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ANALOG COMPUTER ANALYSIS
OF
MAGNETIC TAPE SEISMOGRAMS

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January 20, 1965

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OF
MAGNETIC TAPE SEISMOGRAMS

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SUMMARY

Analog computer programs for analysis of magnetic tape seismograms recorded by a National NC-21 transducer-L-5 amplifier seismometer system are described. These programs extend the analyzable frequency range by compensating for non-linear transducer response, derive displacement and acceleration from recorded velocity, determine two and three component vector amplitudes, provide amplitude-frequency information by band-pass filtering, and aid in the identification of wave modes.

INTRODUCTION

Recording of seismic motion from underground nuclear explosions in the form of magnetic tape velocity seismograms permits direct analysis with analog computer equipment. Several operations can be performed simultaneously, in real time, on three components of ground motion to derive important information from the primary data. This report describes these operations and their development.

The analog programs derive displacement and acceleration from recorded velocity and provide visual analogs of variation of components and vector amplitudes with time. Frequency information is provided by the band-pass filter technique. Individual seismic arrivals are identified from phase relationships among the components.

The seismometer used is the National NC-21 transducer having a natural period of one second. The seismometer output is a low level voltage which is fed to a National L-5 amplifier that gives four high level outputs. These are recorded with a Honeywell 14-channel tape recorder. Currently three transducers and amplifiers are used to

measure radial, transverse and vertical components of ground motion. In the frequency range 0.9 to 55 cycles per second this system has linear response (within 3 db) to ground velocity.

An important consideration in the analysis of seismograms is the linear response of the seismometer with respect to the frequency spectrum of ground motion. There is direct evidence from seismograms of the NC-21 and seismograms from both long and short period seismometers that ground motion from underground nuclear explosions has considerable energy below 1.0 cycle per second but negligible energy above 30 cycles per second at the distances and yields which have been used. Therefore the NC-21 transducer-L-5 amplifier system is deficient only in low frequency response. An analog computer program is developed that compensates for response of the NC-21 transducer to allow analysis below 1.0 cycles per second.

A PACE TR-48 (Electronic Associates, Inc.) analog computer with 48 operational amplifiers, 12 integrators, 7 multipliers and other non-linear devices is used. Auxilliary equipment includes 7 and 14-channel Sanborn tape recorders and a Brush 8-channel strip chart recorder.

I. CORRECTION FOR INSTRUMENT RESPONSE

The analysis required for instrument response compensation involves several steps. The response of the total system is examined to determine the extent to which compensation can reasonably be applied. An analysis of the NC-21 transducer is made to determine the equations governing its motion. From the resulting equations the analog compensation circuit is developed. The response of the L-5 amplifier is examined to evaluate its effect. The final response after compensation is then determined.

a. Frequency Response of the System

The frequency response of the NC-21 transducer-L-5 amplifier seismometer system to constant amplitude ground velocity is linear between 1.6 and 30.0 cycles per second (see Figure 1). At 0.9 and 55.0 cycles per second system response is 3 db down. Response between 0.4 and 1.6 cycles per second is essentially due to the NC-21 transducer. The transducer and amplifier in combination affect the response below 0.4 cycles per second. The L-5 amplifier determines response above 30.0 cycles per second.

Figure 1. Response Curve of the NC-21
Transducer-L5 Amplifier System

Since high frequency response is adequate, only the response below 1.0 cycle per second need be compensated. Although both the NC-21 transducer and L-5 amplifier determine low frequency response, it is only possible to compensate for the NC-21 transducer. The 40 db signal-to-noise ratio of the magnetic tape recorder is the limiting factor. Seismic signals in the frequency range for which compensation of the L-5 amplifier is required are attenuated by the seismometer system to an amplitude approaching the noise level of the tape recorder.

b. Analysis of the NC-21 Transducer

Figure 2 shows the essential elements of the NC-21 transducer. The coil is fixed with respect to the frame which, in turn, is directly coupled to the earth. A displacement of the magnet relative to the coil induces a voltage E in the electrical circuit proportional to the relative velocity between coil and magnet.

Figure 2.

Summation of the voltages in the circuit gives

$$E = G \frac{du}{dt} = L \frac{di}{dt} + (R_w + R_d + R_a)i \quad (1)$$

where

G = Generator constant

$\frac{du}{dt}$ = Velocity of magnet

L = Inductance of coil

i = Current in circuit

R_w = Resistance of coil winding

R_d = Resistor to adjust damping

R_a = Input impedance of L-5 amplifier

Current flowing in the coil produces a magnetic field which exerts a force, F , on the magnet.

$$F = Gi \quad (2)$$

Summation of forces acting on the magnet yields

$$M \frac{d^2u}{dt^2} + ku = - M \frac{d^2x}{dt^2} - Gi \quad (3)$$

where:

M = Mass of magnet

k = Spring constant

$\frac{d^2x}{dt^2}$ = Ground acceleration

Equations (1) and (3) are more conveniently expressed using the following substitutions:

$$R_a i = e_a \equiv \text{Voltage at input to L-5 amplifier}$$

$$R_w + R_d + R_a = R_s \equiv \text{Total resistance}$$

$$\frac{k}{M} = w_s^2 \equiv \text{Angular frequency squared.}$$

Then

$$G \frac{du}{dt} = \frac{L}{R_a} \frac{de_a}{dt} + \frac{R_s}{R_a} e_a \quad (4)$$

$$\frac{d^2 u}{dt^2} + w_s^2 u = - \frac{d^2 x}{dt^2} - \frac{G}{MR_a} e_a \quad (5)$$

Equations (4) and (5) describe the relationship between ground motion and voltage input to the L-5 amplifier. Analysis of these equations is facilitated by Laplace transform theory. Also, analog computer circuits can be directly programmed from the transformed equations.

Let the Laplace transforms of x , u , and e_a be, respectively, $\bar{x}(P)$, $\bar{u}(P)$ and $\bar{e}_a(P)$. Initial conditions are assumed to be zero. Then equations (4) and (5) transform to

$$GP \bar{u}(P) = \left[\frac{L}{R_a} P + \frac{R_s}{R_a} \right] \bar{e}_a(P) \quad (6)$$

$$(P^2 + w_s^2) \bar{u}(P) = -P^2 \bar{x} - \frac{G}{MR_a} \bar{e}_a(P). \quad (7)$$

The relationship between the Laplace transforms of ground velocity, $P\bar{x}(P)$, and output voltage, $\bar{e}_a(P)$, is obtained by eliminating $\bar{u}(P)$.

Thus

$$\bar{e}_a(P) = -F(P) [P\bar{x}(P)] \quad (8)$$

where

$$F(P) = \frac{P^2}{\frac{R_s}{GR_a} \left\{ P^2 \left(1 + \frac{LP}{R_s} \right) + \frac{G^2}{MR_s} P + w_s^2 \left(1 + \frac{LP}{R_s} \right) \right\}} \quad (9)$$

$F(P)$ is the instrument transfer function relating output to input.

Equation (9) can be simplified to obtain a transfer function which has equivalent characteristics in the frequency range of interest. With the value of inductance ($L = 1.01$ henry) and total resistance ($R_s = 915.0$ ohms) used by the manufacturer, the term $\frac{LP}{R_s}$ represents a high frequency roll-off in response above 130 cycles per second and can

thus be omitted. Eliminating $\frac{LP}{R_s}$ from equation (9) gives

$$F_o(P) = \frac{P^2}{\frac{R_s}{GR_a} \left\{ P^2 + \frac{G^2}{MR_s} P + w_s^2 \right\}} \quad (10)$$

Equation (8) then becomes

$$\bar{e}_a(P) = - F_o(P) P\bar{x}(P) \quad (11)$$

To verify that equation (11) is equivalent to equation (8) in the frequency range 0.1 to 60.0 cycles per second, both equations were programmed on the analog computer for simulation. Figure 3 gives the computer diagram for the analog of the actual NC-21 transducer and Figure 4 gives the computer diagram for the analog of the equivalent NC-21 transducer. Under both transient and steady state conditions, the two circuits proved to have identical response over the indicated frequency range. Figure 5 shows the frequency response curve of the analog NC-21 transducer.

Equations (10) and (11) will be used to represent the NC-21 transducer.

Figure 3. Computer Diagram for the Analog NC-21 Transducer

Figure 4. Computer Diagram for the Analog Equivalent NC-21 Transducer

Figure 5. Response Curve of the Analog NC-21 Transducer

c. Development of the Transducer Compensation Circuit

If equation (11) is solved for $P\bar{x}(P)$, then

$$P\bar{x}(P) = - \left[\frac{1}{F_o(P)} \right] \bar{e}_a(P) \quad (12)$$

where

$$\frac{1}{F_o(P)} = \frac{R_s}{GR_a} \left\{ P^2 + \frac{G^2}{MR_s} P + w_s^2 \right\} / P^2 \quad (13)$$

Equation (12) states that an output, $P\bar{x}(P)$ (Laplace transform of ground velocity), is related to an input, $\bar{e}_a(P)$, by the transfer function $1/F_o(P)$. The transfer functions $F_o(P)$ and $1/F_o(P)$ are mathematical operators: $F_o(P)$ operates on $P\bar{x}(P)$ to give $\bar{e}_a(P)$ as output; $1/F_o(P)$ operates on $\bar{e}_a(P)$ to give $P\bar{x}(P)$ as output. Using an analog computer circuit with a transfer function $1/F_o(P)$ and applying a voltage $\bar{e}_a(P)$ to its input, an output $P\bar{x}(P)$ is obtained.

Transformation of equations (12) and (13) back to the time domain yields:

$$v(t) = \frac{R_s}{GR_a} \left[e_a(t) + \frac{G^2}{MR_s} \int_0^t e_a(t) dt + w_s^2 \int_0^t dt \int_0^t e_a(t) dt \right] \quad (14)$$

$\equiv$ Compensated velocity

Figure 6 shows the computer diagram that performs the operations on $e_a(t)$ indicated in equation (14). The circuit is slightly modified by providing a small amount of feedback around the integrators to give D.C. stability (unmodified it is an unstable equilibrium circuit). The frequency response curve for this compensation circuit is shown in Figure 7.

The ability of the compensation circuit to correct for the response characteristics of the NC-21 transducer is shown in Figure 8. This curve was obtained by simulating the transducer on the computer and then correcting its output with the compensation circuit. The roll-off at 0.2 cycles per second and the slight deviation from 1.0 in relative sensitivity is caused by stability feedback in the compensation circuit.

d. The L-5 Amplifier Response

As the voltage output from the NC-21 transducer is normally in the millivolt range, a National type L-5 amplifier is used to provide a voltage level compatible with the requirements of the magnetic tape recorder. The frequency response of this amplifier is shown in Figure 9. The

Figure 6. Computer Diagram for the NC-21 Transducer Compensation Circuit

Figure 7. Response Curve of the NC-21
Transducer Compensation Circuit

Figure 8. Response curve of
Compensated NC-21 Transducer

Figure 9. Response Curve of
the L-5 Amplifier

low frequency roll-off was chosen by the designer to be below that of the NC-21 transducer and thus outside the usable frequency range of the system. However, when the compensation circuit is used, the L-5 amplifier ultimately determines the low frequency response.

e. System Response After Compensation.

The final system response is obtained by modifying the response of the compensated NC-21 transducer (Figure 8) by the response of the L-5 amplifier (Figure 9). Figure 10 gives the final result. Response is 3 db down at 0.25 and 55.0 cycles per second.

An example of two seismic signals before and after correction for transducer response is shown in Figure 11.

Figure 10. Final Response Curve
After Compensation

Figure 11. Seismic Signals Before and After Compensation

II. DISPLACEMENT AND ACCELERATION

Displacement and acceleration are obtained by integrating and differentiating velocity. Figure 12 gives the computer diagram. An example of displacement and acceleration derived from velocity is shown in Figure 13.

Tape noise proves to be a problem for both displacement and acceleration. Low frequency noise is reduced by a high-pass filter with a corner frequency of 0.1 cycles per second. At this corner frequency the filter has negligible effect on frequencies above 0.25 cycles per second — the lower limit of analysis. High frequency noise (60 cycles) is reduced by a low-pass filter with a corner frequency chosen to have negligible effect on the highest frequency in the seismic signal. Noise cannot be completely eliminated, but its effects can be reduced to acceptable levels.

The circuit used for either high-pass or low-pass filtering is given in Figure 14.

Figure 12. Computer Diagram for Acceleration and Displacement

Figure 13. Acceleration and Displacement Derived from Compensated Velocity

Figure 14. Computer Diagram for a High-Pass or Low-Pass Filter

III. VECTOR AMPLITUDE

Vector amplitude is calculated to determine peak ground motion from the recorded components. The total vector amplitude of velocity is given by

$$V_{RTZ} = + \sqrt{v_R^2 + v_T^2 + v_Z^2}$$

where the subscripts refer to the radial, transverse and vertical directions. Note that the plus value of the square root is given. Vector amplitude in the planes defined by the components is given by:

$$V_{RT} = + \sqrt{v_R^2 + v_T^2}$$

$$V_{ZT} = + \sqrt{v_Z^2 + v_T^2}$$

$$V_{RZ} = + \sqrt{v_R^2 + v_Z^2}$$

Analogous formulas hold for acceleration and displacement.

An example of vector amplitude from radial, transverse and vertical components of velocity is shown in Figure 15. Peak amplitudes must be visually determined from the records.

Figure 15. Vector Amplitude from Three Components

The computer operations required for vector amplitude are square, sum and square root. Figure 16 gives the circuit for two component vector amplitude. The extension to three components is straightforward.

Figure 16. Computer Diagram for Vector Amplitude of Two Components

IV. BAND-PASS FILTERING

The Laplace transform of a band-pass filter that has been developed on the analog computer is given by

$$\frac{\text{voltage output}}{\text{voltage input}} = \left(\frac{w_c P}{P^2 + 2hw_c P + w_c^2} \right)^2$$

where

w_c = center angular frequency

h = damping constant. (set at 0.5)

Figure 17 shows the computer diagram for this filter.

To improve the band-pass characteristics of the basic filter, a signal is filtered and the output recorded on magnetic tape. It is then filtered a second time. The response curve using this procedure for a band-pass filter with a center frequency of 1.0 cycle per second is shown in Figure 18.

In practice, the basic filter is set to a center frequency, the signal filtered twice, and the output recorded. The filter is adjusted to a new center frequency and the procedure repeated. This is continued until the frequency range is covered.

Figure 17. Computer Diagram of the Band-Pass Filter

Figure 18. Response Curve for Double Band-Pass Filtering

The center frequencies were chosen such that response is down by a factor of 0.5 at the center frequency of adjacent filters. The center frequencies to cover the range of 0.3 to 11.3 cycles per second are 0.3, 0.41, 0.55, 0.75, 1.00, 1.35, 1.85, 2.50, 3.40, 4.60, 6.20, 8.40 and 11.3 cycles per second. These frequencies are linearly spaced on a log-frequency scale.

Examples of band-pass filtering of a velocity component are shown in Figures 19 and 20.

Figure 19. Double Band-Pass Filtering
of a Seismic Signal

f_c = Center Frequency

Figure 20. Double Band-Pass Filtering of a Seismic Signal

f_c = Center Frequency

V. WAVE MODE IDENTIFICATION

In a paper by Sutton and Pomeroy* a method is developed for identifying compressional (P), shear vertical (SV), and Rayleigh waves by the characteristics of the product of the vertical (Z) and radial (R) components. The ZR product for a P-wave is positive, for an SV-wave it is negative, and for a Rayleigh wave it is alternately positive and negative.

The method employs the relationship between direction of propagation and particle motion for the three wave types. For a P-wave the particle motion is parallel to the direction of propagation, while for an SV-wave the particle motion is perpendicular. A Rayleigh wave is a surface wave with vertical and horizontal components 90° out of phase, giving an elliptical motion.

Let vertical (Z) and radial (R) seismometers be oriented so that the Z-axis is positive upwards, and the R-axis is in the direction of the epicenter and positive away.

* Sutton, G. and Pomeroy, P., "Analog Analysis of Seismograms Recorded On Magnetic Tape," J. Geophys. Res., 68, 2791-2815 (1963).

Then, in general, a P-wave arrival causes motion in the first and third quadrants resulting in components that are in phase, while an SV-wave arrival causes motion in the second and fourth quadrants resulting in components that are 180° out of phase. The components for Rayleigh waves are 90° out of phase. Now, the product of components in phase is positive, the product of components 180° out of phase is negative, and the product of components 90° out of phase is alternately positive and negative. Thus, by observing the sign of the product, the wave type can be identified.

The method is extendable to include products involving the transverse component. Given the epicenter location the T-axis is given an orientation which is positive to the right when facing the epicenter. In addition, it is necessary to specify the general direction from which energy arrives. Direction 1 is defined as right of the RZ plane (when facing the epicenter), direction 2 is defined as left of the RZ plane, and direction 3 is defined as being within the RZ plane. Then each of the three wave types gives a unique combination for the RT, ZT, and RZ product. This is summarized in Figure 21.

Wave Mode	Component Product	Direction		
		1	2	3
P	RT	-	+	0
	ZT	-	+	0
	RZ	+	+	+
SV	RT	-	+	0
	ZT	+	-	0
	RZ	-	-	-
Rayleigh	RT	-	+	0
	ZT	+-	+-	0
	RZ	+-	+-	+-

Figure 21. Wave Identification Table

The method, as described by Pomeroy and Sutton, is modified in that the components are band-pass filtered before the product is formed. Although filtering introduces phase distortion, a particular frequency present in both components is shifted by the same amount, thus preserving the original phase relationship.

Figure 22 gives the computer diagram for forming the product of two components. Analogous circuits are used for the other products.

An example of filtered components and the RT, ZT, and RZ product is shown in Figure 23.

Figure 22. Computer Diagram for Forming the RZ Product for Wave Mode Identification

Figure 23. Wave Identification of
Band-Pass Filtered Seismic Signals
Filter Center Frequency = 3.4 Cycles Per Second