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Numerical Calculation of Surface Waves:
A Modified ZUNI Code with
Surface Particles and Partial Cells

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by

Anthony A. Amsden

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Anthony A. Amsden

ABSTRACT

Extensions to the original Simplified Marker-and-Cell (SMAC) method computing program, ZUNI, are described. These include Nichols and Hirt's improved free-surface treatment and, in addition, a partial-cell treatment that allows an obstacle face to pass through cell diagonals. Applications of the modified program are shown by calculations of wave runup on both submerged and exposed sloping beaches. The validity of the approach for studies of actual tsunami waves has been demonstrated.

I. INTRODUCTION

The Marker-and-Cell (MAC) method¹ was the first successful numerical technique for calculation of viscous, incompressible flows with a free surface. After its introduction, much attention was given to devising more accurate treatments of the free-surface boundary conditions, which originally used only simple approximations. Hirt and Shannon² incorporated improved approximations to the normal stress conditions, and showed the necessity for satisfying the tangential stress conditions as well. The improved stress conditions were included in the ZUNI code,³ which was the original Simplified-Marker-and-Cell (SMAC) method computing program. In the original MAC and SMAC versions, all pressures or pseudopressures were specified at cell centers. Chan et al.⁴ developed a technique for more accurate delineation of the free surface. This permitted the pressure equation to be adjusted so that the free-surface pressure could be specified at the surface itself, rather than at the center of the surface cell. Nichols and Hirt⁵ modified the Chan procedure, and devised a technique for defining the fluid surface by a set of surface-marker particles that move with the local fluid velocity. These particles allow surface-cell pressures in MAC to be accurately specified by means of linear interpolation (or

extrapolation) between the known values of pressure in the nearest full cell and the desired fluid surface pressure. In addition to providing improved treatment of cell variables at the surface, Nichols and Hirt have clarified the use of the various possible free-surface stress conditions.

This report describes a modified version of the original ZUNI code which includes the Nichols and Hirt free-surface improvements. A partial-cell treatment that allows a rigid free-slip obstacle to be placed through cell diagonals has also been included. The desired boundary slope is obtained by choosing the appropriate aspect ratio for the cells of the mesh.

Validity of the free-surface techniques had previously been demonstrated by the accurate, smooth, and stable calculation of large-amplitude surface motions, but the main objective in this modified version of ZUNI is to incorporate the highly accurate treatment now available for low-amplitude free-surface motions, in particular, to follow the growth of waves as they interact with a beach or continental slope. The success of the project has been demonstrated by the numerical simulation of tsunami waves with much greater accuracy than was possible previously.⁶ Since such waves may have width-to-height ratios on the order of 10^5 to 1,

for which the surface would appear to be entirely flat, the free-surface treatment must respond to the very slight variations that exist in the vertical direction.

Section II of this report describes the modifications to ZUNI to include, first, the surface-marker particles and then the partial-cell treatment. Section III illustrates the capabilities of the modified ZUNI code by means of a number of representative calculations. Some discussion of graphics for displaying calculational results is presented, especially concerning techniques for visualizing the extreme low-amplitude, long-wavelength flows. Detailed discussion of the numerical results and comparison with experimental data are provided by Mader.⁶ In the rest of this report, familiarity with the basic SMAC technique³ is assumed.

II. ZUNI CODE MODIFICATIONS

A. Surface-Marker Particles

1. Adding the Surface-Marker Capability. The Nichols and Hirt scheme uses a set of special marker particles spaced at intervals along the surface, the exact surface configuration being given by line segments joining the particles in sequential order. These surface markers are moved with the local fluid velocity, just as ordinary marker particles are. The value of some field variable at the center of a surface (SUR) cell is based upon a linear interpolation or extrapolation using the known values at the free surface and the nearest full (FUL) cell center. The interpolated value is a function of the distance from the actual surface to the center of the nearest FUL cell, along a line connecting the centers of the SUR and FUL cells. This distance is denoted by d in Fig. 1. The general form of the interpolation is

$$q_{SUR} = (1 - \eta)q_{FUL} + \eta q_{free\ surface}$$

where q is some cell-centered variable and η is the ratio of the distance between cell centers to the distance d . When the SUR cell has more than one FUL neighbor, as in Fig. 1, the interpolation process chooses the neighbor whose intersection point lies nearest the center of the SUR cell.

Once every cycle, values of η are calculated and stored for every SUR cell in the mesh. In addition, a special flag, NF , that indicates which of the four neighboring cells is the interpolation neighbor, is stored for every SUR cell. NF is 1, 2, 3, or 4 for a nearest FUL neighbor on the right, top, left, or bottom, respectively. $NF = 5$ is used to indicate the SUR cell has no FUL neighbor, a case that requires special treatment. Cell-storage arrays must be created for η and NF , and a separate particle-storage array must be created for the surface-marker coordinates.

When the surface-marker-particle feature is incorporated in SMAC, the code must be modified and expanded in a number of places. Particular attention must be given to the following ten points:

- (1) A single, new, input variable specifying the approximate surface-marker spacing controls the use of the surface-marker feature. We use a quantity DSP (δ spacing), defined as the reciprocal of the desired average number of surface particles across a cell. In the examples in Sec. III, DSP = 0.3 is a typical value; it will maintain approximately 10 particles for every $3\delta x$ of surface. By specifying DSP = 0 in the input, we omit the surface-marker option from a calculation. When DSP > 0, the setup must generate a set of surface markers ordered from left to right along the free surface, with the end particles just slightly outside the mesh to "tie" the set into the boundaries. Failure to do this will allow the particle line to recede from the walls.
- (2) In the Reflagging section, the surface markers must be allowed to participate just as fully as the regular markers.
- (3) Once every cycle, the set of surface markers must be edited. First, delete any surface markers in FUL cells. Second, remove surface particles

Fig. 1. The interpolation neighbor of the SUR cell is the FUL cell below it.

from coalescing regions whenever they become too closely spaced to be efficient. The minimum spacing we allow in cell units is DSP/2. Third, allow for the opposite situation in expanding regions; we create additional particles whenever the spacing exceeds 1.1 times DSP, regardless of the type of cell in which the new particle is placed. Finally, the Reflagging (step (2) above) is repeated with this revised surface-marker configuration.

(4) Following completion of step (3), a sweep is made through the mesh, calculating new values of η and NF for every SUR cell encountered.

(5) In the θ calculation, set

$$\theta_{\text{SUR}} = (\theta_{\text{SUR}} - \theta_{\text{FUL}})\eta + \theta_{\text{FUL}}.$$

(If NF = 5, $\theta_{\text{SUR}} = 0$.)

(6) It is crucial that the FUL-cell interpolation neighbors be underrelaxed to prevent an oscillating instability in the ψ field from arising in the iteration. The reason for this is described by Nichols and Hirt.⁵ We define a relaxation parameter α_{FUL} as a function of η and the input α , now called α_0 :

$$\alpha_{\text{FUL}} = \min \left\{ \alpha_0, \alpha_{\text{FUL}}, \left[\frac{4(1 + \alpha_0)}{4 - (1 + \alpha_0)(1 - \eta)} - 1 \right] \right\}.$$

This selection process causes α_{FUL} to respond to the most stringent case when the FUL cell is serving as the interpolation neighbor of more than one SUR cell simultaneously. The simplest way to handle the varying α_{FUL} values, which must be adjusted every cycle, is to create a cell-storage array for α_i^j values, initializing $\alpha_i^j = \alpha_0$ for all cells, and then selectively resetting α_i^j in all FUL cells.

(7) In the iteration procedure, after every sweep of the FUL cells, perform a second sweep, setting $\psi_{\text{SUR}} = (1 - \eta)\psi_{\text{FUL}}$, assuming that ψ at the free surface is zero. This loop is inserted at the place where exterior ψ values are set. When NF = 5, set $\psi_{\text{SUR}} = 0$.

(8) The surface-marker particles are moved and plotted in nearly the same manner as the regular marker particles. When plotting the surface markers, we represent them by an asterisk rather than the usual dot, to provide a definite visual distinction between the two types of particles.

(9) The condition of vanishing tangential stress,

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0,$$

is satisfied in ZUNI in setting the tangential velocities just outside the free surface. However, this modified version of the code is intended primarily for high-Reynolds-number bore and wave studies, for which it is not possible to resolve boundary layers, as they are much thinner than a cell dimension. Accordingly, we have found that a "free-slip" condition ($u_{\text{out}} = u_{\text{in}}$) gives a much better approximation to the physical velocity profile.⁵ In ZUNI, this change simply involves the deletion of the $\delta r / \delta z$ and $\delta z / \delta r$ terms from the four equations used in setting the just-outside tangential velocities in statement No. 330.

(10) Several possible cases must be considered in setting normal velocities on empty (EMP) cell faces of SUR cells for bore and wave calculations, the treatment depending upon the number and location of the EMP neighbors:

(a) If the SUR cell has one empty face, the velocity on that face should be chosen such that $\nabla \cdot \vec{u} = 0$ for the cell, which is the usual treatment.

(b) If the SUR cell has either three empty faces or two empty faces opposite one another, no general prescription can be stated and the standard ZUNI treatment is recommended.

(c) If the SUR cell has two adjacent empty faces, however, the standard ZUNI treatment is inappropriate. Consider a wave front propagating from left to right. As indicated in Fig. 2, the original treatment sets the u-velocity component across

Fig. 2. Original ZUNI velocity treatment for SUR cell with EMP neighbors on right and above.

laterally. For bore and wave applications, however, this allows momentum to be transferred forward too rapidly, causing a noticeable "skimming" of the surface. The wave front must be retarded somewhat to compare favorably with experimental data for flows of this type. This can be accomplished by setting $u = u_{FUL}$ from the nearest FUL neighbor. In general, this will be the cell below, which should also be the interpolation neighbor. Finally, v_{TOP} is chosen to ensure that $\nabla \cdot u = 0$ for the SUR cell. Fig. 3 shows the recommended treatment for the configuration of Fig. 2.

In ZUNI, this change in setting velocities of SUR cells with adjacent empty faces is made in two places for the geometries we are considering:

SUR cell with EMP faces on right and above:

$$u_{i+\frac{1}{2}}^j = u_{i+\frac{1}{2}}^{j-1},$$

SUR cell with EMP faces on left and above:

$$u_{i-\frac{1}{2}}^j = u_{i-\frac{1}{2}}^{j-1}.$$

These changes occur at statement No. 653, which becomes " $U(I,J) = U(I,J-1)$," and at statement No. 655, which becomes " $U(I-1,J) = U(I-1,J-1)$."

2. Using Surface Markers as the Only Particles.

With the free-surface marker-particle treatment added to the SMAC code as described above, the regular marker particles generally cease to be useful, and it is practical to modify the code to run without them. This will not only enhance computing efficiency, but will also allow greater freedom in specifying conditions at the inflow boundary.

Fig. 3. Modified ZUNI treatment appropriate for bores and waves.

Consider that a true physical wave oscillates, and to model this oscillation requires the specification of negative as well as positive fluid velocities at an "inflow" boundary. This, in turn, would require special treatment for any regular marker particles at this boundary, as they would attempt to leave the system and subsequently reenter.

In the ZUNI code, changes need to be made at only four places to delete the use of regular marker particles:

- (1) Turn off regular marker-particle generation with a "GØ TØ 90" inserted just before statement No. 50. Note that the "NX, NY, ---" etc. data card remains in use, however, to define the region to be surrounded by surface-marker particles.
- (2) Insert a "Pre-flagger" just after statement No. 262. This routine consists of a reversed double DØ loop that sweeps the grid from top to bottom. The logic is to flag all cells as EMP in each column of cells until a cell that contains surface markers is reached. This cell is flagged SUR, below which all remaining cells in the column are flagged FUL. Be careful, however, not to destroy any ØB (obstacle) flags. The Pre-flagger prevents the subsequent, ordinary Reflagger from reflagging the FUL cells to EMP, which it would otherwise do because they contain no particles.
- (3) The UL (inflow-boundary velocity) can now be a periodic function, allowing the flow to truly oscillate. The ordinary limitation of a minimum inflow velocity of zero (given by $\sin^2 \omega t$, for example) is removed.
- (4) The creation of columns of regular marker particles at the inflow boundary is bypassed by re-writing statement No. 735 as "NP = NPN \$ GØ TØ 770." (Formerly, the "GØ TØ 770" was done only if TYPE = 0, which meant that the problem contained no inflow boundary.)

B. The Partial-Cell Treatment

For simplicity, we restrict discussion of the partial (PAR) cell boundary condition to allow only a rigid, free-slip boundary that follows cell diagonals. The boundary slope is adjusted by selecting the appropriate aspect ratio of the cells, $\delta y / \delta x$, as shown in Fig. 4. Furthermore, the lower-right half of the cell is the obstacle side, and the upper-left half is the fluid side. A sloping boundary in this particular orientation is convenient for

Fig. 4. The partial (PAR) cell as described here.

use with the inflow boundary along the left side of the mesh in ZUNI. From the expressions described below for this case, one can derive similar expressions for the other three possible orientations of this diagonal boundary.

In ZUNI, we set TYPE = 2 in the input data to distinguish the partial-cell obstacle setup from the original rectangular (TYPE = 1) obstacle. We then use L5 and L6 as the two dimensions required to specify the location and extent of the sloping wall, as indicated in Fig. 5. These dimensions are given in integer numbers of cells, and we limit $\bar{I} \geq L5 > 1$ and $\bar{I} \geq L6 > L5$. The cells that lie below the PAR cell in each column are $\emptyset B$ (obstacle) cells, as are the cells in the level portion of the obstacle to the right of L6. The PAR cells are given a flag [F(I,J)] of 0.5 in ZUNI, regardless of whether the fluid side of the cell is submerged or exposed. This fits in as well as possible with the original flagging scheme, allowing most of the flag tests on ".GE." and ".LE." to remain unchanged.

It is convenient to describe the fully submerged and the partially exposed or "partial-surface" cells separately. First, consider the simpler case:

Fig. 5. The diagonal boundary in ZUNI, created from the partial cells of Fig. 4.

1. The Fully Submerged PAR Cell.

(a) Zeroes are permanently stored in the appropriate PAR-cell velocity locations (Fig. 6), just as in the $\emptyset B$ cells.

(b) $\theta = 0$ in the PAR cells.

(c) The $\tilde{u}$ and $\tilde{v}$ equations must be modified to allow for the presence of the (free-slip) diagonal boundary. In calculating $\tilde{u}_{i+\frac{1}{2}}^j$, sense whether cell $\left(\frac{j}{i+1}\right)$ is a PAR cell. If it is, the $\tilde{u}$ equation to be used is:

$$\begin{aligned} \frac{\tilde{u}_{i+\frac{1}{2}}^j - u_{i+\frac{1}{2}}^j}{\delta t} = & \frac{u_{i+\frac{1}{2}}^j u_{i-\frac{1}{2}}^j - (u_{i+\frac{1}{2}}^j)^2}{\delta x} \\ & + \frac{u_{i+\frac{1}{2}}^j v_{\frac{1}{2}}^{j-\frac{1}{2}} - (uv)_{i+\frac{1}{2}}^{j+\frac{1}{2}}}{\delta y} + \frac{\theta_{\frac{1}{2}}^j - \theta_{i+1}^j}{\delta x} \\ & + g_x + v \left[\frac{u_{i+\frac{1}{2}}^{j+1} - u_{i+\frac{1}{2}}^j}{\delta y^2} - \frac{v_{i+1}^{j+\frac{1}{2}} - v_{\frac{1}{2}}^{j+\frac{1}{2}}}{\delta x \delta y} \right]. \end{aligned}$$

If cell $\left(\frac{j}{i}\right)$ is itself a PAR cell, its $\tilde{v}$ (at $\frac{j+1}{i}$) is given by:

$$\begin{aligned} \frac{\tilde{v}_{\frac{1}{2}}^{j+\frac{1}{2}} - v_{\frac{1}{2}}^{j+\frac{1}{2}}}{\delta t} = & \frac{(v_{\frac{1}{2}}^{j+\frac{1}{2}})^2 - v_{\frac{1}{2}}^{j+\frac{1}{2}} v_{\frac{1}{2}}^{j+3/2}}{\delta y} \\ & + \frac{(uv)_{i-\frac{1}{2}}^{j+\frac{1}{2}} - u_{i+\frac{1}{2}}^{j+1} v_{\frac{1}{2}}^{j+\frac{1}{2}}}{\delta x} + \frac{\theta_{\frac{1}{2}}^j - \theta_{\frac{1}{2}}^{j+1}}{\delta y} \\ & + g_y + v \left[\frac{u_{i-\frac{1}{2}}^{j+1} - u_{i-\frac{1}{2}}^j}{\delta x \delta y} - \frac{v_{\frac{1}{2}}^{j+\frac{1}{2}} - v_{\frac{1}{2}}^{j-1}}{\delta x^2} \right]. \end{aligned}$$

(Although a freeslip diagonal boundary is appropriate for the modified program described here, the velocity equations could be adjusted straightforwardly to account for the presence of a noslip boundary instead.)

Fig. 6. The two velocities that are permanently zeroed in any PAR cell.

(d) The $\tilde{D}$ for the partial cell is given by:

$$\tilde{D}_1^j = \frac{\tilde{v}_1^{j+\frac{1}{2}}}{\delta y} - \frac{\tilde{u}_1^{j-\frac{1}{2}}}{\delta x}.$$

With the appropriate velocities zeroed, as in item (a), the regular $\tilde{D}$ formula will automatically give this result.

(e) In the iteration, submerged PAR cells, in addition to the FUL cells, participate in the relaxation process. The ψ of such a PAR cell $\left(\frac{j}{i}\right)$ is given by:

$$h^{+1}\psi_1^j = \frac{1 + \alpha_1^j}{\left(\frac{1}{\delta x^2} + \frac{1}{\delta y^2}\right)} \left(-\tilde{D}_1^j + \frac{h^{+1}\psi_{i-1}^j}{\delta x^2} + \frac{h\psi_i^{j+1}}{\delta y^2} \right) - \left(\alpha_1^j\right) \left(h\psi_1^j\right).$$

(f) In the particle movement, the zero velocities stored on the bottom and right faces of PAR cells in item (a) must be effectively ignored. Failure to do so will cause a false drag along a wall that is meant to be truly free slip. The weighting scheme should use the calculated u and v velocity components from the fluid side for the other one or three velocities, as required. The location of a velocity component is tested to see whether or not it lies on the fluid side of the obstacle, and the calculated u or v is used if it does not, for the components indicated in Fig. 7.

Next, consider the extensions of the basic treatment to allow for the more complex case.

2. The Partial-Surface Cell

(a) The line of surface-marker particles must cut off at the diagonal interface. As in the basic surface-marker usage described in Sec. A, the final

Fig. 7. Velocity-weighting adjustment for particles near the free-slip diagonal boundary.

marker must be constrained to remain just inside the obstacle to "tie" the particle line in, whereas all other surface markers must be kept on the fluid outside the obstacle, as shown in Fig. 8. Furthermore, it is crucial that the final marker be kept from moving too deeply into the obstacle, or it will eventually move to an area of zero velocities and become frozen. If regular marker particles are in use, and one moves into the obstacle, it should simply be deleted from the system. More specifically, ZUNI has the following special particle treatments:

If a regular marker particle moves into the obstacle, it is deleted from the system.

If a surface marker (not the final one) moves into the obstacle, nothing is done at particle-movement time, but the particle is deleted later in the added region where spacing is checked and surface markers are normally deleted or created.

If the final surface marker moves out of the obstacle and into the fluid region, its x or y coordinate is adjusted to place it back in the obstacle.

If the final surface marker moves too deep inside the obstacle (we consider any distance greater than 0.1 cell, in unit-cell dimensions, as too deep), the nearest surface-marker particle lying outside the obstacle is located, and a line is drawn connecting these two particles (Fig. 9). The final surface marker is moved along this line to a position 0.05 cell inside the obstacle. The constraint of the final surface marker to lie within this 0.05-cell belt applies equally along the level face of the obstacle.

(b) $\theta = 0$ in partial-surface cells, just as in submerged partial cells.

(c) Velocities must be specially set near partial-surface cells. Since the double $D\theta$ loops advance from left to right and then upward through the mesh, having the diagonal boundary on the lower right

Fig. 8. The final surface-marker particle should lie in the obstacle.

Fig. 9. The final particle is moved back up closer to the surface of the obstacle whenever it moves in too deep.

Fig. 11. The PAR cell is included in the ψ iteration.

allows special velocity adjustments to be easily propagated upward along the diagonal near the intersection of surface and boundary. Velocities are set in the order indicated for the three cases shown in Fig. 10.

(d) For the ψ iteration, there are a number of important considerations.

(1) For the PAR cells, the following cases are possible.

(a) $NF = 0$ in the PAR cell, meaning that it has no interpolation neighbor. This will happen if:

- The PAR cell is fully submerged (the cell to its left is FUL), and the surface particle line does not intersect the line connecting cell centers, as shown in Fig. 11. The treatment here is to include the PAR cell in the iteration, as mentioned in 1(e) above.

- The cell to the left of the PAR cell is SUR or EMP, as in Fig. 12. In either case, the PAR cell is excluded from the iteration. If the neighbor on the left is EMP, the PAR cell lies entirely above

Fig. 12. The PAR cell is excluded from the ψ iteration if the cell to the left is EMP or SUR.

the surface, and ψ for the PAR cell is not needed. If the cell to the left is a SUR cell, however, set $\psi_{PAR} = \psi_{SUR}$.

(b) $NF > 0$ in the PAR cell if the line of surface-marker particles passes through the PAR cell, and a (FUL) interpolation neighbor can be found on the left (Fig. 13).

Fig. 10. Propagation of velocities near the surface.

Fig. 13. The ψ in the PAR cell is interpolated from the FUL-cell neighbor.

The treatment of this partial-surface cell is the same as that of a regular SUR cell; the intersection point of the particle line with the line connecting cell centers provides the η_{PAR} value. The PAR cell is not iterated; instead,

$\psi_{\text{PAR}} = (1 - \eta_{\text{PAR}}) \psi_{\text{FUL}}$, which is the same formula as that used for ψ_{SUR} .

(2) For the SUR cells with PAR cells below, several cases must be considered. The basic principle is that PAR cells can serve as interpolation neighbors for SUR cells, and that they have the same power as a FUL cell in this respect.

(a) If the PAR cell is fully submerged and $\eta_{\text{SUR}} < 2$, as in Fig. 14, then the PAR cell can serve as the interpolation neighbor for the SUR cell. The PAR cell, which

Fig. 14. The PAR cell is the interpolation neighbor for the SUR cell.

participates in the iteration, must be underrelaxed in the same manner as a FUL-cell interpolation neighbor. Set

$$\psi_{\text{SUR}} = (1 - \eta) \psi_{\text{PAR}}.$$

(b) If, however, the point of intersection of the line of surface markers and the line joining cell centers lies within the PAR cell, as in Fig. 15, then the partial cell is not considered to be fully submerged. In this case, $\eta_{\text{SUR}} > 2$, and it is crucial that this η not be used in setting ψ_{SUR} , as the ψ iteration would diverge even with underrelaxation of ψ_{PAR} . When a SUR cell has a PAR cell below as an interpolation neighbor and $\eta > 2$, set η to 2/3.

(c) The third possibility is that the particle line does not intersect the line joining cell centers, as in Fig. 16. Unless the cell to the left of the SUR cell is a FUL cell, the SUR cell has no interpolation neighbor, and $\psi_{\text{SUR}} = 0$.

III. CALCULATIONAL EXAMPLES

In this section are examples from several calculations made with a modified version of the ZUNI code which includes the sloping bottom boundary and improved free-surface treatment. This code version has shown that it is, indeed, possible to make numerical studies of tsunami waves. These waves had previously proved impractical to calculate because of the severe resolution problems that arise when dealing with waves of such an extreme wavelength-to-amplitude ratio, although successful calculations

Fig. 15. The PAR cell can still serve as the interpolation neighbor for the SUR cell, but η must be reduced.

Fig. 16. The PAR cell cannot serve as an interpolation neighbor for the SUR cell. Unless the cell to the left of the SUR cell is FUL, $\psi_{\text{SUR}} = 0$. See also Fig. 13.

of waves of milder proportions have been reported.⁷

Tsunami waves are generally of little concern when they are travelling over deep water, but when such a wave approaches an island or a continent, the decreasing ocean depth causes the speed of the wave front to decrease and the amplitude to increase drastically. Many disasters have resulted from shoaling waves, and there has, accordingly, been much interest in understanding the interaction of tsunami-type waves with continental shelves and beaches in order to devise more effective means of combating their destructive effects.

The principle of decreasing wavelength with a corresponding increase in amplitude is illustrated in the idealized example of Fig. 17, in which the wave proportions are exaggerated for clarity. The figure shows a sequence of particle plots from the calculation of a single solitary wave passing down a channel and reflecting off a rigid vertical wall at the right boundary. The incoming wave passes over a submerged sloping boundary, and the resulting decrease in channel depth produces the above described change in ratio of wavelength to amplitude. The free-surface-marker particles are represented by heavy asterisks to distinguish them from the regular marker particles. The aspect ratio of the computing cells is 2:1, giving a slope of $26^{\circ} 34'$ on the diagonal boundary.

The first frame in Fig. 17 shows the initial particle configuration before the wave has been introduced, and the succeeding frames show the progress of the wave and its reflection. The eleven

frames in Fig. 18 are velocity vector plots for this problem, with times that correspond to those of the final eleven frames of Fig. 17.

The more realistic calculation of a tsunami wave requires a much longer mesh with a more gradual slope. A typical mesh for this purpose is shown in Fig. 19, which is drawn far from true proportion to make its dimensions readable. Note that the aspect ratio of the cells is 15:1; the resulting slope of the diagonal boundary is $3^{\circ} 50'$. For a detailed study of the surface configuration, a numerical listing of the coordinates of the surface-marker particles is important, but it is also extremely useful to visualize the wave dynamics graphically. A standard particle plot of this grid in its true proportion is useless in illustrating the wave. As shown in Fig. 20, the grid appears practically as a line. Even with the vertical scale exaggerated as much as the film frame will allow, the free surface still appears flat throughout the entire calculation (Fig. 21), because the amplitude variation of the wave occurs over a very small fraction of a cell height.

An effective visual plot must, therefore, not only exaggerate the vertical scale, but must also eliminate its lower part. This was done for the sequence of x:y plots of the free-surface-marker particles in Fig. 22. The plotting region is square, but whereas the horizontal scale covers 459,000 m of ocean surface, the vertical scale covers only 3.5 m near the surface (from 4549.5 m up to 4553 m) and the ocean floor (at 0 m) lies far below the plotted region. The time increment, δt , is 3 seconds. This selection of 13 frames is abstracted from a 16-mm motion picture of an actual ZUNI calculation, in which one such plot was drawn each cycle. The first frame of Fig. 22 shows the initial flat ocean surface. The second frame, at time $t = 735$ sec, shows the profile of a single solitary-like wave 140,000 m long and 1 m high after it has been introduced through the left boundary. The succeeding frames dramatically show the decrease in wavelength and increase in amplitude as the wave passes over shallower, 950-m-deep water. Finally, the wave is shown as it interacts with the rigid vertical right boundary of the computing mesh.

The 13 frames of Fig. 23 are abstracted from a calculation differing in that a continuing series

Fig. 17. A single solitary wave interacting with a submerged sloping beach. Surface markers are represented by heavy asterisks, regular markers by light dots. Sequence of frames is down, then across.

Fig. 18. Velocity-vector plots for the problem shown in Fig. 17.

Fig. 19. Schematic setup for calculating a tsunami wave, drawn out of true proportion for visual clarity.

Fig. 20. The particle plot for the problem of Fig. 19 drawn in true proportion.

Fig. 21. The particle plot of Fig. 20 re-proportioned to exaggerate the vertical scale as much as possible.

of solitary-like waves are introduced at the left boundary and the sloping wall is extended upward to penetrate the surface, indicated by the termination of the surface-particle line. Note that the second wave rises to a higher amplitude than the first upon reaching shallow water, in agreement with observations of shoaling waves. This is evident in a comparison of the fifth frame ($t = 2172$ sec) with the seventh frame ($t = 2727$ sec). The frames in Figs. 22 and 23 correspond in time. Comparison with similar calculations of the wave height above the

undisturbed surface level shows excellent agreement with the best available shoaling-wave data,⁶ offering a strong testimonial for the accuracy of the free-surface treatment.

The final set of examples is concerned with flows up a sloping beach that is only partially exposed. In these examples, the incoming waves have a truly oscillating form. The code modifications that allow exposed or submerged partial cells were made primarily to study the detailed interaction of a wave front with the beach surface. Surface markers are the only particles used in these examples, allowing the necessary negative inflow velocities. The plots in Figs. 24-26 are drawn in true proportion, but they exclude the region covered by the leftmost two columns of interior cells, because use of a simple, fixed, inflow port whose size does not vary with the amplitude of the wave produces a misleading free-surface appearance in the immediate vicinity of the left boundary.

Figure 24 shows highs and lows of the wave front from a calculation in which the inflow velocity, u_L , is given by $u_L = 0.2 \sin(\omega t)$, where the frequency $\omega = 0.90$. This frequency allows the oscillating wave to run up the beach twice, but the wave breaks at the outset of the third run, at a time $t = 20.8$, terminating the calculation. (Further code modifications than described here would be required to allow calculation of a fully breaking wave with the free-surface-marker particle treatment.)

Fig. 22. Selected frames from a motion picture of the calculation shown in Fig. 19. These plots of surface-marker particles cover a horizontal scale of 459,000 m and a vertical scale of 3.5 m (from 4549.5 to 4553.0 m). Times are 0, 735, 1845, 1971, 2172, 2403, 2727, 3225, 3420, 3750, 3876, 4071, and 4386 sec.

Fig. 23. A calculation similar to that of Fig. 22 except that the sloping obstacle is extended upward to penetrate the surface. Multiple waves are introduced at the left boundary. Times correspond to those of Fig. 22.

Fig. 24. An oscillating wave breaks on the third run up the beach. The inflow velocity at the left boundary provides a wave frequency $\omega = 0.90$. Sequence of frames is down, then across.

Fig. 25. Increasing the frequency to $\omega = 1.14$ delays the breaking until the fifth run up the beach. Sequence of frames is down, then across.

Fig. 26. Increasing the frequency still further ($\omega = 1.30$) prevented the wave from breaking at all in this calculation. Sequence of frames is down, then across.

If the frequency of the incoming wave is increased slightly, however, we observe that the wave's tendency to break is decreased or delayed. This is illustrated by the calculation shown in Fig. 25, in which the frequency was increased to $\omega = 1.14$, and the wave did not break completely and terminate the calculation until the beginning of the fifth run up the beach, at a time $t = 30.0$.

Increasing the wave frequency further to $\omega = 1.30$, delayed wave breaking further, as shown in Fig. 26. The wave form at time $t = 37.60$ differs little from that at $t = 23.04$. The oscillating wave was well behaved throughout the entire calculation, which was stopped after $t = 41.0$ when the allotted computing time was exhausted.

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