MASTER

A Quark Model for Leptons*

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Abstract

The possibility is explored that, in analogy with the hadrons, leptons are made up of leptonic quarks. A model is constructed that is consistent with all existing weak interaction data, including a possible mechanism for e , μ and ν mass differences. The close parallel between the leptonic and the hadronic quarks leads to speculations about a modified model that may serve to unify the hadrons and the leptons.

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Since the introduction of the quark model for hadrons by Gell-Mann¹ and Zweig,² the possibility that leptonic quarks also exist has been considered.³ The continuing success of the hadronic quark model⁴ suggests it worthwhile to examine how the leptons can be made of leptonic quarks and the consequences of this point of view.

In this note, we begin by assigning to the leptonic quarks electric charges and spin as suggested by the hadronic quarks. We then consider what the origin of the strong binding force, responsible for the formation of leptons, between these quarks might be. The magnetic charges suggested by Schwinger in his dyon model⁵ are very convenient for this purpose. Next we identify the leptons in terms of their leptonic quark constitutions. Intermediate bosons can also be identified in a natural way. The question of the mass differences between the leptons is considered. It turns out that the observed mass differences are consistent with this model if we assume ^{that} the observed leptons have zero magnetic charge and belong to an octet of spin $\frac{1}{2}$. The mass breaking term transforms as the electric charge in analogy with the usual SU(3) model for hadrons⁶ where the mass breaking term transforms as the hypercharge.

We next turn to the selection rules for weak interactions.⁷ Conservation of lepton numbers can be satisfied if we assign lepton numbers not only to leptonic quarks but also to ^{the} hadronic quarks. Other selection rules of weak interactions can also be satisfied by assuming that those are mediated through the intermediate bosons identified in this scheme. Finally, the close parallel between the leptonic quarks and the hadronic quarks leads us to speculate on a modified quark model which may unify the leptons and the hadrons.

The quantum numbers of the leptonic quarks are suggested by the spin and the electric charges of the hadronic quarks, namely, spin $\frac{1}{2}$ and fractional electric charges $\pm \frac{2}{3}, \pm \frac{1}{3}$ in units of electronic charge e_0 . Since all leptons have spin $\frac{1}{2}$, the simplest possibility is that they are made of three leptonic quarks. One way to obtain this is to assign magnetic charges to the leptonic quarks as proposed by Schwinger⁵ for the hadronic quarks. We have $p_L(\frac{2}{3}e_0, \frac{1}{3}g_0)$, $n_L(-\frac{1}{3}e_0, \frac{1}{3}g_0)$ and $\lambda_L(-\frac{1}{3}e_0, -\frac{2}{3}g_0)$, where e_0 and g_0 are the units of the electric and magnetic charges. The leptonic antiquarks are $\bar{p}_L(-\frac{2}{3}e_0, -\frac{1}{3}g_0)$, $\bar{n}_L(\frac{1}{3}e_0, -\frac{1}{3}g_0)$ and $\bar{\lambda}_L(\frac{1}{3}e_0, \frac{2}{3}g_0)$. The observed leptons they form should have zero magnetic charge.⁵ We have only two possible combinations that will have electric charge $-e_0$ and zero magnetic charge, namely $e^-(n_L n_L \lambda_L)$ and $\mu^-(\bar{p}_L \bar{p}_L \bar{\lambda}_L)$, with the leptonic quark contents as exhibited in the parentheses.⁸ We also have $e^+(\bar{n}_L \bar{n}_L \bar{\lambda}_L)$ and $\mu^+(p_L p_L \lambda_L)$. The electrically neutral states are $(p_L n_L \lambda_L)$ and $(\bar{p}_L \bar{n}_L \bar{\lambda}_L)$. To identify them with neutrinos we should also specify their helicities. We have $\nu_e(p_L n_L \lambda_L; -1)$, $\nu_\mu(\bar{p}_L \bar{n}_L \bar{\lambda}_L; -1)$, $\bar{\nu}_e(\bar{p}_L \bar{n}_L \bar{\lambda}_L; +1)$ and $\bar{\nu}_\mu(p_L n_L \lambda_L; +1)$, where the ± 1 in the parenthesis refer to the helicities. The intermediate bosons can be formed from one leptonic quark with one leptonic antiquark. We have, using the notation of Reference 7:

$W^+(p_L \bar{n}_L)$, $W_a^0(\frac{1}{\sqrt{2}}(p_L \bar{p}_L - n_L \bar{n}_L))$, $W^-(n_L \bar{p}_L)$, $W_b^0(\frac{1}{\sqrt{2}}(p_L \bar{p}_L + n_L \bar{n}_L))$ and $W_5(\lambda_L \bar{\lambda}_L)$. Thus we have one more intermediate boson (W_5) than the necessary minimum.⁹

To discuss the mass differences of the leptons in terms of their different quark contents, we have to make some assumptions about the strong binding force which we take to be the force between magnetic charges. It is quite clear that in the present scheme the mass differences arise from the different electric charges which the quarks carry (we ignore the mass

differences between n_L and p_L , which we assume is of the order of $m_e - m_{\nu_e}$. This suggests the mass breaking mechanism in the usual SU(3) scheme for baryons,⁶ where the mass breaking term transforms like the hypercharge. We therefore assume (λ_L, n_L) form an isospin (I_M) doublet, p_L a singlet and all three together form a basic triplet in the magnetic charge space.¹⁰ The electric charge plays the role of hypercharge in mass breaking. The SU(3) decomposition gives

$$\{3\} \times \{3\} \times \{3\} = \{1\} + \{8\} + \{8\} + \{10\}.$$

Let us assume the observed leptons belong to the same octet.¹¹ We have: e^- belonging to an $I_M = \frac{1}{2}$ doublet, μ^+ belonging to another $I_M = \frac{1}{2}$ doublet, and two electrically neutral states ν_8^T belonging to an $I_M = 1$ triplet and ν_8^S an $I_M = 0$ singlet, all having magnetic charge zero. Other states of the same octet have nonzero magnetic charges. In other words, we assume that a superstrong magnetic charge interaction¹² breaks up the "octet leptons" into very heavy (unobserved) leptons with magnetic charge g_0 consisting of $(p_L p_L n_L)$, $(n_L n_L p_L)$, $(p_L \lambda_L \lambda_L)$, and $(n_L \lambda_L \lambda_L)$ and the light observed leptons $(e, \bar{\mu}, \nu_e, \bar{\nu}_\mu)$ with zero magnetic charge. Moreover an interaction transforming in SU(3) like the electric charge (in fact it could be the [usual] electromagnetic interaction [without magnetic charges] itself) causes the observed mass splitting among the physical leptons.¹³ When considering the mass breaking term, we should include the possibility of mixing¹⁴ between ν_8^S and the SU(3) singlet ν_1 . Let us denote the mass eigenstates after mixing by $\nu_8^{S'}$ and ν_1' . The observed ν_e (or $\bar{\nu}_\mu$, having the opposite helicity) is a linear combination of $\nu_8^{S'}$ and ν_8^T . Hence we demand the masses of these latter states be equal to the observed neutrino masses which are taken to be zero.

The mass differences of the observed leptons are given by the Gell-

Mann-Okubo formula¹⁵ with charge E replacing hypercharge Y and I_M replacing I_E (electric isospin):

$$M_8(E, I_M) = M_8 \left[1 + aE + b [I_M(I_M + 1) - \frac{1}{2}E^2] \right]$$

We have

$$m_e = M_8 \left(1 - a + \frac{b}{2} \right)$$

$$m_\mu = M_8 \left(1 + a + \frac{b}{2} \right)$$

$$m_{\nu_8^s} = M_8$$

$$m_{\nu_8^T} = M_8 (1 + 2b)$$

and

$$m_{\nu_1} = M_1$$

Since we ignored the mass differences between n_L and p_L , it is consistent to put $m_e = 0$. In our model of course $m_{\nu_e} = m_{\nu_\mu} = m_\nu$, and we set $m_\nu = 0$.

Thus we have

$$M_8 = \frac{2}{3} m_\mu$$

$$a = \frac{3}{4}$$

$$b = -\frac{1}{2}$$

and $m_{\nu_8^s} = \frac{2}{3} m_\mu \neq 0$. The mixing between ν_1 and ν_8^s is necessary to bring the mass of ν_8^s equal to the mass of ν_8^T . The parameter M_8 is related to the quark masses and the "binding energy" E_0 (taking $m_{n_L} = m_{p_L} = m$):

$$M_8 = m_{\lambda_L} + 2m - E_0.$$

Let us now consider the selection rules of weak interactions in this scheme. The results of lepton number (L_e and L_μ) conservation can be obtained, if we (a) assign L_e and L_μ to both the leptonic quarks and the hadronic quarks p , n and λ , and (b) assume that the weak interaction coupling is of the V-A type.¹⁶ The following assignments of lepton numbers are adequate: p_L, p have $L_\mu = -\frac{1}{2}$, $L_e = 0$; $n_L, \lambda_L, n, \lambda$ have $L_e = \frac{1}{2}$, $L_\mu = 0$; $\bar{p}_L, \bar{p}$ have $L_\mu = \frac{1}{2}$, $L_e = 0$; and $\bar{n}_L, \bar{\lambda}_L, \bar{n}, \bar{\lambda}$ have $L_e = -\frac{1}{2}$, $L_\mu = 0$. Both points mentioned above are crucial in enabling unique association between e, μ and their neutrinos. For example, in β decay the neutrino accompanying e^- has to be $\bar{\nu}_e$: it cannot be ν_e or $\bar{\nu}_\mu$ (point (a) above) and it cannot be ν_μ (point (b) above). We have the leptonic process

$$n_L \rightarrow p_L + W^-(n_L \bar{p}_L) \rightarrow p_L + e^-(n_L n_L \lambda_L) + \bar{\nu}_e(\bar{p}_L \bar{n}_L \bar{\lambda}_L; +1) \quad (1)$$

As an example of forbidden processes, we have

$$\nu_\mu(\bar{p}_L \bar{n}_L \bar{\lambda}_L; -1) + N(pnn) \not\rightarrow e^-(n_L n_L \lambda_L) + P(ppn). \quad (2)$$

Other selection rules of weak interactions can also be satisfied if we assume the weak interactions are mediated by the intermediate bosons⁹ identified in this scheme. The fact that only charged W can have leptonic interaction can be understood on the quark level as a consequence of magnetic charge conservation, namely, in contrast to (1) above, λ_L cannot go into n_L (or p_L for that matter) plus a neutral intermediate boson which has zero magnetic charge.

We give in the following more examples of allowed weak interaction processes. For leptonic decay, we have, for example

$$\pi^+(p\bar{n}) \rightarrow W^+(p_L \bar{n}_L) \rightarrow \mu^+(p_L p_L \lambda_L) + \nu_\mu(\bar{p}_L \bar{n}_L \bar{\lambda}_L; -1) \quad (3)$$

For strangeness conserving semi-leptonic decay, we have

$$\pi^+(p\bar{n}) \rightarrow W^+(p_L \bar{n}_L) \rightarrow \pi^0(\frac{1}{\sqrt{2}}(p\bar{p} - n\bar{n})) + e^+(\bar{n}_L \bar{n}_L \bar{\lambda}_L) + \nu_e(p_L n_L \lambda_L; -1) \quad (4)$$

and for strangeness changing non-leptonic decay, we have

$$\lambda \rightarrow p + W^-(n_L \bar{p}_L) \rightarrow p + \pi^-(n\bar{p}). \quad (5)$$

All other types of weak interaction reactions can be formulated in similar ways. We can conclude that all existing weak interaction data are consistent with this scheme. The fact that process (5) and the similar process involving λ and n are allowed implies λ , p and n have ^{the} same magnetic charge, a result essential for the discussions in the next paragraph.

The fact that it is necessary to assign lepton numbers to the hadronic quarks in this scheme, and the close parallel between the leptonic quarks and the hadronic quarks, suggest the intriguing possibility that they may be closely related. The recent electron nucleon scattering experiments¹⁷ are consistent with the parton model. One possible interpretation is that hadronic quarks are point like objects. We want to speculate on another possible interpretation that hadronic quarks themselves are composites. In particular, each hadronic quark is made of one leptonic quark carrying all its electric charge and spin, and one spin zero boson¹⁸ to be denoted M for "mediator." First, M has to be magnetically charged so that it will be attracted to the leptonic quarks strongly. But p_L , n_L have magnetic charges opposite to that of λ_L . If we assume M has magnetic charge g , its antiparticle $\bar{M}$ will have $-g$. Therefore p_L , n_L will be attracted to $\bar{M}$ while λ_L will be attracted to M . Thus M cannot carry baryon number. Let us identify p as $p_L \bar{M}$, n as $n_L \bar{M}$ and λ as $\lambda_L M$. Now as discussed above p , n , λ should have same magnetic charges which could be the reason why they interact similarly except for the possible difference in mass (arising from mass differences between λ_L and p_L , n_L). Hence we demand $g = \frac{1}{2}g_0$, and p , n , λ all have magnetic charge $-\frac{1}{6}g_0$. In analogy with atomic physics, all three hadronic quarks may be treated as "orbital electrons."

The corresponding "nucleus," to be called the core particle and denoted C, will have magnetic charge $\frac{1}{2}g_0$ and baryon number 1. We list the quantum number assignments of the five "fundamentons" C, M, p_L , n_L , λ_L and all other low lying states in Table I. Conservation of strangeness can be satisfied in either one of two ways (i) assign $S = -1(+1)$ to $\lambda_L(\bar{\lambda}_L)$ and zero to all other particles, or (ii) assign a quantum number, say $S' = +1(-1)$ to $M(\bar{M})$ and zero to all other particles. Neither S nor S' are, of course, conserved in weak interactions. The second of the two ways seems more natural than the first. Notice that the octet of baryons arises if we assume each hadronic quark moves approximately in an independent orbit¹⁹ and use the Pauli exclusion principle, which for instance puts the third p of Δ^{++} (Cp $\bar{p}$ p) in a higher orbit. One might argue the exclusion principle is also responsible for the fact that the observed leptons have spin $\frac{1}{2}$.²⁰ The fact that the leptons do not have strong interaction in this scheme can be interpreted²¹ as being a result of their forming an "inert core." Incidentally, the fact that the intermediate bosons have spin 1 could be related to the fact that in parallel spin states of the leptonic quark and antiquark system annihilation (through lowest order electromagnetic interaction) is prevented just as in the case of positronium. The possible existence of "fundamentons" is, of course, not inconsistent with the bootstrap idea, as the fundamentons may bootstrap among themselves.

We realize that many more important questions must be answered before this scheme can be taken seriously. However, we hope that the discussions in this note indicate that it may be worthwhile to pursue this point of view further.

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References and Footnotes

1. M. Gell-Mann, Phys. Letters 8, 214 (1964).
2. G. Zweig, CERN Preprint TH401, 412 (1964) (unpublished).
3. T. Massam and A. Zichichi, Nuovo Cimento 43, 227 (1966).
4. See, for example, J. J. J. Kokkedee, The Quark Model (W. A. Benjamin, Inc., New York, 1969).
5. J. Schwinger, Science 165, 757 (1969).
6. See, for example, M. Gell-Mann and Y. Ne'eman, the Eightfold Way (W. A. Benjamin, Inc., New York, 1964).
7. T. D. Lee and C. S. Wu, Annual Review of Nuclear Science 15, 381 (1965).
8. The assignment of e^- and μ^- as to their leptonic quark contents is not completely unambiguous as long as we are not able to break up the leptons experimentally. One can switch the assignments of e^- and μ^- if one switches at the same time the assignments of e^+ , μ^+ ; ν_e , ν_μ ; and $\bar{\nu}_e$, $\bar{\nu}_\mu$. Our assignment seems to be the natural one if we take e^- to be a "particle" rather than an "antiparticle."
9. Assuming the weak interactions are mediated by the intermediate bosons, then, in order to generate the V-A type of weak interaction coupling, there must exist, at least, two charged spin 1 bosons: W^+ and W^- . In order to satisfy the $|\Delta I| = \frac{1}{2}$ and $|\Delta S| \neq 2$ rules for all observed strangeness-nonconserving weak interactions, there must exist besides the W^+ and W^- at least two different neutral W particles. Furthermore, only the charged W can have leptonic interactions. We refer to Reference 7 (Chapter 3) for systematic analysis of the possible existence of intermediate bosons.
10. In the usual SU(3) analysis, I_M is referred to as the U-spin and the magnetic charge space the U-spin space.
11. In the SU(6) classification scheme, we also have the multiplet

$[[8], \frac{1}{2}]$ which belong to the $[20]$ representation from $[6] \times [6] \times [6]$.

12. Using Dirac's argument (P. A. M. Dirac, Proc. Roy. Soc. London Ser. A133, 60 (1931); Phys. Rev. 74, 817 (1948)) that the existence of magnetic charges leads to the quantization of electric charges, Schwinger (Ref. 5) showed that $\frac{g_0}{4\pi} = 36 \times (137)$. Thus the magnetic charges are superstrong.
13. The author would like to thank Prof. M. Suzuki for clarifying comments on this point.
14. For simplicity we do not consider the more complicated case, where we mix ψ_8^S , ψ_1 and ψ_8^T which will be allowed if the mass breaking term does not conserve I_M .
15. S. Okubo, Progr. Theoret. Phys. 27, 949 (1962).
16. Since μ^+ (rather than μ^-) and e^- are "particles" in this scheme. We should have for electron V-A type coupling and for muon V+A type coupling. We refer to E. J. Konopinski, "The Theory of Beta Radioactivity" (Oxford, 1966) p. 351 for more detailed analysis.
17. The experimental results are clearly illustrated in the article by H. W. Kendall and W. Panofsky, Scientific American 224, 60 (1971).
18. The idea of attaching a boson to leptons to create baryons has been considered by Taketani and his collaborators in connection with the Sakata model. See, for example, M. Taketani, Lecture at the "Symposium on the Weak Interaction" held at the Research Institute for Fundamental Physics (Kyoto) in December 1959. Z. Maki, M. Nakagawa, Y. Ohnuki, and S. Sakata, Progr. Theoret. Phys. 23, 1174 (1960). The author wishes to thank Professors M. Suzuki and W. K. Tung for informing him of these references.
19. If this approximation is valid, a "shell model" type calculation may

become feasible for obtaining approximate baryon mass spectra.

20. The question naturally arises where one should look for the spin³₂ (and zero magnetic charge) leptons if these exist. A possible way to estimate their masses $M_{3/2}$ may be considered. If the "independent orbit" assumption referred to in Footnote 19 is valid, and we assume the usual formula for atomic energy levels to be approximately applicable here, then from the Δ and nucleon mass difference, we estimate $M_{3/2} \gtrsim 2 \text{ GeV}$ (ignoring reduced mass corrections).
21. The possibility of this interpretation is suggested to the author by Professor E. J. Konopinski.

Table I

List of particles in this scheme. The electric charges are in units of e_0 (the electronic charge) the magnetic charges are in units of $g = \frac{1}{2}g_0$ (see text). H denotes the helicities for neutrinos,
When not differentiating among the three hadronic quarks, we denote them by the same symbol q.

	PARTICLES							ANTI-PARTICLES						
	B	E	G	J	I	L_μ	L_e	B	E	G	J	I	L_μ	L_e
FUNDAMENTONS	C	1	0	1	0	0	0	$\bar{C}$	-1	0	-1	0	0	0
	M	0	0	1	0	0	0	$\bar{M}$	0	0	-1	0	0	0
	p_L	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\bar{p}_L$	0	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0
	n_L	0	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\bar{n}_L$	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
	λ_L	0	$-\frac{1}{3}$	$-\frac{4}{3}$	$\frac{1}{2}$	0	0	$\bar{\lambda}_L$	0	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{1}{2}$	0	$-\frac{1}{2}$

(continued)