initial density at which the thickness of the sheath about half-way through the first compression becomes half as thick as tube radius. It was shown experimentally in agreement with bounce model calculations⁵ that at about this point the ion heating starts to become inefficient and to lag behind the snowplow or similar models assuming a sharp boundary of the plasma. Ionization during compression is not taken into account. At low filling densities, when no ionization takes place during compression, the density of the ionized part of the gas should be taken as the initial density n_0 .¹⁰⁻¹² At high densities nearly all neutrals are ionized during compression, and so the filling density should be taken as an initial density. The ionization energy can be included in the energy balance of the sheath, but this is not done here.

Altogether four different cases, assuming four different dominant limits of the diffusion process, will be considered in the following. It is assumed that unstable Bernstein waves, ion-acoustic waves ($T_e > T_i$) or two-stream instabilities ($T_e < T_i$) are responsible for limiting the drift velocity. Furthermore, Spitzer's resistivity is assumed to determine the diffusion. In Table 1 the different assumptions are compiled.

TABLE 1. SUMMARY OF ASSUMPTIONS USED TO DETERMINE SHEATH DIFFUSION PROCESSES.

A	B	C	D
(1) $v_D = v_{th} \frac{\omega_{c,e}}{\omega_{p,e}}$	$v_D = c_s$	$v_D = v_{th}$	$\eta = \eta_{Spitzer}$
(2)	$E_{th} = E_{mag}$		
(3)	$t_c = \left(\frac{r_t}{B_0} \right)^{1/2} (\omega_{01} n_0)^{1/4}$		
(4)	$n_{0,c} : \Delta_r = \frac{r_t}{2}$		

v_D : Drift velocity of electrons against ions in the sheath

v_{th} : Thermal velocity of the electrons

$\omega_{c,e}$: Cyclotron frequency of the electrons

$\omega_{p,e}$: Electron plasma frequency

c_s : Ion sound speed

η : Resistivity

E_{th} : Thermal energy of the electrons in the sheath

E_{mag} : Magnetic field energy in the sheath

t_c : Time of first compression

$n_{0,c}$: Critical initial density

Δ_r : Sheath thickness

r_t : Inner tube radius

T_e : Electron temperature

II. PHYSICAL QUANTITIES OF THE SHEATH

In Fig. 1 the assumed simplified profiles for the magnetic field B , density n_e and temperature T_e are shown as a function of r for the case where

$\Delta_r = \frac{r_t}{2}$ just half-way through compression. B_1 is assumed to be equal to $t \cdot B_0$ and $B_1 \gg B_{1,0}$.

according to the assumed linear drop of B inside the sheath the drift velocity is

$$v_D \propto \frac{B_1}{\Delta_r n_s} \quad n_s = 2 n_0$$

Fig. 1

Fig. 1. Diagram of the sheath parameters.

$$v_D \left[\frac{\text{cm}}{\text{s}} \right] = 2.5 \cdot 10^5 \frac{B_1 [\text{kG}]}{\Delta_r [\text{cm}] n_0 [10^{16} \text{cm}^{-3}]}$$

The thermal velocity of the electrons in the sheath is

$$v_{th} \left[\frac{\text{cm}}{\text{s}} \right] = 2.3 \cdot 10^9 \sqrt{T_e [\text{keV}]}.$$

The electron cyclotron frequency averaged over the sheath is

$$\omega_{c,e} [\text{s}^{-1}] = 8.8 \cdot 10^9 B_1 [\text{kG}].$$

Plasma frequency

$$\omega_{p,e} [\text{s}^{-1}] = 5.6 \cdot 10^{12} \sqrt{n_s [10^{16} \text{cm}^{-3}]}$$

$$\omega_{p,e} [\text{s}^{-1}] = 8 \cdot 10^{12} \sqrt{n_0 [10^{16} \text{cm}^{-3}]}.$$

Ion sound speed:

$$c_s = \sqrt{\frac{2kT_e}{m_i}},$$

which in the case of deuterium is:

$$c_s \left[\frac{\text{cm}}{\text{s}} \right] = 3.1 \cdot 10^7 \sqrt{T_e [\text{keV}]}.$$

The magnetic energy inside the sheath is

$$E_{\text{mag}} = \frac{1}{2\mu_0} B^2 S = \frac{1}{6\mu_0} B_1^2$$

III. CRITICAL DENSITIES UNDER DIFFERENT ASSUMPTIONS

Case A

Assuming the drift velocity to be limited by

$$v_D = v_{th} \frac{\omega_{c,e}}{\omega_{p,e}}, \quad (1A)$$

the sheath thickness turns out to be

$$\Delta_r [\text{cm}] = \frac{10^{-1}}{\sqrt{n_0 [10^{16} \text{cm}^{-3}]} T_e [\text{keV}]}.$$

The energy relation gives (2):

$$B_1^2 [\text{kG}] = 3.6 \cdot 10^3 n_0 [10^{16} \text{cm}^{-3}] T_e [\text{keV}] \quad (2A)$$

and together with (1A):

$$\Delta_r [\text{cm}] = \frac{6}{B_1 [\text{kG}]}.$$

It should be noted that this result is very close to the assumption

$$\Delta_r = \frac{c}{\omega_{c,e}} \quad +)$$

Actually, in this case one gets

$$\Delta_r [\text{cm}] = \frac{1.7}{B_1 [\text{kG}]}.$$

Assuming that the compression time t_c is given by the time scale of the snowplow¹³

$$t_c \approx \left(\frac{r_t}{B_0} \right)^{1/2} (\mu_0 m_i n_0)^{1/4}, \quad (3)$$

a typical B_1 during compression is taken to be

$$B_1 = B \left(\frac{t_c}{2} \right) = B_0 \frac{t_c}{2}.$$

This gives a sheath thickness for deuterium of

$$\Delta_r [\text{cm}] = \frac{15}{r_t [\text{cm}]^{1/2} B_0 \left[\frac{\text{kG}}{\mu\text{s}} \right]^{1/2} n_0 [10^{16} \text{cm}^{-3}]^{1/4}}. \quad (4)$$

If one sets $\Delta_r = \frac{r_t}{2}$

as a limit, one gets as a critical lower density limit

$$n_{0,c} [10^{16} \text{cm}^{-3}] = \frac{0.8 \cdot 10^6}{r_t [\text{cm}]^6 B_0 \left[\frac{\text{kG}}{\mu\text{s}} \right]^2}.$$

Expressed in terms of U_t , the induced voltage around the inner surface of the tube (vacuum case),

$n_{0,c}$ is:

$$(A) \quad n_{0,c} [10^{16} \text{cm}^{-3}] = \frac{0.8 \cdot 10^3}{r_t [\text{cm}]^2 U_t^2 [\text{KV}]}.$$

This result will be compared with experimental results later. First however, derivations of critical densities under other assumptions will be given.

Case B

Now the drift velocity is assumed to be limited

by

$$v_D = c_s = \sqrt{\frac{2kT_e}{m_i}}$$

$$v_D \left[\frac{\text{cm}}{\text{s}} \right] = 3.1 \cdot 10^7 \sqrt{T_e [\text{keV}]} \quad (1B)$$

Together with the expression for the drift velocity, the energy relation (2) and the definition of the critical density (4) one gets:

+) The general dependence of Δ_r from the ratio $\frac{v_D}{v_{th}}$ is derived by Chodura.²⁷

$$(B) \quad n_{o,c} [10^{16} \text{ cm}^{-3}] = \frac{1}{r_t [\text{cm}]^2}$$

It shall be pointed out that it is possible to get essentially the same result by assuming the sheath thickness to be of the order of

$$\frac{c}{v_{pi}} : \quad \Lambda_r = \frac{c}{v_{pi}} \quad (1 B')$$

Together with relation (4) this yields:

$$(B') \quad n_{o,c} [10^{16} \text{ cm}^{-3}] = \frac{0.4}{r_t [\text{cm}]^2}.$$

Case C

In an analogous way the assumed limit

$$v_D = v_{th} \quad (1 C)$$

gives a critical density a factor $\frac{2}{3} \frac{m_e}{m_i}$ lower than case B

$$(C) \quad n_{o,c} [10^{16} \text{ cm}^{-3}] = \frac{1.75 \cdot 10^{-4}}{r_t [\text{cm}]^2}.$$

Case D

Finally, the case with only classical diffusion is treated. Neglecting any dynamics and assuming a constant resistivity η in the sheath the diffusion is described by:

$$\frac{\partial^2 B_z}{\partial r^2} = \frac{\mu_0}{\eta} B(r).$$

For simplification it is assumed that

$$\frac{\Delta B_z}{(\Delta r)^2} = \frac{\mu_0}{\eta} B(r)$$

$$\Delta B_z = \frac{1}{2} t_c \dot{B}_0$$

$$\frac{1}{B(r)} = \frac{1}{2} \dot{B}_0$$

Spitzer's resistivity is:¹⁴

$$\eta_s [\Omega \text{ m}] = 4 \frac{10^{-8}}{T_e [\text{keV}]^{3/2}} \quad (\text{for } \lambda = 12, Z = 1)$$

which gives:

$$\Delta r [\text{cm}] = 1.8 \cdot 10^{-2} \sqrt{\frac{t_c [\mu \text{ s}]}{T_e [\text{keV}]^{3/2}}} \quad (1 D)$$

Together with relations (2), (3), and (4) one finally gets:

$$\begin{aligned} (D) \quad n_{o,c} [10^{16} \text{ cm}^{-3}] &= 3 \cdot 10^{-4} B_0 [\frac{\text{kG}}{\mu \text{ s}}]^2 r_t [\text{cm}]^3 \\ &= 0.3 U_t^2 [\text{kV}] r_t^{-1} [\text{cm}]. \end{aligned}$$

In contrast with cases A, B, and C, this critical density represents an upper limit of the density. In nearly all experiments the filling density is well below this limit. This means that classical diffusion is not mainly responsible for the observed strong sheath broadening.

IV. COMPARISON WITH EXPERIMENTAL RESULTS

The density limit $n_{o,c}$ of cases A, B, and C are now compared with experimental results.

Figure 2a shows a comparison of experimental results with case A--the assumption of a drift velocity limit near $\frac{\omega_{ce}}{\omega_{pe}} \cdot v_{th,e}$.

Experiments number 1 to 4 are older θ -pinch experiments^{3-6, 19} carried out at Los Alamos and Garching. By reducing the filling density it was clearly shown in these experiments that the ion heating deviated from snowplow scaling, and that diffusion of the magnetic field occurred at the same time. The density range where this deviation takes place is indicated as the experimental result for the critical density.

Experiment 5 has been proposed to be carried out with the Scyllac bank. The scaling law of the critical density should be found by varying the bank voltage up to a factor of three. In the experiments 6 and 7^{11, 12} the critical density was not systematically varied. But under these experimental conditions the sheath thickness was comparable with the tube radius, which means that relation (4), the definition of the critical density, was just satisfied. As further proof the ion

Fig. 2a

Fig. 2b

Fig. 2a. Critical density, comparison of experimental results with case A.

- (1) Isar IV⁵
- (2) Isar I, 1.5 m coil⁴
- (3) Scylla IV^{3, 25, 26}
- (4) Isar I, 5.4 m coil¹⁹
- (5) Scyllac, proposed experiment
 - (a) $U_t = 40$ kV
 - (b) $U_t = 120$ kV
- (6) Jülich shock experiment¹¹
- (7) Maryland shock experiment¹²

heating of these experiments was compared with the snowplow model and relatively good agreement was found as well.

All experimental results are quite close to the estimated curve A and also show the same relative dependence $r_t^2 \cdot U_t^2$ and the same absolute value.

The comparison of the experimental results with case B (Fig. 2 b) show less good agreement. The experimental values of $n_{o,c}$ do not seem to scale

Fig. 2b. Critical density, comparison with cases B and C.

with r_t^2 , which means that the sheath thickness does not scale as $\frac{c}{\pi}$.

Case C deviates from the experimental results by more than three orders of magnitude. This clearly shows that the plasma is unstable, even if the drift velocity is much less than the thermal electron velocity. This result is in contrast with one-dimensional simulation results^{17, 18} showing the drift velocity and the thermal electron velocity close together in the nonlinear range. Summarizing, we can state that the existing experimental results concerning the critical density fit best with the assumption of a limit for the drift velocity near the onset of the electron cyclotron instability, which leads to a dependence of $n_{o,c} \sim U_t r_t^2$. But it must be pointed out that more experiments are necessary to prove this dependence conclusively.

V. SOME FINAL REMARKS

A. Some special points have to be observed in analysing experimental results to get the critical density. Reference has already been made to the influence of the ionization energy, which was not taken into account in the preceding estimates. Up to 70 eV can be needed for one ionization process.²⁰ Knowing the degree of preionization and assuming complete ionization during compression, one can compare the ionization energy with the thermal energy of the electrons according to relation (2). If the ionization energy is an appreciable part of it, the electron energy is lowered and the diffusion of the magnetic field inside the sheath is increased.

Another effect which could give misleading results is sheath broadening after the first compression especially if one determines the ion energy at the time of maximum adiabatic compression and compares it with a model for the ion heating, a strong reduction compared with this model need not be a sign of strong diffusion during the first compression only. Short circuiting of the electric field in the boundary sheath of the plasma over the ends 21, 22 leads to sheath broadening and a reduction of the ion temperature²¹.

A more technical point is the fact that the voltage around the coil may have a rise time comparable with the compression time. In this case U_t in the above estimates has to be a value averaged over compression. The experimental results of Fig. 1 include corrections for this effect as far as it is known.

B. The relations for the critical density derived above only show a dependence on the initial voltage and tube radius. In every case the gas was assumed to be deuterium. Regarding only the most interesting cases A and B a different scaling with the ion mass m_i was found. The ion mass enters the estimates of $n_{o,c}$ by relation (A3) and (B1). Similarly to the preceding derivations, one gets:

$$\text{Case A:} \quad n_{o,c} \sim \frac{1}{m_i}$$

$$\text{Case B:} \quad n_{o,c} \sim m_i$$

This different scaling of the critical density with the ion mass should be used for an experimental investigation. A change from deuterium to hydrogen

gives a relative variation of the critical densities in both cases by a factor of 4. Experiments with Argon and Xenon at Julich¹¹ do not show the dependence of Case B.

C. It is possible to derive a ratio of the electron temperature T_e inside the sheath to the ion temperature T_{i1} after the first compression from assumptions (2) and (3). This ratio does not depend on any of the pinch parameters r_t , U_t , n_o and m_i . There is also no dependence on the diffusion mechanism (relation (1)). Taking the same factors to average over the sheath as above, one gets an estimate of the electron temperature of the sheath from relation (2):

$$T_e [\text{keV}] = 4.5 \cdot 10^{-5} \frac{B_o [\frac{\text{kg}}{\text{ms}}] r_t [\text{cm}]}{n_o [10^{16} \text{cm}^{-3}]^{1/2}}.$$

Setting the ion velocity after the first compression:

$$v_i \approx \frac{r_t}{t_c}$$

and the ion temperature:

$$kT_{i1} = \frac{1}{2} m_i v_i^2$$

one gets:

$$T_{i1} [\text{keV}] = 1.6 \cdot 10^{-3} \frac{B_o [\frac{\text{kg}}{\text{ms}}] r_t [\text{cm}]}{n_o [10^{16} \text{cm}^{-3}]^{1/2}}.$$

This yields a ratio:

$$\frac{T_e}{T_{i1}} \approx \frac{1}{30}$$

Experiments carried out in the collision-free range 3, 4, 6 show ratios between $\frac{1}{10}$ and $\frac{1}{20}$. Bearing in mind the number of simplifications, this is a reasonable agreement.

ACKNOWLEDGMENTS

The author would like to thank Dr. R. Chodura, Dr. B. M. Marder, Dr. R. L. Morse, Dr. J. Neuhauser, Dr. F. L. Ribe, Dr. R. E. Siemon, and Dr. H. Weitzner for fruitful discussions and R. Wunderlich for computer calculations.

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APPENDIX

It is assumed that outside a conductor with a plane surface and constant resistivity ($\eta = \text{const.}$ if $x > 0$, $\eta = 0$ if $x \leq 0$) a homogeneous magnetic field rises with constant $\dot{B}$. The magnetic field is parallel to the surface ($\vec{B}(t) = (0, 0, \dot{B}_0 \cdot t)$, $x < 0$, $t \geq 0$). At the beginning the magnetic field is zero everywhere ($\vec{B}(t=0) \equiv 0$).

B_z inside the conductor follows the equation:

$$\dot{B}_z = D \frac{\partial^2 B_z}{\partial x^2} \quad \text{with } D = \frac{\eta}{\mu_0}.$$

The substitution

$$B_z(x, t) = \dot{B}_0 t f(\xi) \quad \xi = \frac{x^2}{Dt}$$

leads to

$$4\xi f'' + (2 + \xi) f' - f = 0 \quad \xi \geq 0$$

with the boundary conditions:

$$f(0) = 1 \text{ and } \lim_{\xi \rightarrow \infty} f(\xi) = 0.$$

The solution is:

$$f(\xi) = A(2 + \xi) \int_{\xi}^{\infty} \frac{1}{(2+s)^2} e^{-\frac{s}{4}} \frac{ds}{s^{1/2}}$$

Calculating now the magnetic energy E_{mag} and the dissipated energy E_{dis} inside the conductor, one finds:

$$\begin{aligned} E_{\text{mag}} &= \frac{1}{2\mu_0} \int_0^{\infty} B_z^2 dx \\ &= \frac{\dot{B}_0^2 t^2}{2\mu_0} \int_0^{\infty} [f'(\xi(x,t))]^2 dx \\ &= \frac{1}{4\mu_0} I_1 \dot{B}_0^2 D^{1/2} t^{5/2} \end{aligned}$$

$$\text{with } I_1 = \int_0^{\infty} \xi^{-1/2} f'^2(\xi) d\xi \approx 0.64 \text{ A}^2$$

$$\begin{aligned} E_{\text{dis}} &= \frac{D}{\mu_0} \int_0^t \int_0^{\infty} \left(\frac{\partial B_z}{\partial x} \right)^2 dx d\tau \\ &= \frac{4\dot{B}_0^2}{\mu_0 D} \int_0^t \int_0^{\infty} x^2 f' [f'(\xi(x,t))]^2 dx d\tau \end{aligned}$$

$$\text{with } = \frac{4}{5\mu_0} I_2 \dot{B}_0^2 D^{1/2} t^{5/2}$$

$$I_2 = \int_0^{\infty} \xi^{1/2} f'(\xi)^2 d\xi \approx 0.25 \text{ A}^2.$$

As a time independent ratio of both energies one gets:

$$\frac{E_{\text{dis}}}{E_{\text{mag}}} = 1.25$$