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CRITICAL REVIEW OF THE DESIGN OF THE HALLAM MODERATOR ELEMENT

PHASE II

**TASK 1 - THE CONCENTRATION OF STRESS IN THE CAN AT A JOINT
IN GRAPHITE CAUSED BY TEMPERATURE GRADIENTS ACROSS
A DIAMETER OF THE MODERATOR ELEMENT**

TASK 2 - POSSIBILITY OF MECHANISM OF THERMAL STRESS RATCHET

by

K. R. Stearns
L. Berkowitz

December 14, 1964

Prepared for

**UNITED STATES ATOMIC ENERGY COMMISSION
Chicago Operations Office**

Contract No. AT(11-1)-1402

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CRITICAL REVIEW OF THE DESIGN
OF THE
HALLAM MODERATOR ELEMENT, PHASE II

- TASK 1 THE CONCENTRATION OF STRESS IN THE CAN AT A JOINT IN
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SUMMARY

The second phase of the critical review of the design of the HNPF moderator can was an attempt to obtain approximate solutions of the three stress analyses recommended in the Phase I Report (Ref. 1).

Task I of Phase II was the analysis of the concentration of stress in the can at a joint in graphite caused by temperature gradients across a diameter of the moderator element. The stiff graphite logs will not bow perceptibly and would be expected to compel a considerable distortion of the can at the gaps between graphite sections. Because the can has an undulating periphery with diameter-to-thickness ratio of 1000 the major effort was toward solution of Donnell's equation modified to include lateral, thermal, and friction loading for each of the flat and curved plates of a half periphery. The compatibility matrix would then supply the redundant edge loadings of each member. Because of the time and labor required this solution proved beyond the scope of Phase II. The theory of a beam on elastic foundation is appropriate for the study of a single flat with a temperature gradient across its width. Solution by computer indicated that high stresses would be expected near the joint between logs.

The second task of Phase I was the consideration of the probability of a thermal stress ratchet mechanism that would tend to increase the separation between the graphite sections. The cause would be differences in the temperature distributions and slipping patterns during scram as opposed to startup.

At a level of intensity of friction indicated by the tests (Ref. 2) it is shown here that neither temperature dependence nor kinetic variation of friction would result in sufficient displacement of the sections of graphite to be likely to indicate damage to the sheaths in the small number of scrams from high power levels logged at HNPF if the coolant temperature at any given point on a moderator element drops monotonically during scram. The violent temperature fluctuations noted in the Reference 2 test, however, suggest that a number of successive waves of ratcheting might conceivably accompany a single scram-from-power incident.

Although progressive collapse of a can into a graphite joint might aggravate a tendency toward ratchet, such ratcheting would become an effect of the buckling studied in the first task of Phase II rather than its primary cause.

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TASK I: STRESS CONCENTRATION AT JOINT CAUSED BY BOWING

INTRODUCTION

Under Phase II of the review of the design of the HNPF moderator cans, a study to obtain solutions of the three stress analyses recommended from the work compiled in Phase I was undertaken. The first task was the concentration of stress in the can at a joint in graphite caused by temperature gradients across a diameter of the moderator element. The stiff graphite logs will not bow perceptibly and would be expected to compel a considerable distortion of the can at the gaps between graphite sections.

Assume a temperature difference of ΔT across a diameter of a can for the entire height of can. If distribution is linear, gradient of $\frac{\Delta T}{2r}$ would cause a uniform curvature of the can:

$$\frac{1}{\rho_S} = \frac{\frac{L}{2} \alpha_S \frac{\Delta T}{2}}{\frac{L}{2} r} = \frac{\alpha_S \Delta T}{2r},$$

if the can stood free. Assuming that graphite log sections have the same linear gradient of temperature, their curvature is

$$\frac{1}{\rho_G} = \frac{\alpha_G \Delta T}{2r},$$

giving a difference in curvature:

$$\frac{1}{\rho_S} - \frac{1}{\rho_G} = \frac{(\alpha_S - \alpha_G) \Delta T}{2r}$$

For moderate $\Delta T/2r$, the external pressure will clamp the can tightly against each graphite log, so that the steel can tends to be straightened by the difference in curvature, remembering that $F_G I_G \sim 6 (E_S I_S)$.

By the irregular nature of its periphery, the can appears easily deformed by lateral loads, so that the foregoing straightening

might be accomplished with little stress, if the restraint of the external pressure clamping can to the surfaces of the logs were absent. The logs thus act as a sort of prestressed sandwich filling or stabilizing bulkhead, in effect continuous except for their joints, tending to make the considerable moment of inertia of the can (24.7 in^4) * effective against straightening until the clamping action of ΔP is overcome at larger ΔT 's.

$$I_{R \text{ graphite}} = \frac{\pi r^4}{4} = \frac{\pi (7.6)^4}{4} = 2600 \text{ in}^4$$

$$E_S = 24(10)^6$$

$$E_G = 1.2(10)^6$$

Remote from the ends of a log segment conditions would approach pure bending with the mechanical straightening of the can and further bending of graphite overcoming the thermal discrepancy in curvature:

$$\frac{M}{E_S I_S} + \frac{M}{E_G I_G} = \frac{(\alpha_S - \alpha_G) \Delta T}{2r},$$

or

$$M = \frac{E_S I_S E_G I_G (\alpha_S - \alpha_G) \Delta T}{2r(E_S I_S + E_G I_G)} \quad (1)$$

Using equation (1) axial bending stresses may be evaluated by ordinary beam theory, provided ΔT is not sufficient to separate can and segment at interface. However, the actual segments are only two diameters long, so that slippage, buckling, and warping of cross-sections might extend over a considerable part of the span.

*Perimeter of can = 53.4 in

Estimated radius of gyration = 7.6 in

Thickness = 0.016 in

Nominal area cross-section of metal = 0.855 in²

Estimated rectangular moment of inertia = 24.7 in⁴

Continuing with simple beam theory treatment to get a physical concept of the problem it is found that the constant moment of Equation (1) cannot be produced exactly near the ends of a segment by any physically possible system of lateral restraining systems. To show this, in Figure 2.1.1(a) the half-span cantilever is being straightened by a uniformly distributed load. Such restraint is obviously incompatible with the problem, because the parabolic moment diagram would produce a curvature 3 times the mean at the center of the log and approaching zero at its end, whereas a constant curvature is required by the thermal strain. If the can were capable locally by sustaining a concentrated load at the end of the segment the improvement of Figure 2.1.1(b) would be obtained with the peak curvature twice the mean. It is obvious that uniform curvature imposed by the thermal strain would produce interference on the concave side of the can, Figure 2.1.1(b). If Q is considered the magnitude of this interference reaction acting at its centroid, it is apparent that $Q = P$ is the necessary condition to produce constant moment and curvature between Q and midspan in Figure 2.1.1(c). If the peripheries of can and log were perfectly rigid, in the limit $P = Q \rightarrow \infty$ and $e \rightarrow 0$, while Pe remains equal to M of Equation (1).

In reality it would be expected that membrane extension, shear, bending buckling and suppage compliances would spread the interference reactions P and Q along a considerable fraction of the span and thus lessen their peak values. However, these local deformations may themselves be potentially damaging to the sheath, particularly at the corners of the logs at the gapping joints. The foregoing considerations impelled recommending an analytical investigation of stress concentration at joints resulting from diametral temperature gradients.

It is clear that beam theory does not have the sophistication to deal with so complex a problem. A suitable choice of attack appeared to be writing modified Donnell equations for the flat and curved plates, including terms to accommodate all the types of existing loadings for a half circumference obtaining approximate solutions of these and forming a compatibility matrix for joints between plates. This analysis is outlined

Fig. 2.1.1(a)

Fig. 2.1.1(b)

Fig. 2.1.1(c)

in Appendix A. It was found that numerical solutions would require considerably more time and effort than were available. It appears that such solutions are outside the scope of Phase II. It should be noted that foregoing solution for Task I would inherently contain the solution of Task 3.

Restraint of a Single Flat

It is possible to discuss qualitatively the behavior of a single flat under a peripheral temperature gradient by beam theory. An example may serve to indicate that the can might be damaged by such thermal strain input. In Figure 2.1.2 suppose that deficient heat generation below Z-axis or embarrassed cooling convection above it results in a diametral temperature gradient along Y-axis rising 100°F from J to A. As a result the flat E - F is $100 (3.27)/13.9 = 23.5^\circ$ hotter at E than at F with linear distribution. This gradient is treated as constant in the x direction for the entire length of log.

The can is assumed nearly incapable of concerted action as a beam, so that the elastic bending of the log is negligible. Although the transitions E - E' and F - F' are partially effective as flanges of the flat bending as a beam about the Z-axis they will be neglected (not conservatively). Then the moment that would be required at the gap to restrain the difference in thermal distortion would be

$$M = \frac{(\alpha_S - \alpha_G) I_S E_S \Delta T}{h}$$

$$M = \frac{(10.2 - 1.2) \frac{.016(3.27)^3}{12} (24)23.5}{3.27}$$

$$= 72.4 \text{ in lb}$$

Thermal stress (complete restraint)

$$\sigma_{th} = \frac{M}{Z} = \frac{72.4}{.0285} = 2540 \text{ psi (max. axial)}$$

Figure 2.1.2 - CROSS SECTION OF CAN

Taking Y.S. at 900°F of 304 as 15,000 psi, the flat would withstand a pull of

$$r = 15,000 (.016) = 240 \text{ lb/in}$$

The restraining couple at the end of the segment may be approximated by two loads in Figure 2.1.3 of intensity r and distributed over distance C along edges of flat. The flat is deep enough to support the shear on sect. m-n in diagonal tension.

Equating restraining couple to thermal moment

$$M = C^2 r,$$

or

$$C^2 = \frac{M}{r} = \frac{72.4}{240} = 0.302 \text{ in}^2,$$

whence

$$C = 0.55 \text{ in}$$

Inasmuch as the curved panels of the can would tend to shirk such loads by straightening, circumferential sliding and creasing might result at corner of log along transition radius $E E'$. Under cyclic bowing, with column and lateral loading arising from other sources superposed, and recalling that unsteady creep tends to aggravate deformation, the foregoing effect may be significant.

Critical Span of Moderator Sheath at Joint between Log Segments

In Figure 2.1.4 an axial section of the sheath at a gap of width ℓ between segments is shown. Consider a unit-width strip at the middle of a flat. The pressure difference across the sheath causes an inward distributed load of q lb/in. The axial loads caused by friction approach zero near the gap. The axial compressive load P

Figure 2.1.3 - COUPLE TO PREVENT THERMAL STRAIN OF FLAT

is composed of end pressure, weight, and thermal stresses such as that caused by flat being hotter than adjacent scallops or bowing from a diametral gradient restrained by adjacent moderator elements.

Figure 2.1.4 - STABILITY OF SHEATH AT JOINT BETWEEN LOG SEGMENTS

Across the gap the support of the graphite is lost, so that the strip becomes a column under lateral load. The moment restraint at the edges of the gap will be intermediate between cases A and B, Table 2.1.1. It is noteworthy that any finite lateral load will tend to collapse the strut into the gap when the stability parameter

$$u = \frac{l}{2} \sqrt{\frac{P}{D}}$$

approaches $\frac{\pi}{2}$ in Case A (hinged ends) or π in Case B (ends prevented from rotating).

Under elastic conditions such an inward buckling would relieve the local thermal strain, leaving the curved parts of the can to carry pressure, gravity, and bowing loads. At elevated temperature sustained load may result in creep buckling of scallop so that transition radii would be carrying major parts of column load at high stress and creep rate.

Table 2.1.1
STRUT WITH LATERAL LOAD

Case A. Simply Supported:

$$M_{\max} = \frac{ql^2}{8} \left(\frac{(1 - \cos u)}{u^2 \cos u} \right) \quad (2)$$

Case B. Ends Prevented from Rotating:

$$M_{\text{middle}} = \frac{ql^2}{24} \left(\frac{u - \sin u}{u^2 \sin u} \right) \quad (6)$$

$$M_{\text{ends}} = \frac{ql^2}{12} \left(\frac{\tan u - u}{u^2 \tan u} \right) \quad (3)$$

where

$$u = \frac{kl}{2}$$

$$k^2 = \frac{P}{D}$$

$$D = \frac{Eh^3}{12(1 - \nu^2)}$$

If $E = 24(10)^6$ psi

$h = .016$ in

$$D = \frac{Eh^3}{12(1 - \nu^2)} = \frac{24(.016)^3 10^6}{12(1 - .09)} = 9.00 \text{ in lb/in}$$

The critical stress would be

$$\sigma_{cr} = \frac{P}{h} = \frac{4 u^2 D}{h l^2}$$

Critical Stress:

Case A:

$$u = \pi/2, u^2 = 2.467$$

$$4 u^2 D/h = 4(2.467)(9.00)/.016 = 5,550$$

l	l^2	σ_{CR}
.2	.04	139,000
.4	.16	34,700
.6	.36	15,400
.8	.64	8,680
1.0	1.0	5,550

Case B:

$$u = \pi, u^2 = 9.87$$

$$4 u^2 D/h = 4(9.87)(9.00)/.016 = 22,200$$

l	l^2	σ_{CR}
.2	.04	555,000
.4	.16	139,000
.6	.36	61,700
.8	.64	34,700
1.0	1.0	22,200

It appears that a gap well over a half-inch would be prerequisite to a collapse of the can into a joint as a direct result of axial and lateral loading. Rapid surface chilling during transit

of a cold step of coolant during scram might cause a thermal buckling with a smaller span.

Diametral Thermal Gradient

Consider a flat section of the sheath and assume that the temperature variation is linear along the height of the flat and increases at 1°F per inch of flat. This will give rise to a thermal moment:

$$(1) \quad M^* = E \alpha \frac{h^3}{12}$$

which does not vary along the length, x of the flat.

Model

Due to the overall diametral gradient we expect a "hinge" effect at the log joint due to the sudden drop in moment inertia. We consider the local effects of this behavior by assuming that the flat of the sheath is prevented from assuming the natural circular arc shape by the transition radius hanging up on the graphite log as shown in Figure 1. The log then offers resistance to the flat deflection and provides an "elastic foundation" to the flat which may be considered as a beam of height, h and length l .

Figure 1

Foundation Modulus

It is difficult to establish a foundation modulus for this problem. As a rough estimate we have considered a 1 in length of graphite and assumed that the 3/4 inch projection with 4.72 inch depth offers resistance as a deep beam. From Roark:

$$(2) \quad w_s = \frac{1}{2} F \left(\frac{Wb}{AG} \right)$$

$$F = \frac{6}{5} \quad G = \frac{E_G}{2(1 + \nu)} = \frac{1.2 \times 10^6}{2(1 + .27)} = .472 \times 10^6$$

$$A = 1 \times 4.72 = 4.72, \quad b = .75$$

$$\frac{W}{w_s} = \frac{2AG}{Fb} = 2 \times 4.72 \times \frac{.472 \times 10^6}{.75} \times \frac{5}{6} = 4.96 \times 10^6 \text{ \#/in} \quad (3)$$

Since the modulus may vary considerably from this valve we consider a range of moduli from $.5 \times 10^6$ to 7×10^6 .

Formulation of the Problem

The governing equation for a beam on an elastic foundation is given by:

$$EI \frac{d^4 w}{dx^4} = -kw + q \quad (4)$$

where k is the modulus foundation and q is the applied load. For our case $q = 0$ so that (4) becomes a homogeneous equation.

Boundary requirements:

$$@ x = 0: \quad w = \frac{dw}{dx} = 0 \quad (5)$$

$$@ x = l: \quad \begin{cases} \frac{d^2 w}{dx^2} = -\frac{M^*}{EI} \text{ (Thermal Moment)} \\ \frac{d^3 w}{dx^3} = 0 \text{ (Shear)} \end{cases} \quad (6)$$

Solution

Since $q = 0$ (4) is satisfied by:

$$w = e^{\lambda x} (C_1 \cos \lambda x + C_2 \sin \lambda x) + e^{-\lambda x} (C_3 \cos \lambda x + C_4 \sin \lambda x) \quad (7)$$

where

$$\lambda = \sqrt[4]{\frac{k}{EI}} \quad (8)$$

and C_1 -- C_4 are determined by (5) and (6):

$$\begin{aligned} \frac{1}{\lambda} \frac{dw}{dx} = e^{\lambda x} & \left[C_1 (\cos \lambda x - \sin \lambda x) + C_2 (\cos \lambda x + \sin \lambda x) \right] \\ & - e^{-\lambda x} \left[C_3 (\cos \lambda x + \sin \lambda x) - C_4 (\cos \lambda x - \sin \lambda x) \right] \end{aligned} \quad (9)$$

$$\frac{1}{2\lambda^2} \frac{d^2 w}{dx^2} = -e^{\lambda x} (C_1 \sin \lambda x - C_2 \cos \lambda x) + e^{-\lambda x} (C_3 \sin \lambda x - C_4 \cos \lambda x) \quad (10)$$

$$\begin{aligned} \frac{1}{2\lambda} \frac{d^3 w}{dx^3} = -e^{\lambda x} & \left[C_1 (\cos \lambda x + \sin \lambda x) - C_2 (\cos \lambda x - \sin \lambda x) \right] \\ & + e^{-\lambda x} \left[C_3 (\cos \lambda x - \sin \lambda x) + C_4 (\cos \lambda x + \sin \lambda x) \right] \end{aligned} \quad (11)$$

Due to (5):

$$C_1 + C_3 = 0 \quad (12)$$

$$C_1 + C_2 - C_3 + C_4 = 0 \quad (13)$$

Due to (6):

$$\begin{aligned} \left[-e^{\lambda \ell} \sin \lambda \ell \right] C_1 + \left[e^{\lambda \ell} \cos \lambda \ell \right] C_2 + \left[e^{-\lambda \ell} \sin \lambda \ell \right] C_3 + \left[-e^{-\lambda \ell} \cos \lambda \ell \right] C_4 \\ - \frac{M^*}{4\lambda^2 EI} \end{aligned} \quad (14)$$

$$\begin{aligned} \left[-e^{\lambda \ell} (\cos \lambda \ell + \sin \lambda \ell) \right] C_1 + \left[e^{\lambda \ell} (\cos \lambda \ell - \sin \lambda \ell) \right] C_2 \\ + \left[e^{-\lambda \ell} (\cos \lambda \ell - \sin \lambda \ell) \right] C_3 + \left[e^{-\lambda \ell} (\cos \lambda \ell + \sin \lambda \ell) \right] C_4 = 0 \end{aligned} \quad (15)$$

Note that (12) - (15) provide sufficient equations to determine C_1 -- C_4 . w is therefore known. The tension in the flat may be computed by:-

$$\sigma_z = \frac{-k w(x)}{t} \quad (16)$$

where $t = .016$ is the web thickness.

The analysis was programmed and the computed results show significant stresses near the log point ranging from 16,000 psi for $k = .5 \times 10^6$ to 61,000 psi for $k = 7 \times 10^6$. Note that the computed results should be multiplied by $\frac{22.8}{30}$ since $E = 30 \times 10^6$ was erroneously used. However if the gradient is higher than 1°F/inch the stresses are proportionately higher.

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

BEAM LENGTH	=	16.250 INCHES
BEAM THICKNESS	=	0.016000 INCHES
BEAM HEIGHT	=	3.270000 INCHES
YOUNGS MODULUS	=	30.000 (10) ⁶ PSI
END MOMENT	=	0.874145E-03 IN-LB
END CURVATURE	=	0.100000E-04 1/(IN)

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

LOAD VECTOR

0.00000000E 00 0.00000000E 00 -0.52265685E-03 0.00000000E 00

SYSTEM MATRIX

0.10000000E 01 0.00000000E 00 0.10000000E 01 0.00000000E 00
 0.10000000E 01 0.10000000E 01 -0.10000000E 01 0.10000000E 01
 0.32993285E 03 0.28642100E 06 -0.40217518E-08 -0.34913595E-05
 -0.28609107E 06 0.28675093E 06 0.34953812E-05 0.34873377E-05

COEFFICIENT VECTOR

-0.18268849E-08 -0.18226810E-08 0.18268849E-08 0.54764508E-08

MATRIX INVERSE

0.10000000E-08 0.00000000E 00 0.34953812E-05 -0.34913595E-05
 -0.99950000E-08 0.00000000E 00 0.34873378E-05 0.40216836E-08
 0.10000000E 01 0.00000000E 00 -0.34953812E-05 0.34913595E-05
 0.10000000E 01 0.10000000E 01 -0.10478100E-04 0.69786972E-05

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

FOUNDATION REACTION = 0.500000E 06 PSI
 LAMBDA X LENGTH = 0.125652E 02

DISTANCE ALONG BEAM (INCHES)	BEAM DEFLECTION (INCHES)	NORMAL STRESS (PSI)
0.0000	0.000000E 00	0.000000E 00
16.0000	-0.340005E-03	0.106252E 05
16.0500	-0.373454E-03	0.116704E 05
16.1000	-0.408431E-03	0.127635E 05
16.1500	-0.444951E-03	0.139047E 05
16.2000	-0.483024E-03	0.150945E 05
16.2500	-0.522657E-03	0.163330E 05

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

LOAD VECTOR

0.00000000E 00 0.00000000E 00 -0.30175607E-03 0.00000000E 00

SYSTEM MATRIX

0.10000000E 01	0.00000000E 00	0.10000000E 01	0.00000000E 00
0.10000000E 01	0.10000000E 01	-0.10000000E 01	0.10000000E 01
0.11203645E 08	-0.10271130E 08	-0.48496895E-07	0.44460345E-07
0.21474775E 08	0.93251480E 06	0.40365500E-08	-0.92957240E-07

COEFFICIENT VECTOR

-0.12180535E-11 0.28050412E-10 0.12180535E-11 -0.25614305E-10

MATRIX INVERSE

-0.10000000E-08	0.00000000E 00	0.40365500E-08	0.44460346E-07
-0.13000000E-07	0.00000000E 00	-0.92957242E-07	0.48496897E-07
0.10000000E 01	0.00000000E 00	-0.40365500E-08	-0.44460346E-07
0.10000000E 01	0.10000000E 01	0.84884141E-07	-0.13741759E-06

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. T(11-1)-1402

FOUNDATION REACTION = 0.150000E 07 PSI
 LAMBDA X LENGTH = 0.165368E 02

DISTANCE ALONG BEAM (INCHES)	BEAM DEFLECTION (INCHES)	NORMAL STRESS (PSI)
0.0000	0.000000E 00	0.000000E 00
16.0000	-0.167557E-03	0.157084E 05
16.0500	-0.191344E-03	0.179385E 05
16.1000	-0.216637E-03	0.203097E 05
16.1500	-0.243460E-03	0.228243E 05
16.2000	-0.271829E-03	0.254840E 05
16.2500	-0.301756E-03	0.282896E 05

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
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LOAD VECTOR

0.00000000E 00 0.00000000E 00 -0.21337377E-03 0.00000000E 00

SYSTEM MATRIX

0.10000000E 01	0.00000000E 00	0.10000000E 01	0.00000000E 00
0.10000000E 01	0.10000000E 01	-0.10000000E 01	0.10000000E 01
-0.25297721E 09	0.23791395E 09	0.20976461E-08	-0.19727440E-08
-0.49089117E 09	-0.15063259E 08	-0.12490211E-09	0.40703901E-08

COEFFICIENT VECTOR

0.26650831E-13 -0.86851447E-12 -0.26650831E-13 0.81521281E-12

MATRIX INVERSE

0.10000000E-08	0.00000000E 00	-0.12490210E-09	-0.19727440E-08
-0.13000000E-07	0.00000000E 00	0.40703901E-08	-0.20976461E-08
0.10000000E 01	0.00000000E 00	0.12490210E-09	0.19727440E-08
0.10000000E 01	0.10000000E 01	-0.38205859E-08	0.60431342E-08

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

FOUNDATION REACTION = 0.300000E 07 PSI
 LAMBDA X LENGTH = 0.196656E 02

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DISTANCE ALONG BEAM (INCHES)	BEAM DEFLECTION (INCHES)	NORMAL STRESS (PSI)
0.0000	0.000000E 00	0.000000E 00
16.0000	-0.103530E-03	0.194118E 05
16.0500	-0.122474E-03	0.229638E 05
16.1000	-0.142902E-03	0.267941E 05
16.1500	-0.164847E-03	0.309088E 05
16.2000	-0.188332E-03	0.353123E 05
16.2500	-0.213374E-03	0.400076E 05

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

LOAD VECTOR

0.00000000E 00 0.00000000E 00 -0.16527861E-03 0.00000000E 00

SYSTEM MATRIX

0.10000000E 01 0.00000000E 00 0.10000000E 01 0.00000000E 00
 0.10000000E 01 0.10000000E 01 -0.10000000E 01 0.10000000E 01
 0.17505546E 10 -0.47466654E 10 -0.68393667E-10 0.18545085E-09
 0.64972200E 10 -0.29961108E 10 -0.11705719E-09 -0.25384452E-09

COEFFICIENT VECTOR

0.19347048E-13 0.41955069E-13 -0.19347048E-13 -0.80649166E-13

MATRIX INVERSE

-0.70000000E-08 0.00000000E 00 -0.11705719E-09 0.18545085E-09
 -0.10000000E-08 0.00000000E 00 -0.25384452E-09 0.68393669E-10
 0.10000000E 01 0.00000000E 00 0.11705719E-09 -0.18545085E-09
 0.10000000E 01 0.10000000E 01 0.48795889E-09 -0.43929538E-09

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

FOUNDATION REACTION = 0.500000E 07 PSI
 LAMBDA X LENGTH = 0.223445E 02

DISTANCE ALONG BEAM (INCHES)	BEAM DEFLECTION (INCHES)	NORMAL STRESS (PSI)
0.0000	0.000000E 00	0.000000E 00
16.0000	-0.708423E-04	0.221382E 05
16.0500	-0.867314E-04	0.271036E 05
16.1000	-0.104084E-03	0.325264E 05
16.1500	-0.122941E-03	0.384191E 05
16.2000	-0.143333E-03	0.447915E 05
16.2500	-0.165279E-03	0.516496E 05

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

LOAD VECTOR

0.00000000E 00 0.00000000E 00 -0.13968592E-03 0.00000000E 00

SYSTEM MATRIX

0.10000000E 01	0.00000000E 00	0.10000000E 01	0.00000000E 00
0.10000000E 01	0.10000000E 01	-0.10000000E 01	0.10000000E 01
0.26464082E 11	0.24330338E 11	-0.20478075E-10	-0.18826970E-10
0.21337441E 10	0.50794421E 11	0.39305044E-10	-0.16511048E-11

COEFFICIENT VECTOR

-0.54903614E-14 0.23063613E-15 0.54903614E-14 0.10750087E-13

MATRIX INVERSE

-0.10000000E-07	0.00000000E 00	0.39305046E-10	-0.18826970E-10
0.00000000E 00	0.00000000E 00	-0.16511051E-11	0.20478075E-10
0.10000000E 01	0.00000000E 00	-0.39305046E-10	0.18826970E-10
0.10000000E 01	0.10000000E 01	-0.76958986E-10	0.17175866E-10

STRESS AND DISPLACEMENT NEAR LOG JOINTS OF MODERATOR CAN
 FIL PROJECT NO.31G-B2254-01 FOR USAEC CONTRACT NO. AT(11-1)-1402

FOUNDATION REACTION = 0.700000E 07 PSI
 LAMBDA X LENGTH = 0.243054E 02

DISTANCE ALONG BEAM (INCHES)	BEAM DEFLECTION (INCHES)	NORMAL STRESS (PSI)
0.0000	0.000000E 00	0.000000E 00
16.0000	-0.543607E-04	0.237828E 05
16.0500	-0.684486E-04	0.299463E 05
16.1000	-0.839842E-04	0.367431E 05
16.1500	-0.101014E-03	0.441936E 05
16.2000	-0.119573E-03	0.523134E 05
16.2500	-0.139686E-03	0.611126E 05

TASK 2: PROBABILITY OF THERMAL STRESS RATCHET MECHANISM

INTRODUCTION

Subject task was the second to be undertaken during the Phase 2 study to obtain approximate solutions of the 3 stress analyses recommended as a result of the critical review of can design during Phase 1 (Ref. 1). The intent was to consider the probability of a thermal stress ratchet mechanism that would tend to increase the separation between the graphite sections caused by differences in the space and time distributions of temperature in the can along a log section during scram as opposed to startup.

The input of differential thermal-expansion strain between stainless steel can and graphite log in passing from no load to full-load operation approaches 0.1 inch overall for the uppermost segment. However, it was shown in Reference 2 tests that the major part of this strain would be accommodated by slipping so that the consequent direct axial stresses and corresponding strains would be moderate.

The study reported here attempts to estimate the upper limit of net displacement of a log section during one cycle of normal startup to full power followed by scram. Only the effects of dependence of friction intensity on temperature and on motion are considered. The periphery of the can appears too compliant to produce significant shifts in the normal pressure distribution when non-uniformly heated.

Friction Forces on Can:

In considering the friction forces on the can acting at its contact with a log segment in Figure I, the following notation will be used:

A_S = cross-sectional area of
metal in can

A_G = cross-sectional area of log

α_S = thermal coefficient of
expansion of Type 304 SS

Fig. I - FRICTION BETWEEN CAN AND LOG

α_G = thermal coefficient of
expansion of graphite

ΔT = temperature rise

μ = coeff. friction

Δp = pressure difference on can wall

ϵ_S, ϵ_G = unit strain

s = perimeter of interface

E_S, E_G = moduli of elasticity

L = length of log

F = total force on a cross-section

σ_S, σ_G = axial stresses

Sheath is incapable of supporting any material fraction of Δp , so that friction force up to $\mu \Delta p$ is available at interface. At rise ΔT , difference in free thermal expansion would be $(\alpha_S - \alpha_G) \Delta T$ per unit length. Without slip, stretching of log and compression of can together would equal differential expansion

$$\epsilon_S - \epsilon_G = (\alpha_S - \alpha_G) \Delta T \quad (1)$$

and the axial forces in steel and graphite caused by the foregoing strains would be in equilibrium

$$F_S = - F_G \quad (2)$$

or

$$A_S E_S \epsilon_S + A_G E_G \epsilon_G = 0$$

$$\epsilon_S = - \frac{A_G E_G}{A_S E_S} \epsilon_G \quad (3)$$

$$\epsilon_G = \frac{\alpha_S - \alpha_G}{\frac{A_{G^E G}}{A_{S^E S}} + 1} \Delta T \quad (4)$$

$$F_S = -F_G = \frac{A_{S^E S} (\alpha_S - \alpha_G) A_{G^E G}}{E_G A_G + A_{S^E S}} \Delta T \quad (5)$$

$$F_S = \frac{\alpha_S - \alpha_G}{\frac{1}{A_{S^E S}} + \frac{1}{A_{G^E G}}} \Delta T \quad (6)$$

and

$$\epsilon_S = \frac{A_{G^E G} (\alpha_S - \alpha_G) \Delta T}{A_{S^E S} + A_{G^E G}} \quad (7)$$

until slip occurs (mutual restraint force and strain)

If the log segment and contiguous sheath are evenly heated to ΔT causing greater thermal forces than can be restrained by the available friction, can will begin to slip off segment at ends, up to point where integrated friction force just equals thermal force:

$$\int \mu \Delta p \, s \, dx = E_S \quad (8)$$

If Δp and μ are constant with x and T , and ΔT is increased far enough, entire interface would be slipping outward from middle of section where force would be

$$F_S \left(x = \frac{L}{2} \right) = \frac{\mu \Delta p s L}{2} \quad (9)$$

Consequences of Temperature Dependence of Friction:

Reference 3 attributes the following statement to P. Wagner (Ref. 4): "The static coeff. friction (of graphite) rises as the test temperature rises, whereas the dynamic coeff. is only slightly affected.

"The last statement is perhaps the most difficult to substantiate since it involves the use of controlled atmospheres to prevent oxidation, the effect of which is not understood."

In view of the large temperature changes in the HNPf moderator cans, and the 350°F ΔT of the coolant through the core, a rather small increase in friction per $^{\circ}\text{F}$ temperature rise in the range $600 - 950^{\circ}\text{F}$ might produce significant difference in distributions of friction forces in heating vs cooling that would ratchet log sections relative to sheath. The most likely cycle would include a scram from power. The canned graphite is in a helium atmosphere, with zirconium getters to limit oxygen contamination. During startup the cans are brought up to temperature at rate not exceeding 25°F/hr . The axial temperature rise upward through the core is nominally 350°F distributed somewhat like $\sin x$ for $-\pi < x < \pi$. During heating, a can slips both ways off its graphite sections with the upward friction force F_2 from a given segment less than the downward F_1 by the weight of segment, W .

$$F_1 - F_2 = W \quad (10)$$

or

$$s \int_{x=L}^{x=x_0} \mu \Delta p \, dx - s \int_{x=0}^{x=x_0} \mu \Delta p \, dx = W \quad (11)$$

where

x = distance from bottom of segment
 x_0 = point of no slip or reversal of friction force
 s = perimeter of interface

The Δp varies with x because of the changing head of Na and varies with time when:

- (a) head of coolant above core changes,
- (b) equilibrium of outgassing vs gettering changes, or if
- (c) thermodynamic state of backfill gas changes

These effects are neglected here.

From Wagner's statement that $\mu = f(T)$ we infer also that $\mu = f(x, t)$ in foregoing equation of equilibrium of vertical forces. This could lead to ratcheting in the following way: during scram from power the sudden reduction in heat generation together with the momentum of the coolant causes what amounts to a step drop in coolant temperature convected upward along the can. In the way of any segment other than the lowermost, the difference in average temperature of the upper and lower parts of the contiguous can may be greater than during any normal operating, startup or standby condition for a considerable range of such scram cooling. As a result, if μ increases monotonically with T , there would be a relative deficiency of intensity of friction force at the lower part of the segment, leading to an upward displacement of x_0 during scram as compared to any distribution found during the heating part of the thermal cycle. A net upward displacement of segment in can would result from each scram cycle.

Example:

In the numerical illustration that follows the assumed values of geometrical and property parameters were:

$$\begin{aligned} s &= \text{perimeter of graphite} = 12 (4.48) = 53.8^{\text{in}} \\ L &= 33.66 \text{ in} \\ \alpha_S &= 10.2 (10)^{-6} \text{ in/in } ^\circ\text{F} \\ \alpha_G &= 1.4 (10)^{-6} \text{ in/in } ^\circ\text{F} \\ \Delta p &= 15 \text{ psi} \\ W &= 383 \text{ lb/segment} \end{aligned}$$

The elastic deformation of steel and graphite are small compared to thermal expansion strains. It is conservative to neglect them.

Thermal lag of graphite interface is neglected in the example.

Postulate μ linear with T :

$$\mu = .25 + a (T - 600) \quad (12)$$

where

a = positive constant, $^{\circ}\text{F}^{-1}$

T = interface temperature, $^{\circ}\text{F}$

Consider segment just above midplane of core. At no load it is at 600°F throughout and presumably the contiguous sheath has a stretching superimposed on its column loading of end pressure and weight by the shrinking that occurred during the previous shutdown. Variation in Δp will not be considered here. At full temperature and sinusoidal temperature variation the fourth segment would be $\frac{950 + 600}{2} = 775^{\circ}\text{F}$ at bottom and at $\bar{x}$ temperature would be

$$T = 775 + 175 \sin \left(\frac{x}{L} \cdot \frac{\pi}{6} \right) \quad (13)$$

If η is the fractional amount of full $\Delta T = 350^{\circ}$ warmup above a standby 600°F isothermal condition, in general temperature distribution of 4th segment is:

$$T = 600 + 175\eta + 175\eta \sin \left(\frac{x}{L} \cdot \frac{\pi}{6} \right) \quad (13a)$$

Table I

$\frac{x}{L}$	$30 \frac{x}{L}$	$\sin 30 \frac{x}{L}$	$175 \sin \frac{30x}{L}$	T	F
0					
	1.5	.0262	4.6	779.6	.25 + 179.6a
.1					
	4.5	.0785	13.7	788.7	.25 + 188.7a
.2					
	7.5	.1305	22.8	797.8	.25 + 197.8a
.3					
	10.5	.1822	31.9	806.9	.25 + 206.9a
.4					
	13.5	.2334	40.9	815.9	.25 + 215.9a
.5					
	16.5	.2840	49.7	824.7	.25 + 224.7a
.6					
	19.5	.3338	58.4	833.4	.25 + 233.4a
.7					
	22.5	.3827	67.0	842.0	.25 + 242.0a
.8					
	25.5	.4305	75.3	850.3	.25 + 250.3a
.9					
	28.5	.4772	83.5	858.5	.25 + 258.5a
1.0					

$$T_{av.} = 819.8^{\circ}\text{F}$$

On a per-inch-of-perimeter basis:

$$\Delta p = 15$$

$$dx = 3.366$$

$$\Delta p \, dx = 50.49$$

$$\frac{W}{s} = \frac{383}{53.8} = 7.119$$

Let $a = .0001$ per $^{\circ}\text{F}$ in range $600^{\circ}\text{F} < T < 950^{\circ}\text{F}$, corresponding to linear variation μ from .25 at R.T. to 0.70 at 4575°F^*

Table I(a)

$\frac{x}{L}$	$\mu \Delta p dx$	$\int_{x=0}^x \mu \Delta p dx$	$\int_{x=L}^x \mu \Delta p dx$
0		0	137.321
	13.529		
.1		13.529	123.792
	13.575		
.2		27.104	110.217
	13.621		
.3		40.725	96.596
	13.667		
.4		54.392	82.929
	13.713		
.5		68.105	69.216
	13.757		
.6		81.862	55.459
	13.801		
.7		95.663	41.658
	13.844		
.8		109.507	27.814
	13.886		
.9		123.393	13.928
	13.928		
1.0		136.321	0

* $\mu = 0.70$ reported by Wagner for graphite on graphite at 2450°C

Equation (11) may now be solved for x_0 by assuming linear interpolation to be adequate in interval $.4 \leq \frac{x}{L} \leq .5$ (1st diff. in $\mu \Delta p \, dx$ nearly constant over L)

$$F_2 = m_2 x + b_2 = \frac{13.713}{.1} \left(\frac{x_0}{L} - .4 \right) + 54.392 \quad (14)$$

$$F_1 = m_1 x + b_1 = - \frac{13.713}{.1} \left(\frac{x_0}{L} - .4 \right) + 82.929 \quad (15)$$

For equilibrium

$$F_1 - F_2 = W \quad (16)$$

$$82.929 - 137.13 \left(\frac{x_0}{L} - .4 \right) - \left[54.392 + 137.13 \left(\frac{x_0}{L} - .4 \right) \right] = 7.119$$

$$\text{or } \frac{x_0}{L} = .4 + \frac{21.418}{274.26} = .47809$$

$$x_0 = .47809 (33.66) = 16.093^{\text{in}}$$

Now consider a cold step transient moving up the can so that T of each element of length $dx = \frac{x}{10L}$ successively drops to 600°F . The corresponding coefficient friction would be $\mu = .25$ and friction force available per element of length per unit circumference:

$$\mu \Delta p \, dx = .25 (15) (3.366) = 12.622 \text{ lb/in}^2$$

On following pages x_0 is calculated for arrival of step at each tenth of L of segment.

Until transient approaches x_0 , note that intercept b_2 is the only parameter affected in the interpolation, by the progressive lessening of friction force below x_0 . As it passes the element of length containing x_0 , slopes become $m_2 = -m_1 = 126.22$, and thereafter b_1 decreases while b_2 remains constant. Last two columns of Table II display net values of intercepts at $x/L = 0.4$ as transient passes corresponding tabular values of $\frac{x}{L}$.

Table II

ELEMENT FRICTION FORCES, $\mu \Delta p \, dx$, DURING PASSAGE OF COLD TRANSIENT

$\frac{x}{L}$	<u>Initial</u>	<u>Final</u>	<u>Loss</u>	Integrated Loss in Intercept at $\frac{x}{L} = .4$		Net Intercept at $\frac{x}{L} = .4$	
				$\Sigma \Delta b_2$	$\Sigma \Delta b_1$	b_2	b_1
0				0	0	54.392	82.929
	13.529	12.622	.907				82.929
.1		12.622		.907	0	53.485	82.929
	13.575	12.622	.953				82.929
.2		12.622		1.860	0	52.532	82.929
	13.621	12.622	.999				82.929
.3		12.622		2.859	0	51.533	82.929
	13.667	12.622	1.045				82.929
.4		12.622		3.904	0	50.488	82.929
	13.713	12.622	1.091	3.904		50.488	
.5		12.622		3.904	1.091	50.488	81.838
	13.757	12.622	1.135	3.904		50.488	
.6		12.622		3.904	2.226	50.488	80.703
	13.801	12.622	1.179	3.904			
.7		12.622		3.904	3.405	50.488	79.524
	13.844	12.622	1.222	3.904		50.488	
.8		12.622		3.904	4.627	50.488	78.302
	13.886	12.622	1.264	3.904		50.488	
.9		12.622		3.904	5.891	50.488	77.038
	13.928	12.622	1.306	3.904		50.488	
1.0		12.622		3.904	7.197	50.488	75.732

NOTE: a. Columns 2, 3, 4 are forces associated with values of x/L in Col. 1.

b. For Cols. 5-8, Col. 1 signifies location of cold step transient.

Transient step in temperature arrives at $\frac{x}{L} = 0.1$:

$$-137.13 \left(\frac{x_o}{L} - .4 \right) + 82.929 - \left[53.485 + 137.13 \left(\frac{x_o}{L} - .4 \right) \right] = 7.119$$

$$\frac{x_o}{L} = .4 + \frac{22.325}{274.26} = .48140, x_o = 16.204 \text{ in}$$

at $\frac{x}{L} = 0.2$:

$$\frac{x_o}{L} = .4 + \frac{23.278}{274.26} = .48488, x_o = 16.321$$

$$0.3: \quad \frac{x_o}{L} = .4 + \frac{24.277}{274.26} = .48852, x_o = 16.444$$

$$0.4: \quad \frac{x_o}{L} = .4 + \frac{25.322}{274.26} = .49233, x_o = 16.572$$

At $\frac{x}{L} = 0.5$,

$$m_2 = -m_1 = 126.22$$

$$-126.22 \left(\frac{x_o}{L} - .4 \right) + 81.838 - \left[126.22 \left(\frac{x_o}{L} - .4 \right) + 50.488 \right] = 7.119$$

$$\frac{x_o}{L} = .4 + \frac{24.231}{252.44} = .49599, x_o = 16.695$$

0.6:

$$-126.22 \left(\frac{x_o}{L} - .4 \right) + 80.703 - \left[126.22 \left(\frac{x_o}{L} - .4 \right) + 50.488 \right] = 7.119$$

$$\frac{x_o}{L} = .4 + \frac{23.096}{252.44} = .49149, x_o = 16.544$$

0.7:

$$\frac{x_o}{L} = \frac{21.917}{252.44} + .4 = .48682, x_o = 16.386$$

$$0.8: \quad \frac{x_o}{L} = .48198, x_o = 16.223$$

$$0.9: \quad = .47597, x_o = 16.055$$

$$1.0: \quad = .47180, x_o = 15.881$$

Can is now in 600°F isothermal condition in way of 4th segment.
Startup from 600°F.

Since centroidal abscissa of a sine segment is not affected by amplitude in μ is assumed linear with temperature, x_o would be linear with temperature. Spot check at $\eta = .5$ by equation 13a:

$$T = 600 + 87.5 + 87.5 \sin \left(30 \frac{x}{L} \right)$$

is carried out in Table II (a) following bears out linearity of x_o .

Table II(a)

$\frac{x}{L}$	$.875 \sin 30 \frac{x}{L}$	T	$\mu = .25 + .0001(T-600)$	$\mu \Delta p \frac{dx}{L} = 50.49\mu$	F_2	F_1
0					0	131.774
	2.3	689.8	.25898	13.076		
.1					13.076	118.698
	6.9	694.4	.25944	13.099		
.2					26.175	105.599
	11.4	698.9	.25989	13.122		
.3					39.297	92.477
	15.9	703.4	.26034	13.145		
.4					52.442	79.332
	20.4	707.9	.26079	13.167		
.5					65.609	66.165
	24.8	712.3	.26123	13.190		
.6					78.799	52.975
	29.2	716.7	.26167	13.212		
.7					92.011	39.763
	33.5	721.0	.26210	13.233		
.8					105.244	26.530
	37.7	725.2	.26252	13.255		
.9					118.499	13.275
	41.8	729.3	.26293	13.275		
1.0					131.774	0

$$T_{av.} = 709.9$$

$\eta = .5$, sine temperature distribution

At $x/L = .4$:

$$b_2 = 52.442 \text{ lb/in}$$

$$b_1 = 79.332 \text{ lb/in}$$

$$m_2 = -m_1 = 131.67 \quad \text{in interval } \left(.4 < \frac{x}{L} < .5 \right)$$

Equilibrium equation (16):

$$- 131.67 \left(\frac{x_o}{L} - .4 \right) + 79.332 - \left[131.67 \left(\frac{x_o}{L} - .4 \right) + 52.442 \right] = 7.119$$

$$\frac{x_o}{L} = .4 + \frac{19.771}{263.34} = .47508, x_o = 15.991$$

In Figure II the indicated excursion of x_o/L during a warmup-to-full-power-and-scrum cycle is plotted vs fraction of full ΔT and position of cold step transient for the 4th segment from bottom of can.

FIG. II EXCURSION OF POINT OF NO SLIP UNDER ASSUMPTION THAT μ IS TEMPERATURE DEPENDENT (NUMERICAL EXAMPLE)

Table III

AVERAGE TEMPERATURE OF SEGMENT-CAN INTERFACE DURING SCRAM, 4th SEGMENT

$\frac{x}{L}$	T_{initial}	T_{final}	T_{av} as cold step passes indicated $\frac{x}{L}$	$T_{\text{av}} - 75$	L Free length of can in way of 4th segment	ΔL
0			819.8	744.8	33.8806	
	779.6	600				.0054
.1		600	801.8	726.8	33.8752	
	788.7	600				.0055
.2		600	782.9	707.9	33.8697	
	797.8	600				.0059
.3		600	763.1	688.1	33.8638	
	806.9	600				.0061
.4		600	742.4	667.4	33.8577	
	815.9	600				.0064
.5		600	720.8	645.8	33.8513	
	824.7	600				.0067
.6		600	698.3	623.3	33.8446	
	833.4	600				.0069
.7		600	675.0	600	33.8377	
	842.0	600				.0071
.8		600	650.8	575.8	33.8306	
	850.3					.0074
.9		600	625.8	550.8	33.8232	
	858.5					.0077
1.0			600	525	33.8155	

where

$$L = 33.66 + 33.66 (8.8)(10)^{-6} (T_{\text{av}} - 75)$$

N.B.: Mechanical strain and thermal lag of graphite are neglected

Table IV

DISPLACEMENT OF CAN RELATIVE TO SEGMENT AT LOWER EDGE OF 4th SEGMENT
DURING SCRAM

$\frac{x}{L}$	$\frac{x_o}{L}$	$\frac{x_o}{L}$	L	$\frac{x_o}{L} (\Delta L)$ (-upward)
0	.47809			
		.47974	-.0054	-.00259
.1	.48140			
		.48314	-.0055	-.00266
.2	.48488			
		.48670	-.0059	-.00287
.3	.48852			
		.49042	-.0061	-.00299
.4	.49233			
		.49416	-.0064	-.00316
.5	.49599			
		.49374	-.0067	-.00331
.6	.49149			
		.48916	-.0069	-.00338
.7	.48682			
		.48440	-.0071	-.00344
.8	.48198			
		.47948	-.0074	-.00355
.9	.47697			
		.47438	-.0077	-.00365
1.0	.47180			
Total =				<u>-.02960</u> in

Change in length of can during warmup:

Increase in $T_{av} = 819.8 - 600 = 219.8^\circ F$

$$\Delta L = L(\alpha_S - \alpha_G)\Delta T = 33.66 (8.8) (10)^{-6} (219.8) = .06511$$

Displacement of can relative to lower edge of 4th segment during warmup:

$$\text{Av. value } \frac{x_o}{L} = \frac{.47809 + .47180}{2} = .47494$$

Displacement (+ downward) at $x = 0$:

$$\Delta L \left(\frac{x_o}{L} \right) = .06511 (.47494) = .03092$$

Net displacement per full-power scram:

$$.03092 - .02960 = .00132^{\text{in}} \text{ downward for can (upward for graphite).}$$

This is an estimated order-of-magnitude effect of increase in coefficient friction of .0001 per °F. It is far too small to have been significant in the small number of scrams at Hallam that would have produced cold steps of the magnitude assumed here.

Kinetic Dependence of Friction:

Another possible upset of the friction force balance during scram suggests itself from study of Nightingale's Table 6.17, loc. cit. that shows reductions in dynamic compared to static friction as follows (from Wagner, ref. 4)

<u>Material</u>	<u>Test Temp °C</u>	<u>Atmos.</u>	<u>Coefficient of Static</u>	<u>Friction Dynamic</u>	<u>Reduction %</u>
Graph-i-tite					
G on itself	25	air	0.35	0.25	29
same	2450	helium	0.65	0.40	38
ATJ on					
itself	25	air	0.33	0.24	27
same	2450	helium	0.70	0.28	60
H 4 IM					
on itself	25	air	0.34	0.26	24
same	2450	helium	0.48	0.27	44
ATJ on					
H 4 IM	25	air	0.33	0.26	21

It would be expected that the rising layer of colder coolant during scram would tend to shift progressively the behavior of the can-to-graphite interface from the static to the dynamic friction regime for these reasons:

1. Rapid cooling of can would cause a band of relatively rapid displacement to travel up can that would inhibit reestablishment of contact conditions required to develop static friction.
2. Peripheral shrinkage of can would destroy adhesion of interface asperities and thereby help maintain the dynamic condition of friction.
3. The decreasing periphery of can relative to graphite would shift part of the bearing pressure from flats and scallops to transition radii, affecting the grip of axial static friction.
4. The temperature distribution between flats and scallops would probably be upset, tending to change friction force directions locally and break up static friction patterns.
5. There is evidence from the large component test loop experiments that during the thermal shocking (and therefore presumably during scram in service) that cross convection causes large temperature swings and rapid reversals of local temperature differences. The resulting thermal expansion response of the thin can would tend to maintain the portion immersed in the rising cold wave of sodium well into realm of dynamic friction.

Wagner reported losses of friction coefficient in entering the moving regime in a range of 21 to 29 percent in air at room temperature. In helium at 4442°F range was 44-60 percent loss.

The calculations presented for the effect of friction coefficient increasing by .0001 per deg. F indicate the result of imposing a reduction to $\mu = .2719$ at 819.8°F, or an 8 percent loss, that caused an estimated thermal ratchet of $.0013^{\text{in}}$ upward per full-power scram. Wringing a top estimate of ~ 32 percent loss of friction (static to dynamic transition) from Wagner's results would be expected to yield

$\sim .005^{\text{in}}$ net upward shift of 4th segment per full-power scram, hardly enough to cause early difficulties of HNPF cans.

Complete loss of friction in leading part of scram wave by intermittent local loss of contact through buckling as successive cold and hot striae of coolant pass plus the local annihilation of ordered direction of friction forces by rapid reversal of hot and cold areas might not be a completely fantastic concept. From the foregoing estimates, the ratchet displacement per scram would then approach the order of $.015^{\text{in}}$ for the fourth segment and perhaps $.023^{\text{in}}$ for the sixth (top) segment. Such hypothetical values would approach one quarter of the relative thermal expansion of graphite segment and contiguous part of can between 600°F and temperature distribution corresponding to full power.

If the lack of knowledge of actual flow patterns in the quasi-stagnant sodium in contact with the housings of the process tubes and moderator elements warrants further speculation, it might be that the superposition of local vorticity around the rapidly cooling process tubes on the overall vortex induced by the more rapid fuel coolant flow at the center of the core would result in an ordered direction of travel of alternations of temperature along the sheath of an element. These might lead to a number of increments of unilateral (ratcheting) displacement during a single scram incident.

Conclusions

The study reported here indicates that, with the magnitude of pressure-induced friction forces that could be expected, strong dependence of friction intensity on temperature or motion would not induce sufficient ratcheting to cause the early moderator can difficulty at HNPF if the sheaths are cooled monotonically during scram. However, the violent and rapid fluctuations of temperature during cyclic tests noted in Reference 2 and the small thermal lag of the 16 mil sheath in sodium prevent ruling out ratchet as a damaging factor.

REFERENCES

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3. Nuclear Graphite, R. E. Nightingale, Editor, Academic Press, N.Y. 1962, p. 160.
4. P. Wagner et al., Some Mechanical Properties of Graphite in the Temperature Range 20 to 3000°C, in Proc. 2nd UN Internat. Conf. on the Peaceful uses of Atomic Energy, Geneva, 1958, v 7, pp 379-388 United Nations, N.Y., 1959.

APPENDIX A

Analysis of Elastic Buckling with Lateral Pressure
and Bowing of the Moderator Sheath by an Appropriate
Solution of Donnell's Equation Utilizing the Galerkin
Technique.

1. Summary

The objective of the analyses performed was to predict by means of analysis the behavior of the metal sheath that envelops the graphite logs resulting from an environment which includes temperature variations and lateral pressure. A portion of the moderator element is considered, rather than the entire length as shown in Figure A-1. Two specific problems have been considered; the first one investigates the tendency to buckle and the second deals with the tendency to bow under a diametral thermal gradient. Donnell's Equations are modified to include the effect of a thermal gradient and an approximate solution is obtained by application of the Galerkin Technique. The analysis of the buckling problem needs to be developed in greater detail, however the analysis of the diametral thermal gradient problem has been developed in detail and the analytic results obtained.

1.1 Buckling Problem

Two specific problems are posed. The first problem, concerned with buckling, considers the thermal gradient to be symmetric about any diameter through the center line of the graphite log and it is assumed that separation of two logs has taken place due to the combination of the sheath hugging the logs due to external pressure and axial expansion due to an overall rise in temperature from start-up. There result surface tractions on the sheath due to friction. These surface tractions are idealized into axial compressive edge loads assumed to be acting on the sheath at the mid-height of two adjacent graphite logs. Due to the symmetry of the thermal gradient we are able to consider a one-twelfth segment of the sheath cross-section and we are to find that value of axial compressive edge load to cause buckling of the sheath under the influence of a thermal gradient which may vary circumferentially and axially, but not through the thickness, together with the lateral pressure, and under the non-holonomic kinematic restraint that requires the buckling to be outward.

SECTION A-A

BUCKLING PROBLEM-MODEL FIG. A1

1.2 Diametrical Thermal Gradient Problem

The second problem assumes that the circumferential gradient is symmetric about only one diameter of the sheath cross-section such that the gradient is linear along this diameter. The axial compressive edge load is assumed fixed. The temperature along the length of the sheath is assumed not to vary. The symmetry of this problem demands that we consider one half the sheath cross-section. We are to find the circumferential stresses in the sheath due to an increasing diametral thermal gradient under the influence of lateral pressure and the same kinematic restraints of the first problem.

2. Models

2.1 Buckling Problem

In order to solve the problems posed in 1.1 and 1.2 we consider two models. The model for the Buckling Problem is shown in Figure A-1. We consider a one-twelfth segment of the sheath cross-section consisting of half of one flat and half of one scallop on either side of one transition radius. The axial distance is indicated by x and the circumferential distances are given by y_i where i is the plate designation. The thermal gradient along the circumference is assumed to be expressed as a Fourier expansion and along the axial distance by a linear function. Plate phenomena only (i.e., transverse deflections only) are considered significant.

2.2 Diametral Thermal Gradient Problem

The model for the Diametral Thermal Gradient Problem is similar to the one shown in Figure A-1 except that here we must consider one-half of the sheath cross-section consisting of 13 plates, i.e., two half scallops at each end and intermediate scallops, transition radii and flats. The temperature is assumed linear across the diameter. The circumferential

temperature variation is obtained by projection which leads to a linear temperature variation in the flats, but a trigonometric temperature variation in the transition radii and scallops. No temperature variation is assumed in the axial direction.

3. Method

3.1 Simplifications

We are primarily interested in the "w" deflections of the sheath transverse to the sheath mid-surface since these deflections are accompanied by high bending-stresses due to the low flexural stability of the sheath. In keeping with this objective and in order to simplify our problem we consider only the plate behavior of the sheath segment together with the assumption that the transverse deflection is sufficiently large so as to cause coupling between transverse displacement and membrane forces but not so large as to violate the assumption of linear stress-strain relations. In addition we introduce the approximations in the governing plate equations due to Donnell and are led to an eighth order governing equation for the curved plate and a fourth order equation for the flat plate, both with respect to the unknown transverse displacement, w.

3.2 Boundary Requirements

Consider a curved plate segment whose curvilinear coordinates are given by x in the axial direction and y in the circumferential direction such that:

$$0 \leq x \leq L$$

$$0 \leq y \leq b$$

Note that our models consist of a portion of a closed shell which in turn is broken up into flat and curved plate segments. For completeness we should specify the following:

$$\begin{array}{ll}
 \text{I} & \left\{ \begin{array}{l} \text{at } x = 0, L \\ 1 \quad N_x \text{ or } u \\ 2 \quad N_{xy} \text{ or } v \\ 3 \quad Q_t \text{ or } w \\ 4 \quad M_x \text{ or } \frac{\partial w}{\partial x} \end{array} \right. \\
 \text{II} & \left\{ \begin{array}{l} \text{at } y = 0, b \\ 1 \quad N_y \text{ or } v \\ 2 \quad N_{yx} \text{ or } u \\ 3 \quad Q_y \text{ or } w \\ 4 \quad M_y \text{ or } \frac{\partial w}{\partial y} \end{array} \right.
 \end{array}$$

Note that we have not specified "effective" shears Q_x , Q_y , N_{xy} , N_{yx} because the edges are portions of a closed shell rather than a real edge. In our problem however we have specified only the boundary conditions II at $y = 0, b$, and the membrane force N_x due to the axial compression. The remaining conditions of I have not been specified because we are primarily interested in the behavior of the moderator sheath at the graphite log joint which occurs at $x = \frac{1}{2} L$ and L is large as compared to b .

At each end ($x = 0, L$) of the segment we might require the transverse deflection be zero so as to introduce the axial compression loads; this condition, however, was not used for reasons given later.

At a plane of symmetry we specify at $y = \text{constant}$:

$$\begin{array}{ll}
 1 & N_y = \text{constant} \\
 2 & N_{yx} = 0 \\
 3 & Q_y = 0 \\
 4 & \frac{\partial w}{\partial y} = 0
 \end{array}$$

At a juncture between two plate segments we specify compatibility conditions:

1 - equal displacement, w

2 - equal rotation

Note that u and v need not be specified since they don't enter into the first of Donnell's equations involving only w.

3.3 Galerkin Technique of Solution

If the governing plate equation is represented by an inhomogeneous linear equation:

$$(1) \quad L(w) = 0$$

then

$$\Omega = \sum_{j=1}^N a_j \phi_j(x,y) \text{ will satisfy } L(w) = 0$$

approximately if we require that:

$$(2) \quad \iint [L(\Omega)] \phi_i dx dy = 0 \quad i = 1, \dots, N$$

Note that (2) provides N equations with N unknowns a_j so that this technique due to Galerkin reduces the problem to one of algebra.

Note that since $L(w) = 0$ is inhomogeneous (2) may be rewritten:

$$(3) \quad [A_{ij}] \{a_j\} = k_j$$

For the stability problem we require that the determinant, $[A_{ij}] = 0$. Since the expressions A_{ij} contain the axial compressive stresses.

For the diametral thermal gradient problem $|A_{ij}| \neq 0$ so that $\{a_j\}$ may be found directly from (3) for a given thermal gradient.

In both cases we are at liberty to choose $\phi_j(x,y)$. Since for both problems we require that $w \geq 0$ we have chosen

$$(4) \quad \phi(x,y) = (\cos q_j x + 1) (\cos r_j y + 1)$$

where

$$(5) \quad q_j = \frac{m_j \pi}{L} \quad r_j = \frac{n_j \pi}{b} \quad (m_j, n_j \text{ are integers})$$

Note that (4) does not guarantee that w will be non-negative since the sign of a_j will influence that final outcome. The consequence of a coefficient a_j being negative is that the non-holomorphic kinetic restraint $w \geq 0$ will be violated. The log itself, however, will provide a lateral pressure which would change the loading picture. This is not accounted for in our analytic model, but for the purposes of this investigation, we would merely suppress any negative a_j that results. Note further that ϕ_j as given by (4) also satisfy the boundary conditions at $y = 0, b$ of zero slope and zero effective shear (involving the first and third derivatives with respect to y respectively) but that they do not satisfy the requirement of zero deflection at $x = 0, L$. The function was not discarded, however, since it was felt to be more important that the boundary conditions at $y = 0, b$ be satisfied together with the requirement of being non-negative.