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WHY DOES THE POMERON LOOK SO SIMPLE
IF IT IS REALLY SO COMPLICATED?

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ABSTRACT

The absorbed multiperipheral model is suggested as a vehicle for understanding the approximate constancy with energy of total cross sections that ultimately rise to saturate the Froissart bound. Two approximation schemes, which share as their lowest approximation a simple, factorizable Pomeron, are discussed. In a streamlined version of the model, an estimate of the squared energy s at which this simple picture breaks down is given by $\ln(s/s_0) = [\sigma_{\text{tot}}/(4\pi\alpha')]^{1/3} \cong 4.3$.

1 Introduction

We now know that the Pomeron is not a simple, isolated pole with an intercept of one. Theoretically, the last link in the chain of argument showing that such a pole would be incompatible with unitarity has now been forged.¹ Experimentally, there are indications² that total cross sections may not have finite limits as the energy increases, or, if they do, that the values of those limits may be quite different from what had been previously expected. On the other hand, much of the phenomenology based on treating the Pomeron as a simple pole, for example the constancy of total cross sections and the factorization properties of inclusive distributions, seems to be at least approximately valid over a wide range of energy³. In this paper we present a discussion of the question of how the Pomeron can look like a simple pole and yet really be more complicated.

Several authors, for example those listed in Ref. 4, have suggested that it is useful to perform an expansion in powers of $\xi \ln s$, considered as a small parameter, where ξ is some measure of the Pomeron coupling to inelastic channels; to zeroth order in this expansion, the Pomeron would be a simple pole, while unitarity would be satisfied when all orders were taken. It might seem that this would require that at least the zeroth-order Pomeron had an intercept less than one; otherwise, if a constant zeroth-order σ_{tot} were increased by more than two powers of $\ln s$, it would violate the Froissart bound. We may ask the following two questions: Is it possible that such an expansion can make sense, with both the zeroth-order and the exact Pomeron having an intercept of one? and, How can we understand why this expansion parameter is in fact small? In this paper we answer the first of these questions affirmatively, and suggest a

tentative answer to the second. We will consider a model which satisfies unitarity and the Froissart bound, has diffraction (Pomeron exchange) in inelastic channels even at vanishing momentum transfer, and in which at intermediate energies the Pomeron appears as a simple pole with unit intercept. We then use this model as a laboratory to study the transition between simple Pomeron behavior and ultimate asymptotic behavior; in particular, in a very approximate version of the model, we relate the smallness of the expansion parameter to the smallness of the ratio of the Pomeron slope to the total cross section, as measured at intermediate energy. After this work was completed, we received a preprint by Amati, Caneschi, and Ciafaloni⁵ in which a similar model was considered, but applied to somewhat different questions than the ones which we discuss.

The model we shall take will be a version of the absorbed multiperipheral model^{6,7}. We take the amplitude T_n for producing n particles to be

$$T_n(s, b) = S^{\frac{1}{2}}(s, b) T_n^{MP}(s, b), \quad (1)$$

where S is the elastic S -matrix, which we will take to be real, and T_n^{MP} is what a multiperipheral model would give for T_n ; b is the impact parameter, and we have suppressed the final-state variables in T_n and in T_n^{MP} . As emphasized in Ref. 7, there are many ways one could do absorption, all of which are, for our purposes, essentially equivalent; we adopt multiplication by $S^{\frac{1}{2}}$ as in Eq. (1) as perhaps the most convenient and certainly the most conventional. The unitarity equation for the elastic amplitude $T = D + iA$ is

$$A(s, t) = \int d\phi_2 T T^* + \sum_{n=3}^{\infty} \int d\phi_n T_n T_n^*, \quad (2)$$

so that, from Eqs. (1) and (2) ,

$$\begin{aligned} A(s, b) &= |T(s, b)|^2 + S(s, b) \sum_n \int d\phi_n T_n^{MP} (T_n^{MP})^* \\ &= |T(s, b)|^2 + S(s, b) A^{MP}(s, b). \end{aligned} \quad (3)$$

A^{MP} is thus the unabsorbed multiperipheral expression for the overlap function. If we ignore the real part of T , then Eq. (3) can be solved for A once A^{MP} is given.

We will allow into T_n^{MP} both ordinary Regge (or particle) exchange and also exchange of the Pomeron singularity; schematically we write

$$T_n^{MP} = \mathcal{R} + \mathcal{P}, \quad (4)$$

where we include multiperipheral diagrams with any Pomeron exchange in $\mathcal{P}$.

Because of the presence of this term $\mathcal{P}$, A^{MP} will have, as its leading j -plane singularity, a moving pole with an intercept greater than one. This means that $\sigma_{tot} = s^{-1} A(s, t=0)$ as obtained from Eq. (3) will grow like $(\ln s)^2$, thus satisfying the Froissart bound. We assume also that the Regge exchanges by themselves are just strong enough to produce a Pomeron pole with unit intercept; that is, if we took, instead of (4),

$$T_n^{MP} = \mathcal{R}, \quad (4')$$

then A^{MP} and hence A would contain a Pomeron pole with intercept one.

We shall refer to the leading j -plane singularity in the solution of Eqs. (3) and (4) as the "dressed" Pomeron and that in the solution of Eqs. (3) and (4') as the "bare" Pomeron⁸. We will see that if the term $\mathcal{P}$ in (4) is small, then

phenomenology based on the bare Pomeron will be valid except at very high energy.

In the next section of this paper we discuss the bootstrap solution of this model introduced in Ref. 7 and our extension thereof; we point out that, at least within an approximate version of the model, the term P in Eq. (4), and hence the dressing of the Pomeron, can be shown to lead to small effects except at extremely high energy. If we let $\alpha'_{(0)}$ be the slope of the bare Pomeron, and $\sigma_{\text{tot}}^{(0)}$ the constant bare total cross section, and say that asymptotically $\sigma_{\text{tot}} \rightarrow 2\pi R^2 (\ln s)^2$, then the estimate we obtain is

$$\frac{2\pi R^2}{\sigma_{\text{tot}}^{(0)}} \approx \left[\frac{4\pi\alpha'_{(0)}}{\sigma_{\text{tot}}^{(0)}} \right]^{4/3} \approx .003 . \quad (5)$$

In the third section we discuss two approximation schemes based on the assumption that the P term is small; one is an expansion in powers of $\ln s$, and the other involves systematically making the P term more complete. Within these approximation schemes, we comment on "two-component" models of multiplicity distributions and find some rather unexpected behavior of partial and inclusive cross sections. In the final section we summarize the main results we have obtained.

11 Calculation of R and ξ in a Simplified Model

In this section we will make a series of simplifying approximations to the multiperipheral input into our model which will make calculations particularly simple and will enable us to relate the parameters of the eventual growth of the total cross section to the parameters of the bare Pomeron. Hopefully our results will be relevant also to more general multiperipheral models.

The calculations in Ref. 7 were done assuming a highly-factorized expression for the multiperipheral amplitudes T_n^{MP} , which we for the time being now adopt. We assume that T_n^{MP} , when written in terms of t -channel partial waves, satisfies

$$\int d\phi_n T_n^{MP} (T_n^{MP})^* = C^{n-2} (A_{FS})^{n-1}, \quad (6)$$

where C is a constant and A_{FS} is given by

$$A_{FS} = \int d\phi_2 T T^*. \quad (7)$$

Under this assumption, the multiperipheral overlap function is

$$A^{MP}(t, j) = \frac{C [A_{FS}(t, j)]^2}{1 - C A_{FS}(t, j)}. \quad (8)$$

In Ref. 7, A_{FS} was taken to have the form

$$A_{FS}(t, j) = \frac{2\pi R_s^2}{[(j-1)^2 - R_t^2]^{3/2}}, \quad (9)$$

which corresponds to scattering from a black disk of radius $R \ln(s/s_0)$, and so $A = 2A_{FS}$. This form was shown to be asymptotically self-consistent - if (9) were used in Eq. (8), and then in Eq. (3) to obtain an output amplitude A , then this output amplitude would have the same form as A_{FS} given by (9). More generally, A_{FS} should have contributions from Pomeron exchange, ordinary Regge exchange, and the interference between the two; denoting these three terms respectively by p , r , and i , we can write

$$A_{FS}(t, j) = p(t, j) + r(t, j) + i(t, j). \quad (10)$$

We will suppose that at least that part of p most singular at $t=0$, $j=1$ will be given by (9); both r and i are regular at this point. The asymptotic

behavior of the output cross sections with A_{FS} given by (10) will be essentially the same as if A_{FS} were given by (9), which has been discussed by Caneschi and Schwimmer⁹; we shall concentrate our attention on the approach to this asymptotic behavior.

We now consider a further simplified form of this model, obtained by, in the region near $t = 0$, $j = 1$, ignoring the interference term i , using only the most singular part of p , and making a linear approximation for r .

Thus we write

$$A_{FS}(t, j) = \frac{2\pi R_s^2}{[(j-1)^2 - R_t^2]^{3/2}} + r(0) + (j-1)\left(\frac{\partial r}{\partial j}\right)_0 + t\left(\frac{\partial r}{\partial t}\right)_0. \quad (11)$$

A^{MP} will have a j -plane pole which, at $t = 0$, is to the right of $j = 1$ arising from the vanishing of the denominator in (8), with A_{FS} given by (11); call the position of this pole $\alpha_{MP}(t)$. Its intercept satisfies the relation

$$\frac{2\pi R_s^2}{\xi^3} + r(0) + \xi\left(\frac{\partial r}{\partial j}\right)_0 = \frac{1}{C}, \quad (12)$$

where we have defined

$$\xi \equiv \alpha_{MP}(0) - 1.$$

In a linear approximation to this trajectory function, $\alpha_{MP}(t) \approx \xi + 1 + t \alpha_{MP}'$, we can see by following the argument advanced in Ref. 7 that the output amplitude from (3) will asymptotically correspond to scattering from a black disk of radius $\bar{R} \ln s$, where $\bar{R}$ is proportional to the product of ξ and α_{MP}' . Since for $t \neq 0$ the position of the singularity in the j -plane of the output amplitude is determined by $\bar{R}$, we can demand $\bar{R} = R$; this condition will enable us to

calculate R in terms of the Regge-exchange function $r(t, i)$.

To determine R , we can use the fact [see Ref. 7] that, for any given value of R , if we were to use Eq. (9) for A_{FS} we would obtain an output $\bar{R}$ which was equal to R . Therefore the product $\xi \alpha_{MP}'$ obtained from Eq. (11) must be the same as would be obtained from Eq. (9), for the same value of R . Now $\alpha_{MP}(t)$ is the solution of $A_{FS}(t, i) = C^{-1}$; from this we calculate

$$\xi \alpha_{MP}' = \frac{3\pi R^2 s_o + \xi^5 \left(\frac{\partial r}{\partial t}\right)_o}{6\pi R^2 s_o - \xi^4 \left(\frac{\partial r}{\partial t}\right)_o}, \quad [A_{FS} \text{ given by (11)}];$$

$$\xi \alpha_{MP}' = \frac{R^2}{2}, \quad [A_{FS} \text{ given by (9)}]. \quad (13)$$

Equating the two expressions in (13), we obtain

$$R^2 = 2\xi \left(\frac{\partial r}{\partial t}\right)_o \left[\left(\frac{\partial r}{\partial i}\right)_o\right]^{-1}. \quad (14)$$

We are assuming that if we had only Regge exchange terms [i.e., if $A_{FS} = r$] we would obtain the bare Pomeron pole with intercept one, slope $\alpha'_{(o)}$, and corresponding total cross section $\sigma_{tot}^{(o)}$. [σ_{tot} is s_o^{-1} times the residue of the pole in $A(t=0, i)$]. Thus we can identify

$$r(0) = C^{-1} = s_o \gamma \sigma_{tot}^{(o)}; \quad \left(\frac{\partial r}{\partial i}\right)_o \left[\left(\frac{\partial r}{\partial t}\right)_o\right]^{-1} = -\alpha'_{(o)}, \quad (15)$$

where we have defined

$$\gamma \equiv -\frac{1}{r(0)} \left(\frac{\partial r}{\partial i}\right)_o.$$

Then from Eqs. (12) and (14) we have

$$\frac{2\pi R^2}{\sigma_{\text{tot}}^{(0)}} = \gamma^{-2/3} \left[\frac{4\pi\alpha'_{(0)}}{\sigma_{\text{tot}}^{(0)}} \right]^{4/3} \quad (16)$$

To estimate the value of γ , we use the fact that $r(t=0, j)$ may be expected to have its rightmost singularity near $j=0$; thus we expect $\gamma \approx 1$. As discussed in the next section, $\sigma_{\text{tot}}^{(0)}$ and $\alpha'_{(0)}$ are the cross section and slope observed at intermediate energy, where inelastic diffractive effects are not important. Our crude model does not specify for what process $\sigma_{\text{tot}}^{(0)}$ is the total cross section, and the value of α' is not well determined; we take $\sigma_{\text{tot}}^{(0)} = 40 \text{ mb}$ and $\alpha'_{(0)} = 0.1 \text{ GeV}^{-2}$ (which is the largest value compatible with the analysis of ϕ photoproduction of Anderson et al.¹⁰) and obtain $R \approx 0.22 \text{ GeV}^{-1}$. The contribution to σ_{tot} of $2\pi R^2 \ln^2(s/s_0)$ then is, at $\ln(s/s_0) = 8$ [ISR energy if $s_0 = 1 \text{ GeV}^2$] about 8 mb.

As discussed in the next section, inelastic diffractive effects are unimportant if $\xi \ln(s/s_0) \ll 1$; therefore an estimate of the energy at which the bare Pomeron is no longer adequate is given by

$$\xi \ln(s/s_0) \approx 1 \quad (17)$$

From Eqs. (12) and (16),

$$\xi = \left[\frac{4\pi\alpha'_{(0)}}{\gamma\sigma_{\text{tot}}^{(0)}} \right]^{1/3}; \quad (18)$$

thus with the values of γ , $\sigma_{\text{tot}}^{(0)}$, and $\alpha'_{(0)}$ we have assumed, we get $\xi \approx 0.23$. This estimate does not really set an energy scale, since s_0 is not determined by our considerations; however, if we take $s_0 = 1 \text{ GeV}^2$, we get, from (17), $s \approx 80 \text{ GeV}^2$.

Of course, the real world is not nearly as simple as we have assumed

in writing Eq. (11) ; furthermore, we have not even attempted to explain how the (ordinary) multiperipheral model can succeed in producing a small slope for the bare Pomeron. We would not attach any great significance to the numerical values we have attributed to R and to ξ . However, we do consider as interesting this suggestion of a relation between the small values of the high-energy parameters R [Eq. (16)] and ξ [Eq. (18)] on the one hand, and of the ratio $\alpha'_{(0)}/\sigma_{\text{tot}}^{(0)} \approx 10^{-3}$, which is measured at intermediate energy, on the other.

III Approximation Schemes

A. Expansion in Powers of $\xi \ln s$

In the previous section we defined ξ to be the amount by which the intercept of the leading pole in A^{MP} exceeded unity, and, in a simplified version of our model, were able to calculate a rather small value for ξ (≈ 0.2). Somewhat more generally, we can rewrite Eq. (10) as

$$A_{\text{FS}}(t, j) = g^2 \bar{p}(t, j) + r(t, j) + g \bar{i}(t, j) , \quad (16)$$

where we have defined $\bar{p} = p/g^2$, and $\bar{i} = i/g$; we can then consider ξ to be a function of g , which vanishes linearly with g if p consists of the bare Pomeron, or which vanishes as $g^{\frac{1}{2}}$ if p contains the more singular dressed Pomeron. Either way, ξ will be small if diffraction in inelastic processes is a small effect, and so we may wish to perform an expansion in powers of ξ .

Since ξ is related to g , this expansion might also be thought of as an expansion in the number of Pomerons that are exchanged in the multiperipheral ladder ⁽¹¹⁾ .

Since A^{MP} has the behavior

$$A^{MP}(s, t=0) \rightarrow \beta s^{1+\xi}, \quad (17)$$

we can write (12), with the definition $Y \equiv \ln s$,

$$A^{MP}(s, 0) = s \sum_{k=0}^{\infty} \xi^k f_k(s), \text{ with } f_k(s) = O(Y^k). \quad (18)$$

We wish to expand the elastic absorptive part A in powers of ξ . From (17), when $\xi Y \ll 1$ a good approximation is given by just the zeroth order in this expansion, which corresponds to not having any diffraction in the inelastic processes. Thus for these energies, it is phenomenologically a good approximation to consider the Pomeron to be bare; hence, ignoring corrections of order Y^{-1} , the total cross section appears to be constant and, (since the bare Pomeron pole is generated by a multiperipheral expansion with A_{FS} regular at $j=1$) factorization should hold. To see what the higher-order terms in ξ are like, we can use Eq. (16), generalized [by replacing ξ with $\alpha_{MP}(t) - \alpha_{(0)}(t)$] to non-zero t , in Eq. (3) to solve for the elastic absorptive part A . If we ignore the real part of T , the solution of (3) is

$$A(s, b) = A^{MP} - \frac{1}{2} \left[(4(A^{MP})^2 + 1)^{\frac{1}{2}} - 1 \right]. \quad (19)$$

Let $\Delta \equiv A - A^{MP}$, and expand (19) in a series in A^{MP} :

$$\begin{aligned} A &= A^{MP} + \Delta \\ &= A^{MP} + \sum_{i=1}^{\infty} C_i (A^{MP})^{2i}, \end{aligned} \quad (20)$$

where the C_i are numerical coefficients. The total cross section, which equals $s^{-1} A(s, t=0)$, has an expression similar to that in (20), if we understand the

products of A^{MP} contained in Δ to be convolutions in t . Suppose we keep a finite number of terms, say those up to $k = k_{\max}$, in the expression (18) for A^{MP} . Then A^{MP} will behave as $Y^{k_{\max}}$, while at this order in ξ no term in Δ will be this large. This is because, although Δ will have terms of the form $(\xi^{k_{\max}} f_{k_1} f_{k_2})$, with $k_1 + k_2 = k_{\max}$, the convolution of f_{k_1} with f_{k_2} will be smaller than $Y^{k_1 + k_2}$ due to Regge shrinkage. Thus we see from (20) that, if we take a finite order k in ξ , and then take the leading term in Y , the absorption has no effect; in this limit $A(s, t=0) = A^{MP}(s, t=0)$ and hence σ_{tot} increases as Y^k with a positive coefficient. On the other hand, if we take all orders in ξ , then σ_{tot} increases as Y^2 .

This apparently paradoxical behavior can perhaps be visualized by considering the function

$$f(s) = \frac{s^\xi}{1 + s^\xi Y^{-1}} \quad (21)$$

This function increases linearly with Y ; also, if one expands separately the numerator and denominator of (21) to finite order in ξ , one again obtains a function that increases linearly with Y . However, if one expanded f to some finite order k in ξ , and then took the limit $s \rightarrow \infty$, one would obtain the incorrect estimate $f(s) \rightarrow (\xi Y)^k / (k!)$. Moreover, although both the exact and the first-order expressions for f increase linearly with Y , the coefficients of Y in these two cases is not the same.

To summarize this section: Our model produces a total cross section which grows as Y^2 ; to zeroth order in the expansion in ξ σ_{tot} is a constant while to order k it grows as Y^k . This expansion is valid if ξY is small, but gives a misleading picture of the limit $Y \rightarrow \infty$.

B. Behavior of Partial and Inclusive Cross Sections: Iteration for P

We may expect there to be three regions of energy in which the behavior of σ_{tot} is particularly simple: very low energy, where Regge exchange dominates and σ_{tot} falls (i.e., the resonance region); intermediate energy, where the bare Pomeron gives a good approximation, and so σ_{tot} is constant; very high energy, where the dressing of the Pomeron is important, and σ_{tot} slowly rises. It is the transition between the last two of these regions that we have discussed above. Even if we do not make the extreme factorization approximation implied by Eq. (6), we may expect qualitatively similar behavior in the subenergies of a production process. At any given energy s the subenergies s_i tend to be considerably smaller than s (especially for large multiplicity), so that even if s is so large that the dressed Pomeron must be used to describe the behavior of the elastic amplitude, it may well be that the behavior in the subenergies is much simpler in the major part of phase space. The sizes of the subenergies depend to some extent on the multiplicities; of the various possible cases, let us consider the following two:

Case i : Bare Pomeron behavior in s , Regge behavior in s_i .

Case ii : Dressed Pomeron behavior in s , Regge and Bare Pomeron behavior in s_i .

The actual production amplitudes T_n as given in Eq. (1) depend on the subenergies s_i through T_n^{MP} and on the overall energy s through the absorptive factor $S^{\frac{1}{2}}$.

These two cases can also be viewed as the first two steps of an iteration for P (as has been advocated by Amati, Caneschi, and Ciafaloni⁵): Start

with pure Regge behavior in T_n^{MP} , and solve Eq. (3) to find the bare Pomeron in the elastic amplitude A (that's case i); now go back and put the bare Pomeron into the subenergies of T_n^{MP} , and see that Eq. (3) produces the dressed Pomeron in A (that's case ii). The next step in this iteration, to put the dressed Pomeron into the subenergies, would be relevant when the subenergies themselves were very large. This next step would leave the behavior of the total cross section essentially unchanged; for examining the behavior of σ_{tot} it suffices to let the subenergy Pomerons be bare. In particular, one could consider making an expansion in powers of $\xi \ln s$ of the total cross section as calculated in case ii, and would find essentially the same results as we had above.

Case i We take T_n^{MP} to be constructed from Regge exchange with no diffraction.

To find the actual production amplitude $T_n = S^{\frac{1}{2}} T_n^{MP}$, we have to solve Eq. (3) so that we know what the absorption factor $S^{\frac{1}{2}}$ is. This factor will affect the distribution of the produced particles but it will leave the energy dependence of the partial cross section σ_n essentially unchanged; that is, (neglecting logarithmic terms) $\sigma_n(s) \rightarrow s^{2\alpha_R - 2}$.

Eq. (3) taken at $t = 0$ gives an expression for σ_{tot} which is similar to that of a two-component model. Low multiplicities (in our simplified discussion this means $n = 2$) are given by a diffractive mechanism and the higher multiplicities by a Regge mechanism; in other words, σ_2 is roughly a constant while for $n > 2$ σ_n disappears with a Regge power. However, the high-multiplicity component is not purely multiperipheral because of the absorption. In a very rough sense, absorption can be thought of as multiplication by the factor $(1 - 2\sigma_{el}/\sigma_{tot})$; in this sense, if we denote by σ_D the diffractive (low-multiplicity)

cross section and σ_{MP} the unabsorbed multiperipheral cross section, we can write ¹³

$$\sigma_{tot} = \sigma_D + \sigma_{MP} (1 - \sigma_{el}/\sigma_{tot}). \quad (22)$$

Thus although σ_{MP} might be a reasonable approximation to σ_{tot} , it seems that σ_D should not be a good approximation to $\sigma_{tot} - \sigma_{MP}$.

Before absorption, events with higher multiplicities tend to take place at lower values of the impact parameter ⁹, and so are absorbed less than are the lower multiperipheral multiplicities; thus absorption increases the average multiplicity. On the other hand, the addition of diffractive events decreases the average multiplicity, and so these two effects together tend to leave the average multiplicity the same as it would be in a purely multiperipheral model, which is in accord with the suggestion advanced by Abramovskii, Kancheli, and Gribov ¹⁴.

Case ii Now we allow into T_n^{MP} the exchange of bare Pomerons. Before absorption, the behavior of σ_n would now be (neglecting logarithmic terms) $\sigma_n(s) \rightarrow \text{constant}$. However, this time absorption does have an effect on the energy dependence. For small values of the impact parameter b (this means b/Y small), $A^{MP}(s, b)$ increases as s^ξ , while $A(s, b)$ satisfies unitarity and so is bounded. From Eq. (3), we can see

$$S(s, b) \rightarrow s^{-\xi} \quad (s \rightarrow \infty \quad \text{with } b/Y \rightarrow 0); \quad (23)$$

the scattering disk gets more nearly black as the energy increases.

The range of b that contributes to T_n^{MP} grows only as $Y^{\frac{1}{2}}$, so

from (23) we can see

$$\sigma_n(s) \rightarrow s^{-\xi} \quad (s \rightarrow \infty \quad \text{with bare Pomerons in } T_n^{MP}) . \quad (24)$$

This estimate does not give the truly-asymptotic behavior of σ_n in our model, which is $\sigma_n(s) \rightarrow \text{constant}$ (see Ref. (9)). What happens is that ultimately we have to allow dressed Pomerons into T_n^{MP} ; they force us to consider values of b of the order of Y , and so we cannot use the estimate (23) for S .

Inclusive distributions can be calculated in our model by doing the multiperipheral calculation, and then absorbing. If M is the missing mass in a single-particle distribution, then in the case we are considering the multiperipheral calculation produces a factor of $(M^2)^\xi$ and then absorption gives a factor of $s^{-\xi}$. Thus scaling is preserved, but if $1-x \equiv M^2/s \rightarrow 0$, then the "triple-Pomeron" behavior is (again ignoring logarithmic terms),

$$\frac{d\sigma}{dx} \rightarrow \left(\frac{1}{1-x} \right)^{1-\xi}, \quad (x \rightarrow 1 \quad \text{with bare Pomerons in } T_n^{MP}) . \quad (25)$$

The truly-asymptotic behavior of this cross section in our model does not⁹ have the factor $(1-x)^\xi$. The estimate in (25) breaks down when the subenergy s/M^2 is so large that the Pomeron in that subenergy must get dressed. However, for values of x rather close to 1 [perhaps $x \lesssim 0.99$ (?)] formula (25) should hold.

IV Summary

(1) The absorbed multiperipheral model, with the input T_n^{MP} containing both Regge and Pomeron exchange, yields total cross sections which at extremely

high energy rise as Y^2 . At intermediate energy, where it is justified to regard T_n^{MP} as constructed from Regge exchange only, total cross sections appear constant. We suggest that the model can be useful for understanding the transition between these two types of behavior.

(2) If we do take T_n^{MP} to contain only Regge exchange, there are still (small) corrections to production amplitudes due to absorption. This effect can provide a correction to the multiperipheral component of the total cross section of roughly the same size as is the diffraction component. The total cross section in this approximation is asymptotically constant, and factorization is approximately valid.

(3) If we allow the exchange of bare Pomerons in T_n^{MP} , the total cross section increases as Y^2 . We also obtain the estimates $\sigma_n(s) \rightarrow s^{-\xi}$, $\frac{d\sigma}{dx} \rightarrow (1-x)^{\xi-1}$; while these estimates do not correspond to the truly-asymptotic limits of our model, they should be valid until one reaches extremely large values of subenergies.

(4) The total cross section can be expanded in powers of ξ ; taking terms of order ξ^k , we obtain the estimate $\sigma_{tot} \rightarrow Y^k$. Nevertheless the Froissart bound is respected.

(5) In a simplified form of the model, we have been able to calculate R and ξ as powers of the small parameter $\alpha'_{(0)}/\sigma_{tot}^{(0)}$. We conjecture that similar relations between the parameters of the high- and the intermediate-energy behavior would hold also in more realistic models, and that these relations could provide a partial understanding of why R and ξ are small.

FOOTNOTES

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6. L. Caneschi, Phys. Rev. Letters 23, 254 (1969).
7. J. Finkelstein and F. Zachariasen, Phys. Letters 34B, 631 (1971).
8. What we call the "bare" Pomeron has as its most singular part a simple pole, but contains also Mandelstam cuts. Perhaps we should have called this the Pomeron "in its underwear."
9. L. Caneschi and A. Schwimmer, Nucl. Phys. B44, 31 (1972)
and Nucl. Phys. B48, 519 (1972).
10. R. L. Anderson et al., Phys. Rev. Letters 30, 149 (1973).
11. This identification is most useful if one ignores the logarithmic singularity of $\bar{p}$ at $t=0$, $j=1$; otherwise, an expansion in g could be quite complicated.
12. Some of the f_k in Eq. (18) may vanish, due to the fact that there can be several poles near the point $j=1$ which contribute to the expansion in (18).
In the simplified example discussed in the previous section, we have (see Eq. (12)) $\xi \sim R^{\frac{1}{2}}$; since only terms in R^2 will appear, the only non-zero f_k are those with k a multiple of 4. Also, we will identify f_0 with the residue obtained in the case $\xi = 0$, which is an excellent approximation for small ξ .

13. A similar argument has been stated in Ref. 6 .
14. V. A. Abramovskii, O. V. Kancheli, and V. N. Gribov, Proceedings of the XVI International Conference on High Energy Physics, Vol. 1, p. 389.