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INFORMAL REPORT

Energy Loss in Superconducting Magnetic Energy Storage Coils

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ENERGY LOSS IN SUPERCONDUCTING MAGNETIC ENERGY STORAGE COILS

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ABSTRACT

A simplified analysis has been carried out to estimate the energy loss in some of the superconducting magnetic energy storage coils constructed or planned in connection with the Los Alamos Scientific Laboratory plasma physics program.

I. INTRODUCTION

Superconducting magnetic energy storage has recently been discussed as a possible energy source for future controlled thermonuclear reaction experiments.¹ At the Los Alamos Scientific Laboratory several energy storage coils have been constructed and others are being planned² as part of the physics program working towards energy storage facilities large enough to be used on a scientific feasibility demonstration of fusion power. Coils which have been built or are under construction at LASL range from 30 to 300 kJ; stored energy of 10^9 J will be required for a feasibility experiment. In storage coils for a thermonuclear feasibility experiment, the magnitude of the energy loss is negligible compared to the dissipative losses which accompany irreversible switching. However, for a pulsed reactor these losses become a matter of overriding economic importance. Because of shortcomings in the theory for predicting energy losses and the importance of these losses for pulsed fusion reactors, it will be very important to measure energy loss characteristics at each stage of storage coil development in order to gain a better understanding of their nature. Losses in small superconducting coils have been customarily measured by connecting the coils to an AC power source of variable voltage and frequency and measuring the steady state power dissipation, but this is hardly possible with most energy storage coils because of the large amount of energy involved or the short time normally taken

to discharge the coils. For a storage coil discharging in a few milliseconds these rates go as high as 10^6 or 10^7 G/sec compared to 5×10^4 G/sec for a superconducting synchrotron magnet. Losses in large coils can easily be measured at different rates of discharge and different energizing currents, but these measured losses must then be related to the various mechanisms responsible for them. One motivation for the calculation presented here is to provide a framework for interpreting the experimental results as they become available. Also since magnetic energy storage seems destined to play an increasingly important role in fusion research, it may be worthwhile to collect together in one place a summary of some of the factors which govern energy loss in superconducting magnetic energy storage coils.

The exact calculation of storage coil energy losses would be a very formidable task because of complex geometry, the composite nature of the conductors, and dependence on mechanical details of construction. In the calculations that follow, the philosophy applied was to put in many of the processes recognized as important but to use simple approximate expressions to make the calculation more tractable. Hopefully this procedure will describe the physical processes well enough to provide all the qualitative features of the energy loss and some rough idea of its order of magnitude and scaling with various coil parameters. More

sophisticated calculations can be made later in those areas where the calculated results are shown to be deficient by the experimental data.

Energy loss in a superconducting magnetic energy storage coil occurs whenever the magnetic field in the coil is changing, and hence there will be loss whenever the coil is energized or discharged. In all that follows the material from which the coil winding is made consists of fine filaments of NbTi superconductor imbedded in a copper matrix. In some commercially available wire this copper matrix may be imbedded in turn in another matrix of copper-nickel and the entire mixed matrix drawn to form the final wire. Only a pure copper matrix is considered here. Wires of the composite material are braided or twisted together to form the final conductor. The individual wires are usually twisted as they are drawn, so the superconducting filaments are helical in shape. Sources of ac losses in superconducting coils are usually listed under three headings: eddy current loss, hysteresis loss, and joule loss.^{3,4} Eddy current loss is caused by a changing magnetic field passing through the conductor which induces a voltage across sections of the non-superconducting matrix material. Currents driven by this voltage are dissipated as in ordinary joule loss. Hysteresis loss is caused by the fact that a type II superconductor above the Meissner limit exhibits a resistance whenever the current is changed. At present there exists no definitive theory of this "flux-flow" resistance, however, an order of magnitude calculation of the loss is obtainable by equating the power loss to the product of the electric field produced through the changing magnetic field times the current flowing along this electric field. Finally, if the coil or some section of it should go normal there will be ordinary joule loss as well as the other two. These three sources of energy loss are of course not completely independent so there is some approximation in calculating each loss separately. Moreover most of the expressions which have been derived in the literature are for particular regimes of operation as far as rates of change of field and conductor construction are concerned. Behavior in transition regions is unclear. Some of these problems will be discussed more in following sections.

II. HYSTERESIS LOSS

A. Simple Estimate of Loss

Hysteresis loss is an energy loss in the superconducting material itself which occurs whenever the magnetic field perpendicular to the winding changes. Loss occurs even though the material remains superconducting to the transport current. An estimate of the loss has been derived in the following way.⁵ Consider a slab of superconductor as in Fig. 1 in which there is a changing magnetic field in the $-z$ direction. The changing B causes a diamagnetic current to be induced in the slab and flow around its perimeter. When the current reaches the critical current density j_c , the slab becomes "resistive" and the magnetic field and diamagnetic current penetrate into the slab. The power loss in watts can be calculated from

$$\frac{dW_H}{dt} = \int j_c E dv,$$

where E is the induced electric field in V/cm, j_c is the critical current density in A/cm², and v is the volume of the slab in cm³. From Maxwell's equations,

Fig. 1. Simple slab geometry for estimating hysteresis loss.

$$\nabla \times \mathbf{E} = 10^{-8} \dot{B}$$

or

$$E = 10^{-8} \dot{B} \zeta,$$

where $\dot{B}$ is in G/sec and the electric field along the center of the slab is taken to be zero. Then

$$\frac{dW_H}{dt} = \int_{-d/2}^{d/2} j_c \dot{B} x_0 y_0 \zeta d\zeta,$$

and finally,

$$\frac{dW_H}{dt} = \frac{10^{-8}}{4} \dot{B} j_c v d. \quad (1)$$

The reason for the term hysteresis loss becomes evident here: If Eq. 1 were to be integrated between two values of B the loss would be independent of the time for change. Integrating Eq. (1) from zero to B gives

$$W_H = \frac{10^{-8}}{4} B j_c v d. \quad (2)$$

Equations 1 and 2 may be extended to the case of a filamentary superconducting wire. One important thing to notice in this extension is that the currents might either be taken to flow in both directions on each filament or to flow in one direction in the filaments on one side of the wire, through the matrix, and then in the opposite direction in filaments on the opposite side. If the diamagnetic current were to shield the entire wire, the diameter of a filament (d) in Eq. 1 would have to be replaced by the diameter of the wire and the loss would be much higher. Basically this is due to the larger electric field which would appear across the entire wire compared to that across a single filament.

B. Twisting of Filaments

It is proper to use the diameter of a filament instead of the diameter of the wire in Eq. 1 if the wire is twisted so rapidly that each filament crosses to the opposite side of the wire (where the induced electric field is in the opposite direction) before the current can flow through the intervening matrix material to an adjacent filament. However,

if the wire is not twisted enough, the diamagnet currents can flow through the matrix material to complete the circuit with filaments on the other side of the wire, and the wire diameter must be used in Eq. 1. The higher $\dot{B}$ becomes the more the wire must be twisted to prevent this coupling of filaments from happening. As an estimate of how much the wire must be twisted, a critical twist length can be calculated. Filaments twisted around once in helix in this distance would be shorted together by the intervening matrix material assuming that the entire current of the filament traveled transversely through the matrix. If L_c is the critical twist length in cm, ρ is the resistivity of the matrix in ohm-cm, j_c is the critical current density in A/cm², d is the filament diameter in cm, $\dot{B}$ is the time rate of change of magnetic field in G/sec, λ_1 is the fraction of the cross section occupied by superconductor, and w is the separation between filaments (surface to surface) in cm, it can be shown that⁵

$$L_c \approx 4 \times 10^4 \sqrt{\frac{j_c \rho d}{B}} \lambda_1 \frac{w}{w+d} \quad (3)$$

Figure 2 gives L_c as a function of $\dot{B}$ for a filament diameter of 0.00064 cm and various values of ρ setting $\lambda_1^{1/2} \frac{w}{w+d} = 1$ for simplicity.

Fig. 2. Critical twist length as a function of $\dot{B}$ for various values of ρ .

C. Improved Estimates of Hysteresis Loss

Wilson, et al.,⁵ list a number of corrections to be made to Eq. (1) if more accurate values for energy loss are needed. These include 2 factors of the order of magnitude of 1 to account for a transport current superimposed on the diamagnetic current, and to correct for the fact that wires are round, not slab shaped. The field dependence of the critical current could also be included in the form of the parametrization,

$$j_c = \frac{j_o B_o}{B + B_o}.$$

Inserting this value of j_c into Eq. (1) and integration from B_1 to B_2 gives for the energy loss,

$$W_H = 10^{-8} \frac{V d j_o B_o}{4} \ln \frac{B_2 + B_o}{B_1 + B_o}.$$

If the magnetic field is below the field required to completely penetrate the superconductor, Eq. (2) is multiplied by a factor

$$\frac{1}{3} \left(\frac{B}{B_p} \right)^2,$$

where B_p is the field at which the wire is completely penetrated by the magnetic field. Finally for the self field of a wire, the filaments are completely coupled even if the wire is twisted, and an additional loss occurs which is calculated from an equation similar to Eq. (2) but using the self-field and the full wire diameter. This loss is usually smaller than that calculated from 2 as long as the wire diameter is small.

D. Assumptions Made in This Calculation

Clearly the calculation of the hysteresis loss can be a complicated matter and to expedite the calculations for storage coils the following assumptions are made:

1. The total winding is divided into sections (typically 10) to account for the varying field from inside to outside of the coil. In each of these sections, the loss rate is calculated separately using Eq. (1). To obtain the value of magnetic field needed to evaluate Eq. (1). The magnetic field is scaled linearly from maximum field at the inside to zero field at the outside, and an

average value of the field in each section of the wire is used in Eq. (1).

2. If L_c is the calculated critical twist length (Eq. (3)) and L_F is the actual twist length of the filaments, the distance (d) used in Eq. (1) is the filament diameter if $1/3 L_c > L_F$. For $L_F > L_c$ the diameter of the wire is used, and for $1/3 L_c < L_F < L_c$ a linear interpolation between the two diameters is arbitrarily used. In Eq. (3), B is calculated for each section and a constant value of j_c equal to 5×10^5 A/cm² is used. More will be said about the resistivity to be used in Sec. IV.

3. The value of j_c used in Eq. (1) is also set equal to 5×10^5 A/cm² in the superconductor.

III. EDDY CURRENT LOSS

A. Eddy Current Losses in Homogeneous Normal Wires.

Eddy current loss in a non-superconducting wire subjected to a changing magnetic field perpendicular to the wire has been given by Welsby⁶ (after conversion of units) as,

$$P = 25 R_{dc} B_{rms}^2 d^2 G, \quad (4)$$

where R_{dc} is the dc resistance of a single wire in ohms, B_{rms} is the rms magnetic field in gauss, d is the wire diameter in cm, P is the power loss in watts, and G is a dimensionless factor which is a function of d and the skin depth δ . From the form of the function G in Fig. 3 it is evident that there are 2 limiting forms of the function which occur when either $\delta \ll d$ or $\delta \gg d$. The equation for skin depth in cm is

$$\delta = 0.5 \times 10^4 \sqrt{\rho \tau}, \quad (5)$$

where ρ is the resistivity of the material in ohm-cm and τ is the period of oscillation in seconds.

1. Skin Depth Greater than Wire Diameter. In the limit $\delta \gg d$, i.e., the skin depth is much larger than the wire diameter, Eq. (4) may be converted to power loss per unit volume of conductor and compared with the formula used to compute eddy current losses in transformer laminae. Let the following quantities be defined:

$\frac{dW_v}{dt}$ = power loss per unit volume of conductor
(watts/cm³)

f = frequency of oscillation (sec⁻¹)

$B_m = B_{rms} \sqrt{2}$ = maximum applied field (gauss)

d = wire diameter or lamina thickness (cm)

ρ = resistivity (ohm-cm).

Also take the limiting value for G when $\delta \gg d$,

$$G = \frac{1}{256} \left(\frac{d}{\delta}\right)^4. \quad (6)$$

Then,

$$\frac{dW_v}{dt} = 10^{-16} \frac{\pi^2}{8} \frac{f^2 B_m^2 d^2}{\rho}, \quad (7)$$

or converting to the MKS system,

$$\frac{dW_v}{dt} = \frac{\pi^2}{8} \frac{B_m^2 f^2 d^2}{\rho} \quad [\text{MKS}] \quad (8)$$

This can be compared with the formula derived by Winch⁷ for transformer laminae,

$$\frac{dW_v}{dt} = K \frac{\pi^2}{8} \frac{B_m^2 f^2 d^2}{\rho} \quad [\text{MKS}]$$

where K is a dimensionless constant between 1 and 2.

Figure 4 shows the skin depth as a function of frequency for various resistivities. The "period" for raising the magnetic field in the pulsed feasibility experiment is in the range of 2 to 10 msec, depending upon how the conversion from the 2 msec exponential decay constant to cyclic variation is made. (The buildup time for the magnetic field in a pulsed reactor is of the order of 10 msec). From Fig. 4 it is clear that even for a resistivity of 1/200 that of copper at room temperature the skin depth is still greater or equal to 0.02 cm (8 mils) for this frequency range. Since for practical purposes copper has a maximum resistance ratio (ratio of resistance at 20°C to resistance at 4K) of 200, the situation $\delta \geq d$ would be the one usually encountered in pulsed fusion experiments. Very thick wires or ultra-pure aluminum matrix material would be required to give $d > \delta$.

Fig. 3. The factor G as a function of d/δ .

Fig. 4. Skin depth as a function of period for materials of various resistivity.

2. Skin Depth Less Than Wire Diameter. In

the limit $\delta \ll d$, the current is limited to the wire surface and loss becomes a surface effect measured in W/cm² rather than W/cm³. In addition to the quantities defined in Sec. 1 above, define the following quantities:

$\frac{dW_A}{dt}$ = power loss per unit area of conductor
(watts/cm²)

$\tau = \frac{1}{f}$ = period of oscillation (sec).

Then inserting the limiting value of G for $\delta \ll d$,

$$G = \frac{1}{8} \frac{d}{\delta}, \quad (9)$$

and converting Eq. (4) to loss per cm^2 gives

$$\frac{dW}{dt} \frac{A}{\text{cm}^2} = 1.26 \times 10^{-4} \rho^{1/2} B_m^2 \tau^{-1/2}. \quad (10)$$

As a check this equation may be compared with the calculation of Oliphant on the energy losses in a theta-pinch coil.⁸ Converting loss in joules per centimeter of length to joules per cm^2 of inside area gives

$$\frac{dW}{dt} \frac{A}{\text{cm}^2} = 3.58 \times 10^{-5} \rho^{1/2} B_m^2 \tau^{-1/2}. \quad (11)$$

There is a difference of a factor of 3 between the coefficients in Eqs. 10 and 11. This difference may perhaps be reconciled. Equation 11 was derived for a heavily damped waveform and $\tau/2$ was defined as the interval between the two times at which the magnetic field was $1/\sqrt{2}$ of maximum in the first half cycle. This shorter value of τ makes the coefficient in Eq. (11) larger than in Eq. (10). Another difference stems from the variation of induced voltage and hence current density at different points on the circumference of the wire compared to constant current density at any point on the inside surface of the θ pinch. For present purposes agreement is adequate.

B. Complications Due to Superconducting Filaments Embedded in Matrix Material.

1. Filaments Superconducting and Twisted. G. H. Morgan⁹ has derived an expression for the eddy current loss in multifilament superconductors in which he assumes the eddy currents flow in a direction normal to the wire axis. This appears to be equivalent to assuming that the superconductor flux flow resistance to changes in current is still much less than the matrix resistance. Under this condition the path of the eddy currents is between filaments all along the length of the filament and consequently much higher than in a solid wire of the same size made of the non-superconducting matrix. Assuming that the twist length is still much shorter than the critical twist length and that the wire is not saturated, Morgan's power loss is larger

than that given by Eq. (7) by a factor

$$F = \frac{4}{\pi^2} \frac{L^2}{D^2} \frac{\rho}{\rho_e} \quad (12)$$

Here L is the full helical twist length, D is the wire diameter, and ρ/ρ_e is a factor usually of order 1 to 0.2. This can easily give losses a hundred times those calculated from Eq. (7) for some regions of operation; although if the superconductor becomes normal or, according to Morgan, if B becomes very high Eq. (7) again becomes a valid estimate for the eddy current losses. The important region for fusion experiments is of course that for which B is extremely high.

2. Resistivity of the Composite Conductor.

Using Eqs. (4, 7, or 11) to estimate the eddy current loss requires that some average value for the resistivity of the composite conductor be found. Finding this average resistivity while the wire remains superconducting requires knowledge of the direction in which the eddy currents flow in the wire as well as evaluation of the flux-flow resistivity. The assumption made here is that for high values of B , eddy currents flow primarily along the wire and only in the matrix material. In other words, resistivity of the superconductor to increments in current is considered so high that eddy currents flow only in the matrix material for the time scale considered. Resistivity of the composite can then be written as

$$\rho = \rho_m \frac{1+r}{r}, \quad (13)$$

where ρ_m is the resistivity of the matrix, and r is the volume ratio of matrix to superconductor. When the conductor is completely normal the resistivity of the composite can be written as

$$\rho = \frac{\rho_m \rho_n (1+r)}{\rho_m + r \rho_n} \quad (14)$$

where ρ_n is the normal state resistivity of the superconductor. Almost invariably $\rho_n \gg \rho_m$ which reduces Eq. (14) to Eq. (13).

C. Assumptions Made in This Calculation.

Eddy current losses are calculated essentially using Eq. (4). However, some modification is necessary since Eq. (4) gives the average power loss

with a magnetic field having a sinusoidal time variation, but the instantaneous power loss is the quantity needed for calculation. It is assumed that the time derivative of the instantaneous loss can be found from Eq. (4) by substituting $d^2 w_E / dt^2$ in J/sec^2 in place of P and $1/2 B_m |dB/dt|$ in place of B_{rms}^2 . Here B_m is the maximum field in gauss, dB/dt is the instantaneous rate of change of field, and $d^2 w_E / dt^2$ is the instantaneous power loss. The new equation will be dimensionally correct and give the time averaged result for P expressed by Eq. (4). The final equation is for the power loss during a time interval Δt

$$\frac{dw_E}{dt} = \int_t^{t+\Delta t} 12.5 d^2 R_{DC} B_m \left| \frac{dB}{dt} \right| G dt. \quad (15)$$

Largest loss occurs when $|dB/dt|$ is largest.

The skin depth δ needed to calculate G is found from Eq. (5) using the instantaneous value of ρ , the wire resistivity, from Eq. (14). For finding R_{DC} the same value of ρ is used. Temperature dependence of the resistivities is included. The calculations to be described in Sec. VI were done for the case of current from a storage coil decaying through an external resistance. The value of τ used in calculating λ comes from $\lambda = \frac{R}{L}$, the instantaneous decay constant for the current. If the coil goes normal, variation of coil resistance with temperature is taken into account. The approximation used for τ is

$$\tau = 4/\lambda. \quad (16)$$

Admittedly the treatment of eddy current loss here is unsatisfactory, and emphasizes the need for an experimental measurement of the losses in energy storage coils.

IV. JOULE LOSSES.

A. Equation of State.

Initially the entire storage coil is in the superconducting state, but as the temperature of the winding rises due to eddy current and hysteresis loss a point may be reached where some part of the coil becomes normal. At that time ordinary joule loss becomes operative.

Since the total hysteresis loss is proportional to B_m , and the eddy current loss is proportional

to B_m^2 , the inner windings would be expected to become normal before the outer part of the coil. The critical temperature for transition to the normal state is also lower in the presence of an externally applied field. For testing whether any portion of the winding is normal the following equation of state is assumed:

$$\frac{B}{B_0} + \frac{j}{j_0} = 1 - \left(\frac{T}{T_0} \right)^2, \quad (17)$$

where

$$j_0 = 5 \times 10^5 \text{ A/cm}^2$$

$$B_0 = 1.48 \times 10^5 \text{ G}$$

$$T_0 = 9.5 \text{ K.}$$

This equation may be somewhat arbitrary, but it is a very convenient analytic equation for testing the state of the superconductor. It also has the merit that in the case where $J = 0$, it reduces to the well known parabolic law for the critical field,

$$\frac{B}{B_0} = 1 - \left(\frac{T}{T_0} \right)^2, \quad (18)$$

which then determines the constants T_0 and B_0 . The constant j_0 is selected as the maximum practical current density in zero external field. For the case $T = 0$, Eq. (17) may be looked upon as a small field expansion of the well known parametrization of the critical current density as a function of magnetic field,

$$j_c = \frac{j_1}{B + B_1}, \quad (19)$$

where j_1 and B_1 are constants.

B. Resistivity.

1. Dependence on Temperature and Magnetic Field.

Figure 5 shows the resistivity of OFHC copper and a NbTi alloy in the normal state as a function of temperature.¹⁰ The actual value of the resistivity of these materials at low temperatures is dependent on purity and metallurgical history, but the values shown are typical. The resistivity ratio is about

140 for the copper shown, but resistivity ratios of only 50 to 70 frequently result from cold working of the wire as it is drawn or from the presence of metallic impurities. Copper has a very large magneto-resistance which is additive to the other sources of resistance rather than multiplicative as in aluminum. Hence it does not pay to go to very high purity copper conductors as it does with aluminum conductors at low temperatures. Figure 6 is a plot of total resistance of OFHC copper at a temperature of 4.2 K as a function of applied magnetic field.¹⁰

2. Assumptions in this Calculation. As long as any section of the storage coil is superconducting its dc resistance is zero and hence the joule loss is zero in that section. If a section of the coil goes normal its resistance is calculated from the parallel combination of NbTi filaments and the copper matrix. Resistivities as a function of temperature used to find the resistance in most of the calculations are given in Fig. 5. This means the resistivity ratio of copper is usually taken to be 140, although in one case it has been scaled up to 50 just for comparison purposes. Figure 6 shows that the magneto-resistivity of copper in a field of 40 kG is comparable to the ordinary resistivity of the copper sample shown in Fig. 5 for temperatures below about 30 K. However, for simplicity the magneto-resistance has been ignored in finding the coil resistance.

C. Thermal Capacity.

The energy loss in the storage coil at each instant of time depends on the current, rate of change of magnetic field, and resistivity of the conductor. Since the equation of state and the resistivity depend on temperature it is necessary to know the temperature at each instant of time to compute the energy loss. The general scheme is to begin the calculation of energy loss at a temperature of 4 K and find the net energy loss in a small time interval Δt . An account is kept of the net energy added to each section of the coil and the temperature of that section is found from specific heat tables. The time is then advanced by Δt and the energy loss, current, magnetic field, etc., are found at the new time with the new temperature. For the sake of simplicity, the heat capacity of the wire is taken as

Fig. 5. Typical values of the resistivity of copper and niobium-titanium alloy as a function of temperature.

Fig. 6. Magneto-resistivity of OFHC copper.

if it were all copper. Table I gives the heat capacities used as a function of temperature. Heat conduction between all parts of one of the given segments of the conductor is taken as instantaneous so that the whole segment is at the same temperature.

TABLE I
Heat Capacity of Copper

Temperature (K)	Heat Capacity $J/(cm^3 K)$
2	2.7×10^{-4}
3	5.4×10^{-4}

TABLE I cont.

Temperature (K)	Heat Capacity $J/(cm^3 K)$
4	1.3×10^{-3}
6	2.1×10^{-3}
8	4.3×10^{-3}
10	7.6×10^{-3}
15	2.4×10^{-2}
25	0.14
50	0.88
100	2.3
150	2.9
200	3.2
300	3.4

V. HEAT TRANSFER AND THERMAL CONDUCTIVITY.

A. Thermal Conduction Along the Wire.

The problems of thermal conduction along the conductor and heat transfer to the surrounding helium bath and coil form are both difficult to take into account exactly. Solving the diffusion equation for heat transfer along the conductor would at least mean dividing the coil winding into many more smaller divisions along its length than has been done. Because this would result in considerably longer computing times, and probably more time than the calculation is worth considering the other approximations made, no account has been made for heat conduction between the segments into which the wire is divided along its length. Immediately after the coil is discharged, the temperature difference between one end of the coil winding and the other as found by the present method is usually small for the cases being investigated; hence ignoring heat conduction along the wire should be an acceptable approximation.

B. Heat Transfer to the Helium Bath.

Heat transfer from the winding to the helium bath is complicated by not knowing what area to allow for contact between the bath and the conductor and to what extent the conductor is covered by electrical insulation or plastic potting materials. Even if the braid were not mechanically stabilized with plastic, bubbles of helium gas between the strands would reduce the area of contact with the bath to an extent which is impossible to calculate a priori. Organic electrical insulation or plastic

potting places material with very low thermal conductivity but high heat capacity in contact with the conductor, possibly in a somewhat irregular geometry. In this simplified calculation heat transfer to the bath is calculated as if it were from the bare metal conductor but the effective area of the braid can be adjusted to take some account of the complications described above. The steady-state heat transfer rate per unit area as a function of the temperature difference between the bath and conductor is shown in Fig. 7.¹¹ (The dip is caused by the transition between nuclear and film boiling.) There is some indication that transient heat flow may be higher than steady state rates, but the steady-state rates are used here. Two different values for the conductor area have been made. The first area of zero means that no heat is transferred to the bath, and the temperature of the conductor is raised adiabatically. The second area is equal to the total area of the two wide faces of the braid ignoring the fact that the braid is made up of individual strands.

VI. CALCULATIONS FOR SPECIFIC COILS.

A. Coil Description.

Energy loss calculations were carried out for the 3 coils described in Table II with currents of 500 and 1000 A flowing in the coils. In all 3 solenoids the dimensions, number of turns, overall braid diameter, wire diameter, and filament diameter in the wire were the same. The principal difference was that the copper to superconductor ratio within the wire which was chosen to be 1:1, 3:1, and 6:1

Fig. 7. Heat transfer rate from metal surfaces to liquid helium bath.

for the different coils. Because the wire and filament diameter were held constant, the number of superconducting filaments in each wire as well as the number of wires in the braids had to be changed in each coil. There was also some variation between them in the current density in the superconductor for a given current flowing through each braid.

In all cases the procedure was to start at a temperature of 4 K with a given current flowing in the coil and then calculate the coil energy loss as it is discharged through different sized external resistances. The losses were then plotted as a function of the load resistor which of course determines the decay time of current in the storage coil. For the 1000 A cases the inner windings of the coil, in fact, started out in the normal state but for the 500 A cases the coils were completely superconducting.

TABLE II

Parameters Common to All 3 Coils

Wire Diameter	0.0203	cm
Filament Diameter	6.3×10^{-4}	cm
Coil Length	20.3	cm
Coil i.d.	14.35	cm
Winding Thickness	2.31	cm
Number of Turns	699	
Inductance	42.6	mH
Stored Energy at 1000 A	21.3	kJ
Maximum Field at 1000 A	43	kG
Braid Width	0.44	cm
Braid Thickness	0.152	cm

Coil 1 Parameters

Matrix to Superconductor Ratio	1.0
Number of Wires per Braid	14
Number of Filaments per Wire	534
Superconductor Current Density at 1000 A	4.3×10^5 A/cm ²

Coil 2 Parameters

Matrix to Superconductor Ratio	3.0
Number of Wires per Braid	42
Number of Filaments per Wire	267
Superconductor Current Density at 1000 A	2.86×10^5 A/cm ²

Coil 3 Parameters

Matrix to Superconductor Ratio	6.0
Number of Wires per Braid	84
Number of Filaments per Wire	152

Coil 3 Parameters cont.

Superconductor Current Density at 1000 A 2.50 A/cm²

B. Results.

1. Case I: I = 1000 A, Adiabatic. Figure 8 shows the coil energy loss due to the 3 mechanisms discussed for the 3:1 copper matrix coil with no heat flow to the helium bath. The total energy stored is given by E_{ST} , hysteresis loss by E_H , joule loss by E_J , eddy current loss by E_E , and total loss by E_T . Hysteresis loss is calculated even after the coil section becomes normal, but this in itself does not create much of an error since the hysteresis loss is much smaller than the joule loss. In calculating the hysteresis loss, it is found that the filaments are effectively coupled for a twist of 2 turns per cm which is about as much as is commonly done. At first it might be thought that reducing the external resistance could slow the discharge rate to a point where the filaments decoupled, but the normal state resistance of the storage coil sets a lower limit to the discharge rate. This rate is high enough to just keep the filaments coupled. Figure 9 shows the total loss for 1:1, 3:1, and 6:1 matrix materials. At high values of external resistance the loss is primarily due to eddy currents; and since the losses are larger in the lower resistance material, the 6:1 material shows higher losses. As the resistance is decreased, a point is reached where the coil resistance becomes much larger than the external resistance if the entire coil is driven normal. Under these conditions most of the energy stored in the coil is lost in the coil itself. Just to give an idea of the magnitudes of the various quantities involved, Fig. 10 shows the final temperature of the innermost coil segment and Fig. 11 shows the final coil resistance after discharge. Figure 12 gives the e-folding time for the current decay.

2. Case II: I = 1000 A, Heat Loss to Helium Bath. Figure 13 shows the total energy loss for the 3 different coils with an initial current of 1000 A and with heat loss to the bath calculated as described in Sec. V-B. Figure 14 shows the heat transferred to the bath during the time taken for the current to decay to 100 A. Twisting of 2 turns per cm is again ineffective in reducing the hysteresis loss.

Fig. 8. Energy loss from the 3 sources discussed in a coil made with 3:1 matrix to superconductor ratio. Initial current is 1000 A and no heat is transferred to the helium bath.

Fig. 11. Final coil resistance under adiabatic conditions for an initial current of 1000 A.

Fig. 9. The energy loss in the storage coils under adiabatic conditions for an initial current of 1000 A.

Fig. 10. Final temperature of inner coil segment under adiabatic conditions for an initial current of 1000 A.

Fig. 12. Time for e-fold decay of current under adiabatic conditions. Initial current is 1000 A.

Fig. 13. Energy loss in the storage coil with initial current of 1000 A and including effects of heat loss to the helium bath.

Fig. 14. Heat transfer to the helium bath during the time taken for current to decay to 100 A.

3. Case III: $I = 500$ A, Adiabatic. Figure 15 gives the total coil loss with an initial current of 500 A in the coil, no heat transfer to the helium bath, and no twist in the superconducting wire. The total stored energy with a current of 500 A is 5.3 kJ. If the smaller stored energy in Case III compared to Case I is taken into account, Fig. 15 is almost identical with Fig. 8. In Fig. 16 a twist of 2 turns per cm has been applied and is seen to be effective in lowering the loss in the 6:1 matrix material. For low current decay rates (e-fold times greater than about 10 sec) the coil even remains superconducting. For comparison purposes Fig. 17 gives the total energy loss with no twist in the superconductor but with the resistivity ratio scaled down to 50 from 140. In the 6:1 and 3:1 material, loss at the fastest discharge rates are lower for resistance ratio 50 than for resistance ratio 140. In the 1:1 material, loss at the fastest discharge rates are slightly higher for ratio 50 than for resistance ratio 140 because the joule loss increases faster than eddy current loss decreases.

4. Case IV: $I = 500$ A, Heat Loss to Helium Bath. In Fig. 18 the total coil loss is shown, again allowing for heat transfer to the helium bath. The plateau region on the left of each curve is almost entirely due to hysteresis loss since the coil does not go normal at all in this region. For values of the external resistance greater than 0.1 ohm, twisting of the superconducting wire is still not effective in decoupling the filaments. If results were calculated for still lower values of the

Fig. 15. Total energy loss in the storage coils under adiabatic conditions for an initial current of 500 A and no twist in the superconducting wire.

Fig. 16. Total energy loss in the storage coils under adiabatic conditions for an initial current of 500 A and a twist of 2 turns per cm in the superconducting wire.

Fig. 17. Total energy loss in the storage coils under adiabatic conditions for an initial current of 500 A, no twist in the superconductor, and a resistance ratio in the matrix of 50 instead of 140.

Fig. 18. Energy loss in the storage coils with an initial current of 500 A and including effects of heat loss to the helium bath.

external resistance, twisting would eventually decouple the filaments and the energy loss would fall by a factor equal to the ratio of filament to wire diameter (0.03).

C. Summary.

For the particular coils studied, the results of the preceding calculations can be summarized in two general conclusions.

1. Even if the storage coil should be driven normal there is still a minimum in the energy loss which is only a few times larger than the hysteresis loss for completely coupled filaments. This minimum occurs for discharge times in the range of 1 to 10 msec. For longer discharge times there is more time for joule loss to occur, and for shorter times eddy current loss increases.

2. With a pure copper matrix it is almost impossible to decouple the superconducting filaments by twisting for discharge times of interest to fusion experiments. Twisting may still be useful while the coil is being charged since this is done at a much slower rate. This by no means precludes the use of pure copper matrix material in experimental devices, since the minimum storage coil loss will usually be negligible compared to the energy stored. For reactors it will of course be necessary to go to mixed matrix materials of more complex construction.

While the preceding graphs of storage coil energy loss are valid only for the particular coils

studied under the given specific conditions, the same general types of phenomena should occur for larger storage coils or coils of different construction. Since the material studied was a braid made from fully insulated 0.008-in.-diameter wires of pure copper with imbedded NbTi filaments, caution should be exercised in extending numerical results to either mixed matrix material or conductors of much larger cross section.

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