GENERAL ATOMIC DIVISION
GENERAL DYNAMICS CORPORATION

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GAMD-3779

AEC RESEARCH AND DEVELOPMENT REPORT

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VIEW FACTORS FOR HEAT RADIATION
IN HTGR CORE CONFIGURATION

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Facsimile Price \$ 2.60
Microfilm Price \$.83

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

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Project No. 32.820
U.S. Atomic Energy Commission
Contract No. AT(04-3)-314

December 10, 1962

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1. Introduction

Radiant heat transmission becomes significant in gas-cooled reactors when the main coolant capabilities wane.

This analysis, which deals with the view factors, is a preliminary step in order to find the over-all interchange factors for heat radiation. The procedure follows the general lines in the matter as can be found in the general textbooks, for instance in the Radiant Heat Transmission Chapter of McAdams' HEAT TRANSMISSION, McGraw Hill Book Co., Inc. Examples concerning both EGCR and Peach Bottom HTGR are considered.

2. Nomenclature

A_i	radiant area of surface i , ft^2
C	center-to-center distance, ft or in
D	rod diameter, ft or in
F_{ij}	view factor (from area i to area j), dimensionless
I	intensity of radiation, $\frac{B}{\text{sec in}^2}$ or $\frac{B}{\text{sec ft}^2}$
q	heat transmission rate, $\frac{B}{\text{sec}}$
r	distance between surface areas, ft or in
α) β) γ)	plane angles
$\rho =$	C/D
ϕ	angle made by a radiant beam with normal to radiant area element
ω	solid angle

Super- and Sub-scripts

-	refers to average value
0, 1, 2, ...	radiant areas
L	limiting condition
r	rods
s	streaming radiation

3. The Local View Factor F for Axial Symmetry

For the radiation from dA_1 to dA_2 , which is indicated as $dq_{1 \rightarrow 2}$

$$dq_{1 \rightarrow 2} = I_1 \cdot dA_1 \cdot \cos \phi_1 \cdot d\omega_1$$

according to the sketch.

Assuming axial symmetry (notice that the α 's are in a plane normal to dA_1 and the axis of symmetry while the ϕ 's are also in a plane normal to dA_1 , but parallel to the axis of symmetry)

$$\begin{aligned} \int_{\omega_1} \cos \phi_1 \cdot d\omega_1 &= \int_{\alpha} \int_{\phi} \cos \phi \cdot \cos \alpha \cdot \frac{r \cdot d\phi \cdot \cos \phi \cdot r \cdot \cos \phi \cdot d\alpha}{r^2} \\ &= \int_{\alpha} \int_{\phi} \cos \alpha \cdot d\alpha \cdot \cos^2 \phi \cdot d\phi = \frac{\pi}{2} \int_{\alpha} \cos \alpha \cdot d\alpha = \frac{\pi}{2} [\sin \alpha_1 - \sin \alpha_2] \end{aligned}$$

being α_1 and α_2 , the limiting angles in the plane normal to the symmetry axis.

Therefore, the total radiation from dA_1 to A_2 , which is indicated as dq_{1-2} becomes

$$dq_{1-2} = I_1 \frac{\pi}{2} [\sin \alpha_1 - \sin \alpha_2] \cdot dA_1$$

The total radiation from dA_1 , i.e., for $\alpha_1 = \frac{\pi}{2}$ and $\alpha_2 = -\frac{\pi}{2}$, which is indicated as dq_1 , is given by

$$dq_1 = I_1 \pi dA_1$$

The local view factor is

$$F = \frac{dq_{1-2}}{dq_1} = \frac{1}{2} [\sin \alpha_1 - \sin \alpha_2]$$

4. Two Typical Configurations

Two cases can be considered, which are determined by the ratio $p = c/d$

The sketch illustrates a typical cluster in which there is not interposition to the straight beams between two adjacent bodies. This case occurs when

$$D \leq C \cos \frac{\pi}{6}$$

or

$$\rho \geq \sec \frac{\pi}{6}$$

The other case, that is when interposition occurs, takes place when

$$D > C \cos \frac{\pi}{6}$$

or

$$\rho < \sec \frac{\pi}{6}$$

Example 1. For the seven-rod cluster in one EGCR sleeve

$$D = 3/4 \text{ "}$$

$$C = 1 \text{ "}$$

$$\rho = \frac{C}{D} = \frac{4}{3} > \sec \frac{\pi}{6}$$

This corresponds to the no interposition case.

Example 2. For a typical group of seven fuel elements for Peach Bottom HTGR

$$D = 3.50 \text{ in.}$$

$$C = 3.57 \text{ in.}$$

$$\rho = \frac{C}{D} = 1.02 < \sec \frac{\pi}{6}$$

This reactor core has a configuration which corresponds to the interposed radiation case. These two different cases are considered separately in the following pages, starting with the most simple, for no interposed radiation.

4.A. Case $\rho \geq \sec \frac{\pi}{6}$

The first problem is to find the average value of the view factor, so considering two adjacent bodies, the view factor is given by

$$\bar{F} = \frac{1}{\pi D} \int_{2\pi} F \frac{D}{2} d\beta = \frac{1}{4\pi} \int_{2\pi} (\sin \alpha_1 - \sin \alpha_2) d\beta,$$

which by symmetry can be written as

$$\bar{F} = \frac{1}{2\pi} \int_0^{\pi/2} (\sin \alpha_1 - \sin \alpha_2) d\beta$$

In order to evaluate this factor it is required to have $\alpha_1 = \alpha_1(\beta)$ and $\alpha_2 = \alpha_2(\beta)$, and to better precise the limits for the integration.

Both expressions for the angles can be obtained by trigonometry. The figure indicates the two limiting angles, α_1 and α_2 for the radiation departing from one elementary surface in one body can intercept the other body.

Projection along the normal to the tangents

For $POO'T_1$

$$\frac{D}{2} (1 + \sin \alpha_1) = C \sin (\alpha_1 - \beta)$$

or

$$1 + \sin \alpha_1 = 2 \rho \sin (\alpha_1 - \beta)$$

For $POO'T_2$

$$\frac{D}{2} (1 - \sin \alpha_2) = C \sin (\beta - \alpha_2)$$

or

$$1 - \sin \alpha_2 = 2 \rho \sin (\beta - \alpha_2)$$

Both $\sin \alpha_1$ and $\sin \alpha_2$ can be cleared out, becoming

$$\sin \alpha_1 = \frac{-\frac{1}{4} + \frac{\rho}{2} \cos \beta + \rho \sin \beta \sqrt{\rho^2 - \rho \cos \beta}}{\rho^2 - \rho \cos \beta + \frac{1}{4}}$$

and

$$\sin \alpha_2 = \frac{\frac{1}{4} - \frac{\rho}{2} \cos \beta + \rho \sin \beta \sqrt{\rho^2 - \rho \cos \beta}}{\rho^2 - \rho \cos \beta + \frac{1}{4}}$$

Regarding the limits for integration in the range 0 to $\frac{\pi}{2}$, it can be seen from the figure that $\sin \alpha_2$ is valid for that range, but $\sin \alpha_1$ is only valid from 0 to β_L , being $\sin \alpha_1 = 1$ from β_L to $\frac{\pi}{2}$.

The limiting angle β_L is given by

$$\beta_L = \arccos \frac{D}{C} = \arccos \frac{1}{2\rho}$$

$$\text{A.a.} \int_0^{\pi/2} \sin \alpha_2 \cdot d\beta$$

In order to carry out this integration, it is convenient to rearrange terms in $\sin \alpha_2$, so

$$\sin \alpha_2 = \frac{1}{2} - \frac{1}{2} \frac{\rho^2 - \frac{1}{4}}{\rho^2 - \rho \cos \beta + \frac{1}{4}} + \frac{\rho \sin \beta \sqrt{\rho^2 - \rho \cos \beta}}{\rho^2 - \rho \cos \beta + \frac{1}{4}}$$

Thus

$$\begin{aligned}
 \int_0^{\pi/2} \sin \alpha_2 d\beta &= \frac{\pi}{4} - \frac{1}{2} \left(\rho^2 - \frac{1}{4} \right) \int_0^{\pi/2} \frac{d\beta}{\rho^2 + \frac{1}{4} - \rho \cos \beta} - \int_0^1 \frac{\rho \sqrt{\rho^2 - \rho \cos \beta}}{\rho^2 + \frac{1}{4} - \rho \cos \beta} d \cos \beta \\
 &= \frac{\pi}{4} - \left[\arctan \frac{\left(\rho^2 - \frac{1}{4} \right)}{\left(\rho - \frac{1}{2} \right)^2} \tan \frac{\beta}{2} \right]_0^{\pi/2} - \rho \int_{1 + \frac{1}{4\rho^2}}^{1 + \frac{1}{4\rho^2} - \frac{1}{\rho}} \frac{\sqrt{x - \frac{1}{4\rho^2}}}{x} dx \\
 &= \frac{\pi}{4} - \arctan \frac{\rho + \frac{1}{2}}{\rho - \frac{1}{2}} - \rho \left[2 \sqrt{x - \frac{1}{4\rho^2}} - \frac{1}{\rho} \arctan \sqrt{\frac{x - \frac{1}{4\rho^2}}{\frac{1}{4\rho^2}}} \right]_{1 + \frac{1}{4\rho^2}}^{1 + \frac{1}{4\rho^2} - \frac{1}{\rho}} \\
 &= \frac{\pi}{4} - \arctan \frac{2\rho + 1}{2\rho - 1} - 2\rho \left(\sqrt{1 - \frac{1}{\rho}} - 1 \right) + \arctan 2\rho \sqrt{1 - \frac{1}{\rho}} - \arctan 2\rho \\
 &= \frac{\pi}{4} - \arctan \frac{1 + \frac{1}{2\rho}}{1 - \frac{1}{2\rho}} + 2\rho \left(1 - \sqrt{1 - \frac{1}{\rho}} \right) + \arctan 2\rho \sqrt{1 - \frac{1}{\rho}} - \arctan 2\rho
 \end{aligned}$$

A.b. $\int_0^{\arccos \frac{1}{\rho}} \sin \alpha_1 \cdot d\beta$

A similar arrangement of the terms in the expression of $\sin \alpha_1$ renders

$$\sin \alpha_1 = -\frac{1}{2} + \frac{1}{2} \frac{\rho^2 - \frac{1}{4}}{\rho^2 - \rho \cos \beta + \frac{1}{4}} + \frac{\rho \sin \beta \sqrt{\rho^2 - \rho \cos \beta}}{\rho^2 - \rho \cos \beta + \frac{1}{4}}$$

and the integration

$$\begin{aligned}
 \int_0^{\arccos \frac{1}{\rho}} \sin \alpha_1 \cdot d\beta &= -\frac{1}{2} \arccos \frac{1}{\rho} + \arctan \frac{1 + \frac{1}{2\rho}}{1 - \frac{1}{2\rho}} \tan \frac{\arccos \frac{1}{\rho}}{2} + 2\rho \left[\sqrt{1 - \frac{1}{\rho^2}} - \sqrt{1 - \frac{1}{\rho}} \right] \\
 &\quad - \arctan 2\rho \sqrt{1 - \frac{1}{\rho^2}} + \arctan 2\rho \sqrt{1 - \frac{1}{\rho}}
 \end{aligned}$$

$$\begin{aligned} \text{A.c. } \int_{\arccos \frac{1}{p}}^{\pi/2} d\beta \\ \int_{\arccos \frac{1}{p}}^{\pi/2} d\beta = \frac{\pi}{2} - \arccos \frac{1}{p} \end{aligned}$$

Replacing all the integrals in

$$\begin{aligned} J = \int_0^{\arccos \frac{1}{p}} \sin \alpha_1 d\beta + \int_{\arccos \frac{1}{p}}^{\pi/2} d\beta - \int_0^{\pi/2} \sin \alpha_2 d\beta = \frac{\pi}{4} - \frac{3}{2} \arccos \frac{1}{p} - 2p \left[1 - \sqrt{1 - \frac{1}{p^2}} \right] \\ + \arctan \frac{1 + \frac{1}{2p}}{1 - \frac{1}{2p}} \tan \frac{\arccos \frac{1}{p}}{2} + \arctan \frac{1 + \frac{1}{2p}}{1 - \frac{1}{2p}} - \arctan 2p \sqrt{1 - \frac{1}{p^2}} + \arctan 2p \end{aligned}$$

This expression can be simplified as follows

$$\arctan \frac{1 + \frac{1}{2p}}{1 - \frac{1}{2p}} = \arctan \tan \left(\frac{\pi}{4} + \arctan \frac{1}{2p} \right) = \frac{\pi}{4} + \arctan \frac{1}{2p} = \frac{\pi}{4} + \frac{\pi}{2} - \arctan 2p$$

and

$$J = \pi - 2p \left(1 - \sqrt{1 - \frac{1}{p^2}} \right) - \frac{3}{2} \arctan \sqrt{p^2 - 1} + \arctan \frac{1 + \frac{1}{2p}}{1 - \frac{1}{2p}} \sqrt{\frac{1 - \frac{1}{p}}{1 + \frac{1}{p}}} - \arctan 2p \sqrt{1 - \frac{1}{p^2}}$$

A second simplification can be done by considering the last two terms, so calling

$$\begin{aligned} \arctan x &= \arctan 2p \sqrt{1 - \frac{1}{p^2}} - \arctan \frac{1 + \frac{1}{2p}}{1 - \frac{1}{2p}} \sqrt{\frac{1 - \frac{1}{p}}{1 + \frac{1}{p}}} \\ x &= \frac{2p \sqrt{1 - \frac{1}{p^2}} - \frac{1 + \frac{1}{2p}}{1 - \frac{1}{2p}} \sqrt{\frac{1 - \frac{1}{p}}{1 + \frac{1}{p}}}}{1 + 2p \frac{1 + \frac{1}{2p}}{1 - \frac{1}{2p}} \sqrt{1 - \frac{1}{p^2}} \sqrt{\frac{1 - \frac{1}{p}}{1 + \frac{1}{p}}}} = \frac{(2p - 1) \sqrt{1 - \frac{1}{p^2}} - (1 + \frac{1}{2p}) \sqrt{\frac{1 - \frac{1}{p}}{1 + \frac{1}{p}}}}{1 - \frac{1}{2p} + 2p \left(1 + \frac{1}{2p} \right) \left(1 - \frac{1}{p} \right)} \\ x &= \frac{\sqrt{\frac{1 - \frac{1}{p}}{1 + \frac{1}{p}}}}{2p - \frac{3}{2p}} \left[\left(1 + \frac{1}{p} \right) (2p - 1) - 1 - \frac{1}{2p} \right] = \sqrt{\frac{1 - \frac{1}{p}}{1 + \frac{1}{p}}} = \tan \frac{\arccos \frac{1}{p}}{2} \end{aligned}$$

$$x = \tan \frac{1}{2} \arctan \sqrt{p^2 - 1}$$

Finally $J = \pi - 2(\rho - \sqrt{\rho^2 - 1}) - 2 \arctan \sqrt{\rho^2 - 1}$

and the average value for the view factor for two adjacent bodies is

$$\bar{F}_r = \frac{J}{2\pi} = \frac{1}{2} - \frac{1}{\pi} (\rho - \sqrt{\rho^2 - 1} + \arctan \sqrt{\rho^2 - 1})$$

which is valid for $\rho \geq \sec \frac{\pi}{6}$

When reference is made to the average view factor for every one of the six streaming beams, then

$$\bar{F}_s = \frac{1 - 6\bar{F}_r}{6} = \frac{1}{6} - \bar{F}_r = \frac{1}{\pi} (\rho - \sqrt{\rho^2 - 1} + \arctan \sqrt{\rho^2 - 1}) - \frac{1}{3}$$

for the same range of validity, i.e., $\rho \geq \sec \frac{\pi}{6}$

4.B. Case $\rho < \sec \frac{\pi}{6}$

In order to find the expressions for $\sin \alpha_1$ and $\sin \alpha_2$ for the first steps in solving the problem, reference is now made to the view factor for the direct radiation escaping through the gap between two adjacent bodies, as indicated by the sketch.

Projection along the normal to the tangents will render the required relationships for the tangent angles, so

For $P O_1 T_1$

$$\frac{D}{2} (1 - \sin \alpha_1) = C \sin \left(\frac{\pi}{6} + \beta - \alpha_1 \right)$$

or

$$1 - \sin \alpha_1 = 2\rho \sin \left(\frac{\pi}{6} + \beta - \alpha_1 \right)$$

For $P O_2 T_2$

$$\frac{D}{2} (1 + \sin \alpha_2) = C \sin \left(\frac{\pi}{6} + \alpha_2 - \beta \right)$$

or

$$1 + \sin \alpha_2 = 2\rho \sin \left(\frac{\pi}{6} + \alpha_2 - \beta \right)$$

Manipulating the two equations, both $\sin \alpha_1$ and $\sin \alpha_2$ can be cleared out, obtaining

$$\sin \alpha_1 = \frac{\frac{1}{4} - \frac{p}{2} \cos\left(\frac{\pi}{6} + \beta\right) + p \sin\left(\frac{\pi}{6} + \beta\right) \sqrt{p^2 - p \cos\left(\frac{\pi}{6} + \beta\right)}}{p^2 - p \cos\left(\frac{\pi}{6} + \beta\right) + \frac{1}{4}}$$

and

$$\sin \alpha_2 = \frac{-\frac{1}{4} + \frac{p}{2} \cos\left(\frac{\pi}{6} - \beta\right) - p \sin\left(\frac{\pi}{6} - \beta\right) \sqrt{p^2 - p \cos\left(\frac{\pi}{6} - \beta\right)}}{p^2 - p \cos\left(\frac{\pi}{6} - \beta\right) + \frac{1}{4}}$$

The limiting angle for the integration can be easily obtained by trigonometry. The range of integration for β extends from $-\beta_L$ to β_L (β_L is the angle when $\alpha_1 = \alpha_2 = \alpha_L$).

The limiting angle comes from

$$1 - \sin \alpha_L = 2p \sin\left(\frac{\pi}{6} + \beta_L - \alpha_L\right)$$

$$1 + \sin \alpha_L = 2p \sin\left(\frac{\pi}{6} + \alpha_L - \beta_L\right)$$

$$2 = 2p \cdot 2 \sin \frac{\pi}{6} \cdot \cos(\alpha_L - \beta_L)$$

$$\frac{1}{p} = \cos(\alpha_L - \beta_L)$$

$$\beta_L = \alpha_L - \arccos \frac{1}{p}$$

$$\frac{\sin \alpha_L}{\frac{\sqrt{3}}{2} c} = \frac{\sqrt{1 - \frac{1}{p^2}}}{\frac{D}{2}}$$

$$\sin \alpha_L = \sqrt{3} \sqrt{p^2 - 1}$$

$$\tan \alpha_L = \frac{\sqrt{3p^2 - 3}}{\sqrt{4 - 3p^2}}$$

$$\therefore \beta_L = \arctan \sqrt{\frac{p^2 - 1}{\frac{4}{3} - p^2}} - \arctan \sqrt{p^2 - 1}$$

In a different way

$$\frac{\sin \beta_L}{\sqrt{p^2 - 1} \left\{ \sqrt{\frac{3}{4 - 3p^2}} - 1 \right\}} = \frac{1/p}{\sqrt{3} \frac{p^2 - 1}{4 - 3p^2} + 1}$$

$$\sin \beta_L = \sqrt{1 - \frac{1}{p^2}} (\sqrt{3} - \sqrt{4 - 3p^2})$$

$$\beta_L = \arcsin \sqrt{1 - \frac{1}{p^2}} (\sqrt{3} - \sqrt{4 - 3p^2})$$

When $\rho = 1$ $\beta_L = 0$

When $\rho = \frac{2}{\sqrt{3}}$ i.e., when $\alpha_L = \frac{\pi}{2}$

$$\beta_L = \arcsin \sqrt{1 - \frac{3}{4}} \cdot \sqrt{3} = \arcsin \frac{\sqrt{3}}{2} \quad \therefore \beta_L = \pi/3$$

$\sin \alpha_1$ and $\sin \alpha_2$ can be written as

$$\sin \alpha_1 = \frac{1}{2} - \frac{1}{2} \frac{\rho^2 - \frac{1}{4}}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} + \beta)} + \frac{\rho \sin(\frac{\pi}{6} + \beta) \sqrt{\rho^2 - \rho \cos(\frac{\pi}{6} + \beta)}}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} + \beta)}$$

$$\sin \alpha_2 = -\frac{1}{2} + \frac{1}{2} \frac{\rho^2 - \frac{1}{4}}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} - \beta)} - \frac{\rho \sin(\frac{\pi}{6} - \beta) \sqrt{\rho^2 - \rho \cos(\frac{\pi}{6} - \beta)}}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} - \beta)}$$

B.a. $\int_0^{\beta_L} \sin \alpha_2 \cdot d\beta$

Integrating the terms separately

$$\int_0^{\beta_L} -\frac{1}{2} d\beta = -\frac{\beta_L}{2}$$

$$\frac{\rho^2 - \frac{1}{4}}{2} \int_0^{\beta_L} \frac{d\beta}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} - \beta)} = -\frac{\rho^2 - \frac{1}{4}}{2} \int_0^{\beta_L} \frac{d(\frac{\pi}{6} - \beta)}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} - \beta)}$$

$$= -\arctan \frac{\rho + \frac{1}{2} \tan \frac{\pi}{6} - \beta}{\rho - \frac{1}{2}} \Bigg|_0^{\beta_L}$$

$$= \arctan \frac{\rho + \frac{1}{2} \tan \frac{\pi}{6}}{\rho - \frac{1}{2}} - \arctan \frac{\rho + \frac{1}{2} \tan \frac{\pi}{6} - \beta}{\rho - \frac{1}{2}}$$

$$\begin{aligned} -\rho \int_0^{\beta_L} \frac{\sin(\frac{\pi}{6} - \beta) \sqrt{\rho^2 - \rho \cos(\frac{\pi}{6} - \beta)}}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} - \beta)} d\beta &= -\rho \int_{\cos \frac{\pi}{6}}^{\cos(\frac{\pi}{6} - \beta_L)} \frac{\sqrt{\rho^2 - \rho \cos(\frac{\pi}{6} - \beta)}}{\rho^2 + \frac{1}{4} - \rho \cos(\frac{\pi}{6} - \beta)} d \cos(\frac{\pi}{6} - \beta) \\ &= -\rho \int_{\cos \frac{\pi}{6}}^{\cos(\frac{\pi}{6} - \beta_L)} \frac{\sqrt{\rho^2 - \rho y}}{\rho^2 + \frac{1}{4} - \rho y} dy \end{aligned}$$

Where

$$d \cos \left(\frac{\pi}{6} - \beta \right) = -\sin \left(\frac{\pi}{6} - \beta \right) d \left(\frac{\pi}{6} - \beta \right) = \sin \left(\frac{\pi}{6} - \beta \right) d\beta$$

$$y = \cos \left(\frac{\pi}{6} - \beta \right)$$

$$\beta = 0$$

$$y = \cos \frac{\pi}{6}$$

$$\beta = \beta_L$$

$$y = \cos \left(\frac{\pi}{6} - \beta_L \right)$$

$$1 + \frac{1}{4p^2} - \frac{y}{p} = x$$

$$\beta = 0$$

$$x_0 = 1 + \frac{1}{4p^2} - \frac{\cos \frac{\pi}{6}}{p}$$

$$\beta = \beta_L$$

$$x_L = 1 + \frac{1}{4p^2} - \frac{\cos \left(\frac{\pi}{6} - \beta_L \right)}{p}$$

$$dy = -p dx$$

$$\begin{aligned} -p \int_0^{\beta_L} \frac{\sin \left(\frac{\pi}{6} - \beta \right) \sqrt{p^2 - p \cos \left(\frac{\pi}{6} - \beta \right)}}{p^2 + \frac{1}{4} - p \cos \left(\frac{\pi}{6} - \beta \right)} d\beta &= \int_{x_0}^{x_L} \frac{\frac{p}{x} \sqrt{x - \frac{1}{4p^2}}}{x} dx \\ &= p \left[2\sqrt{x - \frac{1}{4p^2}} - \frac{1}{p} \arctan 2p\sqrt{x - \frac{1}{4p^2}} \right]_{x_0}^{x_L} \\ &= 2p \left[\sqrt{1 - \frac{\cos \left(\frac{\pi}{6} - \beta_L \right)}{p}} - \sqrt{1 - \frac{\cos \frac{\pi}{6}}{p}} \right] \\ &\quad - \arctan 2p\sqrt{1 - \frac{\cos \left(\frac{\pi}{6} - \beta_L \right)}{p}} + \arctan 2p\sqrt{1 - \frac{\cos \frac{\pi}{6}}{p}} \end{aligned}$$

Thus

$$\int_0^{\beta_L} \sin \alpha_2 d\beta = -\frac{\beta_L}{2} + \arctan \frac{p + \frac{1}{2}}{p - \frac{1}{2}} \tan \frac{\pi}{12} - \arctan \frac{p + \frac{1}{2}}{p - \frac{1}{2}} \tan \frac{\pi}{6} - \frac{\beta_L}{2}$$

$$-2p \left[\sqrt{1 - \frac{\cos \frac{\pi}{6}}{p}} - \sqrt{1 - \frac{\cos \left(\frac{\pi}{6} - \beta_L \right)}{p}} \right] + \arctan 2p\sqrt{1 - \frac{\cos \frac{\pi}{6}}{p}} - \arctan 2p\sqrt{1 - \frac{\cos \left(\frac{\pi}{6} - \beta_L \right)}{p}}$$

B.b. $\int_0^{\beta_L} \sin \alpha_1 d\beta$

Proceeding similarly as before

$$\int_0^{\beta_L} \frac{1}{2} d\beta = \frac{\beta_L}{2}$$

$$-\frac{p^2 + \frac{1}{4}}{2} \int_0^{\beta_L} \frac{d\beta}{p^2 + \frac{1}{4} - p \cos(\frac{\pi}{6} + \beta)} = -\frac{p^2 + \frac{1}{4}}{2} \int_0^{\beta_L} \frac{d(\frac{\pi}{6} + \beta)}{p^2 + \frac{1}{4} - p \cos(\frac{\pi}{6} + \beta)} = -\arctan \frac{p + \frac{1}{2}}{p - \frac{1}{2}} \tan \frac{\pi}{6} + \beta \Big|_0^{\beta_L}$$

$$= \arctan \frac{p + \frac{1}{2}}{p - \frac{1}{2}} \tan \frac{\pi}{12} - \arctan \frac{p + \frac{1}{2}}{p - \frac{1}{2}} \tan \frac{\pi}{6} + \frac{\beta_L}{2}$$

$$p \int_0^{\beta_L} \frac{\sin(\frac{\pi}{6} + \beta) \sqrt{p^2 - p \cos(\frac{\pi}{6} - \beta)}}{p^2 + \frac{1}{4} - p \cos(\frac{\pi}{6} + \beta)} d\beta = -p \int_{\cos \frac{\pi}{6}}^{\cos(\frac{\pi}{6} + \beta_L)} \frac{\sqrt{p^2 - p y}}{p^2 + \frac{1}{4} - p y} dy$$

Where

$$d \cos(\frac{\pi}{6} + \beta) = -\sin(\frac{\pi}{6} + \beta) d\beta$$

$$y = \cos(\frac{\pi}{6} + \beta)$$

$$p \int_0^{\beta_L} \frac{\sin(\frac{\pi}{6} + \beta) \sqrt{p^2 - p \cos(\frac{\pi}{6} - \beta)}}{p^2 + \frac{1}{4} - p \cos(\frac{\pi}{6} + \beta)} d\beta = p \int_{x_0}^{x_L} \frac{\sqrt{x - \frac{1}{4p^2}}}{x} dx$$

$$1 + \frac{1}{4p^2} - \frac{y}{p} = x$$

$$dy = -p dx$$

$$\beta = 0 \quad x_0 = 1 + \frac{1}{4p^2} - \frac{\cos \frac{\pi}{6}}{p}$$

$$\beta = \beta_L \quad x_L = 1 + \frac{1}{4p^2} - \frac{\cos(\frac{\pi}{6} + \beta_L)}{p}$$

$$p \int_0^{\beta_L} \frac{\sin(\frac{\pi}{6} + \beta) \sqrt{p^2 - p \cos(\frac{\pi}{6} - \beta)}}{p^2 + \frac{1}{4} - p \cos(\frac{\pi}{6} + \beta)} d\beta = p \left[2\sqrt{x - \frac{1}{4p^2}} - \frac{1}{p} \arctan 2p\sqrt{x - \frac{1}{4p^2}} \right]_{x_0}^{x_L}$$

$$= 2p \left[\sqrt{1 - \frac{\cos(\frac{\pi}{6} + \beta_L)}{p}} - \sqrt{1 - \frac{\cos \frac{\pi}{6}}{p}} \right] - \arctan 2p\sqrt{1 - \frac{\cos(\frac{\pi}{6} + \beta_L)}{p}} + \arctan 2p\sqrt{1 - \frac{\cos \frac{\pi}{6}}{p}}$$

Thus

$$\int_0^{\beta_L} \sin \alpha_1 d\beta = \frac{\beta_L}{2} + \arctan \frac{\rho + \frac{1}{2}}{\rho - \frac{1}{2}} \tan \frac{\pi}{12} - \arctan \frac{\rho + \frac{1}{2}}{\rho - \frac{1}{2}} \tan \frac{\pi + \beta_L}{2} \\ + 2\rho \left[\sqrt{1 - \frac{\cos(\frac{\pi}{6} + \beta_L)}{\rho}} - \sqrt{1 - \frac{\cos \frac{\pi}{6}}{\rho}} \right] + \arctan 2\rho \sqrt{1 - \frac{\cos \frac{\pi}{6}}{\rho}} - \arctan 2\rho \sqrt{1 - \frac{\cos(\frac{\pi}{6} + \beta_L)}{\rho}}$$

Replacing in the integral

$$\mathcal{J} = \int_0^{\beta_L} (\sin \alpha_1 - \sin \alpha_2) d\beta = \beta_L - \arctan \frac{\rho + \frac{1}{2}}{\rho - \frac{1}{2}} \tan \frac{\pi + \beta_L}{2} + \arctan \frac{\rho + \frac{1}{2}}{\rho - \frac{1}{2}} \tan \frac{\pi - \beta_L}{2} \\ + 2\rho \left[\sqrt{1 - \frac{\cos(\frac{\pi}{6} + \beta_L)}{\rho}} - \sqrt{1 - \frac{\cos(\frac{\pi}{6} - \beta_L)}{\rho}} \right] \\ - \arctan 2\rho \sqrt{1 - \frac{\cos(\frac{\pi}{6} + \beta_L)}{\rho}} + \arctan 2\rho \sqrt{1 - \frac{\cos(\frac{\pi}{6} - \beta_L)}{\rho}}$$

Similarly as before $\bar{F}_s = \frac{\mathcal{J}}{2\pi}$

and $\bar{F}_r = \frac{1}{6} - \bar{F}_s$

for the range $1 \leq \rho < \sec \frac{\pi}{6}$
of validity

Example 3. For the EGCR the view factor for two adjacent rods ($\rho = 4/3$) is

$$\bar{F}_r = \frac{1}{2} - \frac{1}{\pi} \left(\frac{4}{3} - \frac{\sqrt{7}}{3} + \arctan \frac{\sqrt{7}}{3} \right) = \frac{1}{2} - \frac{1}{\pi} (0.451 + 0.722)$$

$$\bar{F}_r = \frac{1}{2} - 0.374 = 0.126,$$

and the average view factor for the radiation from the central rod streaming between two adjacent rods

$$\bar{F}_s = \frac{1}{6} - \bar{F}_r = 0.040$$

Example 4. The limiting angle β_L for Peach Bottom HTGR configuration ($\rho = 1.02$)

$$\beta_L = \arcsin \sqrt{1 - \frac{1}{\rho^2}} (\sqrt{3} - \sqrt{4 - 3\rho^2}) = \arcsin \frac{0.201}{1.02} \cdot 0.795 = \arcsin 0.157$$

$$\beta_L \approx 0.157 \approx 9^\circ$$

$$2\beta_L \approx 18^\circ$$

So $6 \times 18^\circ = 108^\circ$ or $108/360 = 0.3$ of the rod surface can produce streaming radiation.

5. The View Factors

Figure 1 shows both view factors $\bar{F}_r$ and $\bar{F}_s$ as function of the dimensionless ratio $\rho = C/D$.

The EGCR reference is to the central rod of any typical seven-rod cluster. The Peach Bottom HTGR reference is to any rod of the core configuration, except for those along the core periphery or those surrounded by different diameter poison rod(s).

Table 1 gives information relevant to that fraction of the radiant area which contributes to streaming radiation. As it can be inferred, those fractions are greater than the respective view factor at the same ρ .

Table 1

$\rho = C/D$	Fraction of Total Area Contributing to Streaming	$6\bar{F}_s$
1.0000	0.0000	0.0000
1.0200*	0.3003	0.0050
1.0400	0.4571	0.0142
1.0600	0.6047	0.0258
1.0800	0.7588	0.0394
1.1000	0.9305	0.0545
1.1200	1.0000	0.0711
1.3333**	1.0000	0.2424

* Peach Bottom HTGR

** EGCR

Example 5. View factors for the central rod of the seven-rod cluster in EGCR

Calling A_0 = A area of central rod ($3/4$ " diameter)

A_1 = 6A area of the six outer rods (also $3/4$ " diameter, 1" pitch)

A_2 = inner area of sleeve (3" diameter)

VIEW FACTORS FOR GAS-COOLED REACTOR CORE CONFIGURATION

FIGURE 1

Using numerical values already obtained (Figure 1) and according to Figure 2

$$F_{00} = 0$$

$$F_{01} = 6 \bar{F}_r = 0.758$$

$$F_{02} = 6 \bar{F}_s = 0.242$$

$$F_{00} + F_{01} + F_{02} = 1.000$$

In order to complete the whole set of view factors, that is, including the view factors for the outer rods and inner sleeve, an estimate of the collimated beams contributing to F_{22} passing between outer and central rods can be done.

Example 6. View factors for Peach Bottom HTGR configuration

Calling $A_0 = A$ area of one typical rod

$A_1 = 6A$ area of the six surrounding adjacent rods

$A_2 = 6A$ area of the farthest six rods receiving direct radiation from A_0 , etc., according to Figure 3

Employing numerical values already obtained for such configuration:

a. Area A_0 (one rod)

$$F_{00} = 0$$

$$F_{01} = 6 \bar{F}_r = 0.995$$

$$F_{02} = 6 \bar{F}_s = 0.005$$

$$F_{00} + F_{01} + F_{02} = 1.000$$

b. Area A_1 (six rods)

$$F_{10} = \bar{F}_r = 0.166$$

$$F_{11} = 2 (\bar{F}_r + \bar{F}_s) = 0.333$$

$$F_{12} = 2 \bar{F}_r = 0.332$$

$$F_{13} = \bar{F}_r + 2 \bar{F}_s = 0.167$$

$$F_{14} = 2 \bar{F}_s = 0.0017$$

$$F_{10} + F_{11} + F_{12} + F_{13} + F_{14} = 1.000$$

Rod Diameter: $3/4"$

Center-to-center
Distance: $1"$

Sleeve Diameter: $3"$

Scale 1:1

EGCR TYPICAL CLUSTER

FIGURE 2

Diameter: 3.50 in.

Center-to-center
Distance: 3.57 in.

PEACH BOTTOM HTGR CONFIGURATION AROUND A TYPICAL ROD
(Lower Part Omitted by Symmetry)

FIGURE 3

c. Area A_2 (six rods)

$$F_{20} = \bar{F}_s = 0.00084$$

$$F_{21} = 2 \bar{F}_r = 0.3317$$

$$F_{22} = 2 \bar{F}_s = 0.0017$$

$$F_{23} = 2 \bar{F}_r = 0.3317$$

$$F_{24} = 2 \bar{F}_r = 0.3317$$

$$F_{25} = 2 \bar{F}_s = 0.0017$$

$$F_{26} = \bar{F}_s = 0.00084$$

$$F_{20} + F_{21} + F_{22} + F_{23} + F_{24} + F_{25} + F_{26} = 1.000$$

etc.

Consequently, no serious error is incurred if, by simplification, different concentric hexagonal layers of rods around A_0 are considered, and such that

Area A_0 : area of one typical rod (same as before)

Area $A_1 = 6A$: area of six surrounding adjacent rods (same as before)

Area $A_2 = 12A$: area of twelve rods surrounding A_1 (previous rods No. 2 and Rods No. 3)

etc.

So a. Area A_0 (one rod)

$$F_{00} = 0$$

$$F_{01} = 1$$

$$F_{02} = F_{03} = \dots = 0$$

b. Area A_1 (six rods)

$$F_{10} = 1/6$$

$$F_{11} = 1/3$$

$$F_{12} = 1/2$$

$$F_{13} = F_{14} = \dots = 0$$

c. Area A₂ (twelve rods)

$$F_{20} = 0$$

$$F_{21} = 1/4$$

$$F_{22} = 1/3$$

$$F_{23} = 5/12$$

$$F_{24} = F_{25} = \dots = 0$$

d. Area A₃ (eighteen rods)

$$F_{30} = F_{31} = 0$$

$$F_{32} = 5/18$$

$$F_{33} = 1/3$$

$$F_{34} = 7/18$$

$$F_{35} = F_{36} = \dots = 0$$

etc.

6. References

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 Peach Bottom HTGR Preliminary Hazards Summary Report, August 1961.
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