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ZERO ORBITAL ANGULAR MOMENTUM TRANSFER
REACTIONS INDUCED BY ${}^6\text{Li}$ IONS

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Abstract

An example of such a reaction is the mutual excitation of ${}^6\text{Li}$ ions to their $T = 1, 0^+$ states at 3.56 MeV. An experimental angular distribution for this reaction is fitted using a distorted wave theory and also an extension of the adiabatic theory of Austern and Blair. Reasonable fits are obtained in terms of one parameter, the strength of the spin-flip, isospin-flip component of the effective nucleon-nucleon force. Applications of the theory to similar reactions and the inclusion of exchange terms in the amplitude are considered.

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1. Introduction.

A special class of reactions induced by ${}^6\text{Li}$ ions consists of those in which ${}^6\text{Li}$ is transformed into a member of the $T = 1$ multiplet, ${}^6\text{He}$, ${}^6\text{Li}^*$ ($T = 1$, $J^\pi = 0^+$ state at 3.56 Mev) or ${}^6\text{Be}$. These reactions have been referred to as 'quasielastic' scattering reactions¹⁾ or double spin-isospin-flip reactions²⁾. The reason for these descriptions is that one might expect the reactions to be dominated by a zero orbital angular momentum transfer direct process. There is then no change in the internal space wave functions of the participating ions, and the reaction consists in the spin and isospin of a nucleon in ${}^6\text{Li}$ and one in the target being flipped by the $\underline{\sigma}_1 \cdot \underline{\sigma}_2 \quad \underline{1}_1 \cdot \underline{1}_2$ component of the effective nucleon-nucleon force. Here we apply distorted wave theory to such reactions and also develop a simpler adiabatic theory by extending the theory of Austern and Blair³⁾. Calculations are performed for the case of the mutual excitation of ${}^6\text{Li}$ ions and result in reasonably good fits to experimental data for the reaction²⁾.

Two cases are considered theoretically corresponding to experiments already performed^{1,2)}. The first experiment²⁾ involves the mutual excitation of ${}^6\text{Li}$ ions to their $T = 1$ states and, in this case, the theory should also apply for a reaction leading to the final state ${}^6\text{He} + {}^6\text{Be}$. In fact, this experiment has been performed and the mutual excitation experiment repeated by other workers⁴⁾ and is presently subject to analysis by them⁵⁾. The direct amplitude for these

reactions arises solely from the $v_{\sigma\tau}$ component of the nucleon-nucleon potential. However, the exchange amplitude, which we discuss but do not calculate, involves all the components of the force. In the short-range δ -function limit the two amplitudes have the same form but the magnitude of the combined amplitude is a linear combination of the strengths of the components. Since the experiment²⁾ was not performed at a high enough energy to ensure that the exchange term is negligible, the strength of the 'effective' $v_{\sigma\tau}$ force that we obtain by fitting the data may reflect this linear combination. We stress this point because analyses of inelastic scattering data in terms of effective forces do not always consider it.

In the second type of experiment¹⁾ the reactions $^{13}\text{C}(^6\text{Li}, ^6\text{He})^{13}\text{N}$ and $^9\text{Be}(^6\text{Li}, ^6\text{He})^9\text{B}$ were observed, and it was found that the main peaks in the spectra corresponded to final states which could be reached by zero orbital angular momentum transfer. We concentrate on the mutual excitation experiment as elastic scattering data and an angular distribution for the reaction are available, but also indicate how the theory could deal with this more general type of experiment.

In the second section we write down the amplitudes for the reactions in the distorted wave formalism and establish our notation. The discussion of the exchange terms in the mutual excitation amplitude is reserved for the Appendix. The extension of the Austern-Blair theory, which is closest to the distorted wave theory in the adiabatic limit where the kinetic energy loss in the reaction is neglected, is considered

in Section 3. In Section 4 the results of calculations for the mutual inelastic scattering reaction are presented and the strength of $v_{\sigma\tau}$ obtained is compared to that from other reactions.

2. Distorted Wave Theory.

We adopt the standard distorted wave approach to the reactions we are considering. Elastic scattering data is to be fitted by an optical model potential to give the parameters necessary to define the elastic scattering wave functions for the reaction calculation. For the latter, we require a form factor which we also need for the elastic scattering potential, so that our main purpose here is to discuss these form factors.

In general we consider a reaction initiated by ${}^6\text{Li}$ on an A-particle nucleus, quantum numbers, $J_i M_i T_i T_{3i}$, leading to a member of the mass 6, $T = 1$ triplet with the third isospin quantum no., T_3 , and a final nucleus, $J_f M_f T_f T_{3f}$. The initial and final internal wave functions of the particles are

$$\psi_i = \phi_i(1\dots 6; 1M_1 00; \underline{R}_1) \times_i(7\dots A+6; J_i M_i T_i T_{3i}; \underline{R}_2)$$

$$\psi_f = \phi_f(1\dots 6; 001T_3; \underline{R}_1) \times_f(7\dots A+6; J_f M_f T_f T_{3f}; \underline{R}_2)$$

(1)

where the quantum nos., $JMTT_3$, have been displayed in order and $\underline{R}_1$, $\underline{R}_2$ are the centers of mass of ${}^6\text{Li}$ and the target nucleus, respectively.

With the neglect of all exchange terms, the effective potentials giving the diagonal elastic scattering matrix element and the quasielastic scattering matrix elements are

$$\begin{aligned}
 V &= \sum_{i=1 \dots 6, j=7 \dots A+6} v(\underline{r}_i - \underline{r}_j) \\
 V' &= \sum_{i=1 \dots 6, j=7 \dots A+6} v_{\sigma\tau}(\underline{r}_i - \underline{r}_j) \underline{\sigma}_i \cdot \underline{\sigma}_j \underline{\tau}_i \cdot \underline{\tau}_j
 \end{aligned}
 \tag{2}$$

The leading term in the elastic scattering potential for center of mass motion is obtained from

$$\begin{aligned}
 U_0(R) &= \langle \Psi_i, V, \Psi_i \rangle \\
 &= \int \rho_T^{Li}(|\underline{r}_1 - \underline{R}_1|) \rho_T^A(|\underline{r}_2 - \underline{R}_2|) v(|\underline{r}_1 - \underline{r}_2|) d^3(\underline{r}_1 - \underline{R}_1) d^3(\underline{r}_2 - \underline{R}_2)
 \end{aligned}
 \tag{3}$$

where ρ_T^{Li} ρ_T^A denote the total nucleon densities of ${}^6\text{Li}$ and the target, respectively and the relative center of mass coordinate is $\underline{R} = \underline{R}_1 - \underline{R}_2$.

In practice (see Section 4) this equation was used in defining the form factor, the magnitude of U_0 being determined by fitting the elastic scattering data. One does not really expect U_0 to have this simple form for heavy ion scattering since exchange terms in the elastic scattering potential must become very important once R gets inside the surface region. The strong energy variation found in the magnitude of U_0 (see Table 3 in Section 4) may reflect this fact.

It is now convenient to treat our two cases separately. For the mutual excitation of ${}^6\text{Li}$ we assume that only the two

p-shell nucleons contribute to the matrix element and that, in the initial and final states, they are coupled to $L = 0$, $S = 1$, $T = 0$ and $L = 0$, $S = 0$, $T = 1$, respectively, their space wave function being unchanged. This assumption, which will result in a slightly overestimated matrix element, is discussed in Section 4. The transition matrix element is

$$\begin{aligned}
 \langle \psi_f | V' | \psi_i \rangle &= \langle \phi_f(1\dots 6; 00 \ 1T_3; \underline{R}_1) \phi_f(7\dots 12; 00 \ 1-T_3; \underline{R}_2) | V' | \\
 &\quad \phi_i(1\dots 6; 1M_1 00) \phi_i(1\dots 6; 1M_2 00) \\
 &= (-1)^{M_1+T_3} 3\delta_{M_1, -M_2} U_1(R)
 \end{aligned} \tag{4}$$

where

$$\begin{aligned}
 U_1(R) &= \int \rho_p^{Li}(|\underline{r}_1 - \underline{R}_1|) \rho_p^{Li}(|\underline{r}_2 - \underline{R}_2|) v_{\sigma\tau}(|\underline{r}_1 - \underline{r}_2|) \\
 &\quad d^3(\underline{r}_1 - \underline{R}_1) d^3(\underline{r}_2 - \underline{R}_2) .
 \end{aligned} \tag{5}$$

The matrix element of $\underline{\sigma}_1 \cdot \underline{\sigma}_2 \ \underline{I}_1 \cdot \underline{I}_2$ has been evaluated explicitly and ρ_p^{Li} is the density of p-shell nucleons in ${}^6\text{Li}$.

In this mutual excitation case we must include the exchange terms in the elastic and inelastic amplitudes and cross-sections which completely interchange the two ions.

The whole question of nucleon exchange terms for the inelastic amplitude is discussed in the Appendix. It is shown that, in the short range force limit at low ion energies, their inclusion is equivalent to renormalizing the strength of the 'effective' $v_{\sigma\tau}$ potential. We assume forces for the elastic scattering which do not depend on the spin of ${}^6\text{Li}$. Then for the elastic case

$$T_{M_1 M_2, M_3 M_4}^{(\theta)} = \delta_{M_1 M_3} \delta_{M_2 M_4} T_{EL}(\underline{k}_f, \underline{k}_i) + \delta_{M_1 M_4} \delta_{M_2 M_3} T_{EL}(-\underline{k}_f, \underline{k}_i) \quad (6)$$

and

$$\begin{aligned} \frac{d\sigma(\theta)}{d\Omega} &= \frac{1}{9} \left(\frac{\mu}{2\pi\hbar^2} \right)^2 \sum_{M_1 M_2 M_3 M_4} |T_{M_1 M_2, M_3 M_4}^{(\theta)}|^2 \\ &= \left(\frac{\mu}{2\pi\hbar^2} \right)^2 [|T_{EL}(\theta)|^2 + |T_{EL}(\pi-\theta)|^2 + \frac{2}{3} \Re(T_{EL}(\theta) T_{EL}(\pi-\theta))] \end{aligned} \quad (7)$$

where

$$T_{EL}(\underline{k}_f, \underline{k}_i) = \langle \exp(i\underline{k}_f \cdot \underline{R}), U(\underline{R}), \psi^{(+)}(\underline{k}_i, \underline{R}) \rangle \quad (8)$$

Here, $\psi^{(+)}(\underline{k}_i, \underline{R})$ is the elastic scattering wave function determined by an optical model potential $U(R)$, and μ is the reduced mass. The optical model potential, $U(R)$, contains a real part with form factor given by $U_0(R)$.

The reaction amplitude for observing a mass 6 ion with third isospin component, T_3 , at $\underline{k}_f$ is

$$T_{T_3-T, M_1 M_2}(\underline{k}_f, \underline{k}_i) = (-1)^{M_1+T_3} \delta_{M_1, -M_2} [T_{IN}(\underline{k}_f, \underline{k}_i) + T_{IN}(-\underline{k}_f, \underline{k}_i)] \quad (9)$$

where in the DWBA approximation,

$$T_{IN}(\underline{k}_f, \underline{k}_i) = \langle \psi_f^{(-)}(\underline{k}_f, \underline{R}), U_1(R), \psi_i^{(+)}(\underline{k}_i, \underline{R}) \rangle \quad (10)$$

The initial and final distorted wave functions are defined in the optical model potential, $U(R)$, and $U_1(R)$ is given by Eq. (5).

The mutual inelastic cross-section is

$$\begin{aligned} \frac{d\sigma(\theta)}{d\Omega} \bigg|_{IN} &= \frac{1}{9} \left(\frac{\mu}{2\pi\hbar^2} \right)^2 \frac{k_f}{k_i} \sum_{M_1 M_2} \left| T_{T_3-T, M_1 M_2}(\theta) \right|^2 \\ &= \frac{1}{3} \left(\frac{\mu}{2\pi\hbar^2} \right)^2 \frac{k_f}{k_i} \left| T_{IN}(\theta) + T_{IN}(\pi-\theta) \right|^2 \end{aligned} \quad (11)$$

The results of calculations of these cross-sections are

presented in Section 4.

The amplitude for the second type of reaction can be evaluated in terms of fractional parentage coefficients for the initial and final states.

$$\begin{aligned} \chi_i(7\dots A+6; J_i M_i T_i T_{3i}) &= \sum_{\lambda j_1 \ell_1} f_{\lambda i} (J_\lambda T_\lambda j_1 \ell_1) \\ &\quad (T_\lambda T_{3\lambda} \frac{1}{2} t_{3i} | T_i T_{3i}) [\chi_\lambda(7\dots A+5; J_\lambda T_\lambda T_{3\lambda}) \phi_i(A+6; j_1 \ell_1 t_{3i})]_{M_i}^{J_i} \\ \chi_f(7\dots A+6; J_f M_f T_f T_{3f}) &= \sum_{\mu j_2 \ell_2} f_{\mu f} (J_\mu T_\mu j_2 \ell_2) \\ &\quad (T_\mu T_{3\mu} \frac{1}{2} t_{3f} | T_f T_{3f}) [\chi_\mu(7\dots A+5; J_\mu T_\mu T_{3\mu}) \phi_f(A+6; j_2 \ell_2 t_{3f})]_{M_f}^{J_f} \end{aligned}$$

(12)

The square brackets denote vector coupling of angular momenta, the $\phi(A+6)$ are single particle wave functions and $f_{\lambda i}$, $f_{\mu f}$ are the fractional parentage coefficients. The sums run over states λ, μ of the nucleus with one less nucleon but, for ${}^6\text{Li}-{}^6\text{He}$ reactions, there will be no sum over the isospin third components, t_{3i} , $T_{3\lambda}$, t_{3f} and $T_{3\mu}$.

Clearly all the terms in the sum over j in V' given by Eq. (2) give the same contribution so that we evaluate the contribution of $v_{\sigma\tau}(\underline{r}_i - \underline{r}_{A+6})$. Also, as before, we assume that only the two p-shell nucleons from ${}^6\text{Li}$ contribute

to the reaction. For a transition with zero orbital angular momentum transfer, which we expect to dominate possible reactions, we must have $\ell_2 = \ell_1$. We assume that the radial wave functions in ϕ_i and ϕ_f are very similar, even for $j = \ell \pm 1/2$, and their combination in the matrix element gives an effective density, $\rho_\ell(r)$ (normalized to unity) for the interacting target nucleon. The matrix element of V' between the initial and final internal states can then be evaluated by standard methods. We obtain the following result which applies to ${}^6\text{Li}-{}^6\text{Li}^*$ reactions as well as to ${}^6\text{Li}-{}^6\text{He}$ reactions.

$$\langle \psi_f, V', \psi_i \rangle = (1 M_1 J_i M_i | J_f M_f)$$

$$\sum_{\lambda j_1 j_2} (T_\lambda T_{3\lambda} \frac{1}{2} t_{3i} | T_i T_{3i}) (T_\lambda T_{3\lambda} \frac{1}{2} t_{3f} | T_f T_{3f})$$

$$(1 T_3 \frac{1}{2} t_{3f} | \frac{1}{2} t_{3i}) [3(2J_i+1)(2j_1+1)]^{\frac{1}{2}} C_{j_1 j_2} \left\{ \begin{matrix} j_2 & j_1 & 1 \\ J_i & J_f & J_\lambda \end{matrix} \right\}$$

$$A f_{\lambda i} (J_\lambda j_1 \ell) f_{\lambda f} (J_\lambda j_2 \ell) U_\ell(R)$$

(13)

where a 6-j symbol has been used and the coefficients are

$$\begin{aligned} C_{j_1 j_2} &= (-1)^{J_i + J_\lambda + j_2 + 1} \left[\frac{(j_1 + j_2 + 2)(j_2 - j_1 + 1)}{2j_2} \right]^{\frac{1}{2}}, \quad j_2 = \ell + \frac{1}{2} \\ &= (-1)^{J_i + J_\lambda + j_2} \left[\frac{(j_1 + j_2)(j_1 - j_2 + 1)}{2(j_2 + 1)} \right]^{\frac{1}{2}}, \quad j_2 = \ell - \frac{1}{2} \end{aligned}$$

(14)

The spatial integral has been expressed in terms of the form factor

$$U_{\ell}(R) = \int \rho_p^{\text{Li}}(|\underline{r}_1 - \underline{R}_1|) \rho_{\ell}(|\underline{r}_2 - \underline{R}_2|) v_{\sigma\tau}(|\underline{r}_1 - \underline{r}_2|) d^3(\underline{r}_1 - \underline{R}_1) d^3(\underline{r}_2 - \underline{R}_2) \quad (15)$$

The factor A , the number of nucleons in the target, arises because of the normalizations assumed for the fractional parentage coefficients and the density, $\rho_{\ell}(r) = (4\pi)^{-1} |\phi_{\ell}(r)|^2$, where $\phi_{\ell}(r)$ is the radial wave function.

The cross-sections to different possible final states may now be written down in terms of a DWBA cross-section. As an example we consider the $^{13}\text{C}(^6\text{Li}-^6\text{He})^{13}\text{N}$ reactions¹⁾ and assume the fractional parentage coefficients connect only to the 0^+ and 2^+ states of ^{12}C . The cross-sections are

$$\frac{d\sigma}{d\Omega} = C_f \frac{d\sigma}{d\Omega} \text{ DW} \quad (16)$$

where the distorted wave cross-section is

$$\frac{d\sigma}{d\Omega} \text{ DW} = \frac{k_f}{k_i} \left(\frac{\mu}{2\pi\hbar^2} \right)^2 \left| \langle \psi^{(-)}(\underline{k}_f, \underline{R}), U_{\ell}(R), \psi^{(+)}(\underline{k}_i, \underline{R}) \rangle \right|^2 \quad (17)$$

where μ is the reduced mass and $\psi^{(+)}$, $\psi^{(-)}$ are the initial and final distorted waves.

The coefficients C_f are given by

$$C_f = \frac{2(2J_f+1)}{3} A^2 \left| \sum_{\lambda j_1 j_2} (2j_1+1)^{1/2} \begin{Bmatrix} j_2 & j_1 & 1 \\ J_i & J_f & J_\lambda \end{Bmatrix} \right|^2 \quad (18)$$

where we have assumed $\ell = 1$ only.

From the ^{13}C ground state, $J = 1/2^-$, the reaction with $\Delta L = 0$ should reach the analogue ^{13}N ground state and the $^{13}\text{N}^*$, $J = 3/2^-$ state at 3.51 MeV. The two ground states are assumed to have identical fractional parentage coefficients to the 0^+ and 2^+ states of ^{12}C .

$$f_{0i}(0, 1/2, 1) = \alpha \qquad f_{2i}(2, 3/2, 1) = \beta$$

If there are no other significant parent states for a p-shell nucleon we have

$$\alpha^2 + \beta^2 = 1/A$$

The ^{13}N , $3/2^-$ state is taken to have the f.p.c.'s.

$$f_{0f}(0, 3/2, 1) = \gamma, \quad f_{2f}(2, 1/2, 1) = \delta, \quad f_{2f}(2, 3/2, 1) = \epsilon.$$

In terms of the f.p.c.'s the coefficients, C_f , become

$$C_{1/2} = 2/9$$

$$C_{3/2} = 8/9 A^2 (\beta\delta - \sqrt{2}\alpha\gamma - \beta\epsilon)^2$$

If the states were single particle states, i.e. $\alpha \approx \gamma \approx A^{-1/2}$,
and $\beta \approx \delta \approx \epsilon \approx 0$, we would have

$$C_{3/2} \approx 8 C_{1/2}$$

From the raw data at two angles¹⁾ we can guess that $\sigma(1/2) > \sigma(3/2)$, showing that this situation is very far from being the case. This analysis has been given to indicate that improved data and a more thorough theoretical analysis of such reactions would possibly yield useful spectroscopic information.

3. An Extension of the Austern-Blair Theory.

The difficulty with working directly with the distorted wave amplitude for heavy ion reactions is that the form factor and the elastic scattering wave functions may not be well defined. It is desirable to have a simpler theory that goes over into the diffraction theory expressions in the limit of small energy transfer in the reaction (the adiabatic limit) and strong absorption. The relation between the distorted wave theory and diffraction theory was explored by Austern and Blair³⁾ for the case of orbital angular momentum transfer. Here we wish to extend the theory to our case of zero orbital angular momentum transfer.

The basic approximation of the theory is that the energy transfer in the reaction is small relative to the incident kinetic energy. This is not a very good approximation for the experiments^{1,2)} performed thus far, in which the ⁶Li ions have only about 30 MeV laboratory energy and where the excitation energies are not small. However, it is worth exploring and will be more applicable to experiments with higher incident energies.

Let $U(\alpha, R)$ be the elastic optical model potential, where α represents the parameters specifying the potential. The elastic amplitude, given by Eq. (8) may be written as

$$T_{EL}(\underline{k}_f, \underline{k}_i) = \langle \exp(i\underline{k}_f \cdot \underline{R}), U + U \frac{1}{E - T - U + i\epsilon} U, \exp(i\underline{k}_i \cdot \underline{R}) \rangle$$

If one of the parameters, α , is changed by $\delta\alpha$ so that $U \rightarrow U + \delta U$ it is easy to see that³⁾, to the first order in

δU , the change in T_{EL} is

$$\Delta T_{EL}(\underline{k}_f, \underline{k}_i) = \langle \psi^{(-)}(\underline{k}_f, R), \delta U(\alpha, R), \psi^{(+)}(\underline{k}_i, R) \rangle \quad (19)$$

where $\psi^{(+)}$ and $\psi^{(-)}$ are initial and final distorted wave functions (at the same energy) defined by the potential, $U(\alpha, R)$.

This expression may be identified with a distorted wave Born approximation amplitude for the perturbation, $\delta U(\alpha, R)$, provided that the actual energy change is small. The optimum choice is to choose the mean of the initial and final energies so that

$$T_{IN}(\underline{k}_f, E_f; \underline{k}_i, E_i; \delta U) \approx \Delta T_{EL}(\underline{k}_f, \frac{1}{2}(E_i + E_f); \underline{k}_i, \frac{1}{2}(E_i + E_f)) \quad (20)$$

where the energies shown are those specifying the magnitudes of $\underline{k}_i$ and $\underline{k}_f$.

In Ref. 3, α was chosen to be the radius parameter of the potential, whose change, $\delta\alpha$, is a dynamical variable connecting the ground state to an excited state, for example by a Y_2 deformation. In our case we want to choose α to be the depth of the real part of the optical potential. In order for this choice to be valid the form factors associated with the actual perturbation and the optical potential must be identical. For the mutual excitation of ${}^6\text{Li}$ ions this requires that $U_1(R)$, given by Eq. (5) and $U_0(R)$, given by Eq. (3) have the same form factor. The requirement should at least be

satisfied for the large R region which mainly contributes to the inelastic amplitude. The results of calculations which examine this point are presented in Section 4 and are shown on Figure 1.

Defining $u_0(R)$ and $u_1(R)$ as the elastic and inelastic potentials of Eq.(3) and (5) calculated with two-body potentials of unit strength, we take their relative normalization to be given by

$$u_0(R) = N u_1(R) \quad (21)$$

The effective change in the depth of the real part of the optical model potential V_{EL} , is then

$$\delta\alpha = \delta V_{EL} = V_{11} v_0 / N$$

where V_{11} is the strength of the $\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$ component of the effective two-body potential, and v_0 the strength of the equivalent Saxon-Woods form of $u_0(R)$ as in Eq. (25).

From Eqs. (19) and (20) the Austern-Blair inelastic scattering amplitude is

$$T_{IN}(\underline{k}_f, \underline{k}_i) = \delta\alpha \frac{\partial}{\partial V_{EL}} T_{EL}(\underline{k}_f, \frac{1}{2}(E_i + E_f); \underline{k}_i, \frac{1}{2}(E_i + E_f)) \quad (22)$$

This expression is used in fitting the data in the next section. We could continue to derive a diffraction model expression for the amplitude on the lines of Austern and Blair³⁾ or a more elaborate diffraction theory,⁶⁾ but

fits to experiment would involve an analysis of the elastic data with diffraction model parameters that we have not carried out. It may well be worthwhile to perform this analysis for our type of reactions at higher incident energies.

4. Calculation for ${}^6\text{Li}$ Mutual Inelastic Scattering.

In this section we report on fits obtained to the data of Ref. 2 using the amplitudes derived in the previous two sections.

The evaluation of $u_0(R)$, given by Eq. (3) with unit two-body interaction strength, requires the total nucleon density $\rho_T(r)$ for ${}^6\text{Li}$.

This was calculated by generating the nucleon single-particle wavefunctions in a Saxon-Woods well of the form

$$V_{SM}(r) = -V_N f(r, R_N, a_N) + \left(\frac{\hbar}{m_\pi c} \right)^2 \frac{V_S}{r} \frac{d}{dr} f(r, R_S, a_S) \underline{l} \cdot \underline{\sigma} + V_C(R_N) \quad (23)$$

where

$$f(r, R, a) = \left[\exp\left(\frac{r-R}{a}\right) + 1 \right]^{-1}$$

and $V_C(R)$ is the Coulomb potential arising from a uniformly charged sphere of radius R .

The standard geometry parameters of Batty and Greenlees⁷⁾ were used, and the central potential well-depths V_N were adjusted so as to correctly reproduce the binding energies, B , of the last proton and neutron in ${}^6\text{Li}$.⁸⁾ The parameters defining $V_{SM}(r)$ are given in Table 1. With these parameters the $1s^{1/2}$ and $1p^{3/2}$ proton and neutron wave functions were generated, and hence the density, $\rho_T(r)$.

The p-shell nucleon density, $\rho_p^{\text{Li}}(r)$, required for the calculation of $u_1(R)$ given by Eq. (5) with unit two-body interaction strength, was also obtained. $u_0(R)$ and $u_1(R)$ were calculated by numerical integration with an effective nucleon-nucleon interaction of Yukawa form

$$v(r) = \exp(-\alpha r) / \alpha r \quad (24)$$

where α was taken to be 0.8 fm^{-1} . These form-factors are shown in Fig.1, where $u_1(R)$ has been arbitrarily normalised to $u_0(R)$ at $R = 4.8 \text{ fm}$ to give a normalization factor $N=8$ (see Eq. (21))

In the asymptotic region the form factors are not too dissimilar, thus satisfying the requirement of Section 3, although N is not well-defined, probably lying in the range $5 \leq N \leq 10$. Functional forms for $u_{0,1}(R)$ were obtained by fitting their numerical values with Saxon-Woods forms,

$$u_{0,1}(R) = v_{0,1} f(R, R_{0,1}, a_{0,1}) \quad (25)$$

The form factors were reproduced well using the parameters defined in Table 2.

To obtain the elastic scattering wave functions, $\psi^{(\pm)}(\underline{k}, R)$, the elastic scattering data of Fortune et.al.⁹⁾ at 10 MeV and 14 MeV relative centre-of-mass energies was fitted using an optical model potential of the form

$$U(R) = - (V_{EL} + iW_{EL}) f(R, R_0, a_0) + V_C(R_0) \quad (26)$$

The elastic data of Ref.2 at 16 MeV centre-of-mass energy unfortunately could not be used because of errors in it.¹⁰⁾

The geometrical parameters, R_0 and a_0 in V_{OPT} were taken from Table 2 and the well-depths V_{EL} and W_{EL} were varied to fit the data. The results are shown in Fig. 2 and it is evident that our potential form gives very good fits. The well-depths are strongly energy-dependent, possibly representing a highly non-local realistic potential which includes exchange effects. To obtain the well-depths required for the mutual excitation calculations we assumed a linear dependence on energy which resulted in the parameters listed in Table 3. The mean energy of 12.5 MeV is required for the Austern-Blair calculations.

The experimental data for the mutual inelastic scattering amplitude at 16 MeV center-of-mass incident energy was analyzed using the DWBA and the Austern-Blair model. Distorted waves were generated by the optical potential (26) with well-depths taken from Table 3. The inelastic amplitude given by Eqs. (9) and (10) was calculated with $u_1(R)$ defined by Eq. (25) and Table 2, using $U_1(R) = V_{11} u_1(R)$. Fig. 3 shows the good agreement obtained between the DWBA cross-section, given by Eq. (11), and the experimental cross-section. It has been found¹⁷⁾ that some of the observed sharp structure can be eliminated by the inclusion of a tensor term in the interaction. The strength, V_{11} , of the effective $v_{\sigma\tau}$ potential needed to fit the data was

4.5 MeV. The Austern-Blair amplitude (22) was calculated by taking the derivative of the elastic amplitude at 12.5 MeV. The approximate fit obtained to the data, also shown in Fig. 3, required an effective potential strength of 3.3 MeV. It is certainly encouraging that the Austern-Blair model should work reasonably well at such a relatively low incident energy and large energy transfer.

Other reactions involving the $v_{\sigma\tau}$ component of the effective nucleon-nucleon force include ${}^7\text{Li}(p,n){}^7\text{Be}$ (G.S. and 0.431 MeV state) and ${}^6\text{Li}(p,p'){}^6\text{Li}$ (3.56 MeV state). Results obtained from such reactions have been summarized by Austin et.al.¹¹⁾ using an effective force of the form

$$v_{\sigma\tau}(\underline{r}) = V_{11} \exp(-\alpha r) / \alpha r$$

with a range $\alpha^{-1} = 1$ fm.

The results for the ${}^7\text{Li}$ reaction show values of V_{11} decreasing slowly from 16 MeV to 13 MeV through an energy range of incident protons of 20 to 50 MeV. They are consistent with the more recent results of Clough et.al.¹²⁾ who used a range $\alpha^{-1} = 1.4$ fm. Results for the inelastic ${}^6\text{Li}$ reaction, which are directly relevant to our mutual excitation experiment, show a gradual increase of V_{11} from 11 MeV to 13 MeV over the same incident energy range. A recent summary of many experiments by Austin¹³⁾ gives a value of $V_{11} = 12 \pm 3$ MeV for the 20 - 60 MeV nucleon energy range. To compare to our result of $V_{11} = 4.5$ MeV with $\alpha = 0.8 \text{ fm}^{-1}$ we assume the force scales

with its integrated strength i.e. V_{11}/a^3 . Then our value would be $V_{11}=9$ MeV with $a^{-1} = 1$ fm which is somewhat lower than that derived by Austin. There is, however, an estimated error of a factor of 1.5 in the absolute magnitude of the experimental cross-section¹⁷⁾. Taking this into account we obtain $V_{11} = 11.0$ MeV. The results of the appendix indicate that the inclusion of the exchange term is important and would reduce our value of V_{11} . However, this term was also neglected in the earlier analyses.

We have assumed pure L-S coupled wave functions for ${}^6\text{Li}$. Although this may be a good approximation¹⁴⁾ any deviations from the L-S limit will have the effect of lowering the theoretical amplitude. This reduction will be more important in our case, as opposed to inelastic scattering on ${}^6\text{Li}$,¹¹⁾ since two ${}^6\text{Li}$ ions are involved. An elaborate wave function for the ground state has been constructed in an attempt to fit the ${}^6\text{Li}$ quadrupole moment¹⁵⁾ but it is hardly worth attempting to use it in our calculation because of the other uncertainties.

In conclusion our attempt to fit the mutual inelastic scattering cross-section for two ${}^6\text{Li}$ ions has been fairly successful. This is rather surprising in view of the considerable uncertainties in optical model potentials and exchange effects but encourages us to believe that detailed analysis of similar ${}^6\text{Li}$ reactions, such as those of Ref. 1, would be fruitful.

Acknowledgements

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Appendix. Exchange Terms for the Mutual Inelastic Scattering Amplitude.

According to Ekstein¹⁶⁾ the complete amplitude for the reaction can be obtained by completely antisymmetrizing the final product wave function and V' of Eq. (2). We can write it in the following form.

$$T = \langle (H-E) \Lambda_{6,6}(1\dots 6, 7\dots 12) [\phi_{fT_3}(1\dots 6) \phi_{f-T_3}(7\dots 12) \psi_f(\underline{k}_f, \underline{R}_1 - \underline{R}_2)] , \phi_{iM_1}(1\dots 6) \phi_{iM_2}(1\dots 6) \psi_i(\underline{k}_i, \underline{R}_1 - \underline{R}_2) \rangle \quad (A1)$$

where we have slightly abbreviated the notation for the internal ion wave functions, ψ_i and ψ_f are the initial and final relative wave functions for the ions, and H and E are the total Hamiltonian and energy of the system.

The operation $\Lambda_{6,6}(1\dots 6, 7\dots 12)$ completely antisymmetrizes the product wave function for the final state whose components are already antisymmetrized in co-ordinates $1\dots 6$ and $7\dots 12$. It takes the form

$$\begin{aligned} \Lambda_{6,6} = & 1 - \sum_{i < k \dots 6, j < c \dots 12} P_{ij} \\ & + \sum_{i < k \dots 6, j < n \dots c \dots 7 \dots 12} P_{ij} P_{kn} - \sum (P)^3 + \sum (P)^4 \\ & - \sum (P)^5 + P_{17} P_{28} P_{39} P_{410} P_{511} P_{612} \end{aligned} \quad (A2)$$

where P_{ij} is the permutation operator which interchanges all the co-ordinates of i and j in the wave functions. The terms with products of 3, 4 and 5 permutation operators are constructed by analogy with the second term.

We want to consider the p- and s-shell nucleons separately so we write the antisymmetrized internal wave functions as, for example,

$$\begin{aligned}\phi_{iM_1}(1\dots 6) &= (15)^{-\frac{1}{2}} \Lambda_{2,4}(12,3\dots 6) \phi_{pM_1}(1,2) \phi_{\alpha}(3\dots 6) \\ \phi_{iM_2}(7\dots 12) &= (15)^{-\frac{1}{2}} \Lambda_{2,4}(7,8,9\dots 12) \phi_{pM_2}(7,8) \phi_{\alpha}(9\dots 12)\end{aligned}$$

(A3)

where ϕ_{pM} , ϕ_2 are the antisymmetrized wave functions of the two p-shell nucleons and the ' α -particle' core respectively. The operator $\Lambda_{2,4}$ is constructed by analogy to $\Lambda_{6,6}$.

The operators $\Lambda_{2,4}$ in the initial state may be acted on the antisymmetric final state when each term they contain is identical, thus cancelling the normalization factors. The amplitude becomes

$$\begin{aligned}
T = & \langle (H-E) \Lambda_{6,6}(1\dots 6, 7\dots 12) \{ \Lambda_{2,4} [\phi_{pT_3}(1,2) \\
& \phi_\alpha(3\dots 6)] \Lambda_{2,4} [\phi_{p-T_3}(7,8) \phi_\alpha(9\dots 12)] \\
& \psi_f(\underline{k}_f, \underline{R}_1 - \underline{R}_2) \} , \phi_{pM_1}(1,2) \phi_\alpha(3\dots 6) \phi_{pM_2}(7,8) \\
& \phi_\alpha(9\dots 12) \psi_i(\underline{k}_i, \underline{R}_1 - \underline{R}_2) \rangle
\end{aligned} \tag{A4}$$

The final state wave function is completely anti-symmetric and it may be verified that we can re-express it in the following more convenient form

$$\begin{aligned}
T = & \langle (H-E) \Lambda_{4,8}(1278; 3\dots 69\dots 12) \Lambda_{4,4}(3\dots 6; 9\dots 12) \\
& \Lambda_{2,2}(12; 78) [\phi_{pT_3}(1,2) \phi_{p-T_3}(7,8) \phi_\alpha(3\dots 6) \\
& \phi_\alpha(9\dots 12) \psi_f(\underline{k}_f, \underline{R}_1 - \underline{R}_2)] , \phi_{pM_1}(1,2) \phi_\alpha(3\dots 6) \\
& \phi_{pM_2}(7,8) \phi_\alpha(9\dots 12) \psi_i(\underline{k}_i, \underline{R}_1 - \underline{R}_2) \rangle
\end{aligned} \tag{A5}$$

The operator $\Lambda_{4,8}$ antisymmetrizes between the s-shell and p-shell wave functions, $\Lambda_{4,4}$ antisymmetrizes between the two α cores and $\Lambda_{2,2}$ exchanges the p-shell nucleons. We first neglect all s-p exchanges so that $\Lambda_{4,8} = 1$. Next we reject all terms in $\Lambda_{4,4}$ which mix, but do not completely exchange the α -particles. Then

$$\Lambda_{4,4} = 1 + P_{39} P_{410} P_{511} P_{612}$$

It may now be verified that the complete α -exchange, together with $\Lambda_{2,2}$ results in a final wave function of the form

$$\Lambda_{2,2}(12;78) [\phi_{p-T_3}(1,2) \phi_{pT_3}(7,8) \phi_\alpha(3\dots6) \\ \phi_\alpha(9\dots12) \psi_f(-\underline{k}_f, \underline{R}_1 - \underline{R}_2)]$$

This wave function gives the amplitude obtained from the 1 term in $\Lambda_{4,4}$ with $\underline{k}_f$ replaced by $-\underline{k}_f$. For the direct unit operator term in $\Lambda_{2,2}$ it gives the exchange term included in Eq. (9) of the text. We are left with the p-shell exchanges arising from $\Lambda_{2,2}$ which is explicitly

$$\Lambda_{2,2} = 1 - P_{17} - P_{18} - P_{27} - P_{28} + P_{17} P_{28} \quad .$$

The operator H-E acting on the final state gives the sum of the two-body potentials between the separated mass 6 ions. We could have included this sum directly with the wave function being operated on by the Λ operators since H commutes with any Λ . For the reaction we consider, only the potentials between the p-shell nucleons can contribute and they involve

one p-shell nucleon from each ion. The term $P_{17}P_{28}$ must exchange another pair of p-shell nucleons in addition to the interacting pair and so we neglect it. It is probably invalid to include exchanges between non-interacting pairs of nucleons in a Born approximation treatment of the amplitude.

The direct term in the amplitude corresponding to Eqs. (9) and (10) arises from the unit operator in $\Lambda_{2,2}$ and is written here as

$$T_D = 4 < \phi_{pT_3}(1,2) \phi_{p-T_3}(7,8) \psi_f(k_f, \underline{r}_1 - \underline{r}_2) , \\ V_{17}, \phi_{pM_1}(1,2) \phi_{pM_2}(7,8) \psi_i(k_i, \underline{r}_1 - \underline{r}_2) > \quad (A6)$$

where we have integrated over the internal variables of the α -cores.

By relabelling the co-ordinates we find that the two nucleon exchange operators in $\Lambda_{2,2}$ give identical amplitudes. Thus, the exchange term arising from them with the two-nucleon potentials inserted explicitly is

$$T_{EX} = 4 < \phi_{fT_3}(7,2) \phi_{f-T_3}(1,8) \psi_f(k_f , \\ \frac{1}{6} [(\underline{r}_7 + \underline{r}_2 + 4\underline{r}_{\alpha 1}) - (\underline{r}_1 + \underline{r}_8 + 4\underline{r}_{\alpha 2})] , V_{17} + V_{12} + V_{78} \\ + V_{28} , \phi_{iM_1}(1,2) \phi_{iM_2}(7,8) \psi_i(k_i, \frac{1}{6} [(\underline{r}_1 + \underline{r}_2 + 4\underline{r}_{\alpha 1}) \\ - (\underline{r}_7 + \underline{r}_8 + 4\underline{r}_{\alpha 2})] >$$

where we have written out explicitly the co-ordinates of relative motion, $\underline{r}_{\alpha 1}$ and $\underline{r}_{\alpha 2}$ being the centres of mass of the α -particle cores.

Now we reject the terms arising from V_{12} and V_{78} as involving bound particles in the initial states. Also we neglect the V_{28} term as nucleons 1 and 7 are then exchanged without an interaction between them. We have left only the 'natural' V_{17} term involving the exchange of interacting particles and write

$$V_{17} = V_0 + V_\sigma \underline{\sigma}_1 \cdot \underline{\sigma}_7 + V_\tau \underline{\tau}_1 \cdot \underline{\tau}_7 + V_{\sigma\tau} \underline{\sigma}_1 \cdot \underline{\sigma}_7 \underline{\tau}_1 \cdot \underline{\tau}_7$$

All the components of V_{17} can contribute to T_{EX} and a straightforward evaluation of the spin and isospin matrix elements results in

$$T_{EX} = (-1)^{M_1 + T_3} \delta_{M_1 - M_2} < \phi(\underline{r}_7, \underline{r}_2) \phi(\underline{r}_1, \underline{r}_8)$$

$$\psi_f(\underline{k}_f, \underline{R}_f), V_0 + V_{\sigma\tau} - V_\sigma - V_\tau, \phi(\underline{r}_1, \underline{r}_2) \phi(\underline{r}_7, \underline{r}_8)$$

$$\psi_i(\underline{k}_i, \underline{R}_i) >$$

(A8)

where ϕ is the space wave-function of the two-p-shell nucleons, $\underline{R}_i$ and $\underline{R}_f$ are the relative co-ordinates given explicitly in Eq. (A7) and the potentials are functions of $|\underline{r}_1 - \underline{r}_7|$.

In a similar form we have from Eqs. (A6) and (9)

$$T_D = (-1)^{M_1+T_3} \delta_{M_1-M_2} < \phi(\underline{r}_1, \underline{r}_2) \phi(\underline{r}_7, \underline{r}_8)$$

$$\psi_f(\underline{k}_f, \underline{R}_1 - \underline{R}_2) , 4 V_{\sigma\tau} , \phi(\underline{r}_1, \underline{r}_2) \phi(\underline{r}_7, \underline{r}_8)$$

$$\psi_i(\underline{k}_i, \underline{R}_1 - \underline{R}_2) > \quad (A9)$$

The total amplitude is

$$\begin{aligned} T(\underline{k}_f, \underline{k}_i) = & T_D(\underline{k}_f, \underline{k}_i) - T_{EX}(\underline{k}_f, \underline{k}_i) \\ & + T_D(-\underline{k}_f, \underline{k}_i) - T_{EX}(-\underline{k}_f, \underline{k}_i) \end{aligned} \quad (A10)$$

Now let us consider the short range limit in which V is a δ -function. It is clear that the spatial integrals in the direct and exchange terms are identical but that the strength of the effective force should be

$$3 V_{\sigma\tau} - V_0 + V_\sigma + V_\tau$$

This result is just as expected as it is the sum of a Wigner and a Majorana force which give the same results in the short range limit. We can crudely estimate the effect of a finite short range Yukawa force as follows. The relative co-ordinate in ψ_f in T_{EX} is $\underline{R}_f = \underline{R}_1 - \underline{R}_2 - \frac{1}{3}(\underline{r}_1 - \underline{r}_7)$.

Apart from the potential, the dependence of the integrand on $\underline{r} = \underline{r}_1 - \underline{r}_7$ arises from the bound state wave functions, a

factor $\exp[i \frac{1}{6}(\underline{k}_i - \underline{k}_f) \cdot \underline{r}]$ from $\underline{R}_1 - \underline{R}_2$ and $\exp[i \frac{1}{3} \underline{k}_f \cdot \underline{r}]$ from the final factor in $\underline{R}_f$. The directions of $\underline{k}_i$ and $\underline{k}_f$ may be changed from their asymptotic values by distortion. Now we assume that, apart from the force range, the dominating short range r dependence arises from the final exponential. This may not be a bad approximation where $1/3 k_f$ is greater than both $1/6 |\underline{k}_i - \underline{k}_f|$ and the inverse range of the asymptotic exponential fall-off of the nuclear p-shell density. If we neglect all r -dependence other than this exponential and the Yukawa force the amplitudes have roughly the same form in the forward direction and

$$F_{EX} \approx \frac{(V_0 + V_{\sigma\tau} - V_\sigma - V_\tau) F_D}{4 V_{\sigma\tau}} \int \frac{\exp(-\alpha r)}{\alpha r} \exp[\frac{1}{3} i \underline{k}_f \cdot \underline{r}] d^3 \underline{r}$$

$$/ \int \frac{\exp(-\alpha r)}{\alpha r} d^3 \underline{r}$$

$$\approx \frac{(V_0 + V_{\sigma\tau} - V_\sigma - V_\tau)}{4 V_{\sigma\tau}} F_D \frac{\alpha^2}{\alpha^2 + \frac{1}{36}(\underline{k}_i + \underline{k}_f)^2}$$

(A11)

where we have symmetrized the dependence on $\underline{k}_i$ and $\underline{k}_f$.

With $\alpha = 1 \text{ fm}^{-1}$ values of $V_0 \approx -27 \text{ MeV}$ and $V_{\sigma\tau} \approx 12 \text{ MeV}$ have been extracted from inelastic scattering data.¹³⁾ From the nucleon isospin potential we have $V_\tau \approx -16 \text{ MeV}$, (subject to considerable uncertainty¹³⁾). Also V_σ is small and not well determined, say 5 MeV when the potential values correspond to a realistic mixture. Then

$$\frac{V_0 + V_{\sigma\tau} - V_\sigma - V_\tau}{4V_{\sigma\tau}} \approx -\frac{3}{4}$$

At the centre-of-mass energy of 16 MeV of the experiment we analyse, the final factor of Eq. (All) is $5/6$ with $\alpha = 1 \text{ fm}^{-1}$, indicating that the approximations we have used in deriving this equation are invalid. All we can assert at this energy is that F_{EX} is almost certainly comparable with F_D and that it has the opposite sign. However, we have felt it useful to include this Appendix as showing how to derive F_{EX} with the various approximations involved, and also to show that, at low incident energies, the 'effective' $V_{\sigma\tau}$ strength found by fitting the data will not be the same as, but probably greater than, that of the original potential.

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Table 1. The proton and neutron potential well parameters.

Orbit	B MeV	V_N MeV	V_S MeV	R_N fm	R_S fm	a_N fm	a_S fm
1p3/2 proton	4.66	61.3	6.0	2.19	1.86	0.76	0.60
1p3/2 neutron	5.67	56.2	6.0	2.32	2.15	0.73	0.60

Table 2. Parameters of the equivalent Saxon-Woods form (25) for the elastic and inelastic form-factors.

Form-factor	$V_{O,1}$ MeV	$R_{O,1}$ fm	$a_{O,1}$ fm
$u_0(R)$	4.80	2.36	1.15
$u_1(R)$	0.28	3.10	1.30

Table 3. The real and imaginary well-depths defining the optical model potential (26).

C.M. Energy MeV	V_{EL} MeV	W_{EL} MeV
9.0	88	20
12.5	66	24
16.0	44	28

Figure Captions

- Fig. 1: The elastic and inelastic form factors, $u_0(R)$ and $u_1(R)$, calculated from Eqs. (3), (5) and (24) of the text.
- Fig. 2: Optical model fits to elastic scattering data for ${}^6\text{Li}({}^6\text{Li}, {}^6\text{Li}){}^6\text{Li}$.⁹⁾ The form of the optical potential used is given by Eq. (26) with $R_0 = 2.36$ fm and $a_0 = 1.15$ fm, and V_{EL} and W_{EL} are the values obtained for the real and imaginary well depths, respectively.
- Fig. 3: DWBA and Austern-Blair model fits to data on the mutual excitation of ${}^6\text{Li}$ ions to their 3.56 MeV levels at 16 MeV centre-of-mass energy. This data has been taken from an unpublished correction¹⁰⁾ to ref.²⁾ The form of the optical potential used is given by Eq. (26) with $R_0 = 2.36$ fm and $a_0 = 1.15$ fm and the well-depths were taken from Table 3 as described in the text. The values obtained for the spin-isospin flip part, V_{11} , are of the effective nucleon-nucleon interaction are shown.

OVERLAP IN MEV

FORM - FACTORS

U_0

$8.0 \times U_1$

RADIUS IN FERMI

10^1

10^0

10^{-1}

2

4

6

8