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Measurement of the $K^- \pi^-$ Elastic Cross Section
and Study of the $K^- \pi^+ \pi^-$ Mass Spectrum
in the Reaction $K^- p \rightarrow K^- p \pi^+ \pi^-$ at 2.0 GeV/c

Abstract

Alan Richard Kirschbaum

We present a pole-extrapolation measurement of the $K^- \pi^-$ elastic scattering cross section for two intervals in $K^- \pi^-$ invariant mass from threshold to 0.84 GeV. These measurements were obtained from the reaction $K^- p \rightarrow K^- \pi^- \Delta^{++}(1238)$ at 2.05 and 2.63 GeV/c. We show how the effect of the background from the competing one-pion-exchange process $K^- p \rightarrow (K^- \pi^+)(\pi^- p)$ can be approximately accounted for, and find that the extrapolated cross sections are relatively insensitive to the presence of this background. In addition, using only the 2.63 GeV/c data, we study the enhancements which occur in the $K^- \pi^+ \pi^-$ invariant mass in the Q region, including a description of their t-dependence. The one-pion-exchange model is again used to account for the background behavior under these enhancements.

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INTRODUCTION

This thesis consists of two separate topics: a measurement of the elastic $K^-\pi^-$ cross section near threshold; and a study of resonant structure in the Q region of the $K\pi\pi$ mass spectrum. However, both these subjects make use of a one-pion exchange model which has found wide application in describing the dynamics of multi-particle production processes such as that considered here. Consequently, considerable space is devoted to a discussion of this model and of its applicability to the reaction of interest. This model is used to describe the invariant mass and momentum transfer dependences of both $\Delta^{++}(1238)$ and $K^*(890)$ production.

We then find the $K^-\pi^-$ cross section using the pole extrapolation method. The off-shell cross section is first normalized to the calculated one-pion exchange cross section, and this ratio is plotted as a function of $-t$, the momentum transfer squared to the $K^-\pi^-$ system. This ratio is extrapolated to the unphysical point $t = M_\pi^2$ to give the on-shell $K^-\pi^-$ cross section. In addition, we discuss other similar measurements of this quantity and compare our results with these.

In the final section we investigate possible enhancements in the $K^-\pi^+\pi^-$ mass and compare our results with those found in the $\bar{K}^0\pi^0\pi^-$ and $K^0\pi^+\pi^-$ mass spectra. The one-pion exchange model is used here also to separate the effects of the K^* background from the mass and t dependence of these enhancements.

I. EXPERIMENTAL

A. Field Corrections and Beam Averaging

The data for this experiment were obtained using a separated K^- beam from the Bevatron at momenta of 2.1, 2.45, 2.58, 2.63, and 2.70 GeV/c incident on the 72-inch hydrogen bubble chamber. The details of this run have been described elsewhere,¹ and will not be mentioned here except for the following. During the 2.63 GeV/c run, one of the magnet coils in the bubble chamber shorted and remained shorted for the duration of the experiment, necessitating the remeasurement of the magnetic field. In addition to changing the overall magnitude of the field, we have found it necessary to adjust the z-component of the field (perpendicular to the plane of the chamber) by performing a translation of the magnetic field coordinate system in the z-direction. The amount of this correction was determined by examining a sample of tau decays of the K^- , $K^- \rightarrow \pi^+ \pi^- \pi^-$. The rest mass of the K^- was calculated from the momenta of the three outgoing tracks, and the field was adjusted so as to give the correct result for the K^- mass.

We also used the tau decays to determine accurately the incident momentum of the K^- beam. Because of the beam optics used in this experiment, there is a correlation between the momentum of a beam track and the azimuthal angle at which it enters the chamber. Since at the incident momenta of this experiment an angle can be measured rather more precisely than can the momentum, this correlation can be used to find the beam momentum of an individual track more accurately than would be expected from the width of the beam momentum distribution. Our sample of taus was used to determine the amount and nature of this correlation, and the resulting parameters were incorporated into the reconstruction and fitting programs in order to process the events used in this analysis. The details of this procedure are given in the appendix, in which the relevant formulas are derived.

B. Measurement and Kinematic Fitting

A total of 53,411 four-prong events at all momenta were measured on the spiral reader, which in addition to digitizing track positions also provides pulse height (i.e., ionization) information for individual

tracks. Of these events, 8272 were rejected either by the spiral reader operator or by the subsequent filtering process performed by computer. The largest subset, some 5400 events (66%), are events for which the filtering program either was unable to match one or more tracks in all three views or could not find enough tracks in some view(s). Based on the experience of other people who have examined such events from another experiment, these rejects are probably bona fide four-prongs. Whether they constitute a biased sample of four-prongs is difficult to answer without re-scanning or re-measuring them. One possible bias might be against events having two close tracks of the same charge curving in the same direction. This would result in a scarcity of events at low invariant $K^- \pi^-$ or $\pi^+ p$ mass, which could affect the measurement of the $K^- \pi^-$ cross section. The magnitude of this effect would depend on the momentum transfer dependence of these events, which is not known a priori. The remaining 33% of the events in this reject class are due to obviously unbiased effects such as spliced-out film, obscured vertex, etc.

A further 5075 events which passed the filtering process were failed by the kinematic reconstruction program (TVGP). Most such failures were due to large point scatters on one or more tracks. To see whether these failures might result in a bias against events having two close similarly charged tracks, the distribution in the cosine of the angle between the two negative tracks and between the two positive tracks was examined for a sample consisting of 7% of the total events. Separate distributions were made for the failing events and for events which were successfully reconstructed and fitted. The negative-track distribution for failing events shows a slight excess of events at small angles ($\cos \theta > 0.95$), but the effect is only marginally significant. The positive-track distributions for failing and passing events appear identical. With this simple test and with these statistics, any bias that may exist is not readily apparent.

Events which were successfully reconstructed were then processed using the Group A program SQUAW, which attempts to fit each event to a series of reaction hypotheses. Table I lists the reactions tried for the four-prongs. Although a separate ionization confidence level was computed for each fit, this information was not used at this stage to reject events or to decide between hypotheses. No satisfactory fit

could be found (i.e., kinematic confidence level (CL) was less than 10^{-5}) for 3411 events. Based on examination of a small subset of these events, fully half of them had an outgoing track which was nearly coincident in momentum and direction with the beam track. Such events could easily be tau decays with an extra beam track through the vertex. In about 20% of the SQUAW failures there was nothing strikingly amiss, other than that the χ^2 was too high. The remainder of the failures were largely a result of bad beam tracks or bad outgoing tracks (unmeasured or badly measured momentum).

Passing events were required to fit at least one four-constraint (4C) hypothesis. Examination of the confidence level (CL) distribution for these events revealed a sharp peaking at low values of the confidence level. The confidence level is essentially an integral distribution of $P(\chi^2)$ and ranges from zero to one. This distribution has the property of being flat if the χ^2 for the events is properly distributed. A peak near zero may indicate that the wrong event hypothesis has been selected for these events (i.e., that the events are contaminated). Or that the errors used in the calculation of χ^2 have been underestimated.⁴³

To investigate possible contamination of the 4C K-induced events due to both π^- and K -induced 1C events having a missing π^0 , we studied the missing mass and missing momentum distributions for events fitting both 4C and 1C hypotheses. Essentially all of these events were found to be consistent with the 4C hypothesis. We conclude that 1C contamination is not a problem.

When errors have been underestimated, the confidence level distribution, instead of being flat, rises as the CL decreases and approaches infinity as the CL goes to zero.⁴³ Based on a comparison of this calculated distribution with the actual one for a small sample of the events, it was estimated that the χ^2 should be scaled by 0.7 for each event (equivalent to scaling the errors by 1.2). This was done and the CL was re-calculated for this small sample. The resulting distribution was only slightly peaked at low CL and showed no evidence for a singularity at zero. The number of events having $CL < 0.001$ was reduced by a factor of seven. It appears that incorrect error estimation can account in large measure for the observed CL distribution. All the 4C events were required to have confidence levels greater than 1%. Based on the study of the small sample, this cut is equivalent to

a 7% cut confidence level for events having correctly estimated errors.

Since the π^- pathlength at the upper three momenta is nearly 20% of that for K^- , a potentially serious problem is the misidentification of real $4C \pi$ events as $4C K$ events and vice versa. Computer-generated samples of "real" π -induced and "real" K -induced four-prongs complete with measurement errors were processed through TVGP and SQUAW, just as were the actual events. In this way one can determine the probability of $4C \pi$ and $4C K$ misidentification. At a confidence level of 10^{-5} , the fraction of events of each type that were erroneously identified was on the order of 2%.

After discarding all $4C$ events which failed to fit at least one $4C K$ hypothesis, events at the upper three momenta were combined into a single sample having mean momentum 2.63 GeV/c (the "2.63 sample"), and the events at 2.1 GeV/c were combined with an additional 4518 events at 2.0 GeV/c which were measured and analyzed at UCLA.² These events, having mean momentum 2.05 GeV/c, constitute the "2.05 sample". Table II lists the number of events at each momentum. It was decided not to use the 2.45 GeV/c events, since (a) the beam energy is not sufficiently high to study the $K\pi\pi$ mass spectrum in the Q region, and (b) there are not enough events to warrant using this sample separately in the determination of the $K^-\pi^-$ cross section.

Most of the events in both samples - 90% or so - are unambiguous. An event was considered ambiguous if the confidence level for the most probable $4C$ hypothesis was less than a factor of 3 greater than that for the next most probable $4C$ hypothesis. Events satisfying both a $4C$ and a $4C$ hypothesis were assumed to be true $4C$ events. The 2.05 and 2.63 samples had respectively about 400 and 1000 ambiguous events, roughly 60% of which were due either to a permutation ambiguity between the K^- and π^- for a given $4C$ reaction, or to an ambiguity with the $4C \pi$ reaction resulting from the inability to distinguish π 's from K 's. Therefore, an additional constraint was applied for these events such that the ionization confidence levels (supplied by SQUAW) for the two hypotheses were required to differ by at least a factor of three. This cut eliminated essentially all ambiguities except the π^- - K^- ambiguities, of which about half remained. The remaining ambiguous events, less than 3% of the total, were assigned to that hypothesis having the higher kinematic confidence level. In fact, it turned out that if we

had simply assigned all events to the most probable hypothesis and not considered ambiguities at all, we would have misidentified only about 30 events for the 2.05 sample and about 100 events for the 2.63 sample, less than 1% of the total.

Since failing events were not remeasured, it is not possible to determine an absolute cross section for the reaction of interest

$$K^- p \rightarrow K^- p \pi^+ \pi^- \quad (1.1)$$

However, Dauber et al.² have found this cross section at 2.0 GeV/c, and Mortara³ has measured it at 2.63 and 2.7 GeV/c. See Table II. From a calculation of the number of events of reaction (1.1) we should have, based on these cross sections plus the known K^- pathlength in this experiment¹, we conclude that $34 \pm 3\%$ of the events have been missed. The quoted error is due to the error in the cross sections. The number of failing events is sufficient to account for a 25% discrepancy between the number of expected events and the number of events actually seen, assuming that the remeasured failing events would have the same passing rate as events measured only once. To this 25% should be added an additional 7% to account for the number of true events excluded by the confidence level cut. The resulting numbers are in reasonable agreement. For the purpose of determining the $K^- \pi^-$ cross section, which requires a knowledge of the cross section for reaction (1.1), we assigned equal weights to our events such that the sum of the weights was the same as the number of expected events based on the known cross section and pathlength.

C. Resonance Production

Although two-body resonance production is not in itself of primary concern here, it is useful to know at least the amounts of the various resonant states occurring in reaction (1.1). Figures 1 and 2 show histograms of the six possible two-body invariant masses for each sample, and Fig. 3 shows the $K^- \pi^+ \pi^-$ invariant mass for the 2.63 GeV/c sample only. Significant production of $Y^*(1520)$, $K^*(890)$, $\Delta^{++}(1238)$, and $\Delta^0(1238)$ occurs. The Δ^0 seems to be produced entirely in association with $K^*(890)$. $\rho(765)$ may be produced, but it is not evident on the basis of Figs. 1 and 2. For the purposes of the rather elementary

treatment done here, we assume that no ρ is present. A more complex analysis including angular distributions and correlations would be necessary to do this problem justice. We assume instead that the final state consists of non-interfering production of the resonances mentioned above plus phase space background. The matrix element squared is

$$|M|^2 = 1 + a_1 f(Y^*(1520)) + a_2 f(\Delta^{++}(1238)) + a_3 f(\Delta^0(1238)) f(K^*(890)) + a_4 f(K^*(890)) \quad (1.2)$$

where the numbers a_i describing the relative amounts are determined from a maximum likelihood fit. The resonances are described using the standard Breit-Wigner prescription of Jackson.⁴ The resonant amplitude squared is of the form

$$f = \frac{M_0 \Gamma_0 q_0}{q} \frac{M \Gamma(q)}{(M^2 - M_0^2)^2 + [\Gamma(q) M_0]^2} \quad (1.3)$$

where the energy dependence of the width is

$$\Gamma(q) = \Gamma_0 (q/q_0)^{2\ell+1} \left[\frac{q_0^2 + m_\pi^2}{q^2 + m_\pi^2} \right] \quad (1.4)$$

Strictly speaking, the factor in brackets in Eq. (1.4) should be used only for the two Δ 's, and a different energy dependence should be used for the K^* . In practice, however, the form of this energy dependence does not significantly affect the fitted resonance fractions.

Here, M_0 is the central value of the resonant mass, $q(q_0)$ is the magnitude of the momentum of one of the two particles in their rest frame at an invariant mass equal to $M(M_0)$, Γ_0 is the resonant width at M_0 , and ℓ is the orbital angular momentum of the resonant decay. We have taken $\ell = 1$ for the $K^*(890)$ and the two Δ 's, and $\ell = 0$ for the $Y^*(1520)$, since for such a narrow resonance the energy dependence of the width becomes insignificant. The best fit values of M_0 and Γ_0 , and the amount of each resonance (corresponding to a_i), were found from a maximum likelihood fit using the Group A program OPTIME. The results are presented in Table III. Using these values and the matrix element of Eq. (1.2) we have calculated the predicted mass spectra of Figs. 1 and 2 and have plotted the curves over the real data, normalized to the

total number of events. Agreement is generally good for both the 2.05 and 2.63 samples, with the exception of the $\pi^+\pi^-$ mass, for which the curve overestimates the data below 0.5 GeV or so. This discrepancy may be due to ρ production.

As shown in Table III, $\Delta^{++}(1238)$ and $K^*(890)$ are copiously produced in both samples. The consequences of this will be discussed in the following sections.

II. ONE PION EXCHANGE

A. Introduction

For the purpose of studying $K^-\pi^-$ scattering as well as structure in the $K^-\pi^+\pi^-$ mass spectrum, the $K^*(890)$ produced in the reaction

$$K^-\bar{p} \rightarrow (K^-\pi^+)(\pi^-\bar{p}) \quad (2.1)$$

may be considered as background. It is desirable to be able to calculate the effects of this background explicitly. In this section we will try to motivate the choice of one-pion exchange (OPE) as the production process for $K^*(890)$, and in addition, explicit formulas for calculating the OPE contribution will be given. We wish to stress at the outset, however, that we do not intend to prove conclusively the existence of OPE in our data, nor do we intend to prove the validity of the specific formulation of this model that we use. It suffices for our purposes to show that this OPE model gives a reasonable ad hoc description of $K^*(890)$ production in our data.

B. Description of OPE Model and Tests for OPE

A convenient way to describe strong interaction processes is in terms of the singularities of the scattering amplitude. Considerable simplification arises if it can be shown that, due to the particular values taken by the energy and momentum transfer variables describing a particular reaction, only one of these singularities dominates the scattering amplitude. For example, although the nucleon-nucleon strong interaction arises from the virtual exchange of many particles (π , ρ , etc.), in fact, only pion exchange is important provided that the momentum transfer between initial and final nucleons is sufficiently small. Not only does this one-pion exchange model serve to describe the nucleon-nucleon interaction, but it also can yield valuable information about the pion-nucleon interaction. In fact, an early measurement of the pion-nucleon coupling (i.e., the residue of the pion pole) was found by applying the OPE model to np charge exchange scattering.⁵

Because of the simplicity of this scheme, both conceptually and computationally, many attempts have been made in recent years to extend it to the production of multi-body final states, particularly three- and

four-body final states. Moreover, in addition to initial states consisting only of nucleons, it seems that OPE can successfully describe processes in which one of the incident particles is a K or π meson. As shown schematically in Fig. 4, multi-body processes in the context of OPE are due to elastic scattering of the virtual pion with one or both of the incident particles. As in the nucleon-nucleon case, it is expected that the pion pole will dominate the scattering amplitude only for small values of $-t$, the four-momentum transfer squared between the incident nucleon and the outgoing nucleon-pion system. To the extent that the exchanged pion behaves like a physical pion, OPE can be used to study indirectly meson-meson elastic and charge exchange scattering. A corollary to this statement is that the scattering of the virtual pion off the incident nucleon should likewise resemble real pion-nucleon scattering. To test this, one can compute the spherical harmonic moments $\langle Y_l^0(\theta) \rangle$ (θ is the angle between the directions of the initial and final nucleons in the center of mass of the outgoing pion-nucleon system), which can be compared with the measured moments for the scattering of real particles. This is in some sense a test for the dominance of OPE relative to whatever other processes occur. In fact, for some reactions the agreement of real elastic scattering data is very good. Bingham et al.,⁶ using data from the International K⁺ Collaboration, have studied the reactions

$$K^+p \rightarrow K^*(890)\pi^+p \rightarrow (K^+\pi^-)(\pi^+p) \quad (2.2)$$

$$K^+p \rightarrow K^*(890)\pi^+p \rightarrow (K^0\pi^0)(\pi^+p) \quad (2.3)$$

$$K^+p \rightarrow K^*(890)\pi^0p \rightarrow (K^0\pi^+)(\pi^0p) \quad (2.4)$$

$$K^+p \rightarrow (K^+\pi^+)(\pi^-p) \quad (2.5)$$

over a wide range of beam energies. They find that the off-shell π^+p moments from reactions (2.2) and (2.3) agree well with the real π^+p scattering moments below about 1.5 GeV, and that the extrapolated moments are in even better agreement. The extrapolated π^0p elastic scattering moments from reaction (2.4) are likewise compatible with real π^0p scattering. However, presumably due to reflections from other channels, the π^-p moments from reaction (2.5) are in poor agreement with the real particle data. Matison et al.⁷ have also studied reaction (2.2) at 12 GeV/c and likewise find good agreement with moments obtained from real particle scattering.

It is worth mentioning, however, that comparison with real πp moments may not be a sufficient criterion for the presence of OPE. Colton and Kirschbaum⁸ have studied $\Delta(1238)$ production at 6.6 GeV/c in the four reactions

$$pp \rightarrow \pi^+ pn \quad (2.6)$$

$$pp \rightarrow p\pi^+\pi^-p \quad (2.7)$$

$$pp \rightarrow p\pi^+\pi^-\pi^0 \quad (2.8)$$

$$pp \rightarrow p\pi^+\pi^-\pi^-n \quad (2.9)$$

If the π^-n system in the latter reaction is produced by some exchange mechanism, the exchanged particle must be doubly charged, and therefore OPE cannot be responsible for the production. As expected, at small production angles (less than 10°), the π^-n spherical harmonic moments do not agree well with the real elastic scattering data. However, at angles between 10° and 20° agreement is actually better than at small angles, indicating that the moments by themselves are not a reliable test for OPE. In any case, this test would not be meaningful in the case of reaction (2.1) since there is a large kinematic overlap with the $\Delta^{++}(1238)$, which is still produced rather strongly, even when only peripheral K^* events are used. The applicability of OPE to K^* production in our data must be inferred from the fact that reactions (2.2) and (2.3) are nicely consistent with real π^+p scattering. If OPE works in this case, it should also work for a different charge state of the πp system. In addition, the OPE invariant mass and momentum transfer predictions, which are discussed in the following pages, agree well with our data.

C. The OPE Formalism

The general form of the OPE amplitude looks like

$$A_{\text{OPE}} \approx \frac{V_a V_b}{t - m_\pi^2} \quad (2.10)$$

which directly exhibits the singularity structure. V_a, V_b are the "vertex functions" which describe the scattering process (i.e., the strength of the coupling) at each vertex. For multi-body processes,

V^2 contains $d\sigma/d\Omega$ for the scattering process occurring at the particular vertex. In this simple picture, V^2 is a function only of the mass of the outgoing system. We know that this cannot really be true, though, since the mass of the virtual pion is a function of t . Equation (2.10) is correct only when t is very close indeed to m_π^2 . To describe OPE at some distance from the pole, one needs to use t -dependent vertex functions.

Specifically, for the four-body production shown in Fig. 4, the OPE differential cross section is given by⁹

$$\frac{d^7\sigma}{dt dm dM d\Omega_m d\Omega_M} = \frac{1}{4\pi^3 m_p^2 P_L^2} \frac{m^2 q M^2 Q}{(t - m_\pi^2)^2} \frac{d\sigma(m, t)}{d\Omega_m} \frac{d\sigma(M, t)}{d\Omega_M} \times G(t) \quad (2.11)$$

Here, m_p = proton mass, P_L = lab momentum, $m = K^- \pi^+$ mass, $M = \pi^- p$ mass, $q(Q)$ = magnitude of the momentum of the outgoing $K^- (p)$ as seen in the $K^- \pi^+ (\pi^- p)$ rest frame, and t = four-momentum transfer squared from the incoming proton to the $\pi^- p$ system. $G(t)$ is an additional ad hoc function weakly dependent on t which will be described later.

There are various schemes one can devise which incorporate off-shell corrections to $d\sigma/d\Omega_m$ and $d\sigma/d\Omega_M$. The one used here, proposed by Durr and Pilkuhn,¹⁰ uses separate correction factors for each partial wave cross section. That is, we can write the total cross section for, say, $\pi^- p$ elastic scattering as a sum of partial wave cross sections multiplied by functions whose form depends on ℓ :

$$\sigma(M, t) = \sum_{\ell J} (J + \frac{1}{2}) |A_{\ell J}^{\pi^- p}(M)|^2 f_{\ell J}(M, t) \quad (2.12)$$

$$\text{where } A_{\ell J}^{\pi^- p}(M) = \frac{1}{5} \sum_{\ell J} [A_{\ell J}^{\frac{3}{2}}(M) + 2A_{\ell J}^{\frac{1}{2}}(M)] ,$$

the superscripts being values of total isotopic spin. In addition to a factor resulting from summing and averaging spins of the initial and final nucleons, there is an angular momentum "barrier penetration factor" which is derived from potential theory and which is familiar in studies of the energy dependence of resonant widths. This factor is unity for S-waves. Defining the spin averaging factor as W ,

$$W = \left[\frac{(M + m_p)^2 + t}{(M + m_p)^2 - m_\pi^2} \right] \quad (2.13)$$

the correction factors for the first few partial waves in π^-p elastic scattering can be written¹⁵

$$\begin{aligned}
 f_{s\frac{1}{2}} &= W \\
 f_{p\frac{1}{2}} &= \left(\frac{Qt}{Q}\right)^2 \frac{1}{W} \left[\frac{1 + (R_{p\frac{1}{2}}Q)^2}{1 + (R_{p\frac{1}{2}}Q_t)^2} \right] \\
 f_{p\frac{3}{2}} &= \left(\frac{Qt}{Q}\right)^2 W \left[\frac{1 + (R_{p\frac{3}{2}}Q)^2}{1 + (R_{p\frac{3}{2}}Q_t)^2} \right] \\
 f_{d\frac{3}{2}} &= \left(\frac{Qt}{Q}\right)^4 \frac{1}{W} \left[\frac{9 + 3(R_{d\frac{3}{2}}Q)^2 + (R_{d\frac{3}{2}}Q)^4}{9 + 3(R_{d\frac{3}{2}}Q_t)^2 + (R_{d\frac{3}{2}}Q_t)^4} \right]
 \end{aligned} \tag{2.14}$$

where Q_t is the magnitude of the momentum of the incoming proton in the rest frame of the π^-p system. The numbers R are the widths of the angular momentum barriers, though in practice they are simply parameters to be determined. Since we in fact work with the differential cross section and not the total cross section, it is convenient to write

$$\begin{aligned}
 \frac{d\sigma(M,t)}{d\Omega} &= \frac{d\sigma(M)}{d\Omega} f(M,t) \\
 f(M,t) &= \frac{\sum_{\ell J} (J + \frac{1}{2}) |A_{\ell J}^{\pi^-p}(M)|^2 f_{\ell J}(M,t)}{\sum_{\ell J} (J + \frac{1}{2}) |A_{\ell J}^{\pi^-p}(M)|^2}
 \end{aligned} \tag{2.15}$$

since integration of Eq. (2.15) gives Eq. (2.12). Note that the function $f(M,t)$ reduces to unity at the pion pole. This function is strictly a kinematical correction to the OPE formula due to the angular momentum at each vertex, since it is assumed, rightly or wrongly, that the dynamics of the pion-nucleon interaction are independent of whether the pion is off the mass shell.

The partial wave amplitudes used to calculate Eq. (2.15) are taken from the compilation of Donnachie et al.¹¹

The $K^-\pi^+$ vertex can be treated in a similar way:

$$\frac{d\sigma(m,t)}{d\Omega} = \frac{dN(m)}{d\Omega} \times g(m,t) \tag{2.16}$$

where $dN/d\Omega$ is the angular distribution (normalized to unity) for elastic $K^-\pi^+(K^+\pi^-)$ scattering constructed from the $K^+\pi^-$ spherical harmonic moments of Trippe et al.¹² using the reaction (2.2).³³ One assumes that these moments reflect the behavior of real $K^-\pi^+$ scattering, since the π^+p moments closely resemble those for real π^+p scattering. Of course, this can be only an approximation.

$g(m, t)$ in Eq. (2.16) is the off-shell total cross section for $K^-\pi^+$ scattering, and as in the π^-p case can be written in terms of the on-shell cross section plus a correction for each partial wave. The on-shell cross section is

$$g(m) = \frac{4\pi k^2}{9} \sum_{\ell} (2\ell + 1) \left[\sin^2 \delta_{\ell}^{3/2}(m) + 4 \sin^2 \delta_{\ell}^{1/2}(m) + 4 \cos(\delta_{\ell}^{3/2}(m) - \delta_{\ell}^{1/2}(m)) \sin \delta_{\ell}^{3/2}(m) \sin \delta_{\ell}^{1/2}(m) \right] \quad (2.17)$$

where again the superscripts denote I-spin amplitudes. Since we wish only to develop a formula which provides a reasonably accurate description of the invariant mass and momentum transfer dependence of $(K^-\pi^+)(\pi^-p)$ production, it is sufficient to retain only the dominant terms in Eq. (2.17). The largest contribution to the cross section comes from the $p_{1/2}$ partial wave, corresponding to the $K^*(890)$. Abundant evidence exists showing that the $s_{3/2}$ cross section is negligible compared with this term. The purpose of Chapter III is to measure this cross section, and the results of Chapter III together with several independent measurements of this quantity summarized therein prove that indeed the $s_{3/2}$ term can be dropped from Eq. (2.17). We also omit the $p_{3/2}$ term, since no evidence exists which indicates this term is non-zero. The $s_{1/2}$ contribution, first seen by Trippe et al.,¹² has since been more accurately determined using phase shift analysis.^{6,7} Because the newer results were published after the completion of our work, our parameterization of the $s_{1/2}$ cross section is that of Trippe et al. Use of the latest numbers would not likely change the conclusions of this or the following chapters.

Thus, Eq. (2.17) becomes

$$g(m) = \frac{16\pi k^2}{9} \sum_{\ell} (2\ell + 1) \sin^2 \delta_{\ell}^{1/2}(m) \quad (2.18)$$

In the energy region of concern to us, only the s and p waves are significant. The higher moments $\langle Y_3 \rangle$, $\langle Y_4 \rangle$ etc, are all consistent with zero up to about 1 GeV. The p wave term in Eq. (2.18) is parameterized using an $\ell = 1$ relativistic Breit-Wigner shape for the $K^*(890)$, with central mass 0.892 GeV and width 0.050 GeV.

The Durr-Pilkahn factor for the p-wave is, as before,

$$f_p = \left[\frac{1 + (R_{K^*} q)^2}{1 + (R_{K^*} q_t)^2} \right] \quad (2.19)$$

where $q(q_t)$ is the magnitude of the momentum of the final (incident) K^- in the $K^- \pi^+$ rest frame.

The constants R in Eqs. (2.14) and (2.19) have been determined by others using maximum likelihood fits of Eq. (2.11) to various four-body reactions, and also fits to certain three-body reactions using an analogous form of this equation pertinent to three-body OPE production. Wolf¹³ has performed a fit to experimental data on the processes

$$\begin{aligned} pp &\rightarrow N^*(1238)N^*(1238) \\ pp &\rightarrow N^*(1238)n \\ \pi^- p &\rightarrow np^0 \\ \pi^+ p &\rightarrow N^*(1238)\rho^0 \end{aligned} \quad (2.20)$$

As is customary, the function $G(t)$ of Eq. (2.11) is taken to be

$$G(t) = \left[(c - m_\pi^2)/(c - t) \right]^2, \quad (2.21)$$

a form originally suggested by Goldhaber.¹⁴ Wolf thereby found that $R_{\Delta^{++}} (= R_{p3/2}$ in Eq. (2.14)) = $3.97 \pm 0.11 \text{ GeV}^{-1}$. In the same fit, he also found that $c = 2.29 \pm 0.27 \text{ GeV}^2$. The size of c is a measure of the adequacy of the Durr-Pilkahn formalism. In a more nearly correct theory, c would be larger. Nevertheless, the t-dependence of Eq. (2.21) is not extreme.

To determine the constants $R_{p1/2}$ and $R_{p3/2}$, Colton et al.¹⁵ have performed a maximum likelihood fit to the reaction $pp \rightarrow p\pi^+p\pi^-$ at 6.6 GeV/c in which they vary simultaneously c, $R_{p1/2}$, and $R_{p3/2}$, and in addition allow the $R_{p3/2}$ parameters at each vertex to vary independently. They find that

$$\begin{aligned}
c &= 1.2 \pm 0.5 \text{ GeV}^2 \\
R_{p\frac{1}{2}} &= 0.3 \pm 0.9 \text{ GeV}^{-1} \\
R_{d\frac{3}{2}} &> 30 \text{ GeV}^{-1} \\
R_{p\frac{3}{2}}^{\pi^+ p} &= 3.5 \pm 0.7 \text{ GeV}^{-1} \\
R_{p\frac{3}{2}}^{\pi^- p} &= 2.3 \pm 0.3 \text{ GeV}^{-1}
\end{aligned} \tag{2.22}$$

The large value of $R_{d\frac{3}{2}}$ simply indicates that for the range of πp mass used in the analysis, no off-shell correction is required.

The fact that the $p\frac{3}{2}$ parameters at the two vertices are not equal, and that c is different from the value found by Wolf, indicates that the model is not completely accurate in describing these reactions. A simple OPE model with no absorption, as is used here, requires that each vertex behave independently of the other. However, using these parameters as found from the 6.6 GeV/c data, Colton et al.¹⁵ find that Eq. (2.11) accurately describes both momentum transfer and invariant mass distributions for data of the same reaction at 28 GeV/c.

Trippe et al.¹² have used the reaction $K^+ p \rightarrow K^+ \pi^- \Delta^{++}(1238)$ to find R_K^* . Having fixed $c = 2.29 \text{ GeV}^2$ and $R_\Delta = 3.97 \text{ GeV}^{-1}$, they have fitted Eq. (2.11) over incident momenta from 3.0 to 13 GeV/c. They obtain a best value of $R_K^* = 1.25 \pm 0.20 \text{ GeV}^{-1}$. We use these same values of c , R_Δ , and R_K^* , as well as the values quoted above for $R_{p\frac{1}{2}}$ and $R_{p\frac{3}{2}}$. ($R_{d\frac{3}{2}}$ is taken to be zero)

D. Comparison with Data

Figure 5 shows plots of invariant mass and momentum transfer distributions for the 2.05 and 2.63 samples plotted together. The curves are obtained by integration of Eq. (2.11) over the invariant mass and momentum transfer intervals indicated. The $\Delta^{++}(1238)$ and $Y^*(1520)$ have been excluded by deleting those events having $1.14 < M(\pi^+ p) < 1.31 \text{ GeV}$ and $1.50 < M(K^- p) < 1.55 \text{ GeV}$ in order to better exhibit the extent to which the OPE model fits reaction (2.1). It should be emphasized that there is no arbitrary normalization or constants involved. Equation (2.11) gives an absolute prediction for the OPE cross section in this reaction. The discrepancy between the curves and the data is largely due to the presence of non-resonant background which is not removed by

cuts. The total K^* production cross section based on the cross sections in Table II and the resonance percentages in Table III is $0.16 \pm .01$ mb for the 2.05 sample and $0.34 \pm .03$ mb for the 2.63 sample. The OPE model predicts respectively 0.15 mb and 0.32 mb, which is in good agreement.

III. $K\pi$ SCATTERING

A. Introduction and Selection of Data

It was mentioned in Section II that information on real meson-meson scattering processes can be obtained if one of the initial mesons is virtual. In this section we will describe how the total cross section for the process

$$K^-\pi^- \rightarrow K^-\pi^- \quad (3.1)$$

can be measured using the formalism of one-pion exchange developed in the previous section. Reaction (3.1) is of interest for several reasons. It is a doubly charged (exotic) channel, and since no exotic resonances have yet been found anywhere, it would be very interesting indeed if some resonant structure were present in this reaction. Due to our limited statistics, however, this question is of academic interest only. It is also important for partial wave analyses of the $K^-\pi^-$ system to know the $s_{3/2}$ cross section so as to eliminate ambiguities in the solution. This cross section is directly obtainable by extrapolating the off-shell cross section for reaction (3.1) to the pion pole, provided that the $p_{3/2}$ contribution is negligibly small. Analysis of the $K^-\pi^-$ spherical harmonic moments can show whether in fact this term can be neglected, although all $K\pi$ partial wave analyses have assumed it to be zero.

In this study we use the reaction

$$K^-p \rightarrow K^-\pi^-\Delta^{++} \quad (3.2)$$

to determine the $K^-\pi^-$ elastic cross section, since the OPE process for this reaction involves the scattering of a virtual π^- with the incident K^- . Since this channel is pure I-spin = $3/2$, we denote the cross section as $\sigma_{3/2}$. In principle, there is no reason to restrict ourselves to Δ^{++} events, but because the Δ^{++} dominates the low mass π^+p region and because the $K^*(890)$ background is a definite problem, it is hoped that the signal-to-noise will be improved by using only Δ^{++} events. To further increase the probability of detecting whatever OPE contribution may be present, we limit the four-momentum transfer squared from the target proton to the Δ to $-t < 0.3 \text{ GeV}^2$ (t is negative in the physical

region). In addition, we restrict ourselves to low mass $K\pi$ systems, since the higher the mass, the higher is the minimum kinematically allowed value of $-t$, $-t_{\min}$, and we wish to stay as close to the pion pole as possible. Unless specifically indicated, m denotes the $K^-\pi^-$ mass; M , the π^+p mass; and t , the momentum transfer to the Δ^{++} . Accordingly, we restrict our study to those events satisfying the cuts

$$\begin{aligned} 1.14 < M < 1.31 \text{ GeV} \\ -t < 0.3 \text{ GeV}^2 \\ m < 0.84 \text{ GeV} \end{aligned} \quad (3.3)$$

There are 836 events of both samples which survive these cuts. Table IV gives the amounts of the various resonances present in this selected sample, determined from the same sort of maximum likelihood fit described in Sections I and II. Figure 6 shows histograms of m , M , and $-t$. Also shown in Fig. 6 are the π^-p and $K^-\pi^+$ invariant masses and the four-momentum transfer squared from the target proton to the π^-p system for the events in the selected sample. The superimposed curves are the predictions of a model to be discussed below. It is evident from Fig. 6 as well as from Table IV that most of the background to reaction (3.2) in our sample comes from the reaction $K^-p \rightarrow K^{*0}\pi^-p$.

B. Pole Extrapolation

Let us suppose for a moment that we have a model which can describe the OPE process $K^-p \rightarrow (K^-\pi^-)(\pi^+p)$ exactly everywhere in the physical region, and let us further assume that no background is present. As seen in the previous section, our model would look like

$$\left(\frac{d^3\sigma}{dt dm dM} \right)_{\text{OPE}} = \frac{\sigma(m)\sigma(M)}{(t-m_\pi^2)^2} F(t,m,M) \quad (3.4)$$

where $\sigma(m)$ is the real $K^-\pi^-$ elastic cross section and $\sigma(M)$ is the real π^+p elastic cross section. The function $F(t,m,M)$ is some off-shell form factor. We now take the ratio of the experimentally measured differential cross section for reaction (3.2) and the OPE differential cross section, Eq. (3.4), and define

$$\sigma(t, m, M) = \frac{d^3\sigma/dt dm dM}{(d^2\sigma/(\hat{t} dm dM))_{OPE}} \quad (3.5)$$

This ratio is in general a function of t , m , and M . With the two assumptions noted above, if $\sigma(m)$ is set to unity, then this function is the real $K^-\pi^-$ elastic cross section as a function of m .

Of course, things are not so simple. It is clear that background is substantial; furthermore, no model is perfect. To see how this affects the measurement of $\sigma_{3/2}(m)$ and to spell out more clearly some of the assumptions in our analysis, we can break down the t -dependence of the data as follows. First, pick some particular values for m and M (in practice, this means selecting only those events falling within certain bins in m and M). Then the differential cross section is a function only of t . This t -dependence will have two contributions, one from OPE and one from whatever background process may be present:

$$d\sigma/dt = (d\sigma/dt)_{OPE}^{REAL} + (d\sigma/dt)_{BACKGROUND} \quad (3.6)$$

We distinguish here the real OPE differential cross section, $(d\sigma/dt)_{OPE}^{REAL}$ with that calculated according to our model, $(d\sigma/dt)_{OPE}$. Now evaluate Eq. (3.4) at the same values of m and M . The resulting differential cross section is also a function only of t . Provided that our OPE model exactly describes the real OPE process,

$$\sigma(t) = \frac{(d\sigma/dt)}{(d\sigma/dt)_{OPE}} = C + \frac{(d\sigma/dt)_{BACKGROUND}}{(d\sigma/dt)_{OPE}} \quad (3.7)$$

where $C = \sigma_{3/2}$. If the OPE model does not exactly describe the real OPE process, then there is an additional t -dependence

$$\sigma(t) = C[1 + a_1(t - m_\pi^2) + \dots] + \frac{(d\sigma/dt)_{BACKGROUND}}{(d\sigma/dt)_{OPE}} \quad (3.8)$$

Equation (3.8) reflects the fact that any model becomes exact at $t = m_\pi^2$. The better the model describes OPE in the physical region, the more gentle the t -dependence in the brackets.

There remains the question of the t -dependence of the background. In studies of this type, it is tacitly assumed that the closer t becomes to m_π^2 , the smaller the background becomes relative to OPE. With this assumption, $\sigma_{3/2}$ is obtained by evaluating Eq. (3.8) at the pole, $t = m_\pi^2$. Even if this assumption is reasonable, which is not obvious at this point, background could still lead to systematic errors if the number of events is not large. Fortunately, in this study we are able to account for the background (or at least a large part of it) explicitly, so that we can see in detail what effect it has on our measurement. We shall see later, though, that background can present special problems in pole extrapolation studies done with three-body reactions.

Although in principle one OPE model is as good as another, since all models become exact at the pole and therefore the result of the extrapolation should be independent of the form of the extrapolating function, in practice statistics are limited and $t_{\min}$ is some distance from m_π^2 . We therefore will obtain the most accurate answer when the t -dependence of Eq. (3.8) is minimal, i.e., when our model is most accurate. Our model is that of Durr and Pilkuhn, as discussed in the previous section. The function $F(t, m, M)$ of Eq. (3.4) contains a p -wave form factor for the $\Delta(1238)$. Since we are ignorant of the angular behavior of the $K^- \pi^-$ vertex, we assume that, at least near threshold, the amplitude is purely s -wave and that therefore no off-shell factor is necessary.¹⁶ In fact, partial wave analyses using other reactions seem to bear this out.

Although we are encouraged by the success of the Durr-Pilkuhn OPE (DP-OPE) model in describing a wide range of pion-exchange differential cross sections,¹⁷⁻¹⁹ there is one additional check on the model we can make, which is also a check of the pole extrapolation method. By extrapolating to the pion pole in the reaction $pp \rightarrow p\pi^+n$ at 6.6 GeV/c, Ma et al.²⁰ have shown that by using the DP-OPE prescription one can obtain values for the elastic π^+p cross section which agree completely with the known values, whereas a simple OPE model with $F(t, m, M) = 1$ gives incorrect results. Matison et al.⁷ and Bingham et al.,⁶ using the reaction $K^+p \rightarrow K^+\pi^-\pi^+p$, also find their extrapolated π^+p cross sections in good agreement with the accepted values. This work indicates that one can obtain meaningful results from a pole extrapolation, and also that the Durr-Pilkuhn form factor adequately describes the $\Delta^{++}(1238)$ vertex.

C. Fitting Procedure and Results

The extrapolation is performed in two 0.1 GeV intervals in $K^-\pi^-$ invariant mass m , starting at 0.64 GeV. Each of these intervals is subdivided into intervals in $-t$. For each of these subintervals the quantity

$$\sigma(\bar{t}, \bar{m}) = \frac{\int_{\Delta m, \Delta M, \Delta t} \left(\frac{d^3 \sigma}{dt dm dM} \right) dt dm dM}{\int_{\Delta m, \Delta M, \Delta t} \left(\frac{d^3 \sigma}{dt dm dM} \right)_{\text{OPE}} dt dm dM} \quad (3.9)$$

is evaluated, where the integral is over the regions defined by the intervals in t , m , and M , and $\bar{t}$ and $\bar{m}$ are the mean values of $-t$ and m in each of these intervals. This procedure is the way in which Eq. (3.5) is evaluated in practice. The numerator is essentially a sum of events. The integral in the denominator is evaluated using a Monte Carlo technique.

Figure 7 plots $\sigma(\bar{t}, \bar{m})$ as a function of t for each of the intervals in m (square data points). As can be seen, these measurements show a strong dependence on $\bar{t}$. This is not surprising, because of the large amount of K^* background known to be in the sample. However, we can use the results of chapter II to find the contribution of this background to the kinematic region defined by Δm , ΔM , and Δt by simply integrating Eq. (2.11) over this region. The result obtained by subtracting this background contribution from the measured cross section is then divided by the OPE cross section for reaction (3.2), again with $\sigma(m)$ set to unity:

$$\sigma(\bar{t}, \bar{m}) = \frac{\int_{\Delta m, \Delta M, \Delta t} \left[\left(\frac{d^3 \sigma}{dt dm dM} \right) - \left(\frac{d^3 \sigma}{dt dm dM} \right)_{\text{BACKGROUND}} \right] dt dm dM}{\int_{\Delta m, \Delta M, \Delta t} \left(\frac{d^3 \sigma}{dt dm dM} \right)_{\text{OPE}} dt dm dM} \quad (3.10)$$

Figure 7 also plots $\sigma(\bar{t}, \bar{m})$ as measured for this background-subtracted sample (triangular data points). This background subtraction has dramatically reduced the dependence of $\sigma(\bar{t}, \bar{m})$ on t .

Linear fits of the form $a-bt$ were attempted for both sets of $\sigma(\bar{t}, \bar{m})$ and in each case the points at $-t = -0.02 \text{ GeV}^2$ represent the extrap-

olated value and the associated statistical error. Only the points with $-t < 0.3 \text{ GeV}^2$ were used in the fits. Table V summarizes the results of these fits, including fit parameters, extrapolated cross sections, and the associated statistical errors. The $\sigma_{3/2}(m)$ is obtained by averaging the background-subtracted results for the 2.05- and 2.63 GeV/c samples for each bin in m .

In order to check that the background-subtraction is reasonable, we have calculated the differential cross section $d\sigma/dM$, $d\sigma/dt$, $(d\sigma/dt)_{\pi^-p}$, $(d\sigma/dM)_{\pi^-p}$, and $(d\sigma/dm)_{K^-\pi^+}$ for our selected sample of data, assuming that only reaction (3.2) and the K^* background contribute incoherently to the sample. The contribution from reaction (3.2) is calculated by inserting our extrapolated value of $\sigma_{3/2}(m)$ in Eq. (3.4). We assume for this purpose that $\sigma_{3/2}(m)$ is 0 in the range 0.64 - 0.74 GeV, since the extrapolated cross section is slightly negative in this region. The K^* background contribution again was evaluated by integrating Eq. (2.11) over the appropriate regions in m , M , and t . These differential cross sections are plotted as the solid lines superimposed upon the corresponding experimentally measured quantities in Fig. 6. Apart from the normalization being somewhat low, this simple calculation gives a reasonably good description of these data. This is a reflection of the small slopes of $\sigma(\bar{t}, \bar{m})$ as a function of $\bar{t}$ for the subtracted extrapolations. The discrepancies between the calculation and the data could be due to the inadequacy of the calculation, the presence of some remaining non- K^* background, or both. We emphasize that we deduce nothing directly about the $K^-\pi^-$ elastic cross section from the extent to which the curves of Fig. 6 describe the data. These figures illustrate only that K^* background is a large contributor to the cross section, and that our procedure for calculating this contribution is reasonable. In fact, whether the t -dependence of the background is explicitly accounted for or not has little effect on the extrapolated cross sections. Within errors, the two sets of data in Fig. 7 when extrapolated to $t = m_\pi^2$ give the same results. Therefore, the implicit assumption made in all such extrapolation measurements, namely that the background becomes negligibly small near the pion pole, is seen to be true for our data.

Figure 8 shows the Jackson frame polar angle distributions in both the $K^-\pi^-$ and π^+p rest frames for those events used in the extrapolation.

olation. The curves are the model predictions calculated in the same fashion as the curves of Fig. 6. An OPE model for reaction (3.2) would predict a flat distribution at the $K^-\pi^-$ vertex and approximately a $(1 + 3\cos^2\theta)$ distribution at the π^+p vertex. The actual distributions are significantly different from these, again reflecting the presence of the large K^* background. The model predicts roughly the correct shape for the angular distribution at the $K^-\pi^-$ vertex, but does noticeably less well at the π^+p vertex.

The $K^-\pi^-$ angular distribution can in principle be used to determine the phase shifts $\delta_s^{3/2}$ and $\delta_p^{3/2}$ which are related to the moments $\langle Y_1^0 \rangle$ and $\langle Y_2^0 \rangle$. These moments were calculated in the same t bins and m bins as were used for the cross section extrapolation, the K^* background contribution having first been subtracted by constructing an appropriate weight for each event.²¹ These moments can then be extrapolated to $t = m_\pi^2$ to get the moments for real $K^-\pi^-$ scattering. In our case, the moments show no appreciable t dependence and in fact are consistent with zero, with the possible exception of $\langle Y_1^0 \rangle$ in the range $m = 0.74 - 0.84$ GeV. $\langle Y_1^0 \rangle$ is slightly positive in all t bins, but the extrapolated value is zero within errors. $\langle Y_2^0 \rangle$ is directly related to the ratio of the $I = 3/2$ p-wave cross section to the total $I = 3/2$ cross section (see, e.g., Ref. 7). Our measurements of $\langle Y_2^0 \rangle$ imply that this ratio is less than 20%.

In Fig. 9 the results of our background-subtracted extrapolations are presented along with the results of other attempts to measure the $I = 3/2$ $K\pi$ elastic scattering cross section. In interpreting our results it is important to keep in mind the uncertainty inherent in this extrapolation procedure. The errors indicated on our measurements are just the statistical ones that are propagated from the least-squares fit to the linear form for $\sigma(\bar{m}, \bar{t})$. Although it might be reasonable to assume the $\sigma(\bar{m}, \bar{t})$ is a well-behaved function of t , there is nothing that requires it. In particular, to get a correct answer for $\sigma_{3/2}^{(m)}$, we have to assume even with infinite statistics that $\sigma(\bar{m}, \bar{t})$ is a continuous function of $\bar{t}$ with continuous derivatives. In fact, we would hope that the function is monotonic as well. In addition, we assume that whatever background remains must be less forward-peaked than $(t - m_\pi^2)^{-2}$ near the boundary of the physical region. These considerations are tacitly assumed but seldom stated in most pole extrapolation studies. To the

extent that these assumptions are not true there will be systematic errors in interpreting the extrapolated results as the $K^-\pi^-$ elastic cross section. The sceptical reader may wish to refer to the work of Kane, which is an excellent treatment of the problems and pitfalls of this sort of analysis.²²

It is worthwhile to mention briefly how others have measured $\sigma_{3/2}$. Cho et al.²³ obtain this cross section by performing a pole extrapolation using the reaction $K^-n \rightarrow K^-\pi^-p$. For this reaction the equation for $\sigma(m, t)$ analogous to Eq. (3.9) has a pole at $t = 0$ unless $d^2\sigma/dt dm$ has a zero there. Any contribution to $d^2\sigma/dt dm$ other than simple one-meson exchange in general gives a non-zero contribution to the amplitude at $t = 0$. Cho et al. assume no such contribution and therefore constrain $\sigma(m, t)$ to be finite at $t = 0$.

Jongejan et al.²⁴ perform a pole extrapolation using 669 events of the type $K^-p \rightarrow K^-\pi^-\Delta^{++}$, where the momentum transfer squared to the Δ^{++} is less than 0.4 GeV^2 . They use a maximum-likelihood polynomial fit to describe the t dependence of their extrapolating function. They do not account for the $K^*(890)\pi^-p$ background in their sample. They point out that the values they quote for the highest two bins in mass may be unreliable, since $-t_{\min}$ for these bins is rather large, 0.2 GeV^2 .

Bakker et al.²⁵ perform a pole extrapolation using 1009 events of the type $K^-n \rightarrow K^-\pi^-p$ having a momentum transfer squared to the nucleon less than 0.32 GeV^2 . They find that their results are not substantially different whether or not they use a form factor to describe the t -dependence of their extrapolating function, which is well fitted by a straight line.

Antich et al.²⁶ have used the reaction $K^-n \rightarrow K^-\pi^-p$ at $12 \text{ GeV}/c$ to determine $\sigma_{3/2}$. With limited statistics, they find that $\sigma_{3/2} = 5 \pm 0.4 \text{ mb}$ over the range $M(K^-\pi^-) < 1.5 \text{ GeV}$. They use a pole extrapolation with no form factors.

In addition, phase shift analyses^{27,28} have been performed by using the high-statistics data of the International K^+ Collaboration on the reaction $K^+p \rightarrow K^0\pi^0\pi^+p(K^+\pi^-\pi^+p)$ in which both $K^-\pi^-$ elastic and charge-exchange scattering were studied. In each case there is a solution for $\sigma_{3/2}$ which is compatible with our results on $\sigma_{3/2}$ and the summary in Fig. 9.

Yuta et al.²⁹ have analyzed the final states $K^-p \rightarrow K^-\pi^+n$ and

$\bar{K}^- p \rightarrow \bar{K}^0 \pi^- p$ at 5.5 GeV/c. They perform a partial-wave analysis in which the angular moments are parametrized by using a non-absorptive one-particle-exchange model involving both scalar and vector exchanges. Taking the absolute value of $\delta_{3/2}$ from measurements in $\sigma_{3/2}$, Yuta et al. find the sign of $\delta_{3/2}$ to be negative.

The solid curve on Fig. 9 is the $I = 3/2$ cross section as predicted by the Veneziano model of Lovelace.³⁰ Although $\delta_{3/2}$ is predicted to be negative, as found by Yuta et al., the calculated cross section overestimates our measurements as well as the others shown in the figure.

Current algebra³¹ predicts s-wave $K\pi$ scattering lengths of $a_{1/2} = 0.26F$, $a_{3/2} = -0.13F$, corresponding to a threshold cross section for the reaction $\bar{K}^- \pi^- \rightarrow \bar{K}^0 \pi^-$ of 2.1 mb. This number is consistent with the results of Fig. 9.

Clearly reaction (3.2) is not an ideal approach to the study of the $I = 3/2$ $K\pi$ cross section, especially at low momentum. However, neither can any experiment thus far done be considered ideal. Most recent experiments involve the use of three-body final states, which have the advantages of minimal background and low values of $-t_{\min}$. As mentioned earlier one must assume the absence of a singularity in the extrapolating function at $t = 0$, or at least that its presence can be ignored. It has been shown²² that considerable systematic errors can thereby arise.

On the other hand, a four-body final-state analysis is complicated by the presence of considerable background. Whether meaningful results can be obtained depends to a large degree on the nature and amount of this background. For example, one could not put much faith in an extrapolation using the reaction $K^+ p \rightarrow (K^+ \pi^+) (\pi^- p)$, since the background $K^{*0}(890) \Delta^{++}(1236)$ is exceedingly strong.³² We have attempted to show that for reaction (3.2), in which the background is considerable but not overwhelming, one can explicitly calculate the effect of the background on the determination of $\sigma_{3/2}$. Our confidence in our results is strengthened by the fact that $\sigma_{3/2}$ is essentially the same both before and after background subtraction.

IV. THE $K\pi\pi$ SYSTEM

A. Introduction

The $K\pi\pi$ system produced in reactions of the type $KN \rightarrow K\pi\pi N$ has been widely studied at a variety of incident beam momenta, particularly for $K\pi\pi$ invariant masses in the region 1.1 - 1.5 GeV. It is generally agreed that a large, broad enhancement exists in this region, the so-called Q enhancement. Furthermore, it may be that there is some substructure within the Q, since many experiments report the existence of one or several smaller, narrower peaks superimposed on the broad enhancement. Nevertheless, since the details of the structure vary from experiment to experiment and energy to energy, there is no universally agreed upon interpretation for the Q. It has been variously suggested that the Q is a resonant structure, a kinematic enhancement, or several resonances superimposed on a kinematic enhancement.

At the higher beam momenta (2.63 sample) of this experiment, the $K\pi^+\pi^-$ mass ranges from threshold up to about 1.6 GeV, just spanning the Q region. Accordingly, we can use the reaction

$$K^-p \rightarrow K^-p\pi^+\pi^- \quad (4.1)$$

to study possible substructure within the Q. For this purpose we exclude those events for which the π^+p mass lies in the $\Delta^{++}(1238)$ region, and those events for which the $K^-\pi^+$ mass lies outside the $K^*(890)$ region. The events used in the analysis therefore have a large amount of $K^*(890)$ present. The OPE model for $K^*(890)$ production developed in chapter II gives quite a reasonable description of the gross features of the $K\pi\pi$ system, and, when the momentum transfer to the $K\pi\pi$ system is small, this model can be used to describe the background under the fine structure in the Q region.

B. Summary of Previous Experiments

This section is a very brief summary of recent experiments reporting structure in the $K\pi\pi$ invariant mass. The summary is intended only to put the results of this chapter in perspective and is not intended as an exhaustive bibliography on the Q. The reader can refer to Davis et al.³⁴ and the references contained therein for a more complete view of the ex-

perimental situation for the Q region as a whole.

Shen et al.,³⁵ in a K^+p experiment at 4.6 GeV/c, find two distinct peaks in $K^*(890)\pi$ mass when $\Delta^{++}(1238)$ events are excluded. These peaks occur at 1320 MeV and 1420 MeV, and have widths of about 80 MeV and 60 MeV respectively. The $K^*(1420)$ accounts for most, but not all, of the peak at 1420 MeV.

Goldhaber et al.³⁶ analyzed K^+p data at 9 GeV/c and found evidence for three peaks, at 1250, 1360, and 1420 MeV, having respective widths of 50, 80, and 80 MeV. Again, $K^*(890)$ events were selected and $\Delta^{++}(1238)$ events were removed. They note that interference effects between the resonances and background might account for the different central value of the lower peak reported by Shen et al.

With three times the data of Goldhaber et al. and at the same momentum, Alexander et al.³⁷ find a peak at 1260 MeV having a width of 40 MeV, and another at 1380 MeV with a width of 120 MeV. Part of the upper peak is due to $K^*(1420)$.

In a similar experiment at 10 GeV/c, Barnham et al.³⁸ find two peaks having about the same masses and widths as those found by Alexander et al. Again, there is some contribution to the upper peak from $K^*(1420)$.

In a K^+p experiment at 5 GeV/c, Bassompierre et al.³⁹ report three peaks, in addition to the $K^*(1420)$, whose central masses are 1230, 1280, and 1320 MeV and whose widths are respectively 60, 80, and 60 MeV. These peaks are seen only when peripheral events (i.e., $-t_{pp} < 0.1 \text{ GeV}^2$) are excluded.

Bishop et al.,⁴⁰ using K^+p data at 3.54 GeV/c, find peaks at 1300 and 1430 MeV having widths of 40 and 90 MeV respectively. These peaks roughly correspond to those found by Shen et al. at a slightly higher beam momentum, although the widths are noticeably different.

Finally, in a K^+d experiment at 9 GeV/c, Garfinkel et al.⁴¹ find evidence for two peaks in $K^+\pi^+\pi^-$ mass, at 1243 MeV and 1344 MeV. The corresponding widths are reported as 70 MeV and less than 60 MeV. Both peaks correspond to a five standard deviation effect above the smooth background.

C. $K\pi\pi$ Enhancements at 2.63 GeV/c

1. $\bar{K}^0\pi^+\pi^-$ and $\bar{K}^0\pi^0\pi^-$ Mass Spectra.

In addition to reaction (4.1), data from the same experiment also exist for the companion reactions

$$K^- p \rightarrow p K^0 \pi^0 \pi^- \quad (4.2)$$

$$K^- p \rightarrow n K^0 \pi^+ \pi^- \quad (4.3)$$

which were analyzed by Friedman.¹ The motivation for the present study derives from the work of Friedman, which is briefly summarized as follows.

Two peaks were observed in the $\bar{K}^0 \pi^0 \pi^-$ mass distribution of reaction (4.2), each having a width of about 50 MeV and occurring at roughly 1280 and 1400 MeV (see Fig. 10). Hereafter, these peaks will be denoted A and B respectively, so they can be identified without referring to their position, which is not always exactly the same in the three reactions. These peaks are clearly inconsistent with a maximum likelihood solution using only $K^*(890)$, $N^{*0}(1238)$, and $N^{*+}(1238)$. To fit the $K\pi\pi$ mass distribution, the two peaks must be explicitly included in the fit.

Also shown in Fig. 10 are the $\bar{K}^0 \pi^0 \pi^-$ mass spectra for those events in which the $\bar{K}^0 \pi^0$ and $\bar{K}^0 \pi^-$ masses lie in the $K^*(890)$ region. Both peaks are clearly strengthened by these selections. They appear as four or five standard deviation effects above background and are cleanly separated by several bins. Both enhancements seem to be more pronounced when the neutral K^* is selected, which is consistent with an isospin assignment $I = \frac{1}{2}$ for both peaks. On this basis the upper peak can be tentatively identified with the $K^*(1420)$, and in fact other evidence exists to substantiate this assignment. If allowance is made for a two-body decay of peak B into $\bar{K}^0 \pi^-$ in the reaction $K^- p \rightarrow \bar{K}^0 \pi^- p$, the likelihood solution gives an amount consistent with that predicted using the branching ratios of the two- and three-body decays of the $K^*(1420)$.⁴²

The situation is rather different for reaction (4.3). A histogram of the $K\pi\pi$ mass (Fig. 11) shows only one statistically significant peak, at 1300 MeV. Figure 11 also indicates that, unlike reaction (4.2), the signal is not strengthened by requiring the $\bar{K}^0 \pi^-$ mass to be in the $K^*(890)$ region. The K^* selection results in two narrow peaks, at about 1350 and 1450 MeV, of very marginal statistical significance. In any case, the large peaks A and B found in reaction (4.2) do not occur, at least not for $K^*(890)$ events. Since reaction (4.3) is not promising

from the point of view of studying $K\pi\pi$ enhancements, it will not be considered further.

We now subdivide the $K^{*0}\pi$ events of reaction (4.2) into three regions of $-t$, the four-momentum transfer squared between the incident kaon and the outgoing $K\pi\pi$ system. The events are those for which the $\bar{K}^0\pi^0$ invariant mass is in the K^{*0} region, and the $\pi^0 p$ mass is outside the $\Delta^{++}(1238)$ region (1.14 - 1.31 GeV). The three ranges of $-t$ are respectively $-t > 1.0 \text{ GeV}^2$, $1.0 < -t < 0.5 \text{ GeV}^2$, and $-t < 0.5 \text{ GeV}^2$. The corresponding plots are shown in Fig. 12. Peak A occurs to some degree in all three ranges of t , though the peak position seems to shift to a slightly higher value for the more peripherally produced $K\pi\pi$ systems. These t cuts do not significantly improve the signal for either peak A or B; in fact, the latter does not appear except in the range $0.5 < -t < 1.0 \text{ GeV}^2$, if at all. If one selects those events for which the $K^0\pi^-$ mass is in the K^{*-} region and the $p\pi^-$ mass is outside the $\Delta^0(1238)$ region, the situation is even worse, since the $K\pi\pi$ spectrum is consistent with no structure at all.

Figure 13 is a scatterplot of the $K^{*0}\pi^-$ mass vs. $-t$ (Δ^+ excluded). There is a clustering of events near the forward direction, however this plot shows no evidence for the bumps seen in the mass plots.

2. $K^-\pi^+\pi^-$ Mass Spectrum

Reaction (4.1), which has over twice as many events as reaction (4.2), affords an opportunity to supplement Friedman's work. Figure 14 is a histogram of the $K^-\pi^+\pi^-$ mass showing (a) all events and (b) all events except for those in which the π^+p mass falls in the $\Delta^{++}(1238)$ region (1.14 - 1.31 GeV) or in which the pK mass falls in the $Y^*(1520)$ region (1.50 - 1.55 GeV). These cuts have been made in all subsequent $K\pi\pi$ plots for reaction (4.1), even if not explicitly stated in the text. There are no significant enhancements present. Figure 14(c) shows those events for which the $K^-\pi^+$ mass lies in the $K^*(890)$ region (Δ^{++} , Y^* excluded). There is but scant evidence for any peaks in this plot, though there is a possible excess of events at around 1300 MeV and also at 1400 MeV.

Figure 15 shows the $K^{*0}\pi^-$ events subdivided into the same three regions of $-t$ as with reaction (4.2). These three regions exhibit

strikingly different behavior. In the backward region, $-t > 1.0 \text{ GeV}^2$, there are three peaks: at 1150 MeV (60 MeV wide); at about 1290 MeV (60 MeV wide); and at 1400 MeV (about 40 MeV wide). Each is roughly a three standard deviation excess above the estimated background. The latter two peaks closely correspond in mass and width to the peaks A and B observed in reaction (4.2). The first peak has no counterpart in that reaction, though it does occur at the position of an $I = 3/2 \text{ K}\pi$ bump reported some years ago which has since been discredited.

In the region $0.5 < -t < 1.0 \text{ GeV}^2$ these features are absent, and the mass spectrum appears smooth within statistical uncertainty.

Finally, in the forward direction, $-t < 0.5 \text{ GeV}^2$, there is a large peaking (at least five standard deviations above any reasonable background) in the region 1260 - 1380 MeV. This peak is roughly twice as wide as either peak A or B of reaction (4.2), with some evidence to suggest that in fact it is a sum of two separate peaks at 1280 and 1320 MeV. This large peripheral enhancement will subsequently be investigated in more detail. In addition, there is a significant shoulder at about 1400 MeV, again corresponding to the position of peak B.

Figure 16 is a scatter plot of $K^* \pi^-$ mass vs. $-t$. As with reaction (4.2), events predominantly cluster in the forward direction, though the structure evident in the mass plots is not seen.

These $K^* \pi^-$ events were examined to see whether the ambiguity between K^- and π^- tracks discussed in chapter I could have any effect on the shape of the $K^* \pi^-$ mass plots. Using the criteria for ambiguousness noted in chapter I, about 1% of these events are ambiguous with another $4C \text{ K}$ hypothesis involving a $K^- - \pi^-$ permutation. Furthermore, as nearly as one can tell, these ambiguous events show no tendency to occur preferentially at any particular value of $K^* \pi^-$ mass, and therefore are unlikely to constitute a problem. In fact, there are only five such events in the forward direction, $-t < 0.5 \text{ GeV}^2$.

There are a greater number of events which are ambiguous with a $4C \pi$ hypothesis involving a $K^- - \pi^-$ ambiguity. There are about 5% of the events which are potential π -induced $4C$'s, and even in the forward direction there are still almost 4%. With one interesting exception, these events appear to be randomly distributed in $K^* \pi^-$ mass, the exception being that a number of ambiguous events occur around 1100 - 1160 MeV. This is the region of the small anomalous peak observed for the

backward-going $K^0 \pi^-$ events in Fig. 15(a). If half of these ambiguous events were actually $4C \pi$'s, a quarter of this peak could be due to real π -induced events. However, in the region from about 1200 to 1450 MeV, which is the primary concern of this chapter, it seems unlikely that real π events masquerading as K's could be a serious problem.

D. Further Analysis of the t-dependence of the Enhancements

It would be useful to examine the t-dependence of the $K^* \pi$ mass distributions in more detail. However, lack of statistics precludes simply selecting smaller intervals of t. Hence, in Figs. 17 and 18 we present a series of plots for reactions (4.1) and (4.2) respectively, for several overlapping intervals of t, each shifted by 0.1 GeV^2 . The interval width is 1.0 GeV^2 , except for very large t (where all events greater than a certain value of t are plotted) and very small t (all events less than a certain value of t are plotted). Δ events have been removed. One must bear in mind that these histograms are highly correlated, since most of the events appearing in any given histogram also appear in the adjacent ones. The advantage is that it is more readily apparent where any given peak first appears and where it achieves its maximum signal-to-noise.

Looking first at Fig. 17 (events from reaction (4.1)), a substantial bump centered just below 1300 MeV and about three bins (60 MeV) wide, corresponding to peak A, occurs at fairly large momentum transfers and is most prominent in the range $1.4 - 2.4 \text{ GeV}^2$, Fig. 17(f). It is at this point also that peak B, at about 1400 MeV, first appears. Note also the third bump at about 1.2 GeV, which was mentioned earlier and about which no more will be said. As t decreases, peak B steadily gets larger while peak A gets smaller until in Figs. 17(i) and 17(j) both are present in roughly equal amounts. Both peaks are at most three bins wide and are clearly distinct from one another. At smaller t, A disappears entirely and B persists, reaching its maximum size in the range $0.3 - 1.3 \text{ GeV}^2$ (Fig. 17(q)). At this point, B decreases rapidly and thereafter is present only as a slight shoulder on the high side of the mass plot. Its existence could be doubted at all were it not for Fig. 15 (c), which shows a clear excess of events over background in this region. In Figs. 17(t) through 17(v) it is not clear just what is happening, but finally at the smallest values of t it is obvious that a

very substantial new bump has appeared, six bins wide, centered at 1300 MeV. The last plot of this series, Fig 17(x), is essentially the same as Fig. 15(c). The interpretation of this bump is ambiguous. One possibility is that it is actually two separate bumps, one being peak A which has somehow reappeared at small t and one being an entirely new bump, again about three bins wide, centered at 1340 MeV. This is difficult to prove since the only evidence for a two-bump structure is the depletion of events in the 1280 and 1300 MeV bins. If, say, the 1300 MeV bin had ten more events, which is not statistically improbable, this interpretation would be untenable. The other possibility is that this bump is a single new enhancement, distinct from both peaks A and B and twice as wide as either. It does not seem possible to decide between these two possibilities using our data.

Figure 18 shows plots of the $K^*{}^0\pi^-$ mass for reaction (4.2) using the same overlapping t bins. At large t the behavior is roughly similar to reaction (4.1). Peaks A and B both occur together at about the same t values as in Fig. 17, although the amount of signal above background is relatively greater than in reaction (4.1). Thereafter, peak B persists for a while and then dies away near the forward direction, just as in reaction (4.2). Peak A, however, persists down to much smaller values of t ; see Figs. 18(o) and 18(p). In fact, it is not clear that it ever disappears entirely. Certainly it is present near the forward direction (Figs. 18(w) and 18(x)), although its central value has shifted upward by roughly 30 MeV. Compare, e.g., Figs. 18(n) and 18(x). The height of the peak above background is certainly not any greater than at larger t , as contrasted with the behavior in reaction (4.1), in which the signal is much stronger in the forward direction. There is no evidence for any double-peak structure centered at 1300 MeV, nor does peak A appear significantly wider than it did at smaller t .

In short, the $K^*{}^0\pi^-$ structure in reaction (4.2) can be interpreted in terms of only two peaks over the whole t range. In reaction (4.1), on the other hand, it seems necessary to invoke the presence of a third peak in the forward direction. The detailed behavior of the peaks as a function of t is rather different in each reaction and is not easily interpretable.

E. The CPE Model Applied to Peripheral $K^-\pi^+\pi^-$ Events

1. Fits to the Mass Plot

The Durr-Pilkahn OPE model can be used to calculate the shape and amount of the K^* "background" contribution to the $K^{*0} \pi^-$ invariant mass and momentum transfer distributions of reaction (4.1). However, we do not expect the model to be applicable at large $-t_{K\pi}^*$, since for such events the momentum transfer to the K^* is also predominantly large. Accordingly, we consider only events having $-t_{K\pi}^* < 0.5 \text{ GeV}^2$.

The curve on Fig. 15(c) is the model prediction for the $K\pi$ invariant mass. Again, no arbitrary normalization has been introduced. Agreement is very good outside of the region where the resonant production occurs, which indicates that the model probably gives an accurate description of the background under the enhancement. Note also that there is considerably more $K^*(1420)$ present than might otherwise have been supposed due to the steeply falling background.

This mass distribution was then fitted to a sum of two incoherent Breit-Wigner resonance shapes plus the OPE background:

$$f = B + a_1 BW_1 + a_2 BW_2 \quad (4.4)$$

One of these resonance shapes corresponds to the $K^*(1420)$, and the other corresponds to the large bump at 1300 MeV. The $K^*(1420)$ parameters were held fixed at $M_0 = 1420 \text{ MeV}$ and $\Gamma = 50 \text{ MeV}$. The parameters of the second resonance were simultaneously varied in a fit which minimized the χ^2 between Eq. (4.4) and the histogram of Fig. 15(c). The best fit values of these parameters were found to be

$$\begin{aligned} M_0 &= (1305 \pm 40) \text{ MeV} \\ \Gamma &= (106 \pm 30) \text{ MeV} \end{aligned} \quad (4.5)$$

The distribution calculated using these values gave a satisfactory fit to the histogram, except that the function could not accurately reproduce the steep rise on the lower side of the 1300 MeV bump.

However, if this bump is assumed to consist of two separate resonances and a new fit is tried, (again holding the $K^*(1420)$ parameters fixed), agreement is much better. The fitted values for the central masses and widths turn out to be

$$\begin{aligned} M &= (1280 \pm 20) \text{ MeV} & \Gamma &= (23 \pm 40) \text{ MeV} \\ M &= (1330 \pm 16) \text{ MeV} & \Gamma &= (68 \pm 24) \text{ MeV} \end{aligned} \quad (4.6)$$

These central values correspond rather well with those reported for $K^* \pi$ enhancements in other experiments, and in addition, the lower peak corresponds to peak A seen in reactions (4.1) and (4.2) at large t . However, while this may be an indication of a two-peak structure near 1300 MeV, it should not be considered proof that such structure exists.

2. OPE Momentum Transfer Distribution

The OPE model also predicts what the $K^* \pi$ momentum transfer squared distribution should look like. It is useful here to form two subsamples of events, those having a $K^* \pi$ mass in the range 1.25 - 1.45 GeV (which includes the enhancements), which is denoted Sample 1, and the remaining events, denoted as Sample 2. Figure 19 shows the t distributions of these two samples, with the OPE model predictions superimposed. One expects the model to agree well with the Sample 2 data, since the invariant mass distribution predicted by the model agrees well in this region. In fact, the data are somewhat more highly peaked than is indicated by the model. Moreover, restricting the events to those with low $t_{K\pi}$, where the model is expected to be more accurate, does not improve the agreement. The source of the discrepancy may lie in the failure of the OPE model to predict accurately the angular distributions at the two vertices, as shown in Sec. II. The momentum transfer to the $K\pi$ system can be written in terms of the momentum transfer to the $K\pi$ system as

$$t_{K\pi} = t_{K\pi} + m_{\pi}^2 + M_{K\pi}^2 E_{\pi} - \vec{p}_{\pi} \cdot \vec{p}_K \quad (4.7)$$

where E_{π} is the energy of the lower vertex pion in the $K\pi$ rest frame, $M_{K\pi}$ is the $K^* \pi$ invariant mass, and p_{π} and p_K are the four-vectors of the lower vertex pion and the incident kaon, respectively. Since the $t_{K\pi}$ distribution agrees rather well with the model and since the last two terms of Eq. (4.7) depend on the angular distribution of the lower vertex, it is certainly possible for this angular distribution to account for the discrepancy in the t distribution.

For the Sample 1 events shown in Fig. 19(b), there is a large excess of events over the OPE prediction. Near the forward direction,

$t < 0.4 \text{ GeV}^2$, this excess is something like a factor of two; in the intermediate range $0.4 < t < 0.6 \text{ GeV}^2$ the excess is fifty percent or so; and for $t > 0.6 \text{ GeV}^2$, the excess expressed as a fraction of the OPE prediction is very large. Since the overwhelming number of events in this latter region also have rather large $t_{K\pi}$, greater than 0.6 GeV^2 or so, this is perhaps not surprising. In fact, if one estimates the number of $K^*(890)$ events in this region in the simplest way possible, i.e., by plotting the $K^-\pi^+$ invariant mass, drawing a smooth background by hand, and counting the number of K^* events above this background, it is clear that the OPE model underestimates the number of K^* events. OPE predicts about 144 events, but there are actually about 200 K^* events. However, even 200 events is considerably less than the actual background in Fig. 19 ($0.6 < t < 1.6 \text{ GeV}^2$), for $K\pi\pi$ masses between 1.25 and 1.45 GeV. Therefore the excess of events at large t is only marginally due to $K\pi\pi$ resonant production, and mostly due to K^* and other background. By contrast, the excess of events at small t is almost entirely due to resonant production, since Fig. 15(c) shows that OPE fit: the $K\pi\pi$ spectrum very well outside the region 1.25 - 1.45 GeV.

F. Conclusions

We will now try to summarize the results of this chapter. The $K^-\pi^+\pi^-$ mass spectrum (from reaction (4.1)) has been studied and compared with the $\bar{K}^0\pi^0\pi^-$ spectrum (from reaction (4.2)) in an effort to understand the nature of the fine structure in the Q region. In particular, special attention has been given to the t -dependence of the fine structure in these two reactions. We have found that at large t both reactions exhibit similar behavior, namely, two distinctly separate peaks about 30 MeV wide, occurring at 1280 and 1400 MeV. Although the upper peak dies away near the forward direction in both cases, the remaining behavior of the two reactions is markedly different at small t . The $K^-\pi^+\pi^-$ mass exhibits a prominent peak about 60 MeV wide centered at about 1300 MeV. It is not possible to say whether this peak is a single enhancement appearing only in the forward direction or whether it is a combination of the 1280 peak (which would have to disappear, then reappear at small t) with a new one of about the same width centered at 1320 MeV. By contrast, the $\bar{K}^0\pi^0\pi^-$ mass at small t is characterized by

the presence of the 1280 peak alone, though reduced in size and with its central value slightly shifted.

In addition, we have applied the predictions of the OPE model to the $K^-\pi^+\pi^-$ spectrum at low t . The invariant mass distribution of those events outside the region of the enhancements agrees well with the predicted distribution, and the momentum transfer distribution is in fair agreement. Those events within the enhancement region show a large number of events in excess of the prediction at small t , as expected.

Appendix: Beam Averaging

By using the correlation between the incident beam momentum and the azimuthal angle of the beam as it enters the fiducial volume, the momentum of an individual beam track can be determined to an accuracy better than that indicated by the width of the incident momentum distribution. This section develops the formulas relevant to "beam averaging".

In the space of the two variable p and φ ,

$$x^2 = (\underline{x} - \underline{a})^+ \cdot G^{-1} \cdot (\underline{x} - \underline{a}) \quad (A.1)$$

$$\text{where } x_1 = p_1 \quad x_2 = \varphi$$

$$a_1 = p_0 \quad a_2 = \varphi_0,$$

p_0 and φ_0 being the average values of p and φ .

$$G = \begin{bmatrix} \langle (\delta p)^2 \rangle & \langle \delta p \delta \varphi \rangle \\ \langle \delta p \delta \varphi \rangle & \langle (\delta \varphi)^2 \rangle \end{bmatrix}$$

$$\text{or } G^{-1} = \frac{1}{\det G} \begin{bmatrix} \langle (\delta \varphi)^2 \rangle & -\langle \delta p \delta \varphi \rangle \\ -\langle \delta p \delta \varphi \rangle & \langle (\delta p)^2 \rangle \end{bmatrix}$$

$$\text{where } \delta p = p - p_0$$

$$\delta \varphi = \varphi - \varphi_0$$

For a given φ , x^2 is minimized by setting $\frac{\partial x^2}{\partial (\delta p)} = 0$, giving

$$\delta p = \frac{\langle \delta p \delta \varphi \rangle}{\langle (\delta \varphi)^2 \rangle} \delta \varphi \quad (A.2)$$

The error to be assigned to Δp is found by noting that the value of x^2 one standard deviation from the minimum x^2 is

$$x^2 = x_{\min}^2 + 1 \quad (A.3)$$

where $x_{\min}^2$ is found by substituting Eq. (A.2) into Eq. (A.1):

$$x_{\min}^2 = \frac{(\delta\phi)^2}{\langle(\delta\phi)^2\rangle}$$

Now, solving Eq. (A.3) for Δp ,

$$\delta p = \frac{\delta\phi\langle\delta p\delta\phi\rangle \pm \sqrt{(\delta\phi)^2 \langle\delta p\delta\phi\rangle^2 - \langle(\delta\phi)^2\rangle \langle(\delta p)^2\rangle - \det G x^2}}{\langle(\delta\phi)^2\rangle} \quad (\text{A.4})$$

Note that the first term is just the most probable value of Δp as derived above, the error being given by the second term. Simplifying, the error in Δp can be written as

$$\Delta(\delta p) = \left[\langle(\delta p)^2\rangle - \frac{\langle\delta p\delta\phi\rangle^2}{\langle(\delta\phi)^2\rangle} \right]^{\frac{1}{2}} \quad (\text{A.5})$$

Thus, p is given by Eq. (A.2) and the error in p is given by Eq. (A.5). In the absence of any correlation, $p = p_0$ and the error in p is simply the rms spread about this value. Note that the correlation must always act so as to make the error smaller.

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Table I. Reaction hypotheses attempted for the 4-prong events. Corresponding missing mass fits were also tried. The numbers quoted are for those events having as their best fit ($CL > 10^{-5}$) the given hypothesis, except that events fitting both 4C and 1C hypotheses are assumed to be 4C. These numbers are only approximate, since ambiguities between various 1C hypotheses were not resolved.

Reaction	Number of events
$K^- p \rightarrow K^- p \pi^+ \pi^-$	12,803
$\rightarrow K^- p \pi^+ \pi^- \pi^0$	5,616
$\rightarrow \bar{K}^0 p \pi^+ \pi^- \pi^-$	1,297
$\rightarrow K^- n \pi^+ \pi^+ \pi^-$	2,135
$\rightarrow \Lambda \pi^+ \pi^+ \pi^- \pi^-$	1,027
$\rightarrow \Sigma^0 \pi^+ \pi^+ \pi^- \pi^-$	141
$\pi^- p \rightarrow p \pi^+ \pi^- \pi^-$	3,320
$\rightarrow p \pi^+ \pi^0 \pi^- \pi^-$	2,167
$\rightarrow n \pi^+ \pi^+ \pi^- \pi^-$	982

Table II. Cross section and number of events at each momentum.

Sample	Cross section for $K^- p \rightarrow K^- p \pi^+$	Number of events
2.05 GeV/c	$630 \pm 30 \mu\text{b}^a$	6620 ^c
2.63 GeV/c	$960 \pm 100 \mu\text{b}^b$	7552

^a Taken from Ref. 2

^b Taken from Ref. 3

^c Includes 4518 UCLA events.

Table III. Resonant fractions determined from a maximum likelihood fit described in the text.

Sample	Nonresonant	$Y^*(1520)\pi^-p$	$\Delta^{++}(1238)K^-\pi^-$	$\Delta^0(1238)K^{*0}(890)$	$K^{*0}(890)\pi^-p$ ^a
2.05 GeV/c	14 ± 2 %	13 ± 2 %	46 ± 2 %	18 ± 2 %	9 ± 1 %
2.63 GeV/c	30 ± 2 %	7 ± 1 %	27 ± 2 %	5 ± 1 %	31 ± 2 %

Parameters used in resonant fits	$Y^*(1520)$	$\Delta^{++}(1238)$	$\Delta^0(1238)$	$K^{*0}(890)$
M_0 (GeV)	1.518	1.219	1.227	0.894
(GeV)	0.017	0.135	0.099	0.053

^a Does not include $\Delta^0 K^{*0}$

Table IV. Resonant fractions after making the following cuts:

$M_{K^- \pi^-} < 0.84 \text{ GeV}$, $-t_{\pi^+ p} < 0.3 \text{ GeV}^2$, and $1.14 < M_{\pi^+ p} < 1.31 \text{ GeV}$.

Resonant masses and widths are those of Table III.

Sample	Nonresonant	$\Upsilon^*(1520)\pi^- p$	$\Delta^{++}(1238)K^- \pi^-$	$\Delta^0(1238)K^{*0}(890)$	$K^{*0}(890)\pi^- p$ ^a
2.05 GeV/c	0	$4 \pm 1 \%$	$71 \pm 3 \%$	$14 \pm 2 \%$	$11 \pm 2 \%$
2.63 GeV/c	0	0	$57 \pm 2 \%$	$14 \pm 2 \%$	$29 \pm 2 \%$

^a Does not include $\Delta^0 K^{*0}$

Table V. Results of fits of the experimental points shown in Fig. 7 to the assumed form $\sigma(t) = a - bt$.

Data set	Fit quantities	$K^-\pi^-$ mass range (GeV)			
		0.64-0.74		0.74-0.84	
		2.05 GeV/c	2.63 GeV/c	2.05 GeV/c	2.63 GeV/c
Background in	Confidence level (%)	35	66	26	17
	a(mb)	-0.1 ± 1.1	1.7 ± 1.0	5.5 ± 1.9	1.7 ± 1.4
	b(mb/GeV ²)	43.0 ± 8.7	27.8 ± 8.8	14.5 ± 9.8	38.6 ± 9.2
	σ_{extrap} (mb)	-1.0 ± 1.2	1.1 ± 1.1	5.2 ± 2.0	0.9 ± 1.4
Background out	Confidence level (%)	48	65	33	46
	a(mb)	-1.5 ± 1.1	1.2 ± 1.1	4.0 ± 1.9	1.3 ± 1.6
	b(mb/GeV ²)	22.6 ± 8.7	-2.5 ± 10.1	5.4 ± 9.9	12.9 ± 11.1
	σ_{extrap} (mb)	-1.9 ± 1.2	1.2 ± 1.2	3.9 ± 2.0	1.1 ± 1.7
Avg. σ_{extrap} (mb)		-0.3 ± 0.9		2.3 ± 1.3	

FIGURE CAPTIONS

1. Two-particle invariant mass distributions for all events of the 2.05 GeV/c sample. Curves are the result of a maximum likelihood fit to an incoherent sum of resonant states as described in the text.
2. Two-particle invariant mass distributions using the 2.63 GeV/c sample. The curves are the result of a maximum likelihood fit to an incoherent sum of resonance states as described in the text.
3. $K^-\pi^+\pi^-$ invariant mass distribution for the 2.63 sample only. The curve is the result of the maximum likelihood fit described in the text.
4. One-pion exchange process describing the reactions $K^-\bar{p} \rightarrow (K^-\pi^-)(\pi^+p)$ and $K^-\bar{p} \rightarrow (K^-\pi^+)(\pi^-p)$.
5. (a) $(d\sigma/dM)_{K^-\pi^-}$ for those events having $|t_{\pi^-p}| < 0.5 \text{ GeV}^2$.
 (b) $(d\sigma/dM)_{\pi^-p}$ for those events satisfying $|t_{\pi^-p}| < 0.5 \text{ GeV}^2$ and $0.84 < M_{K^-\pi^-} < 0.98 \text{ GeV}$.
 (c) $(d\sigma/dt)_{\pi^-p}$ for those events satisfying $0.84 < M_{K^-\pi^-} < 0.98 \text{ GeV}$.
 $\Delta^{++}(1238)$ and $Y^*(1520)$ events have been excluded in all three plots. The Δ^{++} cut is defined by $1.14 < M(\pi^+p) < 1.31 \text{ GeV}$, and the Y^* cut is defined by $1.50 < M(K^-\bar{p}) < 1.55 \text{ GeV}$.
6. (a) $(d\sigma/dM)_{K^-\pi^-}$ for those events satisfying $1.14 < M_{\pi^+p} < 1.31 \text{ GeV}$ and $|t| < 0.3 \text{ GeV}^2$.
 (b) $(d\sigma/dM)_{\pi^+p}$ for those events satisfying $M_{K^-\pi^-} < 0.84 \text{ GeV}$ and $|t| < 0.3 \text{ GeV}^2$.
 (c) $d\sigma/dt$ for those events satisfying $M_{K^-\pi^-} < 0.84 \text{ GeV}$ and $1.14 < M_{\pi^+p} < 1.31 \text{ GeV}$.
 (d), (e), (f) $(d\sigma/dM)_{K^-\pi^-}$, $(d\sigma/dM)_{\pi^+p}$, $(d\sigma/dt)_{\pi^+p}$ for those events satisfying $M_{K^-\pi^-} < 0.84 \text{ GeV}$, $1.14 < M_{\pi^+p} < 1.31 \text{ GeV}$, and $|t| < 0.3 \text{ GeV}^2$. The curves are the predictions of the model described.
7. Plots of $\sigma(t,m)$ vs. t . Square data points are for no background subtraction, and triangular points are with background subtracted as described in the text.
8. Polar angle distributions in the Jackson frame (a) between the outgoing and incident kaons in the $K^-\pi^-$ rest frame, (b) between the outgoing and incident nucleons in the π^+p rest frame for those

- events used in the cross section extrapolation. The curves are the predictions of the model described in the text.
9. Summary of measurements of the $I = 3/2$ $K\pi$ cross section. Curve is a Veneziano model of Lovelace.
 10. Plots of $\bar{K}^0\pi^+\pi^-$ mass for events of reaction (4.2).
 - (a) all events.
 - (b) events having $0.84 < M(\bar{K}^0\pi^+) < 0.98$ GeV
 - (c) events having $0.84 < M(\bar{K}^0\pi^0) < 0.98$ GeV
 11. Plots of $\bar{K}^0\pi^+\pi^-$ mass for events of reaction (4.3).
 - (a) all events.
 - (b) events having $0.84 < M(\bar{K}^0\pi^-) < 0.98$ GeV
 12. Plots of $K^{*0}\pi^-$ mass, with Δ^+ excluded, for events of reaction (4.2).
 - (a) $-t(K^*\pi) > 1.0$ GeV²
 - (b) $1.0 < -t(K^*\pi) < 0.5$ GeV²
 - (c) $-t(K^*\pi) < 0.5$ GeV²
 13. Scatterplot of $K^{*0}\pi^-$ mass vs. $-t(K^*\pi)$, Δ^+ excluded, for events of reaction (4.2).
 14. Plots of $\bar{K}^0\pi^+\pi^-$ mass for events of reaction (4.1).
 - (a) all events.
 - (b) $\Delta^{++}(1238)$, $Y^*(1520)$ excluded. Cuts are defined in the text.
 - (c) $\Delta^{++}(1238)$, $Y^*(1520)$ excluded; $0.84 < M(K^-\pi^+) < 0.98$ GeV.
 15. Plots of $K^{*0}\pi^-$ mass (Δ^{++} , Y^* excluded) for events of reaction (4.1).
 - (a) $-t(K^*\pi) > 1.0$ GeV²
 - (b) $1.0 < -t(K^*\pi) < 0.5$ GeV²
 - (c) $-t(K^*\pi) < 0.5$ GeV²
 16. Scatterplot of $K^{*0}\pi^-$ mass vs. $-t(K^*\pi)$, Δ^{++} and Y^* excluded, for events of reaction (4.1).
 17. Plots of $K^{*0}\pi^-$ mass, Δ^{++} and Y^* excluded, for events of reaction (4.1) for various overlapping intervals of $t(K^*\pi)$.
 18. Plots of $K^{*0}\pi^-$ mass, Δ^+ excluded, for events of reaction (4.2) for various overlapping intervals of $t(K^*\pi)$.
 19. Distributions in $-t(K^{*0}\pi^-)$ for events of reaction (4.1), Δ^{++} and Y^* excluded.
 - (a) $M(K^*\pi)$ outside the region 1.25 - 1.45 GeV.
 - (b) $1.25 < M(K^*\pi) < 1.45$ GeV.

Fig. 1

Fig. 2

XBL 734-373

XBL 734-355

Fig. 3

XBL 734-359

Fig. 4

XBL 717-1210

Fig. 5

Fig. 6

XBL 712-2900

Fig. 7

XBL 734-360

Fig. 8

XBL 712-2901

Fig. 9

XBL 734-361

Fig. 10

EVENTS/.02

$M(K^0 \pi^+ \pi^-)$ GeV

XBL 734-356

Fig. 11

Fig. 12

XBL 734-357

Fig. 13

XBL 734-362

Fig. 14

XBL 734-375

Fig. 15

XBL 734-358

Fig. 16

$M(K^-\pi^+\pi^-) \text{ GeV}$

XBL 734-368

Fig. 17

Fig. 17 (cont.)

XBL 734-366

Fig. 17 (cont.)

XBL 734-365

Fig. 17 (cont.)

Fig. 18

Fig. 18 (cont.)

Fig. 18 (cont.)

$M(K^0 \pi^0 \pi^-) \text{ GeV}$

XBL 734-369

Fig. 18 (cont.)

$-t_{K\pi\pi, K} \text{ GeV}^2$

XBL 734-363

Fig. 19