DUAL-MODEL ANALYSIS OF SINGLE-PARTICLE PRODUCTION AT INTERMEDIATE ENERGIES*

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ABSTRACTNOTICE

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A systematic numerical analysis of the single-particle distribution is presented in a dual-resonance model at energies presently accessible. The results are in reasonably good agreement with experiments. In addition, a criteria for the scaling limit in energy is suggested and the pion production rate is predicted to be much slower in the pionization limit than at intermediate energies.

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It has been reported previously that^{1,2} the single-particle inclusive cross section obtained from the six-line dual amplitude via the Mueller method³ exhibits the desired limiting properties and predicts, among others, a universal cutoff in transverse momentum in qualitative agreement with experiments. Prediction of such experimentally observed feature may be an indication that the dual-resonance model has qualitatively sound properties of multi-Regge behavior and analyticity to describe the multiparticle production despite the lack of unitarity. This view has been further elucidated in the subsequent successful analyses⁴ of two-particle distributions.

In this paper, we would like to present a systematic quantitative analysis of the single-particle distribution functions obtained from the dual resonance

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model and compare with experiments. Due to much concentrated experimental efforts in the last few years, the accumulated data⁵ of the inclusive processes are readily available for this purpose.

However, the distribution function of Ref. 1 cannot be directly applied to the present accelerator data which correspond to a few tens GeV^2 of s , as they are defined in the fragmentation or central regions for the theoretical asymptotic limit of s and the missing mass variable M^2 . In particular, it is desirable to have a practical representation for the distribution function which can be used for all values of $x = 2p_{11}/\sqrt{s}$ between 0 and 1 and $\kappa = p_1^2 + m_x^2$ at these accelerator energies. For this purpose, we suggest to use

$$f(x, \kappa; s) = \left(\frac{\alpha}{\alpha_{ab}} \right)^{\alpha_v} \left(-\frac{\alpha_{ax}}{\alpha} \right)^{\alpha_{bx}} \left(-\frac{\alpha_{ax}}{\alpha} \right)^{\alpha_{bx}} \int_0^\infty \int_0^\infty dy_1 dy_2 y_1^{-\alpha_{bx}-1} y_2^{-\alpha_{bx}-1} \\ \times u^{\alpha_{bx} + \alpha_{bb} - \alpha_{aax}} v^{\alpha_{bx} + \alpha_{bb} - \alpha_{aax}} \\ \times w^{-\alpha_v - \alpha_{bb} + \alpha_{aax} + \alpha_{aax}} (H)^{\alpha_v} \theta(H) \quad (1)$$

where

$$u = 1 - \frac{\alpha}{\alpha_{ax}} y_1, \quad v = 1 - \frac{\alpha}{\alpha_{ax}} y_2, \quad w = u + v - 1 \quad \text{and} \\ H = 1 - y_1 \left(1 - \frac{\alpha_{aa} - \alpha_{aax}}{\alpha_{ax}} \frac{v}{w} \right) - y_2 \left(1 - \frac{\alpha_{bb} - \alpha_{aax}}{\alpha_{ax}} \frac{u}{w} \right) \quad (2)$$

Notice that (1) reduces to $f_b^x(p_{11}, p_1)$ and $f^x(p_1)$ of Ref. 1 in the respective kinematic limits of fragmentation and pionization.

A. Before we proceed to present the numerical results, we make a practical criteria of the fragmentation (single-Regge) and pionization (double-Regge) limits, on the basis of how the respective limiting distributions are obtained from (1). Our definition is based on the reliability of the usual exponentiation

procedure, $\int_0^N dz (1 - z/N)^N \rightarrow \int_0^N dz e^{-z}$, necessary to achieve the Regge-behavior from the dual amplitude. The largeness of N depends on the confidence of the approximation and we find for a class of test functions relevant to the distributions that $N \geq 20$ insures in general a better-than 10% accuracy. Next we note that the momentum transfer variables, $s_{ax} \simeq -s(\bar{x} + \chi)/2$ and $s_{bx} \simeq -s(\bar{x} - \chi)/2$ where $\bar{x} = (\chi^2 + 4\kappa/s)^{1/2}$, take the value $\sqrt{sk}$ for $\chi \simeq 0$ (pionization) while $s_{ax} \simeq -s\chi$ and $s_{bx} \simeq -\kappa/\chi$ for χ near 1 (fragmentation of b). Together with $M^2 \simeq s + s_{ax} + s_{bx}$, we thus conclude that the fragmentation limit, say, for χ around 0.6 is practically reached at the present accelerator energies, i.e., $s \geq 50 \text{ GeV}^2$ but the pionization limit is obtained at much higher energies, i.e., $s \geq 400/\langle \kappa \rangle \simeq 4000 \text{ GeV}^2$. In addition, the scaling behavior in the triple-Regge region, for example, $\chi \simeq 0.95$, is also expected at higher energies, i.e., $s \geq 400 \text{ GeV}^2$.

B. The numerical results are based on (1) with $\alpha_v = \alpha_{aa} = \alpha_{bb} = \frac{1}{2}$, though the case with $\alpha_v = 1$ is also considered in the way of comparison. In addition, the masses appearing in the other trajectory functions are set to zero and their intercepts are chosen to be zero for non-exotic ones and $-\frac{1}{2}$ if exotic to insure the convergence of the integral. Thus the dependence on m_x of the distribution function (1) appears only through κ . Numerically, we take $m_x^2 = 0.02 \text{ GeV}^2$ for pion and $m_x^2 = 0.25 \text{ GeV}^2$ for kaon. The results are given in Figures 1-3.

In Fig. 1a, we plot $f(\chi, \kappa; s)$ versus κ for fixed s and χ . For $s \geq 4000 \text{ GeV}^2$, we are practically in the central region and (1) gives essentially the lower curve in Fig. 4 of Ref. 1. The decrease of the distribution function with κ is faster for smaller χ , while it approaches to $e^{-4\kappa}$ for large κ independent of χ . Figure 1b shows the χ -dependence of $f(\chi, \kappa; s)$ for fixed s and κ . Both results are in qualitative agreement with the π^- spectrum seen in experiments, in particular,

by Ko and Lander.⁶ The sharp peak of $f(\chi, \kappa; s)$ near $\chi = 0$ for both finite and infinite s (though a little less sharper for the latter) suggests the increase of the slow pion concentration and that the average transverse momentum of pions is smaller for smaller χ as shown in Fig.1c. Such sea-gull shape of the correlation between $\langle p_{\perp} \rangle$ and χ (or equivalently $p_{||}$) as well as that $\langle p_{\perp} \rangle$ is smaller for lower values of s is in general agreement with the trends in the experimental data.⁷ The fact that the sea-gull shape persists at higher energies implies that the sharp peak of $f(\chi, \kappa; s)$ near $\chi = 0$ or equivalently the momentum correlation near $p_{||} = 0$ cannot just be due to generation of pions as decay products of resonances with small Q values as has been suggested in some model studies.⁸ This fact as well as the impossibility of separating the distribution function into a product of functions of $p_{\perp}$ and χ is again in accord with the experimental trends.^{6,7,9} Note that $\langle p_{\perp} \rangle$ is larger for higher mass¹⁰ and the mass dependence of $\langle p_{\perp} \rangle$ is stronger near $\chi = 0$. We find that the maximum $\langle p_{\perp} \rangle$ approaches asymptotically to 340 MeV and 350 MeV for pions and kaons. As shown in Figs.1a and 1c, the distribution for $\alpha_v = 1$ decreases faster and thus $\langle p_{\perp} \rangle$ is smaller than the case of $\alpha_v = \frac{1}{2}$.

In order to make a more quantitative comparison, we plot in Figs. 2a-c,

$$G(\chi) = \int_0^{\infty} \frac{2E}{\sigma_{\text{tot}} \sqrt{s}} \frac{d^2 \sigma}{dx dp_{\perp}^2} dp_{\perp}^2 \quad (3)$$

for the exotic and non-exotic cases of the $ab\bar{x}$ channel for $m_x^2 = 0.02$ and 0.25 and compare with the experimental results¹¹ of the $\pi^+ p$ reaction at $s = 42 \text{ GeV}^2$ for π^+ and K_s^0 productions. In doing so, we have normalized the theoretical area $\int_{-0.4}^{+0.4} G(\chi) d\chi$ to that of experiment, as the ratios of the experimental area to the theoretical one are 0.52, 0.79 and 1.96 for π^- , π^+ and K_s^0 respectively. Matching $G(\chi)$ near $\chi = 0$ gives more or less the same ratios. As

expected, the present model cannot account for the asymmetry of the spectrum around $\chi = 0$, nor for the leading particle effect at these energies. Nevertheless, the result is in surprisingly good agreement with experiments¹¹ for $|\chi| \leq 0.4$.

Having obtained the normalization constants, we then proceed to calculate the average multiplicity $\langle n \rangle$ from (1). As it is shown in Fig. 3a, $\langle n \rangle$ can be approximated by a straight line in $\ln s$ around $s = 10^3 \text{ GeV}^2$ with a slope 1.28 for π and 0.1 for K. Thus with the normalization constants obtained above, we find in this energy region,

$$\langle n \rangle = C \ln(s/\text{GeV}^2) + a \quad (4)$$

where $C = 0.42, 0.64$ and 0.13 for π^-, π^+ and K_s^0 respectively. These coefficients of the $\ln s$ term are certainly consistent with the available data.^{7,12} However, the pion production will be slowed down further after $s = 4000 \text{ GeV}^2$ according to Fig. 3a; the slope of $\langle n \rangle$ approaches 0.26, which is precisely the value predicted by the dual-model "central" distribution,

$$C = \frac{\pi}{2} \int_0^1 \int_0^1 dw dv w^{-\alpha_{aax} - \alpha_v} (1-w)^{-\alpha_{aax} + \alpha_v} v^{\alpha_v} \exp \{-m_x^2 / w(1-w)(1-v)\} \quad (5)$$

To make a further comparison with experiments, we show in Fig. 3b the differential multiplicities $d \langle n \rangle / d\chi = \sigma_{\text{tot}}^{-1} d\sigma / d\chi$ as a function of χ at $s = 42 \text{ GeV}^2$. As expected, the highest multiplicity occurs at $\chi \simeq 0$ and the shape can be described roughly by $e^{-b\chi}$ (or equivalently $e^{-a\chi}$) with $b = 9.6$ (or $a = 3.0$) for π and $b = 6.0$ (or $a = 1.85$) for K. Experimentally, $b = 7 \sim 10$ (or $a = 2 \sim 3$) for pions.^{6,9} Note that the pion multiplicity has a sharper peak than that of kaon, implying the increase of the lower-mass particle concentration near $\chi = 0$. The case $\alpha_v = 1$ gives more or less the same values of b .

In summary, the dual resonance model predicts surprisingly good results and can be used to analyse the single-particle distributions at the accelerator energies, well before the exact limit of scaling is reached. The pionization limit is much further away from fragmentation. In addition, we find that the cutoff behavior of the distribution in $p_{\perp}^2$ at large $p_{\perp}^2$ as well as $\langle p_{\perp} \rangle$, $\langle n \rangle$ and $d\langle n \rangle/dx$ are all more or less independent of $\alpha_{bx}^-(0)$ (or of the target). These results are probably of more general nature than the model.

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FIGURE CAPTION

- Fig. 1a-c a. $f(\chi, \kappa; s)$ vs κ for given χ and s ; b. $f(\chi, \kappa; s)$ vs χ for given κ and s ; c. the momentum correlation between $\langle p_{\perp} \rangle$ and χ .
- Fig. 2a-c $G(\chi)$ vs χ . Experimental points are from Ref. 11.
- Fig. 3a-b a. the average multiplicity vs s ; b. $d\langle n \rangle/d\chi$ vs χ .

FIG. 1a

FIG. 1b

FIG. 1c

FIG. 2b

FIG. 2c

FIG. 3a

FIG. 3b