MASTER

**Identification and Handling of Some Simple
Covariance Effects in Material Balance Accounting**

R. J. Parsick

**Technical Support Organization
Brookhaven National Laboratory
Upton, New York 11973**

Under Contract with the U.S. Atomic Energy Commission

July, 1971

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

**Prepared for the 12th Annual Meeting of the
Institute of Nuclear Materials Management, West
Palm Beach, Florida, June 29-July 1, 1971**

Identification and Handling of Some Simple Covariance Effects in Material Balance Accounting

R. J. Parsick

Introduction

The material balance equation for Special Nuclear Material (SNM) yields a value for the Material Unaccounted For (MUF). The limit of error of MUF is usually calculated under the assumption of statistical independence for the components of the material balance. In this paper some of the simplest sources of covariance between the terms of the MUF equation are identified and treated. The correlation between all components is a complex subject, and the complete resolution of covariance effects is beyond the scope of this paper.

Summary

The covariance effects illustrated in this paper are based on adequate physical protection and surveillance to assure the integrity of most material containers so that only minimal re-measurement of their SNM content is required to close the material balance. With assurance based on physical protection and surveillance, the validity of previously measured values could be tested by random sampling and/or by a "shadow" measurement using a quick but relatively inaccurate measurement method, e.g., a nondestructive technique. Under these circumstances, covariance effects resulting from lack of independence for the terms of the MUF equation must be considered. This paper illustrates a simple source of covariance and presents an illustrative calculation of material balance uncertainty (LEMUF) for a hypothetical mixed oxide facility. The results are summarized in Figure 9, showing that LEMUF could be significantly reduced by these covariance considerations, depending on facility size, inventory policy, and inventory frequency. The source of covariance considered in this paper is especially important for short accounting periods, but other sources of covariance could be significant even for long periods.

A Simple Source of Covariance

Figure 1 illustrates a simple source of covariance between the terms of the MUF equation. The arrowed lines connect terms in which the same material item can be represented for the material balance period in both terms of the MUF equation. For example, the familiar concept of static inventory is represented by the line joining beginning and ending inventory, i.e., some items within the facility at the start of the balance period may still be within the facility in the same form at the end of the balance period. These items are included in the sum of SNM for all items in beginning inventory and for all items in ending inventory. Similarly, some of the items in beginning inventory may be represented in the same form as a shipment or as a measured discard during the inventory interval. In a similar way some receipt material, if not yet used as feed for the process, may also be represented in ending inventory for the balance period.

Note that each of the terms joined are terms of opposite sign. There is no possibility for the same material item to occur in two terms of the same sign. For example, it is impossible for material to have been already in the beginning inventory and also to be received as a receipt during the inventory interval. Similarly, it is impossible for material to have been shipped from the facility and still be in ending inventory at the close of the balance period. Therefore, the terms for which there can be a commonality of material are only terms of opposite sign. This plays an important part in determining how this form of covariance should be treated in the propagation of error equations.

Measured SNM Content

The SNM content of the material which is common to two terms of opposite sign in the MUF equation may be represented by a number of different combinations of measurements. Fig. 2 indicates a few of the possibilities for a single item.

Some items common to two terms may be represented by exactly the same weight measurement and the same concentration measurement in both terms. This is not uncommon when the material represents a hard-to-measure component such as UF_6 cylinders in a low enriched UO_2 fabrication facility.

Some items are represented at the same concentration measurement because of the difficulty of resampling and analyzing even though the material has been reweighed. For example, ground pellets in a fabrication facility are weighed in production batches, sampled, and analyzed. At the time of rod loading the ground pellets are reweighed, but independent measurement of their SNM concentration is often not made.

Some items may not be reweighed but further information has been gathered concerning the concentration value and the concentration is changed in the records without reweighing the material.

Even in the case where the material has been reweighed and reanalyzed there is still a possibility of a correlation between the measurements. For example, there may be a calibration constant for the weighing technique which persists long enough to affect both weight measurements and the same can be true for the concentration measurement.

This paper will deal only with covariance resulting from material which is represented at exactly the same measured value in two terms of opposite sign of the MUF equation. By extension of this analysis, other sources of covariance could be handled, but the example in this paper has been limited for the sake of clarity.

Effect of this Source of Covariance

The effect of this commonality of material at the same measured value is to reduce the overall uncertainty in the material balance (IEMUF). This is the result of the opposite signs for the indicated pairs of terms in Fig. 1. Intuitively, a particular measured value (including its uncertainty due to measurement error) should be entirely cancelled out if it is added and subtracted in the same balance. More rigorously, consider the following example:

Let: I_B = beginning inventory of rods = $x + y$
 I_E = ending inventory of rods = $x + z$
 $V(x)$ = variance of x (i.e., square of uncertainty in x due to measurement uncertainties)

where x, y, z are independent measured values
and x = measured quantity common to both

$$\begin{aligned}
 \text{Then: } V(I_B - I_E) &= V(I_B) + V(I_E) - 2 \text{ COV}(I_B, I_E) \\
 &= V(x+y) + V(x+z) - 2 \text{ COV}(x, x) \\
 &= V(x) + V(y) + V(x) + V(z) - 2 V(x) \\
 V(I_B - I_E) &= V(y) + V(z)
 \end{aligned}$$

Therefore, although the uncertainty in the material common to two terms of opposite sign contributes to the uncertainty of knowing the total material in each term, it does not contribute to the uncertainty in the difference between the two terms or in MUF.

Static Inventory

The quantity of material carried over from one term to another is a function of the facility design, inventory policy, and inventory interval. For a facility with an equilibrium inventory condition, (i.e., material on hand stabilized at n days of receipts) the quantity of static inventory will be small for inventory intervals exceeding n days but will be large for inventory intervals less than n days. The total inventory may be the same regardless of the inventory frequency, but the static fraction will not be the same. This is also true for the fraction of receipts which end up in ending inventory and the fraction of shipments which come from beginning inventory in one balance period.

Figure 3 presents an assumed inventory distribution for a hypothetical coprecipitation facility for conversion and fabrication of Pu-U Oxide fuel. The total inventory assumed represents four month's receipts for a $1/2$ Te (U+Pu)/day facility producing 2.5% Pu fuel rods. The effect of changing this inventory policy will be discussed later. The last column in Figure 3 indicates I_E/P , which is the ratio of the number of units in inventory to the number produced in a 10 day period between inventories. This period is completely arbitrary, except that it will illustrate a significant covariance effect. The effect of changing the inventory frequency will be discussed later.

Static inventory for this illustration is indicated by a quantity of a particular material form in ending inventory which exceeds the amount which could have been produced since the last inventory. For equilibrium conditions, the static fraction of inventory for each material is given by the following:

$$F_{\text{static}} \geq 1 - \frac{1}{I_E/P}$$

where

F_{static} = the static inventory fraction
for a particular material

I_E = the inventory quantity of that
material

P = the production quantity of that
material since the last inventory

Therefore, the covariance effects of static inventory will change if changes occur in the inventory frequency, or in policy concerning the quantity of inventory for a particular material, or in the rate of production of that material.

The covariance effects assumed for this illustration are based on adequate physical protection and surveillance to assure the integrity of each material container so that only minimal remeasurement is required (random sampling or perhaps a "shadow" measurement). With assurance based on physical protection, surveillance, and random testing, few measured values on static items in inventory will be changed, and significant covariance effects can occur.

Covariance Effects for Receipts and Shipment

In a similar way, covariance effects can exist for receipts and shipments. Figure 4 illustrates this effect. Of the 74 $\text{Pu}(\text{NO}_3)_4$ bottles received each 10 days, 44 end up in ending inventory. For this illustration there is no static inventory of $\text{Pu}(\text{NO}_3)_4$ bottles. However, no matter how much static inventory there is for receipt material, all receipt material in ending inventory has to come either from the beginning inventory term or from the receipt term of the MUF equation. Therefore, in eliminating the uncertainty resulting

from the items of receipt material common to I_B , I_E , and R , the uncertainty contribution of receipt material in I_E should always be entirely eliminated if it is not measured independently at the time of each inventory. Thus, accounting for this type of covariance affects unequally the uncertainty contribution of beginning inventory and ending inventory of receipt material.

In a mirror image fashion, the uncertainty contribution for I_B of the material being shipped should be entirely eliminated. No matter how much static inventory there is for product material, all product in beginning inventory must go into the ending inventory term or the shipment term of the MUF equation (except perhaps for a small rejection rate at time of shipment). In this illustration all the product shipped comes from beginning inventory and all beginning inventory for product material goes either into shipments or ending inventory.

Covariance Items for this Illustration

The number of inventory items carried over from one inventory to the next and other materials represented at the same measured value in terms of opposite sign in the MUF equation are summarized in Figure 5. For the materials for which there is an excess of inventory over production, the number of items not remeasured has been computed as equal to that excess. For materials without excess inventory, carry-over from I_B to I_E is assumed to be ~10% over a 10 day inventory interval, producing a minor effect on LEMUF. For Pu nitrate receipts in original containers, all of the units in ending inventory are treated as either carryover from beginning inventory or from receipts. For material contained in rods, all but 15 of the units in beginning inventory are considered to be accounted for in identical form at the same measured value in either ending inventory or as shipments.

Since the integrity of some of the items listed in Figure 5 may not be demonstrable (especially solutions), some may actually be remeasured (perhaps on a random basis) at the time of inventory. Therefore, the values listed in Figure 5 represent a slightly high estimate of the number of units not remeasured during the inventory period. If the random check merely shows that the previous measurements are still valid and there is no need to change the label and records information, covariance items listed in Figure 5 are then correct.

LEMUF Excluding Covariance Items

A sample calculation of LEMUF for the hypothetical facility discussed here was presented in another paper at this meeting.* The final results of that paper are presented in Figure 6, which itemizes the uncertainty contributions to each term of the MUF equation from each material form within the facility. Note that elimination of the covariance items in Figure 5 results in unequal contributions to LEMUF from beginning and ending inventory and eliminates all uncertainty from SNM in rod bundles. The resulting LEMUF, shown at the bottom of Figure 6, is 2.97 kgs Pu for the assumed accounting period of 10 days.

The same calculation without eliminating the uncertainty due to covariance items results in an apparent LEMUF which is 60% larger.

Effect of Inventory Policy on LEMUF

The preceeding calculations were repeated for sixty days of receipts in ending inventory, taking into account the change in the number of weighing units, samples, analyses, and covariance items.

Excluding the covariance items, the LEMUF for the sixty days inventory situation is 2.12 kgs Pu for the ten-day interval. The apparent LEMUF without excluding the covariance items is 25% larger.

The above values of LEMUF have been combined with the limiting case of measurement uncertainties based only on Receipts, Shipments, and Measured Discards to produce Figure 7. The nonlinearity of the two curves in the figure is to be expected. The curvature in opposite directions can be explained by realizing that a large reduction in inventory primarily eliminates difficult-to-measure intermediate materials for which the contribution to LEMUF had been minimized by the covariance effects. From Figure 7, it can be seen that equilibrium inventory conditions can be as important to LEMUF as thruput.

*The Need for Experimental Data for LEMUF Calculations,
L. Green, R. Parsick, C. Sastre and S. Suda.

Effect of Thruput Rate on LEMUF

The thruput rate for a facility affects directly the quantity of material to be accounted for. Within limits, the total inventory and the receipts, shipments and measured discards are all directly proportional to thruput. Taking into account the change in the number of weighing units, samples, analyses, and covariance items, all of the above calculations were repeated for a thruput rate of 1/4 metric tonnes of Receipts/day.

Excluding covariance items, LEMUF is 1.59 kgs Pu for the 10-day accounting period at 1/4 Te (U+Pu)/day and 120 days of inventory. The apparent LEMUF without excluding covariance items is 57% larger.

These values are plotted in Figure 8. The upper curve probably cannot proceed all the way to the origin because some minimum inventory would probably be in the facility even for zero thruput rate for a given balance period. However, the lower curve does pass through the origin since at zero thruput rate every item in ending inventory must come from beginning inventory and covariance effects would eliminate uncertainty in knowing the difference between beginning and ending inventory (provided previous measurements can be demonstrated to be still valid).

Effect of Accounting Period on LEMUF

The level of detail for the calculations presented above is not amenable to generalization for the purposes of indicating interrelationships, sensitivities, or trade-off possibilities. The detailed calculations treated the effects of covariance due to static inventory and to receipts and shipments storage on an individual basis for each type of material and for a specific inventory frequency. Extrapolation to other inventory intervals requires a simple method for including the effects of covariance.

Without the complicating effects of covariance, extrapolation to other inventory intervals would be relatively simple. As the inventory interval decreased, the quantities of receipts, shipments and measured discards would decrease relative to the quantities of beginning and ending inventory. In addition, as the quantities decreased, fewer measurements would be made and the relative contribution from random measurement uncertainties

would increase. As the inventory interval approached zero, the propagated uncertainties would depend entirely on the inventory quantities being measured. Thus, the inventory interval determines the total quantities, the distribution of those quantities between flow and inventory measurements, and the relative contribution to total uncertainty from random and systematic components. As the inventory interval approaches zero, IEMUF would decrease and approach a limit set by the total inventory, its distribution, and the single measurement uncertainties for each type of material.

Covariance effects tend to produce a correction in the opposite direction. For inventory intervals approaching zero, much of the inventory would be unchanged and not remeasured between the beginning inventory and ending inventory. Also, an equivalent physical inventory policy (e.g., using tag values with independent procedures to assure validity) would become imperative. Therefore, many measured items would be represented at the same measured value in both beginning and ending inventory. The covariance introduced in this way would have to be identified and accounted for. The net effect would be recognition that the uncertainty in the difference between beginning and ending inventory would be far less than the uncertainty in either inventory. This effect diminishes as the thruput-to-inventory ratio increases, and the thruput can increase as the inventory interval or plant size increases.

An empirical relationship was used which approximated the above description of covariance effects. At zero thruput, either as a result of plant size or inventory interval, covariance eliminates all uncertainty due to the difference between beginning and ending inventory. For large thruputs compared to inventory on hand, covariance reduces the propagated uncertainty only slightly to represent the first batch of shipments for the inventory and the last batch of receipts which are still in storage at the time of ending inventory.

In addition, the contribution from random uncertainties was ignored. For large thruputs, the random contribution is small compared to systematic effects. For small thruput, although the random contribution can be even larger than the systematic, the covariance effects are just as effective in eliminating random contributions as systematic contributions. For those combinations where the extrapolation model could be checked, the random contributions would increase the total propagated uncertainty by 10 to 25% above the results for systematic contributions only.

The results for the hypothetical mixed oxide facility are presented in Figure 9. The effect on LEMUF of plant size, plant inventory policy, and inventory interval can be clearly seen, even if the magnitude of the resulting LEMUF is not exact.

The source of covariance illustrated in these calculations is especially important for short accounting periods, but other sources of covariance could be significant even for long periods.

Some Covariance Sources

$$\text{MUF} = I_B + R - S - D - I_E$$

The diagram illustrates covariance sources between variables in the equation $\text{MUF} = I_B + R - S - D - I_E$. It features a horizontal line above the equation with several vertical lines extending down to specific terms. Arrows indicate the direction of covariance: a downward arrow from the top line to I_B , a downward arrow from the top line to S , a downward arrow from the top line to D , and an upward arrow from the bottom line to I_E . Additionally, a horizontal arrow points from R to I_E , indicating a covariance between these two terms.

MUF = materials unaccounted for

I_B = measured beginning inventory

R = measured receipts

S = measured shipments

D = measured discards

I_E = measured ending inventory

Figure 1

**Measured SNM Content for an Item
in Two Terms of the MUF Equation**

**Measured SNM Content
in Positive MUF Term**

$$w_1 c_1$$

$$w_1 c_1$$

$$w_1 c_1$$

$$w_1 c_1$$

**Measured SNM Content
in Negative MUF Term**

$$w_1 c_1$$

$$w_2 c_1$$

$$w_1 c_2$$

$$w_2 c_2$$

w = weight measurement

c = concentration measurement

Figure 2

HYPOTHETICAL 1/2 Te (U+Pu)/DAY. MIXED OXIDE FABRICATION FACILITY

AVERAGE ENDING INVENTORY (I_E)*

<u>MATERIAL</u>	<u>% OF I_E</u>	<u>Pu WEIGHT kgs</u>	<u>BULK WEIGHT kgs</u>	<u>NO. OF UNITS</u>	<u>I_E/P</u>
Pu NITRATE RECEIPTS	5	75	568	44	.59
Pu NITRATE BLEND	5	75	893	7	.58
U-Pu NITRATE BLEND	15	225	109223	46	1.59**
MIXED POWDER	5	75	3424	428	.54
MILLED POWDER	15	225	10274	1284	1.64**
GREEN PELLETS	5	75	3424	685	.55
SINTERED PELLETS	5	75	3409	682	.56
GROUND PELLETS	15	225	10227	2045	1.76**
RODS	10	150	6818	1704	1.19**
ROD BUNDLES	10	150	6818	1704	1.20**
DISSOLVED SCRAP	5	75	36410	15	5.00**
IN-PROCESS	<u>5</u>	<u>75</u>	---	---	---
	100	1500			

* TOTAL ENDING INVENTORY RESTRICTED TO FOUR MONTHS' RECEIPTS

** RATIOS GREATER THAN 1.0 INDICATE MATERIAL CARRYOVER FROM ONE INVENTORY TO THE NEXT.

Figure 3

EXAMPLES OF COVARIANCE EFFECTS

Figure 4

	<u>NUMBER OF COMMON ITEMS</u>	<u>IN POSITIVE MUF TERM</u>	<u>ALSO IN NEGATIVE MUF TERM</u>
PU NITRATE RECEIPTS	44	R	IE
PU NITRATE BLEND	1	IB	IE
U-PU NITRATE BLEND	17	IB	IE
MIXED POWDER	43	IB	IE
MILLED POWDER	502	IB	IE
GREEN PELLETS	68	IB	IE
SINTERED PELLETS	68	IB	IE
GROUND PELLETS	885	IB	IE
RODS	270	IB	IE (RODS)
	1419	IB	IE (BUNDLES)
ROD BUNDLES	285	IB	IE
	1419	IB	S
DISSOLVED SCRAP	12	IB	IE

Figure 5

Square of Measurement Errors (kgs² Pu/10 days)
Excluding Covariance Items

<u>Material</u>	<u>I_B</u>		<u>Receipts or Removals</u>		<u>I_E</u>	
	<u>Sys</u>	<u>Ran</u>	<u>Sys</u>	<u>Ran</u>	<u>Sys</u>	<u>Ran</u>
Pu Nitrate Receipts	.0898	.0922	.0418	.0643	.0000	.0000
Pu Nitrate Blends	.0645	.0668	-	-	.0645	.0668
U-Pu Nitrate Blends	.8773	.1189	-	-	.8773	.1189
Mixed Powder	.4104	.1146	-	-	.4104	.1146
Milled Powder	.8118	.1587	-	-	.8118	.1587
Green Pellets	.1700	.0662	-	-	.1700	.0662
Sintered Pellets	.2240	.0513	-	-	.2240	.0513
Ground Pellets	.6065	.0472	-	-	.6065	.0472
Rods	.0001	.0003	-	-	.5932	.0471
Rod Bundles	.0000	.0000	.0000	.0000	.0000	.0000
Scrap Solution	.1109	.0436	-	-	.1109	.0436
Liquid Waste	.0000	.0000	.0000	.0000	.0000	.0000
Solid Waste	.0000	.0000	.0003	.0001	.0000	.0000
	3.3653	.7598	.0421	.0644	3.8686	.7144

$$\begin{aligned}
 \text{IEMUF} &= [(3.3653 + .7598) + (.0421 + .0644) + (3.8686 + .7144)]^{1/2} \\
 &= 2.97 \text{ kgs Pu/10 days}
 \end{aligned}$$

Figure 6

HYPOTHETICAL CO-PRECIPITATION MIXED OXIDE FACILITY

$\frac{1}{2}$ TE U-Pu / DAY

Figure 7

HYPOTHETICAL CO-PRECIPITATION MIXED OXIDE FACILITY
(120 DAYS OF RECEIPTS IN ENDING INVENTORY)

Figure 8

Figure 9