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THE LATERAL STRAIN BEHAVIOR OF GRAPHITE —
AN ANISOTROPIC MATERIAL

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A Paper Presented at the
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Abstract

Seven elastic constants describe the strain behavior of an anisotropic material such as graphite. These constants define three different ratios of lateral to axial strain (Poisson's ratios), two elastic moduli, and two shear moduli. The elastic relationships permit the calculation of two of the constants from the experimental measurement of the other five. The five constants to be measured require careful selection since only one is completely independent. A discussion of the relationships between the constants plus models for a molded and an extruded graphite are presented. Experimental data for CFZ and ATJ graphite to 5500°F are presented and the experimental values of Poisson's ratio, elastic moduli, and shear moduli are compared to the values calculated from the theoretical relationships.

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Also, experimental measurements of the tri-directional strains in the plastic range are analyzed.

Introduction

Graphite materials usually exhibit some degree of anisotropy. Consequently, evaluations of the material are generally performed in two directions which are normally identified as the with grain and across grain orientations. These loadings yield two values for the elastic modulus and ultimate strength of the material; however, a total of 3 values of Poisson's ratio are possible. Consider, for example, a specimen taken from a molded graphite, see Figure 1. For a specimen loaded across grain, lateral strain occurs with grain only and one value of Poisson's ratio exists for this loading direction. For a specimen loaded with grain; however, lateral strain occurs both with grain and across grain; hence two values of Poisson's ratio are obtained for this one loading direction. The theoretical relations for this elastic behavior and their experimental clarification are the subject of this paper.

The Elastic Response of Graphites

Mathematical relations of stress to strain require that a model be chosen for the behavior of the material. The one usually chosen is the generalized

Hooke's Law which assumes that each of the six components of strain at any point in the body is a linear function of the six components of stress at that point. For a Cartesian coordinate system denoted by x , y , and z , the relations can be expressed mathematically as shown in Figure 2. Note that a total of 36 constants are involved in the equations. It can be shown that, if the material deforms either adiabatically or isothermally when loaded,

$$C_{ij} = C_{ji}$$

the 36 constants can be reduced to 21 [1]¹. If the material is isotropic, the 21 constants can be further reduced to only 2 independent ones [2]; however, all 21 constants are independent for the most general case of an anisotropic material. The number of independent constants can be reduced from 21 to 5 for an anisotropic material which possesses one plane of isotropy such that the elastic properties are the same for all directions within the plane [3]. This condition generally is considered valid for either molded or extruded graphites.

For purposes for better visualization, the models shown in Figure 3 can be used to describe the physical structure of a molded graphite. Note that whether the grains are considered to be "needles" or "platelets", the

¹Numbers in brackets indicate references listed in Bibliography.

plane of isotropy is the X-Y plane. The X and Y directions are usually labeled "with grain" while the Z direction is designated as "across grain". Note again that for a specimen loaded across grain the lateral strain is with grain regardless of whether it is measured in the X or Y direction. For a specimen loaded with grain, say the X direction, lateral strain occurs across grain in the Z direction and with grain in the Y direction.

For a molded graphite, the seven constants that describe the strain behavior consist of the two elastic moduli, the two shear moduli, and the 3 values of Poisson's ratio outlined Figure 4. Two of the constants can be related to some of the others by the two equations shown [3] . Consequently, for a molded graphite or any anisotropic material which possesses one plane of isotropy, a complete specification of the constants of the material requires that at least 5 constants be measured. Theoretically, there is some freedom in choosing which constants must be measured and which may be calculated. The only constant which is completely independent of any other constant is G_a ; hence, G_a and at least four of the six others must be measured. Even with this freedom, care must be exercised in selecting the ones to be measured. For example, the determination of G_w , G_a , E_w , ν_{ww} , and ν_{aw} is not sufficient as ν_{wa} and E_a would still be indeterminate. A close

examination of the equations reveals that evaluations for both with grain and across grain loading are necessary. The least number of experimental runs is involved if E_w and ν_{ww} are measured for one run loading with grain, E_a and ν_{wa} are measured for one run loading across grain, G_a is measured for one shear run, and G_w and ν_{aw} are calculated. Thus, the seven constants can be determined from three experimental runs measuring the axial and one lateral strain simultaneously during the runs. One disadvantage of this procedure is that the lateral strain is measured with grain for both loading directions and no direct measurement of the across grain lateral strain is made. At least a few runs to check experimentally the across grain lateral strain probably are needed in most cases.

For an extruded graphite, the material can be visualized as shown in Figure 5. The grains tend to align themselves in the extrusion direction. Within the planes perpendicular to this direction, the orientation of the grains is random; hence, the plane of isotropy is again the X-Y plane. For an extruded graphite (following the same analysis as for the molded graphite), the least number of experimental runs are involved if E_w , E_a , G_w , ν_{aw} , and ν_{aa} are measured and G_a and ν_{wa} are calculated from the theoretical equations.

It should be recalled that inherent in the acceptance and use of Hooke's Law is the assumption that the constants are independent of the direction of

the stress, i. e., the same for tension or compression. Otherwise, a separate law and set of constants have to be specified for each loading direction and neither set would be strictly valid for a combined tensile and compressive loading situation. To some extent, graphites depart from this assumption in that the moduli determined from tension, compression, and flexure are different. Even so, in consideration of the current state-of-the-art, the generalized Hooke's Law appears to be the best model available.

Experimental Procedures

The equipment and procedures utilized for the experimental data presented here are described in detail in several reports including AFML-TR-65-142. A photograph of a tensile system is shown in Figure 6. The system utilizes gas-bearing universals in the load train to provide "frictionless" and concentric loading of the specimen. Two separate optical strain analyzers provided continuous and simultaneous recordings of the axial and the lateral (diametrical) strain to fracture. The uncertainty of the optical instrument was about 20 micro-inches. A small, graphite element furnace provided radiant heating of the specimen and contained sight ports to permit visual access to the specimen by the optical instruments. The elastic constants were calculated directly from the initial slope of the stress-strain recordings.

The configuration of the shear specimen was shown in Figure 7. The configuration is somewhat similar to the "single shear" specimen commonly used for ductile materials; however, considerable effort was expended to develop this specimen for use with graphite. Complete details of its development are provided in TR-65-142. Note that two shear sections were provided both of which were analogous to an extremely short cantilever beam. The extremely narrow (0.010") section provided ultimate shear strength while the wider (0.156") section provided shear modulus. The gas-bearing tensile system was employed to provide concentric loading of the sections and optical strain measurement of the shear deformation.

Experimental and Predicted Results

In Figure 8, experimental data are shown for the elastic constants of CFZ in tension from 70 to 5500°F. The data were obtained in statistical quantities at six temperature levels. Each point represents the average of from 10 to 20 experimental values. The bands about each point represent the confidence intervals at the 95% confidence level.

For both grain orientations, the initial elastic modulus in tension increased with temperature to a maximum at 3500°F and then decreased to 5500°F. E_w was about 1.8×10^6 psi at 70°F, increased to about 2.5×10^6 psi at 3500°F, then decreased to about 1.3×10^6 psi at 5500°F. E_a was about 80% less than E_w at the lower temperatures, then the values tended to converge at the higher temperatures to about 40% difference.

The shear moduli followed about the same trend with temperature as that of the tensile moduli, increasing from about 0.6×10^6 psi at 70°F to a peak of about 0.8×10^6 psi at 3500°F , then decreasing to about 0.3×10^6 psi at 5500°F . G_w was about 10% higher than G_a throughout the temperature range.

The measured values of Poisson's ratio increased continuously with increased temperature from about 0.1 at 70°F to about 0.3 at 5500°F with the rate of increase becoming greater above about 3500°F . Unfortunately the lateral orientation was unknown for the tensile specimens loaded with grain and the ratio was a mixture of ν_{aw} and ν_{ww} . This ratio was referred to as ν_{-w} . The mixed orientation did not effect the measurement of ν_{wa} since only with grain lateral strain occurs for the specimen loaded across grain. From the measured value of ν_{wa} , the value of ν_{aw} was calculated. The calculated value followed about the same trend as the measured values by increasing continuously with increased temperature but at a much higher level. Recall that the calculated value reflects directly the difference in the with grain and the across grain tensile moduli which differed from 40 to 80% over the temperature range. Note also that the calculated ratio exceeded slightly the theoretical limit of 0.5 at 5000 and 5500°F . One contributing factor to this apparent discrepancy is probably the considerable influence of strain rate at the higher temperatures. Considerable plasticity

was encountered at the higher temperatures, as is shown in Figure 9 by the photograph of some compressive specimens that were strained over $\frac{3}{4}$ inches without fracturing at 5500°F and at a stress rate of about 6000 psi/min. (considering the original crosssectional area). Very little, if any, linear portion of axial or lateral stress-strain was observed in both tension and compression at these temperatures and the "slopes" were effected significantly by strain rate. The moduli, from which the calculated ratio was determined, were taken as the initial portion (about 10 to 20%) of the stress-strain curve. The effect of strain rate was investigated and reported in TR65-142 for CFZ graphite.

The elastic constants, as determined in compression (except, of course, the shear data), are shown in Figure 10. Practically the same trends with temperature were established in compression as for tension but the absolute values were somewhat different. The elastic moduli were slightly lower in compression than in tension at the lower temperatures but the reverse was true at 3500°F and above. Again, the effect of strain rate was significant and must be considered at the higher temperatures.

The values of Poisson's ratio determined in compression were slightly lower than for tension at the lower temperatures but were about the same at 3500°F and above. The orientation of the compressive specimens was maintained and the ratio ν_{ww} was isolated; however, a difference between ν_{ww}

and ν_{wa} of an amount greater than the confidence intervals was detected only at the intermediate temperature levels. The calculated ratio ν_{aw} again followed the general trend with temperature as the measured values and was higher than ν_{wa} by the ratio of the with and across grain compressive moduli. The ratio approached 0.5 at the higher temperatures as the moduli decreased and plasticity increased considerably.

While the two measured values of Poisson's ratio (ν_{wa} and ν_{ww}) were about the same value throughout the temperature range, the calculated value for (ν_{aw}) predicted from the theoretical relationships was considerably higher at all temperature levels. The large spread between the values expected for the two directions is shown again in Figure 11 for tension and compression. Note that the theoretical values for ν_{aw} are about 40 to 80% greater than the measured ratios for ν_{ww} and ν_{wa} throughout the temperature range. The predicted difference is considerable and, if true, most certainly would require consideration in many applications. The measurement of the absolute value of the third ratio with statistical certainty would have required another complete series of runs and was beyond the scope of the particular program; however, a study was made in-house to explore whether the large difference in lateral strains actually existed for the two directions in an anisotropic graphite.

Three compressive specimens were loaded with grain to measure the lateral strain that was produced in the with grain and across grain directions by rotating the specimen 90° between successive loading cycles. Three points were obtained at each of three temperature levels (70, 2000, and 3500°F) by loading then unloading each specimen to about 25% of ultimate stress while recording the lateral strains, then proceeding to the next temperature level. This procedure provided an experimentally obtained ratio of the difference between the with grain and across grain lateral strains produced in the specimen loaded with grain.

The strain measured across grain was greater than that with grain by an average of 46% at 70°F, 58% at 2000°F, and 50% at 3500°F, indicating rather firmly that the difference did exist and that the degree of difference was quite significant. These data points are shown on Figure 11. This experimental ratio is actually one of ν_{aw} / ν_{ww} but the previous data strongly suggested that ν_{ww} was about equal to ν_{wa} ; therefore, it's reasonable to apply the ratio to the measured ratio ν_{wa} to provide experimental values of ν_{aw} that are comparable to those predicted theoretically. Note the close agreement with the theoretical in both tension and compression at all three of the temperature levels investigated. A statistically sufficient number of direct measurements of the third Poisson's ratio is needed to establish the absolute value but a significant level of departure from the two other Poisson's ratios certainly was indicated.

For an additional check, strain gages were bonded to a 1" sq. x 2" long specimen of ATJ graphite loaded with grain while the with grain and across grain lateral strains were measured at 90° to each other. Poisson's ratios of 0.09 and 0.14 were obtained again suggesting a difference of about 55% for these two orientations.

Further, an examination was made of the CFZ compressive specimens after they had been loaded to fracture at respective temperatures of 4500, 5000, and 5500°F; see Figure 12. The diametrical dimensions were measured with a micrometer at 90° to obtain the permanent diametrical deformation in both lateral directions. The permanent deformations were plastic in nature, of course, and are not equatable to the elastic equations; however, a reasonable indication of the level of difference between the elastic diametrical strains can be expected from an examination of the plastic diametrical deformations. The measurements showed that the gage section of the specimens loaded across grain (the lateral strain is with grain both at 0° and 90°) deformed symmetrically or round. In both lateral directions, the permanent deformation were practically the same at about 17, 37, and 78 mils at 4500, 5000, and 5500°F, respectively.

In contrast, the gage section of the specimens loaded with grain (the lateral strain was with grain at 0° but across grain at 90°) deformed

diametrically in an elliptical fashion. At each temperature level, the deformation across grain was about 90% greater than that with grain. At 4500, 5000, and 5500°F, respectively, the deformation across grain to with grain was 30 to 16, 69 to 35, and 113 to 60 mils. It's also interesting to note that irregardless of whether the specimen was loaded with grain or across grain that the diametrical strain that occurred with grain was about the same. Observe that the lateral deformation with grain for both modes of loading was about 17 mils at 4500°F, 37 mils at 5000°F, and 70 mils at 5500°F.

It thus appears that the large difference in the Poisson's ratios predicted theoretically does exist and is about equal to the ratio of the with to across grain elastic moduli.

From the other constants measured, recall that the shear modulus G_w , can be calculated by:

$$G_w = \frac{E_w}{2(1 + \nu_{ww})}$$

In Figure 13, a comparison is shown at each temperature level between the measured value of G_w and the value calculated from the measured values of E_w and ν_{ww} . The calculated shear modulus showed about the same variance with increased temperature as the measured values increasing to a peak at 3500°F then decreasing to 5500°F. The calculated

value was greater at all temperature levels being about 12% higher at 70°F and about 20% higher at 3500 and 5500°F. If the constants E_w and ν_{ww} from which G_w were calculated are reliable to within 10%, the statistical reliability of the calculation is about 14%. Fair agreement between the measured and calculated values therefore was indicated at most temperature levels, particularly at 70°F. At 2000°F, the calculated value differed by about 40%. We believe that the agreement would be better in a program designed to study the shear response and which the causes of the variance could be tracked down.

In Figure 14, some other Southern Research Institute data for shear modulus are shown for ATJ graphite. At three temperature levels, the values of shear modulus calculated from the measured values of E_w and ν_{ww} compared extremely well with the measured shear modulus. The calculated value departed less than 10% at 3000°F and was practically identical to the measured value at 70 and 4000°F.

Conclusions

Seven elastic constants describe the strain behavior of a graphite. The seven constants can be determined with three experimental runs by careful selection and measurement of five of the constants and calculation of the other two from the theoretical relationships.

While two of the three Poisson's ratios for graphite are about the same value throughout the temperature range, the third ratio is considerably higher. This higher ratio is the one normally calculated from the theoretical relationships. Experimental data confirmed the level of difference of the third ratio as predicted by the theory; however, the significance of the difference suggests that direct experimental measurement of the absolute value of the third ratio be made with statistical certainty.

Most of the equipment and procedures utilized in this study and some of the data were developed in an extensive program with the Atomic Energy Commission and are reported in detail in ORO-461. The experimental data for the CFZ graphite were obtained in a portion of a program with the Research and Technology Division of the Air Force and are reported in AFML-TR-65-142.

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$$(1) \text{ Poisson's Ratio} = \frac{\epsilon_{\text{lateral With Grain}}}{\epsilon_{\text{axial Across Grain}}}$$

$$(2) \text{ Poisson's Ratio} = \frac{\epsilon_{\text{lateral With Grain}}}{\epsilon_{\text{axial With Grain}}}$$

$$(3) \text{ Poisson's Ratio} = \frac{\epsilon_{\text{lateral Across Grain}}}{\epsilon_{\text{axial With Grain}}}$$

Figure 1. Pictorial Representation of the Three Poisson's Ratios in a Specimen of Molded Graphite.

$$\epsilon_x = C_{11} \sigma_x + C_{12} \sigma_y + C_{13} \sigma_z + C_{14} \tau_{yz} + C_{15} \tau_{xz} + C_{16} \tau_{xy} \quad (1)$$

$$\epsilon_y = C_{21} \sigma_x + C_{22} \sigma_y + C_{23} \sigma_z + C_{24} \tau_{yz} + C_{25} \tau_{xz} + C_{26} \tau_{xy} \quad (2)$$

$$\epsilon_z = C_{31} \sigma_x + C_{32} \sigma_y + C_{33} \sigma_z + C_{34} \tau_{yz} + C_{35} \tau_{xz} + C_{36} \tau_{xy} \quad (3)$$

$$\gamma_{yz} = C_{41} \sigma_x + C_{42} \sigma_y + C_{43} \sigma_z + C_{44} \tau_{yz} + C_{45} \tau_{xz} + C_{46} \tau_{xy} \quad (4)$$

$$\gamma_{xz} = C_{51} \sigma_x + C_{52} \sigma_y + C_{53} \sigma_z + C_{54} \tau_{yz} + C_{55} \tau_{xz} + C_{56} \tau_{xy} \quad (5)$$

$$\gamma_{xy} = C_{61} \sigma_x + C_{62} \sigma_y + C_{63} \sigma_z + C_{64} \tau_{yz} + C_{65} \tau_{xz} + C_{66} \tau_{xy} \quad (6)$$

where

σ = normal stress,

τ = shearing stress,

ϵ = normal strain,

γ = shearing strain, and

C_{ij} = "proportionality constant."

	Number of Independent Constants
Anisotropic material (most general case) -----	36
Adiabatic or isothermal deformation $C_{ij} = C_{ji}$ -----	21
Anisotropic material with one plane of isotropy -----	5 (Graphite)
Isotropic material -----	2

Figure 2. Mathematical Relation of Stress to Strain.

(a) "Platelet" Model

(b) "Needle" Model

Figure 3. Illustrative Model of the Grain Orientation for a
Molded Graphite

Least No. of
Experimental Runs

$$G_w = \frac{E_w}{2(1 + \nu_{ww})}$$

$$\nu_{aw} = \frac{E_w}{E_a} \nu_{wa}$$

Figure 4. The Measurement and Calculation of the Seven Elastic Constants for a Molded Graphite.

Figure 5. Illustrative Model of the Grain Orientation for an Extruded Graphite

Figure 6. PICTURE OF A TENSILE STRESS-STRAIN FACILITY

Figure 7. Shear Specimen Configuration

Figure 8. The Seven Elastic Constants For CFZ Graphite From 70 to 5500°F In TENSION.

CFZ, With Grain Compressive Specimens Loaded at
5500°F at a Stress Rate of 6000 psi/min

Original Size of Compressive
Specimens Before Loading

Figure 9. Photograph of Compressive Specimens Showing the Large Deformations Encountered When
Loading at High Temperatures

Figure 10. The Seven Elastic Constants For CFZ Graphite from 70 to 5500°F in COMPRESSION.

Figure 11. Predicted Difference Between The Poisson's Ratios

ν_{wa} and ν_{aw}

LOADED ACROSS GRAIN LOADED WITH GRAIN

FRACTURE TEMPERATURE (°F)	4500	5000	5500	4500	5000	5500	
NO. OF MEASUREMENTS	7	18	7	9	19	8	
PERMANENT DIAMETRICAL DEFORMATION (MILS)	0°	17	37	78	16	35	60
	90°	18	38	77	30	69	113
RATIO OF THE DEFOR- MATIONS	1.05	1.02	1.01	1.87	1.97	1.88	

Figure 12. Measurements of the Permanent DIAMETRICAL Deformation of Compressive Specimens Fractured at 4500, 5000, and 5500°F.