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ANALYSIS OF ACTIVATION MEASUREMENTS OF
Th²³² RESONANCE CAPTURES IN THE PEACH BOTTOM
(40-MW(E) PROTOTYPE HTGR) CRITICAL ASSEMBLY

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GA-3069
REACTOR TECHNOLOGY

ANALYSIS OF ACTIVATION MEASUREMENTS OF Th^{232} RESONANCE
CAPTURES IN THE PEACH BOTTOM (40-MW(E) PROTOTYPE HTGR)
CRITICAL ASSEMBLY

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SUMMARY

Measurements of thorium resonance-capture activations^{*} relative to those of Au¹⁹⁷ which were made in the Peach Bottom (HTGR) critical assembly are analyzed and compared with thorium resonance activations calculated from tabulated values of the resonance parameters. The new vanadium-subtraction method that has been applied is shown to measure thorium resonance captures relative to those of Au¹⁹⁷ to within several per cent in this geometry. The measurements show that the lowest several resonances of thorium capture less strongly than would be predicted by the listed parameters. This reduced capture is such that the thin-limit, episcadmium 1/E-spectrum resonance integral is about 84 b instead of the 96 b predicted by listed parameters; the thorium resonance captures for the critical assembly are about 6% less. The approximations currently used in calculating resonance captures in annular lattices probably underestimate captures by several per cent. The 300°K resonance-capture rate of ThO₂ in the annulus is indicated to be several per cent larger than that of metallic thorium, presumably because of a solid-state effect.

^{*}The technique of measurement is described in Ref. 1.

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I. INTRODUCTION

A quantitative knowledge of the resonance captures in Th^{232} is essential to the optimum design of thorium converter reactors, particularly those which will be operated for high burnup. It is desirable to determine thorium resonance captures under a broad range of conditions of interest to within the accuracy reasonably attainable by activation techniques, that is, about $\pm 3\%$.

Two types of integral measurements are desirable in order to achieve a thorough understanding of thorium resonance captures in the complex reactor situation. The first measurement is that of the thin-limit, $1/E$ -spectrum value, which is the atomic resonance integral, $\int \sigma_{\text{res}} (dE/E)$, for thorium. Since this quantity is simply related to the resonance parameters, its measurement provides a check of these parameters (resonance-parameter values are required to predict the resonance captures in any reactor condition). The second type of measurement is the resonance-capture rate in a reactor geometry having large self-shielding of the resonances. This second measurement together with the first provides an experimental check of the methods of computation of both the self-shielding and the reactor spectrum.

The thin-limit, $1/E$ -spectrum measurement of the atomic resonance integral of Th^{232} is neither new nor unique to the Peach Bottom HTGR. Integral measurements of this quantity have already been made several times, but with results which differ from one another by nearly a factor of two.* Furthermore, there are significant uncertainties in the values of several important resonance parameters. Because of these uncertainties, the thin-limit, $1/E$ -spectrum case has been remeasured as part of the

* See, for example, the results given in Refs. 2 and 3.

Peach Bottom HTGR development program.

Measurements in the HTGR geometry had not been previously made for either Th^{232} or U^{238} . They were carried out in this program with thorium in oxide form dispersed quasi-homogeneously through about one-quarter of the graphite of the core. Measurements were made with this thorium-graphite mixture in two annular arrangements, one annulus larger than the other, and in one cylindrical arrangement in the HTGR critical assembly.

A new technique of measurement termed the vanadium-subtraction technique, described in Ref. 1, has been principally applied in this work. An essential feature is the determination and subtraction of the activations in the $1/v$ cross-section component of an unknown nuclide through an auxiliary measurement. A nuclide having suitable activation characteristics and a cross section close to a $1/v$ energy dependence is required. The element chosen to provide these characteristics in these measurements was V^{51} contained in natural vanadium.

The principal reason for development of this technique was the difficulty in applying the cadmium-ratio methods of measurement to the HTGR-type quasi-homogeneous dispersal of thorium in graphite. The most satisfactory cadmium-ratio technique⁽⁴⁾ requires a large cadmium shield around the thorium-containing region in the reactor. This method cannot be applied to the HTGR because the graphite moderator would remain inside the shield, which would modify the neutron spectrum and invalidate the interpretation of the measurement. The cadmium-ratio technique can be used in making the HTGR measurements if the thorium detector itself is cadmium-shielded. This has been done in making some auxiliary measurements in the void spaces of the HTGR critical assembly; however, the use of cadmium shields on thin detector foils has been shown by experiment to be difficult. Furthermore, the cadmium shield on the detector itself leaves doubts as to whether all neutron in-streaming through cracks in the shield has been eliminated. Even slight streaming of this sort can cause a significant error in the

results obtained by the cadmium-ratio method. A final difficulty, discussed in detail in Section 2.2, is the inherently low accuracy of the cadmium-ratio method of measurement in the HTGR spectrum. Because of these disadvantages, the vanadium-subtraction technique⁽¹⁾ was developed, and it has proven to be quite successful in the HTGR application.

The vanadium-subtraction technique measures the resonance-capture rate of an unknown nuclide relative to that of a thin foil of Au^{197} , which is taken to be known.

Measurements of the activation of thorium have been made with two types of detector: The principal thorium detector used was a metallic foil. In addition, in several cases, the thorium oxide itself, dispersed in graphite and exposed in the HTGR critical assembly, has been used as a detector. This was done by grinding up and collecting the thorium-containing material after exposure, and measuring the induced radioactivity. This second method of detection is referred to as the "compact-grind" method.

In order to check the technique and to assure internal consistency, several auxiliary measurements have been made. Resonance integrals of U^{238} and Mn^{55} have been measured in the $1/E$ spectrum. In the critical-assembly triffles, which are the voids between fuel tubes, resonance-capture rates of thorium and vanadium have been measured using both the cadmium-ratio and the vanadium-subtraction techniques. The resonance integral of vanadium was studied by the cadmium-ratio technique in order to determine the deviation from $1/v$ cross-section behavior.

Two general classes of calculations have been carried out in this program. The first class of calculations establishes the quantitative information that must be known in order to reduce the measurements to a numerical result for the thorium (or other unknown) resonance-capture reaction rate. This information includes foil-detector resonance self-shielding and neutron-spectra determinations, so that the Au^{197} resonance-capture rate can be determined. The results obtained from the measurements together with these data-reducing calculations are termed the measured values.

The second class of calculations determines the resonance-capture rate in thorium from knowledge of resonance parameters and calculated neutron spectra by means of computational procedures for self-shielded cases currently in use at General Atomic. The value thus obtained is considered to be the calculated value of the resonance-capture reaction rate of thorium (or other element). Comparison with the measured value provides a check of the resonance parameters and the method of calculation.

Section II describes the principle of the measurements and gives results of the data-reducing calculations. Section III describes knowledge of the thorium resonance parameters and methods of calculation of resonance capture which are in use at General Atomic. Sections IV through VIII give results, and Section IX discusses the significance of these results.

II. METHODS OF MEASUREMENT AND REDUCTION OF MEASURED DATA

2. 1. THE VANADIUM-SUBTRACTION TECHNIQUE

The vanadium-subtraction technique⁽¹⁾ is based on carrying out neutron-induced activations on six samples, in the following manner:

1. Simultaneous activation, in the neutron spectrum of interest, of thin foils of Au¹⁹⁷, vanadium, and the nuclide whose resonance captures are to be measured.
2. Simultaneous activation of similar foils of the same three nuclides in a Maxwellian neutron spectrum of a thermal column.

All foils are arranged to integrate the activation over the geometry of the exposure. For example, the foil detectors used in the exposures in a cylindrical annulus in the critical assembly have the shape of a sector of this annulus which is bounded by two radii.

In addition to the foil detector used for thorium, some measurements have been made by exposing the ThO₂ dispersed in graphite in an annulus and observing its action after grinding and collecting. This is the "compact-grind" detection method.

During each exposure, care is taken to keep the flux constant to within a few per cent. Both exposures are simultaneously made and are terminated by shutdowns that occur within a second or less (this time difference is short compared with the decay time of any induced radiation).

Radiation is counted from the six activated samples. The pairs of foils for each nuclide decay in the same manner. The quantity A^S/A^M , which is the ratio of the activations in the spectrum of interest (S) to activations in a Maxwellian spectrum (M), therefore remains constant in time and is determined repeatedly at several convenient times after irradiation. Furthermore, the radiation observed following the activation of Au¹⁹⁷ and the unknowns is emitted in each particular case from a single nuclide whose 2200-meter/sec cross section, σ_{2200} , is known. This follows from the specific nature of the radiation from each material, and from the discrimination properties of the counter that was used in the detection.

The ratio of resonance activation of an unknown, x, to that of Au¹⁹⁷ is determined from the measured activation ratios by the following relationship:

$$\frac{(A_{\text{res}})_x}{(A_{\text{res}})_{\text{Au}}} = \frac{(g \sigma_{2200})_x}{(g \sigma_{2200})_{\text{Au}}} \frac{\frac{A_x^S}{A_x^M} - \frac{K}{g_x} \frac{A_V^S}{A_V^M}}{\frac{A_{\text{Au}}^S}{A_{\text{Au}}^M} - \frac{K}{g_{\text{Au}}} \frac{A_V^S}{A_V^M}}, \quad (1)$$

where A_{res} designates the unnormalized resonance-capture reaction rate,^{*} which is defined to be the total reaction rate reduced by the captures in a $1/v$ cross-section component; g is the Westcott-type⁽⁵⁾ spectrum factor for a Maxwellian spectrum, which is required to obtain the relative reaction rate for a cross section which deviates somewhat from $1/v$ behavior; and K is a spectrum-dependent factor[†] of the order of 0.9 which corrects for the small resonance captures in vanadium. The meaning and derivation of Eq. (1) are described in the remainder of this section.

Let the capture cross section of each nuclide be decomposed into a $1/v$ component, $\sigma_{1/v}$, and a resonance component, σ_{res} . The former is defined to be so normalized as to give the entire 2200-meter/sec (0.0253-ev) cross section. The resonance cross section is therefore defined to be the excess after $\sigma_{1/v}$ is subtracted from the total absorption cross section, σ ; that is,

$$\sigma_{\text{res}}(E) = \sigma(E) - \sigma_{1/v}(E) = \sigma(E) - \sigma_{2200} \sqrt{\frac{0.0253}{E}}. \quad (2)$$

Let ϕ^S designate the neutron flux in a spectrum, S , of interest. ϕ^S can be a spectrum having $1/E$ -dependence above about 0.5 ev, an unshielded reactor spectrum, or a cadmium-shielded reactor spectrum, and can be expressed in terms of either energy, E , or lethargy, u , as an independent

^{*} A_{res} is specifically defined in Eq. (16).

[†] K is defined in Eq. (14).

variable. The conventional relations apply between these two choices; namely,

$$u = \ln \frac{10^7}{E} \quad (E \text{ in ev}) , \quad (3a)$$

$$\phi^S(u) = E \phi^S(E) . \quad (3b)$$

The capture reaction rate in a portion of the spectrum, for each choice of independent variable, is

$$dA = \phi(E) \sigma(E) dE , \quad (4a)$$

or

$$dA = \phi(u) \sigma(u) du . \quad (4b)$$

The capture reaction rate over the spectrum interval E_1 to E_2 is

$$A = \int_{E_1}^{E_2} \phi(E) \sigma(E) dE , \quad (5a)$$

or

$$A = \int_{u(E_2)}^{u(E_1)} \phi(u) \sigma(u) du . \quad (5b)$$

For a $1/E$ spectrum, having $\phi(u)$ constant, Eqs. (5) are

$$A = \int_{E_1}^{E_2} \sigma(E) \frac{dE}{E} \quad (6a)$$

or

$$A = \int_{u(E_2)}^{u(E_1)} \sigma(u) du . \quad (6b)$$

Equations (6) have the form of the resonance integral. The specific definition adopted in this work for the thin-limit resonance integral is that form

of Eqs. (6) with contributions of $\sigma_{1/v}$ removed, or with σ replaced by σ_{res} ; that is,

$$I = \int_{E_1}^{E_2} \sigma_{res}(E) \frac{dE}{E} \quad (7a)$$

$$I = \int_{u(E_1)}^{u(E_2)} \sigma_{res}(u) du \quad (7b)$$

The resonance integral is the capture rate in the resonance cross-section component in the particular spectrum having a $1/E$ energy dependence, and with the spectrum normalized so that $\phi^S(u) = 1$.

In much of the following analysis the arguments of ϕ^S and the σ values are omitted, and the independent variable is understood to be E or u according to whether the differential dE or du is written.

Let ϕ^M designate the Maxwellian neutron-flux spectrum. The activation of a detector foil per unit mass of the i th material in ϕ^M is

$$A_i^M = (N\epsilon)_i \int \phi^M (\sigma_{1/v} + \sigma_{res})_i du \quad (8a)$$

or

$$A_i^M = \frac{\sqrt{\pi}}{2} (N\epsilon g)_i (\sigma_{2200})_i \phi^M du, \quad (8b)$$

where N_i is the number of atoms per unit weight of the foil, and ϵ_i is the efficiency of the counter used to measure the activation of the i th detector. Equation (8b) is the definition of the Westcott's Maxwellian spectrum factor g_i^5 , which is unity if σ_{res} is exactly zero at all energies where the Maxwellian flux is significant. For applications of present interest, all g_i values are shown in Section 2.3 to be unity to within a few tenths of one per cent.

The activation of a detector foil in the spectrum ϕ^S is

$$A_i^S = (N\epsilon)_i \int \phi^S(u) [\sigma_{1/v}(u) + f_{res}(u) \sigma_{res}(u)]_i du, \quad (9)$$

where the resonance self-shielding factor, $f_{\text{res}}(u)$, must be included in the analysis. The detector foils whose characteristics are described in Section 2.3 are sufficiently thin that self-shielding is of negligible concern, except for the big resonances. However, for these big resonances, self-shielding corrections of several per cent are necessary. It is sufficient with detector foils of small self-shielding to treat these self-shielding corrections by a single multiplicative factor for the entire resonance region, which is assumed to be the same in all spectra and which can therefore be evaluated in a $1/E$ spectrum. The second term of Eq. (9) is therefore replaced by

$$(N\epsilon f_{\text{res}})_i \int \phi^S(u) [\sigma_{\text{res}}(u)]_i du ,$$

where

$$(f_{\text{res}})_i = \frac{\int [f_{\text{res}}(u) \sigma_{\text{res}}(u)]_i du}{\int [\sigma_{\text{res}}(u)]_i du} . \quad (10)$$

The first term of Eq. (9) depends only on ϕ^S and σ_{2200} through the relation between $\sigma_{1/v}$ and σ_{2200} for each nuclide, and it is therefore proportional to A_i^M/g_i . Equations (9) and (10) can therefore be written

$$A_i^S = C \frac{A_i^M}{g_i} + (N\epsilon f_{\text{res}})_i \int \phi^S (\sigma_{\text{res}})_i du , \quad (9a)$$

where C is a constant of proportionality which is the same for each nuclide. Division of Eq. (9a) by Eq. (8) eliminates the unknown $(N\epsilon)_i$:

$$(f_{\text{res}})_i \int \phi^S \sigma_{\text{res}} du = \frac{\sqrt{\pi}}{2} (g \sigma_{2200})_i \int \phi^M du \left(\frac{A_i^S}{A_i^M} - \frac{C}{g_i} \right) . \quad (11)$$

According to Eq. (9a), C is

$$C = \frac{A_{1/v}^S}{A_{1/v}^M}, \quad (12)$$

where the A terms are the activations of an absorber having an ideal $1/v$ cross section. Actually, $A_{1/v}^S/A_{1/v}^M$ cannot be measured exactly by activation in any known material, because of the contributions from the resonance component of the cross section. In vanadium, however, this resonance component contributes only a few per cent of the activations in spectrum ϕ^S , and none in ϕ^M . C is therefore determined somewhat indirectly by measuring A_V^S/A_V^M to simulate $A_{1/v}^S/A_{1/v}^M$, and then correcting A_V^S to $A_{1/v}^S$ from knowledge of the relative importance of the resonance and $1/v$ component of the vanadium cross section. Let the required correction factor be defined as follows:*

$$K = \frac{A_{1/v}^S}{A_V^S} = \frac{\int \sigma_{1/v, V} \phi^S du}{\int \sigma_V \phi^S du}. \quad (13)$$

Eq. (13) can be finally expressed in terms of cross sections, as follows:

$$K = \frac{1}{1 + \frac{\int \sigma_{res, V} \phi^S du}{\int \sigma_{1/v, V} \phi^S du}}. \quad (14)$$

C of Eq. (11) is then to be eliminated by the following relation:

$$C = K \frac{A_V^S}{A_V^M}, \quad (15)$$

where the activation ratio of Eq. (15) is to be measured. The second term in the denominator of Eq. (14) is calculated in Section 5 from a combination of measured information concerning vanadium resonance capture, described in Section 4.2, and calculated spectral information concerning ϕ^S . The pertinent spectral quantities are defined in Section 2.5.

*g and the self-shielding factors for the vanadium foil are unity.

Equation (1) is then obtained from the ratio of two equations of the form of Eq. (11), one for an unknown and one for Au¹⁹⁷, with C eliminated by Eq. (15):

$$\frac{(f_{\text{res}})_x \int \phi^S(\sigma_{\text{res}})_x du}{(f_{\text{res}})_{\text{Au}} \int \phi^S(\sigma_{\text{res}})_{\text{Au}} du} = \frac{(g \sigma_{2200})_x \frac{A_x^S}{A_x^M} - \frac{K}{g_x} \frac{A_V^S}{A_V^M}}{(g \sigma_{2200})_{\text{Au}} \frac{A_{\text{Au}}^S}{A_{\text{Au}}^M} - \frac{K}{g_{\text{Au}}} \frac{A_V^S}{A_V^M}}. \quad (1a)$$

This equation is just Eq. (1), since the resonance reaction rate, $(A_{\text{res}})_i$, is

$$(A_{\text{res}})_i = (f_{\text{res}})_i \int \phi^S(\sigma_{\text{res}})_i du. \quad (16)$$

Since ratios are used in Eq. (1), this reaction rate can be normalized to any flux level. The normalization adopted in this work to compare calculations with measurements is given in Section 2.5.

Several points about this vanadium-subtraction activation method are noteworthy:

1. An essential feature is the simultaneous exposure of foils of an unknown, of Au¹⁹⁷, and of vanadium in each of the fluxes, ϕ^S and ϕ^M , so that C is the same for each nuclide.
2. The poorly known 2200-meter/sec activation cross section of vanadium is not required in the analysis.
3. An auxiliary measurement is required to determine K, since the resonance integral of vanadium has not yet been measured. This measurement and the resulting determination of K are described in Section V.
4. For an infinitesimally thin detector in a 1/E spectrum, Eq. (16) gives a resonance reaction rate which is the resonance integral with the "1/v tail" subtracted.

The quantity measured by the vanadium-subtraction technique is that usually quoted by the Swedish group⁽⁶⁾ as the resonance integral. A more common definition of the resonance integral⁽⁷⁾ includes the "1/v tail" above the cadmium cutoff energy. The difference between these alternative definitions is the quantity

$$2 \sqrt{0.0253/E_{\text{Cd}}} (\sigma_{2200}) ,$$

where E_{Cd} is the cadmium cutoff energy. This difference is about 35 b in the case of Au^{197} and more than 3 b* in the case of Th^{232} .

2.2. THE CADMIUM-RATIO TECHNIQUE

The cadmium-ratio technique has been employed to obtain a semi-quantitative measurement of the resonance capture rate of vanadium, Mn^{55} , and Th^{232} foils placed in a void in the HTGR critical assembly. This was done for vanadium because no measurements of its resonance integral have been reported previously. The measurements for Mn^{55} and Th^{232} were made for comparison with results obtained by the vanadium-subtraction technique.

The cadmium-ratio technique can be seen to be inherently less accurate for making measurements in HTGR spectra than the vanadium-subtraction technique. Results obtained for these measurements by the cadmium-ratio method should be considered to have an accuracy of about $\pm 10\%$. Reasons for this relatively poor accuracy are discussed below.

The familiar cadmium-ratio method described by Macklin and Pomerance⁽⁷⁾ measures C. R., the cadmium ratio of two foils, and determines the ratio of epicadmium reaction rates from these measurements by means of the following relation:

$$\frac{(R_{\text{epi}})_1}{(R_{\text{epi}})_2} = \frac{(C. R. - 1)_2}{(C. R. - 1)_1} \frac{\int_0^{E_{\text{Cd}}} \phi S \sigma_1 dE}{\int_0^{E_{\text{Cd}}} \phi S \sigma_2 dE} , \quad (17)$$

* The "deviation from 1/v dependence," discussed in Sections 3.1 and 7.2, reduces this difference to about 2.5 b in the case of Th^{232} .

where R_{epi} is the epicadmium reaction rate above the cadmium cutoff energy, E_{Cd} , and the quantity

$$\int_0^{E_{\text{Cd}}} \phi^S \sigma \, dE$$

is a subcadmium reaction rate. Equation (17) is applied to relate the R_{epi} values of an unknown to that of a known quantity, usually taken to be Au^{197} .

For the HTGR, the C. R. of Au^{197} is 1.20 or less. C. R. can be measured to within about $\pm 1\%$. This leaves an uncertainty of $\pm 5\%$ in the quantity (C. R. - 1) of Au^{197} , which makes R_{epi} of Eq. (17) uncertain by $\pm 5\%$.

Further loss of accuracy can occur in determining the resonance-reaction-rate component of R_{epi} , particularly for vanadium. The $1/v$ -cross-section component absorbs about 95% of the epicadmium captures in the case of vanadium. The determination of the σ_{res} capture component of Eq. (14) for vanadium from Eq. (17) therefore involves two separate instances of small differences of nearly equal numbers, leading to a result having the accuracy of only an estimate.

A further complication in applying Eq. (17) with Au^{197} as a standard is that its subcadmium cross section deviates significantly from $1/v$ behavior. For this reason E_{Cd} for Au^{197} is significantly larger^{(8)*} than that of the other nuclides that have $1/v$ -like cross sections, and it is more sensitive to the subcadmium neutron spectrum. These peculiarities of Au^{197} make the ratio of integrals on the right side of Eq. (17) different by about 5% from the ratio of 2200-meter/sec cross sections. A quantitative application of Eq. (17) requires good knowledge of ϕ^S and careful evaluation of the integrals. This difficulty considerably lessens the usefulness of the technique of measuring resonance captures by comparing the cadmium ratio of an unknown with that of Au^{197} . The principal advantage of the measurement procedure described by Macklin and Pomerance⁽⁷⁾ is that, for materials having cross sections with a $1/v$ dependence up to about 1 ev, the ratio of integrals in Eq. (17) is the same as the ratio of the 2200-meter/sec cross sections, so

* See particularly Table III, p. 344 of Ref. 8.

that evaluation of these integrals can be avoided. However, this advantage is not applicable when ratios are taken relative to Au^{197} .

Because of these complications, the method adopted in this work determines the ratio of resonance captures of an unknown to its subcadmium captures, which are considered to be known. No reference to the properties of Au^{197} is involved. To accomplish this, two quantities characteristic of the low-energy spectrum are required. These have been carefully calculated by a method based on a realistic and thorough treatment of the neutron spectrum.⁽⁹⁾ This method of calculation has shown excellent agreement⁽⁹⁾ with spectra which have been measured for very similar cases. The two spectral quantities chosen are $(\text{C. R.})_{1/v}$ and T . The quantity $(\text{C. R.})_{1/v}$ is the cadmium ratio of a $1/v$ absorber. T is proportional to the absorption rate in a $1/v$ -cross-section component above the cadmium cutoff and is defined as follows:

$$T \sigma_{2200} = \frac{1}{\phi^S(1 \text{ ev})} \int_{E_{\text{Cd}}}^{\infty} \phi^S \sigma_{1/v} dE. \quad (18)$$

T is nearly the same for each spectrum of present interest, so that it is well known. $(\text{C. R.})_{1/v}$ is considered to be determined from the quantitative spectrum calculations to within about $\pm 5\%$.

Consider the relation of C. R. of any particular nuclide, other than gold, to the resonance reaction rate and defined spectral quantities for this nuclide:

$$\text{C. R.} = \frac{\int_0^{E_{\text{Cd}}} \phi^S \sigma_{1/v} dE + \int_{E_{\text{Cd}}}^{\infty} \phi^S \sigma_{1/v} dE + f_{\text{res}} \int_{E_{\text{Cd}}}^{\infty} \phi^S \sigma_{\text{res}} dE}{\int_{E_{\text{Cd}}}^{\infty} \phi^S \sigma_{1/v} dE + f_{\text{res}} \int_{E_{\text{Cd}}}^{\infty} \phi^S \sigma_{\text{res}} dE}, \quad (19)$$

since the nuclides of interest have a negligible σ_{res} component at energies less than E_{Cd} . Upon division of numerator and denominator by $\phi(1 \text{ ev})$, Eq. (19) becomes

$$\text{C.R.} = \frac{\frac{1}{\phi^S(1 \text{ ev})} \int_0^{E_{\text{Cd}}} \phi^S \sigma_{1/v} dE + T \sigma_{2200} + \frac{f_{\text{res}}}{\phi^S(1 \text{ ev})} \int_{E_{\text{Cd}}}^{\infty} \phi^S \sigma_{\text{res}} dE}{T \sigma_{2200} + \frac{f_{\text{res}}}{\phi^S(1 \text{ ev})} \int_{E_{\text{Cd}}}^{\infty} \phi^S \sigma_{\text{res}} dE} \quad (20)$$

For a $1/v$ nuclide, Eq. (20) gives

$$\frac{1}{\phi^S(1 \text{ ev})} \int_0^{E_{\text{Cd}}} \phi^S \sigma_{1/v} dE = \left[(\text{C.R.})_{1/v} - 1 \right] T \sigma_{2200} \quad (21)$$

The combination of Eqs. (20) and (21) enables one to obtain the resonance reaction rate as follows:

$$\frac{\int \phi^S \sigma_{\text{res}}^x dE}{\phi(1 \text{ ev})} = \frac{1}{f_{\text{res}}^x} \frac{(\text{C.R.})_{1/v} - (\text{C.R.})_x}{(\text{C.R.})_x - 1} T \sigma_{2200}^x \quad (22)$$

Equation (17) was found to give results in agreement with those obtained by means of Eq. (22) to within the errors discussed previously.

A limitation to the accuracy of the determination of resonance capture rates by means of Eq. (22) results from the difference of C.R.'s in the numerator, each of which may be in error by several per cent. The accuracy is such that the resonance reaction rates of Mn^{55} and vanadium are determined to within about $\pm 10\%$ and $\pm 50\%$, respectively. The measurements of Section V show the resonance captures of vanadium to be only a few per cent of its $1/v$ -component captures. Since this resonance component of vanadium is measured to be small in any case, it is not necessary to improve the $\pm 50\%$ accuracy in its determination.

In the analysis, a renormalized version of Eq. (22), designated Eq. (22a) and described in Section 2.5, is applied.

2.3. CALCULATION OF DETECTOR-FOIL CHARACTERISTICS

The resonance self-shielding factors, f_{res} , and the Maxwellian-spectrum g factors have been carefully calculated for the detector foils.

The f_{res} were found by first determining the resonance integral of a slab of the same thickness as the detector foil and then comparing this value with the value for an infinitesimally thin slab; the resulting ratio is f_{res} . The best available integral-equation method^{(10-13)*} of calculation was applied. The approximately known values of the resonance parameters which were used are adequate to quantitatively determine the relative captures in a practical foil compared with the limit of a very thin foil. For Au^{197} and Mn^{55} , which have about 90% of captures in the lowest resonance, at 4.91 ev and 337 ev, respectively, the self-shielding of only this dominant resonance was determined and was then applied as appropriate for the entire resonance region. For Th^{232} , the resolved resonances up to about 1000 ev were considered in determining foil self-shielding.

The Maxwellian-spectrum g factors are in close agreement with values that have been given by Westcott.⁽¹⁴⁾ Special attention was given to checking values for Au^{197} and Th^{232} . In the case of Au^{197} , measurements made at Columbia University^{(15)†} of the low-energy cross-section dependence were used. For Th^{232} , the cross-section description of Garrison⁽¹⁶⁾ discussed in Section III was applied.

The characteristics of metallic detector foils, including calculated values of f_{res} and g , are given in Table 1. In the case of Th^{232} , two different thicknesses of detector foils were used, the thicker foil having been used in measurements for the large-annulus geometry and the thinner foil in all other measurements. The characteristics of both foils are given in Table 1.

* In Ref. 10, see Sections 3 and 4, in particular.

† See particularly p. 462 of Ref. 15.

Table 1
CHARACTERISTICS OF DETECTOR FOILS

Principal Nuclide	Mass of Principal Nuclide (mg/cm ²)	σ_{2200} (barns)	Resonance Self-shielding Factor, f_{res}	Maxwellian-spectrum Factor, g
Au ¹⁹⁷	0.17	98.8	0.980	1.003
Vanadium	76	4.5	1.00	1.00
Th ²³²	{ 3.3 ^a 4.24 ^b	7.45	0.966	1.002
		7.45	0.959	1.002
U ²³⁸	0.85	2.75	0.974	1.00
Mn ⁵⁵	3.3	13.2	0.955	1.00

^aUsed for most measurements.

^bUsed for large-annulus measurements only.

All the foils were sufficiently thin that 2200-meter/sec cross-section self-shielding factors were within about 0.1% of unity. Consequently, low-energy self-shielding factors were neglected. The f_{res} for the compact-grind method is determined by the grain size of the thorium oxide particles contained in the compact. For most exposures in which the grain size was 40 microns or less, this grain self-shielding was negligible. The large-annulus exposure, in which the average grain size was 237 microns, was calculated to give a value of 0.94 for f_{res} .

2.4. ABSORPTION CROSS SECTION OF Au¹⁹⁷

The following information has been used to determine absorptions in Au¹⁹⁷, which was taken as the standard of reference in these measurements.

Columbia measurements of the resonance parameters^{*} have been adopted. These values are given in Table 2. For energies up to 37 ev, where the cross section is dominated by the 4.91-ev resonance, $\sigma_{n,\gamma}$ is

^{*}Results of these measurements are reported in Ref. 17.

taken to be a sum of the contribution from the dominant resonance and a residual $1/v$ component arising from higher-energy resonances. This gives, for $0 \leq E \leq 37$ ev,

$$(\sigma_{n,\gamma})_{\text{Au}^{197}} = 32.85 \times 10^3 \sqrt{\frac{4.91}{E}} \frac{1}{\left(\frac{E - 4.91}{0.0698}\right)^2 + 1} + \frac{0.8591}{\sqrt{E}}. \quad (23)$$

Each resolved resonance lying between 47 and 940 ev has been treated as having δ -function^{*} absorption of a strength necessary to give the thin-limit resonance integral predicted by the resonance parameters of Table 2, which uses the standard notation of BNL-325.⁽¹⁸⁾ Since these resonances are narrow and the detector foil self-shielding is negligible for all but the 4.91-ev resonance, this mathematically idealized treatment is correct for these higher resonances. Additional resonances of Au^{197} are listed in Table 3.

In the energy region above 940 ev, $\sigma_{n,\gamma}$ is treated on a statistical basis using the averages of the parameters listed in Table 2, as follows:

$$\bar{D} = 21.1 \text{ ev}$$

$$\bar{\Gamma}_{\gamma} = 0.196 \text{ ev}$$

$$\bar{\Gamma}_n^0 = 0.0051 \text{ ev}$$

$$\bar{g} = 1/2$$

Two approximations are made in treating this region, namely, neglect of the reduction due to the Porter-Thomas distribution and neglect of the increase due to $\lambda > 0$ contributions. The net effect of these approximations is probably an overestimate of the resonance integral by about 2 b, which is a negligible fraction of its total.

^{*}The δ -function is P. A. M. Dirac's mathematical idealization of a narrow, peaked function of given area under the curve. The idealization considers the function to be taken to a limit having infinitesimal width and infinite height in a manner such that the area under the curve, or "strength," remains constant.

Table 2
MEASUREMENTS OF Au¹⁹⁷ RESONANCE PARAMETERS
(Data taken from Ref. 17)

E_0 (ev)	g	Γ_n (10^{-3} ev)	Γ_n^0 (10^{-3} (ev) ^{1/2})	Γ_γ (10^{-3} ev)
4.91	5/8	15.6	7.0	124
46.7		0.11	0.016	
58.2		4.7	0.62	143
60.3	5/8	76.0	9.8	120
78.7		13.0	1.46	145
107.3		9.3	0.9	165
122.6		0.8	0.07	
144.8		6.5	0.54	190
151.8	5/8	23.2	1.9	179
163.6		40.0	3.1	
165.6		12.0	0.9	
190.8	3/8	36.0	2.5	189
209.6		0.72	0.05	
241.0	5/8	68.0	4.4	185
256.2		0.6	0.37	
263.4	3/8	150.7	9.3	136
275.0		5.0	0.3	127
294.5		400.0	23.0	250
330.5		200	11.0	
332.5		60	3.2	
357.0	5/8	40.8	2.2	177
373.0		115	5.9	
377.0		12	0.6	
384.0		90	4.6	
403.0		26	1.3	590
442.0	3/8	306.7	15.0	280
453.0	3/8	120	5.6	250
479.0		300.8	13.6	180
492.0		45	2.0	
496.0		15	0.7	
538.0		39	1.7	280
551.0		36	1.5	282
565.0		3.3	0.14	
583.0		370	15.0	
590.0		30	1.2	

Table 2--continued

E_0 (ev)	g	Γ_n (10^{-3} ev)	Γ_n^0 (10^{-3} (ev) $^{1/2}$)	Γ_γ (10^{-3} ev)
606.0	5/8	198.4	8.0	140
621.0	3/8	110.7	4.4	160
628.0		60	2.4	
632.0		35	1.4	
644.0	3/8	653.3	25.0	270
664.0		6.2	0.24	
691.0		10	0.38	
702.0		400	~ 15.0	
706.0		240	~ 9.0	
721.0	5/8	86.4	3.2	210
743.0		11	0.4	
766.0	3/8	426.7	15.3	154
781.0		148	5.3	
791.0		125	4.4	170
803.0	3/8	285.3	10.0	217
819.0		40	1.4	
825.0		150	5	
831.0		150	5	
871.0		16	0.54	150
886.0		65	2.2	125
940.0	5/8	434.4	14.2	110

Table 3

ADDITIONAL E_0 VALUES OBSERVED FOR Au^{197}

(In ev)

(Data taken from Ref. 17)

968	1137	1297	1465	1660	1905	2220
993	1140	1323	1485	1712	1935	2250
998	1194	1340	1505	1725	1965	2270
1002	1217	1347	1514	1740	1995	2315
1047	1228	1367	1545	1752	2055	2360
1072	1232	1375	1566	1775	2105	2395
1086	1255	1408	1595	1825	2140	2440
1100	1265	1438	1610	1840	2155	2470
1130	1293	1445	1630	1870	2175	

These data lead to the following thin-limit, $1/E$ -spectrum resonance integral:

$$\int_0^{\infty} (\sigma_{\text{res}})_{\text{Au}^{197}} \frac{dE}{E} = 1526 \text{ b} , \quad (24)$$

where

$$(\sigma_{\text{res}})_{\text{Au}^{197}} = (\sigma)_{\text{Au}^{197}} - 98.8 \sqrt{\frac{0.0253}{E}} .$$

Here, σ designates $\sigma_{n,\gamma}$. This 1526-b result is in reasonable agreement with the 1490 ± 40 b result for the same integral from 0.5 ev to infinity which was obtained by Swedish workers⁽⁶⁾ who used a cadmium-ratio technique of measurement. The absorption rate of Au^{197} can be used with confidence that it is accurate to within $\pm 2\%$. The value adopted in this analysis may be slightly high. The same comments about accuracy apply to the thorium and other resonance reaction rates determined from these measurements relative to these Au^{197} absorptions.

2.5. CALCULATION OF REACTOR NEUTRON SPECTRA AND Au^{197} RESONANCE-CAPTURE REACTION RATES

The data-reducing class of calculations includes determination of neutron spectra. These spectra in reactor cores, together with Au^{197} cross-section data, are required for determination of the resonance-capture rate of Au^{197} , which is used as the standard of reference in the vanadium-subtraction method.

Measurements were made in spectra of the following four basic configurations:

1. A position 2.8 cm into the water reflector of the TRIGA reactor, i. e., 2.8 cm beyond the core-reflector interface.
2. HTGR critical assembly core 3.
3. HTGR critical assembly core 6.
4. HTGR critical assembly core 7.

Exposure in the TRIGA reactor was used to produce a $1/E$ spectrum in the resonance region. Measurements in this spectrum using the detector foils described in Table 1 have been made to determine thin-limit, $1/E$ -spectrum resonance integrals.

The activations in the several loadings of the Peach Bottom HTGR critical assembly were carried out near the center of the reactor. The nature of this assembly, the specific core loadings, and the details of the local geometry are described in separate reports. Cores 3 and 6 are versions of the so-called "lattice testing" loadings (described in Ref. 19), in which the annular HTGR geometry was simulated at the center of the core. Core 7 (described in Ref. 20) simulated the neutron events in a simpler geometry having the thorium loaded in a cylindrical arrangement.

In addition, several exposures were made in cores 4B and 5, which had essentially the same spectral characteristics above 1 ev as those of core 6. In core 5, the amount of thorium contained in the six annuli surrounding the central annulus was greater than in cores 3 and 6, so that the C/Th ratio in these particular annuli of core 5 was 48.7 (30 wt-% metal).

Activation exposures were made in the following positions in these assemblies:

1. Central annuli of cores 4B, 5, and 6.
2. Buffer cylinder of core 7.
3. Central triffutes, or void spaces around the central annulus, of cores 3 and 6.

Calculations of two general types to determine ϕ^S were carried out. These calculations for a particular ϕ^S can be designated as initial-spectrum and multigroup-spectrum calculations.

The initial spectrum is determined by computations of the slowing-down and thermalization distribution made at a point in the reactor. In these calculations, leakage is approximated by a constant buckling, which was estimated initially; other significant physical effects are considered

to be adequately described. In these initial-spectrum calculations, the solution is obtained in great detail as a function of energy, the broadest lethargy bandwidth being one-quarter of a unit. The energy range down to about 2 ev is handled by the GAM-I code.⁽²¹⁾ GAM-I treats all scattering and degradation realistically and describes thorium resonance absorption by average cross sections determined for each quarter-unit-wide lethargy group. This treatment in each thorium resonance gives the absorptions determined by either the N. R. or I. M. (N. R. I. A.) Wigner resonance integral approximation.^{(10)*}(22)(23) Below 2 ev, the GATHER-II code⁽²⁴⁾ is used; this code computes thermalization on the basis of an advanced and experimentally verified spectrum model.⁽⁹⁾

Initial spectra for HTGR critical assembly cores 3 and 6 are given in Fig. 1. The spectra of these cores differ to only a negligible extent in the resonance region, above about 2 ev, but in the thermal region the borated core, core 6, has a noticeably harder spectrum.

Multigroup spectra have been determined from the output of one-dimensional multigroup calculations done for an entire assembly.⁽²⁵⁾ These calculations have been carried out for HTGR cores 3, 6, and 7, and for the TRIGA reactor. The results obtained for those multigroup spectra in the energy region above several electron volts for each HTGR assembly were very similar, and were consistent with the curve of Fig. 1 to within a few per cent. Therefore, the resonance-region spectrum of each HTGR core can be taken to be adequately described by the initial spectrum of Fig. 1, although the actual spectra of the final analysis were usually determined by the multigroup calculations.

Figure 2 gives the multigroup-determined spectrum at the position of exposure in the reflector of the TRIGA reactor. These calculations show the energy dependence of this spectrum to be close to $1/E$ over the important part of the resonance-capture region.

The calculated HTGR spectrum in the resonance region, given in

* See p. 67 of Ref. 10.

Fig. 1--Spectra at the center of cores of the critical assembly, as determined by the GAM and GATHER codes

Fig. 2--Spectrum at a location 2.8 cm into the water reflector of the TRIGA reactor

Fig. 1, is that for a completely homogenized reactor composition. In the actual condition of exposure, particularly in the thorium-containing elements, sharp dips occur which are very localized in energy or lethargy, caused by the local depletion of the flux by the resonance captures in thorium. These dips can be ignored in computing the Au resonance-capture rate, since it is quite improbable that any important thorium resonance occurs at the same energy as that of an important Au resonance. The same ϕ^S has also been assumed to apply to other resonance absorbers. The effect of these flux dips in the case of thorium resonance capture, where they are indeed important, is accounted for in the methods of computation of A_{res} or σ_{res} that have been developed. These computation methods include the effect of thorium self-shielding in the resonance region. This self-shielding also includes the resonance flux depletion inside the thorium-containing region. That is, ϕ^S is taken to be the nondepleted flux, in which the thorium resonance-capture rate is given by a self-shielded, effective cross section.

The cadmium cutoff, E_{Cd} , has been defined in terms of the reaction rate of a $1/v$ absorber and spectra such as that of Fig. 1. The following definition was applied to determine E_{Cd} :

$$\int_0^{\infty} \phi^S(E) \tau_{\text{Cd}} \sqrt{\frac{E_0}{E}} dE = \int_{E_{\text{Cd}}}^{\infty} \phi^S(E) \sqrt{\frac{E_0}{E}} dE, \quad (25)$$

where τ_{Cd} is the transmission function of the 20-mil-thick (0.0508-cm) cadmium cover used in these measurements.

Low-energy spectral parameters discussed in Section 2.2 have been determined for the spectra of Figs. 1 and 2 and results are given in Table 4.

The resonance-capture rates of Au^{197} in these spectra are given in Table 5. To be expressed in units which are the same as the resonance integral, this reaction rate must be divided by a flux. The particular flux chosen, the choice of which is arbitrary, gives a normalization factor.

The normalization adopted as "standard" in this work is $\overline{\phi^S}$, where

Table 4
CALCULATED LOW-ENERGY SPECTRAL QUANTITIES

Spectrum	E _{Cd} (ev)	(C. R.) _{1/v}	T
1/E	0.52		0.44
Core 3	0.51	6.95	0.47
Core 6	0.51	5.36	0.47

Table 5
RESONANCE REACTION RATES OF Au¹⁹⁷
IN CHARACTERISTIC SPECTRA

Spectrum	$\frac{\int \phi^S (\sigma_{res})_{Au} du}{\phi^S [u(1 \text{ ev})]}$ (barns)	$\frac{\overline{\phi^S(u)}}{\phi^S [u(1 \text{ ev})]}$	$\frac{\int \phi^S (\sigma_{res})_{Au} du}{\overline{\phi^S}}$ (barns)	$\frac{(f_{res} \int \phi^S \sigma_{res} du)_{Au}}{\overline{\phi^S}}$ (barns)
1/E	1526	1.00	1526	1495
Core 3	1585	1.45	1093	1071
Core 6	1579	1.405	1124	1102

$\overline{\phi^S}$ is $\phi^S(u)$ averaged over the lethargy interval corresponding to the energy interval between 13.7 ev and 86,500 ev:

$$\overline{\phi^S} = \frac{1}{u(13.7 \text{ ev}) - u(86,500 \text{ ev})} \int_{u(86,500 \text{ ev})}^{u(13.7 \text{ ev})} \phi^S du. \quad (26)$$

The normalization over this energy band, which includes nearly all thorium resonance captures, gives a thorium resonance reaction rate in ϕ^S which is close to the capture value for the same self-shielding in a 1/E spectrum. Values of the Au¹⁹⁷ resonance reaction rate in ϕ^S , for this same normalization, are given in the fourth column of Table 5, the self-shielding of the Au¹⁹⁷ foil being neglected. Note that for cores 3 and 6 these values differ considerably from the 1526-b value in a 1/E spectrum. This is so because ϕ^S is less at 4.91 ev, where most of the Au¹⁹⁷ resonance captures occur, than the mean value of ϕ^S in the lethargy band extending from 13.7 ev to 86,500 ev.

An alternative normalization factor, namely, $1/\phi^S[u(1 \text{ ev})]$ is implied in the analysis of Eqs. (18) through (22). This is converted to the standard normalization by dividing by $\overline{S}/\phi^S[u(1 \text{ ev})]$ for the particular spectrum. Values of this ratio for cores 3 and 6 are given in the third column of Table 5. The second column of the table gives the Au^{197} reaction rate under the $1/\phi^S[u(1 \text{ ev})]$ normalization. These values for the HTGR cores are more similar to the $1/E$ spectrum value for Au^{197} . The values for resonance reaction rates stated in Section V, which have been determined by cadmium-ratio measurements, are obtained by dividing Eq. (22) by $\overline{S}/\phi^S[u(1 \text{ ev})]$:

$$\frac{\int \phi^S \sigma_{\text{res}}^x dE}{\overline{S}} = \frac{1}{\overline{S}} \frac{1}{\phi^S(1 \text{ ev})} \frac{1}{f_{\text{res}}^x} \frac{(C.R.)_{1/v} - (C.R.)_x}{(C.R.)_x - 1} T \sigma_{2200} \quad (22a)$$

An additional spectral quantity useful in correlating results is a number that will semiquantitatively convert resonance captures measured in the reactor spectrum to the resonance integral, I , defined by Eq. (7). If all resonance captures occurred at a single energy, E_{res} , this number would be $\overline{S}/\phi^S(E_{\text{res}})$. The connection between the resonance reaction rate and I would be, for the single-resonance case,

$$\int \phi^S(u) \sigma_{\text{res}}(u) du = \phi^S[u(E_{\text{res}})] \int \sigma_{\text{res}}(u) du = \phi^S(E_{\text{res}}) I \quad (27)$$

One thus finds I to be given approximately by

$$\begin{aligned} I_{\text{app}} &= \frac{1}{\phi^S(E_{\text{res}})} \int \phi^S \sigma_{\text{res}} du \\ &= \frac{\overline{S}}{\phi^S(E_{\text{res}})} \frac{\int \phi^S \sigma_{\text{res}} du}{\overline{S}} \quad (28) \end{aligned}$$

where

$$\frac{\overline{\phi S}}{\phi^S(E_{res})} = \frac{\overline{\phi S}}{\phi^S(1 \text{ ev})} \bigg/ \frac{\phi^S(E_{res})}{\phi^S(1 \text{ ev})} . \quad (29)$$

In practice, E_{res} is spread over a broad energy band, but it can be considered to be given by energies of the dominant resonances. E_{res} was estimated in this way, and Eqs. (28) and (29) then applied. Data relevant to Eqs. (28) and (29) are given in Table 6.

Table 6
DATA USED TO SEMIQUANTITATIVELY RELATE I
TO REACTOR-SPECTRUM RESONANCE CAPTURE

Nuclide	Estimated E_{res} (ev)	u_{res}	Core 3			Core 6		
			$\frac{\overline{\phi S}}{\phi^S(1 \text{ ev})}$	$\frac{\phi^S(E_{res})}{\phi^S(1 \text{ ev})}$	$\frac{\overline{\phi S}}{\phi^S(E_{res})}$	$\frac{\overline{\phi S}}{\phi^S(1 \text{ ev})}$	$\frac{\phi^S(E_{res})}{\phi^S(1 \text{ ev})}$	$\frac{\overline{\phi S}}{\phi^S(E_{res})}$
			$\overline{\phi S}$	$\phi^S(E_{res})$	$\overline{\phi S}$	$\overline{\phi S}$	$\phi^S(E_{res})$	$\overline{\phi S}$
Vanadium	6000	7.4	1.45	1.55	0.93	1.40	1.50	0.93
Mn ⁵⁵	350	10.2	1.45	1.45	1.00	1.40	1.43	0.98
Th ²³²	60	12.0	1.45	1.30	1.12	1.40	1.28	1.09

III. METHOD OF CALCULATION OF Th^{232} RESONANCE REACTION RATE FROM CROSS-SECTION DATA

3. 1. STATUS OF KNOWLEDGE OF THORIUM ABSORPTION CROSS SECTIONS

The basis of knowledge of the thorium resonance parameters is somewhat complicated, and some of the data have significant uncertainty. This uncertainty has been incorporated into the present analysis by considering two alternative sets of resonance parameters which predict resonance captures differing by about 10% from one another. A principal object of the present program of measurement and calculation is to determine which set of thorium resonance parameters is most correct.

The following background concerning knowledge of thorium resonance parameters is given for the purpose of orientation.

Recent, careful measurements⁽¹⁷⁾ of resonance parameters have been reported by Columbia University. These measurements give the values for most resonances up to about 1000 ev. However, for the four important lowest resonances below 100 ev, only E_0 was measured at Columbia, and values of Γ quoted in Ref. 17 are actually values previously listed in BNL-325.⁽¹⁸⁾ These Γ values for the lowest resonances are less well known than the resonances above 100 ev. They are based on older neutron-spectrometer measurements made at Brookhaven⁽²⁶⁾ and Argonne National Laboratory. Examination of the data of these measurements led J. D. Garrison⁽¹⁶⁾ to suggest that the data are not self-consistent. Selection of a part of the data would give Γ_n values for the two lowest levels reduced by about 30% from the BNL values.⁽¹⁸⁾ This leads to a similar reduction in resonance integrals for these levels, which are primarily determined by Γ_n rather than Γ , as Γ_n is much the smaller. Russian neutron-

spectrometer measurements⁽²⁷⁾ also have indicated lower values of Γ_n for these two lowest resonances.

Consequently, the following alternative sets of data for thorium resonance parameters have been considered:

1. The set of the Columbia Γ values above 100 ev, Columbia measurements of E_0 at all energies, and BNL-325 Γ values below 100 ev.
2. The same data as in set 1 except that Γ_n values for the two lowest levels are reduced by the factor 0.7.

The specific data used as set 1 have been obtained by consultation with the Neutron Cross Section Evaluation Center of Brookhaven National Laboratory.⁽²⁶⁾ These data are listed in Table 7. Also, the revisions which lead to the data of set 2, above, are given.

The self-explanatory terms "high" and "low" used in Table 7 and throughout the rest of this report refer to the first and second sets of resonance parameters, respectively. In both sets, the 0.030-ev value of Γ_γ for the lowest four resonances has been assumed. This value may be inconsistent with the higher values measured at Columbia above 100 ev. The change of Γ_γ of the lowest four levels from 0.030 ev to 0.040 ev would decrease the thin-limit resonance integral by only 2%, so this possible inconsistency is of little practical importance.

The average values derived from the data of Table 7 are:

$$\begin{aligned}\overline{D} &= 19.8 \text{ ev}, \\ \overline{\Gamma_\gamma} &= 0.041 \text{ ev}, \\ \overline{\Gamma_n^0} &= 0.00132 \text{ (ev)}^{1/2}.\end{aligned}$$

In the energy range from a few tenths of an electron volt to the thorium resonance region above 20 ev, the thorium cross section differs significantly from a $1/v$ dependence. This is primarily caused by a virtual level lying at several electron volts below zero. The reason for this virtual level, and values of resonance parameters suggested for it, have

Table 7
 RESONANCE PARAMETERS OF Th²³²

E_0 (ev)	Γ_n (10^{-3} ev)	Γ_n^0 (10^{-3} (ev) ^{1/2})	Γ_γ (10^{-3} ev)
21.84	2.4 (high) 1.68 (low)	0.51 (high) 0.36 (low)	30
23.48	4.0 (high) 2.86 (low)	0.83 (high) 0.59 (low)	30
59.55	4.6	0.060	30
69.2	39	4.69	30
113.15	11	1.04	42
121.0	18.5	1.68	41
128.5	0.10	0.009	
129.4	3.1	0.26	
146.2	0.10		
154.6	0.22	0.018	
170.8	58	4.45	48
193.0	14	1.01	50
196.8	0.13	0.009	0.01
199.8	9	0.64	
203			
221.8	23	1.54	46
252.3	24	1.51	42
264	15	0.92	
286.8	21	1.22	50
306.5		1.14	
330.4	55	3.02	53
343.3	30	1.62	
366.7	35	1.83	
371.0	24	1.25	
402.8	9	0.45	
413.4			
456.4			
465.0	45	2.09	
491.0	50		
513.0	3.5	0.15	
531.5	15	0.65	
543.0	1.0	0.04	
573.0	25	1.04	
581.0	1.6	0.06	

Table 7--continued

E_0 (ev)	Γ_n (10^{-3} ev)	Γ_n^0 (10^{-3} (ev) $^{1/2}$)	Γ_γ (10^{-3} ev)
601.5	11	0.45	
621.5	5	0.20	
660.5	32	1.24	
669.0	17	0.66	
679.5	125	4.80	
692.0	38	1.48	
705.0	18	0.68	
717.5	25	0.94	
746.0	160	5.85	
783.0	10	0.36	
810.0	125	4.40	
848.0	23	0.79	
872.0	10	0.34	
896.0	30	1.00	
950.0	40	1.30	
970.0	7.5	0.24	
990.0	24	0.76	
998.0	70	2.20	
1020			
1046			
1072			
1085			
1100			
1118			
1130			
1159			
1204			
1215			
1238			
1253			
1259			
1280			
1303			
1314			
1367			
1390			

been given by Cooper, Garrison, and Hines. ⁽²⁸⁾ The residual $1/v$ components from the levels lying at positive energies contribute about 0.6 b to the 2200-meter/sec cross section. A similar contribution is to be expected from distant virtual levels lying at negative energies. There remain over 6 b of the 2200-meter/sec cross section unaccounted for. This remainder must be due to a nearby virtual level. Garrison, et al., ⁽²⁸⁾ determined two sets of values for this level, corresponding to Γ_γ of 30×10^{-3} ev and 40×10^{-3} ev. Garrison required that this cross-section fit meet the following conditions:

1. A 2200-meter/sec absorption cross section of 7.45 b.
2. A σ_{total} at 1 ev of 13.4 b, of which 11.8 b is potential scattering.
3. Adoption of the low values of Γ_n of the 21.84-ev and 23.48-ev levels.

The two sets of parameters so determined are as follows:

	<u>Set 1</u>	<u>Set 2</u>
E_0	- 3.5 ev	- 4.3 ev
Γ_n^0	$0.636 \times 10^{-3} \text{ (ev)}^{1/2}$	$0.703 \times 10^{-3} \text{ (ev)}^{1/2}$
Γ_γ	$30 \times 10^{-3} \text{ ev}$	$40 \times 10^{-3} \text{ ev}$

The set having $\Gamma_\gamma = 30 \times 10^{-3}$ has been published. ⁽²⁸⁾ The set having $\Gamma_\gamma = 40 \times 10^{-3}$ ev has been adopted in the present analysis because this value is more consistent with the best measurements of Γ_γ , namely, the Columbia University data. Unfortunately, this is slightly inconsistent with the value of 30×10^{-3} ev used in Table 7 for the resonance region below 100 ev. The effect of the difference on the low-energy cross-section dependence is trivial.

The adoption of the $\Gamma_\gamma = 40 \times 10^{-3}$ ev set and the reduced Γ_n values of the 21.84-ev and 23.48-ev levels gives the following thorium cross section between 0 and 20 ev:

$$\left(\sigma_{\text{Th}}\right)_{\text{low}} = \frac{1}{\sqrt{E}} \left[1.59 + \frac{18.28}{(E + 4.3)^2} + \frac{9.4}{(E - 21.84)^2} + \frac{15.1}{(E - 23.48)^2} \right] . \quad (30)$$

The second term within the brackets is due to the near virtual level, and the last two terms are due to the two lowest real resonances; the first term is the residual effect due to distant virtual and real resonances. Equation (30) and the resonance parameters of Table 7 are used to calculate $\int_{\phi}^S \sigma_{\text{res}} du$ for Th^{232} . This calculation is discussed in Section VII.

3.2. DETAILS OF TECHNIQUE OF COMPUTATION OF THORIUM RESONANCE CAPTURE FROM THORIUM CROSS-SECTION DATA

The best available methods have been applied to calculate thorium resonance captures under several conditions with the resonance parameters given in Table 7. This procedure yields a calculated value to be compared with the measurement obtained by the vanadium-subtraction technique.

In early 1962, when this analysis was done,^{*} the most quantitative method for calculating the capture in a single resonance in a self-shielded condition was the ZUT coding⁽¹³⁾ of Nordheim's integral equation description of resonance captures.^{(10)†(11)(12)} The ZUT code has been used as the basic quantitative computational tool. It is used to check, or "calibrate," the accuracy of the more generally useful determination of thorium resonance captures by the GAM-I slowing-down code,⁽²¹⁾ and also from the iterated multigroup treatment, based on GAM-I, mentioned on p. 23. The GAM-I code and the multigroup calculations treat the low-lying resonances more approximately than ZUT, since each resonance is treated by either the so-called I.M. (N.R.I.A.) or the N. R. approximation.^{**} Because of this, and because of some minor computational inaccuracies, the thorium resonance-capture rates of the GAM-I and multigroup calculations have been found to differ from those determined by ZUT by a few per cent.

^{*}An improved version of GAM, namely, GAM-II, has since been developed (G. D. Joanou, private communication). GAM-II applies the integral-equation solution and corrects other small computational inaccuracies in GAM-I. Therefore, the checks with ZUT to be discussed are pertinent only to the earlier GAM-I version of the code.

[†]See Sections 3 and 4 of Ref. 10.

^{**}For a discussion of these approximations, see p. 67 of Ref. 10, and Refs. 22 and 23.

The method of analysis adopted has therefore used both of these codes. The capture rate was first computed by GAM-I and then corrected from results of an auxiliary calculation which compares GAM and ZUT results. This comparison is described in Section 7.2. The correction obtained is also applied to final results given by the multigroup calculations.

In both GAM-I and ZUT, the effect of lattice-geometry shadowing in reducing resonance capture, which is the Dancoff-Ginsburg effect,⁽²⁹⁾ is not treated in detail for HTGR geometry and therefore must be indirectly treated in the coding instructions. These codes compute the resonance-capture rate of an isolated cylinder (or sphere or slab) embedded in moderator. The ϕ^S is determined by GAM-I or multigroup calculations. The self-shielding reduction of resonance captures is accurately calculated for this hypothetical, isolated cylinder, but not for the lattice.

The following approximation, suggested by L. Nordheim,⁽³⁰⁾ has been used to simulate the lattice condition. A critical review⁽²³⁾ of this approximation indicates that it should predict resonance-capture rates in the lattice to an accuracy of within a few per cent. An annulus in the lattice has the same capture rate as the single cylinder of the following modified diameter:

d = mean chord length of equivalent cylinder

$$\begin{aligned} &= \frac{4V_{\text{annulus}}}{S_{\text{effective of annulus}}} \\ &= \frac{4V_{\text{annulus}}}{S_o(1 - C_o) + S_i(1 - C_i)} \quad , \end{aligned} \quad (31)$$

$$d = \frac{(d_o^2 - d_i^2)}{d_o(1 - C_o) + d_i(1 - C_i)} \quad , \quad (31a)$$

where d_o and d_i are the outer and inner diameters of the annulus, and C_o and C_i are the outer and inner Dancoff shadowing factors. C_o is computed

by the method of Dancoff and Ginsburg⁽²⁹⁾ as though the lattice were composed of solid cylinders of thorium of diameter d_o . C_i is assumed to be equal to the transmission of the graphite cylinder in the center of the annulus for an impressed flux of isotropic neutrons at the surface of the cylinder; i. e. ,

$$C_i = 1 - \Sigma d_i \left[1 - P_c(\Sigma d_i) \right], \quad (32)$$

where Σ is the macroscopic cross section of the graphite in the central cylinder and P_c is the collision probability in this cylinder.*

*For a definition and tabulation of P_c , see p. 17 ff of Ref. 31.

IV. MEASURED RELATIVE ACTIVATIONS AND CADMIUM RATIOS

4. 1. CONDITIONS OF MEASUREMENT

The cadmium-ratio measurements, using detector foils having the characteristics given in Table 1, were all carried out in the triffutes, or void spaces, adjacent to the central fuel tube. Cadmium-ratio measurements were so made in cores 3 and 6, but not in the 1/E spectrum.

Characteristics of the geometries of thorium in exposures of cores 3 and 6 are stated in Table 8. A more detailed description is given in the Appendix.

Table 8
GEOMETRIC CHARACTERISTICS OF REACTOR
EXPOSURES OF THORIUM

	Small Annulus	Large Annulus	Buffer Cylinder
C/Th ratio	$\left\{ \begin{array}{l} 55.7^a \\ 48.7^b \end{array} \right\}$	125.8	46.8
d_o , in cm	5.715	6.985	3.785
d_i , in cm	3.810	4.445	-----
C_o	0.123	0.273	0.112
C_i	0.248	0.203	-----
d , in cm	2.30	3.37	4.07

^aFor 26.5 wt-% loading.

^bFor 30 wt-% loading.

The Maxwellian spectrum was developed in a thermal column of graphite on the side of the TRIGA reactor. The exposure was made at a point separated from the reactor core by 110 cm of graphite. The cadmium ratio of Au¹⁹⁷ measured in this exposure position was about 200. These

conditions assure a neutron spectrum very nearly Maxwellian with a maximum flux occurring very near to 2200 meters/sec.

4.2. CADMIUM RATIOS MEASURED IN TRIFLUTES

Table 9 gives results of measurements of the cadmium ratios made in the triflutes. The cadmium ratio (C. R.) is the activation of the bare foil divided by the activation of the cadmium-covered foil.

Table 9
CADMIUM RATIOS MEASURED IN TRIFLUTES

Nuclide	Core	No. of Measure- ments	Measured Cadmium Ratio ^a
Au ¹⁹⁷	3	1	1.19
Au ¹⁹⁷	6	3	1.141 ±0.010
Vanadium	3	1	5.85
Vanadium	6	3	4.877 ±0.070
Vanadium	6	1	4.98
Mn ⁵⁵	3	1	3.25
Mn ⁵⁵	6	3	2.811 ±0.060
Th ²³² in triflute	3	1	1.24

^aWhere a C. R. has been measured several times, the mean value and range of reproducibility of experimental values are given.

4.3. ACTIVATION RATIOS MEASURED BY THE VANADIUM-SUBTRACTION METHOD

The activation ratios A^S/A^M , where A^S and A^M are the activations in the spectra ϕ^S and ϕ^M , respectively, are listed in Tables 10 through 12. The A^S/A^M ratio for an unknown whose resonance reaction rate is to be determined, together with the same ratio for the Au¹⁹⁷ standard, is stated in each case. All such ratios are recorded on the basis that the ϕ^S and ϕ^M are normalized with respect to each other in such a way that A_V^S/A_V^M is

unity. This unity ratio, if not explicitly stated, is therefore understood to apply to the third ratio measurement for vanadium in each case.

Table 10

RELATIVE ACTIVATION MEASUREMENTS BY VANADIUM-SUBTRACTION TECHNIQUE
(Foil Detectors Exposed in Core 3 Triflutes; $A_V^S/A_V^M = 1.00$)

Unknown	Foil Condition	Measurement Designation	A^S/A^M	
			Unknown	Au ¹⁹⁷
Mn ⁵⁵	Bare	A-1	1.19	5.45
Mn ⁵⁵	Bare	A-2	1.194	5.496
Mn ⁵⁵	Cadmium-covered	B-1	2.14	26.66
Mn ⁵⁵	Cadmium-covered	B-2	2.201	27.28
Th ²³²	Bare	C-1	4.49	5.45
Th ²³²	Bare	C-2	4.609	5.496
Th ²³²	Cadmium-covered	D-1	21.15	26.66
Th ²³²	Cadmium-covered	D-2	21.67	27.28

Table 11

RELATIVE ACTIVATION MEASUREMENTS BY
VANADIUM-SUBTRACTION TECHNIQUE
(Cadmium-covered Detector Foils Exposed in
1/E Spectrum; $A_V^S/A_V^M = 1.00$)

Unknown	Measurement Designation	A^S/A^M	
		Unknown	Au ¹⁹⁷
Th ²³²	{ E-1	19.07	27.75
	{ E-2	19.05	27.68
U ²³⁸	{ F-1	170.6	27.75
	{ F-2	165.0	27.68
Mn ⁵⁵	{ G-1	1.86	27.75
	{ G-2	1.83	27.68

Table 12

RELATIVE ACTIVATION MEASUREMENTS BY VANADIUM-SUBTRACTION TECHNIQUE
(Th²³² in Self-shielded Reactor Geometry; Foil and Compact Grind Detection; $A_V^S/A_V^M = 1.00$)

Local Geometry	Core	$\frac{A_{Au}^S}{A_{Au}^M}$	Foil Detector		Compact Grind	
			Measurement Designation	$\frac{A_{Th}^S}{A_{Th}^M}$	Measurement Designation	$\frac{A_{Th}^S}{A_{Th}^M}$
Small Annulus, C/Th = 55.7	{ 6	5.794	IF-1	3.118	IG-1	3.289
		6.019	IF-2	3.065	IG-2	3.186
		5.981	JF-1	2.880	JG-1	3.207
Small Annulus, C/Th = 48.7	5	5.539	KF-1	2.729	KG-1	2.763
Large Annulus, C/Th = 125.8	{ 4B	5.838	LF-1	3.414	LG-1	3.58
		5.856	LF-2	3.358	LG-2	3.53
		5.705	LF-3	3.395	----	----
		5.733	LF-4	3.433	----	----
Buffer Cylinder	7	5.793	H-1	2.554	----	----

For a number of the conditions of exposure, the ratios have been measured several different times. Each of these is treated as an individual measurement. A^S/A^M values for the unknown and for Au¹⁹⁷ are recorded in Tables 10 through 12 for each such separate measurement. Different measurements of the same exposure condition are identified by a symbol such as A-1, etc. The letter designates the condition of measurement and the number gives the particular measurement made in this condition. Values for the resonance-capture rates predicted by these separate measurements, labeled by the same identification numbers, are given in Section V.

The vanadium-subtraction technique has been carried out both with and without cadmium covers on the foils. Each measurement was made with all detector foils either bare or cadmium-covered. The presence of a cadmium cover modified the ϕ^S in the low-energy region but not in the

resonance region. The addition of such covers therefore changes the A^S/A^M ratio both of the unknown and of Au^{197} , but should lead to the same measured result for the resonance-capture rate. Exposures in the reactor annulus were made bare; those in the 1/E spectrum were cadmium-covered; those in the triffutes of core 3 were made both ways. Columns of tables labeled "foil condition" tell whether the exposures were made bare or cadmium-covered.

In some of the reactor annulus geometry measurements, the "compact-grind" thorium detector technique (see Section 2.2), as well as the thorium foil detector was applied. In each of these measurements, three foils were used, and the "compact-grind" activation constitutes a fourth observation. Where both detectors were used, separate analysis is carried out for each detector, based on the activation ratios A_{Th}^S/A_{Th}^M measured for that detector and using the common values of A_V^S/A_V^M (unity) and A_{Au}^S/A_{Au}^M .

The 1/E spectrum measurement had foils of U^{238} and Mn as well as Th^{232} , and these three were exposed simultaneously as unknowns to be measured in terms of the data for Au^{197} and V which were exposed at the same time.

For the large-annulus geometry, the compacts, or annuli containing Th^{232} , contained ThO_2 grains with an average size of 237 microns. This introduced additional self-shielding associated with the grains. A correction for this additional self-shielding increases captures 6%.⁽³²⁾⁽³³⁾ This increase in the value measured by the compact-grind detector has been included in Table 17, in Section VI. This corrected value can then be compared with a value calculated on the basis of no self-shielding, given in Table 21, Section VIII. The grains of compact-grind detection cases other than the large HTGR annulus were small enough that grain self-shielding was insignificant.

V. CADMIUM-RATIO-MEASURED RESONANCE CAPTURE RATES

The measurements listed in Table 9 were analyzed by Eqs. (22) and (28), together with pertinent auxiliary data of Tables 1, 4, and 6. The results obtained for both $\int \phi^S \sigma_{\text{res}} du / \overline{\phi^S}$ and an approximate resonance integral, I_{app} , are given in Table 13. These data are the final results of investigations carried out in this work by the cadmium-ratio technique. As was pointed out in Section 2.2, the results were not determined relative to Au^{197} but rather with the aid of computed spectral quantities and Eq. (22). It was also pointed out that the accuracy is not high, and is believed to be about $\pm 10\%$ for the resonance integral of Th^{232} and Mn^{55} , and about $\pm 50\%$ for the resonance integral of vanadium.

Table 13
 RESONANCE CAPTURES DETERMINED FROM CADMIUM-RATIO
 MEASUREMENTS MADE IN TRIFLUTES
 (Reduction of Experimental Data Given in Table 9)

Nuclide	Core	Number of Measure- ments	σ_{2200} (barns)	$\frac{\int \phi^S \sigma_{\text{res}} du}{\overline{\phi^S}}$ (barns)	I_{app} (barns)
Vanadium	3	1	4.5	0.33	0.31
Vanadium	6	3	4.5	0.19	0.18
Vanadium	6	1	4.5	0.15	0.14
Mn^{55}	3	1	13.2	7.4	7.4
Mn^{55}	6	3	13.2	6.4	6.2
Th^{232}	3	1	7.45	60.0	66.3

The results of measurements of the resonance integral of vanadium of Table 13 can be written

$$I_V = I_{\text{app}} \text{ of vanadium}$$

$$I_V = 0.25 \pm 0.15 \text{ b.}$$

The indicated range of uncertainty is written conservatively large to include the possibility of an error in the value calculated for $(C. R.)_{I/V}$. Evidence discussed in Section IX indicates that the higher (core 3) result for the I_V is the more correct, and some weight has been given to this fact in selecting the 0.25-b average value.

This value of I_V is determined for the mixture of nuclides occurring in the natural state, and it is measured relative to a 4.5-b value of the 2200-meter/sec activation cross section. This 2200-meter/sec value is poorly known, and this uncertainty contributes additional uncertainty to I_V . However, this additional uncertainty is of no concern in the analysis of measurements made by the vanadium-subtraction technique, since only the ratio of I_V to σ_{2200} occurs in these analyses, through the occurrence of the ratio in Eq. (14). The 2200-meter/sec cross section of vanadium does not occur in Eq. (1). That is, the result $I_V = 0.25 \pm 0.15$ b is a value measured relative to a 4.5-b value of the 2200-meter/sec cross section of vanadium. But results obtained by Eqs. (1) and (14) are independent of possible revision in the 2200-meter/sec value, and the consequent modification of I_V .

The value obtained for I_V is consistent with the known nuclear data. The measured resonance parameters of BNL-325 and an estimate of $\Gamma_\gamma = 0.5$, which is consistent with nuclear theory and measurements of Γ_γ of other nuclides of about mass 55, would predict I_V to be 0.2 b.

The values of 0.25 ± 0.15 b for I_V and 4.5 b for σ_{2200} of vanadium have been applied to determine the values of K given in Table 14. Other data pertinent to determining K are given in Tables 4 and 6. Equations (14), (18), (21), and (28) have been applied to these determinations. The range of uncertainty in K given in Table 14 is that due to the uncertainty in I_V .

Table 14
VALUES OF K FOR SPECTRA OF VANADIUM-
SUBTRACTION MEASUREMENTS

Spectrum	Foil Condition	
	Bare	Cadmium-covered
1/E	---	0.888 $\pm$ 0.056
Core 3	0.975 $\pm$ 0.015	0.845 $\pm$ 0.072
Core 6	0.967 $\pm$ 0.017	---

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VI. RESULTS OF RESONANCE-CAPTURE MEASUREMENTS BY THE VANADIUM-SUBTRACTION TECHNIQUE

Equation (1), the Au^{197} resonance-capture rates of Table 5, and the pertinent auxiliary data of Tables 1 and 14 are applied to the measurements reported in Tables 10 through 12. Results for resonance-capture rates and resonance integrals so obtained appear in Tables 15 through 17.

Table 15

RESONANCE CAPTURES DETERMINED FROM VANADIUM-SUBTRACTION MEASUREMENTS IN TRIFLUTE OF CORE 3
(Reduction of Experimental Data Given in Table 10)

Unknown	Foil Condition	Measure- ment Designa- tion	$\int \phi^S \sigma_{\text{res}} du$	I_{app} (barns)
			$\overline{\phi^S}$ (barns)	
Mn^{55}	Bare	A-1	7.4	7.4
Mn^{55}	Bare	A-2	7.3	7.3
Mn^{55}	Cadmium-covered	B-1	7.5	7.5
Mn^{55}	Cadmium-covered	B-2	7.7	7.7
Th^{232}	Bare	C-1	65.7	73.5
Th^{232}	Bare	C-2	65.8	73.5
Th^{232}	Cadmium-covered	D-1	67.2	75.4
Th^{232}	Cadmium-covered	D-2	68.5	76.6

The uncertainties in K caused by the uncertainty in I_V lead, in turn, to some uncertainties in the resonance-capture rate measured for the unknowns. The uncertainty so caused in I can be seen to be about ± 0.2 b, or less than 1%, for Th^{232} and U^{238} . The uncertainty is about ± 0.6 b, or $\pm 8\%$, in the case of Mn^{55} . These uncertainties are modest, and that for Th^{232} is quite negligible.

Table 16

RESONANCE CAPTURES DETERMINED FROM VANADIUM-SUBTRACTION MEASUREMENTS IN 1/E SPECTRUM
(Reduction of Experimental Data Given in Table 11)

Unknown	Measurement Designation	$\frac{\int \phi^S \sigma_{res} du}{\bar{\phi^S}}$	I (barns)
		(barns)	
Th ²³²	E-1	78.9	78.9
Th ²³²	E-2	79.0	79.0
U ²³⁸	F-1	269	269
U ²³⁸	F-2	261	261
Mn ⁵⁵	G-1	7.6	7.6
Mn ⁵⁵	G-2	7.4	7.4

Table 17

VANADIUM-SUBTRACTION-DETERMINED RESONANCE CAPTURES
IN Th²³² IN SELF-SHIELDED REACTOR CONDITIONS
(Reduction of Experimental Data Given in Table 12)

Local Geometry	Core	Foil Detector		Compact Grind		
		Measure- ment Designa- tion	$\frac{\int \phi^S \sigma_{res} du}{\bar{\phi^S}}$	Measure- ment Designa- tion	$\frac{\int \phi^S \sigma_{res} du}{\bar{\phi^S}}$	
			(barns)		(a)	(b)
Small annulus:						
C/Th = 55.7	6	IF-1	37.0	IG-1	38.6	38.6
C/Th = 55.7	6	IF-2	35.8	IG-2	36.5	36.5
C/Th = 55.7	5	JF-1	32.8	JG-1	37.2	37.2
C/Th = 48.7	5	KF-1	33.4	KG-1	32.7	32.7
Large annulus:						
C/Th = 125.8	4B	LF-1	43.5	LG-1	44.6	47.5
C/Th = 125.8	4B	LF-2	42.3	LG-2	43.6	46.4
C/Th = 125.8	4B	LF-3	44.3			
C/Th = 125.8	4B	LF-4	44.8			
Buffer cylinder	7	H-1	28.3			

^aBefore correcting for grain self-shielding.

^bAfter correcting for grain self-shielding. (32)(33)

The vanadium-subtraction-technique determination of resonance captures therefore appears to be satisfactorily quantitative, despite the $\pm 50\%$ uncertainty in knowledge of I_V .

The uncertainties in $(\int \phi^S \sigma_{\text{res}} du) / \phi^S$ and I_{app} for the several nuclides which arise from the uncertainty in knowledge of I_V or K have not been stated in Tables 15 through 17. It is to be understood that one of the uncertainties in the results in these tables is ± 0.2 b in the case of Th^{232} and U^{238} and ± 0.6 b in the case of Mn^{55} , resulting from the uncertainty in I_V .

In the case of I of Mn^{55} , very similar results have been obtained from measurements in the $1/E$ spectrum and the core 3 triflute spectrum. This consistency is evidence that the spectra in these cases have been calculated correctly.

The 265-b thin-limit resonance integral of U^{238} measured in the $1/E$ spectrum is close to other good measurements. The resonance parameters of U^{238} measured at Columbia University⁽¹⁷⁾ predict a 265-b resonance integral. This further corroborates the claim that the spectrum developed in the reflector of TRIGA is $1/E$.

On the other hand, anomalously low values of I_{app} for several nuclides result from the core 6 triflute measurements by the cadmium-ratio technique. This probably indicates some computational inaccuracy in the spectrum of core 6 in the thermal-energy region.

It should be noted that the Th^{232} exposed in the triflute has its resonance-capture rate reduced by the Dancoff-Ginsburg⁽²⁹⁾ shadowing effect of the surrounding thorium. This shadowing has not been determined for this complicated exposure condition, but it would reduce thorium resonance captures by 10% to 20%. Because of this uncertainty in shadowing, and because the vanadium-subtraction and cadmium-ratio measurements for Th^{232} foils in the triflute are so different, the significance of these Th^{232} triflute measurements is difficult to determine.

VII. QUANTITATIVE CALCULATION OF THORIUM RESONANCE CAPTURES

The thorium capture rate is determined from data of Table 7, Eqs. (31) and (32), and application, described in Section 3.2, of the GAM-I code and related multigroup calculations, and the ZUT code. In addition, several other small corrections must be made; these are described in the following subsections.

7.1. SPECIAL CORRECTIONS TO CALCULATED RESONANCE-CAPTURE RATE

Two corrections must be made in all cases, * and an additional correction for the large annulus, in order to make the calculations consistent with the conditions of measurement; these measured results are those given in Tables 15 through 17.

The resonance-capture rate is calculated on the basis of a Breit-Wigner single-level model of the cross section. The lower limit of the range of integration is 19.5 ev for Th²³². On the other hand, the vanadium-subtraction technique measures a lesser rate, since Eq. (30) shows that the low-energy cross section is less than the 1/v cross section which has been experimentally subtracted. This difference is

$$\frac{1}{\phi S} \int_0^{19.5 \text{ ev}} \left[(\sigma_{\text{res}})_{\text{Th}} - 7.45 \sqrt{\frac{0.0253}{E}} \right] \frac{dE}{E} ,$$

* The GAM-I-ZUT definition is a more usual definition of resonance-capture rate than that measured by the vanadium-subtraction technique, and the two corrections could therefore more logically be made to the measurements instead of to the calculations. If this alternate correction procedure were followed, the thorium resonance integral would be about 1.5 b larger than the values quoted in this report. However, we have chosen the expedient of correcting the calculations to bring them in accord with the definition of Eq. (3) and the vanadium-subtraction measurements of resonance reaction rate.

where $(\sigma_{\text{res}})_{\text{Th}}$ is given by Eq. (30). The revision due to a change in the low-energy cross section is called the "deviation from $1/v$ effect." This correction is about 1 b in magnitude.

The $1/v$ tail is experimentally subtracted throughout the resonance region in the measurements, removing

$$\frac{1}{\phi \bar{S}} \int_{19.5 \text{ ev}}^{\infty} \left[\phi^S(E) 7.45 \sqrt{\frac{0.0253}{E}} \right] \frac{dE}{E},$$

compared with results of GAM-I-ZUT calculations. This quantity, termed the "high $1/v$ tail" is 0.5 b and 0.4 b, in a $1/E$ and any reactor spectrum, respectively.

The large annulus is embedded in a surrounding lattice of small annuli. It is calculated by GAM-I as though it were in its own environment, with d of Eq. (31) determined accordingly. The smaller-annulus neighbors exert less shadowing in the measured condition because of the difference in C_0 of the two lattices. From results for resonance integral as a function of d , it is estimated that this "environment mismatch" should increase the calculated value of $\left(\int \phi^S \sigma_{\text{res}} dE \right) / \phi \bar{S}$ of the large annulus by 0.4 b.

7.2. COMPUTATIONAL CORRECTIONS TO GAM-I RESULT DETERMINED BY COMPARISON WITH ZUT

The need for this comparison was stated in Section 3.2. The comparison has been made for both the large annulus and for the small annulus having a C/Th ratio of 55.7. These comparisons are made under the following conditions:

1. Fixed resonance parameters, having "low" values.
2. Resolved resonances.
3. $1/E$ spectrum.

Two types of computational inaccuracies occurred in GAM-I as applied to the analysis of these measurements. The larger was that made by the application of the N. R. or I. M. (N. R. I. A.) approximations instead of the

integral equation solution. The second was an error caused by the roughness of interpolation applied by GAM-I to compute the surface component of the Wigner resonance integral. This interpolation is based on tables for which the surface integral had been well determined by the Adler-Nordheim method,⁽³⁴⁾ but only for rather widely spaced intervals of the geometric parameter, z . Calculations of Van Howe⁽³⁵⁾ have shown that interpolation errors occur due to the use of these intervals, which are about 5% of the surface component of the resonance integral, or about 2% or more of the resonance integral in HTGR geometry.

These small computational errors are significant primarily for the resolved resonances, and, in fact, the comparisons to be reported show that most of the corrections occur below 100 ev in energy. It has therefore been assumed that the correction can be adequately obtained by a comparison computation in a $1/E$ spectrum and that the correction in all reactor spectra is adequately given by multiplying this $1/E$ spectrum correction by the factor 0.9 to obtain the correction to $\int \phi^S \sigma_{res} du \phi^S$ in the reactor spectrum. This factor is less than unity because ϕ^S at energies below 100 ev is less than ϕ^S .

Table 18 gives a computational correction, $I_{acc} - I_{app}$, to the resonance integrals (in a $1/E$ spectrum) of the resolved resonance band. This computational correction is that necessary to bring I_{app} determined by the Wigner resonance integral approximations (I. M. or N. R.) into agreement with I_{acc} determined by the integral equation formulation. Both I_{app} and I_{acc} were determined by the ZUT code, which includes an option so that the resonance integral for each resonance can be determined in either an I. M. or N. R. manner, in addition to the integral-equation method. The particular choice of either I. M. or N. R. used to determine I_{app} is chosen to be the same as was coded into the instructions of GAM-I as it has been used. This choice for the lowest six resonances is listed in Table 18; all other resonances were N. R. The results shown in Table 18 indicated that in HTGR geometry the I. M. approximation always overestimates the resonance

integral, and the N. R. approximation always underestimates it. The approximation used in GAM-I is fairly satisfactory for all resonances except for the 69.2-ev resonance. For the 69.2-ev resonance, the overestimate made by the I. M. approximation, in the case of the large HTGR annulus, is 1.51 b. Quite similar results were obtained also for calculations done on the small HTGR annulus having C/Th (carbon to thorium atomic ratio in the annulus) 55.7. For this case, the I. M. approximation overestimates the 69.2-ev resonance integral by about 0.9 b. That is, the magnitude of the error made for the 69.2-ev resonance varies noticeably with the geometric parameter.

Table 18

ZUT-DETERMINED COMPARISON OF ACCURATE AND APPROXIMATE
THORIUM RESONANCE INTEGRAL FOR LARGE ANNULUS

E_{res} (ev)	I_{acc} , Obtained by Integral Equation (barns)	Approximation Used in GAM-I	I_{app} , Obtained by GAM-I Approximation (barns)	$I_{acc} - I_{app}$ (barns)
21.84	7.8146	I. M.	7.8400	-0.0254
23.48	9.3233	I. M.	9.4226	-0.0993
59.55	3.0668	N. R.	3.0189	+0.0473
69.20	4.8500	I. M.	6.3624	-1.5124
113.50	1.9258	N. R.	1.8930	+0.0328
121.00	2.1729	N. R.	2.1065	+0.0664
Total over resolved resonances up to 940 ev	43.32		44.56	-1.24

The 69.2-ev resonance integral was also determined in the case of the large annulus by the N. R. approximation. This gave 4.202 b, which is in better agreement with I_{acc} . However, neither the N. R. nor the I. M. approximation is very good for this resonance. This is not surprising, since it is a borderline case which does not meet the criterion well for either approximation.

Table 19 shows the total computational error made by GAM-I, as coded in this work. This total error was determined by a different procedure, namely, by comparison of a GAM-I and ZUT calculation done for resolved resonances in a $1/E$ spectrum. This was accomplished in the case of the GAM-I code by inserting an input parameter which made the contribution of the unresolved resonances zero. Furthermore, the contribution of the region above about 80,000 ev, which was incorporated into GAM-I by BNL-325⁽¹⁸⁾ unshielded cross sections, was subtracted. This high-energy contribution was determined to be 0.93 b. Table 19 also shows the second component error made by GAM-I--the interpolation error. This was actually found by subtracting the N. R. or I. M. approximation error from the total error, determined in the manner just described.

Table 19
ERRORS IN GAM-I CALCULATION FOR RESOLVED
RESONANCES, $1/E$ SPECTRUM; "LOW"
RESONANCE PARAMETERS

	Large Annulus (barns)	Small Annulus, C/Th=55.7 (barns)
Use of I. M. or N. R. approximation	-1.24	-0.65
Interpolation	<u>-0.13</u>	<u>+0.42</u>
TOTAL	-1.37	-0.23

Table 19 shows that the total error made in GAM-I, which is of the order of 1 b, or 3% of the resonance-capture reaction rate, is unimportant in the case of the small annulus having $C/Th = 55.7$, compared with the case of the large annulus. The values of C/Th for the large annulus and for the small annulus for which GAM-I computational inaccuracies have been evaluated are 125.8 and 55.7. It is desirable to estimate these inaccuracies also for the buffer cylinder having $C/Th = 46.8$, and for the small

annulus that had $C/Th = 48.7$. It can be seen by extrapolation that the GAM-I computational inaccuracy would be quite small for these cases. It has therefore been ignored in these cases.

To recapitulate, a computational error in GAM-I as applied to this analysis has been evaluated to lead to errors in $\int \phi^S \sigma_{res} du / \phi^S$ for the large annulus and for the small annulus having $C/Th = 55.7$ of 1.2 b and 0.2 b, respectively. For other cases, this computational error was indicated to be small and was neglected.

7.3. FINAL CALCULATED VALUES OF VANADIUM-SUBTRACTION-MEASURED VALUES OF THE THORIUM RESONANCE-CAPTURE REACTION RATE

Table 20 gives the final results of calculations, including a recapitulation of the various small corrections, for $\int \phi^S \sigma_{res} du / \phi^S$ for Th^{232} , as measured by the vanadium subtraction technique. This rate is that defined by Eqs. (2) and (6). It is about 1.5 b less than that given by captures in the actual thorium resonances, and about 2.5 b less than the total capture rate above cadmium cutoff energy. Table 20 gives the calculated results, based on the thorium resonance-parameter data of Table 7, for the various geometries for which measurements have been made.

Table 20

CALCULATED RESULTS FOR $\int \phi^S \sigma_{res} du / \phi^S$ FOR Th^{232} AS MEASURED
BY THE VANADIUM-SUBTRACTION TECHNIQUE
(In barns)

	1/E Spectrum, Thin- Limit	Small Annulus, Core 6, C/Th=55.7	Small Annulus, Core 6, C/Th=48.7	Large Annulus, Core 6, C/Th=125.8	Buffer Cylinder, Core 7, C/Th=46.8
Initial GAM-I calculated $\int \phi^S \sigma_{res} du / \phi^S$:					
"Low" parameters	81.0	35.9	33.0	43.6	30.2
"High" parameters	93.2	37.6	35.7	46.2	31.7
Corrections:					
Deviation from 1/v	-0.9	-0.9	-0.9	-0.9	-0.9
High 1/v tail	-0.5	-0.4	-0.4	-0.4	-0.4
Environment mismatch	0	0	0	+0.4	0
Correction to GAM-I	0	-0.2	0	-1.2	0
TOTAL CORRECTION	-1.4	-1.5	-1.3	-2.1	-1.3
Corrected $\int \phi^S \sigma_{res} du / \phi^S$					
"Low" parameters	79.6	34.4	31.7	41.5	28.9
"High" parameters	91.8	36.1	34.4	44.1	30.4

* Obtained from total of Table 19, multiplied by 0.9.

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VIII. COMPARISON OF CALCULATED AND MEASURED RESULTS OF THORIUM RESONANCE CAPTURES

Table 21 presents a final comparison of the calculated results for thorium resonance captures with those measured by the vanadium-subtraction technique. The experimental results have been combined for cases where measurements of the same quantity have been repeated several times. The internal consistency of the results of Tables 15 through 17 show that the measured results should be considered to have an uncertainty, due to variation from measurement to measurement, of about ± 1.0 b.

Table 21
COMPARISON OF CALCULATED AND MEASURED RESULTS FOR Th-232 RESONANCE CAPTURES

Condition of Measurement	Measured Results ^a				Calculated Results	
	Foil Detector		Compact-grind Detector		"Low" Parameters (barns)	"High" Parameters (barns)
	Number of Measure- ments	Resonance Capture (barns)	Number of Measure- ments	Resonance Capture (barns)		
1/E spectrum, thin-limit	2	79.0	-	----	79.6	91.8
Small annulus:						
In core 6; C/Th=55.7	2	36.4	2	37.6	34.4	36.1
In core 5; C/Th=55.7	1	32.8	1	37.2	34.4	36.1
Average with C/Th=55.7	3	35.2	3	37.4	34.4	36.1
In core 5; C/Th=48.7	1	33.4	1	32.7	31.7	34.4
Large grain annulus; C/Th=125.8; compact- grind-detector results corrected for grain self- shielding ⁽³²⁾⁽³³⁾	4	43.7	2	46.0	41.5	44.1
Buffer cylinder in core 7; C/Th=46.8	1	28.3	-	----	28.9	30.4

^aAll measurements made by vanadium-subtraction technique.

IX. DISCUSSION OF RESULTS

The vanadium-subtraction technique has been demonstrated to be a very useful technique for measuring resonance integrals relative to that of Au¹⁹⁷. The experience reported here indicates that it is probably more accurate than the cadmium-ratio technique for measuring the resonance captures under Peach Bottom reactor (HTGR) conditions. For Th²³², the uncertainty due to the non-1/v correction of vanadium is about 0.2 b, and for Mn⁵⁵ it is about 0.6 b. The principal limitation to accuracy in the case of Th²³² is caused by the necessity of making six activation measurements. If each activation is determined to within 1%, the total error in a measured resonance reaction rate due to activation errors is probably 3% to 5%. Repetition of measurements can statistically reduce this error.

Allowing for these errors, we judge the values of the thin-limit, 1/E-spectrum resonance integrals of Mn⁵⁵ and Th²³² determined by these vanadium-subtraction measurements to be

$$I_{\text{Mn}55} = 7.5 \pm 0.8 \text{ b} ,$$

$$I_{\text{Th}232} = 79.0 \pm 5.0 \text{ b} .$$

The first measurement is in fair agreement with the 8.15 ± 0.60 b reported by Swedish workers⁽³⁶⁾ for the same quantity. It is less than a Knolls Atomic Power Laboratory measurement,⁽³⁷⁾ which is 10 b when corrected to the same condition.

In the case of Th²³², thin-limit resonance integrals ranging from 60 to 110 b have been previously reported.⁽²⁾⁽³⁾ The measurement reported here is in good agreement with the ORNL value of 85 ± 5 b above cadmium,⁽³⁸⁾ which is equivalent to 82 b under conditions of this measurement. An activation measurement just reported by the British⁽³⁹⁾ has obtained a resonance integral value of 83 ± 6 b for conditions of measurement quite similar to those

of the vanadium-subtraction method, but with the 1.5 b discussed in the footnote of p. 53 included. This is also in good agreement with our results.

The comparison of measured and calculated values given in Table 21 favors the "low"-resonance-parameter values. This is clearly the case for the thin-detector, $1/E$ -spectrum exposure, and for the cylindrical element. These two, particularly the thin-foil detector, are the geometrically simple forms that involve the least computational uncertainty in the determination of self-shielding. Furthermore, the resonance integral which is measured by the thin detector is the result most sensitive to the difference in the two alternative sets of resonance parameters.

The choice of the parameter set which best agrees with measurements for annuli is not as clear. The latest and therefore presumably the best measurements were carried out on the large annulus. One should compare the calculated results with the values measured using the foil detector, for the reason discussed in the fifth paragraph below. This would favor a choice of parameters intermediate in values between those of the "high" and "low" set. The most reasonable interpretation that can be made, before additional measurements and analyses can be carried out, is as follows:

1. The "low" set of resonance parameters is essentially correct, according to measurements made for geometrically simple conditions.
2. The computation of thorium resonance captures for the annular lattices in core 6 underestimates the capture rate by about 5%.

Two pieces of evidence support the second claim made above, one piece indicating the neutron spectrum assumed in the analysis is slightly incorrect, and the other indicating that self-shielding is slightly overestimated due to an overestimate of the shadowing by adjacent members of the lattice.

Cores 5 and 6 contained a driver, which gives a noticeable multi-dimensional effect on the neutron spectrum, with a trace of this effect remaining noticeable at the center of the core. The evidence for this

effect of geometry on the spectrum is a comparison of the neutron spectrum at the center as determined by two-dimensional calculations with that determined by a one-dimensional calculation. This comparison was made for core 3, and has not yet been done for cores 5 and 6, in which the resonance reaction rates have been measured. For core 3, it was found that two-dimensional calculations predicted a somewhat harder spectrum, such that the ratio of thorium to Au¹⁹⁷ resonance captures would be increased by several per cent. This would also increase the thorium resonance-capture reaction rate by a few per cent when carried out by the methods described above.

This modification is apparently due to the spectrum-hardening effect caused by the innermost parts of the driver. This effect calculated for core 3 can be expected also to apply to cores 5 and 6, which had a driver and had the same dimensions of regions. Core 7, however, in which the measurement of the cylindrical element was made, contained no driver, so the spectrum computed by the one-dimensional model should apply well to that case.

The Dancoff-Ginsburg⁽²⁹⁾ calculation of the external shadowing factor, C_0 , for an annulus probably somewhat overestimates this shadowing. This occurs due to the assumption that a neighboring shadowing element is completely black to resonance neutrons instead of being somewhat grey. Wagner⁽⁴⁰⁾ has carried out preliminary calculations which show that an improved calculation of the external shadowing of an annulus should increase the resonance captures in the lattice by a few per cent.

It is therefore to be expected that an improved calculation of the thorium resonance-capture rate for the conditions of measurement would predict more captures. Such a modification could result in a 5% increase, to bring calculations determined from the "low" set of resonance parameters into good agreement with the measurements for the annuli.

The measured results, stated in Tables 17 and 21, show that thorium resonance captures determined by the "compact-grind" detector are larger

than those determined by the foil detector. Despite the scatter of measurements from case to case, a systematic difference is evident which has a probable magnitude of 5%. Such an increased resonance-capture rate at 300°K in ceramic ThO_2 is to be expected on theoretical grounds connected with the dynamics of the ThO_2 crystal lattice. It is not likely that at 300°K the specific heat of this lattice has saturated to its high-temperature value. No measurements of the specific heat of ThO_2 were found in the literature. However, the similar crystal, UO_2 , has been so studied.⁽⁴¹⁾ In the case of UO_2 , the 300°K value of the specific heat is only about two-thirds of the saturated value, which is reached at a temperature above 900°K .

The following summary of the influence of crystalline binding needs to be recalled* to understand its effect on Doppler broadening.

An atom bound in a crystal lattice has a motion at low temperatures subject to less thermal excitation than a free atom. On the other hand, it is subject to zero-point vibrations, which are a quantum mechanical effect. The low-temperature specific heat increases from a small value to a saturation value at higher temperature as thermal excitation contributes more to the motion of the crystal lattice. Designate the temperature at which the specific heat saturates by T_s .

For temperatures T above T_s , the atoms responsible for resonance absorption can be treated as free, in that the Doppler broadening calculated on this basis is found to agree with that computed on the basis of the more correct crystal dynamics. For $T < T_s$, the free-atom model predicts Doppler broadening which deviates from the more correct treatment. The effect of crystalline binding for $T < T_s$ can be considered to increase the effective temperature to be used in the free-atom-model determination. That is, for $T < T_s$ the Doppler broadening and resonance-capture rate is increased by the effect of crystalline binding. An additional consequence of this binding effect is that the Doppler coefficient, which is the logarithmic derivative of the resonance-capture rate with respect to temperature, is

* A more complete discussion can be found in Refs. 42, 43, and 44.

modified. For $T < T_s$, the Doppler coefficient is reduced by crystalline binding; for $T > T_s$, it approaches the value determined by the free-atom model.

Most practical reactor analyses, including that applied to these measurements, ^(11, 12, 13) determine Doppler broadening on the basis of the free-atom model; that is, by the assumption that the velocity distribution of resonance-absorbing atoms is Maxwellian. This can be expected to be correct for $T > T_s$, and to result in the errors described for $T < T_s$.

For metallic thorium, T_s is known ⁽⁴⁵⁾ to be less than 300°K. Therefore, the resonance captures of the metallic-foil detector, even at 300°K, are well described by the free-atom-model determination of this analysis. However, the thorium contained in the ThO₂ crystal was the detector of the "compact-grind" method, which is indicated ⁽⁴¹⁾ to have T_s above 300°K, but less than the operating temperature of the reactor. The present methods of calculating captures in this oxide detector can be expected to underestimate resonance captures at 300°K. Comparison of results obtained by the metallic foil and "compact-grind" detector, given in Tables 17 and 21, are consistent with this expectation.

Measurements made in this Laboratory on the Doppler coefficient of ThO₂ dispersed in graphite ⁽⁴⁶⁾ provide additional evidence of a deviation of calculations at 300°K based on the free-atom model. These results show a measured temperature dependence of the Doppler coefficient consistent with that predicted above, if modifications due to crystalline-binding effects become important below about 500°K.

Measurements of specific heat versus T for both ThO₂ and ThC would clearly be a great help. That for ThO₂ should provide additional experimental confirmation of these suggestions on resonance-capture properties measured for ThO₂. ThC, which is to be used in the power HTGR, can be expected to have similar specific-heat properties as ThO₂, and therefore lead to similar deviations from the present calculations of resonance capture at low temperatures. Confirmation of these expectations for ThC should be carried out. Meanwhile, these reactor physics measurements on ThO₂ can be taken to provide some solid-state physics information.

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X. CONCLUSIONS

The significant points of these measurements and analyses for the calculation of thorium resonance captures in the HTGR can be stated as follows:

1. The "low" value set of resonance parameters of Table 7 is favored by these measurements. This "low" set is in excellent agreement with the geometrically simple measurements, particularly for the $1/E$ -spectrum, thin-limit case. This set predicts the following values of the resonance integral:

- a. 80 b under the conditions of these measurements, having a $1/v$ component which is responsible for the entire 2200-meter/sec cross section subtracted at all energies.
- b. 81 b for the reaction rate contributed by resonances above about 19 ev.
- c. 84 b for the epicadmium reaction rate.

These quantities all apply in a $1/E$ spectrum.

2. The methods of computing thorium resonance captures used in this analysis, which are consistent with methods contained in GAM-II, probably slightly underestimate thorium resonance captures in the HTGR in the following manner:

- a. A 5% underestimate occurs at 300°K due to neglect of crystalline-binding effects. This error vanishes above about 600°K , and therefore is of no importance at operating temperature.
- b. The treatment of lattice shadowing for the annular-element lattice can be expected to underestimate captures by a few per cent.

3. GAM-I, with an I. M. treatment of the 69.2-ev resonance, contained additional computational inaccuracies which overestimated captures by about 3% for the large annulus, but which resulted in little error in other cases.

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