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EVALUATION OF THE ZGS INJECTOR-BOOSTER AS AN
INTENSE NEUTRON GENERATOR

by

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Solid State Science Division

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This report has been prepared to provide a basis for evaluating the possible applications of the ZGS injector-booster accelerator as an intense pulsed neutron source. We suggest naming the facility "ZING." The acronym refers to the major accelerator system, ZGS, of which the booster accelerator is a part, and alludes to the function of the facility, with a bow to the very ambitious Canadian "ING" project, from which come much of the information and stimulus for the present application.

Briefly described, the facility consists of a neutron-producing spallation target, erected on a separate beam line from the high-energy proton booster accelerator which is to be constructed as injector for the ZGS. It could conceivably materialize in about three years. This report consists of a brief description of the booster accelerator, a description of the characteristics of the spallation neutron source, and an evaluation of the possibilities for applications as a pulsed slow neutron source.

We find that the facility would be one of the most intense pulsed neutron sources in the world, both as a fast and as a slow neutron source, as indicated in Table I.

Table I

Comparison of Various Intense Neutron Sources

Device	Average Fast Neutron Production Rate, $n_f/\text{sec.}$	Peak Fast Neutron Production Rate $n_f/\text{sec.}$	Average Thermal Neutron Flux $n_{th}/\text{cm}^2\text{-sec.}$	Peak Thermal Neutron Flux $n_{th}/\text{cm}^2\text{-sec.}$
ORELA	$1. \times 10^{14}$	$1. \times 10^{19}$	FAST NEUTRON SOURCES	
LAMPF-WNRF	$7. \times 10^{14}$	$3. \times 10^{18}$		
Improved Nevis Synchrocyclotron	2×10^{15}	6.4×10^{20}		
ZING @ 60 Hz	5×10^{15}	6.5×10^{20}		
SORA @ 1. MW		SLOW NEUTRON SOURCES	$1. \times 10^{13*}$	$1. \times 10^{15}$
IBR-30			$1. - 2. \times 10^{13}$	$2. - 4. \times 10^{15}$
HFIR			$6. \times 10^{10}$	1.2×10^{13}
			$1. \times 10^{15}$	$1. \times 10^{15}$

* In the moderator reflector.

I. The Booster Accelerator

In 1969, a proposal was written for construction of a 500-MeV rapid-cycling booster injector for the Zero Gradient Synchrotron (ZGS) at Argonne National Laboratory. By injecting protons into the ZGS at 500 MeV rather than the present 50 MeV, the space charge limit of the synchrotron is increased by a factor of 18.

The first stage of this program consisted of converting the former Cornell electron synchrotron into a 200-MeV proton accelerator. To date, injection, beam coasting, and rf bunching have been studied and during a three-month experimental program beginning May 1, 1972, acceleration studies will take place.

The next stage of the program consists of replacing the Cornell synchrotron with the one designed and discussed in the booster report.¹ The characteristics of the beam extracted from this synchrotron are listed in Table II.

Table II

Extracted Beam Parameters

Proton Energy	500 MeV
Protons Per Pulse	5×10^{12}
Pulse Width	180 nsec (maximum)
Repetition Rate	30 Hz
Availability for Non-ZGS Use	$\approx 5/7$

Table III

Machine Parameters

Magnet Radius of Curvature	148"
Length of π Section	356.57"
No. of π Sections	2
Length of Short Section	24"
No. of Short Sections	2
Circumference	1691.06"
Average Radius	269.14"
Radial Aperture	4" in Magnet 6" in Quadrupoles
Vertical Aperture	2" in Magnet 4" in Quadrupoles
Repetition Rate	$f = 60$ cps
$B(t)$	$\left[6.212 + 3.458 \sin \left(2\pi ft - \frac{\pi}{2} \right) \right] \text{ kG}$
$\dot{B}(t)$	$\left[1304 \text{ cps} \left(2\pi ft - \frac{\pi}{2} \right) \right] \text{ kG/sec}$
Maximum Field	9.67 kG
Injection Energy	50 MeV
Maximum Energy	500 MeV
Harmonic Number	1
rf Frequency Injection	2.19 MHz
rf Frequency Final	5.29 MHz

Table III (Cont.)

Field Index	0.5625
Number of Sectors	2.0
ν_x	1.679
ν_y	1.770
β_x (in magnet)	238.4 - 216.7 in.
β_y (in magnet)	216.3 - 184.6 in.
Max β_x (in π section)	672 in.
Max β_y (in π section)	608 in.
Momentum Compaction	-1.2656
Maximum Energy Error Displacement (in magnet)	$-329 \frac{\Delta p}{p}$ in.
Maximum Energy Error Displacement (in π section)	$380 \frac{\Delta p}{p}$ in.
Energy Gain/Turn	$\left[21.05 \cos \left(2\pi ft - \frac{\pi}{2} \right) \right] \text{ keV}$
Maximum Cavity Voltage	40 kV
Stable Phase Angle (at $2\pi ft = \frac{\pi}{2}$ and 40 kV)	$\phi_{\text{smax}} = 31.75^\circ$
Maximum Acceptable Bucket Area (at 40 kV and max. $\dot{B}$)	$\frac{1.55}{2\pi}$ V-sec
Beam Synchrotron Phase-Space Area (with debuncher)	$\frac{0.87}{2\pi}$ V-sec
Quadrupole Gradient	$\left[2.08 + 1.16 \sin \left(2\pi ft - \frac{\pi}{2} \right) \right] \text{ kG/in.}$
Number of Quadrupoles	8
Quality of Ejected Beam (Area/ π)	Radial 0.67 mrad-in. Vertical 1.32 mrad-in.

Table III lists the machine parameters. Figure I shows a conceptual design of the booster accelerator, including the proposed location of an extracted beam line for ZING. The table assumes 60-cycle operation.

II. Intense Neutron Generator

By allowing intense pulses of protons to interact in targets of heavy nuclei, about 10-30 n/proton are produced due to spallation. A series of measurements of neutron yield were carried out for a variety of target materials for proton energies between 0.47 GeV and 1.47 GeV.^{2,3} These results are listed in Table IV and plotted in Figs. II and III. Figure IV contains a computer calculation of the neutron production from lead and uranium. The calculated values are somewhat lower than the measured values.

From Fig. III, it is obvious that there is a large jump in the yield for uranium. This is due to the contribution of neutrons from fast fissions in U^{238} .

For our calculations, let us assume that we have 500-MeV protons incident upon a uranium target 12 cm long and 10 cm in diameter. The length was chosen to be the range of 500-MeV protons in uranium and increasing the length should not increase the number of neutrons produced. In fact, the length could probably be shortened somewhat without affecting the neutron yield, leaving, say, a 100 MeV proton beam for further use.

Table IV
Neutron Yield (n/proton)

Target	Proton Energy, GeV			
	0.47	0.72	0.96	1.47
Be 10.2 cm x 10.2 cm x 91.6 cm	1.38	1.94	2.90	3.43
Sn 10.2 cm diam x 61 cm	4.72	8.20	11.45	15.87
Pb 10.2 cm diam x 61 cm	7.96	11.76	16.64	26.40
Pb 20.4 cm diam x 61 cm	8.67	13.94	20.29	31.47
U 10.2 cm diam x 61 cm (depleted)	18.50	29.06	40.48	56.75

Figure 1. Conceptual Design of the Booster Showing a Possible Location for ZING.

Figure II Measured neutron yield vs proton energy for various targets.

Figure III

Measured neutron yield vs atomic mass number of the target for various proton energies.

Figure IV Heat production per neutron produced and neutron yield vs incident proton energy.

Linear interpolation of the data indicates that the yield for 500-MeV protons is 19.6 n/protons. Thus, for 5×10^{12} protons/pulse, we obtain

$$(1) \quad N = 9.8 \times 10^{13} \text{ n/pulse} \quad .$$

For a booster operating at 30 pulses/sec and available for ZING 5/7 of the time, the average neutron production is

$$(2) \quad \bar{N} = 2.1 \times 10^{15} \text{ n/sec} \quad .$$

Assuming each pulse of protons is 150-nsec wide, then the peak fast neutron production rate is

$$(3) \quad \hat{N} = 6.5 \times 10^{20} \text{ n/sec} \quad .$$

Heat Production

Figure IV shows the heat produced per neutron as a function of the incident proton energy. For 500-MeV protons incident upon a uranium target, the appropriate number is 70 MeV/n. Thus, the average thermal power released in the target is

$$\bar{P} = 5 \times 10^{12} \text{ protons/pulse} \times 19.6 \text{ n/protons} \\ \times 70 \text{ MeV/n} \times 30 \text{ pulses/sec} \times 5/7$$

$$(4) \quad \bar{P} = 1.47 \times 10^{17} \text{ MeV/sec}$$

or since

$$1 \text{ MeV/sec} = 1.6 \times 10^{-16} \text{ kW}$$

$$(5) \quad P = 23.6 \text{ kW} \quad .$$

The average power in the beam is

$$\bar{P}_{\text{beam}} = 500 \text{ MeV/proton} \times 5 \times 10^{12} \text{ protons/pulse} \\ \times 30 \text{ pulses/sec} \times 1.6 \times 10^{-6} \text{ kW/(MeV/sec)} \times 5/7$$

$$(6) \quad \bar{P}_{\text{beam}} = 8.6 \text{ kW} \quad .$$

The remaining 15 kW is due to energy released in fissions and other reactions.

Thermoelastic Shock Effect

Dynamic thermal stresses in the target material are appreciable, but appear able to be handled. Calculated on the same basis as the heat production, the energy deposited per pulse is

$$\begin{aligned}\Delta E &= 5 \times 10^{12} \text{ protons/pulse} \times 19.6 \text{ n/proton} \\ &\quad \times 70 \text{ MeV/n} \times 1.6 \times 10^{-10} \text{ joules/MeV} \\ &= 1100 \text{ j./pulse}\end{aligned}$$

The mass of U^{238} in the target, is

$$\begin{aligned}M &= 19 \text{ gm/cm}^3 \times 942 \text{ cm}^3 \\ &= 18,000 \text{ gm}\end{aligned}$$

Since the specific heat of U^{238} is about

$$C_p = 0.028 \text{ cal/gm-}^\circ\text{C} = 0.135 \text{ j/gm-}^\circ\text{C}$$

the volume-averaged temperature rise per pulse is

$$\Delta T = \frac{\Delta E}{MC_p} = \left(\frac{1100}{18000} \right) \left(\frac{1}{0.135} \right) = 0.45^\circ\text{C/pulse}.$$

However, the major portion of the energy is deposited within 0.75 cm of the line of travel of an incident proton (see Fig. VIII). The maximum temperature rise in a target of 5 cm radius is therefore approximately

$$\Delta T_{\text{max}} = \left(\frac{5.0}{0.75} \right)^2 \Delta T_{\text{ave}} = 20^\circ\text{C/pulse}$$

This strains the present technology, even of advanced materials such as the U-Mo alloys to be used in the SORA pulsed

fast reactor. Similar temperature rises were met in the design study for the Brookhaven pulsed fast reactor, and these problems were the subject of intensive investigation at the time that project was terminated.

In the present case, if the proton beam is dispersed across, say a 2-cm radius, the maximum temperature rise per pulse would be only

$$\Delta T_{\max} = \left(\frac{5 \text{ cm}}{2 \text{ cm}} \right)^2 0.45 = 2.5^\circ\text{C/pulse}$$

This can probably be tolerated, since it is equal to the temperature rise anticipated in the well advanced SORA pulsed fast reactor design.

Power Density

For the assumed target geometry, 12 cm long and 10 cm in diameter, the power density is

$$(7) \quad \bar{P}/V = 23.6 \text{ kW}/.942 \text{ liter} = 25 \text{ kW/liter} \quad .$$

If this power is transferred to the surface of the target, the power per unit area is

$$\bar{P}/A = 23.6 \text{ kW}/534 \text{ cm}^2$$
$$(8) \quad \bar{P}/A = 44 \text{ W/cm}^2 \quad .$$

The heat removal problems therefore seem easily to be solvable within the capabilities of current fast-reactor technology.

Neutron Energy and Angular Spectra

Calculations have been made at Los Alamos for the neutron energy spectra and angular spectra for 800-MeV protons incident upon uranium and lead targets.⁴ These results are shown in Figs. V, VI, and VII.

The angular distribution shows that the highest energy neutrons are found in the forward direction as might be expected.

The energy spectrum can be seen to be composed of two parts, an evaporation component with an effective temperature of about 3 MeV, and a cascade component of lower amplitude, extending up to the energy of the incident protons. The spectrum is softened somewhat by inelastic scattering in targets of large diameter.

Figure VIII shows the radial and axial distribution of the production of evaporation neutrons in a 7.5-cm-radius, 30-cm-long uranium target with 800-MeV protons incident upon the axis.

Figure V

Comparison of 4π neutron spectra for ^{238}U and Pb cylinders 15-cm diam, 30-cm long with 800-MeV protons incident on axis.

Figure VI

Calculated angular neutron spectrum for 15-cm-diam, 30-cm-long ^{238}U cylinder with 800-MeV protons incident on axis.

Figure VII

Comparison of the calculated angular neutron spectra 800 to 100 MeV for 15-cm-diam, 30-cm-long ^{238}U cylinder and 15-cm-diam ^{238}U solid sphere with 800-MeV protons incident on axis.

Figure VIII

Radial and axial distribution of the production of evaporation neutrons in a 7.5-cm-radius, 30-cm-long ^{238}U target with 800-MeV protons incident on axis.

Options for Increasing Proton Intensity

There are two possible ways of increasing the intensity of protons from the booster to ZING. The repetition rate can be increased to 60 pulses per second. This not only increases the number of pulses by a factor of two, but also makes the booster available for ZING 6/7 of the time rather than only 5/7. In this case, the average neutron rate is

$$(9) \quad \bar{N} = 5.0 \times 10^{15} \text{ n/sec} \quad .$$

The cost for such a modification would be approximately 850 thousand dollars. The calculations in following sections assume 60-cycle operation.

One can also consider modifying the booster to accelerate protons to 700 MeV. At this energy, one obtains 28.2 n/protons. Then, both the neutron yield per pulse and the average rate are increased by a factor of 1.44. Thus,

$$(10) \quad N = 1.41 \times 10^{14} \text{ n/pulse}$$

and

$$(11) \quad N = 3.0 \times 10^{15} \text{ n/sec} \quad .$$

The cost for this modification alone is 435 thousand dollars.

If one intends to make both modifications, at a total cost of 1.285 million dollars, then the neutron yields become

$$(12) \quad N = 1.41 \times 10^{14} \text{ n/pulse}$$

and

$$(13) \quad N = 7.3 \times 10^{15} \text{ n/sec} \quad .$$

The calculations in following sections do not assume the 700 MeV option.

Comparison with Other Pulsed Neutron Sources

When ZING is compared with other accelerator sources, its performance in terms of total neutron production is shown to be impressive. The table below includes some well-known sources for comparison. The real value of ZING for any given application depends on more detailed information but the present comparison indicates that ZING is near the forefront as an intense neutron source.

Table V

Comparison of Various Intense Sources of Fast Neutrons

	Average Production Rate, n/sec	Peak Production Rate, n/sec	Neutrons per Pulse, n/pulse	Frequency Hz	Pulse Width Nanosec.
ORELA	1.0×10^{14}	$1. \times 10^{19}$	1×10^{11}	to 1000	2 to 1000
LAMPF-WNRF	$7. \times 10^{14}$	$3. \times 10^{18}$	6×10^{12}	120	20 to 2000
Improved Nevis Synchrocyclotron	1.8×10^{15}	6.4×10^{20}	6×10^{12}	300	10
ZING	$2.1 (5) \times 10^{15}$	6.5×10^{20}	10^{14}	30 (60)	150

III. SLOW NEUTRON SOURCE

There is considerable difficulty estimating the performance of ZING as a source of slow neutrons, because there has been little experience in applications appropriate to a spallation source. In what follows, the thermal neutron flux has been chosen as a base for comparison because it is a familiar quantity, not because it is a particularly useful quantity in terms of which to characterize the emerging neutron distribution.

Estimates from several previous measurements in pulsed-moderator applications are made below. The results of a brief series of measurements designed as a mockup of the ZING slow neutron source are reported below and compared with the previous measurements.

Steady-state measurements are enough to determine the intensity for pulsed moderators for which the pulse shape and energy distribution, are known, see Appendix B. Appendix A and Ref. 6 contain the required information for a variety of moderators. Further information is found in the work of Fluharty, et al.⁵

Figure IX depicts a spallation source, adjacent to it a moderator, surrounded by a reflector, and viewed by several beam ports penetrating a shield. Several moderators could be served, each viewed by several beam ports. The figure is drawn assuming the proton beam enters the spallation target vertically.

In the following, time averages are designated by the vinculum; e.g. $\bar{\phi}$, spatial averages by the subscript "ave"; e.g. ϕ_{ave} , and spatial maximum values of the subscript "max"; e.g. ϕ_{max} .

Figure IX A possible configuration for ZING showing several beam tube arrangements

III.A. ESTIMATE FROM YIELDS OF UNREFLECTED MODERATORS FOR 14 MeV NEUTRONS

The measurements of Graham and Carpenter⁶ provide yields of moderators pulsed by 14 MeV neutrons. Their results, for a variety of room temperature and cryogenic moderators, are partially summarized in Appendix A. Suppose such a moderator were erected adjacent to the spallation source, as in Fig. X, a conservative 7.5 cm distance away. A fairly efficient, heterogeneously poisoned moderator is chosen for this example; 296°K CH₂, 7.65 x 10.2 x 10.2 cm, with 0.020" Cd, 2.54 cm beneath the viewed surface. The moderator is surrounded on five sides by 1" B₄C. The thermal neutron yield per unit solid angle, specified in terms of the number of neutrons incident on the 7.65 x 10.2 cm face is

$$J_{Th}(\underline{\Omega}) = 1.32 \times 10^{-3} n_{Th}/str/n_f \text{ incident} .$$

Considering spallation neutrons to be isotropically distributed as from a point source, then the yield per neutron produced would be smaller by the ratio of the solid angle subtended by the moderator at the source, to 4π ,

$$\frac{n_f \text{ incident}}{n_f \text{ produced}} = \frac{\frac{7.65 \times 10.2}{(7.5)^2}}{4\pi} = 0.1105$$

$$\bar{I}_{Th}/\bar{S} = 1.46 \times 10^{-4} n_{Th}/str/n_f \text{ produced}$$

Using the result of Appendix B, this translates into an average flux on the surface, per source neutron,

Figure X A spallation neutron source erected adjacent to a moderator.

$$\bar{\phi}_{Th}/\bar{S} = 4.29 \frac{\bar{I}_{Th}/\bar{S}}{A_{Moderator}}$$

$$= 6.26 \times 10^{-6} n_{Th}/cm^2/n_f \text{ produced.}$$

Assuming sinusoidal spatial variation of the flux across the surface, the maximum flux on the surface would be $(\pi/2)^2$ times the average;

$$\bar{\phi}_{max}/\bar{S} = (\pi/2)^2 \bar{\phi}/\bar{S} = 1.55 \times 10^{-5} n_{Th}/cm^2/n_f \text{ produced.}$$

The pulse width for this moderator, from the data of Appendix A, is

$$\tau = 16 \mu\text{sec} .$$

for 0.050 eV.

III.B. ESTIMATE FROM SORA MOCKUP MEASUREMENTS

The recent measurements by Mihalcz, ⁷ on the ORNL Y-12 SORA mockup, shown in Fig. XI provide another basis for estimating performance. Mihalcz measured "the number of thermal neutrons leaking from the outer surface of a polyethylene scatterer" per fission in the reactor, to be

$$\bar{J}_{Th} \max / \bar{F} = 3.3 \times 10^{-5} n_{Th} / \text{cm}^2 / \text{fission}$$

at the center of the moderator surface. We interpret this as the maximum leakage current density rather than the flux. (In question is a factor $\approx \sqrt{3}$, which is assumed below to the advantage of this flux estimate.) The mockup, shown in the attached figure, indicates the moderator 16 cm from the center of the source; in the case of the smaller spallation source, the moderator would be closer to the source.

Assuming all neutrons available for leakage, do leak from the reactor core, then the number of source neutrons produced is

$$v(\eta - 1) / \eta = 1.41 n_f \text{ produced / fission}$$

as for a U^{235} -fueled fast reactor. Here, v = the mean number of neutrons per fission and η = the mean number of neutrons per capture. The leakage current density per source neutron is then

$$\begin{aligned} \bar{J}_{Th} \max / \bar{S} &= \frac{3.3 \times 10^{-5}}{1.41} \\ &= 2.34 \times 10^{-5} n_{Th} / \text{cm}^2 / n_f \text{ produced} \end{aligned}$$

- XXXX Core Matrix (fuel rod locations centered at crosses)
- Beryllium
- Iron
- Boron Carbide Canned in Aluminum (Boral)
- ▨ Polyethylene (10.16 x 10.16 x 3.81, cadmium covered except side facing core)
- ▨ Titanium Alloy
- ⊙ 2.54-cm-diam, 20.64-cm-long BF_3 Proportional Counter
- 0.79-cm-diam, 2.54-cm-long BF_3 Proportional Counter in Fuel Rod Location No. 58
- ⊙ 0.79-cm-diam, 2.54-cm-long BF_3 Proportional Counter Covered with 0.064-cm-Thick Cadmium
- ⊙ 0.79-cm-diam, 2.54-cm-long BF_3 Proportional Counter

The scintillator (5.1-cm-diam, 1.27-cm-long) was located with its axis in a plane 5.5 cm below the horizontal midplane.

Figure XI Cross Section of the Assembly at the Horizontal Midplane of the Core. (All Dimensions in Centimeters)

Assuming the relationship of Appendix B, the maximum flux is then

$$\begin{aligned}\bar{\phi}_{Th \text{ max}}/\bar{S} &= \sqrt{3} \bar{J}_{Th \text{ max}}/\bar{S} \\ &= 4.05 \times 10^{-5} n_{Th}/\text{cm}^2/n_f \text{ produced}\end{aligned}$$

The pulse width for 0.050 eV neutrons leaking from this moderator, estimated from the data of Appendix A, is

$$\tau = 22 \text{ } \mu\text{sec.}$$

III.C. ESTIMATE FROM THE BROOKHAVEN PULSED FAST REACTOR CRITICAL EXPERIMENT

Hoffman and Hendrie⁸ measured the absolute thermal neutron current at the surface of a moderator erected beside the BPFR critical experiment. The configuration is shown in Fig. XII. They found that the ratio of the thermal neutron leakage current density at the center of the moderator, to the reactor power, was

$$\bar{J}_{Th \text{ max}}/\bar{P} = 1.7 \times 10^6 \text{ n}_{Th}/\text{cm}^2\text{-watt-sec}$$

Since 1 watt-sec = 3.25×10^{10} fissions and each fission may be assumed to produce $\nu(\eta - 1)/\eta = 1.41$ neutrons available for leakage, the current per source neutron is

$$\begin{aligned} \bar{J}_{Th \text{ max}}/\bar{S} &= \frac{1.7 \times 10^6}{3.25 \times 10^{10}} \frac{1}{1.41} \\ &= 3.7 \times 10^{-5} \text{ n}_{Th}/\text{cm}^2/\text{n}_f \text{ produced} \end{aligned}$$

and the maximum flux per source neutron is

$$\begin{aligned} \bar{\phi}_{Th \text{ max}}/\bar{S} &= \sqrt{3} \bar{J}_{Th \text{ max}}/\bar{S} \\ &= 6.4 \times 10^{-5} \text{ n}_{Th}/\text{cm}^2/\text{n}_f \text{ produced} \end{aligned}$$

The pulse width for 0.050 MeV neutrons leaking from this moderator, as can be estimated from the data of Ref. 5., is about

$$\tau = 30 \text{ } \mu\text{sec.}$$

Figure XII Plan View of a Typical CEFR-1P Assembly

III.D. ESTIMATE FROM AECL's ING LARGE-MODERATOR CALCULATIONS

Still another estimate of the achievable flux can be obtained from Coleman's⁹ calculations in support of the Canadian ING project. Coleman's data reproduced below as Fig. XIII gives the maximum thermal neutron flux in various proton energy--target material--moderator material combinations. The case of interest to us, namely, 500 MeV-U²³⁸-H₂O, is not represented, but extrapolates to about

$$\bar{\phi}_{Th} \text{ max}/\bar{N}_p = 0.03 \text{ n}_{Th}/\text{cm}^2/\text{proton}$$

Since each proton produces 19.6 neutrons, this gives

$$\begin{aligned} \bar{\phi}_{Th} \text{ max}/\bar{S} &= \frac{0.03}{19.6} \\ &= 1.53 \times 10^{-3} \text{ n}_{Th}/\text{cm}^2/\text{n}_f \text{ produced} \end{aligned}$$

The pulse width for 0.050 eV neutrons leaking from this moderator, may be estimated as the infinite medium lifetime,

$$\tau = 200 \text{ } \mu\text{sec.}$$

Figure XIII Calculated maximum thermal-neutron flux per incident proton per second as a function of incident proton energy for various target materials.

III.E. MEASUREMENTS IN A MOCKUP SOURCE-MODERATOR-REFLECTOR ASSEMBLY

In an attempt to put the estimated performance on a firm basis, a mockup of a realistic source-moderator-reflector arrangement, with a beam port, was erected as shown in Fig. XIV, and the measurements described below were carried out.

A new approach to the design of pulsed moderators has been incorporated into the mockup, that of an energy-decoupled moderator-reflector. The reflector surrounding the moderator serves to return into the moderator, fast neutrons which would otherwise leak out while slowing down, and to deflect into the moderator, neutrons which would not collide if the reflector were not present. The reflector liner is required to decouple the moderator (in which the neutron lifetime is short) from the reflector (in which the slowing-down time and thermal diffusion time may be intolerably long). The liner is chosen to transmit neutrons above a certain energy (it might be called the "decoupling energy") and to absorb neutrons below that energy. The decoupling energy is chosen so that the slowing-down time to the decoupling energy is less than the moderator response time. In this way the moderator pulse can remain little broadened by the reflected neutrons.

The mockup was constructed of Be metal, with a 0.020" Cd liner; this combination seemed to be as near an optimal choice as could easily be constructed. The decoupling energy was then about 0.4 eV. The slowing-down time to 0.4 eV in Be is 14 μ sec. The root-mean-square slowing-down distance from 2 MeV to 0.4 eV is about 25 cm. Liquid D₂O would probably be a better reflector

Figure XIV Mockup of the ZING source, moderator and reflector, with a beam port. The assembly was surrounded by 6" Benelex.

material. Since slowing-down times are smaller in D_2O than in Be, the decoupling energy could be lower with a consequent increase in moderator output.

The moderator was a 4 x 4 x 2" polyethylene block, constructed of material whose density was 0.91 gm/cm^3 . The pulse full width at half maximum, for 0.050 eV neutrons leaking from this moderator is about 30 μsec , as can be estimated from the data of Appendix A.

The 4" x 4" source well was chosen to simulate the anticipated 10 cm dia. U^{238} spallation source. The 4" x 4" moderator was chosen since the mean slowing-down distance in polyethylene is about 10 cm, furthermore the range over which the neutrons are produced in the spallation source is also about 10. cm. The 4" x 4" square beam port followed from the choice of the 4" x 4" square moderator.

The Be reflector block was made smaller than would be optimally desirable, and the asymmetric shape was chosen, to minimize the mass of material to be handled. The Be block was surrounded on five faces with 6" of Benelex, and rested above 3" of Benelex on a concrete floor. Room return, therefore, presented no problem.

Fluxes were measured with the very sensitive solid state track recorder (SSTR) method¹⁰ using asymptotically thick 93.1% enriched U fission foils and mica track recorders. Neutrons were incident only through the track recorder side so that no significant self-shielding occurred. Track densities were 10^3 to 10^5 tracks/cm² and were determined by manual counting. Errors quoted below assume Poisson statistics in the numbers of tracks counted.

Sources which had been previously accurately calibrated¹¹ were used in the measurements. Cf²⁵²¹² with a fission-neutron spectrum and Am²⁴¹-Be¹³ with the harder spectrum characteristic of (α ,n) sources, were used to investigate the effects of source spectral variation.

Both bare and Cd-covered detectors were irradiated. Two classes of measurements were made. The flux on the moderator surface was measured by detectors placed directly against the surface. Detectors placed 36 cm and 100 cm from the moderator, well-shielded against room-scattered neutrons, were used to measure the beam current per unit solid angle.

The net sub-cadmium track densities were converted to thermal flux, assuming a spectrum-averaged fission cross section of 439 b/U²³⁵, which is characteristic of a spectrum whose temperature, 365°K, is that expected for this moderator, as can be inferred from the data of Ref. 6.

The thermal flux on the moderator surface, $\phi_{Th}(x)$, was obtained directly from the measured track density. The centerline flux was computed assuming a two-spatial-mode representation for the x-variation of the flux:

$$\phi_{Th}(x) = \phi_1 \cos \frac{\pi x}{a} + \phi_2 \sin \frac{2\pi x}{a}$$

$$\phi_{Th}(x = 0) = \frac{1}{2}(\phi_{Th}(-a/4) + \phi_{Th}(a/4)) \frac{1}{\cos \pi/4}$$

When the y-variation is assumed to be of the form $\cos \pi y/a$, the average flux across the moderator surface is

$$\phi_{Th \text{ ave}} = \left(\frac{2}{\pi}\right)^2 \phi_{Th}(x = 0)$$

The cadmium ratios measured at various positions are

$$R_{CD} = \frac{\text{Track density for bare SSIR}}{\text{Track density for Cd-covered SSIR}}$$

The thermal-neutron beam current per unit solid angle I_{Th} , was inferred from the track densities of detectors irradiated at a distance down the beam from port from the moderator. The current through an area A_D a distance z from the moderator is

$$A_D \varphi_{Th}(z) = I_{Th} \frac{A_D}{z^2}$$

where A_D/z^2 is the solid angle subtended by area A_D at distance z . Therefore, the current per unit solid angle was computed as

$$I_{Th} = z^2 \varphi_{Th}(z)$$

The average flux φ_{lav} on the viewed moderator surface A_M can be related to the beam current per unit solid angle I , if the flux and angular current density are assumed to be related as in Appendix C.

$$\varphi_{Th \text{ ave}} = 4.29 \frac{I_{Th}}{A_M}$$

The results of the measurement are given in Table VI.

Table VI: Results of Measurements on the ZING Be Reflector Mockup

Source	$\text{Am}^{241}\text{-Be}$	Cf^{252}
Intensity, $\bar{S}$; n_f/sec	1.25×10^7	4.35×10^6 *
$\bar{\varphi}_{\text{Th}}(x = -a/4)/\bar{S}$; $n_{\text{Th}}/\text{cm}^2/n_f$	$2.24 \times 10^{-4} (\pm 5\%)$	$2.86 \times 10^{-4} (\pm 6\%)$
$\bar{\varphi}_{\text{Th}}(x = a/4)/\bar{S}$; $n_{\text{Th}}/\text{cm}^2/n_f$	$2.27 \times 10^{-4} (\pm 5\%)$	$1.79 \times 10^{-4} (\pm 5\%)$
$\bar{\varphi}_{\text{Th}}(x = 0)/\bar{S}$; $n_{\text{Th}}/\text{cm}^2/n_f$	$3.19 \times 10^{-4} (\pm 4\%)$	$3.28 \times 10^{-4} (4\%)$
$\bar{\varphi}_{\text{ave}}/\bar{S} = (2/\pi)^2 \bar{\varphi}(x = 0)/\bar{S}$; $n_{\text{Th}}/\text{cm}^2/n_f$	$1.29 \times 10^{-4} (\pm 4\%)$	$1.33 \times 10^{-4} (4\%)$
R_{CD} on surface	6.5	6.5
$\bar{I}_{\text{Th}}/\bar{S}$; $n_{\text{Th}}/\text{str}/n_f$	$3.95 \times 10^{-3} (\pm 7\%)^a$ $4.8 \times 10^{-3} (\pm 5\%)^b$	$3.7 \times 10^{-3} (\pm 9\%)^a$
R_{CD} in beam	2.1 ^a 4.8 ^b	2.2 ^a
$\bar{\varphi}/\bar{S} = 4.29 \bar{I}_{\text{Th}}/\bar{S}/A_{\text{mod}}$	$1.69 \times 10^{-4} \text{ a}$ $2.06 \times 10^{-4} \text{ b}$	

* Computed for the time of the measurement.

^a Measured at $z = 36$ cm.

^b Measured at $z = 100$ cm.

The results seem to harbor no disturbing inconsistencies. The flux per unit source is about the same for both the Am-Be and Cf sources indicating only mild dependence of the yield on source spectrum. The flux distribution appears to be tilted toward the source in the case of the Cf source, but to be about symmetric about the moderator centerline in the case of the Am-Be source.

The cadmium ratio observed at the moderator face appears to be characteristic of the emerging beam; the cadmium ratio observed is influenced strongly by epicadmium neutrons from the beam port walls. The beam current per unit solid angle seems to be a little greater than would be expected on the basis of the idealization of Appendix C, as seen when the surface flux calculated from the beam current, $4.29 I_{Th}/S/A_{mod}$ is compared with that measured on the moderator surface.

The results may be summarized as follows: the max thermal flux taken as the average of the two measurements at $x = 0$

$$\frac{\bar{\phi}_{Th \max}}{S} = 3.2 \times 10^{-4} n_{Th}/cm^2/n_f ;$$

the beam current per unit solid angle, using the average of the three measurements, is

$$\frac{\bar{I}_{Th}}{S} = 4.1 \times 10^{-3} n_{Th}/str/n_f$$

The pulse width at 0.050 eV for this moderator, obtained by extrapolating the data of Appendix A, is

$$\tau = 30 \mu sec.$$

III.F. ESTIMATED PERFORMANCE ACCOMPLISHABLE

Table X below summarizes the estimated and measured fluxes per unit source intensity. The ratio tabulated is the time average flux at the location of the spatial maximum. Also tabulated is the pulse width τ , at 0.050 eV, chosen as a convenient energy for comparison, and the momentary peak flux at the position of the maximum flux, computed as discussed in Appendix B

$$\hat{\phi}_{Th \max} = \frac{1}{\tau} \frac{\bar{\phi}_{Th}}{\bar{S}} S_0$$

using $S_0 = 9.8 \times 10^{13} \text{ n}_f/\text{pulse}$, as obtained in Section II.

TABLE X

Estimate	$\bar{\phi}_{Th \max}/\bar{S}$ $\text{n}_{Th}/\text{cm}^2\text{-n}_f$	τ (0.05 eV), μsec	$\hat{\phi}_{Th \max}$ $\text{n}_{Th}/\text{cm}^2\text{-sec}$
Sec.III A(ZING-Bare)	1.55×10^{-5}	16	9.5×10^{13}
Sec.III B(SORA)	4.05×10^{-5}	22	1.8×10^{14}
Sec.III C(BPFR)	6.4×10^{-5}	30	2.1×10^{14}
Sec.III D(ING)	1.5×10^{-3}	200	7.4×10^{14}
Sec.III E(ZING-Be)	3.2×10^{-4}	30	1.05×10^{15}

The results indicate that one can expect ZING to provide a thermal flux at the center of a roughly $10 \times 10 \text{ cm}^2$ moderator, which momentarily reaches a peak value of $10^{15} \text{ n}_{Th}/\text{cm}^2\text{-sec.}$, each time the spallation target receives a proton pulse. The pulse width for 0.050 eV neutrons would be about 30 μsec .

Speaking in broad generalities, the peak flux would remain roughly unchanged if the moderator were heterogeneously poisoned

to reduce the pulse width. The energy-decoupled Be reflector is seen here to provide about a 5 or 10 x enhancement of the peak flux compared to that achievable with a bare moderator.

The reflector-moderator combination has not been optimized, so further enhancement of the peak flux is possible. On the other hand, the mockup does not incorporate any engineering necessities such as cooling, which will inevitably detract from the peak flux. To a low order of approximation, it might be anticipated that design optimizations would compensate for required practical changes, leaving the peak flux at about $10^{15} n_{Th}/cm^2\text{-sec.}$

The peak thermal leakage current density estimated in accordance with Appendix C, would be

$$\begin{aligned}\hat{J}_{Th \max} &= \frac{1}{\sqrt{3}} \hat{\phi}_{Th \max} \\ &= 5.8 \times 10^{14} n_{Th}/cm^2\text{-sec.}\end{aligned}$$

The peak thermal leakage beam current, per unit solid angle, integrated over the moderator surface, using the results measured on the mockup, calculated in similar fashion to Appendix B, and again using the pulse width for 0.05 eV neutrons, would be

$$\begin{aligned}\hat{I}_{Th} &= \frac{1}{\tau} (\bar{I}_{Th}/\bar{S}) S_0 \\ &= \frac{4.1 \times 10^{-3}}{30 \times 10^{-6}} \times 9.8 \times 10^{13} \\ &= 1.34 \times 10^{16} n_{Th}/\text{Str-sec.}\end{aligned}$$

This is a little greater ($\approx 25\%$) than would be calculated from the flux, using the idealization of Appendix C. This quantity,

the leakage angular current, is what is required for estimates of performance in beam applications.

III.G. ESTIMATE OF THE TIME-AVERAGE THERMAL FLUX IN THE MODERATOR-REFLECTOR

Coleman's calculations⁹ illustrated in Fig. XIII provide an estimate of the time average flux in the moderator-reflector. Assume that the moderator-reflector would be D₂O. Since Coleman's result for the case 500 MeV-Pb-D₂O is

$$\bar{\phi}_{Th \max} / \bar{N}_p = 0.02 \text{ } n_{Th} / \text{cm}^2 / \text{proton}$$

and since the ratio of the U cases to the Pb cases is about 1.8 or 2.0, as seen from Fig. XIII, we estimate that the maximum thermal neutron flux in the D₂O would be

$$\begin{aligned} \bar{\phi}_{Th \max} &= 0.02 \times 1.9 \times 5 \times 10^{12} \times 60 \times 6/7 \\ &= 10^{13} \text{ } n_{Th} / \text{cm}^2 \text{-sec} \end{aligned}$$

III.H. EPITHERMAL BEAM CURRENT

Assuming the ratio $J(0.5 \text{ eV})/J_{Th}$ is the same for the 2-inch polyethylene slab in the ZING-Be mockup as for the 1.5-inch polyethylene slab in Graham's measurement; namely,

$$\frac{J(0.5 \text{ eV})}{J_{Th}} = 0.49 \frac{n_{.5}/\text{str/eV}}{n_{Th}/\text{str}}$$

the epithermal beam current per unit solid angle can be estimated. The time average beam intensity would be

$$\begin{aligned}\bar{I}(0.5 \text{ eV}) &= 0.49 \left(\frac{\bar{I}_{Th}}{\bar{S}} \right) \\ &= 0.49 \times 4.1 \times 10^{-3} \times 10^{15} \\ &= 1.0 \times 10^{13} \text{ n}_{.5}/\text{str-eV/sec.}\end{aligned}$$

The burst width for 0.5 eV neutrons leaking from an unrefracted 2-inch slab is about 4 μsec , while the slowing-down time in the Be moderator reflector is about 14 μsec with a dispersion about that value, of about 3 μsec . It is therefore difficult to estimate the width of the 0.5 eV neutron burst; however, we estimate the burst should be about 10 μsec wide. The peak value of the epithermal beam current then can be estimated as

$$\begin{aligned}\hat{I}(0.5 \text{ eV}) &= \frac{1}{\tau} \left(\frac{\bar{I}}{\bar{S}} \right) S_0 \\ &= \frac{1}{10 \times 10^{-6}} (0.49) (4.1 \times 10^{-3}) \times 9.8 \times 10^{13} \\ &= 2 \times 10^{16} \text{ n}_{.5}/\text{str-eV-sec}\end{aligned}$$

III.1. COMPARISON WITH OTHER PULSED SLOW NEUTRON SOURCES

It must be realized that there are only a few systems with which ZING can be directly compared as a slow neutron source. Further, the comparisons are rough order-of-magnitude comparisons, since the usefulness of each source depends on many factors.

In the table on the following page, some figures are given describing existing or proposed systems with moderators for slow neutron production.

Table XI

Comparison of Various Intense Pulsed Sources of Slow Neutrons

	Epithermal Beam Current at .5 eV, n/sec-eV-str		Thermal Leakage Flux n/cm ² -sec		Frequency Hz	Pulse Width at .050 eV, μ sec
	Time Average	Peak	Time Average	Peak		
Rensselaer	8.8×10^9					
ORELA	4.8×10^{11}					
SORA			$1. \times 10^{13}$	$2-4 \times 10^{15}$	50	110
IBR-30			$6. \times 10^{10}$	1.2×10^{13}	100	50
ZING	1.0×10^{13}	2×10^{16}	$2. \times 10^{12}$	$1. \times 10^{15}$	60	30

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APPENDIX A:

Moderators for Slow-Neutron Production

Attached are some data on a variety of moderators, reproduced from the work of Graham.⁽⁶⁾ Similar additional data may be found in the work of Fluharty, et al.⁽⁵⁾ The geometrical arrangement in which the measurements were made is shown, as are pulse widths vs. neutron energy, energy spectra which represent the total leakage from the $10 \times 10 \text{ cm}^2$ moderator face and typical pulse shapes measured at several energies.

Figure A-1 Detailed arrangement of moderator, neutron source and shielding.

Figure A-II Measured FWHM vs Energy for 296°K polyethylene moderators of 10 x 10 cm lateral dimensions. Measurements by Fluharty et al(23) also shown.

Figure A-III Measured FWHM vs Energy for five 296°K polyethylene moderators of 10 x 10 cm lateral dimensions.

Figure A-IV Energy spectra emerging from 296°K polyethylene moderators of lateral dimensions 10 cm x 10 cm.

Figure A-V 7.65 x 10 x 10 cm 296°K polyethylene, cadmium poison 2.55 cm beneath surface. $\theta_n = 60^\circ$

Figure A-VI Energy spectra emerging from 77°K moderators 7.3 cm x 10 cm x 10 cm. Units of $J(E, \Omega_e)$ are neutrons per ev per steradian per incident 14.7 Mev neutron.

Figure A-VII Measured FWHM vs Energy for two 7.65 x 10 x 10 cm 77°K Polyethylene moderators. One is poisoned at 2.55 cm with .05 cm Cadmium. The other is unpoisoned. The measurements of Fluharty et al⁽²³⁾ are shown.

APPENDIX B:

Relation Between Steady State and Pulsed Measurements

Steady state measurements are adequate to determine the intensity for pulsed moderators for which the pulse shape is known. If $\phi_R(t)$ is the flux at time t following an instantaneous pulse of one neutron at time zero, then for a general source emitting $S(t')$ neutrons per unit time at time t' , the flux is

$$\phi(t) = \int \phi_R(t-t') S(t') dt' \quad (1)$$

The time averaged flux is then

$$\bar{\phi} = \bar{S} \int_0^{\infty} \phi_R(t) dt \quad (2)$$

while for an instantaneously pulsed source for which $S(t') = S_0 \delta(t')$, the fluence per pulse is

$$\int \phi(t) dt = S_0 \int_0^{\infty} \phi_R(t) dt \quad (3)$$

If the moderator pulse response is considered as a pulse of peak height $\hat{\phi}$ and known width τ such that

$$\int \phi(t) dt = \tau \hat{\phi} = S_0 \int_0^{\infty} \phi_R(t) dt \quad (4)$$

then the peak flux in pulsed mode operation can be obtained from the time average, through the relation

$$\hat{\phi} = 1/\tau(\bar{\phi}/\bar{S})S_0 \quad (5)$$

Recognizing that the peak flux and width are both functions of energy, then the peak flux per unit energy can be determined from the steady state measurements when the pulse shape in time and energy is known. If $\phi_R(E, t)$ is the flux per unit energy at time t following an instantaneous pulse of one neutron at time zero, then for a general source

$$\phi(E, t) = \int \phi_R(E, t-t') S(t') dt' \quad (6)$$

The measured time averaged flux per unit energy is then

$$\bar{\phi}(E) = \bar{S} \int_0^{\infty} \phi_R(E, t) dt \quad (7)$$

Suppose, for example, that the measurement produces the thermal flux,

$$\bar{\phi}_{Th} = \int W_{Th}(E) \bar{\phi}(E) dE \quad (8)$$

$$= \bar{S} \int_0^{\infty} W_{Th}(E) \phi_R(E) dE \quad (9)$$

$$= \bar{S} \phi_{RTh} \quad (10)$$

where $W_{Th}(E)$ is the flux weighting function describing the measurement.

For an instantaneously pulsed source $S(t') = S_0 \delta(t')$, the fluence per unit energy, in each pulse is

$$\begin{aligned} \int \phi(E, t) dt &= S_0 \int_0^{\infty} \phi_R(E, t) dt \\ &= S_0 \phi_R(E) \end{aligned} \quad (11)$$

where $\phi_R(E)$ is the flux per unit energy, characteristic of the moderator.

Say the moderator pulse response is considered as a pulse of known width $\tau(E)$ and peak height $\hat{\phi}(E)$ at energy E , such that

$$\int \phi(E, t) dt = \tau(E) \hat{\phi}(E) = S_0 \phi_R(E) \quad (12)$$

Then if the total thermal flux for the moderator is known so that $\phi_R(E)/\phi_{RTh}$ is known, then writing

$$\phi_R(E) = (\phi_R(E)/\phi_{RTh})\phi_{RTh} \quad (13)$$

then $\hat{\phi}(E)$ is easily seen to be

$$\hat{\phi}(E) = 1/\tau(E)S_0\phi_R(E) \quad (14)$$

$$= 1/\tau(E)(\phi_R(E)/\phi_{RTh})(\bar{\phi}_{Th}/\bar{S})S_0 \quad (15)$$

It is implicit in equations 5 and 15 that pulse shapes and energy spectra do not vary between the configuration in which the steady state and the energy and time dependent measurements are made. Thus specific external geometric effects are removed in the ratio $(\phi_R(E)/\phi_{RTh})$. Intensity in relation to specific geometric effects is contained in the ratio $(\bar{\phi}_{Th}/\bar{S})$, which can be determined by steady-state measurements. Time dependent spectrum measurements such as those of references 5 and 6, establish $\hat{\phi}(E)$ which is roughly independent of geometrical effects external to the moderator. Energy spectra, for which $\phi(E)/\phi_{Th}$ is given, are similarly roughly independent of geometrical effects external to the moderator.

APPENDIX C:

Angular Current Density, Flux, and Net Current at a Moderator-Vacuum Interface

When neutrons emerge from a surface to stream into a vacuum, as in neutron beam applications, the angular distribution is markedly anisotropic. Here we briefly discuss the relationships among the flux, the current and the angular current density. This situation is an example of the classic Milne problem. Used here are the results for a planar, source-free, nonabsorbing, isotropically-scattering medium, in the sense of a one-speed theory.

In the diffusion-theory approximation, which is recalled here only for reference,

$$J_x \approx -D \frac{d\phi}{dx} \quad (1)$$

the net current is contrived to approach the exact net current deep in the medium,

$$\lim_{x \rightarrow -\infty} J_x = J_\infty \quad (2)$$

and the flux vanishes at the extrapolation distance, d , from the boundary. The theory gives a poor account of the flux and angular current density at the interface.

The exact angular current density at the interface (and beyond) is shown in Figure C-1. Also shown is Fermi's linear (in μ) approximation, which is both simple and accurate, and also the form implicit in diffusion theory. Fermi's expression for the angular current density is

$$\psi_0(\mu) = \begin{cases} \frac{1}{2\pi} \phi_0 \frac{1+\sqrt{3}\mu}{1+\sqrt{3}/2} & ; 0 \leq \mu \leq 1 \\ 0 & ; -1 \leq \mu < 0 \end{cases} \quad (3)$$

where $\phi_0 = \int_{4\pi} \psi(\mu) d\Omega$ is the flux at the interface.

so that the current density is

$$J = \int_{4\pi} \mu \psi_0(\mu) d\Omega = \frac{1}{\sqrt{3}} \phi_0 \quad (4)$$

Using equation 3, it is easy to see that the actual flux ϕ_0 and the normally-emerging angular current density $\psi_0(\mu=1)$ are related

$$\begin{aligned} \phi_0 &= 2\pi(1+\sqrt{3}/2)\psi_0(\mu=1)/(1+\sqrt{3}) \\ &= 4.29\psi_0(\mu=1) \end{aligned} \quad (5)$$