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MASTER

BUBBLE DENSITY MEASUREMENT WITH THE HOUGH POWELL DIGITIZER

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I. HPD Parameters for Bubble Density Measurement

The HPD employs an intense spot of light that is swept regularly over the useful surface of bubble chamber film; when a bubble image sufficiently obscures the spot, the location of the bubble image is digitized with a precision of a few microns. In order to discuss HPD bubble density measurements we will combine the finite spot size and the bubble size into one equivalent "scan width" for which single bubbles can be treated as points.

The HPD raw output consists of the orthogonal film coordinates of every intercepted bubble image or bubble image complex. We will assume that all the digitizations or "hits" belonging to one track have been isolated.

The linear structure of a track is determined by its bubble density. Due to the finite bubble size, adjacent bubbles frequently coalesce into continuous complexes or "blobs". If we consider one bubble to be the smallest possible blob, then a track consists of many blobs which are separated by an equal number of "gaps". It is also useful to consider a track as a sequential repetition of a basic cell which is composed of one gap followed by one blob.[3]

The HPD representation of linear track structure is composed of the binary elements "hit" or "miss" on one interception of the track by the flying spot. The HPD linear track structure is analogous to the physical track structure. A blob is represented as a string of one or more hits on sequential scan lines and a gap is represented as a string of one or more misses. The HPD basic cell is composed of "m" misses followed by "h" hits where m and h are positive intergers. Between the first miss string and last hit string on a track there are a total of "C" data cells. The jth data cell contains m_j misses followed by h_j hits. The following definitions are useful.

$$M \equiv \Sigma_1^c \; m_j \quad = \text{ total number of misses}$$

$$H \equiv \Sigma_1^c \; h_j \quad = \text{ total number of hits}$$

$$T \equiv M + H \quad = \text{ total number of scan lines}$$

Since each scan of the track is displaced a uniform amount, s, from the preceding scan the product of T and s is the total projected length of the track.

In order to represent the bubble images as individual points we will fold the bubble size into the flying spot size. We think of the following experiment. The scan line separation is made much less than one bubble diameter so that a bubble is digitized many times as the scan lines move over it. As the scan lines advance to the right the first digitization occurs on a scan which has its center line a/2 microns to the left of the bubble center. The last digitization occurs one the scan which has its center line a/2 microns to the right of the bubble center. The distance, a, represents the effective width of the HPD scan. When the center of a bubble lies inside the scan width a hit is recorded. We can now replace finite bubble images by points located at their centers. This mathematical model for the HPD has been suggested by Benot.[4]

We observe that the size of the scan width relative to the scan separation determines whether or not all partions of the track are crossed by the flying spot. If $s > a$ then parts of the track are missed and some bubbles will not be digitized. If $s < a$ then no part of the track is missed and some bubbles may be digitized on several sequential scans.

The size of the scan width is determined by the size of the bubble images and by the HPD noise rejection level. Phototube pulses which are lower than the noise rejection level arc rejected. From photographic density measurements we find that the average bubble has a gaussian density distribution with a full width at half maximum of about sixteen microns. Hence, the average bubble becomes smaller as the noise rejection level is increased.

The variation in scan width is expected to be largely due to the variation in bubble image size. When the variation in bubble size is random, it does not influence the bubble density measurements. We will have more to say about this in the next section.

In order to simplify the discussion of the bubble density measurement we make the following assumption about the track:

A. The direction of the track is orthogonal to the direction of the HPD scan lines.

B. The dip angle of the track is zero.

C. Energy loss causes no significant change in bubble density over the track.

D. There are no background bubbles near the track.

A high momentum beam track is often in accord with these assumptions.

Extension of this method to all tracks is expected to be uncomplicated. We must correct for the increased effective scan width when the scan lines are not orthogonal to the track, and for the turning angle of the track. Bubble densities must be corrected for dip angles and for the variation of chamber to film magnification as a function of depth in the chamber. The conical projection associated with each view must be taken into account. A systematic study of these corrections will be made.

For convenience the bubble density parameters are summarized in Table I.

TABLE I

Parameters used to describe the linear structure of a bubble chamber track.

Parameter	Dimension	Definition
k	μ^{-1}	Mean bubble density on the film.
λ	μ	Mean bubble formation length on the film ($k = 1/\lambda$).
L	μ	Film length of track segment that is measured.
m		Number of misses in a miss string.
h		Number of hits in a hit string.
T		Total number of FSD scans made on the track segment of length, L.
M		Number of misses for the track segment.
H		Number of hits for the track segment ($H + M = T$).
C		Number of data cells for the track segment.
s	μ	HPD scan line separation. ($L = T\ s$)
a/2	μ	Maximum separation of a bubble image center and the flying spot center line for a hit or digitization to occur. The flying spot has a total mathematical width of "a" microns, and bubbles are effectively points located at their image centers.

II. HPD Scan Sequence Probabilities

1. The Probability for a Finite Gap.

The likelihood function is the joint probability for the sequence of hits and misses, that represent a track, expressed is a function of the unknowns k, the bubble density and a, the scan width. The probability that one scan of a track will yield a hit depends on the bubble density of the track. The probability for a miss on the track is just one minus the probability for a hit because of the binary relation between a hit and a miss.

The linear structure of a bubble chamber track is determined by the random process of bubble formation. Ideally, the chance for a bubble image to appear is the same at all positions of the film, and the formation of a bubble image does not influence the chance for another image to be formed at any other position on the film. For a given set of bubble chamber pictures it should be possible to select a fiducial volume in which the bubble formation process is effectively random. If enough bubbles are available on a track then averages are significant and statistics of a random process is applicable.

We consider an interval of length, x, along the track. The statistically significant number of bubbles found in the interval is given by kx where k is the mean bubble density of the track. The statistical fluctuations in the number of bubbles found in the interval, x, is given by P_m, the Poisson distribution,

$$P_m = (kx)^m e^{-kx}/m!$$

which is the probability that m bubbles will be found in the interval x where kx bubbles are found on the average. When the length of the track, L, is sufficiently long with respect to λ, the bubble formation length, ($\lambda = 1/k$) the probability, P_m, is normalized to one:

$$\Sigma_1^{kL} P_m \cong 1$$

This condition is met under normal bubble chamber operating conditions.

The probability that no bubbles will be found in x is given by

$$P_o = e^{-kx}$$

and the probability that some bubbles will be found is given by

$$1 - P_o = 1 - e^{-kx}$$

We have previously defined the HPD scan width, a, and we see now that the probability for a hit on one scan line is

$$1 - e^{-ka} ,$$

and the probability for a miss is

$$e^{-ka} .$$

The Poisson distribution of bubbles has the property that the mean value of the physical gap between bubble edges is just the reciprocal of the mean bubble density. Furthermore, this mean gap is independent of the bubble diameter! (see Appendix I.) Since the HPD scan width depends on the bubble diameter, we expect to be able to make absolute bubble density measurements independent of the scan width. A random variation in bubble size should not change the mean gap.[5]

2. Sequence Probability without Scan Overlap: $s > a$.

We consider first the case without scan overlap when some bubbles can be missed by the flying spot. In this case the result of a given scan has no influence on the probability for the result of the next scan. The probability that a bubble is found in the scan width, a, namely

$$1 - e^{-ka}$$

is the probability for a hit on any scan. The probability for a miss is

$$e^{-ka}$$

Since the probabilities are independent of the scan line separation, s, no measure of the bubble density alone can be made in this case because the likelihood function does not contain a known linear dimension. The product ka can be measured. It is the average number of bubbles found within the scan width.

3. Sequence Probability with Scan Overlap: $s < a$.

Some bubbles will be digitized repeatedly when there is scan overlap ($s < a$) and no part of the track will be missed. We will restrict the discussion to the range of scan line separations,

$$a/2 < s < a.$$

This range is of practical interest because closer scan line spacing would consume more HPD scanning time without improving the process of finding the tracks in the raw HPD data.

We are considering the case when some bubbles will be digitized twice whereas others will be digitized once. In other words, the probability that a hit will follow a hit can never equal one.

We denote by Q (MM), the sequence probability that a miss will follow a miss. Given a miss, the probability that it will be followed by a miss is just the probability that the unscanned interval s will contain no bubbles, which is given by the expression

$$Q\ (MM) = e^{-ks}.$$

It also follows that $Q\ (MH) = 1 - e^{-ks}$ is the probability that a hit will follow a miss.

In appendix II, we show that the probability for a hit to be followed by a hit is given by

$$Q\ (HH) = 1 - e^{-ka}\ (1 - e^{-ks})\ /\ (1 - e^{-ka}).$$

It follows that

$$Q\ (HM) = e^{-ka}\ (1 - e^{-ks})\ /\ (1 - e^{-ka}).$$

is the probability that a hit will be followed by a miss. We see that with scan overlap the likelihood function contains the linear scale s. The bubble density and scan width will be measured in units of s.

III. Maximum Likelihood Solutions

1. Solution without Scan Overlap: $s > a$.

We have observed that the probability for a miss is given by e^{-ka} and the probability for a hit is given by $1 - e^{-ka}$. We let $f_i = 1$ signify a hit on the $i^{\underline{th}}$ scan and $f_i = 0$ signify a miss. Then the likelihood function is given by the product

$$\mathcal{L}\,(ka) = \pi_1^T\,(e^{-ka})^{1-f_i}\,(1 - e^{-ka})^{f_i}$$

$$= (e^{-ka})^M\,(1 - e^{-ka})^H \quad .$$

The maximum likelihood solution, $(ka)^*$, is obtained from the relation

$$\partial\,\mathcal{L} / \partial(ka) = 0 \;,$$

and this solution is

$$(ka)^* = \ln\,(T/M).$$

When the track is long enough to produce good statistics the likelihood function approaches a Gaussian shape. In this case the variance, due to statistics, is given by

$$\overline{\delta\,(ka)^*}^{\,2} = -\left[\,\partial^2 W / \partial(ka)^2\,\right]^{-1} \quad .$$

where

$$W = \ln\,\mathcal{L}.$$

We find in this case that

$$\overline{\delta\,(ka)^*}^{\,2} = H / (MT) \;.$$

We notice that if there are no misses (i.e. $M = 0$) then the variance diverges.

2. Solutions with Scan Overlap: $a/2 < s < a$.

We have observed that

$$Q\ (MM) = e^{-ks}$$

is the probability that a miss will follow a miss and that

$$Q\ (HH) = 1 - e^{-ka}\ (1 - e^{-ks})\ /\ (1 - e^{-ka})$$

is the probability that a hit will follow a hit. Since the likelihood function will now depend on the known scan line separation, s, it will be possible to solve for both k and a in units of s. Again we get $f_i = 1$ imply a hit and $f_i = 0$ imply a miss. The likelihood function can be written as the following product:

$$\mathcal{L}(k,a,s) = \pi_1^{T+1} \left\{ \left[Q\ (MM)\right]^{1-f_{i+1}} \left[Q\ (MH)\right]^{f_{i+1}} \right\}^{1-f_i} \text{ times}$$

$$\left\{ \left[Q\ (HH)\right]^{f_{i+1}} \left[Q\ (HM)\right]^{1-f_{i+1}} \right\}^{f_i}$$

$$= \left[Q\ (MM)\right]^{M-C} \left[Q\ (MH)\right]^{C} \left[Q\ (HH)\right]^{H-C} \left[Q\ (HM)\right]^{C}$$

where we note that

$$Q\ (MH) = 1 - Q\ (MM)$$

$$Q\ (HM) = 1 - Q\ (HH)\ .$$

In this case we must solve for both k and a from the set of equations

$$\partial W / \partial a = 0$$

and

$$\partial W / \partial k = 0$$

where $W = \ln \mathcal{L}$.

Solving this set we find that

$$k^* = \ln \left[M/(M - C)\right] / s$$

and

$$a^* = \left\{ \ln (T/M) \right\} / k^*$$

If sufficient statistics are available then the likelihood function approaches a Gaussian shape and the variances and covariances are obtained as elements of a two by two matrix, H^{-1}_{ij} .

The matrix H_{ij} is given by

$$H_{ij} = - \partial^2 W / \partial q_i \, \partial q_i$$

where $q_1 = k$

$q_2 = a$

We find that the variances are

$$\overline{\delta k^*}^{\,2} = C \Big/ \left[s^2 M (M-C) \right]$$

$$\overline{\delta a^*}^{\,2} = \left[H (H-C) + \frac{\left[aCT/s - (M-C) H \right]}{M (M-C)} \right] \Big/ \left[k^{*2} T^2 C \right] .$$

The covariance is given by

$$\overline{\delta k^* \; \delta a^*} = \left[\frac{a}{m} - \frac{sM}{TH} \right] \left[TC \right] \Big/ \left[M^2 s^2 k^{*2} \right] .$$

IV. Experimental Results for one Beam Track

From the first event processed by the HPD Mark I we plotted all of the road coordinates for the beam track. In this case the road extended about two hundred microns on either side of the previously rough digitized track. The beam track agreed with the simplifying assumptions made in the first section of this report. We selected the hits which belonged to the track, and then we determined whether a hit or a miss occurred on each scan of the track. Using the maximum likelihood solutions obtained in the third section of this report we find that the scan width is

$$a = 40 \pm 4 \ \mu$$

and the bubble formation length is

$$\lambda = 86 \pm 11 \ \mu \ .$$

We see that the scan line spacing, 28 microns, was less than the scan width. The same track was measured on one of the Bubble Chamber Group's measuring projectors using the procedure developed here.[6] An upper limit on the gap length was applied in obtaining the maximum likelihood solution for the mean bubble formation length. It was found to be

$$\lambda = 83 \pm 8 \ \mu \ .$$

V. Conclusions

We have made an absolute bubble density measurement with the HPD Mark I at Brookhaven. The answer has been checked by a direct projection table measurement of the same track, and the two measurements agree.

We have also measured the HPD Mark I scan width. Our result is consistent with the twenty micron spot size and the sixteen micron (full width at half maximum) bubble size.

We plan a systematic study of bubble densities and scan widths for many tracks. We expect the measurements to fail for steep tracks, short tracks, and tracks near the edges of the chamber. Specifically, we will want to locate the fiducial volume over which bubble densities and scan widths can be measured. It is of particular interest to determine the behavior of the scan width, a. As Barkas has shown,[3] knowledge of the bubble size and hence, "a" in our case, permits a more precise bubble density measurement.

Appendix I. The Effect of Poisson Statistics on the Mean Gap

The probability for a gap of length, y, is the product of the probability, e^{-ky}, that no bubble occurs in a distance y and kdy, the probability that a bubble is formed in the infinitesimal interval, dy. We denote the bubble density by k. The average value for y is given by the expression

$$< y >_{AVG} = \int_{o}^{L} y\, e^{-ky}\, kdy \Big/ \int_{o}^{L} e^{-ky}\, kdy$$

which equals the bubble formation length, λ, provided L, the total track length is much greater than λ. ($\lambda = 1/k$) If we subtract a constant, b, from every gap then the average shortened gap is given by

$$< y-b >_{AVG} = \int_{b}^{L} (y-b)\, e^{-ky}\, kdy \Big/ \int_{b}^{L} e^{-ky}\, kdy$$

which also equals λ if $L >> \lambda$. The distance between bubble centers minus the bubble diameter gives the gap between bubble edges. We conclude that the mean gap equals λ and that the mean gap is independent of the bubble diameter.

Appendix II. The Probability That a Hit Will Follow a Hit When $a/2 < s < a$.

When the scan width, a, is used to bound the scan line separation, s, in the following inequality

$$a/2 < s < a,$$

the probability that a hit will follow a hit is always less than one. However, the probability for a hit or miss depends upon the outcome of the previous scan so that the scans must be considered two at a time.

We denote by Q (HH) the probability that a hit will follow a hit and by Q (HM) the probability that a miss will follow a hit. We observe that the binary relation

$$Q\ (HH) = 1 - Q\ (HM)$$

is true.

We obtain the sequence probability Q (HH) by observing that each data cell contains one "hit - miss" and one "miss - hit" sequence. Thus the average value for the number of data cells is given by the average number of misses times the miss - hit sequence probability

$$< C >_{AVG} = < M >_{AVG}\ Q\ (MH)$$

or the average number of hits times the hit - miss sequence probability

$$< C >_{AVG} = < H >_{AVG}\ Q\ (HM)$$

the average number of misses is given by

$$< M >_{AVG} = T e^{-ka},$$

and the average number of hits is given by

$$< H >_{AVG} = T (1 - e^{-ka})$$

where T is the total number of scan lines, k is the bubble density, and a is the HPD equivalent scan width.

We conclude that

$$Q (HM) = \frac{< M >_{AVG} Q (MH)}{< H >_{AVG}}$$

$$= \frac{T e^{-ka} (1 - e^{-ks})}{T (1 - e^{-ka})}$$

where it was shown in Section II - 3.

that $Q (MH) = 1 - e^{-ks}$.

Finally we find that

$$Q (HH) = 1 - e^{-ka} (1 - e^{-ks}) / (1 - e^{-ka}) .$$

This result, as well as the results for each of the four sequence probabilities, can be obtained from the resolution of two overlapping scans into three equivalent non-overlapping scans.

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