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Engineering and
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MASTER

MIXING AND SAMPLING ENRICHED U-235
FLUIDS IN CYLINDRICAL STORAGE CONTAINERS

(Final Report)

J. E. Harrell

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT

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P. O. Box Y
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Y-12 PLANT

Contract W-7405-eng-26
With the US Atomic Energy Commission

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ABSTRACT

A study was performed that combined the measurement of some safe-tank mixing and sampling characteristics with a theoretical analysis for the generalization of mixing characteristics for recirculation in mixed-tank systems.

Safe-tank mixing was simulated in an experimental facility that consisted of both horizontally and vertically orientated tankage that was equipped with flow-rate and fluid-concentration measurement components. Both single and parallel tanks were used. Fluid was mixed in the horizontal systems by recirculation and in the vertical tanks by air sparging. The mixing time required to disperse and mix a relatively small volume of tracer with the bulk fluid was measured. This mixing time was correlated with the flow rate. For single-tank systems, dispersion-intensity data were collected. Both mixing-time and dispersion-intensity data were found to fit reasonably well into a full logarithmic plot as a function of the recirculation rate by linear correlation.

The theoretical treatment utilized a combination of the tanks-in-series model and the dispersion model, and required either analog or digital-computer solutions. For single-tank systems, a generalized and dimensionless solution for mixing-time prediction was obtained and plotted. A similar treatment was made for parallel-tank systems, but a solution to this complex problem cannot be presented as simply. However, an alternate scheme for predicting the upper limit of parallel-tank mixing time using the single-tank solution has been developed.

Guidelines for mixing miscible fluids or mixing miscible fluids containing fine solids prior to representative sampling have been established.

The effect of the piping arrangement of the various tankage systems upon mixing, sampling, and fuel-inventory uncertainties has been studied and recommendations made for the design and operation of a tankage system.

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APPENDIX C (DATA FROM AN INVESTIGATION OF AIR-SPARGE VERTICAL SYSTEMS)	87
APPENDIX D (DATA FROM THE INVESTIGATION OF MIXING-TUBE SYSTEMS)	117
APPENDIX E (DATA FROM THE EXPERIMENTAL SAMPLING OF SOLID- LIQUID MIXTURES).	121

SUMMARY

A program requested by the Division of Nuclear Materials Management of the United States Atomic Energy Commission for study of the mixing and sampling characteristics of cylindrical safe-geometry tankage has been completed at the Oak Ridge Y-12 Plant. The program combined the measurement of some safe-tank mixing and sampling characteristics with a theoretical analysis for the generalization of mixing characteristics for recirculation in mixed-tank systems.

Safe-tank mixing was simulated in an experimental facility that consisted of both vertically and horizontally orientated tankage that was equipped with flow-rate and fluid-concentration measurement components. The facility included both single and parallel tanks mixed by fluid recirculation and vertical tanks mixed by air sparging. For each tankage system, the mixing time required to disperse and mix a relatively small volume of tracer with the bulk fluid contained in the system was measured. This mixing time was correlated with the flow rate by a least-squares treatment of the data and presented graphically. For many systems, the effect of the degree of fullness upon mixing time was observed. For single-tank systems mixed by fluid recirculation, dispersion-intensity data were collected, treated by least-squares methods, and presented graphically. Both mixing-time and dispersion-intensity data were found to fit reasonably well into a full logarithmic plot as a function of the recirculation rate by linear correlation.

A theoretical treatment of single and parallel-tank systems mixed by fluid recirculation was made. The treatments utilized a combination of the tanks-in-series model and the dispersion model, and required either analogue or digital-computer solutions. For single-tank systems, a generalized and dimensionless solution for mixing-time prediction was obtained and presented graphically. Further, a comparison of experimental and theoretical mixing-time data has been made. A similar treatment was made for parallel-tank systems, but a solution to this complex problem cannot be presented as simply. However, an alternate scheme for predicting the upper limit of parallel-tankage mixing time using the single-tank solution has been developed.

Guidelines for mixing miscible fluids or mixing miscible fluids containing fine solids prior to representative sampling have been established. The effects of the piping arrangements of the various tankage systems upon mixing, sampling, and fuel-inventory uncertainties have been reviewed and recommendations have been made for tankage system design and operation.

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INTRODUCTION

In nuclear-fuel processing plants, where a significant quantity of fissile material is contained in solution, the mixing and sampling of fluids in nuclear-safe tankage is necessary for material inventory. Although some safe-geometry slab tankage and some nonsafe-geometry, neutron-poisoned tankage are used for storage of fuel solutions, cylindrical safe-geometry tanks fabricated from small-diameter pipe (usually < 5 " ID) are more commonly used. Mixing and sampling of the tankage contents are periodically required to determine the fuel inventory. Due to the nature of the processing operations, a mixture of solutions is generated over a period of time and transferred to a storage facility to await final processing. Since the composition and concentration of the collected fluid mixture varies with feed-material and process-control variables, most tankage facilities must be mixed to homogeneity prior to sampling in order to withdraw a representative sample from this system.

In most cases, tankage systems are mixed by an established procedure that is outlined by trial-and-error experiments. For some systems, where fluid with nearly constant composition is generated, little if any mixing may be required to homogenize the mixture. For other systems having greater initial inhomogeneity, a greater mixing intensity and duration are required. For those systems where mixing procedures have been established for nearly homogeneous fluid mixtures, changes in the chemical-processing variables may cause the established procedures to be inadequate.

Fluid contents of a tankage system can be mixed to any small degree of inhomogeneity and, in most cases, be representatively sampled provided that stagnant pockets do not exist in the system and the mixing characteristics of the system are utilized. Mixtures of miscible solutions need only to be homogenized prior to representative sampling, while similar mixtures that contain immiscible components or some solid material must be thoroughly mixed and sampled with special methods.

A program was initiated at the Y-12 Plant to study the parameters associated with mixing and sampling fluid mixtures contained in cylindrical tankage. This report summarizes the final experimental results and the theory that was developed for safe-tank mixing. Guidelines are suggested for the design and operation of tankage facilities.

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ACCOMPLISHMENTS

GENERAL

An experimental installation, simulating the construction of vertically and horizontally orientated cylindrical, nuclearly safe storage tankage used in uranium and plutonium processing plants, was built, as shown in Figure 1. The installation consisted of modular four and six-inch-diameter Pyrex pipe tanks equipped with flowmeters and concentration measuring instruments for measuring the degree of fluid homogeneity as a function of time for a given set of mixing conditions.

The following types of tankage systems, included in the installation, were evaluated: (1) single tanks mixed by simple recirculation, (2) single tanks adapted with a selectively perforated, coaxial inner inlet tube and mixed by recirculation, (3) two tanks in parallel mixed by recirculation, and (4) a vertical tank mixed by air sparging.

The program was divided into four areas: (1) a comprehensive preliminary literature survey, (2) an experimental measurement of mixing characteristics of some tankage systems, (3) an experimental determination of requirements for representative sampling of thoroughly mixed liquid-solid mixtures, and (4) the development of a mathematical theory for the generalization of mixing characteristics of recirculation-mixed systems. By continuously measuring and recording the concentration of a miscible tracer as a function of time, a defined mixing time was obtained and correlated with the flow rate for each tankage system considered. For the recirculation-mixed single-tank systems, dispersion-intensity data were collected and correlated for use in the developed mixing theory. As a consequence, guidelines for mixing and sampling cylindrical safe tankage were selected which are applicable to existing facilities and for the design of new systems.

Two recognized authorities in the fields of fluid mixing and sampling were retained as consultants: Dr. J. H. Rushton of Purdue University, who has developed the technology for isokinetic sampling of liquid-solid mixtures flowing in a pipeline and for conventional tank mixing by turbine mixers; and Dr. Octave Levenspiel of the Illinois Institute of Technology, who has developed approaches for mathematically representing the mixing of a fluid flowing through processing piping and equipment.

EXPERIMENTAL

Pulse-type tracer-mixing experiments have been conducted in seventeen safe-tank systems constructed of four or six-inch-diameter Pyrex pipe with length-to-diameter ratios between 17 and 122. For these pulse-type experiments, the tracer was initially confined within a relatively small volume of the safe-tank system. Tracer dispersion as a function of time and rate of flow was measured for 673 experimental runs divided among 53 experimental series. From these runs, mixing-time data

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Figure 1. THE EXPERIMENTAL FACILITY.

and/or dispersion-intensity data were collected, correlated, and evaluated. Measurements were made at recirculation rates between ~ 1 and 101 gpm for recirculation-mixed systems and at air-sparge rates between 0.12 and 5.0 scfm for air-sparge-mixed vertical tanks. For most runs, raw data were recorded on strip charts for subsequent evaluation; however, for all air-sparge runs, where evaluation of strip charts would be arbitrary and imprecise, data were collected with a digital tape acquisition unit for subsequent analysis using a digital computer.

Mixing-time determinations were made for most of the experimental series. Although the mixing time was determined by different methods, as required by raw data, in all cases it gives, as a function of the rate of mixing flow, the time duration of mixing to disperse a small quantity of tracer to some defined condition near homogeneity with the system bulk fluid. For recirculation-mixed systems, where recorder strip charts gave tracer-concentration curves similar in form to exponentially decaying sine curves, the periodic character of a curve was used to determine the mixing time. When the ratio of minimum-to-maximum tracer concentration for any complete circulation of fluid contained in the system was equal to or greater than 0.998, the system was considered to be mixed. The mixing time for a recirculation-mixed system signified the mixing period required to reach a maximum concentration deviation throughout the tankage system of about 0.2 percent of the ultimate concentration. Nonperiodic strip-chart curves which appeared to exponentially approach the ultimate tank tracer concentration were obtained for air-sparge-mixed vertical tanks. Due to the difficulty of locating on a strip chart the point where an arbitrary condition of near homogeneity could be accurately and consistently selected, a digital computer analysis was applied to concentration data acquired by a digital tape unit. By this method, mixed conditions of 0.90, 0.95, and 0.99 were determined.

Mixing-time data were obtained for 25 experimental series conducted in nine different single-tank systems as a function of the recirculation rate and the degree of system fullness. For these series, where recirculation rates were varied between 1.87 and 101 gpm, the mixing-time data ranged between 9.6 and 622.0 minutes. Because the data fit a straight line reasonably well when plotted on full logarithmic graph paper, they were treated by a least-squares correlation of the form:

$$\text{Log } T = A - B \log Q, \quad (1)$$

where:

T represents the mixing time, and

Q the recirculation rate.

A and B are the correlation constants.

Values for constants of these data fits, their standard error, and the variance of estimate for the linear fits have been calculated.

Mixing-time data were obtained for eleven experimental series conducted in a parallel two-tank system. Ratio of the flow through tanks placed in a parallel configuration was varied to show the effect upon the mixing time of the flow split at the tank entrances and the subsequent flow combination at the tank outlets. Eleven flow-split ratios were tested at aggregate recirculation rates between 10 and 100 gpm. Under these conditions, the mixing time varied between 5.8 and 115.0 minutes. These data were correlated by a least-squares technique to Equation 1 as a function of the recirculation rate for each flow-split ratio. Values of the correlation constants, their standard error, and the variance of estimate for each fit were calculated for each experimental series. A crossplot of the mixing time as a function of the flow-split ratio was made for several levels of aggregate recirculation rates. The effect of splitting and recombining the flow through two identical parallel tanks was manifest in this plot by a mixing-time minimization within the range of a flow-split ratio from 0.6 to 0.8. Effectively, both time and energy requirements were found to be reduced with this range of flow-split ratios for a two-tank parallel system.

Mixing-time measurements were made for a recirculation-mixed safe tank adapted with a selectively perforated coaxial, inner inlet tube (termed mixing tube). Mixing-time data were collected as a function of the recirculation rate for five experimental series where different mixing tubes were used to obtain different combinations of series and parallel flows. Within the recirculation rate range of 2.15 and 79.8 gpm, mixing time was found to vary between 2.0 and 111.0 minutes. These data were fit to Equation 1 by a least-squares correlation. The correlation coefficients, their standard error, and the variance of estimate of fit were calculated. These data demonstrate that an indiscriminate selection of axial locations for holes in a mixing tube can lead to a relatively poor mixing system.

Mixing-time measurements were made for an air-sparge-mixed vertical tank in 15 experimental series as a function of the air-flow rate, liquid-column height, and end-mixed condition. The tank filled to give liquid-column, height-to-diameter ratios ranging between 17.2 and 71.8 were studied at air-flow rates between 0.12 and 5.0 scfm. Mixing-time data which ranged between 1.3 and 72.3 minutes were fit to Equation 1 by the method of least squares, and correlation coefficients, their standard error, and the variance of estimate of fit were calculated for the end-mixed conditions of 0.90, 0.95, and 0.99.

Dispersion intensity measurements were made for ten recirculation-mixed, single-tank systems. Data for these systems with length-to-diameter ratios ranging from 29 to 122 were correlated on full logarithmic paper with a Reynolds number. For all but one experimental series, these data fit reasonably well by using an equation of the form:

$$\log [Di] = A - B \log [Re], \quad (2)$$

where:

Di represents the dispersion intensity, and

Re the Reynolds number.

A and B are the correlation constants.

Data for the system with the largest length-to-diameter ratio ($L/d = 122$) were found to compare better with the published data that were collected for very long pipelines and to be poorly represented over the full range of Reynolds numbers by the linear correlation of Equation 2. With the exception of data included in this experimental series, the correlation coefficients, their standard error, and the variance of estimate of fit were calculated.

Using turbidimetric methods, the mixing and sampling characteristics of micron-sized particulates in a fluid stream were measured. In a recirculation-mixed, single-tank system, 0.05 and 1.0-micron alumina were mixed at fluid velocities up to about one foot/second. Except at the highest fluid velocities, significant amounts of solids were left settled on the tank bottom and were not dispersed into the fluid stream. Using isokinetic sampling techniques for a set of experiments conducted at two levels of recirculation rate (42.5 or 97.6 gpm) with 0.05 micron alumina, the measured solids concentration at the low and high recirculation rates were about 20 and 80 percent, respectively, of the actual average solids concentration. The gravitational effect upon solids was too great to be overcome by turbulent eddies created by a recirculation flow at these relatively low recirculation rates. As a result, it was not possible to representatively sample the tank system.

Alumina in 0.05, 1.0, and 20.0-micron size ranges was mixed at air-flow rates between 0.29 and 2.09 scfm in an air-sparged vertical tank. Solids concentration was varied from 250 to 2000 ppm by weight. In general, thief samples were found to be representative of the liquid-solid composite at the high air-flow rate; however, the sample withdrawal rate significantly affected the observed solids concentration. By varying the sample withdrawal rate, the observed solids concentration ranged between 68 and 140 percent of the actual average solids concentration.

THEORETICAL

Theoretical analyses for recirculation-mixed single tankage, multiple parallel tankage, and single tanks adapted with a mixing tube have been outlined. The analyses utilize the dispersion model and the tank-in-series model for mathematically treating the dispersion of a tracer pulse throughout a tankage system. For the recirculation-

mixed single-tank system, a numerical digital computer solution was obtained which gives in dimensionless variables the necessary mixing time for a system to achieve a given mixed condition of near homogeneity. The solution, presented graphically, uses a determinable variable calculated from a dispersion intensity (Reynolds number correlation) and gives the mixing time as a function of this variable with the desired end-mixed condition as a parameter. A comparison of the theoretically predicted mixing time with the experimentally acquired data reflected reasonably good agreement. Within the range of recirculation rates that would be recommended (Reynolds number of about 10,000), the relative error of the predicted mixing time is about ± 30 percent. For the single-tank system, relatively simple calculational procedures have been outlined for the mixing-time calculation.

Two analyses were used to treat mixing in parallel tankage. The first, which used the tank-in-series model, was found to require prohibitively large digital computer times for solution. Although it may be more amenable to analogue computer solution, no effort was made to locate a computer large enough to solve the problems. An alternate analysis similar to that successfully applied to recirculation-mixed, single-tank systems was developed. Although it can be used to compute the mixing time for parallel tankage, it was not possible to generalize the mixing-time predictions as has been done for the single-tank analysis.

DISCUSSION

LITERATURE SURVEY

At the outset, a survey was made of the published literature related to the mixing and sampling of fluids contained in cylindrical tankage. In addition, a survey of unpublished literature was made in conjunction with inspection trips to nuclear-fuel processing facilities at eight AEC plants and contacts with personnel concerned with problems in this area. A summary of these surveys of the state of technology for mixing and sampling was presented in a preliminary report.⁽¹⁾ Due to the unique nature of mixing fluids in cylindrical safe tankage and the complexity of most systems used in processing plants, no prior work of general utility was uncovered except that of Rushton and Hillestad⁽²⁾ for isokinetic sampling of well-mixed, two-mixed, two-phase flowing streams.

Numerous articles have appeared in the literature in the past two decades which treat fluid mixing during a single pass through process piping and tankage. The phenomenon of fluid mixing during flow (commonly referred to as dispersion), which reduces the driving potentials for heat and mass transfer and effects significant changes in many chemical reactions, should be taken into account in equipment design. In several articles, the theory was developed and experimentally tested for describing the intermixing or spread of the fluid within itself during flow. Analytical expressions comprise one product of these articles giving the dispersion of a tracer injected into the stream at some upstream point and measured at a point downstream in the tank or pipeline. The expressions include: (1) those giving the tracer concentration distribution at the measurement point as a function of time and a mixing parameter, and (2) those which can be used to extract mixing-parameter correlations from data acquired for various experimental methods on different mixing systems. In most cases, these expressions were developed from either the dispersion model or the tanks-in-series model as reviewed comprehensively by Levenspiel and Bischoff.⁽³⁾ As discussed in this review of the dispersion model, a mixing parameter (termed the dispersion intensity) has been measured by numerous investigators and correlated with the Reynolds number for very long pipelines.

The utility of these advances in mixing technology is subtle for the mixing of safe tanks by recirculation since these advances cannot be directly applied. In a progress report,⁽⁴⁾ a review was given of some properties of the mixing models which can be used for recirculation mixing where many passes of fluid around a tankage loop are needed to mix the tankage contents to near homogeneity.

Several articles^(5 - 8) have been presented on air-sparged mixing of miscible liquids in a vertical cylindrical tank. Solutions have been found for the governing differential equations which have then been used to experimentally measure a suitably defined mixing parameter. The collected mixing data were generally obtained for small tanks. Since tank diameter and height will affect the developed mixing pattern

in air-sparged tanks due to wall-boundary conditions and axial-static pressure effects, the mixing-parameter data might not be applicable to larger operations.

An excellent discussion of representative sampling methods for nuclear-fuel solutions is given by Jones.⁽⁹⁾ Two difficulties arise, however, in applying these methods: (1) the methods assume fluid homogeneity, and (2) the methods may not be generally applicable to two-phase systems. Since the purpose of this program is to provide guidelines for determining the mixing requirements for achieving homogeneity for miscible fluid mixtures and to provide guidelines for the applicability of these sampling methods to well-dispersed, two-phase mixtures, the current mixing and sampling studies will complement the work of Jones. In addition, isokinetic sampling techniques, discussed by Rushton and Hillestad,⁽²⁾ deal with the parameters associated with representative sampling of a well-mixed composite flowing through a pipe in turbulent flow. This article, which treats both liquid-solid composites and immiscible liquid-liquid composites, should find application in the sampling of recirculation-mixed cylindrical storage tankage.

TANKAGE SYSTEMS

Basically, two types of tankage (recirculation-mixed and air-sparge-mixed tanks) were studied. Experimental data were obtained for a number of tankage configurations and geometries. These tankage systems were constructed of four or six-inch-diameter Pyrex pipe tanks and analytical components as previously reported.^(4, 9)

Recirculation mixing experiments were conducted in horizontal and vertical single tanks (some of which were adapted with a coaxial mixing tube) and in a horizontal, parallel two-tank system to obtain basic mixing knowledge for generalization to more complex tankage systems and to provide accurate data for testing the applicability of developed mixing theories. Air-sparge experiments were conducted in a tall tank to obtain mixing-time requirements. Each type of system is presented diagrammatically in Figures 2 - 7. A tabulation of the aggregate piping lengths and component volumes for the various tank systems is given in Table 1.

A summary of the experimental programs giving the series numbers, types of tankage systems, basic tankage dimensions, and the document where the data have been reported is presented in Table 2.

To complement the theory reported for air-sparge-mixed tanks^(5 - 8) and to provide a basis for analytically treating recirculation-mixed tankage, methods of characterizing these systems were developed by using the tank-in-series and dispersion models. Computer analyses were obtained for single-tank and parallel-tank systems. Due to the complexity of the mixing flow patterns in a single tank adapted with a coaxial mixing tube only, an outline of a method of approach was developed.

Figure 2. REPRESENTATIVE ARRANGEMENT FOR A HORIZONTAL, SINGLE, MIXED TANK.

Figure 3. REPRESENTATIVE ARRANGEMENT FOR A HORIZONTAL, SINGLE-TANK LOOP ADAPTED WITH A MIXING TUBE.

Figure 4. REPRESENTATIVE ARRANGEMENT FOR THE VERTICAL AIR-SPARGED TANK.

RECIRCULATION-MIXED, SINGLE-TANK SYSTEMS

Theoretical Development

An analytical expression was obtained for use in predicting the tank output tracer concentration for a tracer pulse injected at the tank inlet into the flowing stream. This expression utilizes a solution for tracer dispersion during flow in an infinitely long pipeline, as suggested by Levenspiel and Smith,⁽¹⁰⁾ and superimposes a set of

Figure 5. REPRESENTATIVE ARRANGEMENT FOR A VERTICAL, SINGLE-TANK EXPERIMENTAL LOOP.

these solutions in such a manner that the transient tracer concentration at a safe-tank outlet is given in dimensionless form. This expression is:

$$C = \sum_i \sqrt{\frac{1}{4\pi P\theta}} \exp \left[-\frac{(i - \theta)}{4P\theta} \right], \quad (3)$$

where:

C represents the dimensionless tracer concentration measured at the tank outlet,

i the index for the exponential term giving the number of tracer circulations around the safe tank loop (1, 2, 3, ...), and

Figure 6. REPRESENTATIVE ARRANGEMENT FOR A VERTICAL, SINGLE, MIXED TANK ADAPTED WITH A MIXING TUBE.

θ the dimensionless time which is the ratio of the time duration of mixing (eg, minutes) to the mean residence time of the system (total volume divided by recirculation rate).

For most applications, the mixing number, P , can be closely approximated by neglecting the dispersive mixing in loop piping, which connects the safe tank outlet through a pump with the inlet, by the following simple expression:

$$P = \left(\frac{d}{L}\right)_{\text{tank}} \left(\frac{D}{Ud}\right)_{\text{tank}} \left(\bar{t}\right)_{\text{tank}}^2 \left(\bar{t}\right)_{\text{system}}^2, \quad (4)$$

Figure 7. REPRESENTATIVE ARRANGEMENT FOR A PARALLEL-TANK EXPERIMENTAL LOOP.

where:

$\left(\frac{d}{L}\right)_{\text{tank}}$ represents the dimensionless ratio of the tank diameter, d , to the tank length, L ,

$\left(\frac{D}{Ud}\right)_{\text{tank}}$ the dispersion intensity for the tank system obtained from the Reynolds number correlation (see Figure 8) and discussion on Pages 31 through 33,

$\left(\bar{t}\right)_{\text{tank}}^2$ the square of the mean residence time for the tank (eg, square of the ratio of the tank volume to the recirculation rate), and

$\left(\bar{t}\right)_{\text{system}}^2$ the square of the mean residence time for the system (eg, square of the ratio of the aggregate system volume to the recirculation rate).

Table 1
SAFE TANK SYSTEM PIPING LENGTHS AND COMPONENT
VOLUMES FOR DESIGNATED EXPERIMENTAL SERIES

Series Number	Tank Diameter (inches)	Tank Length (feet)	Tank Fullness (%)	Tank Volume (gals)	2.0-Inch Pipe		1.5-Inch Pipe		Pump Volume (gals)	Miscellaneous System Volumes (gals)	Total System Volume (gals)
					Length (feet)	Volume (gals)	Length (feet)	Volume (gals)			
1	4	15	100.0	9.7	15.6	2.7	22.1	2.33	0.22	1.16	16.21
2	4	15	97.7	9.5	15.6	2.7	22.1	2.33	0.22	1.16	15.98
3	4	25	100.0	16.3	16.7	2.9	22.1	2.33	0.22	1.08	22.85
4	4	25	98.5	16.0	16.7	2.9	22.1	2.33	0.22	1.08	22.61
5	4	25	79.6	12.9	16.7	2.9	22.1	2.33	0.22	1.08	19.52
6	6	15	100.0	22.0	7.9	1.3	20.4	2.16	0.34	1.81	27.73
7	6	15	98.5	21.7	7.9	1.3	20.4	2.16	0.34	1.81	27.39
8	6	20	100.0	29.3	16.7	2.9	14.6	1.54	0.34	1.81	35.97
9	6	20	98.9	29.0	16.7	2.9	14.6	1.54	0.34	1.81	35.64
10	6	20	85.2	25.0	16.7	2.9	14.6	1.54	0.34	1.81	31.62
11	6	25	100.0	36.7	16.7	2.9	20.4	2.16	0.34	1.81	43.93
12	6	25	99.1	36.4	16.7	2.9	20.4	2.16	0.34	1.81	43.61
13	6	25	88.2	32.3	16.7	2.9	20.4	2.16	0.34	1.81	39.59
14	6	20	100.0	29.3	16.7	2.9	14.6	1.54	0.34	1.81	35.97
15	6	20	100.0	29.3	16.7	2.9	14.6	1.54	0.34	1.81	35.97
16	6	20	100.0	29.3	16.7	2.9	14.6	1.54	0.34	1.81	35.97
17	6	20	100.0	29.3	16.7	2.9	14.6	1.54	0.34	1.81	35.97
18	6	26	100.0	38.1	16.7	2.9	20.6	2.18	0.34	1.81	45.42
19	6	26	100.0	38.1	16.7	2.9	20.6	2.18	0.34	1.81	45.42
22	6	20	100.0	29.3	16.7	2.9	14.6	1.54	0.34	1.81	35.97
23	6	20	100.0	29.3	16.7	2.9	14.6	1.54	0.34	1.81	45.42
24	6	40	100.0	58.7	66.3	11.5	6.3	0.67	0.22	2.51	73.72
25	6	20	100.0	29.3	17.9	3.1	15.9	1.68	0.34	1.81	36.23
26	6	40	100.0	58.7	66.3	11.5	6.3	0.67	0.22	2.51	73.72
27	6	20	100.0	29.3	17.9	3.1	15.9	1.68	0.34	1.81	36.33
28	6	20	100.0	29.3	56.7	9.8	21.8	2.31	0.34	4.53	46.32
29	6	20	75.0	22.0	56.7	9.8	21.8	2.31	0.34	4.53	38.98
30	6	20	50.0	14.6	56.7	9.8	21.8	2.31	0.34	4.53	31.63
31	6	20	100.0	29.3	17.9	3.1	15.9	1.68	0.34	1.81	36.23
32-42	4	25	100.0	32.6	22.4	3.9	25.4	2.69	0.22	3.33	42.78
43	6	20	100.0	29.3	17.9	3.1	15.9	1.68	0.34	1.81	36.23
44	6	20	100.0	29.3	17.9	3.1	15.9	1.68	0.34	1.81	36.23
45	6	20	100.0	29.3	17.9	3.1	15.9	1.68	0.34	1.81	36.23
46	6	20	76.7	22.5	0.0	0.0	0.0	0.00	0.00	0.00	22.53
49	6	20	100.0	29.3	56.7	9.8	21.8	2.31	0.34	4.53	46.32
51	4	40	100.0	26.1	66.3	11.6	6.3	0.67	0.22	1.16	39.75
52(1)	6	35.9	100.0	52.74	0.0	0.0	0.0	0.00	0.00	0.00	52.74
53	6	30	100.0	44.07	0.0	0.0	0.0	0.00	0.00	0.00	44.07
54	6	24.6	100.0	36.14	0.0	0.0	0.0	0.00	0.00	0.00	36.14
55	6	19.8	100.0	29.09	0.0	0.0	0.0	0.00	0.00	0.00	29.09
56	6	16	100.0	23.50	0.0	0.0	0.0	0.00	0.00	0.00	23.50
57	6	12	100.0	17.63	0.0	0.0	0.0	0.00	0.00	0.00	17.63
58	6	8.6	100.0	12.63	0.0	0.0	0.0	0.00	0.00	0.00	12.63
59	6	16.5	100.0	24.24	0.0	0.0	0.0	0.00	0.00	0.00	24.24
60	6	16.5	100.0	24.24	0.0	0.0	0.0	0.00	0.00	0.00	24.24
61	6	16.5	100.0	24.24	0.0	0.0	0.0	0.00	0.00	0.00	24.24
62	6	30	100.0	44.07	0.0	0.0	0.0	0.00	0.00	0.00	44.07
63	6	30	100.0	44.07	0.0	0.0	0.0	0.00	0.00	0.00	44.07
64	6	20	100.0	29.3	17.9	3.1	15.9	1.68	0.34	1.81	36.23
65	4	35.9	100.0	23.44	0.0	0.0	0.0	0.00	0.00	0.00	23.44
66	4	30	100.0	19.58	0.0	0.0	0.0	0.00	0.00	0.00	19.58
67	4	24.6	100.0	16.06	0.0	0.0	0.0	0.00	0.00	0.00	16.06
68	4	19.8	100.0	12.93	0.0	0.0	0.0	0.00	0.00	0.00	12.93
69	4	16	100.0	10.44	0.0	0.0	0.0	0.00	0.00	0.00	10.44
70	4	12	100.0	7.83	0.0	0.0	0.0	0.00	0.00	0.00	7.83
71	4	8.6	100.0	5.61	0.0	0.0	0.0	0.00	0.00	0.00	5.61

(1) Series 52 through 58 and 65 through 71 - The liquid level in the tank is used as the tank height and also indicates 100% fullness.

Table 2
SUMMARY DATA OF THE EXPERIMENTAL PROGRAM

Series Number	System Safe Tank		Type Experiment
	Diameter (inches)	Length (feet)	
1	4	15	Single Tank
2	4	15	Single Tank
3	4	25	Single Tank
4	4	25	Single Tank
5	4	25	Single Tank
6	6	15	Single Tank
7	6	15	Single Tank
8	6	20	Single Tank
9	6	20	Single Tank
10	6	20	Single Tank
11	6	25	Single Tank
12	6	25	Single Tank
13	6	25	Single Tank
14	6	20	Single Tank
15	6	20	Single Tank
16	6	20	Single Tank
17	6	20	Single Tank
18	6	26	Single Tank
19	6	26	Single Tank
22	6	20	Single Tank with Mixing Tube
23	6	20	Single Tank with Mixing Tube
24	6	40	Single Tank
25	6	20	Single Tank with Mixing Tube
26	6	40	Single Tank
27	6	20	Single Tank with Mixing Tube
28	6	20	Single Tank
29	6	20	Single Tank
30	6	20	Single Tank
31	6	20	Single Tank with Mixing Tube
32-42	4	25	Parallel Tanks
43	6	20	Single Tank with Mixing Tube
44	6	20	Single Tank with Mixing Tube
45	6	20	Single Tank with Mixing Tube
46	6	15.3	Air Sparge
49	6	20	Single Tank
51	4	40	Single Tank
52	6	35.9	Air Sparge
53	6	30	Air Sparge
54	6	24.6	Air Sparge
55	6	19.8	Air Sparge
56	6	16	Air Sparge
57	6	12	Air Sparge
58	6	8.6	Air Sparge
59(1)	6	16.5	Air Sparge
60(1)	6	16.5	Air Sparge
61(1)	6	16.5	Air Sparge
62(1)	6	30	Air Sparge
63(1)	6	30	Air Sparge
64(2)	6	20	Solids-Liquid
65	4	35.9	Air Sparge
66	4	30	Air Sparge
67	4	24.6	Air Sparge
68	4	19.8	Air Sparge
69	4	16	Air Sparge
70	4	12	Air Sparge
71	4	8.6	Air Sparge

(1) Series 59, 60, 61, 62, and 63 were solids-liquid air-sparge-mixed experiments.

(2) Series 64 was a solids-liquid recirculation-mixed experiment.

Figure 8. LINEAR CORRELATION FOR DISPERSION INTENSITY AS A FUNCTION OF THE REYNOLDS NUMBER FOR ALL SERIES.

Equation 4 generates a periodic, cyclical function dependent solely upon the value of P . The function is similar in form to an exponentially decaying sine wave in which the wave maxima approach the ultimate steady-state, dimensionless concentration, as shown in Figure 9. When the ratio of the steady-state concentration to the maximum concentration for each peak is compared for increasing maximum times, the ratio approaches the limiting value of 1.0. The ratio:

$$\gamma \geq \frac{C_{\infty}}{C_p}, \quad (5)$$

where:

γ represents the end-mixed condition parameter,

C_{∞} the ultimate or steady-state tracer concentration, and

C_p the maximum tracer concentration for a given tracer peak,

was used as an index of the degree of homogeneity of the fluid in a tank system. Figures 10 and 11 present recorder charts showing the actual sine-wave decay as

Figure 9. EXPONENTIALLY DECAYING SINE WAVE IN WHICH THE WAVE APPROACHES THE STEADY-STATE TRACER CONCENTRATION OR END-MIXED CONDITION. (Theoretical)

mixing progresses. The mixing period to achieve a condition of homogeneity consistent with Equation 5 is a function of two variables—the end-mixed condition, γ , and the system mixing number, P . This period, defined as the mixing time, is the time associated with an integral number of circulations of the fluid contents of a tank loop. It is dimensionless in form and has been denoted by T .

Mixing-time calculations were performed for seven levels of mixing numbers in the range where most systems would be mixed and for seven levels of the end-mixed condition that are of interest. The calculations were made by an IBM 7090 digital computer for given values of γ and P by simultaneously solving Equation 3 subject to Equation 5. For mixing numbers less than about 0.1, a linear correlation between the mixing number and the dimensionless mixing time for a fixed end-mixed condition was indicated by a plot of these data on log-log coordinates. As a consequence of the discrete nature of the computed mixing-time data (data are a function of an integral number of circulations), the data for a given γ and for mixing numbers less than 0.1 were correlated to the following equation:

$$\log T = A - B \log P, \quad (6)$$

where A and B are the correlation constants. Correlation constants, their standard error, and the variance of estimate of fit for each γ are given.^(4, 9) A plot of these correlations is given in Figure 12. Application of these correlations for the calculation of a single-tank mixing time has been reported previously.⁽¹¹⁾

Experimental Investigations

By using an approximation to the pulse-type tracer injection where the tracer is initially confined within a small volumetric section of loop piping between isolation valves, shown in Figure 13, mixing-time and dispersion-intensity data were collected. These data were gathered for four and six-inch-diameter tanks with length-to-diameter ratios ranging between about 29 and 122. The investigation considered vertical and horizontal (inclined about 1/5 inch per foot for drainage) tankage at several degrees of fullness where the recirculation rate was varied over the range from 1 to 100 gpm. Included was the experimental series listed in Table 2.

As indicated in this table, most of the data have been published in earlier reports; however, some supplementary data and further data analysis have been included in this report.

Mixing-Time Data - An analysis of the mixing-time data has demonstrated that the collected data for a given tankage system, when plotted on full logarithmic coordinates, are well represented by a straight line. As a consequence, all mixing-time data were correlated with the recirculation rate by Equation 1 (Page 13).

Although a slight deviation in the linearity of the data has been detected at low recirculation rates for most of the relatively short tanks ($L/d < 75$) and a deviation was manifest within a Reynolds number range of 4000 and 16,000,⁽⁹⁾ the data correlations are good for Series 24 and 26. As a further justification for this data treatment, it was shown that the form of the fit, Equation 1, meets the required limiting conditions for mixing in a safe-tank loop. These conditions are: (1) an infinite mixing time at zero recirculation rate, and (2) a zero mixing time at an infinite recirculation rate. These conditions are met when the antilogarithm of Equation 1 is tested; thus:

$$T = 10^{A Q^{-B}},$$

where it is seen that:

$$T \longrightarrow \infty \quad \text{as} \quad Q \longrightarrow 0, \text{ and}$$

$$T \longrightarrow 0 \quad \text{as} \quad Q \longrightarrow \infty.$$

Figure 10. STRIP CHART OBTAINED FROM A RECIRCULATION-MIXED, SINGLE, VERTICAL-TANK SYSTEM. (Series 49, Run 482)

Figure 12. LINEAR CORRELATIONS TO PREDICTED DIMENSIONLESS MIXING TIME-MIXING NUMBER DATA FOR SEVEN MIXED CONDITIONS.

Basically, without recirculation in a tankage system, mixing is achieved between the stratified liquid composite solely by liquid diffusion which, for a practical matter, is a negligible contributor; yet, at a very high recirculation rate, the dispersion of tracer by virtue of the fluid radial velocity distribution and the accompanying turbulent eddies reduce the mixing time in the limit to zero.

Details of the mixing-time analysis were previously given, (4, 9) where the exponentially decaying sine-curve concentration distribution, as recorded for each experimental run on a strip chart, was analyzed to select a mixed condition consistent with an end-mixed condition of $\gamma > 0.998$. In addition to a tabulation of the mixing-time-data correlations in these reports for the majority of the systems studied, it was shown experimentally that while pulse-type experiments give the maximum time required to mix a system, only at high recirculation rates do the pulse-type mixing times differ from those where a tracer is added directly to a safe tank and allowed to stratify by gravity before mixing. In that test case, the pulse-type experimental mixing time was about 25 percent greater at the highest recirculation rate (about 95 gpm); at the lowest recirculation rates, where most tankage systems will probably be operated, the difference in mixing time as observed for both methods is small.

Dispersion-Intensity Data - Since dispersion intensity embodies the effective dispersion or intermixing of fluids during the flow in a pipe, the relation of this variable

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Figure 13. TRACER-ADDITION PIPING SECTION OF A FOUR OR SIX-INCH-DIAMETER SAFE-TANK FACILITY.

to the Reynolds number and the effects of tank end-flow conditions upon this variable must be known for the application of a developed theory to any recirculation-mixed tankage systems. As a consequence, data were collected in 16 experimental series conducted in 9 safe-tank systems with length-to-diameter ratios between 29 and 122 at recirculation rates ranging between 1 and 100 gpm.

The calculational method previously discussed^(4, 9) requires a determination of the spread of tracer about a point moving at the mean fluid speed through a tank. From the first tracer peak recorded on a strip chart, the second movement of the tracer concentration distribution was computed from a set of data taken from each chart using an IBM 7090 computer. These data have been obtained for each experimental series as a function of the recirculation rate, and are given in tabular form and in graphs in Appendix A.

The variance data for pulse-type experiments were found, with one exception, to be well represented by a correlation of the form:

$$\log \sigma_t^2 = A - B \log Q, \quad (7)$$

where:

σ_t^2 represents the measured tracer variance in minutes squared, and

Q the recirculation rate in gallons per minute.

A and B are the correlation coefficients of the data fits.

It was shown⁽⁴⁾ that if this correlation can be used to fit a set of variance-recirculation rate data and if the appropriate transformation of variables is performed on these data, a dispersion intensity-Reynolds number correlation results of the form:

$$\log [Di] = A - B \log [Re], \quad (8)$$

where:

Di represents the dispersion intensity, and

Re the Reynolds number.

A and B are correlation coefficients of the fit.

For the series previously correlated with Equation 8, it was shown by two analyses of variance tests that a single slope, $\bar{B}$, and a single intercept, $\bar{A}$, were statistically acceptable at a confidence level of 95 percent to represent data included in all series by a single correlation. Data used in these tests included those for Series 6, 8, 16, 17, 18, and 19 and were restricted to horizontal tanks with length-to-diameter ratios between 29 and 41.5. A later and more extensive data analysis was made which included both horizontal and vertical tanks where length-to-diameter ratios ranged between 29 and 122.0. This analysis demonstrated three points: (1) data for very long tank systems should not be correlated by Equation 8 for Reynolds numbers less than about 20,000; (2) vertical tanks exhibit a significantly different correlation than do comparable horizontal tanks; and (3) a single generalized correlation for all tanks is not possible.

RECIRCULATION-MIXED, PARALLEL-TANK SYSTEMS

Two methods of treating the general problem of recirculation in mixing-in-parallel tankage have been developed. The methods utilize either the tanks-in-series model or the dispersion model with appropriately determined parameters obtained by using the equivalence relationship between the models to account for all components of a

tankage system. Since the prediction of mixing characteristics for parallel tanks having unequal lengths or through which unequal rates of fluid flow circulate is necessary to optimize the mixing characteristics, the developed methods treat any arrangement of tank length and fluid circulation.

Tanks-in-Series Method

One method⁽⁴⁾ treated the parallel mixing problem by the tanks-in-series model by determining the number of equally sized, ideally mixed tanks in series which have the same mean residence time and tracer-variance characteristics as a single safe tank under the same flow conditions. It was shown that the number of ideally mixed tanks for the Jth safe tank can be calculated from the following relationship between the mixing models:

$$N(J) = \frac{1}{2P(J)}, \quad (9)$$

where:

$N(J)$ represents the number of ideally mixed tanks representative of the Jth safe tank, and

$P(J)$ the mixing number for the Jth safe tank (eg, product of safe-tank dispersion intensity and diameter-to-length ratio).

In this manner a parallel tank system such as diagrammed in Figure 7 was represented by an array of ideally mixed tanks, as shown in Figure 14.

A set of equations was obtained for the transient tracer concentration of each ideally mixed tank in a safe tank system. To account for the flow-split condition at the inlet to each safe tank and the subsequent combination of flow from each safe tank, equations were given for determining the tracer concentration in the first ideally mixed tank of each safe tank and for the combined effluent stream tracer concentration from the safe tanks in the loop piping. An expression for an end-mixed condition similar to that used for single-tank systems was suggested.

This set of linear, first-order differential equations was programmed for solution by an IBM 7090 digital computer. Due to the small residence time associated with each ideally mixed tank, very small time increments were required to achieve stability of the numerical computer solution to the set of equations. As a result, a prohibitively large computational time was required to effect a single solution. It was estimated, for example, that over an hour of computer time would be needed to get a single mixing-time solution for a relatively simple two-tank problem.

Figure 14. REPRESENTATIVE ARRANGEMENT OF A PARALLEL SAFE-TANK SYSTEM BY USING A TANKS-IN-SERIES MODEL.

Although this set of equations was found not to be amenable to solution by numerical methods on a digital computer, solution by analog computation which directly solves the differential equations is recommended.

Dispersion Method

Mixing time for a parallel-tank system mixed by recirculation of the system contents through loop piping can be predicted by using the dispersion model. As in the case of recirculation-mixed single-tank systems, application of the dispersion relationship obtained by Levenspiel and Smith⁽¹²⁾ for the spread of an injected pulse (Dirac-delta function tracer pulse) can be made. Although this relationship was derived for the center test section of a doubly infinite pipe, it can be applied to recirculation mixing where, in place of the center test section, an equivalent section of a safe-tank system is considered and an appropriately selected mixing number is used.

For the single-tank problem, the dimensionless relationship was made more general with respect to the location of the tracer measurement point. The relationship given by Levenspiel and Smith:

$$C = \frac{1}{\sqrt{4\pi P\theta}} \exp \left[-\frac{(1-\theta)^2}{4P\theta} \right], \quad (10)$$

where:

C represents the dimensionless tracer concentration,

P the mixing number, and

θ the dimensionless time,

for the single-tank problem gives the tracer distribution as might be measured for a single circulation at safe-tank outlet. It was shown that, in general, tracer distribution for the Jth circulation (eg, measured at an equivalent axial position to J dimensionless tank lengths) is given by:(4, 9)

$$C = \frac{1}{\sqrt{4\pi P\theta}} \exp \left[-\frac{(J - \theta)^2}{4P\theta} \right]. \quad (11)$$

A check of Equation 10 can be made for the first pass of the tracer pulse around a loop by substituting $J = 1$ into Equation 11. By imposing superposition upon each of the concentration distributions where J takes on positive integral values, $J = 1, 2, 3, \dots$, etc, the concentration distribution that would be observed at the tank outlet as a function of time is the sum of these individual distributions, as given by Equation 3 (Page 21). Essentially, the intermixed spread of tracer pulses periodically injected into a stream flowing through an infinitely long pipeline is given by Equation 3 for axial displacements downstream from the injection point equivalent to the product of pass number times the equivalent tank length.

To apply superposition to the mixing of parallel streams which, in general, have different tank residence times, a different approach must be taken. In the first place, dimensionless time has no physical significance for a parallel-tank system as it did for the single-tank system. For the single-tank system, a positive unit value for dimensionless time could be associated with a tracer concentration maxima.

To treat the mixing of fluids in a parallel-tank system, a modification of the application of superposition is necessary. Rather than generating individual tracer concentration distributions, as was done for the single-tank problem by essentially re-locating the point of measurement to some equivalent tank length position, J, a different approach was used which is based upon the time location of the maxima of a pulse-injected tracer. It has been shown by other investigators(2, 10) that a pulse-injected peak moves through a pipeline at the mean speed of flow. The lapsed time between injection and measurement of the maximum tracer concentration at some downstream point is the mean residence time. To modify the dispersion equation of Levenspiel and Smith(10) so that the time lapse associated with a tracer pulse is used in place of the axial length position, J, a transformation was made from dimensionless to dimensioned time. It could be shown that in contrast to the single-tank problem where integral values of dimensionless time could be associated with

tracer concentration maxima, dimensionless time for the general parallel-tank problem where tank lengths or tank flow rates may be unequal has no physical significance.

The modified equation of Levenspiel and Smith which has been made more general with respect to location of the measurement point is:

$$C(\theta) = \frac{1}{\sqrt{4\pi P\theta}} \exp \left[-\frac{(z - \theta)^2}{4P\theta} \right]. \quad (12)$$

To transform this equation to dimensioned time, two relationships must be applied, as given by Levenspiel.⁽¹²⁾ These are the relationship between time:

$$t = \bar{t}\theta, \quad (13)$$

where:

t represents the dimensioned time,

$\bar{t}$ the residence time, and

θ the dimensionless time,

and the relationship between a dimensioned concentration distribution, $C(t)$, and a dimensionless concentration distribution, $C(\theta)$:

$$C(\theta) = \bar{t} C(t). \quad (14)$$

Upon substitution into Equation 12, the following concentration distribution is obtained:

$$C(t) = \frac{1}{\sqrt{4\pi P\bar{t}t}} \exp \left[-\frac{(\bar{t} - t)^2}{4P\bar{t}t} \right]. \quad (15)$$

For a given tracer pulse residence time, $\bar{t}$, and mixing number, P , the dimensioned concentration distribution can be computed as a function of time.

Levenspiel⁽¹²⁾ has pointed out the additive character of tracer dimensioned mean residence time and of tracer dimensioned variance. For a series of mixing regions these are:

$$(\bar{t})_{\text{overall}} = \bar{t}_1 + \bar{t}_2 + \dots + \bar{t}_N = \sum_{i=1}^N \bar{t}_i, \text{ and} \quad (16)$$

$$(\sigma_t^2)_{\text{overall}} = (\sigma_t^2)_1 + (\sigma_t^2)_2 + \dots + (\sigma_t^2)_N = \sum_{i=1}^N (\bar{t})_i^2, \quad (17)$$

where the overall dimensioned residence time and the overall dimensioned variance are $(\bar{t})_{\text{overall}}$ and $(\sigma_t^2)_{\text{overall}}$, respectively. These are readily computed from the following relationships:

$$\bar{t}_i = \frac{V_i}{Q_i}, \text{ and} \quad (18)$$

$$(\sigma_t^2)_i = 2 \left(\frac{d_i}{L_i} \right) (\bar{t}_i)^2 (Di)_i, \quad (19)$$

where:

V_i represents the volume of the i th section of pipe,

Q_i the flow rate through the i th section of pipe,

d_i the diameter of the i th section of pipe,

L_i the length of the i th section of pipe, and

$(Di)_i$ the approximate dispersion intensity.

For a centrifugal pump or other system component that might be better represented by an ideally mixed tank, the mean residence time is computed by an equation of the same form but with the component volume replacing the pipe volume and the variance computed from:

$$(\sigma_t^2)_i = (\bar{t}_i)^2. \quad (20)$$

As a consequence, dimensioned-mean residence times and dimensioned variances for all elements of a safe-tank system can be readily found and used to obtain the mixing number which is computed from:

$$P = 0.5 \frac{(\sigma_t^2)_{\text{overall}}}{(\bar{t})_{\text{overall}}^2}. \quad (21)$$

To apply Equation 15 for calculating the individual concentration distributions, the set of overall mean residence times, $(\bar{t})_{\text{overall}}$, and dimensioned variances, $(\sigma_t^2)_{\text{overall}}$, which are needed to generate the distributions must be determined. It has been found that expansion of a multinomial can be used to determine these quantities and also to determine the number of repetitions of a given set of these variables. The general form of a multinomial⁽¹³⁾ is:

$$(X_1 + X_2 + X_3 + X_i \text{ ----} + X_M)^N = \sum (N; N_1, N_2, \text{ ----}, N_M) X_1^{N_1} X_2^{N_2} \text{ ----} X_M^{N_M}, \quad (22)$$

where:

X_i represents Tank i in a set of parallel tanks ($1 \leq i \leq M$),

M the total number of parallel tanks, and

N the number of passes made by a tracer maxima.

By definition, the following functions are given:

$$(N; N_1, N_2, \text{ ----}, N_M) = \frac{N!}{N_1! N_2! \text{ ----} N_M!}, \text{ and} \quad (23)$$

$$N = N_1 + N_2 + \text{ ----} + N_M, \quad (24)$$

where:

$$N = 1, 2, 3, \text{ ----}.$$

The variable N is to be associated with the number of times a tracer pulse has circulated through the parallel-tank system and is observed to pass the measuring point after flow combination at the tank outlets.

An example of the utility of the multinomial is demonstrated by the following analysis of a three-tank problem (ie, $M = 3$). As the residence times and variances for the first pass are obviously those associated with a given tank, the second-pass analysis is:

$$(X_1 + X_2 + X_3)^2 = X_1^2 + X_2^2 + X_3^2 + 2X_1X_2 + 2X_1X_3 + 2X_2X_3.$$

The coefficients of each term of the multinomial expansion give the number of repetitions of flow-path combinations, for two passes, that exist. The exponent of each X_i gives the number of passes through the i th tank. Therefore, all combinations of flow paths for two tracer passes are obtainable from this expression for three tanks. In addition, the effect of splitting the aggregate recirculating stream at the tank entrances with subsequent flow recombinations at take outlets can be evaluated. Each time a tracer pulse passes out of Tank 1, the pulse is diluted by the aggregate flowing stream by the dilution factor, F_1 :

$$F_1 = \frac{Q_1}{Q_1 + Q_2 + Q_3},$$

and so on for F_2 and F_3 where the Q 's are the respective volumetric flow rates through the three tanks as denoted by the subscripts. The aggregate flow or recirculation rate, Q_T , is the sum of flows through each parallel tank (eg, $Q_T = Q_1 + Q_2 + Q_3$). As a consequence, the dilution factor for each combination of the six flow combinations for two passes is the term given in the multinomial expansion (coefficients excluded) when F_K replaces X_K . Table 3 summarizes the two-pass pulse combinations.

Table 3
SECOND-PASS VARIABLES NEEDED FOR GENERATING
INDIVIDUAL CONCENTRATION DISTRIBUTIONS

	Term of the Expanded Multinomial					
	X_1^2	X_2^2	X_3^2	$2 X_1 X_2$	$2 X_1 X_3$	$2 X_2 X_3$
Mean Residence Times ($\bar{t}_K$)	$2 \bar{t}_1$	$2 \bar{t}_2$	$2 \bar{t}_3$	$\bar{t}_1 + \bar{t}_2$	$\bar{t}_1 + \bar{t}_3$	$\bar{t}_2 + \bar{t}_3$
Variances $(\sigma_t^2)_K$	$2 (\sigma_t^2)_1$	$2 (\sigma_t^2)_2$	$2 (\sigma_t^2)_3$	$(\sigma_t^2)_1 + (\sigma_t^2)_2$	$(\sigma_t^2)_1 + (\sigma_t^2)_3$	$(\sigma_t^2)_2 + (\sigma_t^2)_3$
Repetition Number (R_K)	1	1	1	2	2	2
Dilution Factors (F_K)	F_1^2	F_2^2	F_3^2	$F_1 F_2$	$F_1 F_3$	$F_2 F_3$

The effect of loop piping has been neglected in this table for the sake of simplicity. To account for it, for example, the alteration to the pulse designated by X_1^2 would change the residence time and variance to $[2\bar{t}_1 + 2\bar{t}_l]$ and $[2(\sigma_t^2)_1 + 2(\sigma_t^2)_l]$, respectively. Often the loop variance, $(\sigma_t^2)_l$, is negligible; however, the loop residence time $\bar{t}_l$ should be retained.

For a given concentration distribution, the mean residence time is determined as shown and the mixing number computed as the ratio of the determined variance to the square of the residence time. In general, for the X_1^2 concentration distribution, these are:

$$\bar{t}_{X_1^2} = [2\bar{t}_1 + 2\bar{t}_1], \text{ and}$$

$$P_{X_1^2} = \frac{[2(\sigma_t^2)_1 + 2(\sigma_t^2)_1]}{[\bar{t}_{X_1^2}]^2}.$$

To account for both the effect of the dilution factor and repetition upon the combined concentration distribution, Equation 15 was modified for convenience to:

$$C_K = \frac{R_K F_K}{\sqrt{4\pi P_K \bar{t}_K t}} \exp \left[-\frac{(\bar{t}_K - t)^2}{\sqrt{4P_K \bar{t}_K t}} \right]. \quad (25)$$

Using a multinomial expansion algorithm for computing R_K , F_K , P_K , and $\bar{t}_K$ for individual concentration distributions, the combined concentration distribution that would be observed at a measurement point in the loop piping (eg, at the tank outlets) is given by use of superposition as:

$$C = \sum_K C_K, \quad (26)$$

where:

K represents all sets of pass combination distributions for multinomial exponents,

N equals 1, 2, 3, ----, and

C the combined concentration distribution.

For selection of the time to be associated with an end-mixed condition of near tracer homogeneity throughout a tank, a test criterion similar to Equation 5 (Page 26) for single-tank systems was selected. If over a time increment, Δt , which is larger than the greatest tank residence time, the combined concentration distribution is very nearly equal to the average value of the distribution, C , then the system is defined as mixed. Mixing time exists when:

$$\gamma \geq \frac{C_{avg}}{C} \text{ throughout } \Delta t \text{ for } C_{avg} > C, \text{ or} \quad (27)$$

$$\gamma \geq \frac{C}{C_{avg}} \text{ throughout } \Delta t \text{ for } C_{avg} < C, \quad (28)$$

where

$$\Delta t > [\bar{t}_1 + \bar{t}_1]_{\max}. \quad (29)$$

Based on this developed theory, a digital computer program was written for use on an IBM 7090 computer and Benson-Lehner electroplotter combination to simulate mixing in a parallel-tank facility. The program written in Fortran IV, Version 13 language computes and prints out residence times, variances, and the products of the dilution factor and repetition number for each individual concentration distribution. Subsequently, the program computes the combined concentration distribution and prints out this distribution as a function of time. In addition, the program generates a tape for plotting the concentration distribution for graphical analysis. Appendix B contains the complete program in Fortran statements.

Experimental Investigations

Using the pulse-type tracer injection method, mixing-time data were collected for a two-parallel tank facility consisting of parallel 4-inch-diameter by 25-foot-long Pyrex pipe tanks closed on the ends with 4-by-2-inch reducers. The experimentation consisted of 131 runs conducted at 11 levels of recirculation rate between 10 and 100 gpm, and at 11 levels of flow split.

Flow-split conditions were defined as the ratio of flow through the tanks as:

$$FS = \frac{Q_1}{Q_2}, \text{ and} \quad (30)$$

$$0 < FS < 1.0,$$

where:

Q_1 and Q_2 are, respectively, the flows through Tank 1 and Tank 2, and

FS is the flow-split ratio.

For a flow-split condition of zero, all flow would pass through Tank 2 and the mixing problem essentially would degenerate to that for a single-tank system. When the flow-split ratio is 1.0, equal flows pass through the tanks. By varying the flow-split

ratio between these limiting conditions, the effect of splitting the flow at the tank entrance followed by a recombination of the parallel stream of flow at the tank outlet was studied.

A graphic analysis of the mixing-time data collected for a fixed flow-split ratio was found to be well correlated with the recirculation rate by the relationship used for single-tank data, Equation 1 (Page 13). As a consequence, a correlation between the mixing time and recirculation-rate data collected at a mixed condition of about 0.998 was obtained for each flow-split ratio.⁽⁴⁾ Data tabulation and graphical presentation have also been given along with correlation constants, their standard error, and the variance of estimate of fit for each flow-split ratio data set.

A cross plot was constructed to depict the mixing time as a function of the flow-split ratio with the recirculation rate as a parameter. From this plot, presented in Figure 15, a minimization of the mixing time is seen to exist within the range of flow-split ratios of 0.6 to 0.8.

Figure 15. CROSS PLOT OF MIXING TIME AS A FUNCTION OF THE FLOW-SPLIT RATIO FOR TWO PARALLEL, HORIZONTAL, SAFE TANKS.

AIR-SPARGE-MIXED, VERTICAL-TANK SYSTEMS

Theoretical Development

A theoretical treatment for mixing-time predictions of air-sparge tank systems was not attempted in this program due to the apparently greater need to develop a theory for recirculation-mixed tankage systems. However, interest in gas-sparge mixing of vertical tanks has, in recent years, provided the basis for such a treatment. Siemes and Weiss^(5, 6) obtained a solution for the concentration distribution in a column based upon the dispersion model. They used the concentration distribution function for a pulse-injected tracer to extract from experiment the mixing-coefficient data. It was observed, from these experiments in small columns each equipped with a porous tank-bottom air distributor (pore size between 9 and 55 microns) for uniform air distributions, that the gas-sparge rate had a pronounced effect upon the measured mixing coefficient and that, by analogy with single-phase flow through a pipeline, laminar and turbulent regions could be detected for the flowing air stream. In addition to the effect of the sparging rate upon the mixing coefficient, it was found that the coefficient varies with liquid height and the size of the gas bubbles. Similar work has been performed by Argo and Cova,⁽⁷⁾ by Tadaki and Maeda,⁽⁸⁾ and by Rietema,⁽¹⁴⁾ which generally agrees with that of Siemes and Weiss. Rietema extended the measurement of the mixing coefficient to study the effect of perpendicularity upon the coefficient. He found that for large length-to-diameter ratios (ie, L/d of about 20 or greater) the coefficient increases threefold when the column axis deviates one percent from the vertical.

As has been demonstrated for recirculation-mixed tankage systems, mixing-time predictions can be made for a tankage system provided a concentration distribution function is available which describes mixing as a function of the system dimensions, a predictable mixing coefficient, and time. By using a distribution function, mixing-time calculations could be made on a digital computer for any desired end-mixed condition of small level of inhomogeneity in a column. It would be necessary, however, to investigate the mixing coefficient in tankage systems of the diameter and height ranges that are used in nuclear-fuel processing facilities to uncover any scaleup effects in the relationship for the coefficient which may exist.

Experimental Investigations

An experimental program was designed to evaluate the mixing-time characteristics of a 6-inch-diameter by 40-foot-tall column and a 4-inch-diameter by 40-foot-tall column as a function of the liquid height, air-sparge rate, and end-mixed condition.

The program included 14 experimental series (Numbers 52 through 58 and 65 through 71) that were conducted individually at a fixed liquid level between 8.6 and 35.9 feet while the air-sparge rate for each series was varied between 0.12 and 5.0 scfm. Air-flow rates above 2 scfm in the four-inch-diameter column generally produced too much turbulence for satisfactory measurements.

Mixing data were collected by a digital paper-tape data-acquisition system built at Y-12. The acquisition system combined a liquid conductivity sensing module which used small, pencil-size conductivity cells, mounted as shown in Figure 16; (a) a 1/4-second, full-scale recorder with an 11-inch chart; (b) a four-digit digital voltmeter; (c) and a tape punch. (d) For a given experimental run after the tracer was injected at the column base, data were collected on paper tape beginning at the same time that the air sparging was initiated. The data acquired on tape were conductivity readings taken periodically in time along an exponentially asymptotic curve such as shown by the strip chart in Figure 17. The strip charts were used only to qualitatively analyze the mixed condition of the tank as a function of the air-sparge time; the paper tapes were interpreted by a tape reader and the data punched into data cards for a digital-computer analysis.

A program was written for use on an IBM 7090 digital computer to determine, from the collected input data for a specific run, the mixing time needed to achieve mixed conditions of 0.90, 0.95, and 0.99. To smooth out extreme fluctuations of the data that were caused by air bubbles passing around the conductivity cell electrodes and to reject other spurious data caused by eccentricities within the acquisition system, the program was written to average the data for each card, reject spurious data (data outside the interval from 80 to 120 percent of the average value for that data card), and subsequently recalculate an average for the acceptable data. Since each card contained 20 readings collected at one-second intervals, the data average for each card represented an incremental mixing period of 20 seconds. By computing the average of the last ten data cards for a run which represents a condition of tracer homogeneity with the bulk fluid (demineralized water) in the column, a comparison test was made to extract mixing times to the three end-mixed conditions of near homogeneity.

The card averages, beginning with the first data card, were divided by the average for the last ten data cards. When this ratio of averages was equal to or greater than 0.90, 0.95, and 0.99, the mixing period associated with that card average (ie, 20 seconds times the card number in the series of data cards) was designated as the mixing time for that mixed condition.

By this data analysis, mixing-time determinations were made at each end-mixed condition for each experimental run. When plotted on full logarithmic coordinates as a function of the air-sparge rate, a linear correlation was indicated by these extracted mixing-time data. These data are listed in Tables C-1 and C-2, Appendix C. As a consequence, the mixing-time data were fit to Equation 1 (Page 13) where,

-
- (a) Manufactured by Industrial Instruments, Inc.
 - (b) Elektronik 15 supplied by Minneapolis Honeywell.
 - (c) Model 1234 supplied by Snytron-Donner.
 - (d) Model 420PR supplied by Tally.

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Figure 16. ATTACHMENT OF THE LOWER CONDUCTIVITY CELL AND TEMPERATURE COMPENSATOR WITHIN THE AIR-SPARGED TANK.

in place of the recirculation rate, the flow variable was designated as the air-sparge rate in scfm.

Plots of the mixing-time data for each experimental series at each end-mixed condition, values of the correlation constants, their standard error, and the variance of estimate for each fit are given in Tables C-3 and C-4 of Appendix C.

Figure 17. STRIP CHART OBTAINED FROM AN AIR-SPARGED TANK WITH THE LIQUID LEVEL AT 19.8 FEET AND AN AIR FLOW OF 0.29 SCFM.
(Series 55, Run 61AS)

As indicated by these mixing-time plots, the mixing-time requirement increased with increasing liquid-level height, increased with more nearly homogeneous end-mixed condition, or increased with a decreasing air-sparge rate. These observations agree with intuitive suppositions about an air-sparge column that, with the other variable held fixed: (1) an increase in column height (fixed diameter), a greater L/d , and a greater fluid volume will require a longer mixing period; (2) a more completely mixed end-mixed condition approaching fluid homogeneity will require a longer mixing period; and (3) a decreased sparging rate which determines the intensity of mixing will require a longer mixing period.

In constructing experimental facilities, the vertical column must be constructed several feet higher than is needed for the highest liquid level of interest. As air passes through the system during mixing, the liquid level increases. Table 4 gives some measurements made in a four-inch-diameter column at a steady state and at three air-flow rates.

Table 4
COMPARISON OF THE LIQUID LEVELS OF STATIC
CONDITIONS AND WITH AN AIR FLOW
(For a Four-Inch-Diameter Vertical Column)

Height of Static Liquid (feet)	Air Flow (SCFM)	Liquid Height with an Air Flow (feet)
15.75	0.20	16.42
15.75	0.63	17.50
15.75	2.09	19.83

RECIRCULATION-MIXED, MIXING-TUBE SYSTEMS

Theoretical Development

A method of approach to the mixing problem presented by series and parallel flow patterns in a recirculation-mixed single-tank system adapted with a perforated, coaxial, inner inlet tube has been advanced.⁽⁴⁾ The method relies upon the relationship between the mixing number for a section of pipe and the number of equally sized, ideally mixed tanks which give equivalent mixing. The method provides a set of equations for ideally mixed tanks as an analogue for a single-tank system adapted with a mixing tube.

The developed mixing theory was not programmed for digital computation in view of the calculational difficulties noted in the ideally mixed tank analogy for mixing-in-parallel tankage, and no attempt was made at problem solution by using an analogue computer.

Experimental Investigations

A total of 136 mixing experiments were conducted at recirculation rates between 2.06 and 79.8 gpm in a 6-inch-diameter by 20-foot-long tank adapted with a perforated coaxial, inner inlet tube. For each of the five mixing tubes studied, selected perforations or flow outlets were created by drilling sets of four equally sized holes placed 90 degrees apart at specified axial locations along the tube. To approximate nearly equal flow out of each hole set, all holes were drilled and then reamed to a $9/32$ -inch diameter. A diagram of the mixing tubes that were evaluated is given in Figure 18, and representative recorder charts of the collected data are shown in Figures 19 and 20. Characteristic piping lengths and component volumes for the system have been presented.⁽⁴⁾

Mixing-time data were collected for each experimental series at a mixed condition of about 0.998 over the full range of recirculation rates. The data were observed to be fit by the linear logarithmic relationship given as Equation 1. These data were fit to the equation by a least-squares method and plotted. Values of the constants

Figure 18. REPRESENTATIVE ARRANGEMENT FOR ONE, TWO, THREE, AND FIVE-HOLE SET MIXING TUBES.

of these fits, their standard error, and the variance of estimate of each fit were calculated. The mixing-time data for the series given in Appendix D have not been reported previously.

EXPERIMENTAL SAMPLING OF SOLID-LIQUID MIXTURES

To inventory nuclear fuels contained in solution in a tankage facility, representative samples can be removed by any sampling method after mixing the miscible fluid contents to homogeneity. In those cases, however, where some solid material is contained within the bulk fluid, mixing to near homogeneity followed by conventional sampling methods can lead to nonrepresentative solid-liquid samples and to an inaccurate inventory. To better define the problems of mixing a solid-liquid mixture to near homogeneity and to representatively sample the mixture, an experimental investigation of sampling techniques that might find application for solid-liquid mixtures has been carried out.

This investigation, which complements the liquid mixing studies summarized in this report, considered both mixing in and sampling from a horizontal tank mixed by recirculation and from a vertical tank mixed by air sparging. An extrapolation of results from these two systems can be readily made to other types of tankage systems.

Metallurgical-grade alumina powder^(e) was used as the solid material to be mixed and sampled in demineralized water. Three particle sizes were considered; 0.05, 1, and 20 microns (Linde Types 0.05B, 1.0C, and 40-6620AB, respectively). Since the particle size in this near-colloidal range ($< \text{about } 1 \mu$) affects both the mixing and sampling characteristics of a solid-liquid mixture, the particle-size distribution of the 0.05 and 1.0-micron alumina was measured by electron microscopy. Figures 21 and 22 are electron micrographs of the two materials.

To determine the solids concentration for a sample mixture, a centrifuge method^(f) was used. By this method, a 100-milliliter sample was collected directly in an ASTM cone-shaped centrifuge tube (Figure 23) and was centrifuged^(g) to separate the liquid and solid. After centrifugation, the solid in the bottom of the tube was volumetrically measurable to within about 0.02 milliliter by interpolating along the conically shaped tube. By comparison with previously collected calibration data for each alumina particle size, measured volumes of solids were converted to parts per million on a weight basis. To investigate the mixing characteristics of solids in a recirculation-mixed tankage system, continuously monitored on-stream

(e) Supplied by the Linde Division of Union Carbide Corporation.

(f) Recommended by ASTM Standard D96-57T.

(g) Model UV, International Equipment Company.

Figure 20. STRIP CHART OBTAINED FROM A FIVE-HOLE SET, RECIRCULATION-MIXED, MIXING-TUBE SYSTEM IN A VERTICAL TANK.
(Series 43, Run 542)

EM-1568

Figure 21, ELECTRON MICROGRAPH OF 0.05-MICRON-PARTICLE-SIZE ALUMINA POWDER.

EM-1560

Figure 22. ELECTRON MICROGRAPH OF 1.0-MICRON-PARTICLE-SIZE ALUMINA POWDER.

119175(U)

Figure 23. AN ASTM CONE-SHAPED CENTRIFUGE TUBE.

turbidimetric cells were placed in the tank-outlet piping in a similar manner to the installation of electrical conductivity cells used for mixing an aqueous-salt tracer. As shown in Figure 24, two turbidimetric cells^(h) were mounted into the system for covering the solids concentration ranges of 0 to 300 and 0 to 800 ppm by weight.

(h) Manufactured by the Jacoby-Tarbox Corporation.

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Figure 24. INSTALLATION OF TURBIDITY CELLS AT THE TANK OUTLET.

Recirculation-Mixed, Single-Tank Systems

Solid-liquid sampling experiments relating to the recirculation-mixed single-tank system were conducted in a 6-inch-diameter by 20-foot-long horizontal tank system previously used for Experimental Series 8, modified by the addition of the two turbidimetric cells and the addition of a sampling probe. Due to the gravitational effect on solids in a flowing stream, the isokinetic sampling methods advanced by Rushton and Hillestad⁽²⁾ were used.

The special sampling probe of the pilot design (Figure 25) was accurately positioned at the tank centerline about 30 tank diameters downstream from the inlet reducer. By using this sampling probe, samples were removed from the tank at the same velocity as that of the approaching fluid at the tank centerline. Collected samples were

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Figure 25. INSTALLATION OF THE ISOKINETIC SAMPLING PROBE.

essentially sheared from the flowing stream in such a manner that the composition of the sample was identical with that flowing at the tank centerline. The effect of withdrawing a sample at greater or lesser velocities than that of the approaching stream is to collect too small or too large, respectively, a quantity of solid in the sample. The effect of sample withdrawal velocity upon solids concentration is presented in Appendix E (Table E-1) for Series 63. To closely approximate isokinetic sampling conditions without measuring the velocity profile for these solid-liquid mixtures, all samples were collected at 1.2 times the average stream velocity, as suggested by Rushton and Hillestad.⁽²⁾

After a number of preliminary experiments were conducted at recirculation rates up to 98.3 gpm, it was apparent that significant quantities of solids were left unmixed and deposited upon the bottom of the safe tank. At the higher recirculation rates, a fraction of the solids were streamlined on the tank bottom, as shown in Figure 26, while at lower rates a greater fraction of the solids were accumulated in piles along the tank bottom, as shown in Figure 27. It was observed that much longer mixing periods were required to reach these steady-state, partially mixed conditions than were required to mix an aqueous tracer. For example, a six-hour mixing period was required to reach the nonhomogeneous steady-state mixed condition at a recirculation rate of 98.3 gpm, while for a similar aqueous-tracer mixing experiment, 27.5 minutes were required to reach homogeneity.

A set of experiments, designated as Series 64, were conducted in the system at three levels of solids concentration (250, 500, and 750 ppm by weight) and at two levels of recirculation rate (42.5 and 98.3 gpm) to demonstrate these mixing and sampling difficulties. The smallest particle-size alumina powder, 0.05 micron, which by virtue of its size should be most easily mixed and sampled, was added to the tracer-addition section of the tank-system loop piping in weighed quantities to produce the indicated average solids concentration. After reaching the indicated steady-state turbidity by recirculation mixing, samples were removed isokinetically and analyzed by the ASTM centrifuge method. The measured solids concentrations were as given in Table E-1, Appendix E, Series 64.

In all cases, the measured solids concentration was below the average tank concentration and was observed to be strongly dependent upon the recirculation rate. For the experiments conducted at 42.5 gpm, the withdrawn sample concentrations were only about one fifth the average tank solids concentration. Similarly, the sample concentrations, collected from the tank mixed at 97.6 gpm, were about four fifths the average tank solids concentration.

The difficulty of suspending and mixing solids at an average stream velocity in a safe tank up to about 1 ft/sec is manifest by these acquired sample concentration data. In contrast, the work of Rushton and Hillestad⁽²⁾ demonstrated that mixing and representative sampling could be achieved; however, they considered a higher flow range (from 2 to 15 ft/sec) where generated turbulent eddies mix settled the

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Figure 26. SOLIDS STREAMLINED ON THE TANK BOTTOM AT THE HIGHEST RECIRCULATION RATE.

118793(U)

Figure 27. SOLIDS DEPOSITED ON THE TANK BOTTOM AT A LOWER RECIRCULATION RATE.

solids at a much greater energy input level. As a consequence, it does not appear to be possible to mix and representatively sample a solid-liquid composite at the relatively low recirculation rates (average safe tank velocity much less than 2 ft/sec) commonly used for tankage facilities.

Air-Sparge-Mixed Vertical Tank

Air-sparge mixing and sampling experiments were conducted in a 6-inch-diameter by 40-foot-tall tank previously used for an aqueous salt-tracer mixing experiment in Experimental Series 46. The system was modified to permit sampling about six inches above the column base. This modification, shown in Figure 28, consisted of converting a 1/4-inch-OD aqueous tracer injection line into a sampling line.

Unlike a recirculation-mixed tankage system where samples can be removed isokinetically, samples from a vertical column mixed by air sparging can be collected only by bleeding off a quantity of the mixture. Experiments were conducted by adding a measured quantity of solid and water to the column and mixing at a specified air-flow rate for a period of time greater than that found needed to disperse a salt tracer to near homogeneity. Subsequently, a sample was bled into an ASTM cone-shaped centrifugal tube for solids-concentration measurement via volumetric determination by centrifugation.

119177(U)

Figure 28. SAMPLER FOR AN AIR-SPARGE-MIXED VERTICAL TANK.

After mixing, sampling experiments were conducted at several levels of solids concentration, at varying air-sparging rates between 0.12 and 5.0 scfm, and at eight levels of liquid height. These experiments were undertaken to determine the effect of concentration, air-sparge rate, and liquid level upon the sample solids concentration. In addition, the effect of the sample withdrawal rate upon the sample concentration was evaluated by collecting samples at two levels of withdrawal rate.

For four series of experiments (Series 59 through 62) conducted at varying solids concentrations, air-sparge rates, and liquid heights, and where the sample withdrawal rate was about 72 cc/min, the solids concentration for the collected samples varied about the average mixture concentration. In all cases, these sample concentrations were between 99 and 131 percent of the average, as indicated in Table E-1, Appendix E.

Within the range of variables examined in these experiments, no significant effect upon sample concentration was attributed to the variables.

A set of ten experiments were conducted to study the effect of the withdrawal rate on the measured solids concentration. These experiments were conducted with one-micron-sized alumina solids at a 1000-ppm-by-weight solids concentration at four levels of an air-sparging rate and at a liquid level of 30 feet. The two withdrawal rates considered were estimated to be about 72 and 200 cc/min. Samples withdrawn at these rates indicate, at least qualitatively, the effect of varying the sampling rate between a relatively low and a relatively high withdrawal rate. Results of these experiments (Series 63, Table E-1) indicate that the sampling rate is at least as significant as any of the other factors investigated for mixing and sampling well-mixed solid-liquid composites in an air-sparge column.

CONCLUSIONS AND RECOMMENDATIONS

By using either recirculation or air-sparge methods, fluid mixtures contained in cylindrical safe-geometry tankage can be mixed to near homogeneity. Under reasonable operating conditions, both mixing methods can perform the two requirements of mixing in cylindrical tankage having large length-to-diameter ratios, namely: (1) axially redistribute elements of the fluid contained within a tankage system, and (2) radially mix these fluid elements.

It should be noted that except for the necessity of axially redistributing fluid elements stratified and isolated by gravity, the more conventional propeller-type mixers used in tankage with length-to-diameter ratios of near unity would be effective for safe-tankage systems.

It has been demonstrated experimentally that the effectiveness and efficiency of either air-sparge mixing or recirculation mixing depends upon the energy applied to a system. Below some air-flow or recirculation rate for a tankage system, the gravitational influence on the fluid elements in the system will predominate and greatly retard mixing. At best, under such low-rate conditions, the time requirement for mixing to near homogeneity would be very large. As a further complication to mixing at low levels of energy input, dead pockets are created in which stagnant fluid is retained unmixed. In such cases a physical measurement of fluid homogeneity as a function of the deviation of mixing might indicate prematurely a well-mixed tankage content.

For both air-sparge and recirculation mixing, the mixing period required to achieve a given small level of liquid inhomogeneity for a tankage system was observed to be related to flow rate by the linear logarithmic equation:

$$\log T = A - B \log W, \quad (31)$$

where:

T represents the mixing period needed to achieve a specified final fluid condition of near homogeneity, and

W either the air-flow rate for air-sparge mixing or the fluid-recirculation rate for recirculation mixing.

A and B are constants whose values depend upon the tankage construction, flow distribution, and other mixing conditions.

This relationship was observed to hold for all mixing experiments regardless of the complexity of the flow pattern as, for example, those which exist within recirculation-

mixed parallel tanks, within a single tank adapted with a coaxial, inner inlet tube to create a complex series-parallel fluid-flow path, or within a vertical tank mixed by air sparging. Below the range of the air-sparge rates studied (0.6 to 2.0 scfm/ft²) or the range of recirculation rates studied (expressed as tank Reynolds numbers between 1200 and $55,000$) the relationship could lead to error because of the complexity of the gravitational effects at low mixing flows and dead pockets created by a poor tankage flow design.

To be assured of satisfying the requirements of axial distribution and radial mixing in a system, both energy input, as manifested by the flow rate, and tankage design must be considered. For recirculation-mixed systems, flow through the safe tank or tanks should be in the turbulent range at a Reynolds number of at least $10,000$. For air-sparge mixed systems, the air rate should be at least 2.0 cfm/ft² cross sectional area to provide a turbulent stream of air flowing through the column.

Tankage systems should be designed and constructed to minimize those sections of tankage and extraneous piping that create stagnant fluid regions. For recirculation-mixed systems, these regions are frequently found at tank ends, at interconnecting piping and drain lines, and at sight glasses. With the exception of interconnecting piping and drain lines whose volumes can be reduced by minimizing the length of these lines, the other regions can be wholly removed. For smooth flow transition into and out of a tank during mixing, tank ends should be constructed from piping reducers rather than blind flange-type closures. Sight glasses, used to measure volumetrically the quantity of fluid contained in a tankage system, can collect and retain a stagnant portion of fluid if not placed in the recirculation loop piping. Where a sight glass is installed outside of the loop piping, the measured volume of fluid contained in a tankage system can be in error due to the difference in the specific gravity of the fluid in the tankage system and the specific gravity of the fluid in the sight glass. If, for example, a more dense fluid were collected in the sight glass than was sampled from a well-mixed safe tank to which it was connected, the measured volume of fluid in the tank would be less than the actual volume. Figures 29 and 30 show a recommended and an inferior design for tank ends for a typical horizontal tank installation.

The primary design consideration for an air-sparge column is that of providing good distribution of air flow at the column base. Poor air distribution at the base of a column may allow the more-dense components of the contents of a column to remain stratified and unmixed. To surmount the problem of poor air distribution, a perforated spiral-coil distributor like that used in this experimental program or a porous metallic sparger base with micron-sized pores would be recommended. As for recirculation-mixed systems, piping whose volume is associated with the column and which gives rise to dead pockets should be minimized.

Based upon the acquired experimental mixing data for recirculation and for air-sparge-mixed tankage systems, it is possible to estimate mixing-time requirements

119176(U)

Figure 29. RECOMMENDED DESIGN FOR A TYPICAL HORIZONTAL-TANK END SECTION.

for most tankage systems provided the distribution and rate of flow for a system are known. For single-tank systems, the developed mixing theory can be directly applied for a mixing-time estimation. For parallel-tank systems, mixing-time estimations can be made by application of the developed theory given by the computer program in Appendix B; however, this theoretical analysis has not yet been tested with the acquired two-tank data and further work may be needed before application of this analysis can be confidently used. By viewing a parallel-tank system as a set of separate, single tanks mixed by their separate flows, a conservative estimation of the mixing-time requirement for the parallel-tank system can be made. This analysis, however, neglects the highly effective reduction of the mixing-time requirement for a parallel-tank system by the splitting of flows at the tank entrance through a set of tanks having unequal residence times, and the subsequent flow combination at the tank outlet. Although an accurate prediction of mixing time cannot be obtained for the myriad of parallel-tank mixing problems that could arise without specific experimental measurement or use of a tested computer analysis, an extrapolation of the concept of mixing-time minimization by varying the relative value of the flow through a set of parallel tanks can be made which will improve

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Figure 30. POOR DESIGN FOR A TYPICAL HORIZONTAL-TANK END SECTION.

the mixing efficiency.⁽⁴⁾ Due to the complex nature of mixing in a tank adapted with a mixing tube where a combination of series and parallel flow paths exist, mixing-time estimations for this system cannot now be made. Based on the developed mixing theory, it should be possible to solve this problem either directly by analogue computation or on a digital computer by using a mathematical function which generates the variables needed for a superposition analysis in a similar manner to that proposed for the parallel-tank mixing problems for all possible flow paths. Air-sparge mixing-time estimations should be possible based on the experimental data. Since the time requirements are generally smaller than for single recirculation-mixed tankage systems and are small in an absolute sense, conservative air-sparge mixing-time requirements should be straightforward. Due to the observed significantly greater mixing-time requirements for a single, recirculation-mixed, vertical-tank system compared with the time requirements for a comparable horizontal-tank system,

air entrainment in a recirculation stream apparently retards mixing. As a result, the developed recirculation mixing theory for single-tank systems should be tested for applicability to vertical systems and, if applicable, dispersion-intensity data reflecting the presence of entrained air in a tankage system should be collected and correlated with pertinent variables.

Representative samples can be removed from a cylindrical tankage system after the contents have been thoroughly mixed. When miscible fluids are to be sampled, representative samples can be collected by any withdrawal technique after near homogeneity exists throughout the tankage system. In contrast, a solids-liquid composite can be sampled representatively from recirculation-mixed tankage; however, the mixing energy requires greater fluid velocities throughout the tankage (greater than 2 ft/sec) to achieve suspension and homogeneity than are necessary for fluids alone. Further, isokinetic sampling techniques which have an accuracy of about ± 8 percent or less must be closely approximated for the recirculation-mixed system. Reasonably good sampling results may be obtained from an air-sparge system by simply mixing and bleeding off a sample, provided the specific gravity and particle size of the solids are small; however, solids concentration may vary more widely than that for the alumina-water experiments where the observed solids concentration varied between -10 and +30 percent of the actual concentration. Because of the inherent sampling uncertainty and sampling difficulties, it is recommended that solids be removed from a product stream entering a tankage system.

From the experimental program, the following general comments concerning tankage systems can be made:

1. Horizontal tankage is more easily mixed than vertical tankage since air entrainment in vertical tankage requires greater mixing times.
2. To accommodate large volumes of fluid in storage, a horizontal, parallel-tank facility mixed at flow rates near optimum are advantageous due to their low mixing-time requirement and the necessity of collecting only a single sample for concentration measurement.
3. Vertical air-sparge tanks containing both liquid and solids are in many cases more easily mixed and sampled than recirculation-mixed systems; however, the accuracy of the collected sample with respect to solids concentration may not be as good as for a recirculation-mixed system at high recirculation rates and sampled by isokinetic methods.

There are several areas in which additional effort would benefit the areas of mixing and sampling cylindrical tankage. These investigations include both theoretical and experimental activities dealing with miscible liquid and solid-liquid composites. These studies are as follows:

1. Experimentally measure the mixing times and mixing coefficients for vertical air-spargers in the range of diameter and height that may be found in application. Correlate these data with pertinent variables and generalize the results.
2. Develop a theory for a mixing-time estimation based upon the work of Siemes and Weiss,⁽⁶⁾ experimentally mix coefficient data as published or as determined in Item 1, and establish an end-mixed-condition relationship of near-fluid homogeneity. This development can be numerically compared with the data from Item 1 by digital computation for selected cases.
3. Further develop a mixing theory for the recirculation-mixed parallel-tankage systems and mixing-tube tankage systems by using either digital or analogue computation methods. The theoretical mixing-time data can be compared with the experimental data obtained in this program.
4. Through an experimental program, characterize the mixing generated by tankage ends in a recirculation-mixed system. An accurate prediction of the mixing characteristics of the tankage ends is needed if accurate mixing-time predictions are made for any type of recirculation-mixed system.

The following two items should also be considered:⁽ⁱ⁾

1. Develop representative sampling equipment for use in air-sparger-mixed columns when solids-liquid mixtures must be mixed and sampled.
2. Investigate the relationship between the settling characteristics of solids-liquid mixtures and the recirculation-rate requirements of tankage systems to suspend solids. From a knowledge of these requirements, the minimum recirculation rates for isokinetically sampling a tankage system could be predicted.

(i) These two items were suggested by Dr. J. H. Rushton of Purdue University as areas where extensions of solids-liquid sampling studies previously conducted at Purdue would be fruitful.

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APPENDIX A

DATA FROM THE INVESTIGATION OF SINGLE-TANK SYSTEMS

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Figure A-1. MIXING TIME AS A FUNCTION OF THE RECIRCULATION RATE FOR A COMPLETELY FILLED FOUR-INCH-DIAMETER BY 40-FOOT-LONG HORIZONTAL SAFE TANK.

Figure A-2. TRACER VARIANCE AS A FUNCTION OF THE RECIRCULATION RATE FOR A COMPLETELY FILLED, FOUR-INCH-DIAMETER BY 40-FOOT-LONG HORIZONTAL SAFE TANK.

Figure A-3. MIXING TIME AS A FUNCTION OF THE RECIRCULATION RATE FOR A SIX-INCH-DIAMETER BY 20-FOOT-LONG SAFE TANK WITH A FIVE-HOLE MIXING TUBE WITH EVENLY SPACED OPENINGS.

Figure A-4. TRACER VARIANCE AS A FUNCTION OF THE RECIRCULATION RATE FOR A COMPLETELY FILLED, SIX-INCH-DIAMETER BY 20-FOOT-LONG VERTICAL SAFE TANK.

Table A-1
EXPERIMENTAL DATA FOR SINGLE-TANK SYSTEMS

Run Number	Recirculation Rate (gpm)	Mixing Time (mins)	Variance (min)
<u>Series 49</u>			
627	3.76		0.83054
634	3.76		1.08734
628	5.78		0.41912
633	5.78		0.44069
574	6.70		0.29509
587	6.70		0.22511
629	7.80		0.22584
632	7.80		0.24804
630	10.03		0.21916
631	10.03		0.20542
575	18.20		0.05144
586	18.20		0.05770
576	31.80		0.01347
585	31.80		0.00886
577	46.30		0.00829
584	46.30		0.00540
578	62.20		0.00231
583	62.20		0.00293
579	78.70		0.00040
582	78.70		0.00093
580	92.00		0.00164
581	92.00		0.00124
<u>Series 51</u>			
546	1.87	384.9	0.00000
558	1.87	388.0	0.00000
547	3.62	200.5	0.00000
559	4.33	244.1	0.00000
548	5.45	201.2	0.95609
560	5.45	226.0	0.34687
635	5.45	000.0	0.74521
550	7.35	212.5	0.66306
562	7.35	214.7	0.20644
636	7.35	000.0	0.46138
550	9.36	220.9	0.16603
562	9.36	221.5	0.14171
551	11.60	194.2	0.07238
563	11.60	219.8	0.04903
552	19.90	175.5	0.01147
564	19.90	157.8	0.01100
553	33.20	136.5	0.00311
565	33.20	120.1	0.00312
554	47.40	110.0	0.00120
566	47.40	99.7	0.00124
570	47.40	000.0	0.00145
555	62.80	91.5	0.00082
567	62.80	79.5	0.00078
571	62.80	000.0	0.00078
556	78.80	63.5	0.00046
568	78.80	62.8	0.00049
572	78.80	000.0	0.00047
557	95.10	55.0	0.00041
569	95.10	55.8	0.00000
573	95.10	000.0	0.00029

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APPENDIX B

FORTRAN PROGRAM FOR A THEORETICAL ANALYSIS OF A
PARALLEL-TANK SYSTEM

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$IBFTC TMAIN  NODECK
C   THEORETICAL ANALYSIS OF MIXING TIME FOR A MULTI-PARALLEL
C   CYLINDRICAL TANK SYSTEM.
C
C   THE CONSTANTS A AND B WERE ARBITRARILY SELECTED AND ARE NOT
C   GENERALLY APPLICABLE TO ALL SYSTEMS.
C
C   TANKRT(1) -- TANK RESIDENCE TIME
C   TANKD(1) -- TANK DIAMETER
C   TANKL(1) -- TANK LENGTH
C   TANKQ(1) -- TANK FLOW RATE
C   TANKD2(1) -- DIMENSIONED SIGMA SQUARE
C   ALNF(1) -- LN(TANKQ(1)/TOTQ)
C   TLOOP -- LOOP TIME
C   VLOOP -- LOOP VOLUME
C   TOTQ -- TOTAL FLOW RATE
C   NTANKS -- NUMBER OF TANKS
C   A AND B -- USED TO COMPUTE DISPERSION INTENSITIES FROM REY. NOS.
C   RTMAX -- MAXIMUM RESIDENCE TIME
C   RTMIN -- MINIMUM RESIDENCE TIME
C   NDEGR -- DEGREE OF MODEL MULTINOMIAL
C
C   LOGICAL COND
C   DIMENSION TANKRT(10),TANKD(10),TANKL(10),TANKQ(10),TANKD2(10)
C   ,IPERM(10),ALNF(10)
C
C   COMMON /PULSE/ DSIG2,ALN1,Z
C   LOGICAL LAST
C
C   1  READ(5,21) NTANKS,VLOOP,A,B,GAMMA,NDEGR,NEVALS
C   21  FORMAT(110,4E10.0,2110)
C   READ(5,22) (TANKD(1),TANKL(1),TANKQ(1),I#1,NTANKS)
C   22  FORMAT(3E10.0)
C
C   COMPUTES TANKRT(1),TANKD2(1),TOTQ,TLOOP,RTMAX,RTMIN,ALNF(1)
C
C   WRITE(6,29)
C   29  FORMAT(125H1      TANK NUMBER      DIAM.      LENGTH      FLOW RATE
C   1 RES. TIME      REY. NO.  DISP. IND.  D.SIG. SQ.
C   2      /)
C
C   TOTQ#0.
C   RTMAX#0.
C   RTMIN#.9E+30
C   DO 101 I#1,NTANKS
C   RE#3161.*TANKQ(1)/TANKD(1)
C   DISPI#10.**A*RE**(-B)
C   TVOL#3.1415927*(TANKD(1)/2.)**2*TANKL(1)*7.481/1728.
C   TANKD2(1)#2.*(TVOL/TANKQ(1))**2*DISPI*(TANKD(1)/TANKL(1))
C   TANKRT(1)#TVOL/TANKQ(1)
C   TOTQ#TOTQ+TANKQ(1)
C   WRITE(6,28 ) 1,TANKD(1),TANKL(1),TANKQ(1),TANKRT(1),RE,DISPI
C   1,TANKD2(1)

```

```

28  FORMAT(114,3X,7F12.4)
    IF(RTMAX.LT.TANKRT(1)) RTMAX#TANKRT(1)
    IF(RTMIN.GT.TANKRT(1)) RTMIN#TANKRT(1)
101  CONTINUE
    TLOOP#VLOOP/TOTQ
C
DO 102 I#1,NTANKS
102  ALNF(1)#ALOG(TANKQ(I)/TOTQ)
    WRITE(6,27) NTANKS,TOTQ,TLOOP,VLOOP,A,B,GAMMA,NDEGR
27  FORMAT(1H0/30X,110,18H # NUMBER OF TANKS/
1 28X,F12.4,18H # TOTAL FLOW RATE/ 28X,F12.4,12H # LOOP TIME/
2 28X,F12.4,14H # LOOP VOLUME/ 28X,F12.4,4H # A/
3 28X,F12.4,4H # B/ 28X,F12.4,8H # GAMMA /
5 28X,112,27H DEGREE OF MODEL POLYNOMIAL)
C
C
C
C  GENERATES THE PULSES
    WRITE(6,222)
222  FORMAT(111)
    CALL REWP
    NPUL#0
    DO 204 I#1,NDEGR
C  INITIALIZES FOR A GIVEN II VALUE
    DO 201 I#1,NTANKS
201  IPERM(I)#0
    IPERM(NTANKS)#11
C
C
C  THIS LOOP GENERATES THE PULSES FOR A GIVEN II VALUE
203  ALN1#ALNC(IPERM,NTANKS,11)
    DSIG2#0
    Z#0
    DO 202 I#1,NTANKS
        FIP1#FLOAT(IPERM(1))
        ALN1#ALN1+FIP1*ALNF(1)
        DSIG2#DSIG2+FIP1*TANKD2(I)
202  Z#Z+FIP1*(TANKRT(1)+TLOOP)
        CALL WRITEP
        NPUL#NPUL+1
        CALL NEWPER(11,NTANKS,IPERM,$291)
        GOTO 203
291  CONTINUE
204  CONTINUE
C  END OF PULSE GENERATIONS
C
C
C
C  COMPUTES TINC
    FARPUL#(RTMAX+TLOOP)*FLOAT(NDEGR)
    TINC#FARPUL/FLOAT(NEVALS)
C
C  FUNCTION EVALUATION PORTION

```

```

        DIMENSION F(1500)
        DO 401 I#1,NEVALS
401    F(I)#0.
        CALL REWP
C      RETURNS HERE FOR A NEW PULSE
402    CALL READP(LAST)
        IF(ALN1.LT.-88.) GOTO 409
        ISTART#Z/TINC+1.
        I#1START
403    IF(I.LT.1) GOTO 404
        T1#FLOAT(1)*TINC
        C1#C(T1,EXP(ALN1),DSIG2,Z)
        IF(C1.LT.1.E-18) GOTO 404
        IF(1.LE.NEVALS) F(I)#F(I)+C1
        I#I-1
        GOTO 403
404    CONTINUE
        I#1START+1
405    IF(1.GT.NEVALS) GOTO 406
        T1#FLOAT(1)*TINC
        C1#C(T1,EXP(ALN1),DSIG2,Z)
        IF(C1.LT.1.E-18) GOTO 406
        F(I)#F(I)+C1
        I#I+1
        GOTO 405
406    CONTINUE
409    IF(.NOT.LAST) GOTO 402
C
C
C
C      OUTPUTS THE CONCENTRATIONS
        FMAX#-9.9E+27
        FMIN#9.9E+27
        FGMAX#0.
        NAVE#15
        NOEVS#NEVALS
        DO 501 I#1,NEVALS
            T1#FLOAT(1)*TINC
            IF(1.GT.NEVALS-(NAVE-1)) GOTO 502
            FAVE#0.
            DO 503 JJ#1,NAVE
                J#JJ-1+1
503    FAVE#FAVE+F(J)
                FAVE#FAVE/FLOAT(NAVE)
                FDEL#ABS(F(I)-FAVE)
                FGAM#(FAVE-FDEL)/FAVE
                WRITE(6,522) T1,F(I),FAVE,FGAM
522    FORMAT(F14.3,3F14.7)
                IF(NOEVS.EQ.NEVALS.AND.FGAM.GT..9999) NOEVS#1
                IF(F(I).GT.FMAX) FMAX#F(I)
                IF(F(I).LT.FMIN) FMIN#F(I)
                IF(FGAM.LT.FGMAX) GOTO 504
                FGAM#FGAM
                STEADY#F(I)
504    CONTINUE

```

```

      GOTO 501
502  WRITE(6,522) T1,F(1)
501  CONTINUE
C
C
C   THIS SECTION, WHICH PLOTS THE CONCENTRATION VERSUS TIME
C   CURVE, IS INSTALLATION DEPENDENT AND MAY BE REMOVED.
C
      WRITE(6,521) NAVE,NOEVS
521  FORMAT(1H0,215/)
      DIMENSION X(1500)
      DO 602 I#1,NOEVS
602  X(I)#TINC*FLOAT(1)
      CALL SCALE(X,F,NOEVS,11.,8.,3.,5.,5)
      CALL SCPLLN(X(1),F(1),1)
      DO 601 I#1,NOEVS
      CALL SCPLLN(X(I),F(I),2)
601  CONTINUE
      CALL LEGEND(5.25,-.25,.12,0.,.02,0,15HTIME IN MINUTES)
      CALL LEGEND(0.,3.75,0.,.12,.02,1,13HCONCENTRATION)
      CALL NEWPIC
C
C
C
      GOTO 1
END
$IBFTC ALNDK  NODECK
      FUNCTION ALNC(IPERM,NTANKS,NDEGR)
      DIMENSION IPERM(10)
      ALNC#ALFAC(NDEGR)
      DO 2 I#1,NTANKS
2     ALNC#ALNC-ALFAC(IPERM(I))
      RETURN
      END
$IBFTC ALFDK  NODECK
      FUNCTION ALFAC(I)
      DIMENSION FACTAB(81)
      DATA INIT/1/
      IF(INIT.EQ.0) GOTO 3
      INIT#0
      FACTAB(I)#0.
      DO 4 J#2,81
4     FACTAB(J)#FACTAB(J-1)+ALOG(FLOAT(J-1))
3     CONTINUE
      IF(I.LE.80) GOTO 2
      WRITE(6,21) I
21    FORMAT(12H, ALFAC ERROR,16)
      STOP
2     ALFAC#FACTAB(I+1)
      RETURN
      END
$IBFTC RWREWD  NODECK
      SUBROUTINE REWP
      COMMON /PULSE/ DSIG2,ALNI,Z
      LOGICAL REW

```



```

        DIMENSION AREA(4000,3)
        DATA NTAB/4000/
        REW#.TRUE.
        RETURN
        ENTRY WRITEP
        IF(.NOT.REW) GOTO 5
        I#0
        J#U
        REW#.FALSE.
5       CONTINUE
        IF(I.LT.NTAB) GOTO 3
        WRITE(6,21)
21      FORMAT(22H PULSE WRITE AREA FULL)
        RETURN
        I#I+1
3       AREA(I,1)#DSIG2
        AREA(I,2)#ALNI
        AREA(I,3)#Z
        RETURN
        ENTRY READP(LAST)
        LOGICAL LAST
        IF(.NOT.REW) GOTO 6
        J#0
        REW#.FALSE.
6       CONTINUE
        LAST#.FALSE.
        J#J+1
        IF(J.LT.1) GOTO 4
        LAST#.TRUE.
        IF(J.GT.1) RETURN
4       DSIG2#AREA(J,1)
        ALNI#AREA(J,2)
        Z#AREA(J,3)
        RETURN
        END
$IBFTC NEWDK-  NODECK
        SUBROUTINE NEWPER(N,NS,IPERM,*)
        DIMENSION IPERM(10)
        IF(IPERM(NS).EQ.N) GOTO 2
        IF(NS.NE.2) GOTO 3
        IF(IPERM(NS).EQ.0) RETURN 1
        GOTO 2
3       CALL NEWPI(N-IPERM(NS),NS-1,IPERM,$801)
        RETURN
801     IF(IPERM(NS).EQ.0) RETURN 1
2       NSI#NS-1
        DO 4 I#1,NSI
4        IPERM(I)#0
        IPERM(NS)#IPERM(NS)-1
        IPERM(NS-1)#N-IPERM(NS)
        RETURN
        END
$IBFTC NDEK1  NODECK
        SUBROUTINE NEWPI (N,NS,IPERM,*)
        DIMENSION IPERM(10)

```

```

      IF(IPERM(NS).EQ.N) GOTO 2
      IF(NS.NE.2) GOTO 3
      IF(IPERM(NS).EQ.0) RETURN 1
      GOTO 2
3     CALL NEWP2(N-IPERM(NS),NS-1,IPERM,$801)
      RETURN
801  IF(IPERM(NS).EQ.0) RETURN 1
2     NSM1#NS-1
      DO 4 I#1,NSM1
4     IPERM(I)#0
      IPERM(NS)#IPERM(NS)-1
      IPERM(NS-1)#N-IPERM(NS)
      RETURN
      END
$IBFTC NDEK2  NODECK
      SUBROUTINE NEWP2 (N,NS,IPERM,*)
      DIMENSION IPERM(10)
      IF(IPERM(NS).EQ.N) GOTO 2
      IF(NS.NE.2) GOTO 3
      IF(IPERM(NS).EQ.0) RETURN 1
      GOTO 2
3     CALL NEWP3(N-IPERM(NS),NS-1,IPERM,$801)
      RETURN
801  IF(IPERM(NS).EQ.0) RETURN 1
2     NSM1#NS-1
      DO 4 I#1,NSM1
4     IPERM(I)#0
      IPERM(NS)#IPERM(NS)-1
      IPERM(NS-1)#N-IPERM(NS)
      RETURN
      END
$IBFTC NDEK3  NODECK
      SUBROUTINE NEWP3 (N,NS,IPERM,*)
      DIMENSION IPERM(10)
      IF(IPERM(NS).EQ.N) GOTO 2
      IF(NS.NE.2) GOTO 3
      IF(IPERM(NS).EQ.0) RETURN 1
      GOTO 2
3     WRITE(6,21)
21    FORMAT(13H NS TOO LARGE)
      RETURN
801  IF(IPERM(NS).EQ.0) RETURN 1
2     NSM1#NS-1
      DO 4 I#1,NSM1
4     IPERM(I)#0
      IPERM(NS)#IPERM(NS)-1
      IPERM(NS-1)#N-IPERM(NS)
      RETURN
      END
$IBFTC CDECK  NODECK
      FUNCTION C(T,F1,DSIG2,Z)
      P# .5*(DSIG2/Z**2)
      A#F1/SQRT(Z*12.5663708*P*T)
      B#-((Z-T)**2)/(4.*P*Z*T)
      C#0.

```

```
IF(A.EQ.0.) RETURN  
ALA#ALOG(A)  
IF(ALA+B.LT.-87.) RETURN  
C#A*EXP(B)  
RETURN  
END
```

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APPENDIX C

DATA FROM AN INVESTIGATION OF AIR-SPARGE VERTICAL SYSTEMS

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Figure C-1. MIXING TIME AS A FUNCTION OF THE AIR FLOW IN A SIX-INCH-DIAMETER BY 35.9-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 52)

Figure C-2. MIXING TIME AS A FUNCTION OF THE AIR FLOW IN A SIX-INCH-DIAMETER BY 35.9-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 52)

Figure C-3. MIXING TIME AS A FUNCTION OF THE AIR FLOW IN A SIX-INCH-DIAMETER BY 35.9-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 52)

Figure C-4. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 30.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 53)

Figure C-5. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 30.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 53)

Figure C-6. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 30.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 53)

Figure C-7. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 24.55-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 54)

Figure C-8. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 24.55-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 54)

Figure C-9. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 24.55-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 54)

Figure C-10. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 19.8-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 55)

Figure C-11. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 19.8-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 55)

Figure C-12. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 19.8-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 55)

Figure C-13. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 16.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 56)

Figure C-14. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR SIX-INCH-DIAMETER BY 16.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 56)

Figure C-15. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 16.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 56)

Figure C-16. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 12.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 57)

Figure C-17. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 12.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 57)

Figure C-18. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 12.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 57)

Figure C-19. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 8.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 58)

Figure C-20. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 8.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 58)

Figure C-21. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A SIX-INCH-DIAMETER BY 8.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 58)

Figure C-22. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 35.9-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 65)

Figure C-23. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 35.9-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 65)

Figure C-24. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 35.9-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 65)

Figure C-25. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 30.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 66)

Figure C-26. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 30.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 66)

Figure C-27. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 30.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION (Series 66)

Figure C-28. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 24.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 67)

Figure C-29. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 24.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 67)

Figure C-30. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 24.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 67)

Figure C-31. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 19.8-FOOT-LONG AIR SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 68)

Figure C-32. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 19.8-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 68)

Figure C-33. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 19.8-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 68)

Figure C-34. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 16.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 69)

Figure C-35. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 16.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 69)

Figure C-36. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 16.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 69)

Figure C-37. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 12.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 70)

Figure C-38. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 12.0 FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 70)

Figure C-39. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 12.0-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 70)

Figure C-40. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 8.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.90 MIXED CONDITION. (Series 71)

Figure C-41. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 8.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.95 MIXED CONDITION. (Series 71)

Figure C-42. MIXING TIME AS A FUNCTION OF THE AIR FLOW FOR A FOUR-INCH-DIAMETER BY 8.6-FOOT-LONG AIR-SPARGE COLUMN WITH A 0.99 MIXED CONDITION. (Series 71)

Table C-1
EXPERIMENTAL DATA FOR THE AIR-SPARGE SYSTEMS
(Six-Inch-Diameter Column)

Run Number	Air-Flow Rate (scfm)	Mixing Time (min)		
		0.90	0.95	0.99
<u>Series 52 (liquid level = 35.9 ft)</u>				
116A	0.120	40.3	49.7	72.3
115A	0.200	38.7	48.7	70.0
114A	0.289	38.3	47.3	69.3
113A	0.367	21.7	34.0	50.7
35A	0.456	33.3	41.3	52.7
36A	0.540	30.3	37.7	47.7
37A	0.630	23.3	29.0	35.0
38A	1.350	18.7	25.3	31.3
<u>Series 53 (liquid level = 30.0 ft)</u>				
39A	0.120	26.3	35.3	49.7
40A	0.200	27.3	35.0	44.7
41A	0.289	24.3	30.3	40.7
99A	0.367	24.3	31.7	44.0
100A	0.456	24.7	32.3	42.0
101A	0.540	22.0	27.0	40.0
102A	0.630	18.3	21.3	26.0
103A	2.090	15.0	19.3	25.7
104A	3.550	14.0	18.7	24.3
105A	5.000	14.7	17.7	24.7
<u>Series 54 (liquid level = 24.55 ft)</u>				
106A	0.120	19.3	25.0	36.0
107A	0.200	18.3	22.3	31.3
108A	0.289	17.7	21.7	30.3
109A	0.367	17.0	22.3	30.7
110A	0.456	15.3	19.7	24.0
111A	0.540	14.7	17.7	24.7
112A	0.630	12.0	15.0	22.7
117A	2.090	10.0	13.3	23.7
57A	3.550	9.0	12.7	14.7
58A	5.000	11.0	13.0	15.3
<u>Series 55 (liquid level = 19.8 ft)</u>				
59A	0.120	12.7	16.0	24.3
60A	0.200	13.0	15.0	23.3
61A	0.289	12.3	15.0	21.0
62A	0.367	12.3	15.0	21.3
63A	0.456	11.0	14.7	20.0
64A	0.540	10.0	12.7	22.0
65A	0.630	10.3	14.0	21.7
66A	2.090	6.0	7.7	12.3
67A	3.550	7.0	8.0	11.0
68A	5.000	7.3	8.7	11.0
<u>Series 56 (liquid level = 16.0 ft)</u>				
69A	0.120	23.3	27.7	29.3
70A	0.200	18.0	18.3	21.7
71A	0.289	13.7	15.3	18.7
72A	0.367	14.7	16.0	17.3
73A	0.456	15.0	16.0	18.7
74A	0.540	17.0	18.0	19.3
75A	0.630	11.0	11.7	12.0
76A	2.090	13.0	13.3	14.3
77A	3.550	8.7	10.3	11.3
78A	5.000	8.7	10.0	10.7

Table C-1 (Continued)

Run Number	Air-Flow Rate (scfm)	Mixing Time (min)		
		0.90	0.95	0.99
<u>Series 57 (liquid level = 12.0 ft)</u>				
79A	0.120	3.3	5.0	9.3
80A	0.200	4.0	5.3	8.3
81A	0.289	3.7	4.3	7.0
82A	0.367	4.0	5.3	10.7
83A	0.456	3.3	5.0	7.7
84A	0.540	3.3	4.3	8.3
85A	0.630	2.7	3.7	6.7
86A	2.090	1.7	2.3	4.0
87A	3.550	1.7	2.3	2.7
88A	5.000	1.3	2.0	4.0
<u>Series 58 (liquid level = 8.6 ft)</u>				
89A	0.120	3.3	4.3	7.0
90A	0.200	3.3	4.0	7.0
91A	0.289	2.7	3.7	6.0
92A	0.367	2.7	3.3	5.7
93A	0.456	2.3	3.0	5.7
94A	0.540	2.3	3.0	4.3
95A	0.630	2.3	2.7	3.7
96A	2.090	2.0	2.3	4.3
97A	3.550	2.0	2.7	3.7
98A	5.000	1.7	2.0	2.7

Table C-2
EXPERIMENTAL DATA FOR THE AIR-SPARGE SYSTEMS
(Four-Inch-Diameter Column)

Run Number	Air-Flow Rate (scfm)	Mixing Time (min)		
		0.90	0.95	0.99
<u>Series 65 (liquid level = 35.9 ft)</u>				
118A	0.120	44.0	54.0	63.3
119A	0.200	36.0	41.3	46.7
120A	0.290	38.0	45.0	53.3
121A	0.367	34.3	40.3	45.3
122A	0.456	35.3	42.0	53.7
123A	0.540	31.0	38.0	44.7
124A	0.630	31.3	37.7	44.3
<u>Series 66 (liquid level = 30.0 ft)</u>				
125A	0.120	36.3	42.0	46.3
126A	0.200	33.3	38.3	42.3
127A	0.290	36.0	43.0	51.7
128A	0.367	32.7	39.0	48.7
129A	0.456	27.3	34.3	45.7
130A	0.540	29.0	34.3	41.7
131A	0.630	29.0	34.3	41.0
132A	2.090	20.7	25.0	30.0
<u>Series 67 (liquid level = 24.6 ft)</u>				
133A	0.120	29.0	34.0	42.7
134A	0.200	25.7	30.3	35.3
135A	0.290	25.0	30.0	41.7
136A	0.367	23.3	28.7	37.3
137A	0.456	22.3	27.7	36.7
138A	0.540	19.0	23.7	29.7
139A	0.630	20.3	24.3	31.7
140A	2.090	15.7	19.0	24.3
<u>Series 68 (liquid level = 19.8 ft)</u>				
141A	0.120	20.0	25.0	36.7
142A	0.200	18.0	22.3	27.0
143A	0.290	16.7	20.0	24.7
144A	0.367	16.0	19.3	25.7
145A	0.456	16.3	20.0	27.0
146A	0.540	15.0	17.7	21.7
147A	0.630	14.7	18.0	23.0
148A	2.090	11.0	14.0	18.7
<u>Series 69 (liquid level = 16.0 ft)</u>				
149A	0.120	4.0	10.7	21.7
150A	0.200	6.7	8.3	11.3
151A	0.290	6.0	10.0	16.7
152A	0.367	7.7	12.0	14.3
153A	0.456	5.0	7.3	13.7
154A	0.540	5.3	6.7	12.3
155A	0.630	4.7	5.7	10.3
156A	2.090	2.7	6.3	11.7
<u>Series 70 (liquid level = 12.0 ft)</u>				
157A	0.120	4.7	6.7	9.7
158A	0.200	5.0	6.7	11.7
159A	0.290	4.0	6.0	9.7
160A	0.367	4.0	6.0	10.3
161A	0.456	3.7	5.0	7.7
162A	0.540	3.3	4.7	9.0
163A	0.630	3.0	4.7	11.0
164A	2.090	1.3	1.7	2.3

Table C-2 (Continued)

Run Number	Air-Flow Rate (scfm)	Mixing Time (min)		
		0.90	0.95	0.99
Series 71 (liquid level = 8.6 ft)				
165A	0.120	4.3	5.7	8.0
166A	0.200	4.3	5.7	9.0
167A	0.290	4.3	5.7	8.0
168A	0.367	4.0	5.3	11.3
169A	0.456	3.7	5.0	9.0
170A	0.540	3.3	4.3	7.3
171A	0.630	3.0	4.3	5.7
172A	2.090	2.3	2.7	3.3

Table C-3
 STATISTICAL QUANTITIES OBTAINED FROM THE MIXING
 NUMBER-DIMENSIONLESS MIXING TIME FITS
 (Six-Inch-Diameter Air-Sparged Tank)

Mixed Condition	Parameter A		Parameter B		Variance of Estimate of Fit
	Value	Standard Error	Value	Standard Error	
<u>Series 52</u>					
0.90	1.335	0.047	- 0.329	0.093	0.006
0.95	1.456	0.031	- 0.304	0.061	0.003
0.99	1.549	0.033	- 0.396	0.066	0.003
<u>Series 53</u>					
0.90	1.275	0.012	- 0.200	0.023	0.001
0.95	1.377	0.015	- 0.208	0.027	0.002
0.99	1.504	0.019	- 0.211	0.035	0.003
<u>Series 54</u>					
0.90	1.108	0.015	- 0.205	0.028	0.002
0.95	1.213	0.013	- 0.197	0.023	0.001
0.99	1.346	0.016	- 0.225	0.030	0.002
<u>Series 55</u>					
0.90	0.956	0.017	- 0.208	0.031	0.002
0.95	1.048	0.017	- 0.218	0.031	0.002
0.99	1.210	0.015	- 0.246	0.028	0.002
<u>Series 56</u>					
0.90	1.096	0.022	- 0.223	0.042	0.004
0.95	1.136	0.023	- 0.216	0.042	0.005
0.99	1.176	0.022	- 0.230	0.041	0.004
<u>Series 57</u>					
0.90	0.377	0.024	- 0.305	0.043	0.005
0.95	0.517	0.020	- 0.291	0.037	0.003
0.99	0.744	0.031	- 0.318	0.058	0.009
<u>Series 58</u>					
0.90	0.351	0.011	- 0.165	0.020	0.001
0.95	0.447	0.014	- 0.180	0.026	0.002
0.99	0.640	0.021	- 0.227	0.039	0.004

Table C-4
 STATISTICAL QUANTITIES OBTAINED FROM THE MIXING
 NUMBER-DIMENSIONLESS MIXING TIME FITS
 (Four-Inch-Diameter Air-Sparged Tank)

Mixed Condition	Parameter A		Parameter B		Variance of Estimate of Fit
	Value	Standard Error	Value	Standard Error	
<u>Series 65</u>					
0.90	1.459	0.020	- 0.187	0.037	0.001
0.95	1.538	0.025	- 0.183	0.046	0.001
0.99	1.619	0.038	- 0.162	0.071	0.002
<u>Series 66</u>					
0.90	1.401	0.017	- 0.201	0.032	0.001
0.95	1.483	0.017	- 0.184	0.033	0.001
0.99	1.575	0.026	- 0.151	0.050	0.002
<u>Series 67</u>					
0.90	1.261	0.010	- 0.221	0.020	0.004
0.95	1.348	0.010	- 0.209	0.020	0.001
0.99	1.461	0.021	- 0.196	0.040	0.002
<u>Series 68</u>					
0.90	1.119	0.006	- 0.204	0.012	0.001
0.95	1.209	0.007	- 0.198	0.014	0.001
0.99	1.319	0.020	- 0.210	0.040	0.002
<u>Series 69</u>					
0.90	0.622	0.068	- 0.209	0.131	0.016
0.95	0.826	0.049	- 0.216	0.094	0.009
0.99	1.065	0.046	- 0.179	0.090	0.008
<u>Series 70</u>					
0.90	0.352	0.036	- 0.466	0.070	0.005
0.95	0.499	0.044	- 0.483	0.084	0.007
0.99	0.725	0.081	- 0.486	0.157	0.024
<u>Series 71</u>					
0.90	0.459	0.019	- 0.246	0.037	0.001
0.95	0.566	0.023	- 0.278	0.044	0.002
0.99	0.733	0.060	- 0.338	0.115	0.013

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APPENDIX D

DATA FROM THE INVESTIGATION OF MIXING-TUBE SYSTEMS

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Table D-1
EXPERIMENTAL DATA FOR THE MIXING-TUBE SYSTEMS

Run Number	Recirculation Rate (gpm)	Mixing Time (min)
Series 44 (five-hole mix tube with evenly spaced outlets)		
534	2.06	41.5
533	4.10	21.5
532	6.04	18.0
531	7.90	14.5
529	10.03	10.5
528	15.46	8.0
527	23.96	6.0
526	32.96	4.0
525	47.40	3.5
524	63.20	2.5
523	79.00	2.0

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APPENDIX E

DATA FROM THE EXPERIMENTAL SAMPLING OF SOLID-LIQUID MIXTURES

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Table E-1
EXPERIMENTAL DATA FOR SOLID-LIQUID SAMPLING
IN AN AIR-SPARGE COLUMN

Run Number	Tank Concentration (ppm)	Air-Flow Rate (scfm)	Sampling Results ⁽¹⁾ (ppm)
<u>Series 59 (liquid level = 16.5 ft; 0.05 μ solids)</u>			
1	500	2.09	500
4	250	2.09	250
7	750	2.09	875
8	750	2.09	750
9	750	0.54	750
<u>Series 60 (liquid level = 16.5 ft; 1.0 μ solids)</u>			
2	500	2.09	660
3	250	2.09	300
5	750	0.54	985
6	750	2.09	740
<u>Series 61 (liquid level = 16.5 ft; 20.0 μ solids)</u>			
11	750	2.09	800
12	1250	2.09	1315
13	2000	2.09	1980
<u>Series 62 (liquid level = 30.0 ft; 20.0 μ solids)</u>			
19	750	0.54	800
20	750	2.09	925
21	750	0.63	800
22	1250	2.09	1517
23	1250	0.63	1260
24	1250	0.54	1320
25	2000	2.09	2250
26	2000	1.35	2120
27	2000	0.54	1980
<u>Series 63 (liquid level = 30.0 ft; 1.0 μ solids)</u>			
29	1000	2.09	1410 (1360)
30	1000	1.35	1270 (985)
31	1000	0.54	1100 (1030)
32	1000	0.29	1000 (875)
33	1000	2.09	1175 (1075)
34	1000	1.35	1100 (1035)
35	1000	0.54	980 (700)
36	1000	0.29	1000 (680)
37	1000	2.09	1120 (1060)
38	1000	2.09	1000 (950)

(1) The sampling rate for all series was ~ 200 mls/min; for Series 63, values are given in parentheses for a sampling rate of 72 mls/min.

Table E-2
EXPERIMENTAL DATA FOR SOLID-LIQUID SAMPLING IN A
RECIRCULATION-MIXED HORIZONTAL SAFE TANK
(Series 64; 0.05 μ Solids; Tank Was Six Inches in
Diameter and Twenty Feet Long)

Run Number	Recirculation Rate (gpm)	Tank Concentration (ppm)	Sample Results (ppm)
39	98.3	250	220
40	42.5	250	75
41	98.3	500	415
42	42.5	500	125
43	98.3	750	640
44	42.5	750	240