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Effect of Inner Field Fluctuations

on Plasma Impedance*

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The effective field acting on the electrons in a plasma is studied by methods similar to those employed for analyzing the dielectric properties of neutral gases. The difference between incident and effective field is the polarization field which is associated with the displacement of ions relative to electrons. This latter field is found to be a sensitive function of the ion positions which fluctuates with the fluctuating ion density. The plasma impedance is averaged over these fluctuations and is shown to satisfy a formula of the form

$$Z = Z_1 \left[1 + \sum_{\nu=1}^{\infty} C_{\nu} (n_0 \alpha_0)^{\nu} \right]^{-1}$$
, where n_0 is the average electron density, $\alpha_0 = -\omega_p^2/4\pi\omega^2 n_0$ is the free electron polarizability, ω_p is the electron plasma frequency, and ω is the frequency of the driving field. The coefficients C_{ν} (which are zero if field fluctuations are neglected) have the magnitude $C_1 = 0$, $C_2 = 0.8 \left[1 - 7.6 (\omega/v_0 n_0^{1/3})^2 - i 11.2 (\omega/v_0 n_0^{1/3})^3 \right]$, where v_0 is the average thermal electron velocity. The frequency dependence of Z is quite different from that of the uncorrected impedance Z_1 .

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I. INTRODUCTION

The impedance of electrons in a plasma differs slightly from the free electron impedance (4) because of various interaction effects. The phenomenon which has been studied most thoroughly in the past is the deformation of Debye clouds by the incident radiation field. This deformation generates a restoring force $\underline{F}$ which affects the amplitude and phase of the radiation induced electron oscillations and leads to a correction term in the formula for the free electron impedance.^{1,2} The correction is proportional to the ratio of potential over kinetic plasma energy and is very small in most cases of practical interest. Nevertheless it is of considerable theoretical interest and has been the subject of numerous recent investigations.^{1,14}

In most of these investigations it is implied that the average electronic polarization $\underline{P}$ is linear in the parameter $\alpha_o n_o$, where α_o is the free electron polarizability and n_o the average electron density. This leads to the relations

$$\underline{P} = n_o \alpha_o (\underline{E} + \underline{F}) = n_o \alpha_1 \underline{E}, \quad (1)$$

$$\dot{\underline{P}} = \underline{J} = Z_o^{-1} (\underline{E} + \underline{F}) = Z_1^{-1} \underline{E}, \quad (2)$$

$$\alpha_o = -e^2 / m \omega^2 = -\omega_p^2 / 4\pi \omega^2 n_o, \quad (3)$$

$$Z_o = 4\pi i \omega / \omega_p^2 = (i n_o \alpha_o \omega)^{-1}, \quad (4)$$

where m , $-e$, Z_o are respectively the mass, charge and impedance of free electrons, $\underline{J}$ the average electron current, $\underline{E}$ the average driving

(radiation) field and ω the radiation frequency.

However it is suggested by the work of Kirkwood¹⁶ and others^{16,17} on fluctuations in neutral gases that $\underline{P}$ is not linear in $(\alpha_o n_o)$, and may be represented in the form

$$\underline{P} = n_o \alpha_1 \left[1 + \sum_{\nu=1}^{\infty} C_{\nu} (n_o \alpha_o)^{\nu} \right] \underline{E} = n_o \alpha \underline{E}. \quad (5)$$

Hence, one expects that

$$\underline{Z} \approx 1/\alpha = \underline{Z}_1 \left[1 + \sum_{\nu=1}^{\infty} C_{\nu} (n_o \alpha_o)^{\nu} \right]^{-1}. \quad (6)$$

Equation (5) is easily understood if the electron motion is neglected. Consider a system of N ions with charge e and fixed positions $\underline{r}_j$, and a test electron at position $\underline{r}_e$. The ion field $\underline{\mathcal{E}}^i(\underline{r}_e)$ at the site of the electron changes into $\underline{\mathcal{E}}^i(\underline{r}_e) + \left(\nabla_{\underline{r}_e} \underline{\mathcal{E}}^i(\underline{r}_e) \right) \underline{\rho}$ if the electron is displaced through an infinitesimal distance $\underline{\rho}$. The extra field seen by the displaced electron may be interpreted as being produced by a set of dipoles,

$$\underline{\mu}_j = \underline{\mu}_e = -e \underline{\rho}, \quad (7)$$

located at the ion sites. The change in the ion field can then be written in the form

$$\Delta \underline{\mathcal{E}}^i(\underline{r}_e) = \sum_{j=1}^N (\nabla \times \nabla \times)_{\underline{r}_e} (\underline{\mu}_e / |\underline{r}_e - \underline{r}_j|) = \sum_j T_j(\underline{r}_e) \underline{\mu}_e, \quad (8)$$

where the subscript $\underline{r}_e$ indicates that the space derivatives are to be taken with respect to $\underline{r}_e$.

If the displacement $\underline{\rho}$ is caused by a radiation field $\underline{E}$ the effective polarizing field is

$$\underline{\underline{\epsilon}} = \underline{\underline{E}} + \Delta \underline{\underline{\epsilon}}^i = \underline{\underline{E}} + \sum_j T_j \underline{\underline{\mu}}_e, \quad (9)$$

and

$$\underline{\underline{\mu}}_e = \alpha_o \underline{\underline{\epsilon}} = \alpha_o \underline{\underline{E}} + \alpha_o \sum_j T_j \underline{\underline{\mu}}_e. \quad (10)$$

It follows that

$$\underline{\underline{\mu}}_e = (1 - \alpha_o \sum_j T_j)^{-1} \alpha_o \underline{\underline{E}} = \left[\sum_{v=0}^{\infty} (\alpha_o \sum_j T_j)^v \right] \alpha_o \underline{\underline{E}}. \quad (11)$$

Equation (11) could be obtained from the iteration scheme

$$\underline{\underline{\mu}}_e^{(0)} = \alpha_o \underline{\underline{E}}, \quad (12)$$

$$\underline{\underline{\mu}}_e^{(v)} = \alpha_o \underline{\underline{E}} + \alpha_o \sum_j T_j \underline{\underline{\mu}}_e^{(v-1)}, \quad (13)$$

which shows that the v -th order solution of $\underline{\underline{\mu}}_e$ is of the order $(v+1)$ in α_o . The ensemble average of the operator $\sum_j T_j$ over all ion configurations is zero. However, if a small spherical volume of radius r_o about the electron is considered inaccessible to the ions the ensemble average becomes

$$\left\langle \sum_j T_j (\underline{\underline{r}}_e - \underline{\underline{r}}_j) \underline{\underline{\mu}}_e \right\rangle = (N/V) \int_{V-\sigma} T(\underline{\underline{r}}_e - \underline{\underline{r}}) \underline{\underline{\mu}}_e d\underline{\underline{r}} = (4\pi/3) n_o \underline{\underline{\mu}}_e, \quad (14)$$

where σ is the small volume occupied by the electron. The ensemble average (14) is independent of this volume. We can now take the ensemble average of (11) and find

$$\langle \underline{\underline{\mu}}_e \rangle = \left[1 + (4\pi/3) n_o \alpha_o + \sum_{v=2}^{\infty} (n_o \alpha_o)^v \langle T^v \rangle \right] \alpha_o \underline{\underline{E}}. \quad (15)$$

In the Lorentz-Lorenz theory $\underline{\underline{\mu}}_e$ on the r.h.s. of Eq. (10) is first replaced by $\langle \underline{\underline{\mu}}_e \rangle$ and then an ensemble average of Eq. (10) is taken.

This gives

$$\begin{aligned}
\langle \underline{\mu}_e \rangle_L &= (1 - \alpha_o \langle \sum_j T_j \rangle)^{-1} \alpha \underline{E} = \\
&= [1 + (4\pi/3) n_o \alpha_o + \sum_{\nu=2}^{\infty} (4\pi n_o \alpha_o / 3)^{\nu}] \alpha_o \underline{E}. \quad (16)
\end{aligned}$$

The difference between the correct average (15) over the fluctuating ion field and the Lorentz-Lorenz result (16) has been studied by various authors^{15,16,17} for the case of neutral gases.¹⁸ It is quite significant if the product $n_o \alpha_o$ approaches unity. In neutral gases this product is always smaller than unity and positive if the frequency of $\underline{E}$ is smaller than the dominant absorption frequency. In a system of free electrons $\alpha_o n_o$ is negative and larger than unity if $\omega < (4\pi)^{\frac{1}{2}} \omega_p$. Thus, the difference between (15) and (16) should be even larger in ionized than in high density neutral gases if ω is near ω_p or smaller.

Equation (15) for the average radiation induced dipole moment $\underline{\mu}_e$ has a property which is harmless in the case of neutral gases but requires special consideration for a system of point charges: The terms in (15) with $\nu > 1$ depend on the radius r_o of the sphere occupied by the test particle at $\underline{r}_e$. If the test particle is a neutral atom, r_o may be identified with the atomic radius, or a radial distribution function with repulsive core may be used when performing the ensemble averages. However, if the test particle is an electron a repulsive core does not exist and the terms in (15) with $\nu > 1$ are infinite. Infinities of this type are quite common in systems of point charges and can be removed, without recourse to quantum mechanics, if the orbital motion of electrons about ions is explicitly taken into account.¹⁹ As far as the interaction between an electron and its nearest neighbor ion is concerned, this has been done by Darwin^{20,21,22} whose work will be briefly

reviewed in the next section. However, the motion of the electron affects also its interaction with the more distant ions and, particularly, the ensemble averages required in Eq. (15). For this reason, the dielectric theory of dense neutral gases which neglects the particle motion can not be applied to a plasma, and it is the main purpose of this paper to find the equivalent to Eq. (15) for a system of point charges with arbitrarily high electron temperature.

Throughout this paper only the ion-electron interaction will be considered. The effect of inner field fluctuations on the electron-electron interaction is probably much smaller since, in a first approximation, all electrons oscillate in phase. Consequently the polarization field produced by electrons at the site of a test electron should be nearly zero for almost all particle configurations.

The frequency range of interest in this paper is the neighborhood of the plasma frequency ω_p . For larger frequencies the effect of the fluctuating inner field becomes quite weak, and for smaller frequencies series of the form (15) converge very slowly. Since ω_p is much larger than the collision frequency for most plasmas of practical interest, the effect of hard collisions will be neglected.

II. THE DARWIN EFFECT

When studying the dielectric properties of a plasma Darwin has treated the interaction of a test electron with its nearest neighbor ion as a dynamical problem, and the interaction with all the other ions by the Lorentz-Lorenz method.^{20,21,22} Concerning the nearest neighbor

interaction the essential effect of the radiation induced electron oscillation is a change in the statistics of the electron-ion impact parameter p , as long as the radiation frequency ω is smaller than the reciprocal interaction time $\tau_i = p_{90}/v_o$. (p_{90} is impact parameter corresponding to 90° deflection, $v_o^2 = 2kT/m \approx$ average thermal velocity square). For a specific ion-electron encounter the radiation induced change in p leads to an extra force between ion and electron which is equivalent to an electrical field.

$$\Delta_1 \underline{\underline{\epsilon}}^{\sim} = T_1 \underline{\underline{\mu}}_e, \quad (17)$$

where $\underline{\underline{\mu}}_e$ is the radiation induced dipole moment of the electron before entering the ionic interaction zone. If the tensor T_1 is averaged over all directions and magnitudes of p one obtains an ensemble average which is independent of the electron speed v and satisfies the relation

$$\langle \Delta_1 \underline{\underline{\epsilon}}^i \rangle = \langle T_1 \rangle \underline{\underline{\mu}}_e = - (4\pi/3) n_o \underline{\underline{\mu}}_e. \quad (18)$$

The Darwin effect may be incorporated into the general dielectric theory by replacing the sum $\sum_j T_j \underline{\underline{\mu}}_e$ in the Eqs. (8) to (15) with the expression $(T_1 + \sum_{j=2}^N T_j) \underline{\underline{\mu}}_e$. This leads to a cancellation of the Lorentz-Lorenz term in (15):

$$\langle \underline{\underline{\mu}}_e \rangle = \left[1 + \sum_{\nu=2}^{\infty} (n_o \alpha_o)^\nu \langle (T_1' + T)^\nu \rangle \right] \alpha_o \underline{\underline{E}}, \quad (19)$$

where the prime on T_1' indicates that n_o^ν has been pulled out of the braces. Equation (19), with the sum over ν neglected, provides a justification for the generally accepted belief that the effective polarizing field $\underline{\underline{\epsilon}}$ in a plasma is equal to the average macroscopic field $\underline{\underline{E}}$.

The evaluation of the higher moments of T_1 is quite complicated

and will not be attempted in this paper. For this reason T_1' will be neglected in Eq. (19) and only the effect of the inner field fluctuations on the interaction between a test electron and the ions outside its nearest neighbor zone will be computed. This settles the problem of the radius r_σ of the sphere which is assumed inaccessible to the ions. The volume of this sphere will be assumed equal to the average volume n_0^{-1} available for the nearest neighbor ion which is excluded from consideration. Thus,

$$r_\sigma = (4\pi n_0/3)^{-\frac{1}{3}} \quad (20)$$

The exclusion of the auxiliary sphere σ about the test electron at $\underline{r}_e$ will be formally achieved by the following device: The ion density will be represented as a superposition of δ -functions of the form

$$n(\underline{r}) = (1/V) \sum_{\underline{q}} A_{\underline{q}} \exp(i \underline{q} \cdot \underline{r}), \quad (21)$$

$$A_{\underline{q}} = \sum_{j=1}^N \exp(-i \underline{q} \cdot \underline{r}_j), \quad (22)$$

$$\underline{q}_i = 2\pi h_i V^{-\frac{1}{3}}, \quad h_i = 0, \pm 1, \pm 2, \dots, \quad i = x, y, z. \quad (23)$$

If an upper limit

$$q_m = 1/r_\sigma = (4\pi n_0/3)^{\frac{1}{3}} \quad (24)$$

is imposed on the Fourier series (21) an uncertainty in the ion positions is introduced, i.e. $n(\underline{r})$ is peaked at the ion positions but the peaks are not point like and look like small spheres with effective radius r_σ . The ion charge is smeared out over the volume of this sphere and the electrical ion field is Coulombian for an observer outside the sphere,

but decreases linearly with r from the maximum value e/r_σ^2 for an observer inside σ . Thus, as far as Coulomb forces are concerned, the cut-off (24) effectively prevents an interaction between the test electron and its nearest ion neighbor, without affecting the force law for the more distant ions. A more rigorous method of excluding the nearest neighbor interaction will be outlined in the next section.

The parameter

$$\omega/q_m v_o \simeq \omega_p/q_m v_o \simeq r_\sigma/\lambda_D, \quad \lambda_D = \text{Debye length} \quad (25)$$

which will appear quite often in this work is very small in most plasmas of practical interest, and only the lowest power of this parameter will be kept in the formulas for the impedance. Because of this approximation Debye shielding will be neglected throughout this paper and the radial distribution function of ions about ions and ions about electrons will be put equal to unity.

III. THE POLARIZATION OF A FULLY IONIZED COLLISION FREE PLASMA

We consider a Lorentzian plasma with the ions at rest and the electrons performing a radiation induced oscillation superimposed on thermal motion. A test electron with thermal velocity $\underline{v}$ sees the ions in a time dependent position

$$\underline{r}_j = \underline{r}_j(0) - \underline{v} t, \quad (26)$$

corresponding to a time dependent particle density which may be obtained by substituting (26) into (21) and (22):

$$n(\underline{r}, t) = (1/V) \sum_{\underline{q}} A_{\underline{q}} \exp \left[i \underline{q} \cdot (\underline{r} + \underline{v} t) \right], \quad (27)$$

where $A_{\underline{q}}$ is given by (22) with $\underline{r}_j(0)$ replacing $\underline{r}_j$.

We assume now that the electron performs an oscillation with zero'th order amplitude

$$\underline{p}^{(0)}(\underline{r}_e, t) = (e/m\omega^2) \underline{E} \exp [i(\underline{k} \cdot \underline{r}_e + \omega t)], \quad (28)$$

which is generated, in the spirit of the iteration scheme (12), (13), by the average radiation field $\underline{E} \exp [i(\underline{k} \cdot \underline{r} + \omega t)]$. As explained on page ... the displacement is equivalent to a polarization

$$\underline{p}^{(0)}(\underline{r}, t) = -n_0(\underline{r}, t) e \underline{p}^{(0)}(\underline{r}_e, t). \quad (29)$$

Putting $\underline{r}_e$ equal to zero and utilizing (27) and (28)

$$\underline{p}^{(0)}(\underline{r}, t) = \sum_{\underline{q}} \underline{p}_{\underline{q}}^{(0)} \exp \{i[\underline{q} \cdot (\underline{r} + \underline{v} t) + \omega t]\}, \quad (30)$$

with

$$\underline{p}_{\underline{q}}^{(0)} = (\alpha_0/v) A_{\underline{q}} \underline{E}. \quad (31)$$

The average of (30) over all electron velocities and ion positions,

$$\langle \underline{p}^{(0)}(\underline{r}, t) \rangle_{\underline{v}} = n_0 \alpha_0 \underline{E} \exp(i\omega t), \quad (32)$$

has the frequency of the driving field since all terms in (30) with $\underline{q} \neq 0$ are eliminated by the averaging. (See A 1).

The effective polarizing field,

$$\begin{aligned} \underline{\underline{\epsilon}}(\underline{r}_e, t) &= \underline{E}^i(\underline{r}_e, t) + \Delta \underline{\underline{\epsilon}}_{\Sigma}^i(\underline{r}_e, t) + \Delta \underline{\underline{\epsilon}}^i(\underline{r}_e, t) = \\ &= \underline{E}(\underline{r}_e, t) + \Delta \underline{\underline{\epsilon}}^i(\underline{r}_e, t), \end{aligned} \quad (33)$$

is the sum of the incident radiation field $\underline{E}^i$, the field produced by the surface polarization $\underline{p}_{\Sigma}$, and the field produced by the volume polarization $\underline{p}$. The sum of the first two terms is, by definition, the

radiation field $\underline{E}$ inside the plasma. The polarization field at the electron site $\underline{r}_e$ may be obtained from the source distribution $\underline{P}^{(o)}(\underline{r}, t)$ by the integral

$$\Delta \underline{\underline{\epsilon}}^i(\underline{r}_e, t) + \Delta \underline{\underline{\epsilon}}^i(\underline{r}_e, t) = \int_{\underline{\sigma}} (\nabla \times \nabla \times)_{\underline{r}_e} \sum_{\underline{q}} (\underline{P}_{\underline{q}}^{(o)} / |\underline{r}_e - \underline{r}|) d\underline{r} \exp \{i[\underline{q} \cdot (\underline{r} + \underline{v}t) + \omega t]\}, \quad (34)$$

where $\sum$ and $\underline{\sigma}$ denotes respectively the surface of the macroscopic gas volume V and of the auxiliary sphere around $\underline{r}_e$ which excludes the electron site from the source distribution. It is convenient to interchange the order of differentiation and integration in Eq. (34). Because of the lower limit $\underline{\sigma}$ this interchange requires ²³ to subtract the term

$[8\pi \underline{P}(\underline{r}_e, t)/3]$ from Eq. (34) and one obtains as a first order solution for $\underline{\underline{\epsilon}}$

$$\underline{\underline{\epsilon}}^{(1)}(\underline{r}_e, t) = \underline{E}^i(\underline{r}_e, t) + \sum_{\underline{q}} \left\{ (\nabla \times \nabla \times)_{\underline{r}_e} \int_{\underline{\sigma}} [\underline{P}_{\underline{q}}^{(o)}(\underline{r}, t)] / |\underline{r}_e - \underline{r}| d\underline{r} - (8\pi/3) \underline{P}_{\underline{q}}^{(o)}(\underline{r}_e, t) \right\}. \quad (35)$$

Eq. (35) implies that the radius r_{σ} of the auxiliary sphere tends to zero.

If r_{σ} is given the magnitude (20) the rapidly varying Fourier components with $q > q_m = r_{\sigma}^{-1}$ do not contribute to the quantity, $\sum_{\underline{q}} (8\pi/3) \underline{P}_{\underline{q}}^{(o)}(\underline{r}_e, t)$ and similar cancellations occur in other parts of the following formalism. The net result is that exclusion of the nearest neighbor zone is, in fact, equivalent to the cut-off (24).

The volume integral (35) may be transformed into a surface integral by means of the auxiliary relations

$$\nabla_r^2 (1/|\underline{r}_e - \underline{r}|) = 0, \quad (36)$$

$$\nabla_r^2 \underline{P}_q^{(0)}(\underline{r}) + q^2 \underline{P}_q^{(0)}(\underline{r}) = 0, \quad (37)$$

$$\begin{aligned} & \left[\underline{P}_q^{(0)} / |\underline{r}_e - \underline{r}| \right] = \\ & = q^{-2} \left[\underline{P}_q \nabla_r^2 (1/|\underline{r}_e - \underline{r}|) - (1/|\underline{r}_e - \underline{r}|) \nabla_r^2 \underline{P}_q^{(0)} \right]. \end{aligned} \quad (38)$$

One can now apply Green's theorem to get

$$\begin{aligned} & \int_{\underline{\Sigma}} \left[\underline{P}_q^{(0)} / |\underline{r}_e - \underline{r}| \right] d\underline{r} = q^{-2} \left\{ \int_{\underline{\Sigma}} \left[\underline{P}_q^{(0)} \nabla_r (1/|\underline{r}_e - \underline{r}|) - (1/|\underline{r}_e - \underline{r}|) \nabla_r \underline{P}_q^{(0)} \right] \cdot d\underline{\Sigma} \right. \\ & \left. - \int_{\underline{\sigma}} \left[\underline{P}_q^{(0)} \nabla_r (1/|\underline{r}_e - \underline{r}|) - (1/|\underline{r}_e - \underline{r}|) \nabla_r \underline{P}_q^{(0)} \right] \cdot d\underline{\sigma} \right\}. \end{aligned} \quad (39)$$

The integral over the outer surface $\underline{\Sigma}$ represents the field of the surface polarization which combined with $\underline{E}^i$ gives $\underline{E}$. The integral over the auxiliary surface $\underline{\sigma}$ gives $4\pi \underline{P}_q^{(0)}(\underline{r}_e)$. Again, if r_σ was given its value (20), Fourier components with $q > q_m = r_\sigma^{-1}$ would not contribute to the $\underline{\sigma}$ -integral. Substituting (39) into (35) one obtains a local relation between field and polarization of the form

$$\underline{\Sigma}^{(1)}(\underline{r}_e, t) = \sum_q \{ \nabla_x \nabla_x r_e [4\pi q^{-2} \underline{P}_q^{(0)}(\underline{r}_e, t)] - (8\pi/3) \underline{P}_q^{(0)}(\underline{r}_e, t) \} + \underline{E}(\underline{r}_e, t). \quad (40)$$

The space derivatives can be removed by means of the relations

$$\nabla_x \nabla_x \underline{P}_q^{(0)} = \nabla \nabla \cdot \underline{P}_q^{(0)} - \nabla^2 \underline{P}_q^{(0)}, \quad (41)$$

$$\nabla \nabla \cdot \underline{P}_q^{(0)} = -\underline{q}(\underline{q} \cdot \underline{P}_q^{(0)}) = -(\underline{q} \underline{q}) \underline{P}_q^{(1)}, \quad (42)$$

and Eq. (37). Thus

$$\begin{aligned}\underline{\varepsilon}^{(1)}(\underline{r}_e, t) &= \sum_{\underline{q}} [4\pi(I - \hat{q} \hat{q}) \underline{P}_{\underline{q}}^{(o)}(\underline{r}_e, t) - (8\pi/3) \underline{P}_{\underline{q}}^{(o)}(\underline{r}_e, t)] + \underline{E}(\underline{r}_e, t) \\ &= \sum_{\underline{q}} \underline{\Gamma}(\underline{q}) \underline{P}_{\underline{q}}^{(o)} \exp \{i[\underline{q} \cdot \underline{r}_e + (\underline{q} \cdot \underline{v} + \omega) t]\} + \underline{E}(\underline{r}_e, t),\end{aligned}\quad (43)$$

$$\text{where } \underline{\Gamma}(\underline{q}) = (4\pi/3) (I - 3 \hat{q} \hat{q}) = \underline{\Gamma}(-\underline{q}), \quad (44)$$

I is the unit tensor, $\hat{q}$ is a unit vector in direction of $\underline{q}$, and $\hat{q} \hat{q}$ is a dyadic product.

We shall be dealing later with averages $\langle \underline{\Gamma}(\underline{q}) \rangle_{\Omega}$ and $\langle \underline{\Gamma}^2(\underline{q}) \rangle_{\Omega}$ of the tensors $\underline{\Gamma}(\underline{q})$ and $\underline{\Gamma}^2(\underline{q})$ over all directions in $\underline{q}$ -space, with the magnitude q held constant. Using the familiar relations for the averages of products of direction cosines

$$\begin{aligned}\langle \hat{q}_i \rangle_{\Omega} &= 0, \quad \langle \hat{q}_i \hat{q}_j \rangle_{\Omega} = 0, \quad \langle \hat{q}_i^2 \rangle_{\Omega} = 1/3, \\ \langle \hat{q}_i^4 \rangle_{\Omega} &= 1/5, \quad \langle \hat{q}_i^2 \hat{q}_j^2 \rangle_{\Omega} = 1/15,\end{aligned}\quad (45)$$

one finds

$$\langle \underline{\Gamma}(\underline{q}) \rangle_{\Omega} = 0, \quad (46)$$

$$\langle [\underline{\Gamma}^2(\underline{q})]_{ij} \rangle_{\Omega} = (32\pi^2/9) \delta_{ij}, \quad (47)$$

$$\underline{\Gamma}(0) = 4\pi/3. \quad (48)$$

The average of (43) over all electron velocities and ion configurations is obtained from (31), (32), and (A 1), as

$$\langle \underline{\varepsilon}^{(1)} \rangle_{\underline{v}} = \underline{\Gamma}(0) \langle \underline{P}_o^{(o)} \rangle_{\underline{v}} + \underline{E} = (4\pi/3) \underline{P}_o^{(o)} + \underline{E}; \quad (49)$$

it has the time dependence of the driving field and fluctuation corrections are not yet present.

The first order solution $\underline{P}^{(1)}(\underline{r}, t)$ can be obtained by calculating

the electronic displacement $\underline{\rho}^{(1)}(\underline{r}_e, t)$ produced by $\underline{\xi}^{(1)}$. Since $\underline{\xi}^{(1)}(t)$ has the form (putting $\underline{r}_e$ equal to zero)

$$\underline{\xi}^{(1)}(t) = \sum_{\underline{q}} \underline{\xi}_{\underline{q}}^{(1)} \exp [i(\underline{q} \cdot \underline{v} + \omega) t], \quad (50)$$

with

$$\underline{\xi}_{\underline{q}}^{(1)} = \Gamma(\underline{q}) \underline{p}_{\underline{q}}^{(0)} + \underline{E} \delta(\underline{q}), \quad (51)$$

we write $\underline{\rho}^{(1)}(t)$ as a Fourier series of the form

$$\underline{\rho}^{(1)}(t) = \sum_{\underline{q}} \underline{\rho}_{\underline{q}}^{(1)} \exp [i(\underline{q} \cdot \underline{v} + \omega) t], \quad (52)$$

and insert it into the equation of motion

$$m \ddot{\underline{\rho}}^{(1)}(t) = -e \underline{\xi}^{(1)}(t). \quad (53)$$

This gives

$$\underline{\rho}_{\underline{q}}^{(1)} = [e/m (\omega + \underline{q} \cdot \underline{v})^2] \underline{\xi}_{\underline{q}}^{(1)} \quad (54)$$

and

$$\begin{aligned} \underline{p}_{\underline{r}, t}^{(1)} &= -n(\underline{r}, t) e \underline{\rho}^{(1)}(t) = \\ &\left\{ \sum_{\underline{q}} \sum_{\underline{q}'} \underline{p}_{\underline{q}, \underline{q}'}^{(1)} + \sum_{\underline{q}} \underline{p}_{\underline{q}}^{(0)} \delta(\underline{q}') \right\} \exp \{ i[\underline{q} \cdot \underline{r} + (\underline{q} + \underline{q}') \cdot \underline{v} t + \omega t] \}, \end{aligned} \quad (55)$$

with

$$\underline{p}_{\underline{q}, \underline{q}'}^{(1)} = (\alpha_o/V)^2 \underline{A}_{\underline{q}} \underline{A}_{\underline{q}'} \Gamma(\underline{q}') (1 + \underline{q}' \cdot \underline{v}/\omega)^{-2} \underline{E}. \quad (56)$$

In deriving (56) use has been made of the Eqs. (54, (51), (31), (27) and (3). Taking an ensemble average over all ion positions

and using (A 5) gives

$$\langle \underline{P}^{(1)} \rangle = [(4\pi/3) n_o \alpha_o + 1] n_o \alpha_o \underline{E} \delta(\underline{q}) + (n_o \alpha_o^2 / v) \sum_{\underline{q} \neq 0} \underline{\Gamma}(\underline{q}) (1 - \underline{q} \cdot \underline{v} / \omega)^{-2} \underline{E}. \quad (57)$$

If (57) is averaged over all $\underline{v}$ the quantity $\langle (1 - \underline{q} \cdot \underline{v} / \omega)^{-2} \rangle_{\underline{v}}$ does not depend on the direction of $\underline{q}$ and all terms with $\underline{q} \neq 0$ are eliminated from (57) because of (46). The time dependence of the remaining term ($\underline{q} = \underline{q}' = 0$) is that of the driving field and fluctuation corrections do not yet show up. Thus, we have to investigate the second order polarization $\langle \underline{P}^{(2)} \rangle_{\underline{v}}$.

The step from $\underline{P}^{(1)}$ to $\underline{\varepsilon}^{(2)}$ is done by the same technique as the step from $\underline{P}^{(0)}$ to $\underline{\varepsilon}^{(1)}$. One gets (with $\underline{r}_e = 0$)

$$\underline{\varepsilon}^{(2)}(t) = \sum_{\underline{q}} \sum_{\underline{q}'} \underline{\varepsilon}_{\underline{q}, \underline{q}'}^{(2)} \exp \{ i [(\underline{q}, \underline{q}') \cdot \underline{v} t + \omega t] \}, \quad (58)$$

with

$$\underline{\varepsilon}_{\underline{q}, \underline{q}'}^{(2)} = \underline{\Gamma}(\underline{q}) P_{\underline{q}, \underline{q}'}^{(1)} + \underline{E} \delta(\underline{q}) \delta(\underline{q}') \quad (59)$$

$\underline{\varepsilon}^{(2)}(t)$ generates an electron displacement

$$\underline{\delta}^{(2)}(t) = \sum_{\underline{q}} \sum_{\underline{q}'} \underline{\delta}_{\underline{q}, \underline{q}'}^{(2)} \exp \{ i [(\underline{q} + \underline{q}') \cdot \underline{v} t + \omega t] \}, \quad (60)$$

where

$$\underline{\delta}_{\underline{q}, \underline{q}'}^{(2)} = \{ e/m \omega^2 [1 + (\underline{q} + \underline{q}') \cdot \underline{v} / \omega]^2 \} \underline{\varepsilon}_{\underline{q}, \underline{q}'}^{(2)} \quad (61)$$

and $\underline{\rho}^{(2)}(t)$ is equivalent to a polarization

$$P^{(2)}(\underline{r}, t) = -n(\underline{r}, t) e g^{(2)}(t) = \left\{ \sum_{\underline{q}} \sum_{\underline{q}'} \sum_{\underline{q}''} P_{\underline{q}, \underline{q}', \underline{q}''}^{(2)} + \right. \quad (62)$$

$$\left. + \sum_{\underline{q}} \sum_{\underline{q}'} P_{\underline{q}, \underline{q}'}^{(1)} \delta(\underline{q}'') + \sum_{\underline{q}} P_{\underline{q}}^{(0)} \delta(\underline{q}') \delta(\underline{q}'') \right\} \exp \left\{ i [\underline{q} \cdot \underline{r} + (\underline{q} + \underline{q}' + \underline{q}'') \cdot \underline{v} t + \omega t] \right\},$$

with

$$P_{\underline{q}, \underline{q}', \underline{q}''}^{(2)} = (\alpha_0/v)^3 A_{\underline{q}} A_{\underline{q}'} A_{\underline{q}''} \Gamma(\underline{q}') \Gamma(\underline{q}'') [1 + (\underline{q}' + \underline{q}'') \cdot \underline{v}/\omega]^{-2} E. \quad (63)$$

According to (A 7) the ensemble average of $P_{\underline{q}, \underline{q}', \underline{q}''}^{(2)}$ is zero unless $\underline{q} + \underline{q}' + \underline{q}'' = 0$. Hence, the ensemble average

$$\langle P^{(2)} \rangle_{\underline{v}} = \left\langle \sum_{\underline{q}} \sum_{\underline{q}'} \sum_{\underline{q}''} P_{\underline{q}, \underline{q}', \underline{q}''}^{(2)} \right\rangle_{\underline{v}} + [(4\pi/3) n_0 \alpha_0 + 1] n_0 \alpha_0 E \quad (64)$$

has the same time dependence as the driving field. The triple sum in (61) has the form (See A7)

$$\begin{aligned} \sum_{\underline{q}} \sum_{\underline{q}'} \sum_{\underline{q}''} \langle P_{\underline{q}, \underline{q}', \underline{q}''}^{(2)} \rangle_{\underline{v}} &= (16\pi^2/q) n_0^3 \alpha_0^3 E \\ &+ (\alpha_0^3 n_0^2/v) \left[\sum_{\underline{q} \neq 0} \Gamma(\underline{q})^2 + \sum_{\underline{q}' \neq 0} \Gamma(\underline{q}') \Gamma(0) \langle (1 + \underline{q}' \cdot \underline{v}/\omega)^{-2} \rangle_{\underline{v}} + \right. \\ &\quad \left. + \sum_{\underline{q}'' \neq 0} \Gamma(0) \Gamma(\underline{q}'') \langle (1 + \underline{q}'' \cdot \underline{v}/\omega)^{-2} \rangle_{\underline{v}} \right] E \\ &+ (\alpha_0^3 n_0/v^2) \sum_{\underline{q} \neq 0} \sum_{\underline{q}' \neq 0} \Gamma(\underline{q}') \Gamma(\underline{q} + \underline{q}') \langle (1 - \underline{q} \cdot \underline{v}/\omega)^{-2} \rangle_{\underline{v}} E. \end{aligned} \quad (65)$$

The first term in (65) is a member of the uncorrected Lorentz-Lorenz series (16); all the other terms represent the fluctuation correction. Since $\Gamma(0)$ is a constant the sums over $\underline{q}'$ and $\underline{q}''$ in the square bracket of (65) involve terms which are linear in $\Gamma(\underline{q}')$ and $\Gamma(\underline{q}'')$ respectively. Consequently, these sums vanish for reasons which have been explained in connection with Eq. (57). Similar cancellations occur in the double sum of (65) if the product $\Gamma(\underline{q}') \Gamma(\underline{q} + \underline{q}')$ is expanded according to the rules of tensor multiplication:

$$\Gamma(\underline{q}) \Gamma(\underline{q} + \underline{q}') = \Gamma^2(\underline{q}) - (4\pi/3) \Gamma(\underline{q}) \hat{q}' \hat{q}' - (8\pi/3) \hat{q} \hat{q}'. \quad (66)$$

The second term cancels on summation over $\underline{q}$ and the third on summation over $\underline{q}'$. Thus, the fluctuation correction becomes

$$\begin{aligned} \sum_{\underline{q} \neq 0} \sum_{\underline{q}' \neq 0} \sum_{\underline{q}'' \neq 0} \left\langle \frac{P^{(2)}_{\underline{q}, \underline{q}', \underline{q}''}}{\underline{v}} \right\rangle_{\underline{v}} &= (\alpha_0^3 n_0^2 / V) \sum_{\underline{q} \neq 0} \Gamma^2(\underline{q}) \underline{E} \\ &+ (\alpha_0^3 n_0 / V^2) \sum_{\underline{q} \neq 0} \sum_{\underline{q}' \neq 0} \Gamma^2(\underline{q}) \langle (1 + \underline{q} \cdot \underline{v} / \omega)^{-2} \rangle_{\underline{v}} \underline{E}. \end{aligned} \quad (67)$$

The evaluation of the average over all $\underline{v}$ is familiar from the problem of Landau damping. One is dealing with the integral^{24,25}

$$\begin{aligned} \underline{I}(\underline{q}) &= \omega^2 \int_{-\infty}^{\infty} \frac{f(u) du}{(\omega + \underline{q} u)^2} = \omega^2 \left[-\frac{f(u)}{\underline{q}(\omega + \underline{q} u)} \Big|_{-\infty}^{\infty} + \frac{1}{\underline{q}} \int_{-\infty}^{\infty} \frac{f'(u) du}{\omega + \underline{q} u} \right] = \\ &= P \frac{\omega^2}{\underline{q}} \int_{-\infty}^{\infty} \frac{f'(u) du}{\omega + \underline{q} u} + \frac{i\pi \omega^2}{\underline{q}^2} f'(\omega/\underline{q}), \end{aligned} \quad (68)$$

where

$$f(u) = (\pi^{1/2}/v_0) \exp[-(u/v_0)^2], \quad (69)$$

is Maxwell's velocity distribution for a component u of $\underline{v}$, $\mathcal{P}$ means principal value, and $f'(u) = df(u)/du$. When dealing with Landau damping it is usually assumed that $qu < \omega$. According to Eq. (25) this assumption is not admissible in the present work and the usual expansion of $(\omega + qu)^{-1}$ about $u = 0$ does not converge. Thus, another approximate procedure for evaluating $\mathcal{P}$ must be chosen: Expand

$$\begin{aligned} f'(u) &= -(2\pi^{1/2}u/v_0^3) \exp[-(u/v_0)^2] \approx \\ &\approx -(2\pi^{1/2}/v_0^3)(u - u^3/v_0^2) \quad \text{if } u < v_0 \\ &\approx 0 \quad \text{if } u > v_0, \end{aligned} \quad (70)$$

introduce the variable $\omega + qu = x$, and write

$$\begin{aligned} \mathcal{P} \frac{\omega^2}{q} \int_{-\infty}^{\infty} \frac{f'(u) du}{\omega + qu} &\approx \frac{-2\pi^{1/2}\omega^2}{q^3 v_0^3} \mathcal{P} \int_{\omega - qv_0}^{\omega + qv_0} \frac{[x - \omega - (1/v_0^2 q^2)(x^3 - 3x^2\omega + 3x\omega^2 - \omega^3)] dx}{x} \\ &= -\frac{2\pi^{1/2}\omega^2}{q^3 v_0^3} \left[\left(\frac{\omega^3}{v_0^2 q^2} - \omega \right) \log \frac{(qv_0 + \omega)}{(qv_0 - \omega)} + \frac{4qv_0}{3} - \frac{2\omega^2}{qv_0} \right]. \end{aligned} \quad (71)$$

Equation (71) can be further simplified if the assumption is made, to be justified by Eq. (25), that $qv_0 \gg \omega_p \approx \omega$ for the vast majority of q . In this case, one may employ the expansion

$$\lg \frac{(qv_0 + \omega)}{(qv_0 - \omega)} = \frac{2\omega}{qv_0} - \frac{\omega^3}{q^3 v_0^3} + \dots, \quad (72)$$

and finds

$$P \frac{\omega^2}{q} \int_{-\infty}^{\infty} \frac{\delta'(u) du}{\omega + qu} = - \frac{8\pi^{1/2} \omega^2}{3q^2 v_0^2} \left(1 - \frac{3\omega^2}{q^2 v_0^2} + \dots \right). \quad (73)$$

Keeping only the terms of lowest order in (ω/qv_0) ,

$$I(q) = - \left(8\pi^{1/2} \omega^2 / 3q^2 v_0^2 \right) - i \left(2\pi^{3/2} \omega^3 / q^3 v_0^3 \right) \exp \left[-(\omega/qv_0)^2 \right]. \quad (74)$$

It remains to carry out the summations over $\underline{q}$ and $\underline{q}'$ in Eq. (67).

In a macroscopic system the vectors $\underline{q}$ as defined by Eq. (23) vary almost continuously and $\sum_{\underline{q}}$ may be replaced by the integral $(V/8\pi^3) \int d\underline{q} = (V/2\pi^2) \int q^2 dq$ if the integrand depends only on the magnitude of $\underline{q}$.

Thus, utilizing (47) and (24)

$$(1/V) \sum_{\underline{q} \neq 0} \Gamma_{(\underline{q})}^2 = (16/27) \int_0^{q_m} q^2 dq = 0.8 n_0. \quad (75)$$

The exclusion of $\underline{q} = 0$ is irrelevant for the integral in (75). It is however important in Eq. (65) since the statistics of the Fourier coefficients $A_{\underline{q}}$ depends on whether $\underline{q} = 0$ or $\underline{q} \neq 0$. (See Appendix).

$$\begin{aligned} (1/V) \sum_{\underline{q}} \sum_{\underline{q}'} \Gamma_{(\underline{q})}^2 I(q) &= (8/81\pi^2) \int_0^{q_m} \int_0^{q'_m} q^2 I(q) q'^2 dq dq' = \\ &= 0.8 n_0 \int_0^{q_m} q^2 I(q) dq. \end{aligned} \quad (76)$$

The real part of the last integral is

$$\int_0^{q_m} q^2 \operatorname{Re} I(q) dq = -\frac{8\pi^{1/2}\omega^2}{3v_o^2} \int_0^{q_m} dq = -7.6 n_o (\omega^2/v_o^2 n_o^{2/3}). \quad (77)$$

The imaginary part may be obtained from the expansion

$$\begin{aligned} \exp[-(\omega/q v_o)^2] &\approx 1 - (\omega/q v_o)^2 \text{ if } q > \omega/v_o, \\ &\approx 0 \text{ if } q < \omega/v_o. \end{aligned} \quad (78)$$

Neglecting again the term of second order in $(\omega/q v_o)$

$$\begin{aligned} \int_0^{q_m} q^2 \operatorname{Im} I(q) dq &\approx - (2\pi^{3/2} \omega^3/v_o^3) \int_{\omega/v_o}^{q_m} dq/q = \\ &= -11.2 n_o (\omega^3/v_o^3 n_o). \end{aligned} \quad (79)$$

The integrals (75), (77), and (79) may be substituted into (67), and

(67) may be combined with (65) and (64) to give a formula of the form (5).

It is then seen that the first two coefficients in this expansion are

$$C_1 = 0, \quad (80)$$

$$C_2 = 0.8 [1 - 7.6 (\omega/v_o n_o^{1/3})^2 - i 11.2 (\omega/v_o n_o^{1/3})^3].$$

The expansion (5) converges reasonably fast as long as $\omega > \omega_p/3$.

In this frequency domain the coefficients C_1 and C_2 give a fairly accurate description of the plasma impedance (6) as a function of

frequency, temperature, and ion density. The effect of the inner field

fluctuations is seen to be quite significant and leads to a frequency dependence of the impedance Z which is quite different from that of the uncorrected quantity Z_1 . The magnitude of the correction terms may be obtained from the Eqs. (3) and (25).

APPENDIX

THE STATISTICS OF THE FOURIER COEFFICIENTS OF THE PARTICLE DENSITY

According to Eq. (22) the ensemble average of $A_{\underline{q}}$ over all ion positions $\underline{r}_j$ is

$$\begin{aligned} \langle A_{\underline{q}} \rangle &= (N/V) \int_V \exp(-i \underline{q} \cdot \underline{r}) d\underline{r} = N \text{ if } \underline{q} = 0, \\ &= 0 \text{ if } \underline{q} \neq 0. \end{aligned} \quad (\text{A-1})$$

This is true only in an ideal gas, i.e., for a system for which the probability of finding a particle in the volume element $d\underline{r}$ is $(d\underline{r}/V)$ independent of the position of $d\underline{r}$.

$$\begin{aligned} \langle A_{\underline{q}} A_{\underline{q}'} \rangle &= \left\langle \sum_{\underline{j}} \sum_{\underline{j}'} \exp \left[i \left(\underline{q} \cdot \underline{r}_{\underline{j}} + \underline{q}' \cdot \underline{r}_{\underline{j}'} \right) \right] \right\rangle \\ &= \left\langle \sum_{\underline{j}} \sum_{\underline{j}'} \exp \left\{ i \left[\left(\underline{q} + \underline{q}' \right) \cdot \underline{r}_{\underline{j}} + \underline{q}' \cdot \underline{r}_{\underline{j}, \underline{j}'} \right] \right\} \right\rangle, \end{aligned} \quad (\text{A-2})$$

where

$$\underline{r}_{\underline{j}, \underline{j}'} = \underline{r}_{\underline{j}'} - \underline{r}_{\underline{j}}. \quad (\text{A-3})$$

The sum over j introduces the factor N , and in the sum over j' , the term with $j = j'$ may be written separately. Thus,

$$\langle A_{\underline{q}} A_{\underline{q}'} \rangle = N \left\langle \exp[i(\underline{q} + \underline{q}') \cdot \underline{r}_j] \left[1 + \sum_{j' \neq j} \exp(i \underline{q}' \cdot \underline{r}_{j,j'}) \right] \right\rangle. \quad (\text{A-4})$$

Each term in the sum over j' vanishes on averaging unless $\underline{q}' = 0$.

Hence

$$\begin{aligned} \langle A_{\underline{q}} A_{\underline{q}'} \rangle &= N^2 \text{ if } \underline{q} = \underline{q}' = 0, \\ &= N \text{ if } \underline{q} + \underline{q}' = 0, \underline{q}' \neq 0, \\ &= 0 \text{ otherwise.} \end{aligned} \quad (\text{A-5})$$

$$\begin{aligned} \langle A_{\underline{q}} A_{\underline{q}'} A_{\underline{q}''} \rangle &= \left\langle \sum_j \sum_{j'} \sum_{j''} \exp[i(\underline{q} \cdot \underline{r}_j + \underline{q}' \cdot \underline{r}_{j,j'} + \underline{q}'' \cdot \underline{r}_{j,j''})] \right\rangle \\ &= N \left\langle \sum_{j'} \sum_{j''} \exp\{i[(\underline{q} + \underline{q}' + \underline{q}'') \cdot \underline{r}_{j,j'} + (\underline{q}' + \underline{q}'') \cdot \underline{r}_{j,j''} + \underline{q} \cdot \underline{r}_{j,j''}]\} \right\rangle. \end{aligned} \quad (\text{A-6})$$

Again, one separates the terms with $j' = j$, and $j'' = j'$ from the sums over j' and j'' and finds

$$\begin{aligned} \langle A_{\underline{q}} A_{\underline{q}'} A_{\underline{q}''} \rangle &= N^3 \text{ if } \underline{q} = \underline{q}' = \underline{q}'' = 0, \\ &= N^2 \text{ if } \{ \underline{q} = 0, (\underline{q}' + \underline{q}'') = 0 \}, \\ &= N \text{ if } \underline{q} + \underline{q}' + \underline{q}'' = 0, \underline{q}, \underline{q}', \underline{q}'' \neq 0, \\ &= 0 \text{ otherwise.} \end{aligned} \quad (\text{A-7})$$

The curly bracket in the condition for the ensemble average N^2 indicates

that there are three cases compatible with this condition obtained by cyclic permutation of q, q', q'' .

If the particle system is not ideal the ensemble averages (A-1), (A-4), and (A-6) depend on the Fourier transforms of the molecular distribution functions. For example, let

$$g^{\pm}(r) dr = (1/V) \exp[-U^{\pm}(r)/kT] dr \approx (1/V) [1 - U^{\pm}(r)/kT] \quad (A-8)$$

be the probability that an ion is in a volume element $d\mathbf{r}$ in distance r from the test electron at $\mathbf{r}_e$. For a plasma $U^{\pm}(r)$ is approximately equal to the Debye-Hückel potential:

$$U^{\pm}(r) = -(e^2/r) \exp(-r/\lambda_D). \quad (A-9)$$

The integrand of (A-1) must now be multiplied with the statistical weight $g^{\pm}(r)$ and one gets

$$\begin{aligned} \langle A_q \rangle &= \int_V g^{\pm}(r) \exp(-i\mathbf{q} \cdot \mathbf{r}) d\mathbf{r} = N \text{ if } \mathbf{q} = 0, \\ &= q^2 \lambda_D^2 / (1 + q^2 \lambda_D^2) \text{ if } \mathbf{q} \neq 0. \end{aligned} \quad (A-10)$$

Analogous corrections to (A-5) and A-7) could be readily derived but are not required in this paper.

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