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ENGINEERING REPORT

Thompson Products, Inc.

CLEVELAND, OHIO

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REPORT NO. ER-3234

Dynamic Characteristics and
Equations for a Control System
Analysis of a Mercury Power
Conversion System

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1.0 INTRODUCTION

A conceptual design of a mercury vapor power conversion system has been presented in ER-3099. Included in this design effort was a discussion of the control system required to obtain satisfactory system performance. This discussion was held quite general, and no attempt was made to determine the exact performance, stability or precision required or obtainable with the system discussed. This information has presented the basic requirements for control of the system and two control approaches which appear logical.

The next step in the control study program is to first define as clearly as possible the exact requirements of the system which the control must adhere to. Once this is established, it is then necessary to define the system characteristics which the control must work with. Finally, a description of the system must be obtained in a form which is suitable for control analysis.

Much of the information which is currently available is of a tentative nature since the entire system is still in the early design stages and no actual hardware is available for testing. It is felt, however, that an adequate description of the system can be obtained to make a significant start in the basic control system design.

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2.0 OBJECT

The object of this report is to present the information required for a complete control investigation of a mercury vapor power conversion system. The best data and estimates available will be presented to define the system requirements, characteristics, and dynamic equations pertinent to such a control study.

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3.0 GENERAL SYSTEM DESCRIPTION

3.1 Basic System Requirements

The basic requirements of the mercury power conversion unit are that the system be capable of operating as a self contained unit for a period of not less than sixty days. The system must be capable of producing electrical power from thermal energy in a free space environment and be able to withstand launching accelerations, high vacuum, zero gravity effects, cosmic and heat source radiations, extreme operating temperatures, and possible meteoric bombardment.

The power unit will be required to supply electrical energy to a battery load at the rate of 500 watts for a period of 60 days. The system must be completely reliable and function without benefit of maintenance for the entire period. The system must also be of minimum size and weight.

The net power output of 500 watts is to be supplied by a turbine driven alternator. The alternator voltage will be transformed and rectified by silicon diodes to supply D.C. voltage for charging a battery load bank. These voltages will range from 7.4 volts to 296 volts. The steady power output of the alternator will maintain the batteries at the full charge level for the 60 day period and help supply the system load during the operating cycle.

3.2 Functional Requirements

The power unit will consist of five major parts. These are the boiler or thermal heat source, the turbine, condenser and subcoolers, boiler feed pump, and the alternator. These five components shown

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~ BASIC SYSTEM DIAGRAM ~

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FIGURE 3.2.1

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schematically in figure 3.2.1 are required to function in the following manner:

- 1) Thermal energy from the heat source is transferred to the cycle fluid in the boiler.
- 2) The cycle fluid now in vapor form is directed through the turbine where it releases energy in the form of shaft power.
- 3) This shaft power is converted to electrical power by the alternator and supplied to the battery load bank through a transformer rectifier circuit.
- 4) The working fluid leaving the turbine is returned to the liquid state by the condenser and subcooler.
- 5) The cooled liquid is returned to the boiler by means of the boiler feed pump which like the alternator is driven by the turbine shaft.

3.2.1 The boiler section consists of a heat source, a boiler tube element and a heat rejection control. The heat source in the form of a radio isotope must be capable of delivering the power required over the 60 day period. The boiler tube element may be described as consisting of three parts, a preheater, boiler and super heater. These parts must be designed so the heat transfer characteristics and areas of each permit the desired transfer of thermal energy to the working fluid as it passes through the boiler tube element. The heat input to the system must be maintained at a 5000 watt level by the heat rejection control. This is necessary to produce the required 500 watt output with an overall cycle efficiency

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of 10%.

3.2.2 The alternator is required to deliver the electrical power required by the battery load and the power necessary for control purposes.

3.2.3 The turbine must be designed to convert energy from the cycle fluid to shaft power necessary to drive the alternator, pump, and to overcome friction losses which occur in the bearings and seals.

3.2.4 The condenser must be capable of radiating thermal energy to condense the mercury vapor after it has passed through the turbine. The physical design will be such as to effect the complete transition from the vapor state at the turbine outlet to the liquid state at the subcooler inlet. This will account for about 85% of the boiler input to the system.

Additional cooling of the liquid to the temperature desired at the pump inlet will be accomplished by the subcooler. In addition to the subcooler used in conjunction with the condenser, a second electrical cooler will be provided to dissipate heat generated in the electrical controls and the alternator windings.

The condenser, subcooler, and electrical cooler will account for all energy rejected from the system, other than that which is transferred to the battery load, and the excess energy which is radiated at the heat source itself.

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3.2.5 Finally the pump will be of the centrifugal variety driven by the turbine shaft, and designed to deliver the cycle fluid to the boiler element at the desired pressure and rate. Also the pump must supply enough additional fluid to satisfy the bearing requirements and the electrical cooling demands.

3.2.6 The power conversion system is designed to supply electrical energy at the constant rate of .5 kw to a battery load. However, due to the charging characteristics of such a load there may be some variation in the rate at which power would be absorbed. Also to prevent battery damage as a result of overcharge, some method of switching the alternator output to a resistive parasitic load of similar impedance is required. At present it is felt most likely that some switch mechanism sensitive to battery potential or internal pressure will be utilized.

While Thompson Products is not directly responsible for the operation of this portion of the system, the effects of such load variations must be considered in terms of its influence on the other components of the power conversion system.

Similarly the excess heat rejection which occurs at the heat source must be controlled in some manner to maintain the heat input to the system at a constant 5 KW. While the mode of operation of this control is not of direct concern, the influence it has on the rest of the system is of importance and therefore must be considered.

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3.3 Design Conditions and Performance Estimates

Based on the present evaluation for optimum design condition the following summary presents overall system performance estimates and the design point conditions.

Summary Design Conditions and Performance Estimates

1. Max mercury superheat temperature	1300°F
2. Mercury boiling temperature	1025°F
3. Mercury inlet temperature	333 °F
4. Turbine inlet temperature	1300°F
5. Condenser inlet temperature	500°F
6. Combined subcooler inlet temperature	366°F
7. Subcooler outlet temperature	330°F
8. Electrical cooler inlet temperature	333°F
9. Electrical cooler outlet temperature	200°F
10. Heat source temperature	1335°F
11. Boiler pressure	210 psia
12. Turbine inlet pressure	210 psia
13. Condenser pressure	1.9 psia
14. Subcooler pressure	1.9 psia
15. Pump outlet pressure	250 psia
16. Turbine speed	666 rps-40000 rpm
17. Heat input to system	5000 kw
18. Power output to load	.500 kw
19. Turbine Power output	.667 kw
20. Power required by alternator	.598 kw

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21. Power required by pump	.029 kw
22. Power required by bearings	.040 kw
23. Power required by controls	.038 kw
24. Weight flow to cycle	.0311 #/sec = 112 #/hr.
25. Weight flow to bearings	300 #/hr.
26. Weight flow electrical cooling	67 #/hr.
27. Pump leakage	240 #/hr.
28. Total pump weight flow	.2 #/sec = 720 #/hr
29. Turbine efficiency	40%
30. Mechanical & electrical efficiency	75%
31. Ideal cycle efficiency	33.3%
32. Overall efficiency	10%
33. Alternator efficiency	90%

Note: The above figures neglect small flow losses which would produce slight pressure drops in the system.

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4.0 SPECIFICATION FOR MERCURY POWER CONVERSION SYSTEM

Thompson Products will have responsibility for developing the turbine, pump, alternator, controls, condenser and coolers, and structures required to convert heat to electrical energy. The Martin Company Nuclear Division will have responsibility for the heat source, boiler heat transfer problems and the thermal rejection control. The responsibility of load design and general arrangement and location of various power conversion components will be left to the prime contractors.

SPECIFICATION

It is assumed the power conversion system must meet the following requirements:

4.1 Electrical

A. Net Alternator power output	12 KW H/day
B. Alternator voltage and frequency	Optional
Design values assumed	125 volts - 2000 cps
C. Maximum rectified voltages	7.4, 26.6, 148, 296
D. Voltage regulation (assumed)	1%
E. System peak load	3.0 kw
1. 2.5 kw battery	
2. .5 kw alternator	
F. Duration of Peak load	8 duty cycles per day
of 20 minutes each
G. Continuous system demand on alternator	100 watts
H. Battery charging demand on alternator	400 watts

4.2 Thermal

- A. High energy time degenerating heat source.

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- B. Radiating condenser area less than 100 sq. ft.
- C. Provision for cooling of alternator and electrical controls

4.3 Mechanical

- A. All components must be capable of operating under high acceleration and vibrational load during the launching period.
 - 1. 10 g's parallel to flight direction
 - 2. 3 g's normal to flight direction
 - 3. 25 g's vibratory load over a range of zero to 2000 cps
- B. The operation of the power unit must not affect the vehicle orientation or induce torques about the several axis greater than the following:
 - 1. Yaw axis 5×10^{-4} in.oz.
 - 2. Pitch axis 3×10^{-2} in.oz.
 - 3. Roll axis 5×10^{-2} in.oz.
 - 4. Maximum permissible angular displacement .1 degree

Considering the maximum allowable displacement of .1 degree and using the total mass moment of the rotating section as 7.55×10^{-5} ft #sec² the following speed changes are acceptable. That is to say a step speed change of the magnitude indicated will not cause the vehicle to tip at an angle $> .1$ degree if the duration of the offspeed condition is not greater than that specified.

Step speed change < 1570 rpm - t max - unlimited
 speed change ≥ 1570 rpm - t max ≤ 880 sec.
 speed change = 3000 rpm - t max = 250 sec.
 speed change = 5000 rpm - t max = 175 sec.

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(The above values are determined from equations (5B) TM-971 which deals with Satellite Attitude Stability)

$$(5B) \quad \theta_{max} = .1^\circ = \frac{I_T \Delta \omega}{\sqrt{2 K_0 I_V}} \sin \sqrt{\frac{K_0}{I_V}} t$$

where I_V = vehicle mass moment t = time sec.

$$\Delta \omega = \text{turbine speed change} \quad K_o = \text{restoring const.}$$

4.4 Physical

- A. Weight and size requirements - minimum
- B. Useful life 60 days

The control analysis will be made on the basis of a power conversion designed to meet the preceding specifications. Thompson Products will also make use of reasonable assumptions regarding any important system characteristic not covered by these specifications.

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5.0 PARAMETERS AVAILABLE FOR CONTROL

In defining the system to be controlled, it is necessary to determine all of parameters which are available for control purposes. The effect of these parameters must then be included in the analysis so that any method of control can be investigated.

In regulating the power input to the conversion unit, the following control function can be utilized:

1. Shutters are available to regulate the amount of energy dissipated directly from the heat source.
2. It is possible to insert a pressure regulator in the line from the pump to the preheater and to control pressure within the cycle.

The power output from the conversion unit can be varied in the following ways:

1. Turbine power output can be controlled by regulating boiler pressure.
2. Turbine power output can be controlled by regulating turbine exhaust pressure.
3. Load can be controlled by a small parasitic load varied by a saturable reactor. (This load would be small so as not to effect the overall cycle efficiency).
4. Load current can be controlled by using a saturable reactor in series with the load.

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5. Voltage can be controlled on a flux switching type alternator by inserting an auxiliary field winding.
6. Load could be controlled by a parasitic alternator designed to limit output power at a reduction of cycle efficiency.

5.1 Critical Control Evaluation

Considering the total system as represented by the four simple elements shown in Figure 5.1.1 an evaluation of control parameters is developed. For each element the several inputs and outputs are stated and the properties of the system related to the influencing variables in a tabular manner. The effect of a step change in each of the variables is evaluated by a figure of merit proportionate to the relative influence on the property. The figures of merit, G - good, F - fair, P - poor, N - nil, are indicative of the magnitude and speed of response.

SYSTEM

AR - RADIATION AREA P_2 - INPUT POWER BPO - BRG. POWER OUT
 T_0 - INLET TEMP. T_1 - SUPERHEAT TEMP. PPO - PUMP POWER OUT
 W - WEIGHT FLOW P_C - COND. PRESS. APO - ALT. POWER OUT
 P_1 - BOILER PRESS TPO - TURB. POWER OUT SL - SYSTEM LOAD
 CL - CONTROL LOAD P_L - PARASITIC LOAD

FIGURE 5.1.1.

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HEAT SOURCE POWER INPUT

	Ar Radiated Power	W Weight Flow	p_1 Boiler Pressure	To Inlet Temp.
Power Input	P	G	$\frac{G}{P}$	P
H - Enthalpy	P	N	$\frac{P}{P}$	P
Tl - Superheat	P	P	P	P

TURBINE POWER OUTPUT

	p_1 Boiler Pressure	Tl Superheat Temp.	p_c Condenser Pressure
Power Output	$\frac{G}{G}$	F	F
Speed	$\frac{G}{G}$	F	F
Torque	G	F	F

ALTERNATOR POWER OUTPUT

	Turbine Power	Bearing Power	Pump Power
Power Output	$\frac{G}{G}$	P	P
Voltage	G	P	P
Current	G	P	P
Power Factor	N	N	N
Speed	G	P	P
Torque	G	P	P

ELECTRICAL LOAD POWER INPUT

	Alternator Output	Control Power	Parasitic Load	Electrical Load
Load Input	$\frac{G}{G}$	P	F	G
Voltage	G	P	F	P
Current	G	P	F	G
Power Factor	N	N	P	F

(Eliminate electrical load variable because of difficulty in control.)

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A summary of results quickly shows that all major properties can be related to, and controlled most readily by the pressure variable *pi*. The other variable present themselves as being less suitable as control factors but are not to be disregarded because many factors such as stability and problems in design and fabrication are not accounted for by the figure of merit rating.

Based on the most prominent control variables listed, several complete systems are suggested for a more intensive study. Several of these systems incorporate devices to stabilize the electrical load. Even though it is impractical to attempt direct load control any limitation imposed on load variation, such as would be effected by a current limiter, definitely aid in stabilizing the overall system.

5.2 Tentative Control Systems Under Consideration

The following four systems represent the most feasible approach to the control problem as seen to date. Basically, we are interested in a two loop system which supplies stability in: A. The electrical load, and B. The turbine speed.

SYSTEM 1

- A. Constant current control using a saturable reactor.
- B. A turbine speed control using a boiler pressure regulator.

SYSTEM 2

- A. Constant turbine power output control. (No speed control)
Turbine power controlled by boiler pressure throttle.

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SYSTEM 3

- A. A constant load power control.
- B. A turbine speed control using a boiler pressure regulator.

SYSTEM 4

- A. A parasitic load control.
- B. A fixed boiler pressure regulator.

The following are merits and limitations of these four systems:

MERITS

System 1.

- A. Constant charging current.
- B. Speed regulation.
- C. Accommodates for system deterioration.

System 2.

- A. Constant power to electrical load.
- B. Fewer control components.
- C. Accommodates for system deterioration.

System 3.

- A. Constant power to electrical load.
- B. Speed regulation.
- C. Accommodates for system deterioration.

System 4.

- A. Constant power to electrical load
- B. Speed regulation

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LIMITATIONS

System 1.

- A. Output power is not constant.

System 2.

- A. No speed regulation (attitude instability).

System 3.

- A. None

System 4.

- A. Requires overdesign of system components.
- B. Wastes power.

The final selection of a control system will be determined by the relative merits of the system, as listed above, the dynamic characteristics, as determined by the computer study, the mechanical reliability and ease of fabrication.

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6.0 SYSTEM DISTURBANCES

To achieve a power conversion unit of the smallest size and weight, it is desirable to maintain a constant input and output power. One of the objectives of the control study is to achieve such a system despite known variations in the power source and the load. These variations in source and load can be defined by definite relations. There will be, in addition, disturbances to the system of a random nature which cannot be predicted. It is necessary that the control system be capable of performing properly when subjected to both the normal and random system disturbances.

6.1 General Disturbances

Studies of the normal system disturbances can be handled by applying the defined relations in the control analysis. For very slowly varying disturbances, such as the slow decay of heat source power, it will most likely be sufficient to examine the effect as a variation in steady-state operating point.

Other disturbances which should be examined include the following:

1. Change in alternator output voltage such as might occur due to damage or a momentary overload or underload.
2. Small variations in the heat source power.
3. Torque disturbances on the turbine due to switching in the electrical load circuit.
4. Pressure fluctuation in the system due to changes in condenser temperature.
5. Deterioration of component characteristics, (bearing erosion, deterioration of radiating surfaces, turbine blade erosion, etc.)
6. Variations in incident power due to sun-shade operation.

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6.2 Determination of Electrical Load Variation

In the conduct of determining a suitable control for the power conversion system, it is apparent that the power demanded from the alternator must be stabilized. To accomplish this task it becomes obvious that the electrical load be defined for the two power demand states of the system, that is:

1. Maximum System Power Demand of 3 KW.

(2.5 KW being supplied by the battery bank and 0.5 KW supplied by the alternator)

2. Steady State System Power Demand of 500 watts, 400 watts utilized as battery charging power and 100 watts to the system external load.

By proportioning the 90 minute orbital period to cover the complete electrical cycle, we have developed a typical load program for the turbine-alternator. The complete electrical cycle is 90 minutes with the peak demand time of 10 minutes representing by proportion the 20 minute peak load which occurs 8 times per day (every 3 hours) as stated in the section on system specifications.

The maximum System Power Demand of 3KW occurs at a maximum rate of one peak of 10 minutes duration every ninety minutes. Assuming 10 minutes per cycle as the maximum peak power time, the steady state time is 80 minutes. The battery bank is recharged during this steady power time. (See Fig. 6.2.1. Also for Turbine-Alternator load variations, see Fig. 6.2.1.1)

In accordance with the present plan as set forth by Lockheed, the storage battery is intended to operate in the charge level range

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~ LOAD CYCLE DIAGRAM ~

FIGURE-G.2.1

POWER REQUIRED BY LOAD (WATTS)

TIME (MIN.) — ASSUME ORBITAL CYCLE = 90 MIN.

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FIGURE 6.2.1.1

ALTERNATOR POWER OUTPUT

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of 15% to 75% in order to realize maximum efficiency at a fast recharge rate, avoid gas evolution, and allow an adequate capacity of safety for reliability and to reduce load power variations. In order to avoid possible inequalities in charging, the 4 battery sections it may be necessary that each battery be charged individually with appropriate controls, since charge and discharge power for the four potentials may not be exactly matched. This arrangement requires four individual secondary windings on the charging power transformer with associated rectifiers and charge control components.

A Nickel cadmium battery of 400 volts with taps at 6.3, 28.0, and 150.0 volts has been proposed. Since the charging time per cycle is 80 minutes, the characteristics of this type of battery at a fixed input power are given at the 15% and 75% charge level to be as follows:

TABLE 6.2.1

Voltages

	No. of Cells	15% charge	75% charge
Per Cell	1	1.42	1.48
6.3 volt	5	7.1	7.4
28 volt	18	25.52	26.6
150 volt	100	142.0	148.0
400 volt	200	284.0	296.0

A Typical Battery Charging Power Calculation:

Using the initial estimates in the table above and information obtained from a typical charge curve of a nickel-cadmium battery at a constant $V_t = 1.55$ volts, the battery charging power required at the extremes of the charge level range of 15% to 75% can be

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calculated.

Given $V_t = 1.55$ Volts

EMF/cell at 15% charge = 1.42

EMF/cell at 75% charge = 1.48

Battery Charging Efficiency = 78%

Full charge capacity = 8.25 Amp-Hrs for 2 hrs charge

Charging Current at Full Charge = 0.75 AMPS

15% Charge capacity = 1.24 AMP-Hrs

Charging Current at 15% full charge = 30.0 AMPS

75% Charge Capacity = 6.2 AMP-Hrs

Charging Current at 75% full charge = 3.8 AMPS

R Battery at 15% charge = .00433 OHMS

R Battery at 75% charge = .0184 OHMS

Assuming a constant charging current calculate battery terminal voltage V_t and required charging power at 15% and 75% charge

$$I_c = \text{Charge Input} = \frac{8.250 \text{ AMP-HRS}}{2 \text{ hrs}} = 4.125 \text{ AMPS}$$

$$\text{at 15\% } V_t = (4.125)(.00433) + 1.42 = 1.4378 \text{ volts}$$

$$\text{at 75\% } V_t = (4.125)(.0184) + 1.48 = 1.556 \text{ volts}$$

$$\text{Charge Power at 15\%} = 4.125 \times 1.4378 = 5.94 \text{ watts}$$

$$\text{Charge Power at 75\%} = 4.125 \times 1.556 = 6.42 \text{ watts}$$

$$\% \text{ Power variation} = \frac{6.42 - 5.94}{5.94} = 8.1\%$$

During a peak power demand of 2.5 Kw for a 10 minute duration, the average power delivered by the battery is $2.5 \text{ KW} \times 1/6 \text{ hr} = 417 \text{ Watt-hrs}$. Therefore, the average power required by the batteries during a charge time of 80 minutes is

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$$\frac{417 \text{ Watt-hrs}}{.78 \times 80/60 \text{ hrs}} = 312 \text{ watts}$$

$$\text{Actual Average Charge Power} = \frac{312 \text{ watts}}{.78} = 400 \text{ watts.}$$

Since a total of 400 watts is available for charging the average charge time will be

$$\frac{400}{400} \times 80 = 80 \text{ minutes}$$

It should be noted at this point that a calculation using a constant voltage source for charging the batteries showed an extremely wide charging power variation and this method of charging the batteries was eliminated from further considerations.

6.3 Parasitic Resistor Calculations

Since battery characteristics change with total charge and discharge cycles and since the exact external load distribution is still undefined, it is reasonable to assume that the four battery sections will reach their 75% charge potential at different times. Consequently a charge control device is necessary since overcharge must be prevented.

As shown in Figure 6.2.1, the average charge time of 80 minutes is equal to the steady power demand time of 80 minutes. In the event that a peak power demand is not called for when the batteries have reached 75% of full charge, provision must be made to switch in a parasitic load in each secondary circuit. This will insure an overall constant load (within 10%) to the power conversion system, and safeguard against overcharging.

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Using the final charge potential for each of the battery sections

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ELECTRICAL LOAD SCHEMATIC

V_T = BATTERY TERMINAL VOLTAGE

E_R = RECTIFIED CHARGING VOLTAGE

E_B = BATTERY EMF

R_P = PARASITIC LOAD RESISTOR

Z_B = INTERNAL BATTERY RESISTANCE

Z_{LSP} = STEADY POWER LOAD IMPEDANCE

Z_{LPP} = PEAK POWER LOAD IMPEDANCE

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FIGURE - G.2.2.

as given in Table 6.2.1 and assuming 100 watts charging power for each section, parasitic resistor values can be calculated as follows:

$$R_{6.3v} = \frac{E^2}{P} = \frac{7.4^2}{100} = .548 \text{ OHMS}$$

$$R_{28v} = \frac{26.6^2}{100} = 7.07 \text{ OHMS}$$

$$R_{150v} = \frac{146^2}{100} = 219.0 \text{ OHMS}$$

$$R_{400v} = \frac{296^2}{100} = 878.0 \text{ OHMS}$$

The following methods of sensing the final charge potentials of the battery section have been proposed as charge control possibilities:

1. ON-OFF Pressure Switch
2. Voltage Sensing Device
3. Ampere-Hour Sensing Device

An equivalent circuit for each of the secondary circuits is given in Fig. 6.2.2.

6.4 Some Control Considerations

Since battery characteristics change with the total number of charge and discharge cycles, a voltage and current curve as a function of charge time cannot be accurately determined for constant power.

Therefore, a power variation in charging the batteries does exist. Using the constant current method for charging the batteries maintains the charge power within 10%. This variation in power will cause the alternator to change speed unless this power is dissipated. A speed, frequency, or voltage signal could be used to regulate the output power.

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If a constant power turbine is assumed, a possible regulatory device would be a parasitic resistor in series with the primary winding of the power transformer. The voltage drop across this resistor would indicate the inequality of alternator power load and turbine power.

7.0 DEFINITION OF INDIVIDUAL COMPONENT CHARACTERISTICS

Figure 7.0.1 illustrates in more detail the components in the overall power conversion system, including the associated load banks and system load, heat rejection control and electrical cooling system. In the following sections each component shall be defined in terms of the basic equations of performance.

Components to be discussed in the following sections.

7.1 1. Isotope heat source

7.2 { 2. Pre-heat section
3. Boiler section
4. Superheat section

7.3 5. Turbine

7.4 { 6. Condenser
7. Sub-Cooler
8. Electrical Cooler

7.5 9. Alternator

7.6 10. Bearings and seals

7.7 11. Pump

7.8 12. Heat rejection control 342 033

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~ OVERALL SYSTEM SCHEMATIC DIAGRAM ~

FIGURE - 7.0.1

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- 7.9 { 13. Battery load bank
 { 14. Parasitic load bank
 { 15. System load

7.1 The isotope heat source will be a material possessing a half life of 290 days. The initial power of this heat source will be 5.77 KW. Equation (1) relates the thermal energy available P_h , as a function of time t in days

$$(1) \quad P_h = P_o e^{-2.39 \times 10^{-3} t}$$

Where P_o is the initial power

Figure 7.1.1 is a plot giving the same relation indicated in equation (1).

For comparison purposes a linear plot which approximates equation (1) is shown along with the true exponential expression. It can be seen that an error of only .37% is introduced by this simplification. It is therefore proposed that the linear equation (2) be used for any control analysis purpose.

$$(2) \quad P_h = P_o - .013t$$

It should be noted that the initial power is only 15% in excess of the design input power of 5 KW. It may be possible to utilize this power in the system directly or to dissipate the excess through the condenser rather than by means of the present rejection control at the heat source.

For the sake of simplicity the heat source temperature will be assumed uniform throughout with an operating temperature of 1335°F.

POWER OUTPUT - HEAT SOURCE VS TIME

CURVE OF CE 144
290 DAY 1/2 LIFE

FIGURE 7.1.1

Also for calculating rejected energy the 1335°F temperature will be used.

- 7.2 The complete boiler element consisting of preheater-boiler-superheater will be considered together. The equation for heat transfer will be those applying to a uniform wall temperature case. Also the heat transfer coefficients will be those applying to wetted flow along the boiler tube walls.

Summary of boiler operating conditions:

Fluid flow rate	112 #/hr
Inlet temperature	333°F
Boiling temperature	1025°F
Superheat temperature	1300°F
Total heat added	4.747 BTU/sec - 17080 BTU/hr
Boiler tube size	1/4 O.D. x 3/16 I.D.
Boiler tube temperature (uniform)	1335°F
Heat capacity of entire boiler	6 BTU/°F

For uniform wall temperature the following general equations apply.

- (1) Pre-heat

$$NU = 4.8 + .015 (Re)^{.91} (Pr)^{.91} (Pr)^{.30}$$

- (2) Boiling

Assumed $q/Ab = 100,000 \text{ BTU/Hr-ft}^2$ (for $\Delta T > 40^\circ\text{F}$). This is a conservative estimate - used only because no data is available for boiling heat transfer for $\Delta T > 40^\circ\text{F}$

- (3) Superheat

$$NU = .0243 (Re)^{.8} (Pr)^{.4}$$

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where

$$Nu = \frac{hD}{k} \quad \text{Nusselt Modulus}$$

$$Re = \frac{\rho D U}{\mu} \quad \text{Renolds Number}$$

$$Pr = \frac{C_p \mu}{k} \quad \text{Prandtl Number}$$

The following values are used in (1), (2), and (3).

Symbol	Liquid Mercury	Vapor	Units
D diameter	.0156	.0156	ft
C _p specific heat	.033	.025	BTU/#-°F
ρ density	850	2.245	#/ft ³
μ viscosity	6.4 x 10 ⁻⁴	6.72 x 10 ⁻⁵	#/ft sec
Wt weight flow	.0311	.0311	#/sec
k conductivity	2.08 x 10 ⁻³	1.53 x 10 ⁻⁵	BTU/ft-sec-°F
h transfer coefficient			BTU/ft ² -sec-°F
Q heat flux			BTU/sec

$$\text{Preheat } h_p = \frac{k}{D} \left[4.8 + .015 \left(\frac{4 W_t C_p}{\pi D k} \right)^{.91} \left(\frac{C_p \mu}{k} \right)^{.30} \right]$$

where Wt = #/sec

$$h_p = .64 + .00517 (W_t)^{.91} \quad \text{BTU/ft}^2\text{-SEC-}^\circ\text{F}$$

$$\text{Boiling } h_b = 27.8 \quad \text{BTU/ft}^2\text{-SEC}$$

(FOR ΔT > 40 °F)

$$\text{Superheating } h_s = \frac{k}{D} .0243 \left(\frac{4 W_t}{\pi D \mu} \right)^{.8} \left(\frac{C_p \mu}{k} \right)^{.4}$$

$$h_s = .72 (W_t)^{.8} \quad \text{BTU/ft}^2\text{-SEC-}^\circ\text{F}$$

$$(4) \quad \text{Preheat } Q_p = W_t C_p \Delta T = .710 \quad \text{BTU/SEC}$$

$$(5) \quad \text{Boiling } Q_b = W_t \Delta H = 3.823 \quad \text{BTU/SEC}$$

$$(6) \quad \text{Superheat } Q_s = W_t C_p \Delta T = .214 \quad \text{BTU/SEC}$$

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From the equation

$$Q = A h \Delta T_m$$

we may find the respective areas of the boiler sections, where ΔT_m is average temperature difference. Normally a log-mean temperature value would be used. However, due to the complexity of simulating this expression on the computer it has been considered more practical to use an approximate equation.

Preheater

$$\begin{aligned} (7) \quad A_P &= Q_P / h_P \left(T_w - \frac{(T_o + T_i)}{2} \right) \\ &= .710 / (.64 + .00517 (W_T)^{.91}) (656) \\ &= .00169 \text{ Sq.Ft.} \end{aligned}$$

Boiler

$$\begin{aligned} (8) \quad A_B &= Q_B / h_B \\ &= 3.823 / 27.8 \\ &= .1375 \text{ Sq.Ft.} \end{aligned}$$

Superheat

$$\begin{aligned} (9) \quad A_S &= Q_S / h_S \left(T_w - \frac{(T_o + T_i)}{2} \right) \\ &= .214 / .72 (W_T)^{.8} (172) \\ &= .0276 \text{ Sq.Ft.} \end{aligned}$$

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where

T_w = tube wall temperature

T_o = fluid outlet temperature

T_i = fluid inlet temperature

Q_p = total heat input to preheater

Q_b = total heat input to boiler

Q_s = total heat input to superheater

Total boiler element area

$$\begin{aligned} A_t &= A_p + A_b + A_s \\ &= .1668 \text{ sq ft} \end{aligned}$$

$$\text{Total boiler element length} = \leq \frac{A_N}{\pi D}$$

$$\begin{aligned} L_T &= L_p + L_b + L_s \\ &= .035 + 2.81 + 0.563 \\ &= 3.398 \text{ FT.} \end{aligned}$$

7.3 The turbine has not been defined as to physical size and type.

However, our best estimates indicate the following characteristics.

See Figure 7.3.0.

Summary of Operating Conditions

1. Inlet temperature	1300°F
2. Inlet pressure	210 psia
3. Outlet pressure	1.9 psia
4. Outlet temperature	500°F
5. Total mass moment of turbine rotor, pump,
and Altitude	$7.55 \times 10^{-5} \text{ #ft sec}^2$
6. Ideal enthalpy drop across turbine	51/BTU/#

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TURBINE ALTERNATOR ASSEMBLY

TOTAL INERTIA OF ROTATING MEMBER

$$\begin{aligned}
 I_T &= \frac{1}{2} M_A R_A^2 + \frac{1}{2} M_T R_T^2 \\
 &= 7.55 \times 10^{-5} \text{ FT. LB. SEC}^2
 \end{aligned}$$

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FIGURE 7.3.0

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7. Efficiency	40%
8. Weight flow	112 #/hr.
9. Power output	.667 kw

For a choked nozzle application the performance of a turbine may be expressed in the following manner.

$$P_T = W_T \Delta h_i \eta_T = \dots$$

$$(1) \quad P_T = \frac{K_0 p_1}{\sqrt{T_1}} \Delta h_i \eta_T$$

Where Δh_i may be given as a function of $K_1 \sqrt{T_1} p_1^m$ for a given condenser temperature and/or pressure. Using figure 7.3.1 indicating ideal enthalpy across the turbine as a function of inlet and outlet conditions it is possible to generate a family of curves for various inlet temperatures and pressures with a given condenser which indicates ideal work output as shown in Figure 7.3.2.

From Figure 7.3.2 we may derive an equation (2)

$$(2) \quad \Delta h_i = K_2 p_1^m$$

where K_2 includes $\sqrt{T_1}$

By combining equations (1) and (2)

we have

$$P_T = \frac{K_0 p_1}{\sqrt{T_1}} K_2 p_1^m \eta_T = \frac{K_0 p_1}{\cancel{\sqrt{T_1}}} K_3 \sqrt{T_1} p_1^m \eta_T$$

$$(3) \quad P_T = K p_1^{m+1} \eta_T \quad (K = K_0 \times K_3)$$

The values of K may be determined for a limited range by a trial method which equates a known value (for design conditions) of P_T to equation (3). An overall plot of several design conditions and various turbine efficiencies is shown in Figure 7.3.3 which gives

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ISENTROPIC ENTHALPY

$f(p_1, T_1, p_2)$

$\Delta h_i = K (p_2)^m$

Δh_i BTU./#

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FIGURE 7.3.2

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TURBINE POWER
 VS
INLET PRESSURE

$$P_T = K (\dot{p}_1)^{n+1} / T$$

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FIGURE 7.3.3

P_T as a function of (p_1).

An alternative would be to use equation (1) in the simulation with the term Δh_i defined as a function of p_1, T_1, p_c

This approach may be more acceptable in that it permits feedback of information in terms of p_1, T_1, p_c to other points in the system.

P_T = total power output

η_T = turbine efficiency

Δh_i = ideal enthalpy drop

p_1 = inlet pressure

T_1 = inlet temperature

K = design constants

M = design constants

Figure 7.3.4 gives a graphic evaluation of equation (3) for design conditions.

It should be noted that η_T is a constant term since we are operating over a fairly narrow speed range. It may be seen from Figure 7.3.5 of η_T vs speed that at the design point we will be operating at the peak of the efficiency curve and small speed changes will not appreciably effect the value of η_T .

For our purpose the relationship between torque and speed may be taken from (3)

$$P_T = K (p_1)^{n+1} \eta_T$$

also

$$\Theta_T = P_T / \omega$$

(4)

$$\Theta_T = K_2 (p_1)^{n+1} \eta_T / \omega$$

where

$$\Theta_T = \text{TORQUE FT. LBS.}$$

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TURBINE POWER OUTPUT
VS INLET PRESSURE

$$P_T = K (\rho_i)^{N+1} \eta_T$$

FOR 500° F CONDENSER
 $\eta_T = 40\%$
 $\rho_i = 210 \text{ PSIA}$

(ρ_i) INLET PRESSURE - PSIA

FIGURE - 7.3.4

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FIGURE 7.3.5

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FIGURE 4.3.6

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FIGURE-7.3.7

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$$\omega = 2\pi N \text{ RAD./SEC}$$

K_2 = conversion constant

Figure 7.3.7 presents the relation shown in equation (4) for the total range of operation anticipated and using design conditions

$\eta_T = 40\%$, $T_c = 500^\circ\text{F}$ with p_1 varied between 180 and 230 psia.

Figure 7.3.6 further indicates the speed stability characteristics of the turbine when a load change is felt on the shaft. It can immediately be seen that large speed changes will result with small changes in load for a constant pressure input to the turbine. This would indicate that the system would be dynamically unstable if a constant pressure regulator alone were used for control purpose.

7.4

The condenser, subcooler and electrical cooler shall be considered together. A flat plate construction is assumed in each case.

Summary of Operating Conditions

A. Condenser

Inlet Temperature	500°F
Inlet pressure	1.9 psia
Outlet temperature	500°F
Outlet pressure	1.9 psia
Total heat rejected from system	4.151 kw

B. Subcooler

Inlet temperature	500°F
Inlet pressure	1.9 psia
outlet temperature	330°F

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Outlet pressure 1.9 psia

Total heat rejected from system .262 kw

C. Electrical Cooler

Inlet temperature 333°F

Outlet temperature 200°F

Total heat rejected from system .087 kw

D. Items common to A, B and C

Emissivity of surface ϵ .9

Max. incident power $\propto P_i$.043 kw/ft²

Min. incident power $\propto P_i$.0045 kw/ft²

It should be noted that the value of $\propto P_i$ varies depending on the position of the satellite on its orbit. A plot of average incident power vs. position in time and degrees is shown in Figure 7.4.1. For simulation purpose the sine curve also plotted in Figure 7.4.1 will be used to approximate the change in incident power over the complete orbit with reasonable accuracy.

The following equations are used to determine heat rejection which takes place in the condenser, subcooler, and electrical cooler.

(1) Condenser

$$P_c = [5.07 \times 10^{-13} \epsilon (T_c + 460)^4 - \propto P_i] A_c \quad \text{KW/HR.}$$

(2) Subcooler

$$P_{sc} = [5.07 \times 10^{-13} \epsilon (T_{sc} + 460)^4 - \propto P_i] A_{sc} \quad \text{KW/HR.}$$

(3) Electrical Cooler

$$P_{ec} = [5.07 \times 10^{-13} \epsilon (T_{ec} + 460)^4 - \propto P_i] A_{ec} \quad \text{KW/HR.}$$

INCIDENT POWER VARIATION

TO BE APPLIED TO ALL RADIATING SURFACES

TRUE VARIATION

ASSUMED VARIATION

$$\propto P_{IA} = .0238 + .0198 \sin \frac{3}{2} \lambda$$

$\propto P_I$ INCIDENT POWER KW/FT²

.04
.03
.02
.01

0 30 60 90 120 150 180 210 240 270 300 330 360

λ ORBITAL DEGREES

CYCLE TIME MINUTES

FIGURE-7.4.1

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Where T_{sc} and T_e are average temperatures.

The respective areas are determined as follows from (1) (2) (3)

$$A_c = 12.05 \text{ sq. ft.}$$

$$A_{sc} = 1.74 \text{ sq. ft.}$$

$$A_e = 1.035 \text{ sq. ft.}$$

These areas are determined for the sun condition with $\alpha P_{i2} = .043$
kw/ft²-hr.

The system will tend to maintain these areas constant throughout the range of operation due to inherent stability characteristics.

7.5

The alternator is to be of the permanent magnet type with either axial or radial air gap. The following characteristics and design points are used.

Summary of Operating Characteristics

Alternator efficiency	90%
Power factor	80%
Design voltage	125 volts
Design current	5.4 amps
Power output	.538 kw
Regulation	12.5%
Power losses	60 kw
Z_A apparent armature impedance $(R_A + jX_A)$	2.89 ohms

$$\begin{aligned} (1) P_A &= VI \cos \theta \\ &= [E - I(R_A + jX_A)] I \cos \theta \\ &= [K\phi N - I(R_A + jX_A)] I \cos \theta \end{aligned}$$

For variable speed $E = E_o N/N_o$

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$$E = K \phi N / N_0$$

also

$$I = V / Z_L$$

$$Z_L = \sqrt{R_L^2 + X_L^2}$$

with PF = .8 = R_L / Z_L

at design $Z_L = \frac{125}{5.4} = 23.1 \Omega$

$$R_L = 18.5 \Omega$$

$$X_L = 13.9 \Omega$$

Alternator power $P_A = \eta_A P_{TA}$ turbine power $\times \eta_A$

For our purpose P_{TA} the turbine shaft power will be of value therefore all expression of alternator will be divided by alternator efficiency.

For constant current load

$$P_{TA} = \frac{[E_0 \frac{N}{N_0} - I(R_A + jX_A)] I \cos \theta}{\eta_A}$$

(2) $P_{TA} = .00101 N - .0748 \text{ KW}$

For constant power load

$$P_{TA} = P_A / \eta_A$$

(3) $P_{TA} = .598 \text{ KW}$

For constant impedance load

$$P_{TA} = \frac{[E_0 \frac{N}{N_0} - I(R_A + jX_A)] I \cos \theta}{\eta_A}$$

where

$$I = \frac{E_0 \frac{N}{N_0} - I(R_A + jX_A)}{Z_L}$$

$$I = \frac{E_0 \frac{N}{N_0}}{Z_L + (R_A + jX_A)} = \frac{E_0 \frac{N}{N_0}}{Z_T} \quad 342 \quad 055$$

FIGURE 7.5.1

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$$P_{TA} = \frac{\left[1 - \frac{(R_A + jX_A)}{Z_T}\right] \left(\frac{E_o N}{N_o}\right)^2 \cos \theta}{Z_A}$$

from the regulation factor

we find $\frac{E-V}{V} = 12.5\%$

$$E-V = I Z_A = 15.6 \text{ VOLTS}$$

$$Z_A = 2.89 \Omega$$

$$Z_T = \frac{140.6}{5.4} = 26.0 \Omega$$

check

$$Z_T = Z_A + Z_L = 2.89 + 23.1 = 25.99 \Omega$$

(4)

$$P_{TA} = 3.04 \times 10^{-5} \left(\frac{E_o N}{N_o}\right)^2 \text{ KW}$$

From the basic equation of torque, speed and power we may develop

alternator torque characteristics

$$\theta_A = \frac{K P_{TA}}{2 \pi N} = \text{FT. LBS.}$$

where

$$K = 738$$

For constant current load

$$(5) \quad \theta_A = .119 - \frac{8.82}{N} \text{ FT. LBS.}$$

For constant power load

$$(6) \quad \theta_A = \frac{70.3}{N} \text{ FT. LBS.}$$

For constant impedance load

$$(7) \quad \theta_A = 1.585 \times 10^{-4} N \text{ FT. LBS.}$$

The curves of equations (2) through (6) are plotted in Figure 7.5.1.

7.6

The bearing characteristics are not specifically known but may be reasonably approximated. The general equations of performance do not

FIGURE - 7.6.1

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lend themselves to simple analysis, however, a simplified expression for torque and power may be had from an examination of the following curves in Figure 7.6.1.

Summary Of Operating Conditions

Weight flow	300#/hr.
Total Power Required	.040 kw

From Figure 7.6.1 we see that the linear torque speed relation may be stated as in equation (1).

$$(1) \quad \theta_B = 1.05 \times 10^{-5} N \text{ Ft. LBS.}$$

Also the power speed relationship will be a function of the square of the speed such that equation (2) applies

$$(2) \quad P_B = 9.0 \times 10^{-8} N^2 \text{ kW}$$

It should be noted equations (1) and (2) are based on a fairly constant bearing load and weight flow.

7.7

The pump will be of a centrifugal type operating to supply the cycle fluid to the system the bearing lubricant, and the coolant weight flow to the electrical components.

It is assumed the pump outlet pressure will be throttled to the 210 psia boiler inlet pressure for the cycle consideration. For the determination of pump characteristics however, the total pressure head generated is used to calculate torque and power requirements.

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Summary of pump operating characteristics

Weight flow	.2 #/sec.	720 #/hr.
Inlet pressure		1.9 psia
Outlet pressure		251.9 psia
Efficiency overall		26.1%
Shaft size diameter		.300 in.
Outlet temperature		333°F
Turbine power required		.029 kw

Careful consideration of typical pump characteristics has lead to several conclusions as to the probable appearance of curves dealing with the actual pump characteristics. By design our pump will operate at high speeds at relatively small flow rates, and with a β angle at the impellar outlet of about 90°. Typical curves for a pump of this type would appear as shown in Figure 7.7.1 which illustrates a relatively flat head vs capacity characteristic.

Actual test results to date have indicated this flat head characteristic to prevail for considerable changes in flow rate. It is therefore, felt quite reasonable to assume a constant head dependent on shaft speed only for simulation purpose in the computer analysis. This is indicated by the almost constant head shown in Figure 7.7.1 for flow rate changes between Q_A and Q_B . Present estimates of flow changes seem to indicate changes of less than 3% for the actual system.

It is known that pump head can be expressed as a function of speed squared, flow rate as a function of speed to the first power, and pump power as a function of speed cubed. Therefore, considering the

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TYPICAL HEAD-CAPACITY CURVE

NOTE: ACTUAL HEAD $\approx$ VIRTUAL HEAD $\times \frac{1}{2}$

FIGURE 7.7.1

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constant flow rate assumption we propose the equations (1) and (2) to give a reasonable approximation of power and torque as a function of speed alone.

$$P_{TP} = P_p / \eta_p = W_p H = .029 \text{ KW}_{(\text{at design point})}$$

where η_p = overall efficiency % of pump

W_p = weight flow #/sec. = $K(N)$

H = head ft.

P_p = pump power

P_{TP} = turbine power required kw

and since we are interested in the power required to operate the pump

$$(1) \quad P_{TP} \approx K_2 N^3$$

where N = speed rps

$$K_2 = \text{constant} = 9.83 \times 10^{-11} \text{ KW SEC}^3$$

$$(1) \quad P_{TP} \approx 9.83 \times 10^{-11} N^3$$

Similarly torque may be developed from

$$\theta_p = P_{TP} / 2\pi N$$

$$(2) \quad \theta_p \approx \frac{K N^2}{2\pi} = K_1 N^2$$

where $K_1 = \text{constant} = 1.15 \times 10^{-8} \text{ FT LBS SEC}^2$

θ_p = pump torque - ft. lbs.

N = speed rps

$$(2) \quad \theta_p \approx 1.15 \times 10^{-8} N^2 \text{ FT. LBS.}$$

A plot of equations (1) and (2) is shown in Figure 7.7.2.

Since we have a small and fairly constant inlet pressure we may also indicate the pump head as a function of speed only

$$H \approx \frac{K V^2}{2g}$$

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K&E 10 X 10 TO THE 1/2 INCH 359-11
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FIGURE 7.7.2

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$$(3) \quad p_1 = \Delta p = \frac{850 \text{ m}^2 (2\pi)^2 N^2}{2g} \quad \text{(NEGLECTING INLET HEAD } p_c)$$

$$(3) \quad p_1 = 5.64 \times 10^{-4} N^2 \text{ P.S.I.A.}$$

Figure 7.7.2 shows the relation of power and torque vs speed. For the range considered the near linear characteristics lead to the following simplified equations (3) and (4)

$$(3) \quad P_{TP} = -.0576 + 1.31 \times 10^{-4} N \text{ KW}$$

$$(4) \quad \theta_P = -.0513 + 1.55 \times 10^{-4} N \text{ Ft.LBS.}$$

The weight flow will remain fairly constant since the bulk of the pump output is used to supply nearly constant demands of the bearings and electrical coolers.

7.8

The heat rejection control is not specifically given as a problem area for Thompson Products, however, this control has considerable influence over the rest of the system and therefore, must be considered.

This control must effect the rejection of excess thermal energy of the heat source in the early period of operation. At the present time it is felt the most probable way of doing this is by radiating heat directly from the heat source surface to outer space and controlling the rate of thermal dissipation by regulating the radiating surface area.

The equation for power rejection by radiation is as follows:

$$(1) \quad P_R = [5.07 \times 10^{-13} \epsilon (T_R + 460)^4 - \alpha P_i] A_R \text{ KW/HR}$$

where P_R max. rejected power = .77 kw

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FIGURE-7-8.1

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Ar max. rejection area = .166 sq. ft.

Tr heat source temperature = 1335°F

α Pi max. incident power = .043 kw/ft²

α Pi min. incident power = .0045 kw/ft²

ϵ emissivity = .9

The reduction of radiating area may be closely approximated by the linear relation shown in Figure 7.8.1 for Area vs time. Actually the nature of the heat source would require an exponential reduction in rejection area. However, due to the relatively long half life of the proposed heat sources a linear relation is quite reasonable. Figure 7.1.1 of heat source power has shown linearity within .37%.

The plot in figure 8 may be expressed by the equation

(2)

$$Ar \approx .166 - .00277 t$$

where t is in days

7.9

Typical characteristics for the Ni-cad battery load and the parasitic and system load have been discussed in the preceding sections. For the purpose of analysis the actual loading will be assumed to possess the same characteristics as the typical load. The cycle time is proportioned according to the duty cycles and number of periods per day given in the section 4.0 on specifications.

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8.0 DYNAMIC EQUATIONS OF THE SYSTEM

The purpose of this section is to describe the basic mathematical relationships of the isotope heat source and power conversion apparatus. A schematic of the fundamental system not indicating any particular control scheme is shown in Figure 8.0.1. There are several types of mathematical relations describing the isotope heat source and power conversion system. First, there are the temperature and heat transfer relationships. Second, there are the pressure and mass transfer relationships. Finally, there are the power, speed and torque relationships. All of these are inter-related. Due to these different relations, the mathematical model will be treated as shown in Figure 8.0.2 and in more detail in Figure 8.0.3.

For the system indicated in Figure 8.0.3 the mathematical relationships will be developed for the heat transfer, mass transfer and speed-torque loops.

8.1 A Mathematical Model of the Heat Transfer Loop

8.1.1 Pre-Heater

The two fundamental heat transfer equations for a system are:

$$(1) dA_w U_f (T_w - T) = W C_{p0} \left[\left(\frac{\partial T}{\partial x} \right) dx + \left(\frac{\partial T}{\partial t} \right) dt \right]$$

$$(2) q_p dA_w = dM_w C_{pw} \frac{dT_w}{dt} + dA_w U_f (T_w - T)$$

where

W = mercury flow - #/sec.

T = mercury temperature - °F

dA_w = incremental heat transfer area ft^2

U_f = heat transfer coefficient $Btu/sec-ft^2-°F$

FIGURE 8.0.2

MATHEMATICAL MODEL

FIG. 8.0.2

FIGURE 8.0.3

t = time-sec.

dx = incremental length along tube - ft.

dM_w = incremental weight of heat source and tubine #

C_{PO} = specific heat of liquid mercury = .033 Btu/#-°F.

C_{PW} = specific heat of heat source and tubing

q_p = heat flux - Btu/sec-ft²

The first relation states that whatever heat transferred into the mercury coolant doesn't raise the temperature difference of the inlet and outlet coolant in the section will raise the magnitude of the average temperature with time. The second states that whatever heat isn't transferred into the coolant will raise the temperature level of the heat source with time.

The two fundamental heat transfer equations can be approximated to fit the pre-heater by treating the pre-heater as an incremental unit of length and area.

Rewriting (1) for the pre-heater,

$$(3) \quad A_{WP} U_{fP} (T_w - \tilde{T}_p) = W C_{PO} \left[(T_i - T_o) + \frac{d\tilde{T}_p}{dt} \tau_p \right]$$

but

$$(4) \quad W \tau_p = M_p \quad \text{= mercury inventory in the pre-heater}$$

By the substitution of (4) into (3) and the re-arrangement of terms,

$$(5) \quad \frac{M_p}{W} \frac{d\tilde{T}_p}{dt} = \frac{A_{WP} U_{fP}}{W C_{PO}} (T_w - \tilde{T}_p) - (T_i - T_o)$$

$$(6) \quad \tau_p \frac{d\tilde{T}_p}{dt} = K_p (T_w - \tilde{T}_p) - (T_i - T_o)$$

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where

$$\tau_P = \frac{M_P}{W} = \text{transit time of mercury in pre-heater}$$

$$= 0.18 \text{ sec. at design conditions}$$

$$K_P = \frac{A_{WP} U_{fP}}{W C_{PO}} = \frac{A_{WP} [1.64]}{W \cdot 0.33} = 19.6 \frac{A_{WP}}{W}$$

A_{WP} = heat transfer area of pre-heater ft^2

where A_{WP} = heat transfer area of pre-heater

in general

$$A_{WP} = \frac{W C_{PO}}{U_{fP}} \left(\frac{T_i - T_o}{T_w - \tilde{T}_P} \right) = .0516 W \left(\frac{T_i - T_o}{T_w - \tilde{T}_P} \right)$$

at design conditions

$$A_{WP} = 0.00169 \text{ ft}^2$$

T_P = average mercury temperature in pre-heater

which can be approximated by $\tilde{T}_P = \frac{T_o + T_i}{2} \text{ } ^\circ\text{F}$

U_{fP} = heat transfer coefficient of liquid mercury =

$$.64 + .00517 (W)^{.91} \frac{\text{BTU}}{\text{ft}^2 \cdot \text{SEC} \cdot \text{F}}$$

For the range of $W = .0311$ $U_{fP} \approx .64$

T_i = boiler temperature = $\psi(p_i)$

(see boiler)

Rewrite equations (2) in the form as follows:

$$(7) \tau_{WP} \frac{dT_w}{dt} = \frac{Q_P}{U_{fP}} - (T_w - \tilde{T}_P)$$

where

$$\tau_{WP} = \frac{M_{WP} C_{PW}}{A_{WP} U_{fP}} = \frac{36.4}{.64} = 57.7 \text{ SEC.}$$

at design conditions

T_w = wall temperature of tubing

$\frac{M_{WP}}{A_{WP}}$ = weight of pre-heater wall and heat source per unit
heat transfer area

Q_P = pre-heater heat flux Btu/sec-ft^2

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8.1.2 Boiler

In the boiler, the temperature is approximately constant (small pressure drop) assumed, and changes with pressure changes.

$$T_1 = \psi(p_1)$$

(8)

$$\text{or } T_1 = \frac{1.2895 \times 10^4}{14.0052 - \ln p_1} - 460^\circ$$

where T_1 = boiler temperature $^\circ\text{F}$

p_1 = boiler pressure in psia

A plot of the above relation is shown in Figures 8.1.2.1 and 8.1.2.2.

$$(9) \quad \tau_{wb} = \frac{q_B}{U_{fB}} - (T_w - T_1)$$

$$(10) \quad W h_{fg} = A_{wB} \times \text{heat flux into boiling mercury}$$

$$\text{where } \tau_{wb} = \frac{M_{wB} C_{pw}}{A_{wB} U_{fB}} = \frac{36.9}{.0897 \frac{\text{BTU}}{\text{ft}^2 \text{SEC} - ^\circ\text{F}}} = 412 \text{ SEC}$$

q_B = boiler heat generation per unit heat transfer area
Btu/sec-ft²

$$A_{wB} = \text{boiler heat transfer area} = .138 \text{ ft}^2 \text{ (AT } W = .0311 - \text{ #/SEC)} \\ = \frac{W h_{fg}}{27.8}$$

$\frac{M_{wB}}{A_{wB}}$ = weight of boiler wall and heat source per unit heat transfer area

$$U_{fB} = \text{boiling mercury heat transfer coefficient} = \frac{27.8}{310} = .0897 \frac{\text{BTU}}{\text{ft}^2 \text{SEC} - ^\circ\text{F}}$$

h_{fg} = heat of vaporization of mercury $124 \frac{\text{B.T.U.}}{\text{LB.}}$

Q_B = heat generated in boiler section Btu/sec

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FIG. 8.1.2.1

MERCURY VAPOR PRESSURE VERSES TEMPERATURE

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FIG. 8.1.2.2

MERCURY VAPOR PRESSURE VERSUS TEMPERATURE

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TEMPERATURE (°F)

FIG 8.1.2.2

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$$U_{fB} (T_w - T_1) = \text{heat flux into boiling mercury} = 27.8 \text{ Btu/ft}^2\text{-sec.}$$

8.1.3 Superheater

The superheater equations are developed in an identical manner to those of the pre-heater.

$$(11) \quad \tau_s \frac{d\tilde{T}_s}{dt} = K_s (T_w - \tilde{T}_s) - (T_2 - T_1)$$

where

$$\tau_s = \frac{M_s}{W} = \text{transit time of mercury in superheater}$$

$$= 0.006 \text{ sec. at design conditions}$$

M_s = mercury inventory in superheater

$\tilde{T}_s$ = average temperature of superheater $\approx \frac{T_2 + T_1}{2}$

$$K_s = \frac{A_{ws} U_{fs}}{W C_{p2}} = \frac{A_{ws}}{C_{p2}} \frac{.86}{(W)^2} = 34.4 \frac{A_w}{(W)^2}$$

C_{p2} = heat capacity of superheat fluid Btu/#-°F

U_{fs} = heat transfer coefficient of super heat vapor

$$= .86 (W)^{.8} \text{ Btu/sec ft}^2 - ^\circ\text{F}$$

A_{ws} = superheat heat transfer area

$$A_{ws} = .1628 - A_{WP} - A_{WB} = .0231 \text{ ft}^2$$

$$W = .0311 \text{ \#/sec}$$

also

$$(12) \quad \tau_{ws} \frac{dT_{ws}}{dt} = \frac{q_s}{U_{fs}} - (T_w - \tilde{T}_s)$$

where

$$\tau_{ws} = \frac{M_{ws}}{A_{ws}} \frac{C_{pw}}{U_{fs}} = \frac{42.8}{(W)^8} \text{ SEC} = 692 \text{ SEC}$$

q_s = heat flux of super-heater Btu/sec-ft² - °F

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8.1.4 Condenser and Radiator

The mathematical heat transfer relations of the condenser can be simplified by neglecting the pressure drop in the condenser. Furthermore, it is assumed that the condensing heat transfer coefficients of mercury are high enough such that the mercury temperature can equal the radiator wall temperature. The following heat transfer relationships describe the condenser-radiator portion of the system.

$$(13) \quad C \frac{dT_3}{dt} = KQ [1-\eta] - Q_c(t)$$

where $KQ [1-\eta]$ = heat rejected by condensing fluid

η = actual cycle efficiency

Q = heat added into cycle fluid from heat source.

From thermodynamic considerations, neglecting pump work, the heat into the system either goes into work or is rejected by the condenser plus the subcooler

$$Q = Q_{\text{rejected}} + Q_{\text{work}}$$

$$\text{But } Q_{\text{work}} = \eta Q$$

therefore

$$\begin{aligned} Q_{\text{REJECTED}} &= Q - Q_{\text{WORK}} = Q - \eta Q \\ &= Q(1-\eta) \end{aligned}$$

From another viewpoint, the heat rejected from the condenser fluid = $W h_{fg} (1-y)$

where

$$W = \#/\text{sec.}$$

$$h_{fg} = \text{heat of vaporization Btu/\#}$$

$$y = \% \text{ moisture inlet to condenser}$$

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$$(14) \quad Q_c(t) = A_c \sigma \epsilon (T_3 + 460)^4 - A_c \alpha P_i(t)$$

$$(15) \quad C \frac{dT_3}{dt} = KQ(1-\eta) - A_c \sigma \epsilon (T_3 + 460)^4 + A_c \alpha P_i(t)$$

where

C = heat capacity of condenser structure and interior
in Btu/°F

C = weight of mercury x C_{pmer} + weight of structure
x C_{psir} = 2.54 Btu/°F

η = cycle efficiency

$Q_c(t)$ = rejected heat Btu/sec.

Q = heat into cycle Btu/sec.

P_i = incident power from direct solar radiation, re-
flected solar radiation from the earth, and the
earth's radiation.

ϵ = emissivity = .9

α = absorptivity = .9

σ = Stefan - Boltzmann Constant 4.8×10^{-13} BTU/SEC-FT²-R°

T_3 = condenser temperature - °F

A_c = condenser area = 12.00 ft²

weight of mercury = 1.70 lb.

weight of structural material = 20.6 lb.

K = Fraction of total heat rejected from condenser = .958

8.1.5 Mixing Equations

The entrance to the subcooler comprises the condenser outlet
plus the electrical cooling, bearing flow and pump leakage.

The inlet temperature to the subcooler, T_{3.6} is:

$$T_{3.6} = \frac{W_{EL}}{W_{sc}} T_{EL} + \frac{W_B}{W_{sc}} T_B + \frac{W_P}{W_{sc}} T_P + \frac{W}{W_{sc}} T_3$$

where

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T_{EL} = outlet temperature of electrical coolant loop-°F = 350°F

T_B = outlet temperature of bearing coolant loop-°F = 347°F

T_P = outlet temperature of pump leakage coolant loop
- °F = 333°F

W_{SC} = total weight flow in subcooler #/sec.

$$= W_{EL} + W_B + W_P + W = 2 \text{ LBS./SEC}$$

W_{EL} = Wt. flow through electrical coolant loop = 0.0188 #/sec.

W_B = Wt. flow through bearing coolant loop = 0.0833 #/sec.

W_P = Wt. flow through pump leakage coolant loop = 0.0668 #/sec.

W = Wt. flow through condenser = 0.0311 #/sec.

therefore

$$\begin{aligned} T_{3.6} &= 32.8 + 144.5 + 111.0 + 5 W T_3 \\ &= 288.3 + 5 W T_3 \end{aligned}$$

AT $T_3 = 500^\circ\text{F}$ and $W = .0311 \text{ #/sec.}$

$T_{3.6} = 366^\circ\text{F}$ Inlet Temperature of Subcooler

8.1.6 Subcooler

The subcooler can be treated similarly to the condenser by approximating the rejected heat to occur at the mean average temperature of the subcooler.

$$(16) \quad Q_{SC}(t) = -C_{SUB} \frac{dT_{SC}}{dt} + W_{SC} C_{PMER} (T_{3.6} T_4)$$

$$(17) \quad Q_{SC}(t) = A_{SC} \sigma E (T_{SC} + 460)^4 - A_{SC} \alpha P_L(t)$$

$$(18) \quad A_{SC} \sigma E (T_{SC} + 460)^4 = A_{SC} \alpha P_L(t) - C_{SUB} \frac{dT_{SC}}{dt} + W_{SC} C_{PMER} (T_{3.6} T_4)$$

where

$$T_{SC} = \frac{T_{3.6} + T_4}{2} = \text{mean temperature of subcooler}$$

C_{SUB} = heat capacity of subcooler

= weight of subcooler structure $\times C_{P SUB} + W_{T OF Hg} \times C_{PMER}$

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$$= 0.457 \text{ Btu/}^{\circ}\text{F}$$

$$W_{SC} = \text{flow rate \#/sec. into subcooler} = 6.43 \text{ W}$$

$$T_4 = \text{exit subcooler temperature } ^{\circ}\text{F}$$

$$A_{SC} = \text{subcooler area - ft}^2 = 1.74 \text{ ft}^2$$

$$\text{weight of mercury} = 3.7\#$$

$$\text{weight of structure} = 2.84\#$$

8.1.7 Radiation Shutters

$$Q_L = Q + Q_R$$

where

$$Q_L = \text{heat generated by isotope heat source}$$

$$Q = \text{heat input to mercury Rankine cycle}$$

$$Q_R = \text{heat radiated through shutters}$$

For the mathematical model it is desired to have the following transfer function

$$\frac{Q_R(S)}{T_W(S)} = G(S)$$

or a similar one such as $\frac{Q_R(S)}{T_2(S)}$ if a superheater temperature signal is used instead of the wall temperature (T_W) signal to actuate the shutters. Transfer functions for the other signal " $X(S)$ " of actuation $\frac{Q_R(S)}{X(S)}$ should also be obtained.

8.1.8 Constants and Assumptions for the Heat Transfer Model

Assumptions

- (1) Preheater, Superheater, and Boiler are at a constant wall Temperature as they are surrounded by a liquid metal

$$(2) \frac{M_{WP} C_{PW}}{A_{WP}} = \frac{M_{WB} C_{PW}}{A_{WB}} = \frac{M_{WS} C_{PW}}{A_{WS}}$$

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The above can be calculated by the total heat capacity of the heat source divided by the total heat transfer area.

$$M_W C_{PW} = 6 \text{ BTU/}^\circ\text{F}$$

$$\frac{M_W C_{PW}}{A_W} = \frac{6}{.1628} = 36.9 \frac{\text{BTU}}{^\circ\text{F-Ft}^2}$$

For the case of constant wall temperature, equations (7), (9) and (12) can be grouped into the following relation:

$$M_W C_{PW} \frac{dT_W}{dt} = Q_i - Q$$

where $M_W C_{PW}$ = heat capacity of heat source = 6 Btu/ $^\circ\text{F}$

Q_i = heat generated by the isotope source minus the heat rejected by the radiation shutters.

$$\begin{aligned} Q &= \text{heat supplied to mercury Rankine cycle} \\ &= A_{WP} U_{fP} (T_W - \tilde{T}_P) + A_{WB} U_{fB} (T_W - T_i) \\ &\quad + A_{WS} U_{fS} (T_W - \tilde{T}_S) \end{aligned}$$

T_W = wall temperature $^\circ\text{F}$ if the heat supplied to the mercury Rankine cycle is removed by the mercury much more quickly than the wall temperature would change with time, then it is justifiable to use the following Q in the above relation:

$$\begin{aligned} Q &= A_{WP} U_{fP} (T_W - \tilde{T}_P) + A_{WB} U_{fB} (T_W - T_i) \\ &\quad + A_{WS} U_{fS} (T_W - \tilde{T}_S) \end{aligned}$$

$$Q = W [C_P (T_i - T_o) + h_{fg} + C_{PS} (T_2 - T_i)]$$

8.2 Mathematical Model of Speed-Torque Loop

The turbine, alternator, pump, and bearings are all on one shaft and the speed of each of these components are equal. The basic torque-speed relation is as follows:

$$J \frac{d\omega}{dt} = \text{driving torque} - \text{resisting torque}$$

$$2\pi J \frac{dN}{dt} = \Theta_T - \Theta_A - \Theta_B - \Theta_P$$

where J = moment of inertia of whole rotating system =

$$7.55 \times 10^5 \text{ ft-}\# \text{-sec}^2$$

N = rps

Θ_T = turbine driving torque (lb-ft)

Θ_A = alternator resisting torque

Θ_B = bearing resisting torque

Θ_P = pump resisting torque

ω = rad/sec.

The torques are expressed as a function of the system variables in a previous section of the report.

8.3 Mathematical Model of Mass Transfer Loop

Two approaches to the mathematical model of the mass transfer loop will be indicated.

8.3.1

The first, a simplified approach will assume that an instantaneous change in pressure in one portion of the system will be accompanied by instantaneous change in other portions of the system. This assumption is justified since pressure waves travel with the speed of sound in a particular medium.

The reference pressure of the system is the condenser pressure as this pressure is established by the radiating temperature. An expression for condenser pressure as a function of temperature in the range of 450°F to 550°F is obtained from TM-777, Thermodynamic Properties of Mercury

$$p_3 = \text{ANTILOG}_{10} \left[9.8383 - \frac{6000.56}{T_3 + 459.69} - 1.1531 \text{ LOG}_{10} (T_3 + 459.69)^2 - 2.675 \times 10^{-12} (T_3 + 459.69)^3 + 3.6 \in \frac{-9640}{(T_3 + 459.69)} \right]$$

where P_3 = condenser pressure psia

T_3 = radiating temperature °F

A simplified but less accurate expression is as follows:

$$\ln p_3 = 140052 - \frac{1.2895 \times 10^4}{T_3 + 460}$$

(see Figures 8.1.2.1 & 8.1.2.2)

where $\ln p_3 = \text{LOG}_e p_3$

Pressure drops in the system are negligible and essentially the boiler pressure equals the condenser pressure plus the head developed by the pump.

8.3.2

The second approach is a complex treatment of the pressure loop, in which the fundamental equations of momentum and continuity are applied. The general relations will refer to a given section of tubing of cross-sectional area A and length L.

(1) Momentum Equation

$$\Sigma F = \frac{d}{dt}(MV)$$

where

ΣF = unbalanced force = driving minus resisting forces

M = mass to which unbalanced force is applied

V = velocity of mass

This may be rewritten as:

$$\Delta p - \Delta p_f = \frac{L}{Ag} \frac{dW}{dt}$$

where

L = length of section in ft.

$\Delta p = p_{IN} - p_{OUT}$ = total pressure difference across section
in #/ft²

A = cross section area in ft²

g = 32.2 ft/sec²

W = flow rate #/sec. out of section

Δp_f = friction pressure drop psf

(2) Continuity

$$\frac{dW}{dx} = -A \frac{d\rho}{dt}$$

where

A = cross section area ft²

ρ = avg. density in #/ft³

Rewriting

$$\Delta W = -AL \frac{d\rho}{dt}$$

where

ΔW = change of flow rate in volume AL ft³

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If W were considered to be the weight flow out of section L, then

$$W_{in} = W_{out} + AL \frac{d\rho}{dt}$$

For an incompressible fluid,

$$\frac{d\rho}{dt} = 0 \quad \text{AND} \quad W_{in} = W_{out}$$

in the section.

For a compressible fluid, the continuity equation should be applied in the above form.

$$W_{in} = W_{out} + AL \frac{d\rho}{dt}$$

The boiler of this system for the mass transfer system can be approximated by liquid phase and a vapor phase. A diagram indicating this approach is in Figure 8.3.2.1. The weight of the vapor phase = $\bar{x} M_B$ while the weight of the liquid phase = $(1 - \bar{x}) M_B$

where $\bar{x}$ = average quality of vapor

M_B = mercury boiler inventory

The volume of liquid mercury equals

where

$\rho_{Hg.}$ = density of liquid mercury

The volume of the vapor equals $V_B - \frac{\bar{x} M_B}{\rho_{Hg.}} = \frac{(1 - \bar{x}) M_B}{\rho_g}$

where

V_B = boiler volume

ρ_g = density of mercury vapor

at the boiling temperature and pressure.

The length of the vapor section of the boiler = $\frac{(1 - \bar{x}) M_B}{\rho_g A} = L_B$

The total length of the vapor section of the boiler and superheater

$$L = L_B + L_S$$

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MODEL OF MASS TRANSFER SYSTEM FOR PREHEATER, BOILER AND SUPERHEATER

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FIGURE-8.3.2.1

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where L_B = length of boiler section

L_S = length of superheater section

Divide the vapor section into n sections. Let the length of any

section = ΔX_i

such that

$$L = \sum_{i=1}^n \Delta X_i$$

For a section of length ΔX_i

$$W_{in i} = W_{out i} + A \Delta X_i \frac{dP}{dt}$$

and

$$\Delta p_{vi} - \Delta p_{xi} = \frac{\Delta X_i}{A g} \frac{dW}{dt}$$

For simplicity assume that compression in the vapor can be expressed

by $p v^n = \text{constant}$

$$p n v_g^{n-1} \frac{dv_g}{dp} + v_g^n = 0$$

$$\frac{dv_g}{dp} = -\frac{v_g}{n p} \quad dv_g = -\frac{v_g}{n p} dp$$

$$v_g = \frac{1}{p g}$$

$$dv_g = -\frac{1}{p g^2} dp_g$$

$$\therefore dp_g = -\frac{p_g}{n p} dp$$

$$W_{in i} = W_{out i} - \frac{A \Delta X_i p_g}{n p_{vi}} \frac{dp_{vi}}{dt}$$

Let

$$A \Delta X_i p_g = M_i$$

where

$$W_{out i} - W_{in i} = \frac{M_i}{n p_{vi}} \frac{dp_{vi}}{dt}$$

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M_i = mercury inventory in section i
 n = gas constant = 1.656
 p_i = average pressure in section i

8.3.3

To complete the description of the mass transfer loop, it is desirable to summarize the mass transit time over various portions of the system as follows:

	For W #/sec.	@ W = .0311 #/sec.
Pre-heater	$5.60/W \times 10^{-3}$	0.180 sec.
Boiler	$2.49/W \times 10^{-2}$	0.800 sec.
Superheater	$1.86/W \times 10^{-4}$	0.006 sec.
Condenser	$1.70/W$	54.700 sec.
Subcooler	$0.576/W$	18.500 sec.

In addition the piping to the turbine will have the following dead time for 3/16" I.D. tubing:

Mass transit time per foot of piping:

for W #/sec. = $4.27/W \times 10^{-4}$ sec.

for W = .0311 #/sec. = 0.01375 sec.

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8.4 Summary of Mathematical Model

Each of the various portions of the Mercury Vapor Power Conversion System has been examined separately to obtain the required information for that part of the mathematical model. To tie the complete system together a general information flow diagram is presented in Figure 8.4.1. A more detailed information flow diagram is given in Figure 8.4.2. On this diagram each section is represented by blocks containing the appropriate transfer functions. This flow diagram, along with parameter values given in this report, represent the final step prior to preparation of a detailed computer diagram for simulation of the complete system.

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INFORMATION FLOW DIAGRAM FOR PCU

FIGURE 8.4.1

INFORMATION FLOW DIAGRAM FOR MATHEMATICAL MODEL OF PCU (SEE TM-1002)

FIGURE 8.4.2

NOTE: SEE FOLLOWING PAGE FOR
FLOW DIAGRAM NOTATION.

FIGURE 8.4.2

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FLOW DIAGRAM NOTATION

- A_c = condenser area (ft^2)
- A_{sc} = subcooler area (ft^2)
- C_c = condenser heat capacity ($\text{Btu}/^\circ\text{F}$)
- C_{sc} = subcooler heat capacity ($\text{Btu}/^\circ\text{F}$)
- C_{pp} = pre-heater specific heat ($\text{Btu}/\text{lb}-^\circ\text{F}$)
- C_{ps} = super-heater specific heat ($\text{Btu}/\text{lb}-^\circ\text{F}$)
- E_o = alternator e.m.f. at N_o (volts)
- E_N = alternator e.m.f. at N (volts)
- h_{fg} = mercury heat of vaporization (Btu/lb)
- Δh_i = ideal enthalpy change in turbine (Btu/lb)
- I_a = alternator current (amps)
- K = appropriate constants and conversion factors
- $M_T C_p'$ = power source heat capacity ($\text{Btu}/^\circ\text{F}$)
- N = speed (rpm)
- N_o = rated speed (rpm)
- p_1 = boiler pressure (psia)
- p_c = condenser pressure (psia)
- P_T = turbine work (KW)
- P_B = bearing power (KW)
- P_P = pump work (KW)
- P_{TA} = power delivered to alternator (KW)
- P_{RAD_c} = condenser radiated power (Btu/sec)
- $P_{RAD_{sc}}$ = subcooler radiated power (Btu/sec)
- $P_{INC}(t)$ = absorbed incident power per area ($\text{Btu}/\text{sec}-\text{ft}^2$)

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Flow Diagram Notation (Cont'd)

- Q = isotope heat generation (Btu/sec)
- R_T = electrical load resistance (Ω)
- S = Laplace transform symbol
- $S.R.$ = saturable reactor
- SW = switch
- T_o = subcooler exit temperature ($^{\circ}F$)
- $T_{s,e}$ = subcooler entrance temperature ($^{\circ}F$)
- T_{sc} = average subcooler temperature ($^{\circ}F$)
- T_c = condenser temperature ($^{\circ}F$)
- T_i = boiler temperature ($^{\circ}F$)
- T_w = wall temperature = superheat exit temperature ($^{\circ}F$)
- W = system weight flow (#/sec)
- W_{sc} = subcooler weight flow (#/sec)
- Z = impedance (Ω)
- ϵ = emissivity
- η_A = Alternator efficiency
- η_T = turbine efficiency
- τ = time constant

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