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Galilean Invariance and Green's Functions for Bound Systems*

M. Bolsterli

School of Physics, University of Minnesota, Minneapolis, Minnesota

and

University of California,
Los Alamos Scientific Laboratory, Los Alamos, New Mexico

It is shown that Galilean invariance leads to a natural definition of invariant Green's functions for bound systems. The spectral representation and differential equation for the single-particle Green's function are given.

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Galilean Invariance

Quantum-mechanical Galilean invariance requires, in addition to the existence of the four infinitesimal translation operators $\hat{E}$, $\hat{P}$ and the three rotation generators $\hat{J}$, the existence of a three-parameter set of unitary velocity transformations $U(\underline{V})$ whose action on state vectors is given by¹

$$\begin{aligned} U(\underline{V}) |M_1, \underline{P}_1; M_2, \underline{P}_2; \dots \text{in} \rangle \\ = |M_1, \underline{P}_1 + M_1 \underline{V}; M_2, \underline{P}_2 + M_2 \underline{V}; \dots \text{in} \rangle \end{aligned} \quad (1)$$

(similarly for "out" states) and which transform the second-quantized wave function for a particle field of mass M according to

$$U(\underline{V}) \psi(t, \underline{x}) U^\dagger(\underline{V}) = e^{-iM\underline{V} \cdot (\frac{1}{2} \underline{V}t + \underline{x})} \psi(t, \underline{x} + \underline{V}t) \quad (2)$$

The transformation $U(\underline{V})$ satisfies

$$U(-\underline{V}) = U^\dagger(\underline{V}) = U^{-1}(\underline{V}) \quad (3)$$

Application to Single-Particle Matrix Elements

Consider the matrix element $\langle N \underline{\alpha} \underline{K} | \psi(\underline{x}) | N+1 \beta \underline{K}' \rangle$. Here $|N \underline{\alpha} \underline{K}\rangle$ is the state consisting of N particles in internal state α with total (center-of-mass) momentum $\underline{K}$ and energy $N^{-1} \underline{K}^2 + \mathcal{E}_\alpha$ with $\mathcal{E}_\alpha$ the internal energy of the state α (units $\hbar = 2m = 1$). The symbol $\underline{x}$ represents $t, \underline{x}$. Now (1) - (3), give²

$$\begin{aligned}
\langle N \underline{K} | \psi(0) | N + 1 \underline{K}' \rangle &= \langle N \underline{K} - \frac{N}{N+1} \underline{K}' | U^\dagger \left(\frac{2\underline{K}'}{N+1} \right) \psi(0) U \left(\frac{2\underline{K}'}{N+1} \right) | N + 1 \underline{K}' \rangle \\
&= \langle N \underline{K} - \frac{N}{N+1} \underline{K}' | \psi(0) | N + 1 \underline{K}' \rangle = \langle N \underline{K} | \psi(0) | N + 1 \underline{K}' - \frac{N+1}{N} \underline{K} \rangle \\
&\equiv (2\pi)^{-3/2} \tilde{\psi}_{\alpha\beta} \left(\frac{N}{N+1} \underline{K}' - \underline{K} \right), \quad (4)
\end{aligned}$$

and, with translations,

$$\begin{aligned}
\langle N \underline{K} | \psi(x) | N + 1 \underline{K}' \rangle &= (2\pi)^{-3/2} e^{i \left\{ \left(\frac{K^2}{N} + \epsilon_\alpha - \frac{K'^2}{N+1} - \epsilon_\beta \right) t - (\underline{K} - \underline{K}') \cdot \underline{x} \right\}} \\
&\quad \tilde{\psi}_{\alpha\beta} \left(\frac{N}{N+1} \underline{K}' - \underline{K} \right) \quad (5)
\end{aligned}$$

Application to Green's Functions

The Green's functions considered here will be expectation values of the usual time-ordered products in the ground state g_n of the N-particle system. This state is assumed bound. Because of translation invariance, allowance must be made for momentum to be transferred to this state. The Green's functions therefore involve the matrix elements (with $l = x_1 = (t_1, \underline{x}_1)$, etc.)

$$\begin{aligned}
\langle g_N, \underline{K} - \underline{k}/2 | T \left(\psi(1) \psi(2) \dots \psi(n) \psi^\dagger(n') \dots \psi^\dagger(1') \right) | g_N, \underline{K} + \underline{k}/2 \rangle \\
= \langle g_N, -\underline{k}/2 | U^\dagger \left(\frac{2\underline{K}}{N} \right) T \left(\psi(1) \dots \psi^\dagger(1') \right) U \left(\frac{2\underline{K}}{N} \right) | g_N, \underline{k}/2 \rangle \\
= e^{i \frac{\underline{K}}{N} \cdot (\sum \underline{x}_i - \sum \underline{x}_{i'}) - i \frac{K^2}{N^2} (\sum t_i - \sum t_{i'})}. \quad (6)
\end{aligned}$$

$$\cdot \langle g_N, -k/2 | T \left(\psi(t_1, x_1 - \frac{2K}{N} t_1) \dots \psi^\dagger(t'_1, x'_1 - \frac{2K}{N} t'_1) \right) | g_N, k/2 \rangle$$

Thus, all Green's functions are simply related to the functions

$$\begin{aligned} G_{Nk} (1, \dots n; 1', \dots n') &= (-i)^n \langle g_N, -k/2 | T \left(\psi(1) \dots \psi^\dagger(1') \right) | g_N, k/2 \rangle \\ &= (-i)^n e^{+ik \cdot A} \langle g_N, -k/2 | T \left(\psi(1 - A) \dots \psi^\dagger(1' - A) \right) | g_N, k/2 \rangle \end{aligned} \quad (7)$$

where A is any four-vector. The special choice

$$A_c = (\Sigma x_i + \Sigma x'_i) / 2n \quad (8)$$

explicitly reduces the number of coordinates to just the translation-invariant ones; hence, it follows from Galilean invariance that all Green's functions can be expressed in terms of the invariant Green's functions

$$G_{Nk}^I (1 \dots n; 1' \dots n') = e^{-ik \cdot A_c} G_{Nk} (1 \dots n; 1' \dots n') \quad , \quad (9)$$

where the invariant Green's function G^I depends only on translation invariant combinations of the coordinates.

Spectral Representation of the Invariant Single-Particle Green's Function

The invariant single-particle Green's function is (upper sign for Fermions, lower for Bosons)

$$g_{Nk}(x) = (-i) \langle g_N, -k/2 | T \left(\psi(\frac{x}{2}), \psi^\dagger(-\frac{x}{2}) \right) | g_N, k/2 \rangle$$

$$\begin{aligned}
&= -i\theta(t) S_\alpha \int d\underline{p} \langle \underline{g}_N - \underline{k}/2 | \psi(\frac{\underline{x}}{2}) | N+1 \alpha \underline{p} \rangle \langle N+1 \alpha \underline{p} | \psi^\dagger(-\frac{\underline{x}}{2}) | \underline{g}_N \underline{k}/2 \rangle \\
&\quad \pm i\theta(-t) S_\beta \int d\underline{p} \langle \underline{g}_N - \underline{k}/2 | \psi^\dagger(-\frac{\underline{x}}{2}) | N-1 \beta - \underline{p} \rangle \langle N-1 \beta - \underline{p} | \psi(\frac{\underline{x}}{2}) | \underline{g}_N \underline{k}/2 \rangle \\
&= \int \frac{d\underline{p}}{(2\pi)^3} e^{i\underline{p} \cdot \underline{x}} \left[-i\theta(t) S_\alpha e^{i \left(\frac{k^2}{4N} + \epsilon_g - \frac{p^2}{N+1} - \epsilon_\alpha \right) t} \tilde{\psi}_{g\alpha} \left(\frac{N}{N+1} \underline{p} + \frac{\underline{k}}{2} \right) \tilde{\psi}_{g\alpha}^* \left(\frac{N}{N+1} \underline{p} - \frac{\underline{k}}{2} \right) \right. \\
&\quad \left. \pm i\theta(-t) S_\beta e^{-i \left(\frac{k^2}{4N} + \epsilon_g - \frac{p^2}{N-1} - \epsilon_\beta \right) t} \tilde{\psi}_{\beta g}^* \left(\underline{p} - \frac{N-1}{N} \frac{\underline{k}}{2} \right) \tilde{\psi}_{\beta g} \left(\underline{p} + \frac{N-1}{N} \frac{\underline{k}}{2} \right) \right] \\
&= \int \frac{d\underline{p} d\lambda}{(2\pi)^4} e^{-i\lambda t} e^{i\underline{p} \cdot \underline{x}} \tilde{g}_{Nk}(\lambda, \underline{p}) \\
&\tilde{g}_{Nk}(\lambda, \underline{p}) = S_\alpha \frac{\tilde{\psi}_{g\alpha} \left(\frac{N}{N+1} \underline{p} + \frac{\underline{k}}{2} \right) \tilde{\psi}_{g\alpha}^* \left(\frac{N}{N+1} \underline{p} - \frac{\underline{k}}{2} \right)}{\lambda - \left(\epsilon_\alpha + \frac{p^2}{N+1} - \epsilon_g - \frac{k^2}{4N} \right) + i0} \pm S_\beta \frac{\tilde{\psi}_{\beta g} \left(\underline{p} - \frac{N-1}{N} \frac{\underline{k}}{2} \right) \tilde{\psi}_{\beta g}^* \left(\underline{p} + \frac{N-1}{N} \frac{\underline{k}}{2} \right)}{\lambda - \left(\epsilon_g + \frac{k^2}{4N} - \epsilon_\beta - \frac{p^2}{N-1} \right) - i0} \quad (11)
\end{aligned}$$

Eq. (5) has been used in deriving Eq. (10).³

Reference 2 gives a condition on the weight functions in (11), namely,

$$\begin{aligned}
&S_\alpha \tilde{\psi}_{g\alpha} \left(\frac{N}{N+1} \underline{p} + \frac{\underline{k}}{2} \right) \tilde{\psi}_{g\alpha}^* \left(\frac{N}{N+1} \underline{p} - \frac{\underline{k}}{2} \right) \\
&\quad \pm S_\beta \tilde{\psi}_{\beta g}^* \left(\underline{p} - \frac{N-1}{N} \frac{\underline{k}}{2} \right) \tilde{\psi}_{\beta g} \left(\underline{p} + \frac{N-1}{N} \frac{\underline{k}}{2} \right) = \delta(\underline{k}) \quad (12)
\end{aligned}$$

The delta function comes from the terms with α equal to gq , the internal state consisting asymptotically of $\underline{g}_N$ plus a single particle with relative momentum $\underline{q}$:

$$\tilde{\psi}_{g, gq}(\underline{k}) = \delta(\underline{k} - \underline{q}) + \tilde{\psi}'_{g, gq}(\underline{k}), \quad (13)$$

where $\tilde{\psi}'$ has no delta-function part. Then let

$$\tilde{\psi}_{g\alpha}(\underline{k}) = \tilde{\psi}'_{g\alpha}(\underline{k}) \quad \alpha \neq gq \quad (14)$$

so that

$$\begin{aligned} S_{\alpha} \tilde{\psi}_{g\alpha}(\underline{k}) \tilde{\psi}_{g\alpha}^*(\underline{k}') &= \delta(\underline{k} - \underline{k}') + \tilde{\psi}'_{g, gK}(\underline{k}) + \tilde{\psi}_{g, gK}^{**}(\underline{k}') \\ &\quad + S_{\alpha} \tilde{\psi}'_{g\alpha}(\underline{k}) \tilde{\psi}_{g\alpha}^{**}(\underline{k}') \end{aligned} \quad (15)$$

Since $\epsilon_{gq} = \frac{N+1}{N} q^2 + \epsilon_g$, it follows that

$$\tilde{g}_{Nk}(\lambda, p) = \tilde{g}_0(\lambda, p) \delta(\underline{k}) + \tilde{g}'_{Nk}(\lambda, p) \quad (16)$$

$$g_{Nk}(x) = g_0(x) \delta(\underline{k}) + g'_{Nk}(x)$$

$$\tilde{g}_0(\lambda, p) = (\lambda - p^2 + i0)^{-1} \quad (17)$$

$$g_0(x) = \int \frac{dp \, d\lambda}{(2\pi)^4} e^{-i(\lambda t - p \cdot x)} (\lambda - p^2 + i0)^{-1}$$

$$g'_{Nk}(x) = \int \frac{dp \, d\lambda}{(2\pi)^4} e^{-i(\lambda t - p \cdot x)} \tilde{g}'_{Nk}(\lambda, p) \quad (18)$$

$$\tilde{g}'_{Nk}(\lambda, p) = \frac{\tilde{\psi}'_{g, gK}(s)}{\lambda - (\underline{p} - \frac{\underline{k}}{2})^2 + i0} + \frac{\tilde{\psi}'_{g, gK}(t)}{\lambda - (\underline{p} + \frac{\underline{k}}{2})^2 + i0} + S_{\alpha} \frac{\tilde{\psi}_{g\alpha}(s) \tilde{\psi}'_{g\alpha}(t)}{\lambda - (\epsilon_{\alpha} + \frac{p^2}{N+1} - \epsilon_g - \frac{k^2}{4N}) + i0}$$

$$\pm S_{\beta} \frac{\tilde{\psi}_{\beta g}^* \left(\underline{p} - \frac{N-1}{N} \frac{\underline{k}}{2} \right) \tilde{\psi}_{\beta g} \left(\underline{p} + \frac{N-1}{N} \frac{\underline{k}}{2} \right)}{\lambda - \left(\epsilon_g + \frac{k^2}{4N} - \epsilon_{\beta} - \frac{p^2}{N-1} \right) - i0} \quad (19)$$

$$\underline{s} = \frac{N}{N+1} \underline{p} + \frac{\underline{k}}{2} \quad \underline{t} = \frac{N}{N+1} \underline{p} - \frac{\underline{k}}{2}$$

and (12) can be written, using (15), as

$$\tilde{\psi}'_{g, g'_{\underline{N}\underline{k}}}(\underline{s}) + \tilde{\psi}'_{g, g'_{\underline{N}\underline{k}}}(\underline{t}) + S_{\alpha} \tilde{\psi}'_{g'_{\alpha}}(\underline{s}) \tilde{\psi}_{g'_{\alpha}}^*(\underline{t}) + S_{\beta} \tilde{\psi}_{\beta g}^* \left(\underline{p} - \frac{N-1}{N} \frac{\underline{k}}{2} \right) \tilde{\psi}_{\beta g} \left(\underline{p} + \frac{N-1}{N} \frac{\underline{k}}{2} \right) = 0$$

That is, the integral of the weight functions in the spectral representation of $g'_{\underline{N}\underline{k}}$ is zero for all $\underline{k}$.

The part of g or g' with $+i0$ in the denominator is the "particle" part of g or g' ; the $-i0$ term is the "hole" part. Since g_0 is just the Green's function for $N=0$, it is seen that the $g_0 \delta(\underline{k})$ term is just the free-particle propagation part of $g_{\underline{N}\underline{k}}$. Equation (16) is just the first examples of a general rule for separating out free propagation. It can also be written in terms of $G_{\underline{N}\underline{k}}(1;1')$:

$$G_{\underline{N}\underline{k}}(1;1') = G_0(1;1') \delta(\underline{k}) + G'_{\underline{N}\underline{k}}(1;1') \quad (21)$$

Similarly, for the two-particle Green's function:

$$\begin{aligned} G_{\underline{N}\underline{k}}(12;1'2') &= G_0(12;1'2') \delta(\underline{k}) \\ &+ (1 \mp P_{12}) (1 \mp P_{1'2'}) G_0(1;1') G'_{\underline{N}\underline{k}}(2;2') + G'_{\underline{N}\underline{k}}(12;1'2') \end{aligned} \quad (22)$$

In such reductions, every particle in G'_{Nk} interacts at least once with the N -particle ground state. The terms in (21) and (22) with G_0 as a factor depend in a nontrivial way on fewer relative coordinates than those without G_0 . When n becomes $\geq N$, then the resolution (21), (22), etc., begins to break down, and care must be exercised in writing the analogous equations.

Note that for noninteracting nonidentical particles the G'_{Nk} would all be zero, so that in this case the magnitude of the weight functions in G'_{Nk} is a measure of the strength of the interaction between the single particle and the stable g_N . Because of the zero on the right-hand side of (20), it might seem that the weight functions can be arbitrarily small. However, the identity of the particles, together with the assumption that g_N is a bound state, places a limit on part of (20). This is best seen by using the invariant density defined in reference 2:

$$\langle g_N, -\frac{k}{2} | \psi^+(0\tilde{x}) \psi(0\tilde{x}) | g_N \frac{k}{2} \rangle = (2\pi)^{-3/2} \tilde{n}_{gg}(\tilde{k}) e^{ik \cdot \tilde{x}} \quad (23)$$

$$\begin{aligned} &= S_\beta \int d\tilde{p} \langle g_N - \frac{k}{2} | \psi(0\tilde{x}) | N-1 \beta - \tilde{p} \rangle \langle N-1 \beta - \tilde{p} | \psi(0\tilde{x}) | g_N \frac{k}{2} \rangle \\ &= e^{ik \cdot \tilde{x}} S_\beta \int \frac{d\tilde{p}}{(2\pi)^3} \tilde{\psi}_{\beta g}^* \left(\tilde{p} - \frac{N-1}{N} \frac{\tilde{k}}{2} \right) \tilde{\psi}_{\beta g} \left(\tilde{p} + \frac{N-1}{N} \frac{\tilde{k}}{2} \right) \end{aligned} \quad (24)$$

Hence

$$\tilde{n}_{gg}(\tilde{k}) = S_\beta \int \frac{d\tilde{p}}{(2\pi)^{3/2}} \tilde{\psi}_{\beta g}^* \left(\tilde{p} - \frac{N-1}{N} \frac{\tilde{k}}{2} \right) \tilde{\psi}_{\beta g} \left(\tilde{p} + \frac{N-1}{N} \frac{\tilde{k}}{2} \right) \quad (24)$$

and

$$\begin{aligned}
N\delta(\underline{k}) &= \langle g_N, -\frac{\underline{k}}{2} \rangle \int d\underline{x} \psi^\dagger(0, \underline{x}) \psi(0, \underline{x}) |g_N \frac{\underline{k}}{2}\rangle \\
&= (2\pi)^{3/2} \tilde{n}_{gg}(0) \delta(\underline{k})
\end{aligned}
\tag{25}$$

Therefore

$$N = (2\pi)^{3/2} \tilde{n}_{gg}(0) = S_\beta \int d\underline{p} \tilde{\psi}_{\beta g}(\underline{p}) \tilde{\psi}_{\beta g}(\underline{p}) \tag{26}$$

It follows from (19) and (24) that the weight function of the hole part of $g_{N\underline{k}}^i$ integrated over $\underline{p}$ must give $(2\pi)^{3/2} \tilde{n}_{gg}(\underline{k})$, and this latter is equal to N for $\underline{k} = 0$.

Note that comparison of (23) and (10) gives

$$g_{N\underline{k}}^i(0) = g_{N\underline{k}}(0) = \pm i (2\pi)^{-3/2} \tilde{n}_{gg}(\underline{k}), \tag{27}$$

which also follows from (19) and (24). Equations (16) and (27) indicate that it is highly unlikely that $g_{N\underline{k}}^i$ can be developed as a perturbation series in the interparticle interaction, since the "unperturbed" part of $g_{N\underline{k}}$ is just g_0 , but, owing to (27), the perturbation cannot be arbitrarily small. Some other starting point, such as, for example, a Hartree or Hartree-Fock approximation must be used. Of course, perturbation theory can be used if the starting-point is taken to be free particles, but then the information that the N -particle ground state is bound is difficult to incorporate and the spectral information contained in the momentum-dependent energy denominators in (11) would not be easily obtained. By explicitly incorporating the bound-state nature of g_N it may be possible to avoid such difficulties.

Differential Equations for Invariant Green's Functions

With the assumption of just two-particle interactions, the differential equation for the operator $\psi(x)$ is

$$D_x \psi(x) = (i \frac{\partial}{\partial t} + \nabla_x^2) \psi(x) = \int dy V(x - y) \psi^\dagger(y) \psi(y) \psi(x), \quad (28)$$

where

$$V(x - y) = \delta(t_x - t_y) v(\underline{x} - \underline{y})$$

with $v(x - y)$ the two-particle potential. Eq. (28) is invariant under Galilean transformations and leads to the well-known equation⁵ for the Green's functions

$$D_1 G_{Nk}(1; 1') = \delta(\underline{k}) \delta(1 - 1') + i \int V(1 - 2) G_{Nk}(12; 1'2') d2 \quad (29)$$

$$D_1 G_{Nk}(1 \dots n; 1' \dots n') = (1 + \sum_{i=1}^n P_{1,i}) \delta(1 - 1') G_{Nk}(2 \dots n; 2' \dots n') \quad (30)$$

$$+ i \int V(1 - (n+1)) G_{Nk}(12 \dots n, n+1; 1' \dots n' (n+1)^+) d(n+1)$$

where n^+ stands for $t_n + 0$, $\underline{x}_n$ and is used to insure that $\psi^\dagger(y)$ in (28) stands to the left of $\psi(y)$.⁴

Reductions of the form of (21) and (22) can be used to write (29) and (30) in terms of the G_{Nk}^i functions. In such reductions the $\delta(\underline{k})$ terms cancel. For example (29) becomes

$$D_1 G_{\mathbf{N}\mathbf{K}}^{\mathbf{I}}(1;1') = \mp i \int V(1-2) [G_0(1;1') G_{\mathbf{N}\mathbf{K}}^{\mathbf{I}}(2;2') \mp G_0(2;1') G_{\mathbf{N}\mathbf{K}}^{\mathbf{I}}(1;2') + G_{\mathbf{N}\mathbf{K}}^{\mathbf{I}}(12;1'2')] d2 \quad (31)$$

where $G_0(2,2') = 0$ and $\delta(t_1 - t_2) G_0(1;2') = 0$ have been used. Invariance properties can now be used to reduce (31) to an equation involving only invariant functions. The set of equations thus obtained is what one would like to solve for the invariant Green's functions. However, these equations are not as transparent as Eq. (31), and will not be given here. A subsequent paper will consider some of the approximations that can be used on this equation.

Summary

The effects of Galilean invariance on the properties of Green's functions for bound systems have been explored. Invariant Green's functions have been defined and shown to split naturally into "free" and "interacting" parts. The spectral representation of the invariant single-particle Green's function is given in Eqs. (16) and (19). The separation of the free part leads to the unusual sum rule (20). The boundary condition (27), together with (26), shows the impossibility of an expansion in powers of the interparticle interaction. Finally, the differential equations for the Green's functions have been considered briefly.

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