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PARTICLE GROWTH AND SIZE DISTRIBUTION IN FLUIDIZED
BED PROCESSES -- A MATHEMATICAL MODEL
WITH COMPUTER SOLUTIONS

Earl S. Grimmett

**PHILLIPS
PETROLEUM
COMPANY**

ATOMIC ENERGY DIVISION

**NATIONAL REACTOR TESTING STATION
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ABSTRACT

A mathematical model describing particle dynamics in fluidized bed processes is presented. This model includes the important factors of particle growth, particle attrition, elutriation of particles from the surface of the bed, and the addition of seed particles to the bed. Mathematical equations are presented describing both the steady state and unsteady state particle size distribution in the bed. Digital computer solutions of the unsteady state model are presented to demonstrate its applicability to actual fluidized bed processes, and the computer code for solving the unsteady state equation written for an IBM 7040 computer is also presented.

SUMMARY

Fluidized bed processes involve particles which are in intimate contact with an upward flowing stream of fluid -- usually a gas. Depending upon the process, these particles may grow or disintegrate, enter or leave the system, absorb or reject heat, transfer heat from one part of the system to another, provide surface area for catalytically induced reactions, or react with the flowing stream of fluid. Many of these phenomena occur together in a complex manner. This study is confined to those factors which affect changes in the particle population and size distribution in a fluidized bed system.

The formulation of a mathematical model which describes particle size distribution is based on the assumption of a perfectly mixed bed. For a group of particles within the particle size interval L to $(L + \Delta L)$, the number (or mass) of particles within this interval depends upon the rates of particle growth, particle attrition, elutriation, feed deposition, seed addition, and gross product removal. A material balance around the particle size interval resulted in a differential equation describing the particle size distribution in terms of the significant independent variables and parameters.

The complexity of the partial differential equation describing the unsteady state process renders an analytical solution impossible. An analytical solution is possible only for a simplified steady state system. The steady state solution is useful in studying the quantitative effects of feed rate, seed rate, seed size, and bed weight on the particle size distribution in bed.

The partial differential equation describing the unsteady state particle size distribution was solved by a modification of the Runge-Kutta procedure used in the solution of ordinary differential equations. A general computer code employing the Runge-Kutta technique was written for an IBM 7040 computer. The particle size distribution was calculated for the conditions used in several experimental fluidized bed calcination runs; the simulated and actual results compared favorably.

Simulated calciner runs were compared with actual calciner runs made in 3-inch-diameter, 12-inch-diameter, and two-foot-square calciners. Generally, good agreement between the simulated and the actual particle size distribution of the beds was obtained in each case, even though there were striking differences between the sizes and operating conditions for the three calciners. Based on the simulation of the actual calciner runs studied, one can conclude that the mathematical model is applicable to a wide range of operating conditions and sizes of fluidized bed calciners.

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I. INTRODUCTION

Fluidized bed processes involve particles in intimate contact with an upward flowing stream of fluid -- usually a gas. Depending upon the process, these particles may grow or disintegrate, enter or leave the system, take up or reject heat, transfer heat from one part of the system to another, provide surface area for catalytically induced reactions, or react with the flowing stream of fluid. These phenomena occur together in a complex but interrelated way. This study is confined to those factors which affect changes in the particle population and size distribution in a fluidized bed system.

1. FLUIDIZED BED CALCINATION PROCESS

The fluidized bed calcination process for converting aqueous nuclear reactor fuel reprocessing wastes into granular solids embodies many of the phenomena typical of a fluidized bed process[1 through 5]. This process involves spraying the waste solution into a well fluidized, heated bed of granular solids. The water vapor and gaseous decomposition products leave the fluidizing vessel with the fluidizing gas while the solid portion of the feed is either deposited on the surface of the particles in the bed or is spray dried. Solids in the form of particles are continuously withdrawn from the vessel and combined with the fines from the primary cyclone. The product is transported to undergo tanks for permanent storage.

2. NEED FOR THIS STUDY

The fluidized bed calcination process involves the rate processes of particle growth, particle attrition, transfer of heat from heat source to particle, elutriation of particles from the bed, and withdrawal of particles as product from the bed. Each of these phenomena is related to the size distribution of the particles in the bed. A fluidized bed process can be designed and operated in a more efficient manner if the dependency of the particle size distribution on the above rate processes and operating variables is determined.

The need for controlling the size distribution of the particles in a fluidized bed process can be demonstrated by citing several examples. The mass of particles of a given size range elutriated from a fluidized bed depends upon the concentration of these particles in the bed. The coefficient of heat transfer between a heat source and the bed particles is highly dependent upon the particle size distribution of the bed. The particle size distribution of the solid product from the calciner influences the design of the burial vaults and the design of future processing equipment if the solid wastes are to undergo further treatment.

It is desirable to know the particle size distribution in a fluidized bed process as a function of the various rate processes, and to know the response of the fluidized bed to changes in the operating variables.

If the generation rate of new seed particles suddenly decreases, how long will it be before the particle size distribution in the bed significantly affects the

heat transfer rates and fluidity of the bed? How much time is required for corrective action to bring the particle size distribution back to normal? Quantitatively, what should be the magnitude of the corrective action? These questions can best be answered explicitly through development of a workable mathematical model of the process.

The proposed model of the various rate processes taking place in the fluidized bed is based on phenomena which are understood to varying degrees. Some of the rate processes are based, at best, on debatable premises. However, the basic model as developed herein will serve as a useful starting point, and it is hoped that some of its components can be improved by future thought and research.

3. SCOPE OF THE PRESENT STUDY

A study of the factors affecting the particle size distribution is a study of rate processes. After simplifying assumptions were made concerning the system, mathematical expressions were derived for each of the rate processes which affect the particle size distribution. These expressions were then used in the derivation of the general partial differential equation which describes the bed weight distribution as a function of particle size and time.

The differential equation for the case of steady state established a relationship between the process operating parameters and the steady state particle size distribution. An analytical solution of the differential equation permits the effect of operating parameters on the particle size distribution to be investigated.

Methods of obtaining the solution for the particle size distribution in the transient case were studied and programmed on a digital computer. Comparisons of experimental particle size distributions with those predicted by the mathematical model were used to verify the model.

A short section of the report is devoted to a discussion of the practical applications of the digital computer solution of the mathematical model to the design and operation of fluidized bed calcination processes, fluidized bed dryers, and fluidized bed catalyst systems.

II. BASIC RATE EQUATION

The formulation of a mathematical model of the weight distribution of particles of various sizes in a fluidized bed process is based on the assumption that the bed is perfectly mixed. Particle size distribution in a fluidized bed process is governed by the several individual rate processes occurring within the system. A mathematical representation of the system requires a study of the individual rate processes and the process operating variables. The general partial differential equation describing the unsteady state particle size distribution was derived by making a material balance around an arbitrary particle size interval L to $(L + \Delta L)$. The equivalent solids feed rate, the total bed weight, the seed rate, the particle size distribution of the seeds, the fluidizing velocity, the properties of the fluidizing gas, and the particle shape are specified. Rate processes which influence the weight in any given particle size interval are shown in Figure 1.

Fig. 1 Representation of the rate processes which determine size distribution.

Consider the group of particles within the particle size interval from L to $(L + \Delta L)$. The number (or mass) of particles in the size interval ΔL , at any time, is a function of the several rate processes which move mass and particles into and out of the interval. As shown in Figure 1, these rate processes include particle growth, particle attrition, particle elutriation, feed deposition, product removal, and seed introduction. Only particle growth and attrition actually move particles across the boundaries of the interval, whereas particles enter and leave the interval directly by means of the other processes.

1. PARTICLE GROWTH RATE

Investigators^[6, 12, 13] studying the mechanism of particle growth in fluidized bed processes have concluded that particle growth can occur by two mechanisms. Particles can grow by agglomeration, through collision, of particles having a layer of partially dried feed. However, the most prevalent mechanism involves the layerwise deposition of solidified feed on the surface of the particle.

Dunlop et al^[6] and Lee et al^[12] concluded that the mass rate of growth of a particle in a fluidized bed system is proportional to its surface area. The linear particle growth rate in a fluidized bed process was found by Grimmer^[13] to be a function of the equivalent solid feed rate, the solid density, the particle size, and the total surface area of the bed as given by the following equation:

$$g_i = \frac{2F(a + bL_i)}{\rho_s S_T} \quad (1)$$

where

g_i = the linear growth rate in the i th particle size interval

F = the solids feed rate

ρ_s = the solids feed density

S_T = the total surface area of the bed

a, b = experimentally determined constants.

If the motion of the particles through the feed zone is random, the values for the constants, a and b , will be one and zero, respectively. Since the mathematical model was to be as general as possible, Equation (1) with experimentally determined constants a and b was used to calculate the rate of particle growth.

Linear growth rates calculated from Equation (1) are valid provided the total surface area, S_T , of the bed is constant. If S_T is a function of time, Equation (1) requires modification to prevent the calculation of ambiguous growth rates within a given particle size interval. An expression for the instantaneous growth rate, valid for a bed operating at unsteady state, is derived as follows.

The instantaneous mass rate of feed deposition on the particles of size L_i is given by

$$\frac{dM_i}{dt} = \left(\frac{dL_i}{dt} \right) \left(\frac{dm_i}{dL_i} \right) = g_i \left(\frac{dm_i}{dL_i} \right) N_i. \quad (2a)$$

If the particles are assumed to be spherical and the expression for g_i is substituted from Equation (1), the equation for the total instantaneous mass rate of feed deposition within the i th interval becomes

$$\frac{dM_i}{dt} = \frac{F(a + bL_i) S_i}{S_T} \quad (2b)$$

where

m_i = mass of an individual particle in the interval i

S_i = the total surface area of particles in the interval i

N_i = the total number of particles in the interval i .

The total solids mass rate entering the calciner may be specified as equal to the summation of the mass rates being deposited within each particle size interval as a result of particle growth. Mathematically, this restraint may be stated by the equation

$$F = \sum_{i=1}^{i=N} \frac{dM_i}{dt} = \sum_{i=1}^{i=N} F \left(\frac{S_i}{S_T} \right) (a + bL_i). \quad (3)$$

The total surface area of the bed S_T , must then satisfy the equation

$$S_T = \sum_{i=1}^{i=N} S_i (a + bL_i). \quad (4)$$

The linear particle growth rate, corrected for a time dependent surface area S_T , can be obtained by substitution of Equation (4) into Equation (1). Thus,

$$g_i = \left(\frac{2F}{\rho_s} \right) \frac{(a + bL_i)}{\sum_{i=1}^{i=N} S_i (a + bL_i)} \quad (5)$$

The instantaneous mass rate of particles crossing the particle size boundary at L is a function of the particle growth rate and the number of particles at L . This is shown dimensionally by the following statement:

$$\begin{aligned} \text{Mass input rate at size } L = & \lim_{\Delta L \rightarrow 0} \left[\underbrace{\left(\text{linear growth rate, } \frac{1}{t} \right)}_{g_i} \right. \\ & \left. \underbrace{\left(\frac{\text{mass growth rate}}{\text{linear growth rate}}, \frac{m}{1} \right)} \cdot \underbrace{\left(\frac{\text{total mass in particle}}{\text{mass of growth}}, \frac{m}{m} \right)} \right. \\ & \cdot \frac{\pi}{2} L^2 \rho_s \cdot \frac{L^3}{[3L^2 \Delta L + 3L(\Delta L)^2 + (\Delta L)^3]} \\ & \left. \left(\underbrace{\text{number of particles at } L, t} \right) \right] \\ & \cdot \left. \frac{6\Delta L}{\pi L^3 \rho_s} \cdot \frac{\partial W(L, t)}{\partial L} \right|_{L + \alpha \Delta L} \quad (6) \end{aligned}$$

where α is a constant having any value < 1 .

The terms representing each statement (shown below each statement) are multiplied together to give:

$$\begin{aligned} \text{Mass input rate at size } L = & \lim_{\Delta L \rightarrow 0} \left[\frac{3g_i L^2}{3L^2 \Delta L + 3L(\Delta L)^2 + (\Delta L)^3} \cdot \frac{\partial W(L, t)}{\partial L} \right]_{L + \alpha \Delta L} \Delta L \\ & = g_i(L) \frac{\partial W(L, t)}{\partial L} \bigg|_L \quad (7) \end{aligned}$$

The mass rate at which particles of size L pass into the size interval at L is given by Equation (7).

1.1 Attrition

Considerable kinetic energy is transferred to the particles in the fluidized bed from the fluidizing gas, the air from the feed nozzle, the generated water vapor, and any gaseous reaction products. This energy is dissipated through

numerous collisions between the bed particles. An important result of these collisions is the rate process known as attrition. Physically, attrition is the process wherein the larger particles in the bed break into randomly distributed smaller particles. The rate at which particles change from large sizes into smaller sizes is a complex function of particle size, bed particle size distribution, impact strength of the material from which the particles are formed, fluidizing gas velocity, pressure drop across the bed support plate, feed nozzle air rate and velocity, liquid feed rate, and undoubtedly many other factors.

The formulation of a model for the rate of attrition is based upon the observed behavior of beds which have been subjected to an attrition test. It is known that the cumulative weight fraction of particles up to size L increases during an attrition test. An example of the results of such an attrition test is shown in Figure 2.

Fig. 2 Effect of attrition on product from the fluidized bed calcination process.

The process of attrition can be visualized as a negative growth rate, although not exactly in the same sense as a positive growth rate. Since particle growth is a continuous process, the size at some future time can be predicted. This is not the case when a particle undergoes attrition, because the process of attrition is a discrete rather than a continuous process. The final disposition of a single particle undergoing attrition cannot be predicted from the present state of knowledge.

If all of the particles in a fluidized bed are considered together, the situation becomes more manageable. Although each particle breaks down into smaller particles in discrete steps at a random rate, the particles, when considered in total, decrease in size in an orderly and predictable way as shown by the curve in Figure 2. Thus the particles, through the process of attrition, may be considered as having a negative growth rate. Although not physically true, it is assumed that these particles move smoothly through each size in the bed. This negative growth rate is designated throughout this study as the attrition rate. Consider a system of particles in a fluidized bed as depicted in Figure 1. The rate at which particles enter the size interval at $(L + \Delta L)$ or leave the size interval at L is the product of the particle weight density and the particle attrition rate. Thus,

$$r_a = A(L) \cdot \frac{\partial W(L,t)}{\partial L} \bigg|_{L + \alpha \Delta L}^{\Delta L \Delta t} \quad (8)$$

This follows from the previous section on particle growth.

1.2 Seed Addition

If a fluidized bed process is at steady state, it is obvious that new particles must be introduced into the system at the same rate as that at which particles leave the system as product and elutriated fines. These newly introduced particles

are called seed particles; the rate at which they enter the system is called the seed rate. Seed particles are not restricted in size (only in the practical sense), although their rate of introduction may differ considerably from size to size.

The seed rate is closely related to the attrition rate in most fluidized bed processes, since the new particles generated by the process of attrition are by definition seed particles. Since many fluidized bed processes do not generate enough new seed particles within the system, seeds must be either introduced from an external source or generated by special devices within the system. An excellent example of this situation is described in a paper by Dunlop, Griffin, and Moser[6]. A similar situation sometimes exists in the fluidized bed calcination process. If seeds are introduced into the bed, consideration must be given to the rate at which new seeds are added to the system from external sources.

The rate of seed entering a given particle size interval is a function of the total seed rate and the particle size distribution of the seeds. Let $S(t)$ be the total seed addition rate and $w_s(L, t)$ be the weight fraction of seeds in any particle size interval ΔL . The weight of seeds entering the particle size interval during the time interval Δt , is

$$\text{Mass input of seeds into the particle size interval } \Delta L \text{ during the time } \Delta t = \int_t^{t + \Delta t} S(t) w_s(L, t) dt \quad (9a)$$

The weight fraction of seeds within the interval ΔL may be expressed as a function of cumulative weight fraction by the relationship

$$w_s(L, t) = W_s(L, t) \Big|_{L + \Delta L} - W_s(L, t) \Big|_L \quad (9b)$$

where $W_s(L, t)$ is the cumulative weight fraction of seed particles up to size L . By the Theorem of the Mean of Differential Calculus, Equation (9b) becomes

$$w_s(L, t) = \frac{\partial W_s(L, t)}{\partial L} \Big|_{L + \alpha \Delta L} \Delta L \quad (9c)$$

Substitution of Equation (9c) into Equation (9a) and application of the Mean Value Theorem of Integral Calculus yields the final expression for the mass of seeds passing into the interval ΔL during the time Δt , where the bar represents the average value of the quantity under the integral in Equation (9a).

$$\text{Mass of seeds passing into interval } \Delta L \text{ during time } \Delta t = S(t) \overline{\frac{\partial W_s(L, t)}{\partial L}} \Big|_{L + \alpha \Delta L} \Delta L \Delta t \quad (9d)$$

If the particle size distribution of the seeds, $W_s(L, t)$, is independent of time, $W_s(L)$ can be empirically expressed as a function of L only, by Equation (9e).

$$W_s = S_0 + S_1 L + S_2 L^2 + S_3 L^3 + \dots \quad (9e)$$

1.3 Elutriation

Elutriation from a fluidized bed is a result of the drag force of the fluidizing gas on a particle exceeding the force of gravity. It is assumed in this study that sufficient distance exists between the surface of the fluidized bed and the fluidizing gas outlet to ensure that no particles leave with the fluidizing gas because of kinetic effects^[7].

Elutriation of particles from a fluidized bed has been studied by Leval^[8]; Osberg and Charlesworth^[9]; Thomas, Grey, and Watkins^[10]; and Wen and Hashinger^[11]. All agree that the elutriation rate of small particles of a given size from a fluidized bed is proportional to the concentration of those particles in the bed. The rate of change of concentration of fines of a particular size in a fluidized bed can be expressed by

$$\frac{dC_i}{dt} = -EC_i \quad (10)$$

where C_i is the concentration of a given sized fines in the bed and E is the elutriation rate constant that may be a function of several bed properties and operating conditions.

Wen and Hashinger^[11] have proposed a generalized correlation for evaluating the elutriation rate constant in a multiparticle-sized bed. The variables affecting the elutriation rate constant and the conclusions concerning the magnitude of the effect of each variable are summarized below:

- (1) The difference between the magnitude of the linear gas velocity and the terminal velocity of the particle is the major factor influencing the value of the elutriation constant.
- (2) The bed dimensions and the coarse particle size in large diameter columns have a negligible effect on the value of the elutriation rate constant.
- (3) The initial fines concentration in a two-particle-size system seems to have no effect on the elutriation constant until a concentration of approximately 25 percent is reached; above 25 percent the elutriation constant tends to decrease as the fines concentration increases.
- (4) The elutriation constant varies with the gas velocity approximately as the 0.7 power and inversely with the bed weight. The particle size of the fines, the gas density, and the gas viscosity also influence the value of the elutriation constant.

Equation (10) was modified by Wen and Hashinger^[11] by considering the change in concentration of a given sized fines to be inversely proportional to the bed weight and directly proportional to the cross-sectional area of the bed. For the case of constant bed weight and density, Equation (10) then becomes

$$\frac{dw(L,t)}{dt} = - \frac{A}{w_B} E(L) w(L,t) \quad (11)$$

where A_c is the cross-sectional area of the bed, W_b is the weight of the bed, and $w(L,t)$ is the weight of fines in the particle size interval ΔL .

$$\begin{array}{l} \text{Mass of solids elutriated from} \\ \text{the interval } \Delta L \text{ during time } \Delta t = \frac{A_c}{W_b} E(L) \int_t^{t + \Delta t} w(L,t) dt. \end{array} \quad (12)$$

Finally, the mass of material elutriated from the particle size interval, ΔL , may be expressed in terms of the total bed weight up to the particle size interval under consideration by the equation

$$\begin{array}{l} \text{Mass of solids elutriated from} \\ \text{the interval } \Delta L \text{ during time } \Delta t = \frac{A_c}{W_b} E(L) \frac{\partial W(L,t)}{\partial L} \left| \Delta L \Delta t \right. \\ \left. L + \alpha \Delta L \right. \end{array} \quad (13)$$

Based on dimensional analysis, Wen and Hashinger correlated their data so that the specific elutriation-rate constant, E , could be determined by the following equation:

$$E = (1.52 \times 10^{-5}) (\rho_f) (u_f - u_t) \left[\frac{(u_f - u_t)^2}{gL} \right]^{0.5} \left[\frac{Lu_t \rho_f}{\mu_f} \right]^{0.725} \left[\frac{\rho_s - \rho_f}{\rho_f} \right]^{1.15} \quad (14)$$

subject to the following limits:

$$\begin{aligned} 0.004 \text{ cm} &< L < 0.015 \text{ cm} \\ 0.01 \text{ lb/ft}^3 &< \rho_f < 0.07 \text{ lb/ft}^3 \\ 81 \text{ lb/ft}^3 &< \rho_s < 312 \text{ lb/ft}^3 \\ 0.7 \text{ ft/sec} &< u_f < 4.3 \text{ ft/sec} \end{aligned}$$

where

ρ_f = the density of the fluidizing gas

u_f = the fluidizing velocity

u_t = the terminal settling velocity of the particle size in question

L = the particle diameter

ρ_s = the apparent density of the particle

μ_f = the viscosity of the fluidizing gas

g = the local acceleration of gravity.

If the Reynolds number, $\frac{Lu_t \rho_f}{\mu_f}$, is less than 2, then Stoke's law is used to determine the terminal settling velocity, or

$$u_t = \frac{Y u_f L^2 (\rho_s - \rho_f)}{18 \mu_f}$$

where Y is a factor correcting for the non-sphericity of the particle. At Reynolds numbers greater than 2, a modified Stoke's law is used:

$$u_t = \frac{0.153 Y g^{0.71} L^{1.14} (\rho_s - \rho_f)^{0.71}}{\rho_f^{0.29} \mu_f^{0.43}}$$

Wen and Hashinger's dimensional analysis of elutriation indicated that an additional term should be included in their correlation; however, their experimental data indicated that this term was negligible. The decision to drop this term was justified for the range of particle sizes considered in their study, but it cannot be dropped for smaller sized particles. This is demonstrated in Figure 3 where the relative magnitude of the specific elutriation-rate constant, E, is plotted against the particle size, the values for E being determined by Equation (14). This curve indicates that the elutriation rate of the particles in a fluidized bed increases from zero at the particle size at which the terminal settling velocity is equal to the fluidizing velocity, to some maximum value as the particle size decreases, and then to zero again as the particle size reaches zero. Experimental observation reveals that the elutriation rate continues to increase as the particle size decreases. This is illustrated by the dotted portion of the curve in Figure 3.

Fig. 3 Relative E values as calculated by Equation (14) as a function of particle size.

Indeed the term discarded by Wen and Hashinger is of the proper form so that when used with Equation (14) the dotted line in Figure 3 is obtained. The term they discarded is

$$\left(\frac{u_f - u_t}{u_t} \right)^d$$

Regardless of the values obtained from the other terms in Equation (14), use of this term makes the value of E approach infinity as the particle size approaches zero. Therefore, this term is included with the other terms of Equation (14), even though the correct value for the exponent d is not now known.

1.4 Product Rate

Since the bed is assumed to be perfectly mixed, any material removed from the bed as product has the same particle size distribution as the bed. Therefore, the weight of particles leaving the bed as product from the size interval ΔL , during time Δt , is given by the following expression

$$\begin{array}{l} \text{Mass of solids leaving the interval} \\ \Delta L \text{ as product during the time } \Delta t = \end{array} \int_t^{t + \Delta t} \frac{P(t) w(L,t) \Delta t}{W_B} \quad (15)$$

where $w(L,t)$ is the weight of material in the interval and W_B is the total bed weight. Of course this is not true for the special case where the fines removed from the fluidizing gas by a cyclone are added to the particles removed from the bed and the combined material called the product.

Equation (15) may be expressed in terms of the total bed weight up to particle size L as follows:

$$\begin{array}{l} \text{Mass of solids leaving the interval} \\ \Delta L \text{ as product during the time } \Delta t = \end{array} \frac{\overline{P(t)} \cdot \frac{\partial W(L,t)}{\partial L}}{W_B} \bigg|_{L + \alpha \Delta L}^{\Delta L} \Delta t \quad (16)$$

where the quantity under the bar represents an average value of the quantity under the integral in Equation (15).

1.5 Feed Deposition Rate

The deposition of feed on the surface of the particles within size interval ΔL is the final rate process to be discussed. While it is related to the particle growth rate, there is a distinct difference between the two rate processes when considering the material balance around ΔL . The particle growth rate is used to determine the weight of particles entering and leaving ΔL in an interval of time, Δt , whereas the deposition rate dictates the mass of feed which is deposited on the surface of the particles in ΔL during a given interval of time, Δt .

The mass of material deposited on the particles within the interval, ΔL , during the time, Δt , is given by the following expression:

$$\int_t^{t + \Delta t} \underbrace{\left[g_i \right]}_{\frac{1}{t}} \underbrace{\left[\frac{\pi L^2 \rho_s}{2} \right]}_{\frac{m}{l}} \underbrace{\left[N_i(L,t) \right]}_{\text{Number of particles in interval}} dt \quad (17)$$

The number of particles in the particle size interval can be stated in terms of the function, $W(L,t)$, as follows:

$$N_i(L,t) = \frac{w(L,t)}{\frac{\pi}{6} L^3 \rho_s} = \left[\frac{6}{\pi L^3 \rho_s} \right] \left[\frac{\partial W(L,t)}{\partial L} \right] \bigg|_{L + \alpha \Delta L}^{\Delta L} \quad (17a)$$

Substitution of this expression for $N_i(L,t)$ in the above integral followed by the application of the Mean Value Theorem of Integral Calculus results in

the expression for the mass of material deposited in the particle size interval, ΔL , during the time, Δt . Thus,

$$\text{Mass of material deposited in the interval } \Delta L \text{ during time } \Delta t = \overline{\frac{3g_i}{L} \cdot \frac{\partial W(L,t)}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta L \Delta t \quad (18)$$

where the quantity under the bar represents an average value of the integrand in Equation (17).

Substituting for the quantity, g_i , from Equation (5) yields the final expression for the mass of material deposited in time, Δt .

$$\text{Mass of material deposited in the interval } \Delta L \text{ during time } \Delta t = \overline{\frac{6C_f F(t)}{L \rho_s S_T} \cdot \frac{\partial W(L,t)}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta L \Delta t \quad (19)$$

where $C_f \triangleq (a + bL_i) / \sum_{i=1}^{i=n} S_i(a + bL_i)$.

1.6 General Partial Differential Equation Describing the Particle Size Distribution in the Fluidized Bed

The various rate expressions required to make an unsteady state material balance around the particle size interval ΔL have been derived.

The total input, output, and accumulation of mass in the particle size interval during the time, Δt , is given by the following expressions:

$$\begin{aligned} \text{Total mass input during } \Delta t = & \overline{A(L) \frac{\partial W}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta t + \overline{g_i \frac{\partial W}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta t \\ & + \overline{S(t) \frac{\partial W}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta t \\ & + \overline{\frac{6F(t) C_f}{L \rho_s S_T} \cdot \frac{\partial W}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta L \Delta t \end{aligned} \quad (20)$$

$$\begin{aligned} \text{Total mass output during } \Delta t = & \overline{A(L) \frac{\partial W}{\partial L}} \bigg|_L^L \Delta t + \overline{g_i \frac{\partial W}{\partial L}} \bigg|_{L + \Delta L}^L \Delta t \\ & + \overline{\frac{A_c E(L)}{W_B} \cdot \frac{\partial W}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta L \Delta t \\ & + \overline{\frac{P(t)}{W_B} \cdot \frac{\partial W}{\partial L}} \bigg|_{L + \alpha \Delta L}^L \Delta L \Delta t \end{aligned} \quad (21)$$

Accumulation of mass in the interval ΔL during the time $\Delta t = w(L,t) \Big|_{t + \Delta t} - w(L,t) \Big|_t$

$$= \frac{\partial^2 W}{\partial L \partial t} \Big|_{\substack{L + \alpha \Delta L \\ t + \beta \Delta t}} \Delta L \Delta t \quad (22)$$

Equations (20), (21), and (22) can be combined to give the general partial differential equation describing the weight distribution in the bed as a function of particle size and time.

$$\begin{aligned} \frac{\partial^2 W}{\partial L \partial t} = & \left[A(L) \frac{\partial^2 W}{\partial L^2} + \frac{\partial A(L)}{\partial L} \cdot \frac{\partial W}{\partial L} \right] - \left[g(L) \frac{\partial^2 W}{\partial L^2} + \frac{\partial g(L)}{\partial L} \cdot \frac{\partial W}{\partial L} \right] \\ & + S(t) \frac{\partial W_s}{\partial L} + \frac{6F(t) C_f}{L \rho_s S_T} \cdot \frac{\partial W}{\partial L} - \frac{A_c E(L)}{W_B} \cdot \frac{\partial W}{\partial L} - \frac{P(t)}{W_B} \cdot \frac{\partial W}{\partial L} \quad (23) \end{aligned}$$

III. STEADY STATE PARTICLE SIZE DISTRIBUTION

Equation (23) is the partial differential equation which describes the distribution of bed weight as a function of time and particle size. Simplifying assumptions for the case of steady state make an analytical solution of the governing differential equation possible. The solution of the steady state equation can be used to study the effect of the operating parameters on the particle size distribution.

1. SIMPLIFYING ASSUMPTIONS

Derivation of the differential equation describing the steady state particle size distribution is based on the following simplifying assumptions.

- (1) All of the particles in the system are spherical.
- (2) Attrition is negligible.
- (3) Elutriation is negligible.
- (4) The seed particles enter the system at a constant rate and are the smallest particles in the bed. All of the seed particles are identical in size.

Application of the simplifying assumptions and the following considerations will yield the differential equation describing the steady state weight distribution in the bed.

$$\begin{array}{l} \text{Mass passing any point } L \text{ because} \\ \text{of particle growth during time } \Delta t \end{array} = \int_t^{t + \Delta t} \bar{g} \frac{dW}{dL} \bigg|_L dt. \quad (24a)$$

Mass of the feed deposited on the particles in the interval ΔL during the time Δt

$$= \int_t^{t + \Delta t} \left[\left(\bar{g} \frac{\pi L^2 \rho_s}{2} \right) \left(\frac{\frac{1}{\pi L^3 \rho_s}}{6} \right) \frac{dW}{dL} \bigg|_{L + \alpha \Delta L} \right] dt. \quad (24b)$$

$$\begin{array}{l} \text{Mass leaving the interval } \Delta L \text{ due} \\ \text{to product removal during time } \Delta t \end{array} = \int_t^{t + \Delta t} \frac{P}{W_B} w(L) dt \\ = \int_t^{t + \Delta t} \frac{P}{W_B} \cdot \frac{dW}{dL} \bigg|_{L + \alpha \Delta L} dt. \quad (24c)$$

The differential equation describing the weight distribution as a function of particle size is obtained by evaluating the limit of the following as ΔL approaches zero.

$$\begin{aligned} \text{Limit as } \Delta L \rightarrow 0 \quad \overline{g} \cdot \frac{dW}{dL} \left|_{L} \right. + \frac{3\overline{g}}{L} \cdot \frac{dW}{dL} \left|_{L + \alpha \Delta L} \right. - \overline{g} \cdot \frac{dW}{dL} \left|_{L + \Delta L} \right. \\ - \frac{P}{W_B \overline{g}} \cdot \frac{dW}{dL} \left|_{L + \alpha \Delta L} \right. = 0 \end{aligned} \quad (25)$$

Application of the Theorem of the Mean of Differential Calculus to the function $\overline{g} \cdot \frac{dW}{dL} \left|_{L + \alpha \Delta L} \right.$ yields the following differential equation in the limit as $\Delta L \rightarrow 0$.

$$\frac{d^2 W}{dL^2} - \left(\frac{3}{L} - k \right) \frac{dW}{dL} = 0 \quad (26)$$

where $k = \frac{P}{W_B \overline{g}}$, and is defined as the distribution number.

Equation (26) can be reduced to a first order equation by defining a new dependent variable

$$\phi = \frac{dW}{dL}.$$

The solution of Equation (26) is then readily found to be

$$\ln \left(\frac{\phi}{L^3 C} \right) = -kL$$

or

$$\phi = CL^3 e^{-kL} \quad (27)$$

The integration constant, C, can be evaluated by imposing conditions which must be satisfied at the point, $L = L_s$. Thus,

$$\phi(L_s) = \phi_s = \left(\frac{dW}{dL} \right)_s = \text{slope of weight distribution curve at the point } L = L_s$$

and

$$C = \frac{\phi_s e^{-kL_s}}{L_s^3}.$$

Equation (27) then becomes

$$\phi = \frac{dW}{dL} = \frac{\phi_s L_s^3 e^{-k(L - L_s)}}{L_s^3} \quad (28)$$

Equation (28) can be integrated between the limits L_s and any particle size L to yield the expression for the bed weight, W , up to particle size L .

$$W = \frac{\bar{g}_s W_B}{PL_s^3 k^3} \left[\frac{e^{\frac{k(L - L_s)}{e}} - 1}{e^{\frac{k(L - L_s)}{e}}} \right] [6 + 6k(L_s - L) + 3k^2(L_s^2 - L^2) + k^3(L_s^3 - L^3)] \quad (29)$$

The cumulative weight fraction (CWF) as a function of L can be obtained by dividing both sides of Equation (29) by W_B . Equation (29) can be expressed in terms of the seed rate, S , since the following condition must be satisfied at L_s .

$$\bar{g} \left(\frac{dW}{dL} \right)_{L_s} = S$$

The final expression for the cumulative weight fraction is

$$Q = \frac{S}{PL_s^3 k^3} \cdot \left[\frac{e^{\frac{k(L - L_s)}{e}} - 1}{e^{\frac{k(L - L_s)}{e}}} \right] [6 + 6k(L_s - L) + 3k^2(L_s^2 - L^2) + k^3(L_s^3 - L^3)] \quad (30)$$

This equation is similar in form to those developed by Dunlop^[6] and Saeman^[14] for predicting the crystal size distribution in mixed crystal suspensions, and the particle size distribution in a fluidized coker, respectively.

2. EFFECT OF OPERATING VARIABLES ON THE STEADY STATE PARTICLE SIZE DISTRIBUTION IN A FLUIDIZED BED PROCESS

Although Equation (30) was obtained by making a number of simplifying assumptions, it is useful in studying the effect of various parameters on the steady state particle size distribution in a fluidized bed.

The quantity, k , in Equation (30) has been named the distribution number. The seed rate and distribution number determine the shape and position of the CWF curve. This is illustrated in Figure 4 where CWF is shown as a function of particle size for several values of the distribution number at a constant seed rate and for seed size approaching zero.

The relationship between the seed rate and the various factors in the distribution number will now be discussed. As L in Equation (30) approaches infinity, Q approaches one, and the equation becomes that shown as Equation (31).

$$1 = Q = \frac{S}{PL_s^3 k^3} [6 + 6kL_s + 3(kL_s)^2 + (kL_s)^3] \quad (31)$$

The effects of the seed rate and the product rate on the particle size distribution in the bed are of considerable interest. Assume that the size of the seed particles is 0.001 cm (10 μ). (According to studies made at this laboratory, this is about the smallest sized particle that can act as a seed.) Assume also that the bed weighs 160,000 grams. Equation (31) can be used to calculate the values of the distribution number for various product and seed rates. Figure 5 shows these relationships, using the distribution number as a function of both the product rates and the seed rate.

Fig. 4 Cumulative weight fraction versus particle size with distribution number as a parameter.

Fig. 5 Distribution number and growth rate as a function of product rate with seed rate as a parameter.

The distribution number decreases with an increase in the product rate and increases with an increase in the seed rate. As the distribution number decreases, the range of particle sizes in the bed increases.

The particle growth rate depends on the seed and product rates. From Equation (31) it is seen that changes in the product rate, P , at a constant seed rate and seed size, require the distribution number to change in order that the value of the terms on the right-hand side of the equality sign remains one. The particle growth rate as a function of the product rate is also shown in Figure 5 with the seed rate as a parameter.

The effect of the size of the seeds on the weight distribution was also investigated. Cumulative weight fraction is shown in Figure 6 as a function of particle size at a constant value of the distribution number with seed size as a parameter. Note that the upper parts of the curves almost coincide and that the curves originate at the specified size of the seed particles.

Fig. 6 Calculated effect of the size of the seed particles on the cumulative weight fraction.

3. COMPARISON OF THE EXPERIMENTAL AND CALCULATED STEADY STATE PARTICLE SIZE DISTRIBUTION

The validity of the model for the steady state particle size distribution was established by comparing the CWF predicted by Equation (30) with the experimental CWF obtained from three fluidized bed calcination runs, two in the two-foot-square calciner and one in the six-inch-diameter calciner.

The value of k was determined from the product rate, the bed weight, and the particle growth rate. Particle growth rates were determined by the methods described by Grimmett[13]. Seed diameter was specified as 0.001 cm. Cumulative weight fractions computed from Equation (30) are compared with the experimental CWF in Figure 7.

The agreement between the calculated and the experimentally determined curves is only fair. However, when considering that steady state was difficult to achieve during these runs, that attrition and elutriation were assumed to be negligible, and that the diameter of the seeds was arbitrarily specified at 0.001 cm, the agreement between the model and the experimental results is really quite encouraging.

Fig. 7 Calculated versus experimental CWF at steady state.

IV. THE PREDICTION OF THE TRANSIENT PARTICLE SIZE DISTRIBUTION

1. SOLUTION OF THE PARTIAL DIFFERENTIAL EQUATION

A solution of the partial differential equation, Equation(23), is most desirable. Behnken et al^[16] proposed a model of a system of growing particles, which is similar in concept to the model proposed herein; however, no detailed breakdown of the various rate processes known to take place in a fluidized bed process was presented. The solution of the differential equation based on the model of Behnken et al^[16] is not applicable to the prediction of the weight distribution in a fluidized bed.

Since an analytical solution of Equation (23) is beyond our present ability, the equation must be solved by some other method. A finite difference method which can be used is a modification of the Runge-Kutta method for solving ordinary differential equations. A computer program based on the Runge-Kutta procedure was available for the IBM 7040 at this location. Since the Runge-Kutta method was not programmed to solve partial differential equations, it was necessary to modify Equation (23) so that its solution could be obtained by solving a number of ordinary differential equations at each time step. These modifications will be outlined in the paragraph which follows.

The unsteady state material balance around the particle size interval, ΔL , may be described by the following equation. This follows from the previously derived Equations (20), (21), and (22).

$$w(t) \Big|_{t + \Delta t} - w(t) \Big|_t = \left[\bar{g}_{i-1} \left(\frac{dw}{dL} \right)_t + \frac{6F(t) C_f}{L_i \rho_s \epsilon_T} \left(\frac{dw}{dL} \right)_t \Delta L + \bar{s}(t) \left(\frac{dw_s}{dL} \right)_t \Delta L \right] - \left[\bar{g}_{i+1} \left(\frac{dw}{dL} \right)_t + \frac{A_c E(L)}{W_B} \left(\frac{dw}{dL} \right)_t \Delta L + \frac{\bar{P}(t)}{W_B} \left(\frac{dw}{dL} \right)_t \Delta L \right] \Delta t \quad (32)$$

The mass balance around each particle size interval, ΔL , expressed by Equation (32) results in the weight of the particles in the interval at time $t + \Delta t$. The last term, representing the input of seed particles, is necessary if the seed particles are of varying sizes. Application of the Runge-Kutta method to Equation (32) gives the bed weight, W , up to any particle size, L , as a function of time.

2. IBM 7040 COMPUTER PROGRAM FOR SOLVING THE GENERAL PARTIAL DIFFERENTIAL EQUATION DESCRIBING A MATHEMATICAL MODEL OF PARTICLE GROWTH AND SIZE DISTRIBUTION IN FLUIDIZED BED PROCESSES

The following objectives were established for the computer program:

- (1) The program should have the capability of solving a wide range of problems in fluidized bed technology
- (2) The program should be as flexible as possible by permitting the specification of the size and number of particle size intervals used and the methods of generating the output data
- (3) Economical use of computer time is required for a given problem.

Although the general differential equation permits the feed rate and seed rate to be time dependent, the computer program will permit only step changes in these variables.

2.1 Particle Size Interval

Small particle size intervals result in increased accuracy at the expense of additional computer time. The particle size interval should be chosen as large as possible to reduce computer time and yet yield results within acceptable limits of accuracy.

A number of particle size intervals were tested to determine the largest size which would result in stable solutions. Tests were made by simulating the operating conditions which might be used during a fluidized bed calciner run. Input data for these runs were based on typical values used during the development of the fluidized bed waste calcination process.

The four particle size intervals tested were 0.01, 0.005, 0.001, and 0.0005 cm. Operating parameters were identical for each particle size interval tested. The computer was allowed to run until conditions of steady state were achieved. This was equivalent to approximately 100 hours of actual calciner operation. Cumulative weight fractions based on each particle size interval were compared with the CWF at steady state as predicted by Equation (23).

As shown in Figure 8, particle size intervals larger than 0.005 cm resulted in unacceptable predictions for the CWF. Particle size intervals smaller than 0.001 cm gave results which approach the curve calculated from the steady state equation. A size interval

Fig. 8 Effect of the particle size interval on the predicted steady state CWF.

of 0.0005 cm resulted in a CWF curve which was nearly indistinguishable from the calculated steady state curve. Based on the results of these tests, a particle size interval of 0.0005 cm was used throughout this study.

2.2 Time Interval Used in Making Calculations

The time interval used in Equation (32) affects the accuracy of the predicted CWF in a manner completely analogous to the particle size interval. Effect of the time interval on the stability of the solution of Equation (32) was determined by using several time intervals in the calculation of the steady state CWF. Calculated CWF curves for two runs using time intervals of 0.1 hour and 0.05 hour are shown in Figure 9. Time intervals of 1.0 and 0.5 hour resulted in instability for the particular systems investigated.

Fig. 9 Steady state particle size distributions based on time intervals of 0.1 and 0.05 hour.

No specific rule can be stated concerning the time interval which should be used in a particular problem. If it is known that the particle size distribution in the bed is near steady state, or if new operating conditions do not rapidly change the particle size distribution of the bed, time intervals of 0.5 to 1.0 hour may be used. Otherwise, time intervals in the range of 0.1 to 0.05 hour will be required.

2.3 Simulated Versus Actual Calciner Runs

The usefulness of a mathematical model is judged by the accuracy with which it predicts the response of the process described. Validity of the mathematical model proposed herein was determined by comparing experimental and predicted particle size distributions for various calciners and operating parameters. Operating conditions for the various calciners are listed in Table I, and the required input data for the computer program are listed in Table II.

2.4 Two-Foot-Square Calciner

The bed weight used in the two-foot-square calciner^[5] can vary between 200 and 400 kilograms. Heat is supplied to the bed by circulating hot NaK through a tube bundle placed directly in the bed of solids. The bed of solids occupies a volume in the range of 6 to 10 cubic feet. Liquid feed is introduced directly into the bed through one or more pneumatic atomizing nozzles. The fluidizing gas is air.

The particular run studied in this unit was used for two reasons: (a) It was a typical run of fairly long duration in which it was necessary to add seed particles to control the particle size of the product. (b) Particle attrition was minimal, thereby justifying the assumption of negligible attrition in the model.

TABLE I

OPERATING CONDITIONS FOR EXPERIMENTAL CALCINER RUNS

Operating Parameter	Two-Foot-Square Calciner	Twelve-Inch-Diameter Calciner	Three-Inch-Diameter Calciner
Product Rate, kg/hr	4.59 (0 to 74 hrs) 4.42 (74 to 146 hrs) 4.40 (146 to 222 hrs)	0.568	0.12 ¹
Bed Weight, kg	360.0	63.4	1.0 ^[a]
Particle Density, g/cm ³	1.93	1.91	1.3 ^[b]
Seed Rate, kg/hr	0.180	0.041	0.0
Seed Size (Tyler Mesh)	-80 +155	-80 +115	---
Run Duration, hrs	222.0	148.0	6.0

[a] Bed weight was not given by authors. Estimated from data given for other runs producing uranium trioxide in the three-inch unit.

[b] Particle density was not given. Estimated from calcination temperature and feed concentration. (Effect of feed concentration and calcination temperature on particle density was determined previously^[4].)

TABLE II

INPUT DATA FOR COMPUTER PROGRAM

Operating Data	Two-Foot-Square Calciner	Twelve-Inch-Diameter Calciner	Three-Inch-Diameter Calciner
Particle Size Interval, cm	0.0001	0.0001	0.0001
Time Interval, hrs	0.5	0.5	0.1
Growth Constant, a (dimensionless)	1.0	0.542	1.0
Growth Constant, b, cm ⁻¹	0.006	9.12	0.0
Fluidizing Velocity, cm/sec	100	100	22
Particle Density, g/cm ³	1.93	1.91	1.3
Density of Fluidizing gas, g/cm ³	0.00051	0.00051	0.00051
Viscosity of Fluidizing gas, g/cm-sec	0.00025	0.00025	0.00025

The run was divided into three parts. During the first 74 hours of the run, seeds (-80 +115 Tyler mesh) were added to the bed at a rate of 180 grams per hour. At the end of this period, seed addition was discontinued and the calciner operated another 72 hours. Seeds were then added to the bed at a rate of 180 grams per hour for the remaining 76 hours of the run.

The calculated and experimental CWF curves for the two-foot-square calciner run are shown in Figure 10. Calculated and experimental size distributions are compared at the end of each of the three parts of the run.

The agreement between the CWF based on the mathematical model and the experimental CWF is encouraging when one considers the simplifying assumptions used in the model.

2.5 Twelve-Inch-Diameter Calciner

The twelve-inch-diameter calciner^[17] operates with bed weights in the range of 35 to 55 kilograms and with bed volumes in the range of from 1.2 to 1.5 cubic feet. Heat is supplied to the bed from electrical heaters placed directly in the fluidized bed of particles. Liquid feed is introduced directly into the bed of fluidized particles through a single pneumatic atomizing nozzle.

The 12-inch calciner run used to compare with a simulated run was chosen because it was operated under conditions typical of those used in this unit. It was chosen also because the addition of seed particles to the bed was required to stabilize the particle size distribution of the bed. Under these conditions, attrition of the bed is so slight that when simulating the run, the attrition term in the model can be neglected.

Experimental and calculated CWF curves for the twelve-inch calciner are shown in Figure 11. The agreement between the CWF curve calculated on the basis of the model and that obtained during the calciner run is excellent.

Fig. 10 Calculated versus experimental particle size distributions in the two-foot-square calciner.

Fig. 11 Calculated versus experimental particle size distribution in the twelve-inch-diameter calciner.

2.6 Three-Inch-Diameter Calciner

The experimental data on the three-inch-diameter calciner was obtained in the Chemical Engineering Laboratory of Argonne National Laboratory, Argonne, Illinois [17, 18]. An aluminum nitrate-nitric acid feed similar to that used in the studies at the National Reactor Testing Station was employed. The run in the three-inch-diameter calciner permitted the extension of the range of calciner sizes over which the proposed model could be tested.

The bench-scale unit consisted of a three-inch-diameter stainless steel pipe surmounted by a four-inch-diameter pyrex glass pipe disengaging section. The range of feed rates and bed weights used in their studies was not disclosed in the references cited. It is known, however, that the unit was heated through the walls with electrical heaters, and that a pneumatic-type atomizing nozzle was used to introduce the liquid feed into the bed.

Calculated and experimental CWF curves for the three-inch-diameter calciner are shown in Figure 12.

Fig. 12 Calculated versus experimental particle size distribution in the three-inch-diameter calciner.

3. DISCUSSION

A mathematical model of the relationships between the particle size distribution of the fluidized bed in the calcination process and the various independent variables associated with the process have been tested by the preceding three examples. The results of these tests have ranged from a fair to an excellent simulation of the actual calciner runs. Before the usefulness of this model is judged, a few pertinent facts should be reviewed.

The simulation tests used as examples in this paper were conducted in fluidized bed calciners having a wide range of sizes. Thus the mathematical model was tested with calciners having bed cross-sectional areas of 0.0513, 0.777, and 4.00 square feet, a range of 78 fold. The nominal bed weights used in these calciners ranged 300 fold. These and other interesting comparisons between the three calciners are summarized in Table III.

An examination of Table III reveals that there are striking differences between the sizes and operating conditions for the three units. The closest similarity is the ratio of the product rate to the bed cross-sectional area, and here there is only a three-fold difference. Based on the comparisons between the CWF curves calculated on the basis of the model and the experimental curves, it can be concluded that the model is generally applicable to a wide range of calciner sizes and operating conditions.

TABLE III

A COMPARISON OF SOME OF THE FEATURES OF THE
THREE CALCINERS USED FOR THE TEST RUNS

Feature	Calciner Size		
	24 in.	12 in.	3 in.
Cross-Sectional Area of Bed, ft ²	4.0	0.777	0.051
Nominal Bed Weight, kg	300.0	45.0	1.0
Nominal Product Weight, kg/hr	4.6	0.57	0.12
Product Rate/Bed Weight	0.015	0.013	0.124
Bed Cross Section/Bed Weight	0.013	0.017	0.051
Product Rate/Bed Cross Section	1.14	0.73	2.42

The model as it now stands has two weaknesses: (a) The equation describing the elutriation rate from the fluidized bed has not produced results consistent with actual observations, and (b) a rate equation for the process of particle attrition in the bed is not available. The lack of information describing the attrition process taking place in fluidized bed processes is particularly evident, and it appears to be a field which if studied would yield information valuable to those involved in fluidized bed technology.

4. OTHER APPLICATIONS

No mention has been made of fluidized bed processes other than the calcination process. It would appear that this model can be made applicable to many systems of particles by retaining only those terms in the general equation which describe the particular system under study.

As an example, suppose a fluidized bed calcination process is to be operated so that all of the particles in the bed reach a given size before they can be removed from the system as product. (In actual practice, various means can be devised to accomplish this.) This system can be simulated by removing the product term from each of the particle size interval Equations (32) up to the particle size specified. For each size intervals larger than that, a modified product term is retained. These terms consist of the weight of the particles in each size interval multiplied by a product rate factor, and they give the rate at which particles leave each interval. The values of the product rate factors are determined by the type of classifying equipment used in the bed. The sum of the rates of the product leaving each of the size intervals larger than the specified particle size represents the product rate. At steady state, this rate will be equal to the total theoretical product rate. This option has been made available in the present FORTRAN program.

Another example of the applicability of the model to various systems can be demonstrated. Suppose the particle size distribution in a fluidized bed catalytic cracker is desired. If the attrition rates and the elutriation rates of the catalyst

are known, and if the feed, growth, product, and deposition terms are dropped from Equation (23), a model is obtained which represents this system. The effect on the particle size distribution of the catalyst in the bed can then be determined as a function of changes made in makeup catalyst particle size distribution and input rate, and of changes made in the velocity of the fluidizing medium and its properties. A study of this nature would permit the specification of operating conditions necessary for optimum catalyst particle size.

No doubt many other particle systems could be represented by appropriate changes of the general equation, including those in the field of crystallization. The author hopes that this study will stimulate further work in the field of particle technology.

V. CONCLUSIONS

A mathematical model has been proposed which describes the appearance, the growth, and the disappearance of particles into and out of an initial population of particles. Although the various rate processes taking place in this system have been oriented toward the fluidized bed calcination process developed at this and other laboratories, the basic differential equation, with appropriate alterations, is believed to be applicable to a wide variety of particle systems.

Application of the equation to predicting the particle size distribution (as a function of time) of the beds in the fluidized bed calcination process has given encouraging results. These results were obtained by simulating, with the help of the mathematical model and an IBM 7040 computer, calcination runs made in three calciners of widely varying sizes. The actual run in the smallest calciner was not made at this laboratory, but was conducted at the Argonne National Laboratory.

No doubt many other particle systems other than those previously described including those in the field of crystallization can be represented by making appropriate changes to Equation (23). The author hopes that this study will stimulate further work in this interesting and challenging field of particle technology.

VI. NOMENCLATURE

- A = attrition rate
- A_c = cross-sectional area of the bed
- A_i = surface area of a particle in the i th size interval
- a = constant in particle growth rate equation
- b = constant in particle growth rate equation
- C_f = a factor used to correct for the effect of particle size on particle growth rate
- C_1 = constant in the equation describing particle attrition rates
- C_2 = constant in the equation describing particle attrition rates
- C_3 = constant in the equation describing particle attrition rates
- E = specific elutriation-rate constant
- F = equivalent solid feed rate
- g = gravitational constant
- g = particle growth rate
- $\bar{g}$ = mean particle growth rate in the bed
- g_i = particle growth rate in the i th particle size interval
- k = distribution number
- L = particle size
- L_i = particle size in the i th size interval
- L_s = size of seed particle in steady state equation
- l = dimension of length
- M_i = the mass of feed deposited on the particles in the i th size interval
- m_i = the mass of feed deposited on a single particle in the i th size interval
- N = total number of particle size intervals, ΔL , in the bed
- N_i = total number of particles in the i th size interval
- P = product rate
- Q = cumulative weight fraction of particles in the bed up to size L

- r_a = the rate particles enter or leave a size interval, ΔL , in the bed due to attrition
 r_e = the rate particles are elutriated from a size interval ΔL in the bed
 s_i = the surface area of a single particle in the i th size interval
 S = total seed addition rate into the bed
 S_i = surface area of the particles in the i th size interval
 S_T = total surface area of the particles in the bed
 S_0, S_1, S_2 = constants in the equation describing the particle size distribution of the seeds
 t = dimension of time
 u_f = velocity of fluidizing gas
 u_t = terminal settling velocity of particles
 W = total bed weight up to some particle size L
 W_B = total weight of the bed
 W_S = cumulative weight fraction of seed particles up to some size L
 w = weight of the bed in the size interval, ΔL , at any time
 w_s = weight of seeds in size interval, ΔL
 Y = factor to correct for non-sphericity of particles when determining their terminal velocity.

GREEK LETTERS

- α = an arbitrary constant having any value less than one
 μ_f = viscosity of fluidizing gas
 ρ_f = density of fluidizing gas
 ρ_s = density of solid particles in the bed.

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APPENDIX A -- FORTRAN NOMENCLATURE AND SOURCE LIST

Appendix A shows the FORTRAN source list used in the program. The program was written in FORTRAN IV for an IBM 7040 computer. This list presents the program in detail and permits the potential user a high degree of versatility in solving various problems. The FORTRAN source list is shown on page 37. The nomenclature used in the FORTRAN source list follows:

1. NOMENCLATURE USED IN FORTRAN SOURCE LIST

AA = constant in particle-growth-rate equation which takes into account the effect of particle size on the particle growth rate, dimensionless.

AREA = cross-sectional area of fluidized bed, ft.

PC1

PC2 coefficients to describe attrition rate curve:

PC3 = $GAL = PC1 + PC2 * DIAP + \dots + PC5 * DIAP ** 4$

PC4 (GAL can be thought of as a negative growth rate, cm/hr).

PC5

BB = constant in particle growth rate equation which takes into account the effect of particle size on the particle growth rate, cm^{-1} .

BL = smallest particle size in fluidized bed, cm.

CAYS(I) = part of equation to find surface area of particle in ith size interval

$$\ln \frac{DI(I + 1)}{DI(I)} \cdot \frac{(6XSCF)}{RHO[DI(I + 1) - DI(I)]}$$

CI(J) = particle sizes used to describe the particle size distribution curve of the initial fluidized bed, cm. (No more than 50)

CI(K) = particle sizes used to describe the particle size distribution curve of the seeds, cm. (No more than 50)

DIAP = mean diameter of particle in the ith size interval, cm.

$$\frac{DI(I + 1) + DI(I)}{2}$$

DI(I) = particle sizes extending over the entire particle size range expected in the fluidized bed, cm. (No more than 2001)

DL(I) = interval width at the ith size interval, cm.

$$[DI(I + 1) - DI(I)]$$

DT1 = time step used in solving differential equations, hr.

DT2 = time step used for listing output, hr.

DT3 = number of hours desired for simulated calciner run, hr.

EK(I) = elutriation constants calculated from the equation:

$$\left[\frac{50.83 \cdot \text{RHOG}(\text{FVG}-\text{UTS})}{(980 \cdot \text{DIAP})} \right]^2 \left[\frac{\text{DIAP} \cdot \text{UTS}-\text{RHOG}}{\text{FMU}} \right]^{0.725} \left[\frac{\text{RHO}-\text{RHOG}}{\text{RHOG}} \right]^{1.15} \left[\frac{\text{FVG}-\text{UTS}}{\text{UTS}} \right]^{0.9}$$

(The term $\left[\frac{\text{FVG}-\text{UTS}}{\text{UTS}} \right]$ has been found to be necessary; however, the assigned value of 0.9 as the exponent is arbitrary. Experimental data are required to determine the most probable value for this exponent.)

FMU = viscosity of fluidizing gas, poises, g/cm-sec.

FVG = fluidizing velocity, cm/sec.

G = average particle growth rate, cm/hr, based on the equation:

$$G = \frac{2 \cdot \text{PO}}{\text{RHO} \cdot \text{SS}}$$

HD = number of lines reserved for header cards.

HD(I) =

IKZ = number of primary intervals to be summed for the purpose of the program punch out. These individual sums make up the cumulative particle size distribution curve punched out by the computer. If the weights for each secondary interval are to be listed instead of the cumulative particle size distribution curve, this number must be zero.

J = number of points to be read into program from initial bed particle size distribution curve. (No more than 50)

K = number of points to be read into program from seed particle size distribution curve. (No more than 50) (As initially defined in program)

L Indicates particle size. It can refer to sizes in the bed distribution curve or the seed size distribution curve.

L = cumulative average particle growth since time zero, cm/hr. (Used in program for calculations)

M = number of intervals in initial bed particle size distribution curve plus one. (Right adjusted, maximum of 2000)

- MK = When a product is desired in which only particles larger than a specified size is required, the symbol MK is used to represent this specific size. (Size is determined by the size in the mth size interval)
- N = the number of equally sized primary intervals in each secondary interval. (In order to increase the accuracy of the calculations, the particle size range of interest can be divided into as many as 2000 individual particle size intervals. For convenience these size intervals can be grouped into larger intervals called secondary intervals. Each secondary interval can be defined on one data input card by specifying the number, N, of equally sized primary intervals of width, DL, making up the secondary interval.) The size interval description will require NIC of these cards.
- NIC = number of secondary size intervals required to define the particle size interval description. (See definition of N)
- OPK = product factor. This term is used only when particles larger than a specified size, MK, are removed from the bed as product. This factor when multiplied by the weight in the bed of particles within the size intervals whose particle sizes are greater than the specified size, MK, should be particular interval, not size, will determine the product rate.
- PC(III) = spaces set aside for constants used in calculating elutriation rates and attrition rates. Eleven spaces are provided. The first five spaces are used for the attrition rate constants, which are read into the program. The last six spaces are used for storing the elutriation constants. These constants are a permanent part of the program.
- PO = equivalent solid rate of liquid feed, g/hr.
- PS(K) = a point (weight) on the cumulative size distribution curve for seeds, which corresponds to some particle size. The size distribution curve for the seeds is based on the actual weight in grams per hour of seeds entering the bed.
- RHO = density of the material being deposited on the particles in the bed, g/cc.
- RHOG = density of the fluidizing gas, g/cc.
- RIX The program is constructed so that if needed the number of intervals can be increased beyond the number, M, used initially to describe the particle size range expected in the run. When the weight in the size interval representing the largest particle size in the program exceeds a certain fraction of the weight in the particle size interval just smaller than it, a new and larger size interval is included in the next calculation. This fraction is defined in the program as RIX.

$S(I)$ = surface area of the particles in the i th particle size interval.

SCF = surface correction factor. Used to correct for non-spherical particles.

SP = rate of seed introduction, g/hr.

SPIB = smallest particle in the bed, cm.

SS = total surface area of particles in bed, cm^2 .

TCE = total growth of smallest particle in bed, cm.

TS = cumulative average particle growth since time zero, cm/hr.
(Used in program for calculations)

WB = initial bed weight, grams.

WO(J) = a point (weight) on the cumulative particle size distribution curve of the initial bed which corresponds to some particle size $CI(J)$, grams.

Y = drag factor for non-spherical particles.

2. FORTRAN SOURCE LIST

\$JOB C ESG21610-C422 40.0116 1 CALCINER PREDICTION CONVERSION
 \$IBJOB MAP
 \$IBFTC CPC

CALCINER PREDICTION CONVERSION	FORTRAN SOURCE LIST	03/09/66
ISN	SOURCE STATEMENT	
0	\$IBFTC CPC	
C	CALCINER PREDICTION CODE L.E.REESE FOR E. GRIMMETT	0010
C		0020
1	DIMENSION WO(50),HD(12),CI(50),CAYS(2000),TEMP(2000)	
2	DIMENSION W(2001),DW(2000),PS(2001),S(2000),DI(2001),PC(11),EK(20	
	100)	
3	COMMON PS,S,PO,SS,SP,WB, G,DI,M,AA,BB,PC,AREA,OPK,MK	
4	COMMON EK,GAL	
5	EXTERNAL EAR	0050
C		
C	READ INPUT PAREMETERS AND CALCULATE INITIAL BED CONDITIONS	
C	ALSO PREPARE OUTPUT OF INPUT PARAMETERS	
C		
6	222 READ (5,101)HD	0140
7	WRITE (6,102)HD	0150
10	WRITE (6,114)	
11	WRITE (6,115) HD	
12	READ (5,103)J,K,NIC,BL,DT1,DT2,DT3,M,AA,BB	0160
17	WRITE (6,103) J,K,NIC,BL,DT1,DT2,DT3,M,AA,BB	
20	READ (5,104)(WO(I),CI(I),I=1,J)	
25	WRITE(6,104)(WO(I),CI(I),I=1,J)	
32	SP18=CI(2)	
33	NIN=2	
34	DI(1)=BL	0190
35	I=2	0200
36	N=0	0210
37	TS=0.0	0220
40	TT=0.0	0230
41	DT4=0.0	0240
42	DO 4 I1=1,NIC	0250
43	READ (5,105)DL,N1	0260
45	WRITE(6,105)DL,N1	
46	N=N+N1	0270
47	DO 3 JJ=1,N1	0280
50	DI(I)=DI(I-1)+DL	0290
51	3 I=I+1	0300
53	4 CONTINUE	0310
55	J=2	0320
56	NN=N+1	0330
57	W(1)=WO(1)	0340
60	DO 5 I=2,NN	0350
61	8 J=J	
62	IF (DI(I)-CI(J)) 6,6,7	0360
63	6 W(I)=WO(J-1)+((DI(I)-CI(J-1))/(CI(J)-CI(J-1)))*(WO(J)-WO(J-1))	0370
64	5 CONTINUE	0380
66	GO TO 41	0390
67	7 J=J+1	0400
70	GO TO 8	0410
71	41 DO 9 I=1,N	0420
72	9 W(I)=W(I+1)-W(I)	0430
74	WBCX=0.0	
75	DO 1002 I=1,M	
76	1002 WBCX=WBCX+W(I)	
100	WRITE (6,106) WBCX	
101	READ (5,106) WB,PO,SP,RHO,RIX,IRZ,Y,SCF	

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103	WRITE(6,106) WB,PO,SP,RHO,RIX,IRZ,Y,SCF	
104	READ(5,113) RHOG,FMU,FVG,AREA,OPK,MK	
106	WRITE(6,113) RHOG,FMU,FVG,AREA,OPK,MK	
107	READ(5,1305)(WO(I),CI(I),I=1,K)	
114	WRITE(6,1305)(WO(I),CI(I),I=1,K)	
121	READ(5,113) (PC(III),III=1,5)	
126	WRITE(6,113) (PC(III),III=1,5)	
133	WRITE(6,102)HD	
134	WRITE(6,117)RHO,RHOG,FMU,FVG,AREA,SP,PO,DT1	
135	WRITE(6,107)	0450
136	IF (IRZ) 99,301,303	0460
137	301 WRITE(6,108)(DI(I),I=1,NN)	0470
144	GO TO 302	0480
145	303 WRITE(6,304)DI(1),DI(IRZ+1)	0490
146	IK=IRZ+1	0510
147	305 KI=IK+IRZ	0520
150	IF (KI-NN) 306,306,307	0530
151	306 WRITE(6,304)DI(KI),DI(KI)	0540
152	IK=KI	0550
153	IF (IK-NN) 305,302,302	0560
154	307 KI-NN	0570
155	GO TO 306	0580
156	302 WRITE(6,109) TT,WB,PO,SP,M,DT1	
157	WRITE(6,110)	0600
160	J=2	0630
161	PS(1)=WO(1)	0640
162	DO 14 I=2,NN	0650
163	12 J=J	
164	IF (DI(I)-CI(J)) 13,13,15	0660
165	13 PS(I)=WO(J-1)*((DI(I)-CI(J-1))/(CI(J)-CI(J-1)))*(WO(J)-WO(J-1))	0670
166	14 CONTINUE	0680
170	GO TO 42	0690
171	15 J=J+1	0700
172	GO TO 12	0710
173	42 DO 43 I=1,N	0720
174	43 PS(I)=PS(I+1)-PS(I)	0730
176	X=0.317	
177	PC(6)=-1.4706659E-03	
200	PC(7)=+1.0128452E-02	
201	PC(8)=-3.7994190E-03	
202	PC(9)=-5.2904083E-02	
203	PC(10)=-2.1038709E-01	
204	PC(11)=+9.1332559E-01	
205	DO 10 I=1,N	0740
206	DIAP=(DI(I+1)+DI(I))/2.0	
207	UTS=980.0*DIAP**2*(RHO-RHOG)*Y/(18.0*FMU)	
210	RENNO=DIAP*UTS*RHOG/FMU	
211	IF (RENNO-2972.0) 801,803,803	
212	801 IF (RENNO-X) 806,806,802	
213	802 XP=ALOG10(RENNO)	
214	GO TO 804	
215	803 XP=3.47305	
216	804 COFAC=PC(6)	
217	DO 805 J=7,11	
220	805 COFAC=COFAC*XP+PC(J)	

222	UTS=UTS*COFAC	
223	806 IF (FVG-UTS) 808,808,807	
224	8070EK(I)=50.83*RHO*((FVG-UTS)**2)*(DIAP*UTS*RHO/FMU)**.725*((RHO-RH 80710G)/RHO)**1.15*((FVG-UTS)/UTS)**.60/SQRT(980.0*DIAP)	
225	WRITE (6,3009) EK(I)	
226	3009 FORMAT(E16.7)	
227	GO TO 10	
230	808 EK(I)=0.0	
231	10 CAYS(I)= ALOG(DI(I+1)/DI(I))*SCF*6.0/(RHO*(DI(I+1)-DI(I)))	
233	23 SS=0.0	0760
234	DO 16 I=1,M	0780
235	S(I)=CAYS(I)*W(I)	0790
236	16 SS=SS+S(I)	0820
240	G=2.0*PO/(RHO*SS)	
241	IF (IT) 99,199,207	0840
242	199 IS=TS+G*DT1/2.0	0850
243	200 CALL DEQ(TT,DT1,N,W,DW,TEMP,EAR)	0860
244	IF(TT) 99,201,202	0870
245	201 IF (IRZ) 99,401,402	0880
246	401 WRITE(6,708)(W(I),I=1,M)	
253	GO TO 208	0900
254	402 IRW=1	0910
255	IRX=IRZ	0920
256	XXX=0.0	0930
257	408 DO 405 I=1,10	0940
260	406 DO 404 J=IRW,IRX	0950
261	404 XXX=XXX+W(J)/WB	
263	HD(I)=XXX	0970
264	IRW=IRW+IRZ	0980
265	IRX=IRX+IRZ	0990
266	405 CONTINUE	1000
270	WRITE (6,108)(HD(I),I=1,10)	1010
275	IF (IRX-M) 408,413,413	1020
276	413 IF (IT) 99,208,415	1030
277	202 DT4=DT4+DT1	1040
300	WB=0.0	1050
301	701 IF (SP.GT.0.) GO TO 799	
304	IF (GAL.NE.0.) GO TO 799	
307	ASXW=0.	
310	TSFP = TS + G*DT1/2.	
311	MMM=M/2+1	
312	FUDGE1= (AA*BB*(SPIB))/(AA*BB*(DI(MMM)))	
313	TCE=SPIB*TSFP*FUDGE1	
314	702 IF (DI(NIN),GT,TCE) GO TO 703	
317	ASXW=ASXW+ W(NIN-1)	
320	W(NIN-1)=0.	
321	NIN=NIN+1	
322	GO TO 702	
323	703 DO 704 I=1,20	
324	PITI=I	
325	ITTY= NIN-2+1	
326	704 W(ITTY)=W(ITTY) + .0025*(9.5 + PITI)*ASXW	
330	799 DO 205 I=1,M	1060
331	205 WB=WB+W(I)	1070
333	IF (ABS(DT2-DT4)-.1*DT1) 203,203,204	1080

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334	204	IF(W(M)-W(M-1)*RIX) 23,23,206	1090
335	206	M=M+1	1100
336		GO TO 23	1110
337	203	WRITE (6,109) TT,WB,PO,SP,M,DT1	
340		WRITE (6,110)	1130
341		IF (IRZ) 99,414,402	1140
342	414	WRITE(6,708)(W(I),I=1,M)	
347	415	DT4=0.0	1160
350		GO TO 204	1170
351	207	TS=TS+G*DT1/2.0	1180
352		CALL DEQSET	
353		CALL DEQ(TT,DT1,N,W,DW,TEMP,EAR)	
354		IF(DT4)209,209,210	1190
355	210	IF(DT3-TT) 211,211,199	1200
356	209	WRITE (6,112)G,SS,TS	1210
357		GO TO 210	1220
360	211	CALL DEQSET	1230
361		GO TO 222	1240
362	99	CALL EXIT	1250
363	208	WRITE (6,111)G,SS	1260
364		GO TO 200	1270
	C		1280
	C		1290
	C	FORMAT STATEMENTS	1300
365	101	FORMAT(12A6)	1310
366	102	FORMAT(1H112A6)	1320
367	103	FORMAT(2X12,2(1X,12),4F10,4,6X,14,2F10,5)	
370	104	FORMAT(1XF10,0,0PF10,5)	
371	105	FORMAT(1XF9,4,6X14)	
372	106	FORMAT(1XF10,0,F10,1,F10,2,F9.5,F10,5,6X14,2F10,5)	
373	107	FORMAT(19H0INTERVAL,ENDPOINTS)	1370
374	108	FORMAT(1H 10F12,6)	
375	1090	FORMAT(7H1TIME =F6.1,2X9HBED WT. =F10,0,2X6HFEED =F10,1,2X7HSEED	
	1091	=F10,2,2X3HM =14,2X4HDT =F7,3)	
376	110	FORMAT(24H0BED WEIGHT DISTRIBUTION)	1410
377	111	FORMAT(4H0G =F10,5,5X14HSURFACE AREA =F15,1)	
400	112	FORMAT(4H0G =F10,5,5X14HSURFACE AREA =F15,1,5X3HL =F10,5)	
401	113	FORMAT(1XF9.5,4F10,5,6X14)	
402	1140	FORMAT(12H0INPUT CARDS/35X12HCARD COLUMNS/4X4H1-106X5H11-205X5H21	
	1141-305X5H31-405X5H41-505X5H51-605X5H61-705X5H71-80/ 12H 23456789012		
	1142 62H34567890123456789012345678901234567890123456789012345678901234		
	1143 6H567890)		
403	115	FORMAT(1X12A6)	
404	1170	FORMAT(22H0DENSITY OF MATERIAL =F10,5/22H DENSITY OF GAS =F1	
	1171 0.5/22H VISCOSITY =F10,5/22H FLUIDIZING VELOCITY =F10,5		
	1172 /22H AREA OF EXPOSURE .. =F10,5/22H SEED RATE (HOURLY) =F10,2		
	1173 /22H FEED RATE (HOURLY) =F10,1/22H TIME STEP (HOURS) , =F10,5)		
405	304	FORMAT(5H FROMF10,4,4H TOF10,4)	0500
406	1305	FORMAT(1XF10,2 ,0PF10,5)	
407	708	FORMAT(1X10F10,2)	
410	500	FORMAT(1XF15,5)	
411		END	1440

```

0 $IBFTC EAR
C
C EAR-SUBROUTINE TO COMPUTE DERIVATIVES
1 SUBROUTINE EAR(TT,N,W,DW)
2 DIMENSION W(2001),DW(2000),PS(2001),S(2000),DL(2001),PC(11),EK(20
100)
3 COMMON PS,S,PO,SS,SP,WB, G,DL,M,AA,BB,PC,AREA,OPK,MK
4 COMMON EK,GAL
5 PON=PO
6 GAL=0.0
7 FAC1=0.0
10 SUMEK=0.0
11 RFAC=0.0
12 BF=BB/2.0
13 DO 1 I=1,M
14 RFAC=RFAC+(AA+BF*(DL(I+1)+DL(I)))*S(I)
15 1 SUMEK=SUMEK+EK(I)*W(I)
17 SUMEK=SUMEK+AREA/WB
20 WRITE (6,101) SUMEK
21 101 FORMAT(E16.8)
22 K=M-1
23 IF (OPK) 99,2,3
24 2 PROD=(PON+SP-SUMEK)/WB
25 IF (PROD) 20,3,3
26 20 PROD = 0.0
27 3 DO 10 I=1,K
30 I=I
31 GAL2=GAL
32 DLIA=(DL(I+2)+DL(I+1))/2.0
33 GAL=PC(I)
34 DO 4 J=2,5
35 4 GAL=GAL+DLIA+PC(J)
37 IF (OPK) 99,8,5
40 5 IF (I-MK) 6,6,7
41 6 PROD=0.0
42 GO TO 8
43 7 PROD=OPK
44 8 FAC2=FAC1
45 FAC1=AA+BF*(DL(I+1)+DL(I))
46 WTF1=FAC1*SS/RFAC
47 WTF2=FAC2*SS/RFAC
50 90 DW(I)= PS(I)
91 +(S(I)/SS)*PO+WTF1
92 -G*WTF1*(W(I)/(DL(I+1)-DL(I)))
93 +GAL*W(I+1)/(DL(I+2)-DL(I+1))
94 -GAL2*W(I)/(DL(I+1)-DL(I))
95 -PROD*W(I)
96 -EK(I)*AREA*W(I)/WB
97 +G*WTF2*(W(I-1)/(DL(I)-DL(I-1)))
51 10 CONTINUE
53 FAC2=FAC1
54 FAC1=AA+BF*(DL(M+1)+DL(M))
55 WTF1=FAC1*SS/RFAC
56 WTF2=FAC2*SS/RFAC
57 110 DW(M)= PS(M)

```

0020
0010

```

111 +(S(M)/SS)*PO+WTF1
112 -GAL*W(M)/(DL(M+1)-DL(M))
113 -PROD*W(M)
114 -EK(M)*AREA*W(M)/WB
115 +G*WTF2*W(M-1)/(DL(M)-DL(M-1))
60 J=M+1
61 DO 12 I=J,N
62 12 DW(I)=0.0
64 I=TT
65 RETURN
66 99 CALL EXIT
67 END

```

APPENDIX B -- ARRANGEMENT OF INPUT DATA

The arrangement of the input data is described in this section. The header card plus seven groups of data are required to get the necessary input information into the computer. Some of the groups require only one card while other groups require a number of cards. The sequence in which these cards must be used and the information required on them are described in the following paragraphs. An example of the data input for the 12-inch calciner is given.

1. PUNCH CARD SEQUENCE FOR INPUT DATA

1.1 Header Card

This card must precede the data cards. Any information may be put on this card, but it usually contains the program name or identification.

1.2 Data Card No. 1

Only one card is used. The column numbers used for each bit of input information are shown:

<u>Columns</u>	<u>Input Information</u>
3-4	= J, Number of points to be read in from the original bed particle size distribution (cumulative weight) curve -- right adjusted, maximum of 50 points.
6-7	= K, Number of points to be read from the seed particle size distribution (CW) curve -- right adjusted, maximum of 50 points.
9-10	= NIC, Number of size interval length changes needed.
11-20	= BL, Size of smallest particle in the bed at zero time -- use decimal.
21-30	= DTI, Time interval or "step" used in making the calculations -- use decimal.
31-40	= DT2, Time step used to list bed particle size distribution -- use decimal.
41-50	= DT3, Time of simulated run -- use time constant with time units used throughout program (seconds, minutes, hours, etc). Use only whole number.

<u>Columns</u>		<u>Input Information</u>
57-60	=	M, Number of nonzero intervals in the original bed plus one -- right adjusted, maximum of 3000 intervals
61-70	=	A, Constant in growth rate equation, use decimal.
71-80	=	B, Constant in growth rate equation, use decimal.

Note: See format statement 103 for placement of decimals.

1.3 Data Card(s) No. 2

J cards are required to describe the original bed distribution. Up to 50 data points from a cumulative weight curve can be used.

<u>Columns</u>		<u>Input Information</u>
2-11	=	WO(I), Weight from cumulative weight curve corresponding to same particle size, CI(I).
12-21	=	CI(I), Particle size from cumulative weight curve corresponding to the weight, WO(I).

Note: The next to the last data point should be the total bed weight and should be at the largest sized particle in the bed. The last data point should also be the total bed weight but should be at some particle size (within the limits of the program) larger than that obtained by any particle in the bed. The first data card should show zero weight at size BL. The second card should show zero weight at the smallest sized particle in the original bed. See format statement 104 for details on placement of decimals.

1.4 Data Card(s) No. 3

As many as 2000 particle size intervals can be used in this program. These intervals need not be the same size. Starting at the smallest particle size in the program, BL, the interval size, DL, can be defined and the number of intervals, N1, of that size desired. As many additional size intervals, each with the number of intervals of that size desired, can be specified so long as the total number of intervals of all sizes does not exceed 2000. Each size interval and the number of intervals of that size desired require a separate card. Each card is punched as shown below.

<u>Columns</u>		<u>Input Information</u>
2-10	=	DL, Size of particle size interval.
12-21	=	N1, Number of intervals of size intervals, DL, desired.

Note: See format statement 105 for details on placement of decimals.

1.5 Data Card No. 4

Only one card is used. The column numbers used for each bit of input information are shown:

<u>Columns</u>	<u>Input Information</u>
1-11	= WB, Original total bed weight.
12-21	= PO, Equivalent solid feed rate per unit of time.
22-30	= SP, Seed rate per unit of time.
31-40	= RHO, Particle density.
41-50	= RIX, Fraction of the weight of the bed in the next to the last interval before the number of intervals is increased beyond M.
57-60	= IRZ, Number of particle size intervals to be summed for each succeeding cumulative bed weight punchout. If the bed weight in each particle size interval is to be listed, IRZ should be zero.
61-70	= Y, A factor used to correct for non-spherical particles when calculating the terminal settling velocity of the particles in the bed.
70-80	= SCF, A factor used to correct for non-spherical particles when calculating the total surface area of the particles in the bed.

Note: See format statement 106 for details on placement of decimals.

1.6 Data Card No. 5

Only one card is used. The column numbers used for each bit of input information are shown:

<u>Columns</u>	<u>Input Information</u>
2-10	= RHOG, Density of the fluidizing medium.
11-21	= FMU, Viscosity of fluidizing medium.
22-31	= FVG, Superficial velocity of the fluidizing medium based on the temperature and pressure in the vessel, and the cross-sectional area of the vessel.

<u>Columns</u>	<u>Input Information</u>
32-41	= AREA, Cross-sectional area of the fluidized bed at its surface.
42-51	= OPK, A product rate factor used <u>only</u> when product particles larger than a given size are desired. This factor when multiplied by the bed weight in each size interval determines the amount of product removed from each size interval per unit of time. The sum of all the product removed from each of the size intervals is the product rate.
58-61	= MK, Particle size interval used when the program calls for a product which contains particles larger than a given size. MK is the size interval associated with the smallest particle in the product. It can be any of the particle size intervals from one to M. If this program option is not used, OPK should be 0.00 and MK should be 0.

Note: See format statement 113 for details on placement of decimals.

1.7 Data Card(s) No. 6

K cards are required to describe the particle size distribution of the seeds. Up to 50 points can be used.

<u>Columns</u>	<u>Input Information</u>
2-11	= WL(I), Weight from cumulative weight curve corresponding to some particle size CI(I).
12-21	= CI(I), Particle size from the cumulative weight curve corresponding to the weight WO(I).

Note: The cumulative weight curve for the seeds is actually based on the total seed addition rate. The cumulative weight at the largest particle size actually represents the total seed rate into the bed. The next to the last data point should be the total seed rate into the bed and should be at the largest sized particle in the bed. The last data point should also be the total seed rate into the bed and should be at some particle size (within the limits of the program) beyond that which the largest particle should never exceed. The first data point should show zero seed rate at size BL. The second data card should show zero seed rate at the smallest sized seed particle being introduced into the bed. See format statement 1305 for details on placement of decimals.

1.8 Data Card No. 7

This card provides a method to introduce factors to account for attrition in the bed. The input data consists of five constants. These constants are used in an equation of the form:

$$GAL = PC1 + (PC2)(DIAP) + \cdots (PC5)(DIAP)^{[4]}.$$

GAL is the attrition rate of the particles of size DIAP, and can be considered a negative particle growth rate. This equation is an empirical expression which will simulate attrition in a fluidized bed if the proper constants are used.

<u>Columns</u>	<u>Input Information</u>
2-10	= PC(5), Attrition rate constant.
11-21	= PC(4), Attrition rate constant.
22-31	= PC(3), Attrition rate constant.
32-41	= PC(2), Attrition rate constant.
42-51	= PC(1), Attrition rate constant.

Note: See format statement 113 for details on placement of decimals.

2. EXAMPLE OF 12-INCH CALCINER INPUT-DATA

```

FLUIDIZED BED MODEL (RUN 40-12INCH) EARL GRIMMETT

INPUT CARDS
CARD COLUMNS
1-10 11-20 21-30 31-40 41-50 51-60 61-70 71-80
23456789012345678901234567890123456789012345678901234567890
FLUIDIZED BED MODEL (RUN 40-12INCH) EARL GRIMMETT
14 9 1 0.0001 0.5000 4.0000 152.0000 360 0.54200 9.12000
    0. 0.00010
    0. 0.01300
    165. 0.01470
    2187. 0.02080
    8299. 0.02950
    37089. 0.04170
    43860. 0.04950
    49186. 0.05890
    52654. 0.07010
    56698. 0.08330
    59837. 0.09910
    61202. 0.11680
    63400. 0.17000
    63400. 4.80000
    0.0005 500
    63400.
    63400. 568.0 41.00 1.91000 0.75000 10 1.00000 1.00000
0.00051 0.00025 50.00000 0.77700 0.00000 0
    0.00 0.00010
    0.00 0.01240
    3.32 0.01300
    10.74 0.01400
    19.94 0.01500
    28.17 0.01600
    37.15 0.01700
    41.00 0.01740
    41.00 4.80000
0.00000 0.00000 0.00000 0.00000 0.00000

```

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