BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
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ACCELERATOR DEPARTMENT
Informal Report

MASTER

STUDY OF ELECTROMAGNETIC INTERACTIONS

WITH E-P COLLIDING BEAMS

Jay Orear

November 22, 1971

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BROOKHAVEN NATIONAL LABORATORY
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ISABELLE PROJECT

STUDY OF ELECTROMAGNETIC INTERACTIONS
WITH e-P COLLIDING BEAMS

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November 22, 1971

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Abstract

We will discuss both e-P elastic and inelastic scattering of a 10 GeV electron beam on a moving proton target. We will propose detection schemes for a proton energy of 200 GeV and e-P luminosity of $10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$ and estimate the experimental limits in Q^2 and ν .

Introduction

Some of the staff at Brookhaven National Laboratory are now designing 200 GeV intersecting storage accelerators to be fed by the 30 GeV AGS.⁽¹⁾ As a future option, they are considering placing a 5 to 10 GeV electron storage ring in the same tunnel. Such a scheme was originally proposed by Professor Melvin Schwartz as a way to study e-P weak interactions. 10 GeV electrons colliding with 200 GeV protons has the same cm. energy as 4260 GeV electrons hitting a stationary target.

The purpose of this paper is to estimate cross sections for the e-P elastic and inelastic electromagnetic interactions. Specific detection schemes will be proposed along with estimates of rates assuming a luminosity of $10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$. The present design for ISABELLE (the BNL 200 GeV P-P storage accelerator) should give a luminosity of $\sim 10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$ for collinear collisions over the interacting region without bunching.⁽²⁾ In this paper we are allowing the electron current to be at least an order of magnitude lower than the proton current.⁽³⁾

We will conclude that one could not observe elastic e-P scattering beyond $Q^2 \sim 8 \text{ GeV}^2$ because of the rapid drop of the proton form factor with increasing Q^2 . However, in the case of inelastic scattering one could reach values of $Q^2 \sim 500 \text{ GeV}^2$ and $\nu \sim 1500 \text{ GeV}$.

Elastic Scattering

It is of interest to see if the e-P elastic cross section is truly energy independent. At ISABELLE one could go to an equivalent electron energy $E_0 = \frac{2E_p}{M} E = 2 \times 200 \times 10 / .938 = 4263 \text{ GeV}$. We will now show how to study momentum transfers up to $Q^2 \sim 8 \text{ GeV}^2$.

The approach will be to measure θ , ϕ , and P for both the scattered electron and proton. Proportional chambers gated by e-P coincidences would be used. A BNL bending magnet of 20" x 48" aperture (48D48) would be used for momentum analysis of the scattered electron. The steering magnets used to pull the electron beam away from the proton beam would be used to momentum analyse the scattered protons as shown in Fig. 1. While still colinear, the beams will pass through a hole drilled in one of the polefaces of the 48D48. It is intended that most of the interactions take place over a 3 meter region; hence quadrupoles should be placed at a distance of $\sim 20 \text{ m}$ on either side in order to achieve $\beta \sim 1 \text{ m}$ in the interacting region.⁽²⁾ At $Q^2 = 4 \text{ GeV}^2$ the electron scattering angle is 200 mrad, the proton scattering angle is 10 mrad, and the scattered electron energy is $E' = 10.1 \text{ GeV}$.

We will use the subscript zero to designate quantities

in the rest frame of the target proton. In this frame

$$\frac{d\sigma_o}{d\omega_o} = \frac{\alpha^2}{4E_o^2 \sin^4 \frac{\theta_o}{2}} \cos^2 \frac{\theta_o}{2} \left[1 + \frac{2E_o}{M} \sin^2 \frac{\theta_o}{2} \right]^{-1} G^2(Q^2)$$

Since $\theta_o \approx \frac{1}{2000}$ we have

$$\frac{d\sigma_o}{d\omega_o} \approx \frac{4\alpha^2}{Q^4} E_o^2 G^2(Q^2)$$

The relativistically invariant cross section is $d\sigma/dQ^2$ where $Q^2 = 2EE'(1-\cos\theta)$ in all frames.

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma_o}{dQ_o^2}$$

$$\frac{d\sigma}{d\omega} \frac{d\omega}{dQ^2} = \frac{d\sigma_o}{d\omega_o} \frac{d\omega_o}{dQ_o^2}$$

$$\begin{aligned} \frac{d\sigma}{d\omega} &= \frac{d\sigma_o}{d\omega_o} \left[\frac{\frac{d\omega_o}{dQ_o^2}}{\frac{d\omega}{dQ^2}} \right] \\ &= \frac{d\sigma_o}{d\omega_o} \left[\frac{\frac{1}{2E_o E_o'} + \frac{Q_o^2}{4ME_o E_o'^2}}{\frac{1}{2EE'} - \frac{Q^2}{8EE'^2} \left(\frac{1}{E} - \frac{1}{E_p} \right)} \right] \approx \frac{d\sigma_o}{d\omega_o} \left[\frac{E^2}{E_o^2} \right] \text{ since } E_o' \approx E_o \end{aligned}$$

according to Equation 4.

$$\frac{d\sigma}{d\omega} \approx \frac{4\alpha^2 E^2 G^2}{Q^4} \text{ where } G^2 \approx \left(1 + \frac{Q^2}{.71} \right)^{-4}$$

For $Q^2 = 4$, $G^2 = 5.16 \times 10^{-4}$ and $\frac{d\sigma}{d\omega} = 6.8 \times 10^{-7} \text{ GeV}^{-2}$.

Converting to cm^2 gives

$$\frac{d\sigma}{d\omega} = 2.64 \times 10^{-34} \text{ cm}^2.$$

The solid angle subtended by the 20"x48" aperture is 45 msr.

Hence the event rate is

$$N_{ep} = L \frac{d\sigma}{d\omega} \Delta\omega = 10^{33} \times 2.64 \times 10^{-34} \times 0.045 = 0.012/\text{sec.}$$

$$= 43 \text{ events/hr.}$$

For $Q^2 \gg 1 \text{ GeV}^2$, $d\sigma/d\omega$ is dropping with Q^2 as $1/Q^4$; hence a doubling of Q^2 from 4 to 8 GeV^2 will decrease the event rate by almost a factor of 64. We conclude that $Q^2 \sim 8 \text{ GeV}^2$ is the upper limit obtainable by e-P colliding beams. The interaction rate is $\sim 10^3$ inelastic events/sec.; hence background tracks in proportional chambers P_1 and P_4 would occur less than 1% of the time. As discussed in the next section, the electromagnetic background would also be low.

Inelastic Scattering

As shown in Fig 2 the same electron spectrometer as described for the elastic scattering would be used to detect the inelastically scattered electron. A gas cherenkov counter and a shower counter are used to distinguish $\sim 10 \text{ GeV}$ electrons from other particles. The lab angle θ and lab momentum P of each scattered electron is measured yielding a Q^2 and ν_0 measurement for each electron.

$\nu_0 = E_0 - E'_0$ is the electron energy loss in the rest system of the target proton. As before, the subscript zero is used for quantities in the rest frame of the target proton. Quantities in the ISABELLE lab system have no subscript. We use primed superscripts for quantities after the interaction. The relativistically invariant cross section is

$$\frac{d\sigma}{\frac{d^3p'}{E'}} = \frac{d\sigma}{p' dp' d\omega'} \quad \text{since } p = E \text{ for all electrons under consi-}$$

deration. Hence the lab cross section is

$$(1) \quad \frac{d\sigma}{dp' d\omega'} = \frac{p'}{p_0} \frac{d\sigma_0}{dp'_0 d\omega'_0}$$

In the target rest frame

$$\frac{d\sigma_0}{dE'_0 d\omega'_0} = \frac{4\alpha^2 E_0'^2}{v_0 Q^4} \left[\cos^2 \frac{\theta_0}{2} + 2 \sin^2 \frac{\theta_0}{2} \frac{Q^2 + v^2}{Q^2} \frac{1}{1+R} \right] F_2$$

$$\text{where } F_2 = v_0 W_2 \quad (4)$$

In the region of interest the quantity in the bracket is ≈ 1 (even for Q^2 as high as 100 GeV^2 , $\theta_0 \approx 1/400$). Experimentally the factor F_2 is found to have the value $F_2 \approx 0.3$ whenever

$$\frac{2Mv}{Q^2} \geq 3 \text{ for all values of } E_0 \text{ (up to } 20 \text{ GeV at SLAC). We}$$

will assume this scale invariance continues to hold for E_0 up to 4000 GeV . Then

$$\frac{d\sigma_0}{dE'_0 d\omega'_0} \approx \frac{1.2\alpha^2 E_0'^2}{v_0 Q^4}$$

The lab cross section is obtained by substituting this into Eq. 1:

$$(2) \quad \frac{d\sigma}{dE' d\omega'} = \frac{1.2\alpha^2 E' (E_0 - v_0)}{v_0 Q^4}$$

The value for E_0 can be obtained by evaluating S in the lab system and target system: $S = m^2 + M^2 + 2E(E_p + p_p)$ in all frames.

Hence

$$E_o(E_{p_o} + P_{p_o}) = E(E_p + P_p)$$

$$(3) \quad E_o = \frac{(E_p + P_p)}{M} E = \frac{400}{.938} \times 10 = 4263 \text{ GeV}.$$

We can evaluate E' by Lorentz transforming the scattered electron energy from the rest system of the proton to the lab system:

$$\begin{aligned} E' &= \gamma E'_o - \gamma \beta P'_o \cos \theta_o \quad \text{where } \gamma = \frac{E_p}{M} \text{ and } \gamma \beta = \frac{P_p}{M} \\ &= \frac{E_p E'_o}{M} - \frac{P_p E'_o}{M} \left(1 - \frac{Q^2}{2E_o E'_o}\right) \quad \text{using } Q^2 = 2E_o E'_o (1 - \cos \theta'_o). \\ &= E - \frac{v_o M}{E_p + P_p} + \frac{P_p Q^2}{2(E_p + P_p)E} \quad \text{using Equation 3.} \\ (4) \quad E' &\approx E - \frac{M}{2E_p} v_o + \frac{Q^2}{4E} \quad \text{using } E_p \approx P_p \end{aligned}$$

The spectrometer in one fixed position will accept a large bite in Q^2 and a very large region of v_o . For example the spectrometer position shown in Fig. 2 gives a Q^2 bite of 35 GeV^2 centered about $Q^2 \sim 100 \text{ GeV}^2$. The v_o -region extends from 50 to 2000 GeV. The region covered in Q^2 - v space by this spectrometer position is shown in Fig. 3. The events from such a run would be sorted into bins of ΔQ^2 and Δv . Fig. 3 shows 4 such bins with the estimated number of events per day for each bin.

In order to estimate the event rate for each bin we need to calculate

$$(5) \quad \frac{d\sigma}{dQ^2 dv_0 d\phi} = \frac{d\sigma}{dE' d\omega'} J \quad \text{where the Jacobian } J \text{ is}$$

$$J = \begin{vmatrix} \frac{\partial E'}{\partial Q^2} & -\frac{\partial \cos\theta}{2Q^2} \\ \frac{\partial E'}{\partial v_0} & -\frac{\partial \cos\theta}{\partial v_0} \end{vmatrix} = \frac{M}{4EE'E_p} \quad \text{using Equation 4.}$$

If we substitute this and Eq. 2 into Eq. 5, we obtain

$$\frac{d\sigma}{dQ^2 dv_0 d\phi} = \frac{0.3\alpha^2 (E_0 - v_0) M}{v_0 Q^4 EE_p} = \frac{7.5 \times 10^{-9} (E_0 - v_0)}{v_0 Q^4} \text{ GeV}^{-5}$$

Converting $d\sigma$ to cm^2 gives

$$\frac{d\sigma}{dQ^2 dv_0 d\phi} = \frac{2.9 \times 10^{-36} (4260 - v_0)}{v_0 Q^4} \frac{\text{cm}^2}{\text{GeV}^3}$$

The event rate per day for a bin of width ΔQ^2 and v_0 range from v_1 to v_2 is

$$\begin{aligned} N &= L \int_{v_1}^{v_2} \left(\frac{d\sigma}{dQ^2 dv_0 d\phi} \right) dv_0 \times \Delta Q^2 \Delta\phi \\ &= L \times \frac{2.9 \times 10^{-36} \times (4260 - v_0)}{Q^2} \times \Delta\phi \times \frac{\Delta Q^2}{Q^2} \times \ln \frac{v_2}{v_1} \end{aligned}$$

$$\text{where } L = 8.6 \times 10^{37} \text{ cm}^{-2}/\text{day}, \Delta\phi = 0.43$$

Note that for equal percentage bites in Q^2 , the rate only drops as $1/Q^2$ in contrast with the $1/Q^{12}$ dependence of the elastic scattering. Kinematics are such that different spectrometer positions have almost the same $\frac{\Delta Q^2}{Q^2}$.

For the first bin in Fig. 3 the rate is

$$N = 8.6 \times 10^{37} \times \frac{2.9 \times 10^{-36} (4050)}{100} \times .43 \times .35 \times \ln \frac{290}{160} \\ = 900 \text{ events/day}$$

We see from Fig. 3 that the 54° spectrometer position gives a few thousand events per day in the region $Q^2 \approx 100 \text{ GeV}^2$. A 90° spectrometer position gives ~ 80 events/day/bin in the region $Q^2 \approx 350 \text{ GeV}^2$. Q^2 as high as 600 GeV^2 might be possible depending on background. We see that the area in Fig. 3 covered by ISABELLE might be over 3000 times the region covered by SLAC.

The usual background for such a single arm spectrometer looking at a target in an electron beam consists of low energy electrons produced by low energy bremsstrahlung and by electron-electron scattering in the target. Colliding beams have the great advantage that there are no orbital electrons in the target and hence no e-e scattering, no photoelectrons, and no compton electrons. Also the target density is so low that there will be no pair production from the bremsstrahlung. Some electrons would be produced in the window of the vacuum chamber. If necessary, this source of background could be eliminated by extending the vacuum chamber to include proportional chamber P_1 inside the vacuum. The production cross section for pions of high transverse momentum should be comparable to what we are trying to measure.⁽⁵⁾ Pions and other hadrons are rejected by the gas cherenkov and shower counters shown in Fig. 2.

The length of the interaction region should match the magnet aperture; hence β should be ~ 10 cm. This would require quadrupoles ~ 200 inches from the interaction region.⁽²⁾ A factor of 2.5 could be gained here by using the magnet the other way; i.e., the 48 inch aperture would be along the beam. Either way, the total event rate would be the same.

This paper was inspired in part by a memo written by Karl Berkelman entitled "How Far Can We Go Into The Deep Inelastic?" I would also like to thank Professor Berkelman and others at Cornell for helpful comments.

Footnotes

1. J. P. Blewett, "200 GeV Intersecting Storage Accelerators", paper presented at the 8th International Conference on High Energy Accelerators, Sept. 1971, CERN.
2. Blewett, et. al., Brookhaven National Laboratory report CRISP 71-14.
3. If the 15 amp stored proton beam is bunched at the same frequency as bunching of the electron beam, an electron current of 0.4 amps would give a luminosity of $10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$ for colinear collisions according to the parameters in Ref. 2. Such an electron current requires 2 megawatts of RF to make up for the synchrotron radiation. By comparison, DESY plans to store 10 amps at 3 GeV.
4. Drell and Walecka, *Annals of Physics* 28, 18(1964).

Footnotes(cont.)

5. If pion production continues to drop with $p_{\perp}$ as $\exp(-6.5p_{\perp})$ as it does for pion production by 18 GeV photons (and also by hadrons), then at $p_{\perp} \approx 10$ GeV/c there would be no pion background. The 18 GeV photon result is obtained from Fig. 26 of the paper delivered by Bjorn Wiik at the Cornell 1971 International Symposium on Electron and Photon Interactions (to be published in the Conference Report.)

Figure Captions

Fig. 1. Layout for e-P elastic scattering at $Q^2 = 4 \text{ GeV}^2$.

P_1 to P_6 are proportional wire chambers. Other trigger counters are not shown. M_1 is a BNL analysing magnet with 20 inch gap. M_2 and M_3 are bending magnets to bring the beams together for colinear interactions.

Fig. 2. Layout of e-P inelastic scattering in the region $Q^2 \approx 100 \text{ GeV}^2$. P_1 to P_3 are proportional wire chambers and $\checkmark$ is a gas cherenkov counter which gives pulses for electrons but not for pions.

Fig. 3. Plot of Q^2 vs. ν showing the kinematical regions covered by two spectrometer positions, 54° and 90° . The estimated number of events/day are shown in their corresponding bins.

Fig. 1

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Fig. 2

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Fig. 3