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DYNAMIC ANALYSIS OF LIQUID-METAL-COOLED FAST POWER REACTORS

by

J. A. DeShong, Jr.

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Reactor Physics Division

January 1969

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ABSTRACT

The small-signal dynamic behavior of liquid-metal-cooled fast power reactors is analyzed by means of feedback control theory.

Transfer functions to calculate dynamic feedback reactivity generated by fuel expansion, sodium void effect, and thermal motion of control rods are developed, as well as a computer program to calculate dynamic feedback reactivity produced by thermomechanical distortion and movement of the core fuel elements. A theoretical block diagram of the EBR-II reactor system is constructed from these elements.

I. INTRODUCTION

The dynamic behavior of liquid-metal-cooled fast reactors is sensitive to many factors that influence reactivity, such as sodium void effect, axial thermal expansion of fuel, heat transfer from fuel to coolant, Doppler effect in large cores, and the radial movement and distortion of fuel subassemblies due to thermal gradients in the core. This report describes the results of a study made of a specific reactor for the purpose of developing a theoretical description of the reactor core dynamics.

The following sections develop in dynamic form the linear components of feedback reactivity by using heat-transport considerations. The linear treatment is suitable for everything but the structural distortion, where the structural component movements all interact on each other in a complex way. The latter case is attacked by numerical analysis using a computer program, BOW III, developed to analyze a specific reactor core structure as an example. Static heat-transfer calculations were made by Cushman to develop radial differential temperature profiles of the fuel subassembly.¹ These profiles are used as input to the BOW III program to calculate the static power-reactivity coefficient, which is defined as the reactivity generated by structural movement due to a specific change of reactor operating power from one equilibrium value to another.

The dynamic reactivity change due to structural motion is obtained by making heat-transfer calculations to determine dynamic radial and axial temperature response to a rod-drop reactor power-time function. The calculated dynamic temperatures are used to obtain the differential temperature profiles at specified times over the time range of interest. These profiles then are input to the BOW III program to determine the structural positions at the specified times. The dynamic motion of the structure in response to a rapid change in reactor power could also be influenced by mechanical inertia, spring constants, and damping. However, for the chosen example, the motions and therefore the inertial forces are small, while the spring constants expressed in force per unit distance are very large. Therefore the mechanical movements are assumed to follow the temperature changes with no time lag.

All the foregoing work is combined to develop a system block diagram to represent the theoretical core dynamics of the EBR-II reactor.

II. LINEAR REACTOR CORE DYNAMICS

A. Heat Transfer of Core Fuel Element

The individual fuel element is taken as a right circular cylinder, divided into two annular regions of equal volume, shown as ① and ② in

Fig. 1. Cross Section of Fuel Element

Fig. 1, which is a cross section of the cylinder. These are surrounded by another annular region of coolant, shown as ③. Then D_2 , the outside diameter of the inner volume, is 0.707 times D_4 , which is the outside diameter of the outer volume. The heat capacity of each volume can be lumped at $D_1 = 0.5D_4$ for the inner volume and $D_3 = 0.866D_4$ for the outer volume. The heat generated in each volume is taken as originating at these same diameters. R_1 , then, is the reciprocal conductivity from the inner to the outer volume, and R_2 is the reciprocal conductivity from the outer volume to the coolant surface, including any film effect.

R_3 is the reciprocal conductivity from the fuel-element surface radially to the center of mass of the coolant. These are reciprocal conductivities evaluated using the fuel-element length (L).

Fig. 2. Heat-flow Diagram for Fuel Element

Figure 2 shows the above quantities assembled into a heat-flow diagram. The lumped heat capacity of each half-volume is designated C_1 , and the heat generation rate in each half-volume as $\dot{Q}/2$ calories per second. The coolant heat capacity is represented by C_{Na} , and the heat deposition rate into the coolant by $\dot{q}$. The heat transport

rate out of the core, as a result of coolant flow, determines R_{VR} . All of these are evaluated based on the fuel-element length, L . Average fuel temperature over length L is θ_F ; average coolant temperature over L is θ_C . In this study, heat transport time in the coolant is negligibly small compared to the basic fuel-assembly time constant, since flow velocities are high. The above temperatures, therefore, are those at the core centerline.

B. Coolant Void Effect

The dynamic void reactivity produced by the thermal expansion of the liquid-metal coolant can be expressed in the Laplace transform as

$$\text{Laplace}[F(K\theta_C/\dot{Q})] = \rho_c K_c \times \text{Laplace}[F(\theta_C/\dot{Q})],$$

where ρ_c is the reactivity change per unit density change, and K_c is the coolant density change per unit coolant temperature change. The transform quantity on the right of the above equation is the solution to the heat-flow diagram of Fig. 2. If C_{Na} is small and can be neglected as in the case considered here, the solution is a quadratic expression of the form

$$\frac{\dot{q}(s)}{\dot{Q}(s)} = \frac{1 + \frac{sR_1C_1}{2}}{1 + s(R_1 + 2R_4)C_1 + s^2R_1R_4C_1^2},$$

where

$$R_4 = R_2 + R_3 + R_{VR}.$$

Then,

$$\theta_C = \dot{q}R_{VR},$$

and

$$\text{Laplace}[F(K\theta_C/\dot{Q})] = \frac{\rho_c K_c R_{VR} \left(1 + \frac{sR_1C_1}{2}\right)}{1 + s(R_1 + 2R_4)C_1 + s^2R_1R_4C_1^2}.$$

This expression is used in Appendix A to calculate the dynamic coolant temperature response for rows 1-6, 9, and 10-15 of the EBR-II reactor. Since the value of ρ_c is very small for rows outside the core, the solution for core rows 1-6 was chosen to represent the dynamic coolant void effect in Section IV.A of this report. The solutions for rows 9 and 10-15 are used later in Section III.B to develop the dynamic structural reactivity. The solutions of rows 7 and 8 require consideration of radial heat flow as well, but, since rows 7 and 8 have only minor coolant void effect, their solution will be given in Section III.B, where they have a large influence on the structural reactivity component.

C. Fuel Axial-expansion Effect

The fuel axial-expansion effect is considerable with metallic fuel such as that used in EBR-II and similar reactors. The dynamic fuel expansion can be expressed in Laplace transform as

$$\text{Laplace}[F(K\theta_F/\dot{Q})] = \rho_F K_F \times \text{Laplace}[F(\theta_F/\dot{Q})],$$

where ρ_F is the reactivity change per unit change in core length, K_F is the thermal-expansion coefficient of the fuel, and θ_F is the average fuel temperature, defined as the temperature in the fuel at 0.707 of the fuel outside diameter.

The transform quantity on the right above can be shown to be

$$\frac{\theta_F(s)}{\dot{Q}(s)} = \frac{4R_4 + R_1}{4} \cdot \frac{1 + sC_1 \frac{2R_1R_4}{4R_4 + R_1}}{1 + s(R_1 + 2R_4)C_1 + s^2R_1R_4C_1^2}$$

Then,

$$\text{Laplace}[F(K\theta_F/\dot{Q})] = \rho_F K_F \cdot \frac{4R_4 + R_1}{4} \cdot \frac{1 + sC_1 \frac{2R_1R_4}{4R_4 + R_1}}{1 + s(R_1 + 2R_4)C_1 + s^2R_1R_4C_1^2}$$

The above expression is used in Appendix A to calculate the dynamic fuel expansion response for rows 1-6. This solution appears as the fuel axial expansion in Section IV.A.

D. Axial Expansion of Control Rod and Subassembly

A reactor whose control subassemblies are suspended from above and immersed in the coolant can generate feedback reactivity due to axial thermal expansion of the rods as the coolant temperature changes with

power. In certain cases, the amount of such reactivity plays a significant part in the total feedback. Such is the case with the EBR-II reactor, where the control-rod drives are calibrated at equilibrium powers to correct for this effect.²

The dynamics of such expansion, which forms an important part of the system dynamics, is determined by dividing the length of the rod and the subassembly into sections, each of which possesses characteristic dynamic heat-flow properties. Calculations of the properties of each section are then used in combination to represent the way in which the total drive length, and thereby the feedback reactivity, varies with reactor power.

Fig. 3. Control-rod Suspension

The EBR-II control rod and subassembly can be divided into four sections, which possess distinctly different dynamic heat-transfer characteristics. The sections are shown as L_1 through L_4 in Fig. 3, which is a sketch of the control-rod suspension.³

The first section (L_1) will expand or contract in a way governed by its coolant temperature (θ_c) and by the transport time required for a coolant temperature change to travel the length of the section. The relation of coolant temperature and power can be taken from Section II.A above, and a single time-constant equivalent assumed for the transport lag. The transform of feedback reactivity divided by power change for section L_1 then is

$$\frac{(\delta K/K)(s)}{\Delta P(s)} = \frac{2K_L \left(1 + \frac{sR_1C_1}{2}\right)}{1 + s(R_1 + 2R_3)C_1 + s^2R_1R_3C_1^2} \cdot \frac{C_{L_1}W}{1 + \tau_T s},$$

where $2K_L$ is the equilibrium change in fuel subassembly coolant temperature with power, in units of degrees centigrade per megawatt; C_{L_1} is the expansion of section L_1 , in units of centimeters per degree change in coolant temperature; W is feedback reactivity in units of $\delta K/K$ per centimeter of fuel movement; and τ_T is 0.63 times the transport time of the coolant through L_1 . The remaining R 's and C 's are those defined in Section II.A.

All the remaining sections (L_2 , L_3 , and L_4) respond to plenum coolant temperature, which in EBR-II is delayed with respect to the fuel subassembly coolant temperature by the coolant holdup time of the upper plenum shown in Fig. 3. If a simple lag represents the holdup time, the transform of plenum temperature divided by power change is

$$\frac{\Delta\theta_P(s)}{\Delta P(s)} = \frac{K_p}{1 + \tau_p s},$$

where K_p is the equilibrium change in plenum coolant temperature with power, in units of degrees centigrade per megawatt, and τ_p is 0.63 times the plenum coolant holdup time. This assumes that core coolant transport can be neglected as short compared to plenum holdup time, and that blanket coolant transport time can be neglected because of the small amount of heat it supplies to the plenum. Now that the plenum temperature response is available, the response of control rod section L_2 to power is

$$\frac{(\Delta K/K)(s)}{\Delta P(s)} = \frac{K_p}{1 + \tau_p s} \cdot \frac{C_{L2} W}{1 + \tau_{L2} s},$$

where C_{L2} is the expansion of section L_2 , in centimeters per degree change in coolant temperature, τ_{L2} is the time constant that relates the internal temperature of the 0.1-cm-thick subassembly hexagonal containment shell to the coolant temperature, and the other constants are as previously defined.

The radial heat transfers of sections L_3 and L_4 shown in Figs. 4 and 5 must be calculated to determine their respective expansion characteristics. Both sections have similar alternating annuli of sodium and stainless steel, but of differing thicknesses. Therefore both may be represented by the radial heat-flow diagram in Fig. 6. The lumped heat capacities of the annuli from inner to outermost are C_1 , C_{Na} , and C_2 , respectively. Each is lumped at the diameter which divides the annulus into two equal volumes. The reciprocals of the heat conductivities of the

Fig. 4. Cross Section of Control Drive Rod at L_3

Fig. 5. Cross Section of Control Drive Rod at L_4

Fig. 6. Heat-flow Diagram for the Control Drive Rod

portions of annuli between these points of lumped heat capacity are designated R_1 , R_2 , R_3 , and R_4 (proceeding from inner to outer annuli). The plenum coolant traverses an annulus between the reactor top cover hole and the drive rod that passes through this hole. The quantity $\Delta\theta_{TC}$, which represents the temperature of the coolant in this annulus, is assumed to be identical to

$\Delta\theta_p$ but about 10% lower because of heat loss to the surrounding cover. Also, the practical magnitudes of the heat conductivities and capacities of the inner annuli are such that only R_4 and C_2 are needed to obtain a good approximation for the responses of sections L_3 and L_4 .

The response of section L_3 to power is

$$\frac{(\delta K/K)(s)}{\Delta P(s)} = \frac{0.9K_p}{1 + \tau_p s} \cdot \frac{C_{L3}W}{1 + \tau_{L3}s},$$

where C_{L3} is the expansion of section L_3 , in centimeters per degree change of coolant temperature, τ_{L3} is R_4 times C_2 evaluated for section L_3 , and the others are as defined above. Similarly,

$$\frac{(\delta K/K)(s)}{\Delta P(s)} = \frac{0.9K_p}{1 + \tau_p s} \cdot \frac{C_{L4}W}{1 + \tau_{L4}s},$$

with similar definitions to those above but relating to section L_4 .

All the relations appearing in this section are evaluated in Appendix A for use in Section IV.A.

III. THERMOMECHANICAL DISTORTION AND MOVEMENT OF THE CORE FUEL ELEMENTS

This section is specialized in the sense that any structural treatment must necessarily relate to a specific design, in this case, the EBR-II reactor core design. The effect of the small core is to make reactivity sensitive to fuel subassembly movements as small as one-thousandth of an inch. Small thermomechanical motions of the subassemblies are therefore important to the reactor dynamics and must be calculated to determine the structural feedback reactivity component.

A. BOW III Computer Program for Determination of Equilibrium Reactivity

The fuel elements of the EBR-II core are contained in semi-free-standing hexagonal subassemblies, with lower coolant nozzles fitted into holes in two grid plates so that they stand upright.³ They are spaced about 1/32 in. from each other on all sides of the hexagon, except at the center. At the center, buttons are embossed to reduce spacing to 2 mils to limit movement, but still allow the subassemblies to "flower" outward at the top in response to radial thermal gradients that develop due to heat transfer as reactor power increases. Many combinations of subassemblies such as core and inner and outer blanket types are possible, but the chief interest is in the combination used in reactor run No. 26, where considerable positive structural feedback was observed. This run and its loading are described and analyzed in Refs. 2 and 4, where the bowing problem is discussed in detail.

Bump⁵ later developed a method of numerical analysis, known as the BOW II computer program, to compute the structural distortion of the subassemblies by treating them as loaded flexible structural beams. The output of this program, which used approximations for the beam formulas, showed positive reactivities corresponding roughly to the amount of positive reactivity that could be postulated from reactor measurements from which other known reactivity effects were subtracted. To make dynamic analysis possible, I calculated the actual beam formulas representing the various modes of bending in the subassemblies and incorporated them into a new program known as BOW III. The bending modes and the formulas are given in Appendix B along with the method used to calculate the formulas. The FORTRAN description of BOW III appears in the same appendix. The input required for the program includes data about the subassembly mechanical properties and the equilibrium radial differential temperatures of the subassembly evaluated for 10 axial positions along each subassembly. References 1 and 2 describe the method of determining the differential temperatures.

The output of BOW III is a listing of the equilibrium radial locations of each subassembly at 10 axial positions. The core is influenced chiefly by positions 4, 5, and 6, which represent the bottom, center, and top of the core. The weighted position shift of each row of subassemblies is determined from BOW III data, and then the reactivity worth of the shift evaluated by using constants developed for rows 6, 7, 8, and 9. This evaluation assumes that the structure from row 6 to the core center shrank or expanded radially in proportion to the row 6 movement. Thus the entire core reactivity change is evaluated in terms of row 6. The effects of rows 7, 8, and 9 on reactivity are calculated using the respective constants for each row; rows further out have greatly reduced effect and are neglected.

Fig. 7. Equilibrium Feedback Reactivity of Structure

Appendix B gives the constants for rows 6, 7, 8, and 9, in inhours per mil, together with the differential temperature profiles and subassembly mechanical properties used as input to BOW III to calculate the equilibrium reactivity curve shown in Fig. 7. Also shown in Fig. 7 is an experimental curve taken from Ref. 6. Convergence difficulties cause the lack of smoothness in the calculated curve, although in general the two curves are in good agreement.

B. Heat-transfer Dynamics of Reflector Subassemblies

The stainless steel reflector subassemblies in rows 7 and 8 receive an appreciable amount of heat due to radial heat transfer from row 6 to row 7 and from row 9 to

row 8. This transfer of heat must be taken into consideration, and therefore rows 7 and 8 cannot be treated as the other rows were in Section II.A.

An approximate treatment is used, which includes a number of assumptions and is based on the results of equilibrium calculations contained in Ref. 1. The subassemblies of both rows 7 and 8 contain 19 solid stainless steel rods, which are 1.25 cm in outside diameter and 140 cm long, centered about the core axial centerline. Figure 8 shows a subassembly cross section. In this figure, Q_{RADIAL} is the heat flow between rows 6 and 7 or between rows 9 and 8. Zero heat flow is assumed between rows 7 and 8. The individual axial coolant passages act as effective heat barriers in the radial direction. Therefore the temperature of the subassembly hexagonal can on the radial heat-input side will assume a temperature between that of its own immediately adjacent coolant, $\theta_{7\text{Cin}}$, and that of the coolant in the adjacent subassembly hexagonal

Fig. 8. Cross Section of Reflector Subassembly

can. The adjacent subassembly can is row 6 for row 7 and row 9 for row 8. The relation between the coolant temperature (θ_c) and reactor power for rows 6 and 9 was determined in Section II.A. The coolant temperature of row 6 is controlled almost completely by the heat generation in row 6. Therefore the radial heat flow to row 7 from row 6 is proportional to the coolant temperature of row 6. Rows 9 and 8 are analyzed similarly. The

Fig. 9. Heat-flow Diagram for Reflector Subassembly

reflector coolant temperature, θ_{7Cin} , on the radial heat-input side is dependent on the heat generated in the half of each element in contact with the hexagonal can, such as the average element shown in Fig. 8, and also on Q_{Radial} from the next row. Where Q_{Radial} is zero, the half-elements alone control the reflector coolant temperature. Figure 9 is a simplified heat-flow diagram for the reflector subassembly. The chief influences on conductivity between the nearest coolant passages of two adjacent subassemblies are the hexagonal cans themselves. Then the total reciprocal conductivity of two hexagonal thicknesses in series is $2R_1$. R_{VR} is the effective reciprocal conductivity due to heat removal by coolant flow. The reciprocal heat conductivity of one half-element facing an outer coolant passage is twice that of a full element, or $2R_2$, where R_2 is the value calculated for heat flow between the diameter where the full element heat capacity (C_1) is lumped and the coolant. (The above diameter is 0.707 times the element outside diameter.) The outer temperature of the row 6 hex is $\theta_{6Hexout}$; θ_{7Hexin} and $\theta_{7Hexout}$ are inner and outer temperatures of the row 7 hex. The differential temperature that causes the bowing is designated ΔT_7 . The row 6 coolant temperature (θ_{6C}) and the heats generated internally in half of a row 7 inner element or an outer element (designated Q_0 and Q_{00} , respectively) are all proportional to the reactor power (P).

It can be shown that the Laplace transforms of the inner and outer hexagon temperatures are

$$\frac{\theta_{7Hexin}(s)}{P(s)} = \left[K_1 - \frac{1.5R_1(K_1 - K_2R_{VR})}{2R_1 + R_{VR}} \cdot \frac{1 + \frac{sC_1(2R_2 + R_{VR})}{2 \left(1 - \frac{K_2R_{VR}}{K_1}\right)}}{1 + \frac{sC_1(2R_2 + R_{VR} - R_{VR}^2)}{2}} \right] \cdot \frac{1}{1 + \tau_{6C}s}$$

and

$$\frac{\theta_{7\text{Hexout}}(s)}{P(s)} = \frac{K_3 R_{VR}}{1 + \frac{s C_1 (R_{VR} + 2R_2)}{2}}$$

Then,

$$\frac{\Delta T_7(s)}{P(s)} = \frac{\theta_{7\text{Hexin}}(s) - \theta_{7\text{Hexout}}(s)}{P(s)}$$

Similarly it can be shown that the transforms for the inner and outer hexagon temperatures of row 8 are

$$\frac{\theta_{8\text{Hexin}}(s)}{P(s)} = \frac{K_6 R_{VR}}{1 + \frac{s C_1 (R_{VR} + 2R_2)}{2}}$$

$$\frac{\theta_{8\text{Hexout}}(s)}{P(s)} = \left[K_4 - \frac{1.5 R_1 (K_4 - K_5 R_{VR})}{2 R_1 + R_{VR}} \cdot \frac{1 + \frac{s C_1 (2R_2 + R_{VR})}{2 \left(1 - \frac{K_5 R_{VR}}{K_4} \right)}}{1 + \frac{s C_1 (2R_2 + R_{VR} - R_{VR}^2)}{2}} \right] \cdot \frac{1}{1 + \tau_9 C s}$$

and

$$\frac{\Delta T_8(s)}{P(s)} = \frac{\theta_{8\text{Hexin}}(s) - \theta_{8\text{Hexout}}(s)}{P(s)}$$

Fig. 10. Power Variation with Time for a Rod-drop Experiment.

The above equations are evaluated in Appendix C for the conditions of Run 26, using data from Refs. 1 and 2.

C. Calculation of Structure Dynamic Reactivity

The dynamic feedback reactivity is calculated by inputting a rod-drop reactor power-time function⁷ like that in Fig. 10 to the individual row dynamic equations and evaluating them as a function of time. Each row time function that results will have a specific value at a given time, which may be used to multiply the starting power differential profile (T_Δ) for the row to obtain the

new profile at the given time. The general form then is

$$(T\Delta)_{[t]} = (P/45) \left((T\Delta)_{[P=45\text{MW}, t=0]} \right) [1 + C(t)],$$

where P is the starting power in megawatts, and $C(t)$ is a function that describes the time behavior of a particular row. When the rod-drop reactor power-time function is represented by a ramp plus an exponential, as in Fig. D.1, $C(t)$ has two general forms, which can be used to describe all rows. These are:

1. Rows 1-6 and 9-15

For $t \leq t_0$,

$$C(t) = \left(\frac{\Delta P}{P} \right)_{t_0} \frac{\sigma_1}{t_0} \left[\exp(-t/\sigma_1) + \frac{t}{\sigma_1} - 1 \right].$$

For $t > t_0$,

$$C(t) = K_B + \left(\frac{\Delta P}{P} \right)_{t_0} \frac{\sigma_1}{t_0} \left\{ \exp(-t/\sigma_1) - \exp[-(t - t_0)/\sigma_1] \right\} \\ + K \frac{\tau_K \exp[-(t - t_0)/\tau_K] - (\sigma_1) \exp[-(t - t_0)/\sigma_1]}{\sigma_1 - \tau_K}.$$

2. Rows 7 and 8

For $t \leq t_0$,

$$C(t) = \frac{(\Delta P/P)_{t_0}}{t_0} \left\{ (t - \sigma_1 - \sigma_2 + \sigma_3) + \frac{\sigma_1(\sigma_1 - \sigma_3) \exp(-t/\sigma_1)}{\sigma_1 - \sigma_2} - \frac{\sigma_2(\sigma_2 - \sigma_3) \exp(-t/\sigma_2)}{\sigma_1 - \sigma_2} \right\}.$$

For $t > t_0$,

$$C(t) = K_B + (\Delta P/P)_{t_0} \frac{\sigma_1(\sigma_1 - \sigma_3)}{t_0(\sigma_1 - \sigma_2)} \left\{ \exp(-t/\sigma_1) - \exp[-(t - t_0)/\sigma_1] \right\} \\ + (\Delta P/P)_{t_0} \frac{\sigma_2(\sigma_2 - \sigma_3)}{t_0(\sigma_2 - \sigma_1)} \left\{ \exp(-t/\sigma_2) - \exp[-(t - t_0)/\sigma_2] \right\} \\ + \frac{K\sigma_1(\sigma_3 - \sigma_1)}{(\sigma_1 - \sigma_2)(\sigma_1 - \tau_K)} \exp[-(t - t_0)/\sigma_1] + \frac{K\sigma_2(\sigma_3 - \sigma_2)}{(\sigma_2 - \sigma_1)(\sigma_2 - \tau_K)} \exp[-(t - t_0)/\sigma_2] \\ + \frac{K\tau_K(\sigma_3 - \tau_K)}{(\tau_K - \sigma_1)(\tau_K - \sigma_2)} \exp[-(t - t_0)/\tau_K].$$

Values for the constants in the above functions are obtained from Appendix D, and $C(t)$ is then evaluated by means of a computer program, PROFILE, whose output consists of decks of temperature profiles in BOW III format (see Appendix D). These are then input to BOW III to obtain the dynamic reactivity response to the specified $\Delta P/P$ time function. Initial calculations have shown large fluctuations, which are caused by convergence difficulties in the BOW III program. A new BOW IV program is being developed to eliminate these problems.

Fig. 11. Dynamic Change in Row Differential Temperature Profiles

The above $C(t)$'s can be used to estimate the structure feedback reactivity. The $C(t)$'s plotted in Fig. 11 are for Run 26B (see Table VI in Appendix D) as calculated by PROFILE. The equilibrium starting value for all rows is 1.0, so that the profile differential temperatures for row 7, for example, would all be multiplied by 1.097 at 10 sec. The reactivity worths of rows 6 and 7 dominate rows 8 and 9, and row 6 tends to follow the row 7 position. Therefore, if rows 6 and 7 are assumed to move inward with a position change similar to the temperature changes shown in Fig. 11 for row 7, the reactivity change can be estimated. Figure 7 indicates that the calculated BOW III

curve changes about 54 lh from 0 to 45 MW. This is about 1.2 lh/MW or 3.7×10^{-3} \$/MW. Then the positive structure reactivity change for the rod-drop can be estimated as

$$(\Delta k/k)_t = (3.7 \times 10^{-3})(41.4 \text{ MW})[C(t)],$$

$$(\Delta k/k)_{1\text{sec}} = 3.7 \times 10^{-3} \times 41.4 \times -5.77 \times 10^{-2} = -9.0 \times 10^{-3},$$

$$(\Delta k/k)_{3\text{sec}} = 3.7 \times 10^{-3} \times 41.4 \times -7.00 \times 10^{-2} = -10.8 \times 10^{-3},$$

and

$$(\Delta k/k)_{10\text{sec}} = 3.7 \times 10^{-3} \times 41.4 \times -9.71 \times 10^{-2} = -15.0 \times 10^{-3}.$$

If the above values are added to those in Fig. 13, the total reactivity feedback is

$$(\Delta k/k)_{1\text{sec}} = +3.7 \times 10^{-3} \$,$$

$$(\Delta k/k)_{3\text{sec}} = +5.5 \times 10^{-3} \$,$$

and

$$(\Delta k/k)_{10\text{sec}} = +10.3 \times 10^{-3} \$.$$

The corresponding experimental values were $+5.1 \times 10^{-3}$, $+6.25 \times 10^{-3}$, and $+9.2 \times 10^{-3}$ \$, respectively. The agreement is fortuitously good and indicates that the method is sound.

IV. SYSTEM BLOCK DIAGRAM OF CORE DYNAMICS

A. EBR-II System Diagram

Figure 12 is a system block diagram incorporating all the core reactivity feedbacks treated in the preceding sections. The individual blocks are shown in the simplified forms developed in the various appendixes, where each block was evaluated for the EBR-II Run 26 core configuration. $P(s)$ is the EBR-II zero-power transfer function.^{8,9} The dynamic linear expansions of the control-rod sections are represented by d_{L1} , d_{L2} , d_{L3} , and d_{L4} . The row temperature profiles are designated $(T\Delta)_{N,P,t}$, where N is the row, P is the starting power, and t is the time for which the profile was calculated. The button positions of rows 6-9 as a function of time are shown as d_6 - d_9 . The reactivity coefficients $(\rho_6$ - $\rho_9)$ for the above rows are expressed in reactivity per unit displacement.

Fig. 12. System Block Diagram of EBR-II Core Dynamics

All elements on Fig. 12 are shown in the linearized transfer function form described in Ref. 9, except for the core structure component, which is the nonlinear element shown within the dot-dash lines. The core component could be shown in approximate transfer function form by the use of artificial time constants and coefficients which would not be directly related to physical heat-transfer phenomena. However, every set of reactor test conditions would lead to a new set of arbitrary constants and coefficients that would not be particularly useful for analytic purposes. Therefore, it is left as a nonlinear element, and the time domain is used for analysis in the following sections.

B. Calculated Reactivity Time Response to a Reactor Rod Drop

As mentioned in Section III, the EBR-II reactor experimental run No. 26 was of special interest because of the large positive structure feedback.² Therefore, from here on, the analysis is based on a specific rod-drop experiment done at 41.4 MW during run No. 26. The variation in reactor power with time which occurred during this experiment is shown in Fig. D.1. The continued falloff in reactor power after the fast ramp insertion of the control rod, as shown in Fig. D.1, will persist until the negative reactivity inserted by means of the rod is just compensated by the feedback reactivity regained by reduction in power. The total dynamic feedback reactivity generated by the remaining parts of Fig. 12 was calculated as in Section III.C for the structure by inputting the above rod-drop reactor power-time function into the point marked $\Delta P/P$ after opening the loop by removing $P(s)$, the zero-power function. Time functions were calculated for each part by inverse

Fig. 13. Calculated Reactivity Variation with Time for a Rod-drop Experiment

calculated for each part by inverse Laplace transformation using the DYNAMIC computer program described in Appendix E. The power reactivity coefficients used for sodium expansion and for axial fuel expansion were those derived for Run 26B by Persiani *et al.*⁶ Figure 13 is a plot of the individual functions and their sum.

V. SUMMARY AND CONCLUSIONS

The direct systems approach used in the preceding sections of this report has led to a theoretical description of the power behavior of a small liquid-metal-cooled fast breeder reactor. Although a detailed analysis of a particular reactor

(EBR-II) was made using the theory, it is emphasized that the time domain approach is a very powerful tool for the analysis of any reactor and that the methods described here have general application. These methods are particularly useful for determining the theoretical performance of preliminary reactor designs before construction. This would apply especially to a reactor whose cores are constructed of large numbers of long, slender fuel elements, which are loaded vertically into the core and which interact on each other as a result of the core axial and radial thermal gradients.

The analysis of the EBR-II reactor points up the need to consider carefully the thermal gradients in a core and the resulting thermomechanical core-movement effects on reactivity. The BOW III program permits a systematic evaluation of the effects of thermal and mechanical parameter variations on core reactivity, where the calculated final core subassembly positions are accurate to about 1 mil. This is not sufficient resolution for determining core thermomechanical dynamics, and, in fact, about another factor of ten improvement to 0.1 mil is required. An approach to this problem has been instituted in the BOW IV program presently being developed. In this program, the entire group of core subassemblies is considered as a large three-dimensional interacting array of springs with gradual increase in the number of subassemblies treated as a group as power is raised. This is unlike BOW III, where each subassembly is treated individually as if imprisoned between the subassemblies directly adjacent to it. Interaction information from more distant subassemblies is required, and BOW IV will provide this. The added information entering into the calculation should provide the desired position resolution.

The dynamic heat-transfer calculations delineated in the foregoing sections, while sufficient for preliminary evaluations of designs, could be based on a much more finely divided axial and radial mesh of lumped elements of the type used by Cushman¹ for the equilibrium temperature distributions used for BOW III. It would be necessary to introduce heat capacities and axial heat flow dynamics, and these would materially increase the labor required and would probably be justified only in very specific design cases.

The overall dynamic reactivity results inferred as a result of the calculated row 7 movement at the end of Section III agreed well with the experimental measurements made by Hyndman,⁷ and the improved BOW IV program should lead to even better agreement.

APPENDIX A

Dynamic Heat-transfer Calculations1. Fuel Subassembly; Rows 1-6a. Dynamic Coolant Response

Since the heat generation rate ($\dot{Q}$) is proportional to reactor power (P), the expression for reactor coolant temperature on p. 9 can be rewritten as

$$\frac{\Delta\theta_C(s)}{\Delta P(s)} = \frac{K_L \left(1 + \frac{sR_1C_1}{2}\right)}{1 + s(R_1 + 2R_4)C_1 + s^2R_1R_4C_1^2},$$

where (as before)

$$R_4 = R_2 + R_3 + R_{VR}.$$

The various reciprocal heat conductivities (R 's) and heat capacities (C 's) can be calculated from the physical properties of the materials used. In all the examples given, liquid sodium ($\sim 700^\circ\text{F}$), Type 304 stainless steel ($\sim 700^\circ\text{F}$), or uranium are the main constituents. Their physical properties are listed in Table I.

TABLE I. Physical Properties of Type 304 Stainless Steel, Liquid Sodium, and Uranium

	304 SS	Sodium	Uranium
Density, g/cm ³	7.7	0.97	19.0
Conductivity, cal-cm/sec-cm ² -°C	0.048	0.174	0.082
Specific heat capacity, cal/g°C	0.115	0.305	0.038
Expansion coefficient, cm/cm°C	1.8×10^{-5}	-	-

Since internal generation of heat in the core subassemblies in rows 1-6 occurs chiefly within the core length of 36 cm, the calculations for these rows are based on this length, but the calculations for rows 9-15 are based on a length of 70 cm.¹ The outer diameter of the fuel pin (D_4) is taken to be 0.37 cm (see Fig. 1); the outer diameter of the stainless steel reflector and uranium blanket pins is 1.25 cm. The outside diameter of the effective coolant annulus (D_6) is ~ 0.54 cm for the fuel pin and ~ 1.33 cm for the reflector and blanket pins.

The heat capacity (C_1) of the equal inner and outer volumes (see Fig. 1) is

$$C_1 = \text{volume} \times \text{density} \times \text{specific heat capacity.}$$

For rows 1-6,

$$C_1 = \pi \left(\frac{0.707 \times 0.37}{2} \right)^2 \times 36 \times 19.0 \times 0.038 = 1.40 \text{ cal/}^\circ\text{C.}$$

Since the heat capacities are lumped at diameters D_1 and D_3 , the reciprocal heat conductivity (R_1) between these points is calculated using the mean cylindrical area and the radial distance between the two diameters. The reciprocal conductivity between D_3 and D_1 is

$$R_1 = \frac{\log_e (D_3/D_1)}{2\pi \times \text{conductivity} \times \text{length}}.$$

For rows 1-6,

$$R_1 = \frac{\log_e (0.866/0.50)}{2\pi \times 0.082 \times 36} = 0.0282^\circ\text{C/cal-sec}^{-1}.$$

Similarly, for R_2 ,

$$R_2 = \frac{\log_e (1/0.866)}{2\pi \times 0.082 \times 36} = 0.0078^\circ\text{C/cal-sec}^{-1}.$$

Then, for R_3 , for a clad pin of the type used in EBR-II, the film and cladding total temperature drop can be approximated at power by making $R_3 = (R_1/2) + R_2$, or

$$R_3 = 0.0282/2 + 0.0078 = 0.0219^\circ\text{C/cal-sec}^{-1}.$$

The core coolant temperature (θ_c) is taken as the average temperature or one-half the core outlet temperature, so that R_{VR} is compatible with the previously calculated R 's, which are all mean values.

Then, for R_{VR} ,

$$Q_{\text{Element}} = (509 \text{ kW}/91)(238.9 \text{ cal-sec}^{-1}/\text{kW}) = 1342 \text{ cal-sec}^{-1};$$

$$R_{VR} = (T_{\text{out}}/2)/Q_{\text{Element}}$$

$$= [134/(2 \times 1.8)]/1342 = 0.026^\circ\text{C/cal-sec}^{-1} \text{ (see Ref. 1).}$$

The coolant heat capacity (C_{Na}) is calculated in the same way that C_1 was treated.

For rows 1-6,

$$C_{Na} = (65/500) \times 36 \times 0.97 \times 0.305 = 1.38 \text{ cal/}^\circ\text{C}.$$

The time constant represented by

$$\tau = \frac{R_{VR}(R_2 + R_3)}{R_{VR} + R_2 + R_3} C_{Na} = 0.019$$

is so small that it was possible to neglect C_{Na} in deriving the expression for $\Delta\theta_c(s)/\Delta P(s)$ given on p. 23. The evaluation of this expression for rows 1-6 is

$$\begin{aligned} \frac{\Delta\theta_c(s)}{\Delta P(s)} &= \frac{K_L[1 + s(0.0282 \times 1.4/2)]}{1 + s(0.0282 + 0.1114) + s^2(0.0202 \times 0.0557)(1.4)^2} \\ &= \frac{K_L(1 + 0.02s)}{1 + 0.195s + 0.0032s^2}. \end{aligned}$$

The denominator may be factored to give, for the average coolant in the core,

$$\begin{aligned} \frac{\Delta\theta_c(s)}{\Delta P(s)} &= \frac{K_L(1 + 0.02s)}{(1 + 0.173s)(1 + 0.018s)} \\ &\approx \frac{K_L}{1 + 0.17s}. \end{aligned}$$

b. Dynamic Coolant Void Response

The above expression for $\Delta\theta_c(s)/\Delta P(s)$ may be applied to the sodium expansion effect in the core, which is one-half the total effect. The remaining one-half of the effect comes from the coolant volume between the core and the upper plenum. The transport time through this region is

$$t_u = \frac{25.5 \times 2.54 \text{ cm}}{500 \text{ cm/sec}} = 0.13 \text{ sec, or } \tau_u \approx 0.07 \text{ sec.}$$

Then, as a first approximation, for the coolant directly above the core,

$$\frac{\Delta\theta_u(s)}{\Delta P(s)} \approx \frac{2K_L}{(1 + 0.19s)(1 + 0.07s)}.$$

c. Dynamic Fuel Expansion Response

The fuel expansion effect, which is largely due to rows 1-6, will be considered next before proceeding to the outer rows.

The expression for reactor average fuel temperature on p. 12 can be rewritten as

$$\frac{\Delta\theta_F(s)}{\Delta P(s)} = K_L \frac{4R_4 + R_1}{4R_{VR}} \cdot \frac{1 + sC_1 \frac{2R_1R_4}{4R_4 + R_1}}{1 + s(R_1 + 2R_4)C_1 + s^2 R_1R_4C_1^2}$$

Substituting the values derived above and noting that the right-hand denominator is like that for $\Delta\theta_c(s)$, we obtain, for the average fuel in the core,

$$\begin{aligned} \frac{\Delta\theta_F(s)}{\Delta P(s)} &= \frac{0.0639}{0.026} \cdot \frac{(1 + 0.014s) K_L}{(1 + 0.173s)(1 + 0.018s)} \\ &\approx \frac{2.4K_L}{1 + 0.17s} \end{aligned}$$

2. Blanket Subassembly; Row 9

a. Dynamic Coolant Response

The response of row 9 may be obtained in the same way as for a fuel pin. The main differences are in dimensions. The length of the elements in rows 7-15 requires use of a length of 70 cm to calculate the heat transfer in place of the core length of 36 cm used above for the fuel subassemblies. The results are

$$C_1 = \pi \times \left(\frac{0.707 \times 1.25}{2} \right)^2 \times 70 \times 19.0 \times 0.038 = 30.9 \text{ cal/}^\circ\text{C},$$

$$R_1 = \frac{\log_e (0.866/0.5)}{2\pi \times 0.082 \times 70} = 0.0145^\circ\text{C/cal-sec}^{-1},$$

$$R_2 = \frac{\log_e (1.0/0.866)}{2\pi \times 0.082 \times 70} = 0.0039^\circ\text{C/cal-sec}^{-1},$$

$$R_3 = 0.0145/2 + 0.0039 = 0.0112^\circ\text{C/cal-sec}^{-1},$$

$$Q_{\text{Element}} = (36 \text{ kW}/19)(238.9 \text{ cal-sec}^{-1}/\text{kW}) = 452 \text{ cal-sec}^{-1},$$

$$\text{Average } T_{\text{Out}} = (125 + 145)/(2 \times 1.8) = 75^\circ\text{C},$$

$$R_{VR} = (T_{Out}/2) \div Q_{Element} = 75/(2 \times 452) = 0.083^\circ\text{C}/\text{cal-sec}^{-1},$$

and

$$C_{Na} = (19.1/130) \times 70 \times 0.97 \times 0.305 = 3.04 \text{ cal}/^\circ\text{C}.$$

Again neglecting C_{Na} , we obtain

$$\begin{aligned} \frac{\Delta\theta_c(s)}{\Delta P(s)} &= \frac{(R_{VR}\dot{Q}_9/45)[1 + s(0.0145 \times 30.9/2)]}{1 + s(0.0145 + 0.1962)(30.9) + s^2(0.0145 \times 0.0981)(30.9)^2} \\ &= \frac{(R_{VR}\dot{Q}_9/45)(1 + 0.224s)}{1 + 6.53s + 1.36s^2}, \end{aligned}$$

where $(\dot{Q}_9)$ is the cal/sec output of a row 9 subassembly for a reactor equilibrium power of 45 MW. The denominator may be factored to give

$$\begin{aligned} \frac{\Delta\theta_c(s)}{\Delta P(s)} &= \frac{(R_{VR}\dot{Q}_9/45)(1 + 0.224s)}{(1 + 6.2s)(1 + 0.22s)} \\ &\approx \frac{R_{VR}\dot{Q}_9/45}{1 + 6.2s}. \end{aligned}$$

This equation will be used later for row 8 structural effects, depending on heat transfer between rows 8 and 9. It will also be used for the row 9 structural effect.

3. Blanket Subassembly; Rows 10-15

a. Dynamic Coolant Response

The response of rows 10-15 is like that of row 9, except that R_{VR} is altered because the flow is reduced from 6 to 2.5 gpm, giving

$$R_{VR} = (6/2.5)(0.083) = 0.210^\circ\text{C}/\text{cal-sec}^{-1},$$

and

$$\begin{aligned} \frac{\Delta\theta_c(s)}{\Delta P(s)} &= \frac{(R_{VR}\dot{Q}_J/45)(1 + 0.224s)}{1 + s(0.0145 + 0.449) 30.9 + s^2(0.0145 \times 0.224)(30.9)^2} \\ &= \frac{(R_{VR}\dot{Q}_J/45)(1 + 0.224s)}{1 + 14.4s + 3.11s^2}. \end{aligned}$$

The denominator, when factored, gives

$$\frac{\Delta\theta_c(s)}{\Delta P(s)} = \frac{(RVR\dot{Q}_J/45)(1+0.224s)}{(1+15.5s)(1+0.219s)}$$

$$\approx \frac{RVR\dot{Q}_J/45}{1+15.5s}$$

4. Control Rod Section L_1

The expression for $(\delta K/K)(s)/\Delta P(s)$ for Section L_1 shown on p. 11 may be written directly using the fuel subassembly dynamics already calculated:

$$\tau_T = 0.63 \times 71 \text{ cm} / 500 \text{ cm-sec}^{-1} = 0.09 \text{ sec}$$

$$\frac{(\delta K/K)(s)}{\Delta P(s)} = \frac{2K_L}{1+0.19s} \cdot \frac{C_{L_1}W}{1+0.09s},$$

where

$$2K_L = (225^\circ\text{F} \times 5/9) / 62.5 \text{ MW} = 2.0^\circ\text{C}/\text{MW},$$

$$W = (\Delta K/K)/d = \frac{8 \text{ lh/in.} \times 12 \text{ rods}}{2.54 \times 430 \text{ lh} / 0.01(\delta K/K)} \text{ (see Ref. 2)}$$

$$= 8.8 \times 10^{-4} (\delta K/K) / \text{cm} = 0.118 \text{ } \$/\text{cm},$$

and

$$C_{L_1} = \text{Length} \times \text{expansion coefficient} = L_1 \alpha$$

$$= 128 \text{ cm} \times 1.8 \times 10^{-5} \text{ cm/cm}^\circ\text{C} = 2.3 \times 10^{-3} \text{ cm}/^\circ\text{C}.$$

5. Plenum Effect

The plenum holdup time is approximately 2.37 sec, and $\tau_p = 0.63 \times 2.37 = 1.49 \text{ sec}$. Then,

$$\frac{\Delta\theta_p(s)}{\Delta P(s)} = \frac{K_p}{1+1.49s}.$$

6. Control Rod Section L₂

The quantity τ_{L_2} is the time constant associated with the 0.1-cm-thick hexagonal subassembly containment shell when heated from the outside by plenum sodium with the temperature of the inner side assumed constant. It may be determined by evaluating C_S and R_S using an area of 1 cm². Thus,

$$C_S = \text{volume} \times \text{density} \times \text{specific heat capacity} \\ = 1 \times 1 \times 0.1 \times 7.7 \times 0.115 = 0.089 \text{ cal/}^\circ\text{C},$$

$$R_S = \frac{\text{thickness}/2}{\text{conductivity} \times \text{area}} \\ = \frac{0.1/2}{0.048 \times 1 \times 1} = 1.04^\circ\text{C/cal-sec}^{-1},$$

and

$$\tau_{L_2} = R_S C_S = 0.089 \times 1.04 = 0.093 \text{ sec.}$$

Then,

$$\frac{(\Delta K/K)(s)}{\Delta P(s)} = \frac{K_p}{1 + 1.49s} \cdot \frac{C_{L_2} W}{1 + 0.093s},$$

where

$$K_p = (183^\circ\text{F} \times 5/9)/62.5 \text{ MW} = 1.63^\circ\text{C/MW} \text{ (see Ref. 2),}$$

and

$$C_{L_2} = L_2 \alpha = 46 \text{ cm} \times 1.8 \times 10^{-5} \text{ cm/cm}^\circ\text{C} \\ = 8.3 \times 10^{-4} \text{ cm/}^\circ\text{C}.$$

7. Control Rod Section L₃

Trial calculations of the magnitudes of the inner annuli heat conductivities and capacities for Section L₃ shown in Fig. 4 showed that only R_4 and C_2 need be considered. The outer and inner diameters of the outermost stainless steel annulus are 6.35 and 2.54 cm, respectively. Hence,

$$C_2 = (\pi/4)(D_1^2 - D_2^2) \times L_3 \times 7.7 \times 0.115 \\ = (\pi/4)(6.35^2 - 2.54^2) \times 50.8 \times 7.7 \times 0.115 = 1179 \text{ cal/}^\circ\text{C}.$$

C_2 may be lumped at the mean diameter D_M of the annulus; thus,

$$D_M = \sqrt{\frac{D_1^2 + D_2^2}{2}} = 4.83 \text{ cm},$$

$$R_4 = \frac{\log_e (D_1/D_M)}{2\pi \times 0.048 \times L_3}$$

$$= \frac{\log_e (6.35/4.83)}{2\pi \times 0.048 \times 50.8} = 0.0178^\circ\text{C/cal-sec}^{-1},$$

and

$$\tau_{L_3} = R_4 C_2 = 1179 \times 0.0178 = 21.0 \text{ sec.}$$

Then,

$$\frac{(\delta K/K)(s)}{\Delta P(s)} = \frac{0.9K_p}{1 + 1.49s} \cdot \frac{C_{L_3}W}{1 + 21.0s},$$

where

$$C_{L_3} = L_3 \alpha = 50.8 \times 1.8 \times 10^{-5}$$

$$= 9.14 \times 10^{-4} \text{ cm}/^\circ\text{C}.$$

8. Control Rod Section L_4

Again as in Section 7 of this appendix, only R_4 and C_2 need be considered. The outer and inner diameters of the outermost stainless steel annulus are 6.35 and 4.45 cm, respectively.

$$C_2 = (\pi/4)(D_1^2 - D_2^2) \times L_4 \times 7.7 \times 0.115$$

$$= (\pi/4)(6.35^2 - 4.45^2) \times 50.8 \times 7.7 \times 0.115$$

$$= 725 \text{ cal}/^\circ\text{C}$$

C_2 may be lumped at the mean diameter of the annulus, which is

$$D_M = \sqrt{\frac{D_1^2 + D_2^2}{2}} = 5.48 \text{ cm},$$

to give

$$R_4 = \frac{\log_e (6.35/5.48)}{2\pi \times 0.048 \times 50.8} = 0.0112^\circ\text{C}/\text{cal-sec}^{-1}$$

and

$$\tau_{L4} = R_4 C_2 = 725 \times 0.0112 = 8.1 \text{ sec.}$$

Then,

$$\frac{(\delta K/K)(s)}{\Delta P(s)} = \frac{0.9K_p}{1 + 1.49s} \cdot \frac{C_{L4}W}{1 + 8.1s},$$

where

$$\begin{aligned} C_{L4} &= L_4 \alpha = 50.8 \times 1.8 \times 10^{-5} \\ &= 9.14 \times 10^{-4} \text{ cm}/^\circ\text{C.} \end{aligned}$$

APPENDIX B

BOW III Thermomechanical Structure Analysis1. Theoretical Bending of Subassemblies due to Combined Nozzle, Button, and Top Loading

The structural bending of the subassemblies can be determined by treating them as lightly loaded flexible structural beams. The method by

Fig. B.1. Subassembly as a Loaded Beam

which a subassembly is supported is shown schematically at the left in Fig. B.1. Briefly, the subassembly nozzle is inserted into two grid plates containing axially aligned holes, which support the subassembly in a vertical position. Coolant hydraulic forces thrust downward to hold the subassembly against a spherical pivot support in the upper grid plate. Clearance of the nozzle in the lower plate permits some off-vertical motion, which is limited by contact between adjacent subassemblies at the buttons or at the top. The thermal bowing of the subassemblies due to radial temperature gradients, as explained in Ref. 2, causes mechanical interferences at the lower grid plate, the button, and the top levels. The subassemblies interact and flex as beams until the interference is resolved by the deflection and the balanced forces between subassemblies. The subassembly is

depicted as a loaded beam at the right in Fig. B.1. The nozzle is either free or loaded radially inward or outward. The button and the top may be loaded radially either inward or outward, as shown.

Several different modes of bending can take place because of the different ways that the subassembly can be loaded. I have worked out solutions for radial button displacement in terms of top and button loads for a number of the modes that cannot make use of formulas available in the literature.¹⁰

Table II lists the various modes with the direction of their radial loading indicated by L or R, where L is a force directed radially outward against the subassembly and R is a force directed inward. There is no contact and therefore no force where 0 appears in the table. Interchange

of L's and R's will reverse the direction of bending but not the characteristic mode shape, which is simply a mirror image along the axis.

TABLE II. BOW III Bending Modes

Mode	Force Direction		
	Nozzle	Button	Top
II Top	R	0	R
II	R	R	0
III	R	R	R
IIIA	R	L	R
IV	0	L	R
V	L	L	R

Fig. B.2. Subassembly M/EI Diagram and Radial Displacement

The solution for Mode V is worked out below. The method is general and can be applied to any flexible-beam problems of a similar nature. Figure B.2 shows the radial displacement profile for a subassembly with Mode V loading, along with the corresponding M/EI diagram. In the displacement diagram, forces directed upward are taken as positive and displacement downward is also taken as positive.

The relation between y_B and the constants of the M/EI diagram can be obtained by noting from the figure that

$$y_B = A - B = (l_C C / l_A) - B.$$

The values of B and C may be obtained by using the Second Area Moment Proposition.¹¹

$$\begin{aligned} A &= \frac{1}{EI_A} \cdot \frac{l_C}{l_A} (A_1 \bar{l}_{A_1} + A_2 \bar{l}_{A_2}) \\ &= \frac{1}{EI_A} \cdot \frac{l_C}{l_A} \left(W_A l_A \frac{l_A}{2} \cdot \frac{l_A}{3} + W_B l_B \frac{l_B}{2} \cdot \frac{l_B}{3} \right). \end{aligned}$$

Similarly,

$$B = \frac{1}{EI_A} \left(W_A l_c \frac{l_c}{2} \cdot \frac{l_c}{3} \right).$$

Combining the above equations, we obtain

$$y_B = \frac{1}{EI_A} \left(W_A \frac{l_C l_A^3 - l_C^3 l_A}{6 l_A} + W_B \frac{l_B^3 l_C}{6 l_A} \right).$$

It is necessary to apply the Second Area Moment Proposition again to determine the relation between W_A and W_B in the above expression. Thus, from Fig. B.3 it can be seen that

$$\begin{aligned} F &= \frac{1}{EI_A} (A_1 \bar{l}_{A_1} + A_2 \bar{l}_{A_2}) \\ &= \frac{1}{EI_A} \left[W_A l_A \frac{l_A}{2} \cdot \frac{2 l_A}{3} + W_B l_B \frac{l_B}{2} \left(l_C + \frac{2 l_B}{3} \right) \right] \end{aligned}$$

and

$$\begin{aligned} D &= -\frac{1}{EI_N} A_3 \bar{l}_{A_3} \\ &= -\frac{1}{EI_N} W_N l_N \frac{l_N}{2} \cdot \frac{2 l_N}{3}. \end{aligned}$$

Also,

$$D = \frac{l_N}{l_A} F, \quad W_N l_N = W_A l_A + W_B l_B, \quad \text{and } l_C = l_A - l_B.$$

Fig. B.3

Rotation of Subassembly in
Spherical Pivot Support

Substituting and solving, we obtain

$$W_A = W_B \frac{\ell_B}{\ell_A} \left[\frac{\ell_B^2 - 3\ell_B \ell_A + 2\ell_A^2}{2\ell_A^2 \left(1 + \frac{I_A \ell_N}{I_N \ell_A}\right)} - 1 \right]$$

and

$$W_B = 6EI_{Ay}b \frac{\ell_A}{\ell_B^2(\ell_A - \ell_B)} \left[(2\ell_A - \ell_B) \frac{\ell_B^2 - 3\ell_B \ell_A + 2\ell_A^2}{2\ell_A^2 \left(1 + \frac{I_A \ell_N}{I_N \ell_A}\right)} + \ell_B \right]^{-1}$$

Similar calculations for Modes III and IIIA result in the same solutions as those for Mode V above.

The solution for Mode IV, written in a slightly different form from that used in Ref. 11, is

$$W_A = W_B(\ell_A/\ell_B)(-1)$$

and

$$W_B = 6EI_{Ay}b \frac{\ell_A}{\ell_B^2(\ell_A - \ell_B)} \left[(2\ell_A - \ell_B)(-1) + \ell_B \right]^{-1}$$

The solution for Mode II in the same form is

$$W_B = -6EI_{Ay}b \left[2\ell^3 \left(1 + \frac{I_A \ell_N}{I_N \ell_B}\right) \right]^{-1}$$

The subassembly radial deflection (y_x) at any position (x) measured upward from the pivot in the axial direction is

$$y_x = \frac{1}{6EI_A} \left\{ W_A \left[(\ell_A - x) \ell_A^2 - (\ell_A - x)^3 \right] + W_B \left[(\ell_A - x) \frac{\ell_B^3}{\ell_A} - (\ell_B - x)^3 \right] \right\}$$

The above equation can be evaluated for a specific mode by substituting the relations for W_B and W_A developed earlier for that mode.

2. BOW III Computer Program

The FORTRAN description of the above program is given later in this appendix. It is similar to the BOW II program developed by Bump,⁵ but incorporates the exact relations for the various bending modes as derived

in Part I of this appendix and includes an important mode ignored in BOW II. Also, D. Kucera has added stabilization to produce calculated output positions accurate to about 1 mil. Improvement to about 0.1 mil is necessary for BOW III to be useful in computing accurate dynamic movement of the subassemblies. A new approach conceiving of the subassembly array as a large group of coupled springs is under development by Kucera. This program will be known as BOW IV.

BOW III output includes an input listing as well as the output for chosen iterations. Each quantity printed is identified by its name within the code, and Table III relates program names with physical quantities.

TABLE III. BOW III FORTRAN Quantities

A. <u>Input Quantities</u>	
MI	Number of equally spaced vertical nodes on each subassembly
MJ	Number of radial rows of subassembly
LI	Node number corresponding to button level
QQ	Initial fractional power
QD	Fractional-power increment
QT	Final fractional power
TL	Height of each subassembly (in.)
YFL1	Inmost allowed radial position for top of central subassembly
DELY	Maximum position change permitted by one iteration, at top
DL	Maximum position change permitted by one iteration, at button level
B8	Radial distance between vertical-assembly nozzle and grid- plate obstructions, inward or outward (in.)
B9	Length of nozzle (in.)
ALPHA	Coefficient of thermal expansion of buttons (in./in. °F)
D	Distance across flats of a subassembly (in.)
YR	Distance from outmost row centerline L to ring, at top level (in.)
D1	D1(1)
B1	Initial lean slope for all rows
B2	Free-swing distance at top level due to B8 clearance at nozzle
B3	$B4(1) + 1$
B4	$B4(1)$
DELT(I,J)	Full-power temperature differential across row J at node I level (°F at 45 MW)
YP(J)	Centerline radial position of row J
B4(J)	$I_A \ell_N / I_N \ell_A$ for row J
B5(J)	$3EI_A F(J) / \ell_A^3$ for row J

TABLE III (Contd.)

B6(J)	Input as $3EI_A F(J)$, printed as $\frac{3EI_A F(J)}{\ell_B^2(\ell_A - \ell_B)^2}$ for row J
D1(J)	Material thickness between centerlines of rows J and J + 1, at top level
DD1(J)	Initial material thickness between centerlines of rows J and J + 1, at button level
F(J)	The number of subassemblies in row J
T(J)	Full-power temperature difference between pivot and top level, for row J ($^{\circ}\text{F}$ at 45 MW)
F1(J)	Single-button squashing factor for row J (in./lb)

B. Output Quantities

YT(I,J)	Thermal deflection at node I of row J, for the power level whose iteration values follow
M	Iteration number
Q	Fractional power
Y(I,J)	Radial position of node I of row J
MODE(J)	Most severe beam-bending formula-type required to fit row J into its constraint positions
ZF(J)	Effective maximum motion at top-level for row J
YFR(J)	Top-level constraint location on outside of row J
YFL(J)	Top-level constraint location on inside of row J
ZL(J)	Effective maximum motion at button level for row J
YFLR(J)	Button-level constraint location on outside of row J
YFLL(J)	Button-level constraint location on inside of row J
PL(J)	Load exerted on top level of row J from inside
PR(J)	Load exerted on top level of row J from outside
WL(J)	Load exerted on button level of row J from inside
WR(J)	Load exerted on button level of row J from outside
DD(J)	Button-level material thickness between row J and row J + 1 centerlines at fractional power Q

Some of the input quantities in this table are illustrated in Fig. B.4 with the radial scale greatly enlarged for clarity. Thus, ℓ_A is typically 65 in., and B8 is 4×10^{-3} in. The method of summing DD1(J) and D1(J) between rows J and J + 1 is shown.

All doubly dimensioned arrays are printed so that row number increases with downward motion through each column and node number increases with rightward motion across each column. Thus, values for the top nodes of each row appear in the outermost column.

Fig. B.4
BOW III Module

3. Calculation of Input Quantities

a. Button Stiffness

The button stiffness can be obtained by assuming a concentrated load on a long, flat, rectangular plate with fixed edges.

Then,

$$\Delta_b = 0.00725 P a^2 / D \text{ (see Ref. 12),}$$

where

$$\Delta_b = \text{deflection at load (in.),}$$

$$P = \text{load (lb),}$$

$$a = \text{width of flat} = 1.30 \text{ in.,}$$

$$D = \text{flexural rigidity} = 130 \text{ lb-in. (see Ref. 12),}$$

and

$$E = 24.25 \times 10^6 \text{ lb/in.}^2 \text{ for 304 SS.}$$

Therefore,

$$\Delta_b / P = 0.00725 \times 1.3^2 / 130 = 1 \times 10^{-4} \text{ mil/lb.}$$

Because of the hexagonal configuration of buttons, the effective stiffness per button per subassembly is 1.5 times as much as the actual stiffness.

Thus,

$$(\Delta_b/P)_{\text{eff}} = 1 \times 10^{-4} / 1.5 = 0.7 \times 10^{-5} \text{ mil/lb},$$

and

$$F_1(J) = 2(\Delta_b/P)_{\text{eff}} = 1.4 \times 10^{-5} \text{ mil/lb}.$$

The above value was used for rows 1-6. Rows 7-15 were assigned a value one-tenth that of $F_1(J)$, or 1.4×10^{-5} mil/lb, because the hexagonal can is supported immediately under the button by the rigid elements contained in the can, and therefore its flexure is greatly reduced.

b. Moment-of-inertia Calculations for Nozzles and Upper Parts of Subassemblies

The nozzles for rows 1-6 are all similar. Their moment of inertia is¹³

$$I_{N,(1-6)} = 0.049(D^4 - d^4) = 0.049(1.683^4 - 1.433^4) = 0.187 \text{ in.}^4.$$

The nozzles for rows 7-15 are similar, and their moment of inertia is

$$I_{N,(7-15)} = 0.049(1.468^4 - 1.068^4) = 0.164 \text{ in.}^4.$$

The hexagonal cans for rows 1-6 and 9-15 will determine the moment of inertia of the portion of the subassembly between the pivot and the top. The hexagon can be calculated, since it has a thin wall, by assigning it an average outside diameter of 2.450 in. The wall thickness of 0.040 in. leads to an inside diameter of 2.370 in.

Then

$$I_{A,(1-6,9-15)} = 0.049(2.450^4 - 2.370^4) = 0.218 \text{ in.}^4.$$

For the stainless steel reflector rows 7 and 8 used in Run 26, the stiffness of the upper part of the subassembly above the pivot is increased by the 19 full-length 0.493-in.-diam rods contained in each subassembly. (Such is not the case in rows 9-15, where the elements are made up of stacked short lengths, which are free to flex.)

Then,

$$\begin{aligned} I_{A,(7-8)} &= 0.049(2.450^4 - 2.370^4) + 19[0.049(0.493^4)] \\ &= 0.275 \text{ in.}^4 \quad (\text{elements free to slip}); \end{aligned}$$

$$\begin{aligned}
I_{A,(7-8)} &= 0.22 + 5 \times 0.049(0.493)^4 + (8 \times \pi \times 0.493^2/4)(0.493 \times 0.866)^2 \\
&\quad + (6 \times \pi \times 0.493^2/4)(0.493 \times 0.866 \times 2)^2 \\
&= 1.35 \quad (\text{elements unable to slip}).
\end{aligned}$$

Since some slippage may occur, the following average value was assumed:

$$I_{A,(7-8)} = 0.77 \quad (\text{partial slippage}).$$

Such partial slippage could account for the apparent hysteresis in the EBR-II feedback reactivity, which assumes two values depending upon the direction of power change.

The remaining inputs are combinations of moments of inertia, lengths, numbers of subassemblies per row, and mechanical properties of the materials, all of which are specified in the sample run. The differential temperature profiles for run No. 26B calculated by R. Cushman are used for the input (DELT).

4. BOW III Sample Run

The printed output included on pages 41-59 is that for the computer run that produced the curve plotted in Fig. 7. The data for the curve appear in the last column of Table IV, which is the sum of the reactivity changes in inhours due to rows 6-9. The row displacement from its zero-power position at the button level is represented by d_6 , d_7 , etc., in mils. It is obtained from the printed output by subtracting Y (in column 5) for a subassembly row from that row's YP. The reactivity due to the row displacement is obtained by multiplying d by the row reactivity constant (ρ), which has the units of inhours/mil. The calculated values for ρ_6 , ρ_7 , ρ_8 , and ρ_9 are -1.806, -0.989, -0.500, and -0.289, respectively, for the EBR-II run No. 26B configuration. As power is increased from zero to 45 MW, rows 6 and 7 move steadily toward the core center and rows 8 and 9 move away from the center. The combined effect is to increase reactivity as power is increased, yielding positive feedback.

TABLE IV. Reactivity Evaluation of Sample Output

Q, P/45 MW	Row 6		Row 7		Row 8		Row 9		Total, lh
	d_6	$\rho_6 d_6$	d_7	$\rho_7 d_7$	d_8	$\rho_8 d_8$	d_9	$\rho_9 d_9$	
0.1	0.09	-0.16	0.46	0.45	0.6	-3.0	0.45	-0.13	-0.14
0.2	5.52	9.97	7.68	7.60	7.42	-3.71	5.08	-1.47	12.25
0.3	11.5	20.77	13.53	13.38	11.30	-5.65	9.04	-2.61	25.89
0.4	14.4	25.54	13.61	13.46	17.88	-8.92	15.66	-4.53	25.55
0.5	17.25	31.15	19.25	19.04	20.95	-10.48	18.85	-5.45	34.26
0.6	19.68	35.54	21.72	21.48	25.73	-12.87	23.71	-6.85	37.30
0.7	22.42	40.49	24.53	24.26	28.66	-14.33	26.73	-7.72	42.70
0.8	24.18	43.67	26.30	26.01	34.82	-17.41	32.97	-9.53	42.74
0.9	26.32	49.53	28.56	28.25	37.02	-18.51	35.25	-10.19	47.08
1.0	28.35	51.20	30.75	30.41	38.45	-19.23	36.76	-10.62	51.76

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      PROGRAM BOW III
      DIMENSION CELT(20,20),YP(20),B1(20),D1(20),B3(20),B4(20),B5(20),
1  B6(20),Y(20,20),YI(20,20),YT(20,20),YFR(20),YFL(20),PL(20),
2  PR(20),XDELT(20,20),YY(20),AY(20,20),DD1(20),WR(20),WL(20),
3  F(20),DD(20),T(20),F1(20),A6(20),A7(20),YFLL(20),YFLR(20),
4  YZ(20),MA(30),YPT(20,20),B35(20),XX(20),APL(20),APR(20),
5  AWL(20),AWR(20),PLB(20),PRB(20),WLB(20),WRB(20),B3L(20),B4L(20),
6  B7(20),B36(20),B35L(20),GL(20),HL(20),PRP(20),PLP(20),ZF(20),
7  ZL(20),MODE(20),B37(20),B38(20),B39(20),B40(20)
      TYPE DOUBLE B3,B4,B5,B6,Y,YFR,YFL,PL,PR,YY,AY,WR,WL,DD,A6,A7,
1  YFLL,YFLR,YZ,B35,XX,APL,APR,AWL,AWR,PLB,PRB,WLB,WRB,B3L,B4L,B7,
2  B36,B35L,GL,HL,PRP,PLP,ZF,ZL,B37,B38,B39,B40,B5L,CL,P,C1,C2,C3,
3  C4,PD,DS,DM,DNOZ,PP,C3M,WB,WA,C2M,YW,C4M
      READ1000,MI,MJ,LI,QQ,QD,QT,TL,YFL1,DELY
      READ 5041,MT,DL,B8,B9,B11
5041  FORMAT(1I6,7E10.4)
      READ 5040,(MA(N),N=1,MT)
5040  FORMAT(20I4)
      READ 2000,ALPHA
      READ2000,((DELT(I,J),I=1,MI),J=1,MJ)
      READ 2000,D
      READ2000,(YP(J),J=1,MJ)
      READ 2000,YR
      READ2000,(D1(J),J=1,MJ)
      READ2000,(B1(J),J=1,MJ)
      READ 2000,B2
      READ2000,(B4(J),J=1,MJ)
      READ2000,(B5(J),J=1,MJ)
      READ2000,(B6(J),J=1,MJ)
      READ2000,(DC1(J),J=1,MJ)
      READ2000,(F(J),J=1,MJ)
      READ2000,(T(J),J=1,MJ)
      READ2000,(F1(J),J=1,MJ)
1000  FORMAT(3I6,6E10.4)
2000  FORMAT(8E10.4)
      PRINT5000
5000  FORMAT(6X,2HMI,6X,2HMJ,6X,2HLI,6X,2HQQ,10X,2HQD,10X,2HQT,10X,2HTL,
18 X,4HYFL1,8 X,4HDELY,10X,2HDL)
      PRINT 5010,MI,MJ,LI,QQ,QD,QT,TL,YFL1,DELY,DL
5010  FORMAT(3I8,8E12.4)
      PRINT 5042,B8,B9
5042  FORMAT(6H B8,B9,6X,9E12.4)
      MK=MJ-1
      XMI=MI
      DELX=TL/XMI
      XLI=LI
      A4=XLI*DELX
      A42=A4**2
      A2=TL-A4
      A5=A2**2
      E2=TL/(A42*A5)
      A10=3.*XMI-XLI
      A15=XMI-XLI
      A11=3.*(XMI**2)*A15
      A12=(DELX/XMI)**3/12.
      A17=2.*(XMI**3)

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      A18=A12*(XLI**2)
      A20=2.*XMI-XLI
      DO240 I=1,MI
      XI=I
      XX(I)=(1.5*(XMI/XI)-.5)*((XI/XMI)**2)
      A1=XI*DEIX
      A3=TL-A1
      A32=A3**2
      A8=2.*TL
      A6(I)= A1*(A8*A3-A5-A32)/(2.*A42*A2)
      GL(I)=((XI/XLI)**2)*(1.5*(XLI/XI)-.5)
      HL(I)=(XI-XLI)/XLI
      B38(I)=XI*(XMI-XI)*(2.*XMI-XI)
      IF(I.GT.LI) GO TO 239
      B39(I)=XI*(XI**2-3.*XLI*XI+XLI**2*(3.-XLI/XMI))
      GO TO 240
239  B39(I)=XLI**3*(XMI-XI)/XMI
240  A7(I)= A3*(A8*A2-A5-A32)/(2.*A4*A5)
      DO71 J=1,MJ
      B3(J)=B4(J)+1.
      B35(J)=B5(J)/B3(J)
      B4L(J)=B4(J)*XMI/XLI
      B3L(J)=B4L(J)+1.
      B5L=B5(J)*((XMI/XLI)**3)
      B35L(J)=B5L/B3L(J)
      B6(J)=B6(J)*E2
      B37(J)=A20*A15*.5/(B3(J)*XMI**2) -1.
      B36(J)=2.*B6(J)*A15/(A20*B37(J)+XLI)
      B37(J)=B37(J)*XLI/XMI
71  B40(J)=XLI*A15*(B37(J)*A20+XLI**2/XMI)
      PRINT5001
5001  FORMAT(5X,5HALPHA,7X,1HD,11X,2HYR,10X,2HD1,10X,2HB1,10X,2HB2,10X,
12HB3,10X,2HR4)
      PRINT5011,ALPHA ,D ,YR ,D1(1),B1(1),B2 ,B3(1),B4(1)
5011  FORMAT(10E12.4)
      PRINT 5002,((DELT(I,J),I=1,MI),J=1,MJ)
5002  FORMAT(5H DELT,5X,10E11.4)
      PRINT 5003,(YP(J),J=1,MJ)
5003  FORMAT(3H YP,7X,10E11.4)
      PRINT 5039,(B4(J),J=1,MJ)
5039  FORMAT (3H B4,7X,10E11.4)
      PRINT 5004,(B5(J),J=1,MJ)
5004  FORMAT(3H B5,7X,10E11.4)
      PRINT 5025,(B6(J),J=1,MJ)
5025  FORMAT(3H B6,7X,10E11.4)
      PRINT5035,(D1(J),J=1,MJ)
5035  FORMAT(3H D1,7X,10E11.4)
      PRINT 5027,(DD1(J),J=1,MJ)
5027  FORMAT(4H DD1,6X,10E11.4)
      PRINT 5028,(F(J),J=1,MJ)
5028  FORMAT(2H F,8X,10E11.4)
      PRINT 5029,(T(J),J=1,MJ)
5029  FORMAT(2H T,8X,10E11.4)
      PRINT 5030,(F1(J),J=1,MJ)
5030  FORMAT(3H F1,7X,10E11.4)
      DO80 J=1,MJ

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      DO8C I=1,MI
      XI=I
80    YI(I,J)=YP(J)+B1(J)*DELX*XI
      Q=QC
      GOTC 60
1     ASSIGN 304 TO MM
      GOTC 305
304    Q=Q+QD
      IF(C-QT)60,60,61
61    STOP
60    DO50 J=1,MJ
      ZF(J)=DELY
      ZL(J)=DL
      DD(J)=DD1(J)*(1.+ALPHA*T(J)*Q)
      DO50 I=1,MI
50    XDELT(I,J)=Q*DELT(I,J)
      M=M-M
      K=1
      DO 15 J=1,MJ
      SUM=0.
      SUM2=0.
      DO 15 I=1,MI
      TERM=ALPHA*XDELT(I,J)*DELX/D
      SUM=SUM+TERM
      TERM2=(SUM-TERM/2.)*DELX
      SUM2=SUM2+TERM2
15    YT(I,J)=SUM2
      DO635 J=1,MJ
      DO635 I=1,MI
635    YPT(I,J)=YI(I,J)+YT(I,J)
      PRINT 5043,((YT(I,J),I=1,MI),J=1,MJ)
5043   FORMAT (3H YT,7X,10E11.4)
      DO19 J=1,MK
19     YFR(J)=YP(J+1)-D1(J)
      YFR(MJ)=YP(MJ)+YR      +Q*811
      DO18 J=2,MJ
18     YFL(J)=YP (J-1)+D1(J-1)
      YFL(1)=YFL1
      DO551 J=1,MK
551    YFLR(J)=YP(J+1)-DD1(J)
      YFLR(MJ)=YP(MJ)+1.
      DO552 J=2,MJ
552    YFLL(J)=YP(J-1)+DD1(J-1)
      YFLL(1)=0.
125    M=M+1
      IF(M-MA(MT))103,103,304
C
C      FIT TOP-LEVEL CONSTRAINTS
C
103   DO 607 J=1,MJ
      MSQ=0
      PR(J)=PL(J)=0.
      IF(YFR(J).LT.YFL(J)) GO TO 612
      IF(YPT(MI,J).GE.YFL(J)) GO TO 609
      CL=YFL(J)-YPT(MI,J)
      GO TO 613

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609 IF(YPT(MI,J).GT.YFR(J)) GO TO 611
    DO 610 I=1,MI
610 AY(I,J)=YPT(I,J)
    GO TO 618
611 CL=YFR(J)-YPT(MI,J)
    GO TO 613
612 CL=.5*(YFL(J)+YFR(J))-YPT(MI,J)
    MSQ=1
613 SGN=1.
    IF(CL.LT.0.) SGN=-1.
    B2S=B2-SGN*TL*B1(J)
C
C     MODE 1
C
    C1=CL $ ASSIGN 618 TO M1
    IF(ABSF(CL).LE.B2S) GO TO 614
    C1=SGN*B2S $ ASSIGN 616 TO M1
614 DO 615 I=1,MI
    XI=I
615 AY(I,J)=YPT(I,J)+C1*XI/XMI
    C2=CL-C1
    GO TO M1
C
C     MODE 2
C
616 DO 617 I=1,MI
    XI=I
617 AY(I,J)=AY(I,J)+C2*(B4(J)+XX(I))*XI/(XMI*B3(J))
    P=-C2*B35(J)
    IF(SGN.LT.0.) PR(J)=P
    IF(SGN.GT.0.) PL(J)=P
618 PLP(J)=PL(J)
    PRP(J)=PR(J)
C
C     SQUASHING
C
    IF(MSQ.EQ.0) GO TO 607
    PD=(YFL(J)-YFR(J))*F(J)/F1(J)
    PL(J)=PL(J)-PD
    PR(J)=PR(J)+PD
607 CONTINUE
    DO241 J=1,MJ
    WL(J)=0.
241 WR(J)=0.
C
C     SUPERIMPCSE BUTTON-LEVEL CONSTRAINTS
C
    DO 900 J=1,MJ
    MCDE(J)=0 $ MSQ=0
    IF(YFLR(J).LT.YFLL(J)) GO TO 902
    IF(AY(LI,J).GE.YFLL(J)) GO TO 901
    CL=YFLL(J)-AY(LI,J)
    GO TO 903
901 IF(AY(LI,J).LE.YFLR(J)) GO TO 900
    CL=YFLR(J)-AY(LI,J)
    GO TO 903

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902 CL=.5*(YFLL(J)+YFLR(J))-AY(LI,J)
   MSQ=1
903 YFF=YFR(J) $ SGN=1.
   IF(CL.GT.0.) GO TO 904
   YFF=YFL(J) $ SGN=-1.
C   CHOOSE INITIAL MODE
904 IF(YFR(J).LT.YFL(J)) YFF=AY(MI,J)
   IF(PLP(J).NE.PRP(J)) GO TO 908
C
C   MODE 1
C
   MODE(J)=1
   DS=A4/B9*(SGN*B8+B9*(4.*AY(1,J)-AY(2,J)-3.*YP(J))/(2.*DELX))
   DM=A4/TL*(YFF-AY(MI,J))
   IF(ABSF(DM).LT.ABSF(DS)) GO TO 905
C   1 TO 2
   C1=DS
   ASSIGN 916 TO M1
   GO TO 906
C   1 TO 4
905 C1=DM
   DNCZ=-B9*(DS-DM)/A4
   ASSIGN 922 TO M1
906 IF(ABSF(C1).LT.ABSF(CL)) GO TO 912
   C1=CL $ ASSIGN 931 TO M1
912 DO 907 I=1,M1
   XI=I
907 AY(I,J)=AY(I,J)+C1*XI/XLI
   CL=CL-C1
   GO TO M1
C
C   MODES 3 AND 3A
C
908 IF(PLP(J).EQ.0.) GO TO 909
   PP=PLP(J)
   IF(CL) 911,900,910
909 PP=PRP(J)
   IF(CL) 910,900,911
C   MODE 3
910 MODE(J)=3
   C3M=-PP/(B36(J)*B37(J))
   IF(ABSF(C3M).GE.ABSF(CL)) GO TO 914
   C3=C3M
   ASSIGN 916 TO M3
   GO TO 915
C   MODE 3A
911 MODE(J)=30
   C3M=-PP/(B36(J)*(B37(J)+XLI/XMI))
   IF(ABSF(C3M).GE.ABSF(CL)) GO TO 914
   C3=C3M
   ASSIGN 922 TO M3
   DNCZ=-SGN*2.*B8
   GO TO 915
C   MODES 3,3A
914 C3=CL
   ASSIGN 931 TO M3

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915 DO 932 I=1,MI
932 AY(I,J)=AY(I,J)+C3*(B37(J)*B38(I)+B39(I))/B40(J)
    WB=C3*B36(J)
    WA=WB*B37(J)
    IF(CL.LT.0.) WR(J)=WB
    IF(CL.GT.0.) WL(J)=WB
    IF(PP.LT.0.) PL(J)=PL(J)+WA
    IF(PP.GT.0.) PR(J)=PR(J)+WA
    CL=CL-C3
    GO TO M3

C
C
C    MODE 2

916 MODE(J)=2
    C2M=(YFF-AY(MI,J))/(1.+HL(MI)*(1.5+B4L(J))/B3L(J))
    IF(ABSF(C2M).GE.ABSF(CL)) GO TO 917
    C2=C2M
    ASSIGN 929 TO M2
    GO TO 918
917 C2=CL
    ASSIGN 931 TO M2
918 DO 919 I=1,MI
    XI=I
    IF(I.GT.LI) GO TO 920
    YW=C2*(B4L(J)+GL(I))*XI/(XLI*B3L(J))
    GO TO 919
920 YW=C2*(1.+(1.5+B4L(J))*HL(I))/B3L(J))
919 AY(I,J)=AY(I,J)+YW
    WB=-B35L(J)*C2
    IF(CL.LT.0.) WR(J)=WR(J)+WB
    IF(CL.GT.0.) WL(J)=WL(J)+WB
    CL=CL-C2
    GO TO M2

C
C
C    MODE 4

922 MODE(J)=4
    C4M=-DNOZ*2.*XLI*A2/(B9*A20)
    IF(ABSF(C4M).GE.ABSF(CL)) GO TO 923
    C4=C4M
    ASSIGN 929 TO M4
    GO TO 924
923 C4=CL
    ASSIGN 931 TO M4
924 DO 926 I=1,MI
    IF(I.GT.LI) GO TO 925
    YW=C4*A6(I)
    GO TO 926
925 YW=C4*A7(I)
926 AY(I,J)=AY(I,J)+YW
    WB=-C4*B6(J)
    WA=-WB*XLI/XMI
    IF(CL.LT.0.) GO TO 927
    WL(J)=WL(J)+WB
    PR(J)=PR(J)+WA
    GO TO 928

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927  WR(J)=WR(J)+WB
     PL(J)=PL(J)+WA
928  CL=CL-C4
C
C      MCDE 5
C
929  MCDE(J)=5
     WB=CL*B36(J)
     WA=WB*B37(J)
     DO 933 I=1,MI
933  AY(I,J)=AY(I,J)+CL*(B37(J)*B38(I)+B39(I))/B40(J)
     IF(CL.LT.0.) GO TO 930
     WL(J)=WL(J)+WB
     PR(J)=PR(J)+WA
     GO TO 931
930  WR(J)=WR(J)+WB
     PL(J)=PL(J)+WA
C
C      BUTTON-LEVEL SQUASHING
C
931  IF(MSG.EQ.0) GO TO 900
     MCDE(J)=-MCDE(J)
     CM=YFLR(J)-YFLL(J)
     WL(J)=WL(J)+CM*(F(J)-3.)/F1(J)
     WR(J)=WR(J)-CM*(F(J)+3.)/F1(J)
900  CONTINUE
     DO572 N=1,MT
     IF(M-MA(N))572,320,572
572  CONTINUE
     GOTC 303
320  ASSIGN 303 TO MM
305  PRINT 5023,M,Q
5023  FORMAT(4H M,Q,1I6,1E12.4)
     PRINT 5005,((AY(I,J),I=1,MI),J=1,MJ)
5005  FORMAT(2H Y,8X,10F11.5)
     PRINT 5006,(MCDE(I),I=1,MJ)
5006  FORMAT(5H MCDE20I4)
     PRINT 5024,(ZF(J),J=1,MJ)
5024  FORMAT(* ZF*7X,10E11.4)
     PRINT 5008,(YFR(J),J=1,MJ)
5008  FORMAT(4H YFR,6X,10F11.5)
     PRINT 5009,(YFL(J),J=1,MJ)
5009  FORMAT(4H YFL,6X,10F11.5)
     PRINT 5026,(ZL(J),J=1,MJ)
5026  FORMAT(* ZL*7X,10E11.4)
     PRINT5036,(YFLR(J),J=1,MJ)
5036  FORMAT(5H YFLR,5X,10F11.5)
     PRINT 5037,(YFLL(J),J=1,MJ)
5037  FORMAT(5H YFLL,5X,10F11.5)
306  PRINT 5021,(PL(J),J=1,MJ)
5021  FORMAT(3H PL,7X,10E11.4)
     PRINT 5022,(PR(J),J=1,MJ)
5022  FORMAT(3H PR,7X,10E11.4)
     PRINT 5032,(WL(J),J=1,MJ)
5032  FORMAT(3H WL,7X,10E11.4)
     PRINT 5031,(WR(J),J=1,MJ)

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5031  FORMAT(3H WR,7X,10E11.4)
      PRINT 5033,(CD(J),J=1,MJ)
5033  FORMAT(3H DC,7X,10E11.4)
      GOTC MM
303   DO 107 J=1,MJ
      IF(ABSF(AY(LI,J)-Y(LI,J)).GT.1.E-5) GO TO 119
107   CONTINUE
      GO TO 1
119   IF(K.GT.1) GO TO 131
      DO 132 J=1,MJ
      PLB(J)=PL(J)
      PRB(J)=PR(J)
132   YY(J)=AY(MI,J)
      GO TO 133
131   DO 102 J=1,MJ
      PLB(J)=(PL(J)+APL(J))/2.
      PRB(J)=(PR(J)+APR(J))/2.
102   YY(J)=(Y(MI,J)+AY(MI,J))/2.
133   DO 701 J=2,MJ
      PMAX=(-PLB(J)+PRB(J))/2.
      IF(PMAX.EQ.0.) PMAX=.1
      P=PRB(J)+PLB(J+1)
      IF(J.EQ.MJ) P=0.
      DEL=(PLB(J)+PRB(J-1)+P)/PMAX
      IF(DEL.EQ.0.) GO TO 701
      FAC=1.
      IF(DEL/ZF(J).LT.0.) FAC=-.8
      ZF(J)=FAC*ZF(J)
      DELF=ZF(J)
      DELA=ABSF(DEL)
      IF(DELA.LT.1.) DELF=DELA*DELF
      YY(J)=YY(J)+DELF
701   CONTINUE
750   IF(YY(MJ).GT.YFR(MJ)) YY(MJ)=YFR(MJ)
      DO100 J=1,MK
100   YFR(J)=YY(J+1)-C1(J)
      DO101 J=2,MJ
101   YFL(J)=YY(J-1)+C1(J-1)
      IF(K.GT.1) GO TO 559
      DO560 J=1,MJ
      WLB(J)=WL(J)
      WRB(J)=WR(J)
560   YZ(J)=AY(LI,J)
      GO TO 561
559   DO 562 J=1,MJ
      WLB(J)=(WL(J)+AWL(J))/2.
      WRB(J)=(WR(J)+AWR(J))/2.
562   YZ(J)=(Y(LI,J)+AY(LI,J))/2.
561   DO 801 J=2,MJ
      WMAX=(-WLB(J)+WRB(J))/2.
      IF(WMAX.EQ.0.) WMAX=.1
      W=WRB(J)+WLB(J+1)
      IF(J.EQ.MJ) W=0.
      DEL=(WLB(J)+WRB(J-1)+W)/WMAX
      IF(DEL.EQ.0.) GO TO 801
      FAC=1.

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      IF(DEL/ZL(J).LT.0.) FAC=-.8
      ZL(J)=FAC*ZL(J)
      DELF=ZL(J)
      DELA=ABSF(DEL)
      IF(DELA.LT.1.) DELF=DELA*DELF
      YZ(J)=YZ(J)+DELF
801  CONTINUE
      832  IF(YZ(MJ).GT.YFLR(MJ)) YZ(MJ)=YFLR(MJ)
          DD(1)=DD1(1)*(1.+ALPHA*Q*T(1))+WLB(2)*F1(1)/6.
          DO547 J=1,MK
547   YFLR(J)=YZ(J+1)-DD(J)
          DO548 J=2,MJ
548   YFLL(J)=YZ(J-1)+DD(J-1)
          K=2
          IF(YFR(1).LT.YFL(1)) YFR(1)=YFL(1)
          IF(YFLR(1).LT.YFLL(1)) YFLR(1)=YFLL(1)
          IF(YFL(MJ).GT.YFR(MJ)) YFL(MJ)=YFR(MJ)
          IF(YFLL(MJ).GT.YFLR(MJ)) YFLL(MJ)=YFLR(MJ)
          DO110 J=1,MJ
          APL(J)=PL(J)
          APR(J)=PR(J)
          AWL(J)=WL(J)
          AWR(J)=WR(J)
          DO110 I=1,MI
110   Y(I,J)=AY(I,J)
      GOTC 125
      END

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[illegible]

[illegible]

[illegible]

ZL	1.3744-004	2.0000-003	2.0000-003	2.0000-003	2.0000-003					
YFLR	0.00000	2.21709	4.25481	6.35255	8.45038	10.55825	12.69343	14.79130	16.89902	18.99712
YFLR	21.10479	23.22259	25.34250	27.45250	30.55000					
YFLL	0.00000	2.21771	4.25534	6.35308	8.45092	10.55875	12.66647	14.76414	16.89905	18.99695
YFLL	21.10476	23.22244	25.33979	27.44759	29.54750					
PL	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	-1.4496+002	0.0000+000	0.0000+000
PL	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000					
PR	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	1.4994+002	0.0000+000	0.0000+000	0.0000+000
PR	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000					
WL	0.0000+000	-1.2961+001	-3.2909+001	-5.5123+001	-7.8836+001	-9.3532+001	-1.9902+002	0.0000+000	-1.1619+002	0.0000+000
WL	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000					
WR	0.0000+000	3.8882+001	5.4849+001	8.3005+001	1.2021+002	1.6598+002	0.0000+000	2.1616+002	1.1453+002	0.0000+000
WR	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000					
CD	2.2177+000	2.0380+000	2.0980+000	2.0981+000	2.1081+000	2.1080+000	2.0979+000	2.1077+000	2.0979+000	2.1078+000
DD	2.1175+000	2.1175+000	2.1075+000	2.0975+000	-0.0000+000					
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	1.7175-003	4.4998-003	7.8661-003	1.1232-002	1.4599-002
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	3.4350-004	1.8892-003	4.9463-003	8.6561-003	1.2366-002	1.6076-002
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	4.1220-004	2.2327-003	5.8051-003	1.0133-002	1.4461-002	1.8789-002
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	4.1220-004	2.2327-003	5.8051-003	1.0133-002	1.4461-002	1.8789-002
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	3.4350-004	1.8549-003	4.8089-003	8.3813-003	1.1954-002	1.5526-002
YT	0.0000+000	0.0000+000	0.0000+000	1.3740-004	1.0992-003	4.3967-003	1.1885-002	2.5007-002	4.4380-002	7.0176-002
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	-5.1524-004	-2.7480-003	-8.0035-003	-1.7347-002	-3.1224-002	-4.9635-002
YT	0.0000+000	0.0000+000	1.0305-004	6.1829-004	1.7862-003	3.5380-003	5.4272-003	6.7325-003	6.9386-003	5.9081-003
YT	0.0000+000	0.0000+000	1.3740-004	7.2134-004	2.6449-003	7.5913-003	1.7003-002	3.1430-002	5.1009-002	7.5878-002
YT	0.0000+000	0.0000+000	1.0305-004	4.8089-004	1.7175-003	5.1524-003	1.1885-002	2.1949-002	3.4865-002	5.0185-002
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	2.7480-004	1.5114-003	4.2937-003	8.7248-003	1.4633-002	2.1846-002
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000
YT	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000
M,Q	312	4.0000-001								
Y	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Y	2.21919	2.21839	2.21758	2.21677	2.21627	2.21687	2.21885	2.22141	2.22397	2.22653
Y	4.25866	4.25729	4.25590	4.25450	4.25344	4.25358	4.25523	4.25753	4.25983	4.26213
Y	6.35838	6.35656	6.35459	6.35252	6.35081	6.35050	6.35194	6.35414	6.35634	6.35854
Y	8.45757	8.45514	8.45271	8.45028	8.44826	8.44765	8.44879	8.45069	8.45259	8.45448
Y	10.56788	10.56524	10.56222	10.55893	10.55586	10.55394	10.55346	10.55359	10.55373	10.55387
Y	12.67736	12.67440	12.67103	12.66731	12.66387	12.66216	12.66415	12.67138	12.68462	12.70416
Y	14.78288	14.78612	14.78974	14.79381	14.79784	14.80066	14.80088	14.79733	14.78946	14.77716
Y	16.89220	16.89478	16.89775	16.90133	16.90566	16.91059	16.91565	16.92013	16.92352	16.92566
Y	18.99196	18.99415	18.99662	18.99954	19.00370	19.01070	19.02199	19.03817	19.05942	19.08591
Y	21.10177	21.10372	21.10588	21.10834	21.11161	21.11697	21.12552	21.13733	21.15194	21.16894
Y	23.22158	23.22330	23.22513	23.22707	23.22938	23.23272	23.23766	23.24430	23.25245	23.26191
Y	25.34138	25.34279	25.34424	25.34571	25.34721	25.34873	25.35027	25.35182	25.35337	25.35494
Y	27.45094	27.45189	27.45283	27.45377	27.45471	27.45566	27.45660	27.45754	27.45849	27.45943
Y	29.55084	29.55168	29.55252	29.55336	29.55420	29.55504	29.55588	29.55672	29.55756	29.55840
MODE	0	-1 -2 -2	-1 -2 30	30 -2 -5	5 0 0	0 1 1				
ZF	3.0000-003	3.0000-003	3.0000-003	3.0000-003	3.0000-003	3.0000-003	2.4000-003	5.0332-004	5.0332-004	3.0000-003
ZF	6.2308-007	1.3194-004	-2.1923-008	3.0000-003	3.0000-003	3.0000-003				
YFR	0.03552	2.24812	4.28553	6.38147	8.47086	10.62116	12.70416	14.84263	17.01289	19.08591
YFR	21.16894	23.26191	25.37640	27.48339	29.64500					
YFL	-0.00000	2.19100	4.24052	6.33512	8.43153	10.53747	12.63686	14.77716	16.86016	18.99863
YFL	21.16889	23.26191	25.35494	27.43791	29.53240					
ZL	2.0000-003	-6.5536-004	1.3744-004	1.3744-004	1.3744-004	-1.6000-003	-1.6000-003	8.7961-005	5.6295-005	-1.8093-011
ZL	1.4757-005	1.7014-007	1.4757-005	2.0000-003	2.0000-003					
YFLR	0.00000	2.21531	4.25263	6.34999	8.44741	10.55515	12.69986	14.79784	16.90563	19.00368
YFLR	21.11187	23.22971	25.34920	27.45569	30.55000					
YFLL	0.00000	2.21724	4.25425	6.35163	8.44911	10.55658	12.66387	14.76133	16.90570	19.00372
YFLL	21.11161	23.22910	25.34687	27.45471	29.55420					
PL	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	-3.0035+002	0.0000+000	0.0000+000
PL	0.0000+000	-2.2839+001	-5.9416+000	0.0000+000	0.0000+000					
PR	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	3.0675+002	0.0000+000	0.0000+000	4.6609+001

[illegible]

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Y	2.21849	2.21684	2.21510	2.21328	2.21206	2.21302	2.21673	2.22161	2.22649	2.23137	2.25656	6.35755	8.45713	10.56681	12.67681	14.78466	16.89409	18.95531	21.01070	23.06222	25.11022	27.16404	29.21844	31.27444	33.33222	35.39044	37.44888	39.50777	41.56722	43.62722	45.68722	47.74722	49.80722	51.86722	53.92722	55.98722	58.04722	60.10722	62.16722	64.22722	66.28722	68.34722	70.40722	72.46722	74.52722	76.58722	78.64722	80.70722	82.76722	84.82722	86.88722	88.94722	91.00722	93.06722	95.12722	97.18722	99.24722	101.30722	103.36722	105.42722	107.48722	109.54722	111.60722	113.66722	115.72722	117.78722	119.84722	121.90722	123.96722	126.02722	128.08722	130.14722	132.20722	134.26722	136.32722	138.38722	140.44722	142.50722	144.56722	146.62722	148.68722	150.74722	152.80722	154.86722	156.92722	158.98722	161.04722	163.10722	165.16722	167.22722	169.28722	171.34722	173.40722	175.46722	177.52722	179.58722	181.64722	183.70722	185.76722	187.82722	189.88722	191.94722	194.00722	196.06722	198.12722	200.18722	202.24722	204.30722	206.36722	208.42722	210.48722	212.54722	214.60722	216.66722	218.72722	220.78722	222.84722	224.90722	226.96722	229.02722	231.08722	233.14722	235.20722	237.26722	239.32722	241.38722	243.44722	245.50722	247.56722	249.62722	251.68722	253.74722	255.80722	257.86722	259.92722	261.98722	264.04722	266.10722	268.16722	270.22722	272.28722	274.34722	276.40722	278.46722	280.52722	282.58722	284.64722	286.70722	288.76722	290.82722	292.88722	294.94722	297.00722	299.06722	301.12722	303.18722	305.24722	307.30722	309.36722	311.42722	313.48722	315.54722	317.60722	319.66722	321.72722	323.78722	325.84722	327.90722	329.96722	332.02722	334.08722	336.14722	338.20722	340.26722	342.32722	344.38722	346.44722	348.50722	350.56722	352.62722	354.68722	356.74722	358.80722	360.86722	362.92722	364.98722	367.04722	369.10722	371.16722	373.22722	375.28722	377.34722	379.40722	381.46722	383.52722	385.58722	387.64722	389.70722	391.76722	393.82722	395.88722	397.94722	400.00722	402.06722	404.12722	406.18722	408.24722	410.30722	412.36722	414.42722	416.48722	418.54722	420.60722	422.66722	424.72722	426.78722	428.84722	430.90722	432.96722	435.02722	437.08722	439.14722	441.20722	443.26722	445.32722	447.38722	449.44722	451.50722	453.56722	455.62722	457.68722	459.74722	461.80722	463.86722	465.92722	467.98722	470.04722	472.10722	474.16722	476.22722	478.28722	480.34722	482.40722	484.46722	486.52722	488.58722	490.64722	492.70722	494.76722	496.82722	498.88722	500.94722	503.00722	505.06722	507.12722	509.18722	511.24722	513.30722	515.36722	517.42722	519.48722	521.54722	523.60722	525.66722	527.72722	529.78722	531.84722	533.90722	535.96722	538.02722	540.08722	542.14722	544.20722	546.26722	548.32722	550.38722	552.44722	554.50722	556.56722	558.62722	560.68722	562.74722	564.80722	566.86722	568.92722	570.98722	573.04722	575.10722	577.16722	579.22722	581.28722	583.34722	585.40722	587.46722	589.52722	591.58722	593.64722</
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ZF	8.4442-005	1.3154-004	2.2136-005	3.4588-005	2.4000-003					
YFR	0.04392	2.25476	4.28721	6.37554	8.45970	10.63791	12.72091	14.89331	17.08289	19.15586
YFR	21.23877	23.33167	25.42465	27.52856	29.64500					
YFL	-0.00000	2.19100	4.24892	6.34176	8.43321	10.53154	12.62570	14.79391	16.87691	19.04931
YFL	21.23889	23.33186	25.42477	27.50767	29.58065					
ZL	2.0000-003	-4.1943-004	2.1475-004	8.1920-004	5.2429-004	-1.6000-003	-1.6000-003	8.1920-004	-2.2836-008	-7.0369-005
ZL	-8.7112-008	2.3058-005	1.0141-006	5.6295-005	-1.9807-006					
YFLR	0.00000	2.20604	4.24090	6.33583	8.43061	10.53846	12.72004	14.81845	16.92639	19.02514
YFLR	21.13358	23.25102	25.36849	27.47597	30.55000					
YFLL	0.00000	2.21485	4.24938	6.34435	8.43959	10.54481	12.64925	14.74645	16.92712	19.02564
YFLL	21.13385	23.25119	25.36858	27.47602	29.57349					
PL	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	0.0000+000	-1.9109+003	0.0000+000	0.0000+000
PL	-4.7526+002	-8.4875+002	-1.0099+003	-7.2204+000	0.0000+000					
PR	0.0000+000	0.0000+000	0.0000+000	-1.6544-024	0.0000+000	0.0000+000	1.9359+003	0.0000+000	0.0000+000	1.2297+003
PR	1.2832+003	1.0241+003	6.0444+002	0.0000+000	0.0000+000					
WL	0.0000+000	-1.8352+002	-5.3001+002	-8.8695+002	-1.3102+003	-1.1859+003	-3.7130+003	0.0000+000	-2.6061+003	-4.8813+003
WL	-3.2582+003	-1.5870+003	-4.1804+002	-6.2289+002	-4.0276+002					
WR	0.0000+000	5.5468+002	8.8335+002	1.3268+003	1.8371+003	1.6725+003	0.0000+000	3.4785+003	2.5847+003	1.9784+003
WR	1.1714+003	8.4652+002	9.5798+002	2.5898+002	0.0000+000					
DD	2.2149+000	2.0391+000	2.0992+000	2.0995+000	2.1095+000	2.1092+000	2.0988+000	2.1083+000	2.0989+000	2.1086+000
DD	2.1175+000	2.1175+000	2.1075+000	2.0975+000	-0.0000+000					

APPENDIX C

Reflector Heat-transfer Dynamics1. Reflector Subassembly; Row 7

From Section III.B, the relation between row 7 differential temperature $\Delta T_7(s)$ and power $P(s)$ is

$$\frac{\Delta T_7(s)}{P(s)} = \frac{\theta_{7\text{Hexin}}(s) - \theta_{7\text{Hexout}}(s)}{P(s)}.$$

It is convenient to evaluate the above expression by evaluating the numerator terms separately and then combining them.

From Section III.B,

$$\frac{\theta_{7\text{Hexin}}(s)}{P(s)} = \frac{1}{1 + \tau_{6C}s} \left\{ K_1 - \frac{1.5R_1(K_1 - K_2R_{VR})}{2R_1 + R_{VR}} \frac{1 + sC_1(2R_2 + R_{VR})/[2(1 - K_2R_{VR}/K_1)]}{1 + sC_1(2R_2 + R_{VR} - R_{VR}^2)/2} \right\},$$

where the C's, R's, and K's are defined in Fig. 9.

The calculations will be based on a unit vertical slice of the subassembly, which is one element wide, as shown in Fig. 9A. The length again will be taken as 70 cm, as for the row 9 calculation in Appendix A. Since the element discharges approximately half its heat to the hexagon side and half towards the subassembly interior, one-half the element helps to determine the hexagon temperature.

Then,

$$C_1/2 = \frac{\pi}{2} (1.25/2)^2 \times 70 \times 7.7 \times 0.115 = 38 \text{ cal/}^\circ\text{C},$$

$$R_1/2 = R_{\text{hex}}/2 + R_{\text{film}},$$

and

$$R_{\text{hex}}/2 = \frac{0.1/2}{0.048 \times 1.25 \times 70} = 0.012.$$

Assume $r_{\text{film}} \approx 0.008$. Then,

$$\frac{R_1}{2} \approx 0.012 + 0.008 = 0.020^\circ\text{C/cal-sec}^{-1}$$

and

$$2R_2 = \frac{2 \log_e [1.25 / (0.707 \times 1.25)]}{2\pi \times 0.048 \times 70} = 0.026^\circ\text{C}/\text{cal-sec}^{-1}.$$

From the row 9 calculation in Appendix A, one full element has $R_{VR} = 0.083$. The value for one-half element then is

$$R_{VR} = 2 \times 0.083 = 0.166^\circ\text{C}/\text{cal-sec}^{-1}.$$

The radial constants are considered to be lumped at the average axial condition, which is taken as approximately half the outlet condition.

Then,

$$\theta_{7Cin} = (168/2)(5/9) = 46.7^\circ\text{C} \quad (\text{see Ref. 1, Fig. 5})$$

and

$$Q(R_{VR}) = 46.7/0.166 = 281 \text{ cal/sec.}$$

The heat input per sodium channel in row 7 (six channels on the side of the subassembly in contact with row 6) is

$$Q_{in} = Q_{total}/6 \approx 3 \text{ kW} \times 238.9/6 \approx 119.4 \text{ cal/sec},$$

where Q_{total} is the radial heat flow from row 6 to row 7 per subassembly.

Then,

$$Q_0 = K_2 \times 45 = Q(R_{VR}) - Q_{in}$$

and

$$K_2 = (281 - 119)/45 = 3.60 \text{ cal/MW-sec.}$$

Similarly, for a row 7 average half-element facing row 8,

$$\theta_{7Hexout} = (60/2)(5/9) = 16.7^\circ\text{C},$$

where 60°F is the average of the row 7 outer and row 8 inner channel temperatures of 81 and 43°F , respectively, explained in Ref. 1. Then,

$$Q_{00} = K_3 \times 45 = \frac{\theta_{7Hexout}}{R_{VR}} = 100 \text{ cal/sec}$$

and

$$K_3 = \frac{16.7}{0.166 \times 45} = 2.20 \text{ cal/MW-sec.}$$

(We may treat row 7 outer and row 8 inner channels separately because a check of heat flow between these points indicates a flow of about 14 cal/sec, which is small compared to the value of Q_{00} above and the value for row 8, which will be seen in Section 2 of this appendix.)

Also,

$$\theta_{6C} = Q_{in} \times 2R_1 + \theta_{7Cin} = K_1 P,$$

$$K_1 = \frac{119.4 \times 2 \times 0.04 + 46.7}{45} = 1.24,$$

and

$$\theta_{6C} = \frac{1.24P}{1 + 0.19s},$$

where $\tau_{6C} = 0.19$ sec as in Appendix A, but with added transport time of 0.02 sec. To evaluate,

$$\frac{\theta_{7Hexin}(s)}{P(s)} = \left\{ 1.24 - \frac{0.156(1 + 14.1s)}{1 + 6.3s} \right\} \frac{1}{1 + \tau_{6C}s} = \frac{1.08(1 + 5.1s)}{(1 + 6.3s)(1 + 0.19s)}.$$

From Section III.B,

$$\frac{\theta_{7Hexout}(s)}{P(s)} = \frac{K_3 R_{VR}}{1 + \frac{sC_1(R_{VR} + 2R_2)}{2}}$$

To evaluate,

$$\frac{\theta_{7Hexout}(s)}{P(s)} = \frac{2.2 \times 0.166}{1 + 38s(0.166 + 0.026)} = \frac{0.365}{1 + 7.3s}.$$

Then,

$$\frac{\Delta T_7(s)}{P(s)} = \frac{1}{1 + 0.19s} \left\{ \frac{1.08(1 + 5.1s)}{1 + 6.3s} - \frac{0.365}{1 + 7.3s} \right\}$$

Rewriting results in

$$\frac{\Delta T_7(s)}{P(s)} = \frac{0.72}{1 + 0.19s} \cdot \frac{(1 + 5.8s)(1 + 9.7s)}{(1 + 6.3s)(1 + 7.3s)}$$

Also, to a good approximation,

$$\frac{\Delta T_7(s)}{P(s)} = \frac{0.72(1 + 9.7s)}{(1 + 0.19s)(1 + 7.3s)}$$

2. Reflector Subassembly; Row 8

The calculations for row 8 are made in the same way as for row 7, noting that the heat source for row 8 is row 9, but for row 7 the source is row 6, and also noting that the R's and C's are identical to those above for row 7.

As before, the average temperature between rows 7 and 8 is

$$\theta_{8\text{Hexin}} = (60/2)(5/9) = 16.7^\circ\text{C},$$

$$Q_{00} = K_6 \times 45 = \theta_{8\text{Hexin}}/R_{VR} = 100 \text{ cal/sec},$$

$$K_6 = 16.7/(45 \times 0.166) = 2.22 \text{ cal/MW-sec},$$

$$\theta_{8\text{Cout}} = (110/2)(5/9) = 30.6^\circ\text{C},$$

and

$$Q_{(R_{VR})} = 30.6/0.166 = 184 \text{ cal/sec}.$$

The input per sodium channel in row 8 (six channels on side of subassembly in contact with row 9) is

$$Q_{\text{in}} = Q_{\text{total}}/6 \approx 3 \text{ kW} \times 238.9/6 \approx 119.4 \text{ cal/sec}.$$

Then,

$$Q_0 = K_5 P = Q_{(R_{VR})} - Q_{\text{in}},$$

$$K_5 = (184 - 119)/45 = 1.44 \text{ cal/MW-sec},$$

$$\theta_9\text{C} = Q_{\text{in}} \times 2R_1 + \theta_{8\text{Cout}} = K_4 P,$$

and

$$K_4 = (119.4 \times 2 \times 0.04 + 30.6)/45$$

$$= 0.89 \text{ cal/MW-sec.}$$

From Section III.B,

$$\frac{\theta_{s\text{Hexout}}(s)}{P(s)} = \frac{1}{1 + \tau_{9C}s} \left\{ K_4 - \frac{1.5R_1(K_4 - K_5R_{VR})}{2R_1 + R_{VR}} \frac{1 + sC_1(2R_2 + R_{VR})/[2(1 - K_5R_{VR}/K_4)]}{1 + sC_1(2R_2 + R_{VR} - R_{VR}^2)/2} \right\}$$

To evaluate,

$$\frac{\theta_{s\text{Hexout}}(s)}{P(s)} = \frac{1}{1 + 6.2s} \left[0.89 - \frac{0.159(1 + 10.0s)}{1 + 6.3s} \right]$$

$$= \frac{0.73(1 + 5.5s)}{(1 + 6.3s)(1 + 6.2s)} \approx \frac{0.73}{1 + 6.3s}$$

From Section III.B,

$$\frac{\theta_{s\text{Hexin}}(s)}{P(s)} = \frac{K_6R_{VR}}{1 + \frac{sC_1(R_{VR} + 2R_2)}{2}} = \frac{2.22 \times 0.166}{1 + 38s(0.166 + 0.026)} = \frac{0.37}{1 + 7.3s}$$

From Section III.B,

$$\frac{\Delta T_8(s)}{P(s)} = \frac{\theta_{s\text{Hexin}}(s) - \theta_{s\text{Hexout}}(s)}{P(s)} = - \left(\frac{0.73}{1 + 6.3s} - \frac{0.37}{1 + 7.3s} \right)$$

$$= - \frac{0.36(1 + 8.4s)}{(1 + 7.3s)(1 + 6.3s)}$$

The above results are in a form suitable for display in the log-log frequency domain. However, the time domain is more useful for computing temperature profiles with the PROFILE program. Therefore they are transformed into the time domain in Appendix D; which describes PROFILE.

APPENDIX D

PROFILE Temperature Calculations

The computer program PROFILE calculates the row dynamic temperature-time function $C(t)$ using a specified rod-drop reactor power-time function as the input. An input like that in Fig. D.1 may be represented by an initial ramp with an exponential following the end of the ramp. The Laplace transform of such an input has the form

$$\frac{\Delta P}{P}(s) = \frac{\left(\frac{\Delta P}{P}\right)_{t_0}}{s^2}, \quad t \leq t_0;$$

$$\frac{\Delta P}{P}(s) = \frac{\left(\frac{\Delta P}{P}\right)_{t_0}/t_0}{s^2} - \exp(-t_0 s) \left[\frac{\left(\frac{\Delta P}{P}\right)_{t_0}/t_0}{s^2} - \frac{K}{\tau_K} \cdot \frac{1}{s\left(s + \frac{1}{\tau_K}\right)} \right], \quad t > t_0.$$

Fig. D.1
PROFILE Input Function

The function $C(t)$ resulting when the above input is applied to a row transfer function will have two basic forms as shown in Section III.C. Slightly rearranged, these forms are

$$1. \quad C(t) = C_0[t - \sigma_1 + \sigma_1 \exp(-t/\sigma_1)], \quad t \leq t_0;$$

$$C(t) = K_B + C_0 \sigma_1 [\exp(-t/\sigma_1) - \exp(-t'/\sigma_1)] \\ + K \frac{\tau_K \exp(-t'/\tau_K) - \sigma_1 \exp(-t'/\sigma_1)}{\sigma_1 - \tau_K}, \quad t > t_0$$

$$2. \quad C(t) = C_0 \left[t - \sigma_1 - \sigma_2 + \sigma_3 + \frac{\sigma_1(\sigma_1 - \sigma_3) \exp(-t/\sigma_1) - \sigma_2(\sigma_2 - \sigma_3) \exp(-t/\sigma_2)}{\sigma_1 - \sigma_2} \right],$$

$t \leq t_0$

$$C(t) = K_B + C_0 \frac{\sigma_1(\sigma_1 - \sigma_3)[\exp(-t/\sigma_1) - \exp(-t'/\sigma_1)] - \sigma_2(\sigma_2 - \sigma_3)[\exp(-t/\sigma_2) - \exp(-t'/\sigma_2)]}{\sigma_1 - \sigma_2}$$

$$+ K \left[\frac{\sigma_1(\sigma_3 - \sigma_1)}{(\sigma_1 - \sigma_2)(\sigma_1 - \tau_K)} \exp(-t'/\sigma_1) + \frac{\sigma_2(\sigma_3 - \sigma_2)}{(\sigma_2 - \sigma_1)(\sigma_2 - \tau_K)} \exp(-t'/\sigma_2) \right.$$

$$\left. + \frac{\tau_K(\sigma_3 - \tau_K)}{(\tau_K - \sigma_1)(\tau_K - \sigma_2)} \exp(-t'/\tau_K) \right], \quad t > t_0.$$

Note that $t' = t - t_0$ in the above expressions. Also, the Type 2 row transfer function was

$$F(s) = \frac{1 + \sigma_3 s}{(1 + \sigma_1 s)(1 + \sigma_2 s)}.$$

The program then calculates temperature values for specified times and punches decks of cards for use with BOW III. Thus

$$T_{\Delta}(t) = (P/P_0)[1 + C(t)] T_{\Delta}(0).$$

Table V defines the PROFILE symbols used.

TABLE V. PROFILE FORTRAN Quantities

MI	Number of axial node points per subassembly
MJ	Number of subassembly rows
LT	Number of time values
T(I)	Time values, t
TO	t_0 , time duration of input ramp
CO	C_0 , slope of input ramp
AK	K , asymptotic value of input exponential
TK	τ_K , time constant of input exponential
P	P , starting power
PO	P_0 , base power, = 45 MW
DPP	$(\Delta P/P)_{t_0}$, end value of ramp
NT(I)	Function-type, row I
S(I,J)	$\sigma_1, \sigma_2, \sigma_3$, row I time constants (see $F(s)$ above)
DT(I,J)	Temperature difference across node I of row J for 45-MW equilibrium conditions, $T_{\Delta}(0)$

The following pages list the FORTRAN for PROFILE and the output for a typical run, in this case, the 41.4-MW rod drop made during reactor run 26B. The input quantities used appear in the INPUT REVIEW that follows the FORTRAN. Table VI gives $C(t)$ values for rows 6-10, normalized to a base whose initial value is 1.0.

TABLE VI. $C(t)^a$ for Rows 6-10

Time	Row				
	6	7	8	9	10
0	0	0	0	0	0
0.1	0.4138	0.5492	0.0165	0.0149	0.0060
0.2	1.4194	1.8815	0.0656	0.0594	0.0239
0.4	3.2019	4.2257	0.2160	0.1965	0.0845
0.6	3.9126	5.1388	0.3587	0.3262	0.1383
0.8	4.2439	5.5443	0.5010	0.4559	0.1931
1.0	4.4415	5.7708	0.6429	0.5854	0.2488
2.0	5.0862	6.4428	1.3436	1.2296	0.5397
3.0	5.6492	7.0049	2.0263	1.8635	0.8485
4.0	6.1709	7.5132	2.6865	2.4826	1.1171
5.0	6.6546	7.9735	3.3213	3.0831	1.5048
6.0	7.1029	8.3907	3.9286	3.6625	1.8462
7.0	7.5186	8.7690	4.5073	4.2189	2.1929
8.0	7.9039	9.1126	5.0566	4.7511	2.5425
9.0	8.2611	9.4247	5.5765	5.2585	2.8931
10.0	8.5923	9.7086	6.0672	5.7407	3.2430

^aMultiply tabulated values by 10^{-2} . Values are negative and represent the time variation normalized to a base whose initial value is 1.0.

```

*      PROGRAM PROFILE
      DIMENSION S(3,15),T(20),NT(15),C(15),DT(10,15),DT1(10,15)
      PAUSE
      READ 100,MJ,MJ,LT
100    FORMAT(12I6)
      READ 110,(T(I),I=1,LT)
110    FORMAT(6E12,5)
      READ 130,T0,C0,AK,TK,P,P0,DPP
      AKB=(DPP+AK)
      DO 1 I=1,MJ
1      READ 120,NT(I),(S(J,I),J=1,3)
120    FORMAT(16,3E12,6)
      READ 130,((DT(I,J),I=1,MI),J=1,MJ)
130    FORMAT(8E10,4)
      PRINT 200
200    FORMAT(13H1INPUT REVIEW//6X,1HP13X,2HT012X,2HC012X,1HK11X,
1      6HTAU(K))
      PRINT 201,P,T0,C0,AK,TK
201    FORMAT(1X,5(E12,6,2X))
      PRINT 202
202    FORMAT(/7H NUMBER2X,4HTYPE6X,2HS112X,2HS212X,2HS3/)
      PRINT 203,(I,NT(I),(S(J,I),J=1,3),I=1,MJ)
203    FORMAT(15,17,3(2X,E12,6))
      PRINT 204
204    FORMAT(/28H1INITIAL TEMPERATURE PROFILE/)
      PRINT 205,((DT(I,J),I=1,MI),J=1,MJ)
205    FORMAT(10(1X,E11,5))
      DO 2 I=1,LT
      T1=T(I)
      T2=T1-T0
      DO 3 J=1,MJ
      E11=EXPF(-T1/S(1,J))
      E12=EXPF(-T2/S(1,J))
      E21=EXPF(-T1/S(2,J))
      E22=EXPF(-T2/S(2,J))
      IF(T2) 50,50,52
50    G=0,
      E=0,
      F0=T1*S(1,J)-S(2,J)*S(3,J)
      F1=S(1,J)*E11
      IF(NT(J)=1) 55,55,51
51    F1=(F1*(S(1,J)+S(3,J))-S(2,J)*(S(2,J)-S(3,J))*E21)/(S(1,J)-S(2,J))
      GO TO 55
52    F0=0,
      B=AKB
      F1=S(1,J)*(E11-E12)
      IF(NT(J)=1) 53,53,54
53    G=(TK*EXPF(-T2/TK)*S(1,J)*E12)/(S(1,J)*TK)
      GO TO 55
54    F1=(F1*(S(1,J)+S(3,J))-S(2,J)*(S(2,J)-S(3,J))*(E21-E22))/
1      (S(1,J)+S(2,J))
      G=S(1,J)*(S(3,J)-S(1,J))*E12/((S(1,J)-S(2,J))*(S(1,J)+TK))
1      +S(2,J)*(S(3,J)-S(2,J))*E22/((S(2,J)-S(1,J))*(S(2,J)+TK))
2      +TK*(S(3,J)+TK)*EXPF(-T2/TK)/((TK-S(1,J))*(TK-S(2,J)))
55    C(J)=E+(C0*(F0+F1)+AK*G)
      DO 3 K=1,MI
3      DT1(K,J)=DT(K,J)*P*(1+C(J))/P0
      PRINT 210,T1
210    FORMAT(6H1TIME=F7,2//)
      PUNCH 215,T1
215    FORMAT(9H*** TIME=F7,2)
      PRINT 205,((DT1(K,J),K=1,MI),J=1,MJ)
      PUNCH 130,((DT1(K,J),K=1,MI),J=1,MJ)
2      CONTINUE
      END

```

MEMORY MAP

INTEGER VARIABLES

IDENT	LEVEL	OBJECT CODE LOCATION	UP SUBROUTINE
K	1	1 5244	1 2400
J	1	1 5470	1 2406
I	1	1 5526	1 2426
LT	1	1 5533	NOT USED
MJ	1	1 5535	NOT USED
MI	1	1 5537	NOT USED
IO	0	1 7775	1 2442

FLOATING	POINT	VARIABLES
IDENT	LEVEL	OBJECT CODE LOCATION

F1	1	1 5246
F0	1	1 5251
E	1	1 5254
G	1	1 5262
E22	1	1 5265
E21	1	1 5272
E12	1	1 5275
E11	1	1 5300
T2	1	1 5303
T1	1	1 5306
AKB	1	1 5472
DPP	1	1 5475
P0	1	1 5500
P	1	1 5503
TK	1	1 5506
AK	1	1 5511
CO	1	1 5514
TO	1	1 5517

INTEGER ARRAYS

IDENT	LEVEL	OBJECT CODE LOCATION	DIMENSION	DIM1	DIM2
NT	1	1 7422	1		

FLOATING POINT ARRAYS

IDENT	LEVEL	OBJECT CODE LOCATION	DIMENSION	DIM1	DIM2
DT1	1	1 5541	2	10	
DT	1	1 6443	2	10	
C	1	1 7345	1		
T	1	1 7460	1		
S	1	1 7554	2	3	

CONSTANTS

VALUE	OBJECT CODE LOCATION
0.10000000 E 01	1 5241
0.0	1 5257
2	1 5270
10	1 7763
20	1 7765
15	1 7767
3	1 7771
1	1 7773

STATEMENT NUMBERS

IDENT	LEVEL	OBJECT CODE	LOCATION
-------	-------	-------------	----------

54	1	1	1657
53	1	1	1627
51	1	1	1535
55	1	1	2027
52	1	1	1572
50	1	1	1466
3	1	1	2066
2	1	1	2312
1	1	1	0775
0	1	1	0650

FORMAT STATEMENTS

IDENT	LEVEL	OBJECT CODE	LOCATION
-------	-------	-------------	----------

215	1	1	5220
210	1	1	5231
205	1	1	5311
204	1	1	5320
203	1	1	5342
202	1	1	5354
201	1	1	5403
200	1	1	5413
120	1	1	5462
130	1	1	5456
110	1	1	5522
100	1	1	5530

LIBRARY FUNCTIONS

IDENT	LEVEL	OBJECT CODE	LOCATION
-------	-------	-------------	----------

(P5230		0	6740
(16120		0	7150
(R4051		0	7230
INPUT		1	0001
EXPF		1	0505

ERASABLE STORAGE 1 2453 TO 1 5220.

INPUT REVIEW

P	TO	CO	K	TAU(K)
.414000E 02	.220000E 00	-.186000E 00	-.870000E-01	.132000E 02

NUMBER	TYPE	S1	S2	S3
1	1	.190000E 00	.000000E 00	.000000E 00
2	1	.190000E 00	.000000E 00	.000000E 00
3	1	.190000E 00	.000000E 00	.000000E 00
4	1	.190000E 00	.000000E 00	.000000E 00
5	1	.190000E 00	.000000E 00	.000000E 00
6	1	.190000E 00	.000000E 00	.000000E 00
7	3	.190000E 00	.730000E 01	.970000E 01
8	3	.630000E 01	.730000E 01	.820000E 01
9	1	.620000E 01	.000000E 00	.000000E 00
10	1	.155000E 02	.000000E 00	.000000E 00
11	1	.155000E 02	.000000E 00	.000000E 00
12	1	.155000E 02	.000000E 00	.000000E 00
13	1	.155000E 02	.000000E 00	.000000E 00
14	1	.155000E 02	.000000E 00	.000000E 00
15	1	.155000E 02	.000000E 00	.000000E 00

INITIAL TEMPERATURE PROFILE

.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 01	.23000E 02	.17000E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.10000E 02	.25000E 02	.19000E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.12000E 02	.29000E 02	.22000E 02	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.12000E 02	.29000E 02	.22000E 02	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.10000E 02	.24000E 02	.18000E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.40000E 01	.20000E 02	.48000E 02	.74000E 02	.90000E 02	.92000E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.15000E 02	.35000E 02	.53000E 02	.66000E 02	.66000E 02	.66000E 02
.00000E 00	.00000E 00	.30000E 01	.90000E 01	.10000E 02	.70000E 01	.30000E 01	.14000E 02	.18000E 02	.18000E 02
.00000E 00	.00000E 00	.40000E 01	.90000E 01	.30000E 02	.58000E 02	.72000E 02	.74000E 02	.76000E 02	.78000E 02
.00000E 00	.00000E 00	.30000E 01	.50000E 01	.20000E 02	.44000E 02	.52000E 02	.45000E 02	.38000E 02	.32000E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.80000E 01	.20000E 02	.25000E 02	.23000E 02	.20000E 02	.18000E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00

TIME= .00

.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.82800E 01	.21160E 02	.15640E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.92000E 01	.22999E 02	.17480E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.11040E 02	.26679E 02	.20240E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.11040E 02	.26679E 02	.20240E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.92000E 01	.22080E 02	.16560E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.36799E 01	.18399E 02	.44159E 02	.68080E 02	.82799E 02	.84639E 02	.87399E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.13800E 02	.32200E 02	.48760E 02	.60720E 02	.60720E 02	.60720E 02
.00000E 00	.00000E 00	.27600E 01	.82800E 01	.92000E 01	.64400E 01	.27600E 01	.12880E 02	.16560E 02	.16560E 02
.00000E 00	.00000E 00	.36800E 01	.82800E 01	.27600E 02	.53360E 02	.66240E 02	.68080E 02	.69920E 02	.71760E 02
.00000E 00	.00000E 00	.27600E 01	.46000E 01	.18400E 02	.40480E 02	.47840E 02	.41400E 02	.34960E 02	.29440E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.73600E 01	.18400E 02	.22999E 02	.21160E 02	.18400E 02	.16560E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00

TIME= .10

.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.82457E 01	.21072E 02	.15575E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.91619E 01	.22904E 02	.17407E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.10994E 02	.26569E 02	.20156E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.10994E 02	.26569E 02	.20156E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.91619E 01	.21988E 02	.16491E 02	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.36597E 01	.18298E 02	.43917E 02	.67706E 02	.82345E 02	.84175E 02	.86919E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.13797E 02	.32194E 02	.48751E 02	.60709E 02	.60709E 02	.60709E 02
.00000E 00	.00000E 00	.27595E 01	.82787E 01	.91986E 01	.64390E 01	.27595E 01	.12878E 02	.16557E 02	.16557E 02
.00000E 00	.00000E 00	.36797E 01	.82795E 01	.27598E 02	.53356E 02	.66236E 02	.68075E 02	.69915E 02	.71755E 02
.00000E 00	.00000E 00	.27598E 01	.45997E 01	.18398E 02	.40477E 02	.47837E 02	.41397E 02	.34957E 02	.29438E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.73595E 01	.18398E 02	.22998E 02	.21158E 02	.18398E 02	.16559E 02
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00
.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00	.00000E 00

[illegible][illegible][illegible]

[illegible][illegible][illegible]

[illegible][illegible][illegible]

TIME= 10.00

[illegible]

APPENDIX E

DYNAMIC Reactivity Calculations

The computer program DYNAMIC calculates the reactivity time function $C(t)$ of each component on Fig. 12 with up to two denominator roots in the transfer function of the component and using a specified rod-drop reactor power-time function as the input. An input like that in Fig. D.1, which was used in Appendix D, is also used here. The function $C(t)$ resulting when the Laplace-transformed input is applied to the component transfer function has two basic forms depending upon whether there are one or two denominator roots in the transfer function. These forms are:

Form 1

$$C(t) = PCC_0[t - \sigma_1 + \sigma_1 \exp(-t/\sigma_1)], \quad t < t_0;$$

$$C(t) = PC \left\{ C_0 + \sigma_1 [\exp(-t/\sigma_1) - \exp(-t'/\sigma_1)] + K \left[1 + \frac{\tau_K \exp(-t'/\tau_K) - \sigma_1 \exp(-t'/\sigma_1)}{\sigma_1 - \tau_K} \right] \right\}, \quad t \geq t_0.$$

Form 2

$$C(t) = PCC_0 \left(t - \sigma_1 - \sigma_2 + \frac{\sigma_2^2 \exp(-t/\sigma_2) - \sigma_1^2 \exp(-t/\sigma_1)}{\sigma_2 - \sigma_1} \right), \quad t < t_0;$$

$$C(t) = PC \left\{ C_0 \left[t_0 + \frac{\sigma_2^2 [\exp(-t/\sigma_2) - \exp(-t'/\sigma_2)] - \sigma_1^2 [\exp(-t/\sigma_1) - \exp(-t'/\sigma_1)]}{\sigma_2 - \sigma_1} \right] + K \left[1 - \frac{\tau_K^2 \exp(-t'/\tau_K)}{(\tau_K - \sigma_1)(\tau_K - \sigma_2)} - \frac{\sigma_1^2 \exp(-t'/\sigma_1)}{(\sigma_1 - \sigma_2)(\sigma_1 - \tau_K)} - \frac{\sigma_2^2 \exp(-t'/\sigma_2)}{(\sigma_1 - \sigma_2)(\sigma_2 - \tau_K)} \right] \right\}, \quad t \geq t_0.$$

Note that $t' = t - t_0$ in the above expressions.

Table VII lists DYNAMIC FORTRAN quantities.

TABLE VII. DYNAMIC FORTRAN Quantities

NC	Number of components to be evaluated	AK	K, asymptotic value of input exponential
NS	Number of component subtotals to be calculated	TK	τ_K , time constant of input exponential
LT	Number of time values	P	P, starting power
NS1(I)	Initial number of components in subtotal I	BCD(J,I)	18-character BCD identifier for component I
NS2(I)	Final number of components in subtotal I	NT(I)	Function type for component I
T(I)	Time values	S(J,I)	σ_1, σ_2 component I time constants
TO	t_0 , time duration of input ramp	C(I)	C component I multiplier in \$/MW
CO	C_0 , slope of input ramp		

The following pages list the FORTRAN for DYNAMIC and the output for a typical run, in this case the 41.4-MW rod drop made during reactor run 26B. The input quantities used appear in the INPUT REVIEW which follows the FORTRAN. The various outputs in dollars versus time are plotted in Fig. 13.

```

* PROGRAM DYNAMIC
  DIMENSION S(2,10),T(20),SUM(20),BCD(3,10),NS1(5),NS2(5),NT(10),
1  C(10),DK(10)
  READ 100,NC,NS,LT,(NS1(I),NS2(I),I=1,NS)
100  FORMAT(12I6)
  READ 110,(T(I),I=1,LT)
  READ 110,T0,C0,AK,TK,P
110  FORMAT(6E12.5)
  DO 1 I=1,NC
    1  READ 120,(BCD(J,I),J=1,3),NT(I),(S(J,I),J=1,2),C(I)
120  FORMAT(3A6,I6,3E12.6)
  PRINT 200
200  FORMAT(13H1INPUT REVIEW//6X,1HP13X,2HT012X,2HC012X,1HK11X,
1  6HTAU(K))
  PRINT 201,P,T0,C0,AK,TK
201  FORMAT(1X,5(E12.6,2X))
  PRINT 202
202  FORMAT(//7H NUMBER7X,5HTITLE8X,4HTYPE6X,2HS110X,2HS215X,2HC /)
  PRINT 203,(I,(BCD(J,I),J=1,3),NT(I),(S(J,I),J=1,2),C(I),I=1,NC)
203  FORMAT(3X,I2,3X,3A6,2X,I2,2(2X,E12.6),5X,E12.6)
  DO 2 I=1,LT
    T1=T(I)
    T2=T1-T0
    SUM(I)=0.
    DO 3 J=1,NC
      E11=EXP(-T1/S(1,J))
      E12=EXP(-T2/S(1,J))
      E21=EXP(-T1/S(2,J))
      E22=EXP(-T2/S(2,J))
      IF(T2) 50,50,52
50  G=0.
      F0=T1-S(1,J)-S(2,J)
      F1=S(1,J)*E11
      IF(NT(J)-1) 55,55,51
51  F1=(-S(1,J)*F1+S(2,J)**2*E21)/(S(2,J)-S(1,J))
      GO TO 55
52  F0=T0
      F1=S(1,J)*(E11-E12)
      IF(NT(J)-1) 53,53,54
53  G=1.+(TK*EXP(-T2/TK)-S(1,J)*E12)/(S(1,J)-TK)
      GO TO 55
54  F1=(-S(1,J)*F1+S(2,J)**2*(E21-E22))/(S(2,J)-S(1,J))
      G=1.-TK**2*EXP(-T2/TK)/((TK-S(1,J))*(TK-S(2,J)))
1  -S(1,J)**2*E12/((S(1,J)-S(2,J))*(S(1,J)-TK)) -S(2,J)**2*E22/
2  ((S(2,J)-S(1,J))*(S(2,J)-TK))
55  DK(J)=C(J)*(C0*(F0+F1)+AK*G)*P
3  SUM(I)=SUM(I)+DK(J)
  PRINT 210,T1
210  FORMAT(6H1TIME=F7.2//)
  PRINT 220,(J,(BCD(K,J),K=1,3),DK(J),J=1,NC)
220  FORMAT(14H COMPONENT NO.I2,5X,3A6,E12.6,8H DOLLARS)
  PRINT 230,SUM(I)
230  FORMAT(/15X,15HCOMBINED EFFECT5X,E12.6,8H DOLLARS)
  IF(NS) 2,2,56
56  PRINT 240
240  FORMAT(///10H SUBTOTALS/)
  DO 4 J=1,NS
    N1=NS1(J)
    N2=NS2(J)
    SM=0.
    DO 5 K=N1,N2
      SM=SM+DK(K)
5  SM=SM+DK(K)
4  PRINT 250,N1,N2,SM
250  FORMAT(5X,10HCOMPONENTSI2,3H TOI2,5X,E12.6,8H DOLLARS)
2  CONTINUE
  PRINT 260
260  FORMAT(18H1REVIEW OF RESULTS//)
  PRINT 270,(T(I),SUM(I),I=1,LT)
270  FORMAT(10X,2HT=E12.6,5X,E12.6,8H DOLLARS)
  END

```

INPUT REVIEW

P T0 C0 K TAU(K)
 .414000E 02 .220000E 00 -.186000E 00 -.870000E-01 .132000E 02

NUMBER	TITLE	TYPE	S1	S2	C
1	AXIAL FUEL EXP.	1	.170000E 00	.000000E 00	.365000E-02
2	CORE NA EXP.	1	.190000E 00	.000000E 00	.135000E-02
3	ABOVE-CORE NA EXP.	2	.190000E 00	.700000E-01	.135000E-02
4	ROD-BANK EXP., L1	2	.190000E 00	.900000E-01	.540000E-03
5	ROD-BANK EXP., L2	2	.149000E 01	.930000E-01	.160000E-03
6	ROD-BANK EXP., L3	2	.149000E 01	.210000E 02	.160000E-03
7	ROD-BANK EXP., L4	2	.149000E 01	.810000E 01	.160000E-03

TIME= .10

COMPONENT NO. 1	AXIAL FUEL EXP.	-.685856E-03 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.231277E-03 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.825386E-04 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.274693E-04 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.113584E-05 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.587493E-08 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.164498E-07 DOLLARS

COMBINED EFFECT -.102829E-02 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.313815E-03 DOLLARS
COMPONENTS 4 TO 7	-.286275E-04 DOLLARS

TIME= .20

COMPONENT NO. 1	AXIAL FUEL EXP.	-.231659E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.793319E-03 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.443381E-03 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.154759E-03 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.721110E-05 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.502307E-07 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.130717E-06 DOLLARS

COMBINED EFFECT -.371544E-02 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.123670E-02 DOLLARS
COMPONENTS 4 TO 7	-.162151E-03 DOLLARS

TIME= .40

COMPONENT NO. 1	AXIAL FUEL EXP.	-.504876E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.178507E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.149866E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.561477E-03 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.340613E-04 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.355643E-06 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.914782E-06 DOLLARS

COMBINED EFFECT -.892931E-02 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.328373E-02 DOLLARS
COMPONENTS 4 TO 7	-.596809E-03 DOLLARS

TIME= .60

COMPONENT NO. 1	AXIAL FUEL EXP.	-.603721E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.218228E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.205710E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.801544E-03 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.641313E-04 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.950763E-06 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.243467E-05 DOLLARS

COMBINED EFFECT -.111456E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.423938E-02 DOLLARS
COMPONENTS 4 TO 7	-.869061E-03 DOLLARS

TIME= .80

COMPONENT NO. 1	AXIAL FUEL EXP.	-.647568E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.236745E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.230701E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.912658E-03 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.921391E-04 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.180825E-05 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.460526E-05 DOLLARS

COMBINED EFFECT -.121613E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.467446E-02 DOLLARS
COMPONENTS 4 TO 7	-.101121E-02 DOLLARS

TIME= 1.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.674254E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.247792E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.244079E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.970831E-03 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.117760E-03 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.290059E-05 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.734888E-05 DOLLARS

COMBINED EFFECT -.127600E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.491871E-02 DOLLARS
COMPONENTS 4 TO 7	-.109884E-02 DOLLARS

TIME= 2.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.769190E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.283825E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.281521E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.112342E-02 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.217944E-03 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.111780E-04 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.275755E-04 DOLLARS

COMBINED EFFECT -.147255E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.565347E-02 DOLLARS
COMPONENTS 4 TO 7	-.138012E-02 DOLLARS

TIME= 3.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.854109E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.315290E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.313159E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.125018E-02 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.287864E-03 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.228192E-04 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.547931E-04 DOLLARS

COMBINED EFFECT -.164412E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.628449E-02 DOLLARS
COMPONENTS 4 TO 7	-.161566E-02 DOLLARS

TIME= 4.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.932824E-02 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.344448E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.342473E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.136762E-02 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.340950E-03 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.366669E-04 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.857018E-04 DOLLARS

COMBINED EFFECT -.180284E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.686921E-02 DOLLARS
COMPONENTS 4 TO 7	-.183093E-02 DOLLARS

TIME= 5.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.100579E-01	DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.371479E-02	DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.369648E-02	DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.147648E-02	DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.384166E-03	DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.520456E-04	DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.118447E-03	DOLLARS

COMBINED EFFECT -.195003E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.741128E-02	DOLLARS
COMPONENTS 4 TO 7	-.203114E-02	DOLLARS

TIME= 6.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.107344E-01	DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.396538E-02	DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.394841E-02	DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.157741E-02	DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.421164E-03	DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.685374E-04	DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.151948E-03	DOLLARS

COMBINED EFFECT -.208672E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.791379E-02	DOLLARS
COMPONENTS 4 TO 7	-.221906E-02	DOLLARS

TIME= 7.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.113615E-01	DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.419769E-02	DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.418195E-02	DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.167097E-02	DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.453895E-03	DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.858577E-04	DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.185542E-03	DOLLARS

COMBINED EFFECT -.221374E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.837964E-02	DOLLARS
COMPONENTS 4 TO 7	-.239626E-02	DOLLARS

TIME= 8.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.119429E-01	DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.441304E-02	DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.439846E-02	DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.175770E-02	DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.483438E-03	DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.103804E-03	DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.218806E-03	DOLLARS

COMBINED EFFECT -.233181E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.881151E-02	DOLLARS
COMPONENTS 4 TO 7	-.256375E-02	DOLLARS

TIME= 9.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.124819E-01 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.461269E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.459917E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.183811E-02 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.510416E-03 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.122216E-03 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.251453E-03 DOLLARS

COMBINED EFFECT -.244159E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.921186E-02 DOLLARS
COMPONENTS 4 TO 7	-.272219E-02 DOLLARS

TIME= 10.00

COMPONENT NO. 1	AXIAL FUEL EXP.	-.129815E-01 DOLLARS
COMPONENT NO. 2	CORE NA EXP.	-.479777E-02 DOLLARS
COMPONENT NO. 3	ABOVE-CORE NA EXP.	-.478523E-02 DOLLARS
COMPONENT NO. 4	ROD-BANK EXP., L1	-.191265E-02 DOLLARS
COMPONENT NO. 5	ROD-BANK EXP., L2	-.535216E-03 DOLLARS
COMPONENT NO. 6	ROD-BANK EXP., L3	-.140964E-03 DOLLARS
COMPONENT NO. 7	ROD-BANK EXP., L4	-.283286E-03 DOLLARS

COMBINED EFFECT -.254366E-01 DOLLARS

SUBTOTALS

COMPONENTS 2 TO 3	-.958301E-02 DOLLARS
COMPONENTS 4 TO 7	-.287211E-02 DOLLARS

REVIEW OF RESULTS

T= .100000E 00	-.102829E-02 DOLLARS
T= .200000E 00	-.371544E-02 DOLLARS
T= .400000E 00	-.892931E-02 DOLLARS
T= .600000E 00	-.111456E-01 DOLLARS
T= .800000E 00	-.121613E-01 DOLLARS
T= .100000E 01	-.127600E-01 DOLLARS
T= .200000E 01	-.147255E-01 DOLLARS
T= .300000E 01	-.164412E-01 DOLLARS
T= .400000E 01	-.180284E-01 DOLLARS
T= .500000E 01	-.195003E-01 DOLLARS
T= .600000E 01	-.208672E-01 DOLLARS
T= .700000E 01	-.221374E-01 DOLLARS
T= .800000E 01	-.233181E-01 DOLLARS
T= .900000E 01	-.244159E-01 DOLLARS
T= .100000E 02	-.254366E-01 DOLLARS

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REFERENCES

1. R. A. Cushman, *The Effect of Radial Heat Transfer on the Temperature Distribution in EBR-II Stainless-Steel Reflector Subassemblies*, Trans. Am. Nucl. Soc. 10, 661 (1967).
2. T. R. Bump, *Effect of Reactor Temperatures on Bowing of EBR-II Subassemblies*, Trans. Am. Nucl. Soc. 10, 661 (1967).
R. R. Smith, P. J. Persiani, D. A. Meneley, J. T. Madell, J. K. Long, F. S. Kirn, R. W. Hyndman, R. A. Cushman, T. R. Bump, J. C. Beitel, and D. A. Kucera, *EBR-II Run 25 and 26A Interim Report*, ANL internal memorandum (1968).
3. L. J. Koch, W. B. Loewenstein, and H. O. Monson, *Addendum to Hazard Summary Report, Experimental Breeder Reactor-II (EBR-II)*, ANL-5719 (Addendum) (June 1962).
4. P. J. Persiani, T. R. Bump, R. A. Cushman, J. Long, and D. Kucera, *Analysis of the Variation in the EBR-II Power Coefficient Due to Blanket Change*, Trans. Am. Nucl. Soc. 11, 282 (1968).
5. T. R. Bump, *Bow II: A Code for Calculating Deflections of Closely Spaced Parallel Beams, Each with Limited-pivot Support at One End, Possible Beam Interactions at Other End and at One Intermediate Position, and Arbitrary Temperature Distribution*, ANL-7528 (to be published).
6. P. J. Persiani, J. Kallfelz, F. Kirn, J. K. Long, and R. Smith, *EBR-II Power Coefficient Variations During Operational Run 26*, Trans. Am. Nucl. Soc. 11, 281 (1968).
7. R. Hyndman, private communication, ANL Idaho Division (Rod-drop Power Time Curve).
8. J. A. DeShong, Jr., *Power Transfer Functions of the EBWR Obtained Using A Sinusoidal Reactivity Driving Function*, ANL-5798 (Jan 1958).
9. J. A. DeShong, Jr. and W. C. Lipinski, *Analyses of Experimental Power-reactivity Feedback Transfer Functions for a Natural Circulation Boiling Water Reactor*, ANL-5850 (July 1958).
10. R. J. Roark, *Formulas for Stress and Strain*, McGraw-Hill (1965), Table III, p. 104.
11. P. Laurson and W. Cox, *Properties and Mechanics of Materials*, John Wiley and Sons (1931), p. 255.
12. S. Timoshenko, *Theory of Plates and Shells*, 2nd Edition, McGraw-Hill (1959), p. 206.
13. F. Jones and E. Oberg, *Machinery's Handbook*, The Industrial Press, New York (1937), p. 346.