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KNOLLS
ATOMIC POWER
LABORATORY

***FLOPSY-II —
A Digital Computer Program
for
Calculating Flows
and
Pressure Drops
in
Hydraulic Networks***

***B.L. Strain
June 1, 1965***

Operated for the
United States Atomic
Energy Commission by

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UC-32, Mathematics and
Computers

FLOPSY-II

A DIGITAL COMPUTER PROGRAM FOR CALCULATING FLOWS AND
PRESSURE DROPS IN HYDRAULIC NETWORKS

(Miss) B. L. Strain

June 1, 1965

UNC A. E. Kubetz
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Date

General Electric Company
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Schenectady, New York
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ACKNOWLEDGMENT

This report represents a revision and expansion of the FLOPSY-I computer program written by Mrs. M. Helme, nee Phillips, and therefore draws heavily on the original report (Reference 1). Appreciation is expressed to all those who gave their assistance in writing the FLOPSY-II code and this report, especially to G.J. Habetler for the method of convergence.

FLOPSY-II

A DIGITAL COMPUTER PROGRAM FOR CALCULATING FLOWS AND PRESSURE DROPS IN HYDRAULIC NETWORKS

(Miss) B. L. Strain

INTRODUCTION

The FLOPSY code deals with the fluid flow distribution and pressure drop in any given network. It is capable of handling either an open-loop or closed-loop circuit and variable resistances either in series or parallel. The flow and pressure drop are calculated for every branch of the system.

The code calculates weight flow and pressure drop throughout a network of nodes and branches (a node defined as any intersection of two or more branches and a branch defined as any flow path between two points or nodes with no intervening nodes).

Given the following information:

1. The layout of numbered nodes and branches,
2. An estimated weight flow for each branch,
3. Inlet and outlet flows, pressures and/or head rise,
4. Dimensions of each branch,

FLOPSY will calculate:

1. The weight flow in each branch, with direction of flow,
2. The unrecoverable pressure drop across each branch,
3. The pressure at each node,

to within the specified error criteria.

FLOPSY does not consider: (1) temperature-dependent flow branches, (2) two-phase flow in branches, (3) a variable head rise, and (4) recoverable pressure drop.

For definition of terms used in the sections of the report which follow, see the nomenclature listing (p. 33).

BASIC EQUATIONS

Two basic equations governing fluid flow in a network are analogous to Kirchhoff's laws governing the flow of current in electrical circuits, namely:

1. The algebraic sum of the currents at any point must equal zero, or the currents coming into a point must equal the currents leaving.
2. The algebraic sum of the voltage about a complete loop in a circuit must equal zero, or the voltage between any two points must be the same as determined over each path.

With respect to a network of fluid flow, these laws can be stated:

1. The weight flows coming into a point must equal the weight flows leaving.
2. Between any two points the pressure drop must be the same as determined over each path.

The three basic equations used in FLOPSY are:

$$\sum W = 0 \quad (1)$$

at any node.

$$\Delta P = RW|W| - H \quad (2)$$

in every branch, where H is a head rise and is usually zero except when there is a pump. $W|W|$ is used rather than W^2 so that the sign will be preserved. Thus, if the wrong direction of flow is assumed, FLOPSY will show this by calculating a negative flow and pressure drop.

$$\Delta P = P_{in} - P_{out} \quad (3)$$

in every branch.

The original assumed weight flows must satisfy the first equation. The code then calculates the pressure drops by means of the second equation. By the method discussed in the Appendix, the code then calculates the pressure at each node. From this, the pressure drops are calculated according to Equation 3. Since the weight flows are merely assumed, the value of ΔP in Equations 2 and 3 will be different. Therefore, FLOPSY will iterate until they are the same by means of the following empirical equations (see the Appendix for this justification):

$$\Delta W_k^{(i)} = \alpha \left[P_{in_k}^{(i)} - P_{out_k}^{(i)} - R_k W_k^{(i-1)} |W_k^{(i-1)}| + H_k^{(i-1)} \right] \quad (4)$$

$$W_k^{(i)} = W_k^{(i-1)} + \Delta W_k^{(i)}, \quad (5)$$

where the subscript k refers to the branch number, where the superscript i refers to the iteration number, and where α is a multiplying factor to ensure convergence.

Equations 2, 3, and 5 are repeated until $[P_{in} - P_{out} - \Delta P]$ is sufficiently small for the particular problem. Thus, the final result is:

$$W_k^{(n)} = W_k^{(0)} + \sum_{i=1}^n \Delta W_k^{(i)} \quad (6)$$

DESCRIPTION

Definition of Basic Terms

Node

A node is the intersection of two or more branches. The following terminology is used in this report:

1. Internal node - a node where the pressure is not known and is to be calculated.
2. External node - a node where the pressure is known. These nodes are used to supply the driving force across pressure-controlled networks.
3. Flow nodes - any node, either internal or external, at which a known amount of flow enters or leaves the network. These nodes are the sources and sinks in flow-controlled networks.

Branches

A branch is any flow path between two nodes, or points, with no intervening nodes. There are four kinds of branches.

1. Internal branch - any branch not bounded by a flow node.
2. External branch - a branch connected to a flow node. For example, a branch with a known constant flow is an external branch.
3. Dummy branch - one branch used to represent a number of parallel flow paths between two nodes. All branches which go between the same two end points must be represented as a single dummy branch.
4. Parallel branch - one of many parallel flow paths which make up a dummy branch.

Convergence

α is the multiplying factor to force a convergence of the flows in a system. The code presently uses the Tchebyshev Semiiterative method to calculate new values of α every 10 iterations by the equation:

$$\alpha(\text{IA}) = \frac{1}{\overline{R|W|} + \underline{R|W|} + (\overline{R|W|} - \underline{R|W|}) \cos \left(\frac{\pi(2*\text{IA} - 1)}{2*IPVAR} \right)}$$

where IA goes from one to IPVAR.

IPVAR is an input quantity, and $\overline{R|W|}$ and $\underline{R|W|}$ are, respectively, the maximum and minimum values of all internal branches. It is expected that with use, an optimum value of IPVAR will be found to give fastest convergence for each type of problem. Values of IPVAR between 1 and 10 have been found adequate, with the higher values of IPVAR being used for problems where parallel resistances differ greatly in order of magnitude.

Convergence is assumed to exist where the following condition is satisfied by all branches

$$|[P_{in} - P_{out} - \Delta P]| \leq \gamma \quad \text{for two consecutive iterations.}$$

Variable Resistance

Calculation of Resistances

In FLOPSY II the resistance for each branch is calculated at the end of each iteration according to the formula:

$$R = \frac{f \cdot L/D + K_T}{2g\rho A^2}$$

where f = friction factor

L = branch length (in.)

D = hydraulic diameter (in.) = $\frac{4 \times \text{cross sectional area}}{\text{perimeter}}$

K_T = entrance, exit, or turn loss coefficients or any combination of these, based upon the area for the branch

g = acceleration of gravity ($32.17 \text{ ft/sec}^2 = 386.04 \text{ in./sec}^2$)

ρ = density of fluid involved (lb/in^3)

A = area (in^2)

R = resistance ($\text{sec}^2/\text{lb-in}^2$)

ρ is input for the problem; and L , D , K_T , and A are input for each branch of the problem. f is calculated within the program for each branch.

The friction factor f is calculated during each iteration for each branch. It is calculated as a function of two variables:

ϵ/D = relative roughness, where ϵ is absolute roughness.
(arithmetic average) input for each branch,

Re = Reynolds number $\left(\frac{D}{\mu A}\right) \times W$, where μ is viscosity
(lb/sec-in.) input for the problem.

The values calculated for f are based on the graph of friction factor vs Reynolds number with relative roughness as a parameter as described in Reference 2.

The Moody curve is coded as a matrix $[Z]$ and two vectors, $[X]$ and $[Y]$, where $f = z_{i,j}$ for a given $Re = x_i$ and a given $2 \epsilon/D = y_j$. However, this applies only within certain limits. When values of either $2 \epsilon/D$ or Re lie outside these limits, the code uses a different method of "graph reading":

1. For each $Re = x_i$ there is given $2 \epsilon/D = y_{jmax_i}$ such that for $2 \epsilon/D = y_{jmax_i}$, f is a function of $2 \epsilon/D$ only. Since Re is proportional to weight flow, this means that if the roughness is large enough, the effect of the weight flow is negligible as far as friction is concerned. In this case, the code chooses $f = z_{min_j}$ from the vector $ZMIN$ for the given $2 \epsilon/D = y_j$.
2. For each $Re = x_i$ there is also a given $2 \epsilon/D = y_{jmin_i}$ such that for $2 \epsilon/D = y_{jmin_i}$, f is a function of Re only. In physical terms this means that, if the roughness is small enough and the weight flows are large enough, then the effect of roughness is negligible with respect to friction. In this case, the code chooses $f = z_{max_i}$ from the vector $[ZMAX]$ (which is actually coded as $z_{i,19}$, where $y_{19} = 0$) for the given $Re = x_i$.
3. When $y_{jmin_i} < 2 \epsilon/D < y_{jmax_i}$, f is a function of both Re and $2 \epsilon/D$, as first described. For these three cases, the code interpolates linearly with respect to either Re or $2 \epsilon/D$ or both, depending upon which case is applicable.
4. For $Re \leq 2000$, the equation $f = 64/Re$ is used.
5. For $Re > 10^8$, the code uses the values for $Re = 10^8$ and proceeds as usual.
6. For $2 \epsilon/D > 0.05$, the equation $f = \left(\frac{1.0}{1.14 - 0.86 (2\epsilon/D)} \right)^2$ is used.

The values of x_i , y_j , $z_{i,j}$, etc. are coded separately in a list called "MOOD," with 24 values for x or Re from 2×10^3 to 1×10^8 and 19 values for y or $2 \epsilon/D$ from 0.05 to zero. The instructions to test the values of Re and $2 \epsilon/D$ and to "read" and interpolate from the "graph" accordingly are coded separately as FUNCTION FRICT.

Types of Problems Handled

FLOPSY handles various types of problems. These include:

1. Pressure-controlled problems for which the pressure across a network is known.
2. Flow-controlled problems for which the total flow through a network is known.
3. Flow- and pressure-controlled problems for which the pressure drop is known across one segment of the network while the total flow in some other segment is also known.

The following diagrams illustrate the nodal arrangement used for typical networks of the type listed above. These networks appear later (p. 15) as sample problems:

1. A pressure-controlled network

2. A flow-controlled network

3. A flow- and pressure-controlled network

Key

- $\frac{1}{\bullet}$ internal node
- $\frac{0}{\bullet}$ external node
- $\text{---} \textcircled{1} \text{---}$ branch (internal)
- --- branch (external)
- $\times$ flow node

In the above problems, the code must know either the pressure or the flow at every inlet or outlet. If no pressure is actually known, as in the flow-controlled problem, it is usually convenient to set the pressure at an outlet node equal to zero, since the code is only concerned with relative pressures within the network.

The hydraulic characteristic of a pump in a closed loop may be approximated over a large portion of the pump's operating range in the following manner:

If the dashed line is the pump characteristic, it is approximated over a portion of its range by a quadratic expression in W . This is easily inserted into the code as a branch with a constant head and a fixed hydraulic resistance. Sample Problem Number 4 (p. 27) presents a flow network with a pump in one of the closed loops.

When doing design calculations, it is often desirable to put a "safety" factor on hydraulic resistances. This can be accomplished in FLOPSY with the use of the R multipliers, and is done by indicating branch numbers with their respective resistance multipliers so that the code will then calculate

$$R = R \text{ MULT} \left(\frac{\frac{fL}{D} + K_t}{2 \rho g A^2} \right).$$

The code solves the problem without the resistance multipliers first, and then re-solves the problem with the multipliers. If no resistance multipliers are used, the problem is solved once.

Whenever more than one branch connects the same two nodes and there are no intervening nodes, these branches must be regarded as a set of parallel branches, analogous to electrical circuits in parallel:

In the preceeding example, given the individual resistances R_a , R_b , R_c , and R_d , the code will calculate an equivalent resistance R_A according to the equation:

$$\frac{1}{\sqrt{R_A}} = \frac{1}{\sqrt{R_a}} + \frac{1}{\sqrt{R_b}} + \frac{1}{\sqrt{R_c}} + \frac{1}{\sqrt{R_d}},$$

or

$$R_A = \left(\frac{1}{\sum \frac{1}{\sqrt{R}}} \right)^2,$$

It will then regard the four individual branches as one branch, A. Calculating the correct total weight flow, W_A , it will then calculate the individual weight flows W_a , W_b , W_c , and W_d by means of

$$W_a = \sqrt{\frac{\Delta P_A}{R_a}}.$$

In systems of this sort, a branch such as A is called a "dummy" branch.

Cautions

1. For every inlet and outlet of an open-loop system there must be either an external node or a flow node. When there are flow nodes at all inlets and outlets of the system, one external pressure must be given. (This is usually the node at which pressure is assumed to be the lowest, and is usually set equal to zero.)*
2. Dummy branches must only be internal branches.
3. For every branch there must be an inlet and outlet node, internal or external, depending upon the problem.
4. In an open-loop system, an external node has either an inlet or outlet branch, but not both.
5. A branch may have no more than one external node.
6. Internal branches must be numbered first, then external branches.
7. The sum of the inlet and outlet flows at any node must equal zero. The code will automatically check for this before starting on the problem by checking that the $|\Sigma W|$ at any node is less than the specified value FLOWE.

*In a closed-loop system one node must be specified as an external node for a reference pressure value.

8. When estimating flows for pressure-controlled problems, it is advisable to estimate on the high side for convergence reasons. All zero initial flow estimates are not allowed.
9. FLOPSY cannot handle problems when the pressure drop across a branch is greater than, and opposed to, an inputted head rise across the branch. In other words, negative flow through a pump is not permitted.

Limitations

At present, the code does not consider the following conditions:

1. Temperature-dependent flow branches
2. Two-phase flow in branches
3. A variable head rise
4. Difference between static and total pressure (i.e., FLOPSY calculates unrecoverable pressure drop but ignores recoverable pressure drop).

General Order of Operations

An outline of the basic order of operations in FLOPSY II is given below:

1. Read nodal layout.
2. Calculate matrix B and B^{-1} .
3. Read dimensions and other input.
4. Calculate resistances and other constants for all branches.
5. Set $i = 0$ (i = iteration number).
6. Set $i = i + 1$.
7. Calculate values for α if it is the proper iteration.
8. Calculate ΔP , $[\Delta P - \hat{D}\hat{P}]$, ΔW and W for internal branches; calculate P for internal nodes, for each value of α .
9. Calculate W 's for parallel branches, if any.
10. Calculate friction factors and new resistances for internal branches and for parallel branches, if any.
11. Write output i , R , H , W , ΔP , Re , f , and $DIFFO$ ($[P_{in} - P_{out} - \Delta P]$) for all branches and dummy branches, and P and P for all nodes.
12. Continue Steps 6 through 11 until $[P_{in} - P_{out} - \Delta P] \leq \gamma$ for all internal branches for two consecutive iterations, writing output at iterations where requested, and stopping if $i = I$. (I = total number of iterations.)
13. Test for new problem. If there is one, go back to Step 1, otherwise STOP.

Maximum Size of Problems

1. Total number of internal branches	$1, \dots, K'$	$2 \leq K' \leq 200$
2. Total number of external branches	$K'+1, \dots, K$	$K \leq 200$
3. Total number of internal nodes	$1, \dots, N$	$3 \leq N \leq 100$
4. Total number of external nodes	$1, \dots, M$	$1 \leq M \leq 50$
5. Total number of dummy branches	$1, \dots, L$	$L \leq 30$
6. Total number of parallel types	$1, \dots, T$	$T \leq 35$
7. Total number of printouts	$1, \dots, Z$	$Z \leq 100$

INPUT

The code for the format types listed below may be defined as follows:

- 1 - Integer, i.e., 1, 10, 12
- 2 - Floating point number, i.e., 1.0, 1.0+6
- 3 - Hollerith field

Card No.	Columns	Format Type	FORTTRAN Nomenclature	Description
1	2-72	3		Run identification card
2	1-5	1	NINTN	Number of internal nodes (Max 100)
	6-10	1	NEXTN	Number of external nodes (Max 50)
	11-15	1	NINTB	Number of internal branches (Max 200)
	16-20	1	KTOT	Total number of branches (Max 200)
	21-25	1	NUMDUM	Number of dummy branches (Max 30)
	26-30	1	NUMTYP	Number of parallel types (Max 55)
	31-35	1	NUMPT	Number of printouts requested (Max 100)
	36-40	1	ITOT	Maximum number of iterations
	41-45	1	MULT	Number of resistance multipliers to be read in
	46-50	1	MTX	{+1 calculate and invert matrix 0 do not calculate and invert matrix
	51-55	1	MESS	{+1 read in dimensions of branches 0 do not read in dimensions
	56-60	1	JUMP1	{+1 read in weight flows 0 do not read in weight flows

Card No.	Columns	Format Type	FORTTRAN Nomenclature	Description
3	1-10	2	VISC	Fluid viscosity, lb/sec-in.
	11-20	2	RHO	Fluid density, lb/in ³
	21-30	2	GAMMA	γ - convergence criteria
	31-40	2	FLOWE	Allowable flow error in input values
	41-50	1	IPVAR	Number of values of α to be used in each iteration

Cards 4 and 5 only if matrix is to be calculated and inverted.

4	1-10	1	KBNCH	Branch number
	11-15	1	INODE	Internal inlet node
	16-20	1	JNODE	Internal outlet node
	21-30	1		Branch number
	31-35	1		Internal inlet node
	36-40	1		Internal outlet node
	41-50	1		Branch number
	51-55	1		Internal inlet node
	56-60	1		Internal outlet node

Branch numbers must be consecutive and there must be one card for every three total branches.

5	1-10	1	KNODE	External node number
	11-15	1	IBNCH	Inlet branch number
	16-20	1	JBNCH	Outlet branch number
	21-30	2	PEXT	Pressure at external node, psi
	31-40	1		External node number
	41-45	1		Inlet branch number
	46-50	1		Outlet branch number
	51-60	2		Pressure at external node, psi

External node numbers must be consecutive and there must be one card for every two external nodes.

Cards 6, 7, and 8 if dimensions are to be read in. (Because of space limitation in the program, if matrix is to be inverted then dimensions must be read in.)

6	1-5	1	KTYPE	Parallel branch type number
	6-10	1	IFRIND	<div style="display: inline-block; vertical-align: middle;"> <div style="font-size: 3em; vertical-align: middle; margin-right: 5px;">{</div> <div> -1 no friction factor to be calculated 0 calculate friction factor </div> </div>

Card No.	Columns	Format Type	FORTTRAN Nomenclature	Description
6	11-20	2	ARIND	Area of branch, in ²
	21-30	2	DIMIND	Hydraulic diameter, in.
	31-40	2	BLIND	Length, in.
	41-50	2	RUFIND	Arithmetic roughness
	51-60	2	CONIND	Loss coefficient (other than friction)
	61-70	2	WTEMP	Initial guess of flow (used only for calculating initial value of Reynolds number, lb/sec
7	1-5	1	KDUM	Dummy branch number
	6-10	1	IMAX	Number of parallel types in dummy branch
	11-15	1	NPAR	Parallel type number
	16-20	1	NUMBAN	Number of branches of this type
	21-25	1		Parallel type number
	26-30	1		Number of branches of this type
	31-35	1		Parallel type number
	36-40	1		Number of branches of this type
	41-45	1		Parallel type number
	46-50	1		Number of branches of this type
	51-55	1		Parallel type number
	56-60	1		Number of branches of this type
	61-65	1		Parallel type number
	66-70	1		Number of branches of this type

If there are more than 6 parallel types in a dummy branch, the continuation card starts in column 11.

One set of card 7 for every dummy branch.

Card 8 for each branch that is not a dummy branch.

8	1-5	1	KBNCH	Branch number
	6-10	1	IFR	{ -1 no friction factor to be calculated 0 calculate friction factor
	11-20	2	AREA	Area of branch, in ²
	21-30	2	DIAM	Hydraulic diameter, in.
	31-40	2	BLEN	Length, in.

Card No.	Columns	Format Type	FORTTRAN Nomenclature	Description
8	41-50	2	ROUGH	Arithmetic roughness
	51-60	2	CONK	Loss coefficient (other than friction)
	61-70	2	H	Head rise in branch, psi

Card 9 only if weight flows are to be read in.

9	1-10	2	W	Weight flow
	11-20	2		Weight flow
	21-30	2		Weight flow
	31-40	2		Weight flow
	41-50	2		Weight flow
	51-60	2		Weight flow
	61-70	2		Weight flow

One card for every 7 total branches.

10	1-5	1	ITPT	Iteration to be printed
	6-10	1		Iteration to be printed
	11-15	1		Iteration to be printed
	16-20	1		Iteration to be printed
	⋮			
	61-70	1		Iteration to be printed

One card for every 14 iterations to be printed.

11	1-5	1	KBNCH	Branch number
	6-15	2	RMULT	Multiplier to be used
	16-20	1		Branch number
	21-30	2		Multiplier to be used
	31-35	1		Branch number
	36-45	2		Multiplier to be used
	46-50	1		Branch number
	51-60	2		Multiplier to be used

One card for every 4 branches with multipliers. Branches for which none are supplied are taken to be 1.0.

OUTPUT

FLOPSY will print out the entire input and the symmetric B matrix at the beginning of the output. It will also print out the following information at each requested iteration and at convergence (if attained):

1. Title card (at convergence only).
2. The statement of use of branch multipliers (at first print-out and at convergence).
3. The statement of convergence (if applicable).
4. The iteration number.
5. The value(s) of α .
6. For each regular branch:
 - (a) number
 - (b) resistance, $\text{sec}^2/\text{lb-in}^2$
 - (c) head rise, lb/in^2
 - (d) weight flow, lb/sec
 - (e) pressure drop, psi
 - (f) relative roughness
 - (g) Reynolds number
 - (h) friction factor
 - (i) value of $[P_{\text{in}} - P_{\text{out}} - \Delta P]$
7. For dummy branches:
 - (a) branch number and for each parallel type
 - (b) parallel type number
 - (c) resistance, $\text{sec}^2/\text{lb-in}^2$
 - (d) weight flow, lb/sec
 - (e) relative roughness
 - (f) Reynolds number
 - (g) friction factor
8. Pressure at each internal node, psi
9. Pressure at each external node, psi

SAMPLE PROBLEMSProblem 1. Pressure-Controlled Network

The users network will replace the triangular network shown here. External Nodes 1 and 2 with their adjacent branches control this type of problem. The following is the input and output for this problem.

Sample Problem 1 Input and OutputInput

```

2      CHARGE CARD
LU SHANNON BLDG C-1 RM 128
1      GO TO NEXT
PRES CONTROLLED
  3    2    5    5    0    0    6 600    0    1    1    1
.5972-5 .2853-1 .5-3    1.0    1    3    1    2
  1    2    3    2    1    2    3    1    2
  4    3    0    5    0    1
  1    4    0    0.0    2    0    5    2.25
  1 -1    1.0    1.0    0.0    0.0    1.0    0.0
  2 -1    1.0    1.0    0.0    0.0    1.0    0.0
  3 -1    1.0    1.0    0.0    0.0    1.0    0.0
  4 -1    1.0    1.0    0.0    0.0    0.0    0.0
  5 -1    1.0    1.0    0.0    0.0    0.0    0.0
  50.0 50.0 50.0 100.0 100.0
100 200 300 400 500 600
2      GO TO QUIT

```

OUTPUT FROM FLOPSY II

PRES CONTROLLED

NINTN	NEXTN	NINTB	KTOT	NUMDUM	NUMTYP	NUMPT	ITOT	LAST	MTX	MESS	JUMP1
3	2	5	5	0	0	6	600	0	1	1	1

VISC.	RHO	GAMMA	FLOWE	IPVAR
0.5972+005	0.2853+001	0.5000+003	0.1000+001	1

BRANCH	INLET NODE	OUTLET NODE	BRANCH	INLET NODE	OUTLET NODE	BRANCH	INLET NODE	OUTLET NODE
1	2	3	2	1	2	3	1	3
4	3	0	5	0	1			

THE SYMMETRIC MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	3.0000	1 2	-1.0000	1 3	-1.0000	2 2	2.0000
2 3	-1.0000	3 3	3.0000				

THE INVERSE OF THE MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	0.6250	2 1	0.5000	3 1	0.3750	2 2	1.0000
3 2	0.5000	3 3	0.6250				

EXT. NODE	INLET BRANCH	OUTLET BRANCH	EXTERNAL PRESSURE	EXT. NODE	INLET BRANCH	OUTLET BRANCH	EXTERNAL PRESSURE
1	4	0	0.	2	0	5	2.2500

BRANCH NO.	TYPE OF FR	BRANCH AREA	HYDRAULIC DIAMETER	BRANCH LENGTH	ARITHMETIC ROUGHNESS	BRANCH LOSS COEF.	HEAD RISE
1	-1	1.0000	1.0000	0.	0.	1.0000	0.
2	-1	1.0000	1.0000	0.	0.	1.0000	0.
3	-1	1.0000	1.0000	0.	0.	1.0000	0.
4	-1	1.0000	1.0000	0.	0.	0.	0.
5	-1	1.0000	1.0000	0.	0.	0.	0.

FLOW	FLOW	FLOW	FLOW	FLOW	FLOW	FLOW
50.0000	50.0000	50.0000	100.0000	100.0000		

OUTPUT PRINTED AT THE FOLLOWING ITERATIONS
100 200 300 400 500 600

PRES CONTROLLED

THE WEIGHT FLOWS HAVE CONVERGED

ITERATION NO: 28

ALPHA= 0.3128+001

BRANCH	RESISTANCE	HEAD RISE	WEIGHT FLOW	DELTAP	REL. ROUGHNESS	REYNOLDS NO.	FR. FACTOR
1	0.45398-001	0.	4.9/84	1.125324	0.	0.83363+006	0.
2	0.45398-001	0.	4.9/84	1.125324	0.	0.83363+006	0.
3	0.45398-001	0.	7.0405	2.258575	0.	0.11789+007	0.
4	0.	0.	12.0190	0.	0.	0.20126+007	0.
5	0.	0.	12.0190	0.	0.	0.20126+007	0.

THE PRESSURE AT EACH INTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	DIFF0
1	2.250225	2	1.125000	3	-0.000225			-0.000100
								-0.000100
								-0.000125
								-0.000225
								-0.000225

THE PRESSURE AT EACH EXTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE
1	0.	2	2.250000				

END OF PROBLEM

21115367\$ FLR 0 PP 72021 7204 3.63005 5.032 5.047 .015 3-104-07-64

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Problem 2. Flow-Controlled Network

For the above configuration, the flow in Branches 4 and 5 control the problem. Because Branches 4 and 5 are adjacent to flow nodes, they are called external branches and, therefore, should be the last two branches numbered. Note that the pressure at External Node 1 is set at zero. This is the only pressure specified in the problem.

Sample Problem 2 Input and Output

Input

```

3      CHARGE CARD
LU SHANNON BLDG C-1 RM 128
1      GO TO NEXT
1  SAMPLE PROBLEM      FLOW CONTROLLED
  4      1      3      5      0      0      6      600      0      1      1      1
.5972-5 .2853-1 .5-3      1.0      1      3      1      3
      1      2      3      2      1      2      3      1      3
      4      3      0      5      4      1
      1      4      0      0.0
      1      -1      1.0      1.0      0.0      0.0      1.0      0.0
      2      -1      1.0      1.0      0.0      0.0      1.0      0.0
      3      -1      1.0      1.0      0.0      0.0      1.0      0.0
      4      -1      1.0      1.0      0.0      0.0      0.0      0.0
      5      -1      1.0      1.0      0.0      0.0      0.0      0.0
4.978      4.978      7.042      12.02      12.02
100      200      300      400      500      600
2      GO TO QUIT
END DATA

```

Output

SAMPLE PROBLEM FLOW CONTROLLED

NINTN	NEXTN	NINTB	KTUT	NUMDUM	NUHTYP	NUMPT	ITOT	LAST	MTX	MESS	JUMP
4	1	3	5	0	0	6	400	0	1	1	1

VISC.	RHO	GAMMA	FLOWE	IPVAR
0.5972+005	0.2853+001	1.5000+003	0.1000+001	1

BRANCH	INLET NODE	OUTLET NODE	BRANCH	INLET NODE	OUTLET NODE	BRANCH	INLET NODE	OUTLET NODE
1	2	3	2	1	2	3	1	3
4	3	0	5	4	1			

THE SYMMETRIC MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	3.0000	1 2	-1.0000	1 3	-1.0000	1 4	-1.0000
2 2	2.0000	2 3	-1.0000	2 4	0.	3 3	3.0000
3 4	0.	4 4	1.0000				

THE INVERSE OF THE MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	1.6667	2 1	1.3333	3 1	1.0000	4 1	1.6667
2 2	1.6667	3 2	1.0000	4 2	1.3333	3 3	1.0000
4 3	1.0000	4 4	2.6667				

EXT.	INLET	OUTLET	EXTERNAL	EXT.	INLET	OUTLET	EXTERNAL
NODE	BRANCH	BRANCH	PRESSURE	NODE	BRANCH	BRANCH	PRESSURE
1	4	0	0.				

BRANCH TYPE	BRANCH HYDRAULIC	BRANCH	ARITHMETIC	BRANCH	HEAD		
NO. OF FR	AREA	DIAMETER	LENGTH	ROUGHNESS	LOSS COEF.	RISE	
1	01	1.0000	1.0000	0.	0.	1.0000	0.
2	01	1.0000	1.0000	0.	0.	1.0000	0.
3	01	1.0000	1.0000	0.	0.	1.0000	0.
4	01	1.0000	1.0000	0.	0.	0.	0.
5	01	1.0000	1.0000	0.	0.	0.	0.

FLOW	FLOW	FLOW	FLOW	FLOW	FLOW	FLOW
4.9780	4.9780	7.0420	12.0200	12.0200		

OUTPUT PRINTED AT THE FOLLOWING ITERATIONS
100 200 300 400 500 600

SAMPLE PROBLEM FLOW CONTROLLED

THE WEIGHT FLOWS HAVE CONVERGED

ITERATION NO. 2
ALPHA 0.3128+001

BRANCH	RESISTANCE	HEAD RISE	WEIGHT FLOW	DELTA P	REL. ROUGHNESS	REYNOLDS NO.	FR. FACTOR
1	0.45398-001	0.	4.9785	1.125599	0.	0.83365+006	0.
2	0.45398-001	0.	4.9785	1.125599	0.	0.83365+006	0.
3	0.45398-001	0.	7.0415	2.250401	0.	0.11791+007	0.
4	0.	0.	12.0200	0.	0.	0.20127+007	0.
5	0.	0.	12.0200	0.	0.	0.20127+007	0.

THE PRESSURE AT EACH INTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	DIFFO
1	2.250666	2	1.125333	3	0.000000	4	2.250666	0.000265

THE PRESSURE AT EACH EXTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	DIFFO
1	0.							0.000000

END OF PROBLEM

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Problem 3. Flow- and Pressure-Controlled Network

In this problem, the constant flow in Branch 6 and the known pressure drop between Nodes 2 and 1 control this problem. Branch 6 is an external branch because it is adjacent to Flow Node 4. Therefore, Branch 6 is the last numbered branch. The input and output for this problem follows.

Sample Problem 3 Input and Output

Input

```

3      CHARGE CARD
LU SHANNON BLDG C-1 RM 128
1      GO TO NEXT
1  SAMPLE PROBLEM      FLOW AND PRESSURE CONTROLLED
    4      2      5      6      0      0      6      600      0      1      1      1
    .5972-5      .2853-1      .5-3      1.0      1
    1      2      3      2      1      2      3      1      3
    4      3      0      5      0      1      6      4      2
    1      4      0      0.0      2      0      5      2.25
    1      -1      1.0      1.0      0.0      0.0      1.0      0.0
    2      -1      1.0      1.0      0.0      0.0      1.0      0.0
    3      -1      1.0      1.0      0.0      0.0      1.0      0.0
    4      -1      1.0      1.0      0.0      0.0      0.0      0.0
    5      -1      1.0      1.0      0.0      0.0      0.0      0.0
    6      -1      1.0      1.0      0.0      0.0      0.0      0.0
51.0      50.0      50.0      101.0      100.0      1.0
100      200      300      400      500      600
2      GO TO QUIT

```

Output

24

SAMPLE PROBLEM FLOW AND PRESSURE CONTROLLED

NINTN	NEXTN	NINTB	KTOT	NUMDUM	NUMTYP	NUMPT	ITOT	LAST	MTX	MESS	JUMP1
4	2	5	6	0	0	6	600	0	1	1	1

VISC.	RHO	GAMMA	FLOWE	IPVAR
0.5972E-005	0.2853E-001	0.5000E-003	0.1000E-001	1

INLET BRANCH	OUTLET NODE	INLET BRANCH	OUTLET NODE	INLET BRANCH	OUTLET NODE
1	2	2	1	3	1
4	3	5	0	6	4

THE SYMMETRIC MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	3.0000	1 2	-1.0000	1 3	-1.0000	1 4	0.0000
2 2	3.0000	2 3	-1.0000	2 4	-1.0000	3 3	3.0000
3 4	0.	4 4	1.0000				

THE INVERSE OF THE MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	0.6250	2 1	0.5000	3 1	0.3750	4 1	0.5000
2 2	1.0000	3 2	0.5000	4 2	1.0000	3 3	0.6250
4 3	0.5000	4 4	2.0000				

EXT. NODE	INLET BRANCH	OUTLET BRANCH	EXTERNAL PRESSURE	EXT. NODE	INLET BRANCH	OUTLET BRANCH	EXTERNAL PRESSURE
1	4	0	0.	2	0	5	2.2500

BRANCH NO. OF FR	TYPE	BRANCH AREA	HYDRAULIC DIAMETER	BRANCH LENGTH	ARITHMETIC ROUGHNESS	BRANCH LOSS COEF.	HEAD RISE
1	-1	1.0000	1.0000	0.	0.	1.0000	0.
2	-1	1.0000	1.0000	0.	0.	1.0000	0.
3	-1	1.0000	1.0000	0.	0.	1.0000	0.
4	-1	1.0000	1.0000	0.	0.	0.	0.
5	-1	1.0000	1.0000	0.	0.	0.	0.
6	-1	1.0000	1.0000	0.	0.	0.	0.

KAPL-3030

FLOW	FLOW	FLOW	FLOW	FLOW	FLOW	FLOW
51.0000	50.0000	50.0000	101.0000	100.0000	1.0000	

OUTPUT PRINTED AT THE FOLLOWING ITERATIONS

100 200 300 400 500 600

SAMPLE PROBLEM FLOW AND PRESSURE CONTROLLED

THE WEIGHT FLOWS HAVE CONVERGED

ITERATION NO. 28

ALPHA 0.3128+001

BRANCH	RESISTANCE	HEAD RISE	WEIGHT FLOW	DELTAP	REL. ROUGHNESS	REYNOLDS NO.
1	0.45398-001	0.	5.4533	1.350236	0.	0.91315+006
2	0.45398-001	0.	4.4533	0.900466	0.	0.74570+006
3	0.45398-001	0.	7.8406	2.250620	0.	0.11789+007
4	0.	0.	12.4939	0.	0.	0.20921+007
5	0.	0.	11.4939	0.	0.	0.19246+007
6	0.	0.	1.0000	0.	0.	0.16745+006

FR. FACTOR	DIFFO
0.	0.000108
0.	0.000108
0.	0.000135
0.	0.000243
0.	0.000243
0.	0.000000

THE PRESSURE AT EACH INTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE
1	2.250243	2	1.349885	3	0.000243	4	1.349885

THE PRESSURE AT EACH EXTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE
1	0.	2	2.250000				

END OF PROBLEM

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Problem 4. Pump, Dummy Branch, and Resistance Multipliers

This flow-controlled problem illustrates how a pump may be mocked up in a flow network and also how a dummy branch is used. The pump is in Branch 4 and is mocked up as described on page 7. The dummy branch is Branch 3 with 420 parallel flow paths of two types. Dummy branches must be used whenever there is more than one branch between two nodes. The input and output for the problem follows. Note that the hydraulic resistance of Branch 3 was multiplied by a 1.05 factor. The first set of output is without the factor. The second set is with the factor.

Sample Problem 4 Input and Output

Input

```

3      CHARGE CARD
LU SHANNON BLDG C-1 RM 128
1      GO TO BEGIN
1 FLOPSY SAMPLE PROBLEM WITH PUMP RUN 10 4/13/64
  6 1 6 8 1 2 2 600 1 1 1 1
  3.82-5 3.59-2 .5-3 1.0 5
    1 1 2 2 3 3 4
    4 4 5 5 2 6 5 1
    7 2 0 8 6 1
    1 7 0 0.0
  1 0 0.639 0.902 36.0 .3-4 1.2 1.0
  2 0 0.515 0.810 36.0 .3-4 1.3 1.0
  3 2 1 400 2 20
  1 0 78.7 10.02 52.00 .2-3 0.0 0.0
  2 -1 100.0 1.0 0.0 0.0 5.45 0.0
  4 -1 100.0 1.0 0.0 0.0 5.0 10.0
  5 0 .863 1.049 40.0 .2-4 2.0 0.0
  6 0 78.7 10.02 10.0 .2-3 0.0 0.0
  7 -1 1.0 1.0 0.0 0.0 0.0 0.0
  8 -1 1.0 1.0 0.0 0.0 0.0 0.0
5005.0 10.0 10.0 10.0 5.0 5.0 5000.0
5000.0
200 600
  3 1.05
2 GO TO QUIT

```

Output

FLOPSY SAMPLE PROBLEM WITH PUMP RUN 10 4/13/64

NINTN	NEXTN	NINTB	KTUT	NUMDUM	NUMTYP	NUMPT	ITOT	LAST	MTX	MESS	JUMP1
5	1	6	8	1	2	2	500	1	1	1	1

VISC.	RHO	GAMMA	FLOWE	IPVAR
0.3020-004	0.3590-001	0.5000-003	0.1000-001	5

BRANCH	INLET NODE	OUTLET NODE	BRANCH	INLET NODE	OUTLET NODE	BRANCH	INLET NODE	OUTLET NODE
1	1	2	2	2	3	3	3	4
4	4	5	5	5	2	6	5	1
7	2	0	8	6	1			

THE SYMMETRIC MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	3.0000	1 2	-1.0000	1 3	0.	1 4	0.
1 2	-1.0000	1 6	-1.0000	2 2	4.0000	2 3	-1.0000
2 4	0.	2 5	-1.0000	2 6	0.	3 3	2.0000
3 4	-1.0000	3 5	0.	3 6	0.	4 4	2.0000
4 2	-1.0000	4 6	0.	5 5	3.0000	5 6	0.
6 6	1.0000						

THE INVERSE OF THE MATRIX B

LOCATION	TERM	LOCATION	TERM	LOCATION	TERM	LOCATION	TERM
1 1	1.6364	2 1	1.0000	3 1	1.0909	4 1	1.1818
2 1	1.2727	2 2	1.6364	3 2	1.0000	4 2	1.0000
4 2	1.0000	5 2	1.0000	6 2	1.0000	3 3	1.7273
4 3	1.4545	5 3	1.1818	6 3	1.0909	4 4	1.9091
5 4	1.3636	6 4	1.1818	5 5	1.5455	6 5	1.2727
6 6	2.0364						

EXT. NODE	INLET BRANCH	OUTLET BRANCH	EXTERNAL PRESSURE	EXT. NODE	INLET BRANCH	OUTLET BRANCH	EXTERNAL PRESSURE
1	7	0	0.				

BRANCH TYPE OF BR	BRANCH AREA	HYDRAULIC DIAMETER	BRANCH LENGTH	ARITHMETIC ROUGHNESS	BRANCH LOSS COEF.	ESTIMATED FLOW
1 0	0.6390	0.9020	36.0000	0.3000-004	1.2000	1.0000
2 0	0.5150	0.8100	36.0000	0.3000-004	1.3000	1.0000

DUMMY NO.	OF TYPE	NO. OF TYPE	NO. OF TYPE	NO. OF TYPE	NO. OF TYPE	NO. OF TYPE	NO. OF TYPE
BRANCH	TYPES	BRANCHES	BRANCHES	BRANCHES	BRANCHES	BRANCHES	BRANCHES
3	2	1	400	2	20		

OUTPUT PRINTED AT THE FOLLOWING ITERATIONS

200 600

BRANCH	R. MULT.	BRANCH	R. MULT.	BRANCH	R. MULT.	BRANCH	R. MULT.	BRANCH	R. MULT.
1	1.000	2	1.000	3	1.050	4	1.000	5	1.000
6	1.000	7	1.000	8	1.000				

BRANCH TYPE	BRANCH	HYDRAULIC	BRANCH	ARITHMETIC	BRANCH	HEAD
NO. OF FR	AREA	DIAMETER	LENGTH	ROUGHNESS	LOSS COEF.	RISE
1	0	78.7000	10.0200	52.0000	0.2000-003	0.
2	-1	100.0000	1.0000	0.	0.	5.4500
4	-1	100.0000	1.0000	0.	0.	5.0000
5	0	0.8630	1.0490	40.0000	0.2000-004	2.0000
6	0	78.7000	10.0200	10.0000	0.2000-003	0.
7	-1	1.0000	1.0000	0.	0.	0.
8	-1	1.0000	1.0000	0.	0.	0.
FLOW	FLOW	FLOW	FLOW	FLOW	FLOW	FLOW
5005.0000	10.0000	10.0000	10.0000	5.0000	5.0000	5000.0000
5000.0000						

THE WEIGHT FLOWS HAVE CONVERGED

ITERATION NO. 167

ALPHAS 0.5014+000 0.6162+000 0.9784+000 0.2373+001 0.1999+002

BRANCH	RESISTANCE	HEAD RISE	WEIGHT FLOW	DELTAP	REL. ROUGHNESS	REYNOLDS NO.
1	0.30888+000	0.	5197.8852	8.345232	0.19960+004	0.17324+008
2	0.17663+004	0.	206.0537	0.834764	0.	0.53941+005
3	0.11535+005	0.	206.0537	0.046972	0.	0.
4	0.18039+004	10.0000	206.0537	-9.234162	0.	0.53941+005
5	0.22513+000	0.	8.1684	8.349116	0.19056+004	0.25992+006
6	0.76787+007	0.	197.8852	0.003407	0.19960+004	0.65954+006
7	0.	0.	5000.0000	0.	0.	0.13089+009
8	0.	0.	5000.0000	0.	0.	0.13089+009

FR. FACTOR	DIFFQ
0.010218	0.000438
0.	0.000437
0.	0.000437
0.	0.000437
0.015292	=0.000001
0.013209	0.000438
0.	0.000000
0.	0.000000

DUMMY BRANCH NO. 3

SUB-BRANCH	RESISTANCE	WEIGHT FLOW	REL. ROUGHNESS	REYNOLDS NO.	FR. FACTOR
1	0.19889+000	0.4962	0.33259+004	0.18336+005	0.026332
2	0.34286+000	0.3779	0.37037+004	0.15561+005	0.027461

THE PRESSURE AT EACH INTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE
1	8.345670	2	0.000000	3	-0.835201	4	-0.884610
5	8.347115	6	8.345670				

THE PRESSURE AT EACH EXTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE
1	0.						

END OF PROBLEM

FLOPSY SAMPLE PROBLEM WITH PUMP RUN 10 4/13/64

THE SOLUTION FOR THE PROBLEM USING R. MULT.

THE WEIGHT FLOWS HAVE CONVERGED

ITERATION NO. 3

ALPHA: 0.2015+000 0.6162+000 0.9784+000 0.2373+001 0.1999+002

BRANCH	RESISTANCE	HEAD RISE	WEIGHT FLOW	DELTAP	REL. ROUGHNESS	REYNOLDS NO.
1	0.30888-006	0.	5197.8928	8.345292	0.19960-004	0.17324+008
2	0.19663-004	0.	206.0609	0.834913	0.	0.53943+005
3	0.12112-005	0.	206.0609	0.051429	0.	0.
4	0.10039-004	10.0000	206.0609	9.234025	0.	0.53943+005
5	0.12513+000	0.	8.1680	8.346039	0.19066-004	0.25991+006
6	0.76786-007	0.	197.8929	0.003007	0.19960-004	0.65957+006
7	0.	0.	5000.0000	0.	0.	0.13089+009
8	0.	0.	5000.0000	0.	0.	0.13089+009

FR. FACTOR	DIFFO
0.010219	=0.000127
0.	=0.000121
0.	=0.000121
0.	=0.000121
0.015292	0.000006
0.013209	=0.000127
0.	0.000000
0.	=0.000000

DUMMY BRANCH NO. 3

SUB-BRANCH	RESISTANCE	WEIGHT FLOW	REL. ROUGHNESS	REYNOLDS NO.	FR. FACTOR
1	0.20883+000	0.4963	0.33259-004	0.18338+005	0.026331
2	0.35999+000	0.3780	0.37037-004	0.15563+005	0.027460

THE PRESSURE AT EACH INTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE
1	8.345165	2	0.000000	3	-0.834792	4	-0.856100
5	8.346045	6	8.345165				

THE PRESSURE AT EACH EXTERNAL NODE

NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE	NODE	PRESSURE
1	0.						

END OF PROBLEM

REFERENCES

1. Phillips, M. (Miss). KAPL-M-D1G-TD-11. "FLOPSY-I". April 20, 1960.
2. Moody, L. F. "Friction Factors for Pipe Flow." Trans ASME 66. 1944.
Pp. 671-684.

NOMENCLATURE

<u>This Report</u>	<u>FORTRAN</u>	<u>Description</u>
[A]		A(K x N) matrix showing internal node connections
a	AREA ARIND	Area of the branch (regular and parallel)
[B]	B	An N x N symmetric matrix called the "connection" matrix ($= [A^T] \cdot [A]$)
[B ⁻¹]	BINV	Inverse of the B matrix
[C]	C	An N x N matrix used in matrix inversion
[C ⁻¹]	CINV	Inverse of the C matrix
D	DIAM DIMIND	Hydraulic diameter of the branch (regular and parallel)
	DIV DIVIND	Term $\frac{D}{a}$ for regular and parallel branches
[D]	D	An N x N matrix used in matrix inversion
[$\hat{D}$]		A K x M matrix showing external node connections
[D ⁻¹]	DINV	Inverse of the D matrix
DIFF	DIFFO	[P _{in} - P _{out} + ΔP] value for each branch
	DPEXT	External node pressure effect on branches
	FLOWE	Value for allowable error in calculating flows at any node
f	FR FRIND	Friction factor of the branch (regular and parallel)
H	H	Head rise in the branch
i	ITNUM	Iteration number
I	ITPT	Printout iterations
I'	ITOT	Total number of iterations
	IBNCH	Inlet branch to an external node
	IFR IFRIND	Indicator for calculation of friction factor (regular and parallel)
	INODE	Inlet node to a branch
	IPC	Indicator for use of resistance multipliers

<u>This</u> <u>Report</u>	<u>FORTRAN</u>	<u>Description</u>
	IPVAR	Maximum number of α 's to be used
	ITERCT	Indicator used in determining true convergence
	JBNCH	Outlet branch to an external node
	JNODE	Outlet node to a branch
k	KBNCH	Branch number
K	KTOT	Total number of branches
K_T	CONK CONIND	Loss coefficient of branch (regular and parallel)
	KDUM	Dummy branch number
	KTYPE	Parallel branch type number
K'	NINTB	Total number of internal branches
L	BLN BLIND	Length of branch (regular and parallel)
L	NUMTYP	Total number of parallel branches
	LMAX	Number of parallel types in a specified dummy branch
m	KNODE	External node number
M	NEXTN	Total number of external nodes
n	N	Internal node number
N	NINTN	Total number of internal nodes
	NPAR	Parallel branch type number and
	NUMBAN	Number of such branches in the specified dummy branch
	NUMPT	Total number of printouts requested
p	NUMDUM	Total number of dummy branches
P	PINT	Pressure at internal node
$\hat{P}$	PEXT	Pressure at external node
P_{in}		Pressure at the inlet node of a branch
P_{out}		Pressure at the outlet node of a branch
[P]		An N x 1 matrix of pressures at internal nodes
$[\hat{P}]$		An N x 1 matrix of pressures at external nodes

This Report	FORTTRAN	Description
	PROD	Sum of the pressure drops of all branches connected to a specified node (use in calculating pressure at each node)
R	R	Resistance of regular and dummy branches
	RIND	Resistance of parallel branch type of dummy branches
Re	RE REYIND	Reynolds number of the branch (regular and parallel)
	RMAX	Maximum value of $R W $ } used in calculating α Minimum value of $R W $ }
	RMIN	
	RMULT	Resistance multipliers of regular and dummy branches
	SUM	Pressure drop and external node pressure effect on branches
w	W	Weight flow in regular and dummy branches
	WIND	Weight flow in parallel-branch type of dummy branch
	WTEMP	Initial guess of flow in a parallel-branch type
	XCON	Term $\frac{L/D}{2g a^2}$ for regular and parallel branches
	XIND	
	YCON	Term $\frac{k_T}{2g a^2}$ for regular and parallel branches
	YIND	
x	X	Reynold's number table for calculating friction factor
y	Y	$2 \epsilon/D$ table for calculating friction factor
z	Z	Friction factor associated with x and y
ΔP	DELTAP	Pressure drop in a branch
α	ALPHA	Multiplying factor on ΔP to calculate ΔW
γ	GAMMA	Convergence factor on $[P_{in} - P_{out} - \Delta P]$
ϵ	ROUGH RFIND	Arithmetic roughness of a branch (regular and parallel)
μ	VISC	Average fluid viscosity
ρ	RHO	Average fluid density

APPENDIX A. CALCULATIONAL METHODS

Mrs. M. Helme

DERIVATION OF EQUATIONS

As previously explained, the basic equations used in FLOPSY are:

$$\Sigma W = 0 \quad (1)$$

at any node.

$$\Delta P = RW|W| - H \quad (2)$$

in every branch.

$$\Delta P = P_{in} - P_{out} \quad (3)$$

in every branch.

$$\Delta W_k^{(i)} = \alpha \left[P_{in_k}^{(i)} - P_{out_k}^{(i)} - R_k W_k^{(i-1)} |W_k^{(i-1)}| + H_k^{(i-1)} \right] \quad (4)$$

$$W_k^{(i)} = W_k^{(i-1)} + \Delta W_k^{(i)} \quad (5)$$

$$W_k^{(n)} = W_k^{(o)} + \sum_{i=1}^n \Delta W_k^{(i)} \quad (6)$$

It remains necessary to do two things:

1. To find an equation for the pressures.
2. To show that $\Sigma \Delta W = 0$ so that the final weight flows will satisfy Equation 1.

To do this we need some matrices. In matrix form, Equation 3 becomes:

$$[A] \cdot [P] + [\hat{D}] \cdot [\hat{P}] = [\Delta P] \quad (3')$$

where $[A]$ and $[\hat{D}]$ are rectangular, nonsquare matrices showing nodal connections, where every term is +1, -1, or 0, and where $[\Delta P]$, $[P]$ and $[\hat{P}]$ are vectors of pressure drop, internal pressure, and external pressure, respectively.*

*See "Nomenclature" for definition of "internal" and "external."

The equations this notation represents might look something like this:

$$P_1 - P_5 = \Delta P_1$$

$$P_2 - P_4 = \Delta P_2$$

$$P_3 - \hat{P}_1 = \Delta P_3$$

⋮

In matrix form, Equation 1 becomes

$$[A^T][W] = [0] ,$$

where $[A^T]$ is the transpose of $[A]$, i.e., if $a_{i,j}^T$ is a term in the i th row and j th column of $[A^T]$, and if $a_{j,i}$ is a term in the j th row and i th column of $[A]$, then $a_{i,j}^T \equiv a_{j,i}$ for all i and j .

The equations this notation represents might look something like this:

$$W_1 - W_3 + W_4 - W_6 = 0$$

$$W_2 - W_4 + W_7 = 0$$

$$-W_4 + W_5 = 0 .$$

We now wish to find an equation for $[P]$. Rearranging Equation (3') we get:

$$[A][P] = [\Delta P] - [\hat{D}][\hat{P}] ,$$

where the $\hat{P}$'s are known and the ΔP 's can be calculated by means of Equation (2). Multiplying both sides by $[A^T]$ we get:

$$[A^T] \cdot [A] \cdot [P] = [A^T] \{ [\Delta P] - [\hat{D}][\hat{P}] \} .$$

Let $[A^T][A] = [B]$. Since $[B]$ is a product of a matrix and its transpose, it is square and can therefore be inverted. (It is also symmetric.)

$$\begin{aligned} [B] \cdot [P] &= [A^T] \{ [\Delta P] - [\hat{D}][\hat{P}] \} \\ [P] &= [B^{-1}][A^T] \{ [\Delta P] - [\hat{D}][\hat{P}] \} , \end{aligned} \quad (7)$$

where $[B^{-1}]$ is the inverse of $[B]$.

Now, putting Equation (4) into matrix form we get

$$[\Delta W] = \alpha \{ [A][P] + [\hat{D}][\hat{P}] - [\Delta P] \} . \quad (4')$$

Substituting for $[P]$ and rearranging we get

$$[\Delta W] + \alpha \left\{ [A][B^{-1}][A^T] - [I] \right\} \left\{ [\Delta P] - [\hat{D}][\hat{P}] \right\}, \quad (8)$$

where $[I]$ is the identity matrix, i.e., all terms on the main diagonal are +1 and all other terms are 0.

It now remains necessary to show that

$$[A^T][\Delta W] = [O].$$

Thus, it will be sufficient to show that the following relation holds:

$$[A^T] \left\{ [A] \cdot [B^{-1}] \cdot [A^T] - [I] \right\} \left\{ [\Delta P] - [\hat{D}] \cdot [\hat{P}] \right\} = [O]. \quad (9)$$

Now, considering the first two terms of this product,

$$[A^T][A][B^{-1}][A^T] - [A^T][I] = [B][B^{-1}][A^T] - [A^T][I].$$

Now, since $[B] \cdot [B^{-1}] = [I]$ for any matrix, then the above expression in Equation 9 becomes

$$\begin{aligned} & \left\{ [I][A^T] - [A^T][I] \right\} \left\{ [\Delta P] - [\hat{D}][\hat{P}] \right\} \\ & [O] \cdot \left\{ [\Delta P] - [\hat{D}][\hat{P}] \right\} = [O]. \quad \text{Q.E.D.} \end{aligned}$$

MATRIX INVERSION

For each problem in FLOPSY, the matrix $[B]$ is inverted once. Since $[B]$ is a product of a matrix and its transpose, it is symmetric. This means that $b_{i,j} = b_{j,i}$.

To invert an $n \times n$ matrix, storage locations for two such matrices are usually needed. By the laws of matrix algebra, if the matrix $[B]$ is reduced to the identity matrix $[I]$ by a series of elementary row operations, then, by performing the same series of operations on $[I]$, $[I]$ is converted to $[B^{-1}]$, where $[B^{-1}]$ is the inverse of $[B]$ such that $[B][B^{-1}] = [B^{-1}][B] = [I]$. To do this in FLOPSY for a problem of 100 nodes would require $2(100^2) = 20,000$ storage locations. Thus, the problem is how to make use of the symmetry in $[B]$ and perhaps limit it to half that number.

At first it seemed possible to carry out the inversion process using only half the terms, carrying the terms of $[B]$ in the lower triangle and the terms of $[I]$ in the upper triangle of the same square. But, in order to invert a matrix (even a symmetric one) it is necessary to maintain the entire square during the inversion process because symmetry is not maintained during that process, although the final result $[B^{-1}]$ is, of course, symmetric.

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Thus, the procedure which follows was developed. It can be shown that a symmetric matrix [B] with terms b_{ij} can be set equal to a product of three special matrices:

$$[B] = [C_L] \times [D] \times [C_U] ,$$

where $[C_L]$ is a lower triangular matrix with all the terms on the main diagonal equal to 1, where $[D]$ is a diagonal matrix, and where $[C_U]$ is the transpose of $[C_L]$.

An example in a 4 x 4 matrix is as follows:

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{12} & b_{22} & b_{23} & b_{24} \\ b_{13} & b_{23} & b_{33} & b_{34} \\ b_{14} & b_{24} & b_{34} & b_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ c_{21} & 1 & 0 & 0 \\ c_{31} & c_{32} & 1 & 0 \\ c_{41} & c_{42} & c_{43} & 1 \end{bmatrix} \times \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix} \times \begin{bmatrix} 1 & c_{21} & c_{31} & c_{41} \\ 0 & 1 & c_{32} & c_{42} \\ 0 & 0 & 1 & c_{43} \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} d_1 & 0 & 0 & 0 \\ c_{21}d_1 & d_2 & 0 & 0 \\ c_{31}d_1 & c_{32}d_2 & d_3 & 0 \\ c_{41}d_1 & c_{42}d_2 & c_{43}d_3 & d_4 \end{bmatrix} \times \begin{bmatrix} 1 & c_{21} & c_{31} & c_{41} \\ 0 & 1 & c_{32} & c_{42} \\ 0 & 0 & 1 & c_{43} \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} d_1 & c_{21}d_1 & c_{31}d_1 & c_{41}d_1 \\ c_{21}d_1 & (c_{21})^2d_1 + d_2 & c_{21}c_{31}d_1 + c_{32}d_2 & c_{21}c_{41}d_1 + c_{42}d_2 \\ c_{31}d_1 & c_{21}c_{31}d_1 + c_{32}d_2 & (c_{31})^2d_1 + (c_{32})^2d_2 + d_3 & c_{31}c_{41}d_1 + c_{32}c_{42}d_2 + c_{43}d_3 \\ c_{41}d_1 & c_{21}c_{41}d_1 + c_{42}d_2 & c_{31}c_{41}d_1 + c_{32}c_{42}d_2 + c_{43}d_3 & (c_{41})^2d_1 + (c_{42})^2d_2 + (c_{43})^2d_3 + d_4 \end{bmatrix}$$

Thus,

$$b_{11} = d_1$$

$$b_{12} = b_{21} = c_{21}d_1$$

$$b_{13} = b_{31} = c_{31}d_1$$

$$b_{14} = b_{41} = c_{41}d_1$$

$$b_{22} = (c_{21})^2d_1 + d_2$$

$$b_{23} = b_{32} = c_{21}c_{31}d_1 + c_{32}d_2$$

$$b_{24} = b_{42} = c_{21}c_{41}d_1 + c_{42}d_2$$

$$b_{33} = (c_{31})^2d_1 + (c_{32})^2d_2 + d_3$$

$$b_{34} = c_{31}c_{41}d_1 + c_{32}c_{42}d_2 + c_{43}d_3$$

$$b_{44} = (c_{41})^2d_1 + (c_{42})^2d_2 + (c_{43})^2d_3 + d_4$$

We wish to get the c's and d's in terms of the b's and calculated values of c and d.

$$d_1 = b_{11}$$

$$c_{j,1} = (b_{1,j})/(d_1), \quad j = 2, 3, 4$$

$$d_2 = b_{22} - (c_{21})^2d_1$$

$$c_{j,2} = (b_{2,j} - c_{21}c_{j,1}d_1)/(d_2), \quad j = 3, 4$$

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$$d_3 = b_{33} - (c_{31})^2 d_1 - (c_{32})^2 d_2$$

$$c_{43} = (b_{34} - c_{31} c_{41} d_1 - c_{32} c_{42} d_2) / (d_3)$$

$$d_4 = d_{44} - (c_{41})^2 d_1 - (c_{42})^2 d_2 - (c_{43})^2 d_3$$

To generalize, for an nxn matrix:

$$d_i = b_{i,i} - \sum_{k=1}^{i-1} (c_{i,k})^2 d_k$$

$$c_{j,i} = \left[b_{i,j} - \sum_{k=1}^{i-1} (c_{i,k} c_{j,k} d_k) \right] / (d_i)$$

The code handles it this way:

1. Calculate: $b_{i,j}$ for $i = 1, \dots, n$ and $j = i, \dots, n$.

Thus $1/2(n^2) + n$ storage locations are needed for [B].

2. Calculate d_i for $i = 1, \dots, n$ and $j = i + 1, \dots, n$
 $c_{j,i}$

Thus $1/2(n^2) - n$ storage locations are needed for [C] and n storage locations for [D]. The total number of storage locations needed is, therefore, $n^2 + n$, which, for $n = 100$ is 10,100, which is certainly an improvement over 20,000.

To invert [B]: since $[B] = [C_L] \times [D] \times [C_U]$, then $[B^{-1}] = [C_U^{-1}] \times [D^{-1}] \times [C_L^{-1}]$ by the laws of matrix algebra. The term $[C_L]$ will contain the same terms as $[C_U]$. It will therefore be sufficient to invert $[C_L]$ only.

To invert $[C_L]$ we will perform a series of elementary row operations to reduce it to the identity matrix [I]. By performing the same series of operations on [I] we will get $[C_L^{-1}]$.

	$[C_L]$	$[I]$	
Row 1	$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$	Row 1
Row 2	$\begin{bmatrix} c_{21} & 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$	Row 2
Row 3	$\begin{bmatrix} c_{31} & c_{32} & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$	Row 3
Row 4	$\begin{bmatrix} c_{41} & c_{42} & c_{43} & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$	Row 4

We will not go through the tedious process of inversion here, but the results of it are:

$$c_{12}^{-1} = -c_{21}$$

$$c_{13}^{-1} = -c_{31} - c_{32} \cdot c_{12}^{-1}$$

$$c_{14}^{-1} = -c_{41} - c_{42} \cdot c_{12}^{-1} - c_{43} \cdot c_{13}^{-1}$$

$$c_{23}^{-1} = -c_{32}$$

$$c_{24}^{-1} = -c_{42} - c_{43} \cdot c_{23}^{-1}$$

$$c_{34}^{-1} = -c_{43}$$

To generalize,

$$c_{i,j}^{-1} = -c_{j,i} - \sum_{k=i+1}^{j-1} (c_{j,k} \cdot c_{i,k}^{-1}) \text{ for } i = 1, \dots, n-1 \text{ and } j = i+1, \dots, n.$$

Since the upper triangle of $[C_L]$ is all zeros, it is not necessary to preserve those terms or the terms in the upper triangle of $[C_L^{-1}]$. It is also unnecessary to preserve the terms of the main diagonals since they are all equal to one. Therefore, we need only $1/2(n^2) - n$ for $[C_L]$ and the same amount for $[C_L^{-1}]$. The terms of $[C_L^{-1}]$ will replace those of $[B]$ so that we still are not exceeding 10,000 storage locations.

To invert $[D]$:

$$[D] = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$$

It can be shown that:

$$[D^{-1}] = \begin{bmatrix} 1/d_1 & 0 & 0 & 0 \\ 0 & 1/d_2 & 0 & 0 \\ 0 & 0 & 1/d_3 & 0 \\ 0 & 0 & 0 & 1/d_4 \end{bmatrix}$$

To generalize, $d_i^{-1} = 1/d_i$ for $i = 1, \dots$, where the terms of $[D^{-1}]$ replace those of $[D_i]$ to save storage space.

Now, since $[B^{-1}] = [C_U^{-1}] \times [D^{-1}] \times [C_L^{-1}]$, then, $[B^{-1}] =$

$$\begin{bmatrix} 1 & c_{12}^{-1} & c_{13}^{-1} & c_{14}^{-1} \\ 0 & 1 & c_{23}^{-1} & c_{24}^{-1} \\ 0 & 0 & 1 & c_{34}^{-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} d_1^{-1} & 0 & 0 & 0 \\ 0 & d_2^{-1} & 0 & 0 \\ 0 & 0 & d_3^{-1} & 0 \\ 0 & 0 & 0 & d_4^{-1} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ c_{12}^{-1} & 1 & 0 & 0 \\ c_{13}^{-1} & c_{23}^{-1} & 1 & 0 \\ c_{14}^{-1} & c_{24}^{-1} & c_{34}^{-1} & 1 \end{bmatrix} =$$

$$\begin{bmatrix} d_1^{-1} & c_{12}^{-1} d_2^{-1} & c_{13}^{-1} d_3^{-1} & c_{14}^{-1} d_4^{-1} \\ 0 & d_2^{-1} & c_{23}^{-1} d_3^{-1} & c_{24}^{-1} d_4^{-1} \\ 0 & 0 & d_3^{-1} & c_{34}^{-1} d_4^{-1} \\ 0 & 0 & 0 & d_4^{-1} \end{bmatrix} \times$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ c_{12}^{-1} & 1 & 0 & 0 \\ c_{13}^{-1} & c_{23}^{-1} & 1 & 0 \\ c_{14}^{-1} & c_{24}^{-1} & c_{34}^{-1} & 1 \end{bmatrix}$$

Therefore,

$$b_{11}^{-1} = d_1^{-1} + \sum_{k=2}^4 [(c_{1,k}^{-1})^2 d_k^{-1}]$$

$$b_{21}^{-1} = c_{12}^{-1} d_2^{-1} + \sum_{k=3}^4 (c_{1,k}^{-1} c_{2,k}^{-1} d_k^{-1})$$

$$b_{31}^{-1} = c_{13}^{-1} d_3^{-1} + c_{14}^{-1} c_{34}^{-1} d_4^{-1}$$

$$b_{41}^{-1} = c_{14}^{-1} d_4^{-1}$$

$$b_{22}^{-1} = d_2^{-1} + \sum_{k=3}^4 [(c_{2,k}^{-1})^2 d_k^{-1}]$$

$$b_{32}^{-1} = c_{23}^{-1} d_3^{-1} + c_{24}^{-1} c_{34}^{-1} d_4^{-1}$$

$$b_{42}^{-1} = c_{24}^{-1} d_4^{-1}$$

$$b_{33}^{-1} = d_3^{-1} + (c_{34}^{-1})^2 d_4^{-1}$$

$$b_{43}^{-1} = c_{34}^{-1} d_4^{-1}$$

$$b_{44}^{-1} = d_4^{-1}$$

To generalize,

$$\left. \begin{aligned} b_{i,i}^{-1} &= d_i^{-1} + \sum_{k=i+1}^n [(c_{i,k}^{-1})^2 d_k^{-1}] \\ b_{i,j}^{-1} &= c_{j,i}^{-1} d_i^{-1} + \sum_{k=i+1}^n (c_{j,k}^{-1} c_{i,k}^{-1} d_k^{-1}) \end{aligned} \right\} \begin{aligned} &\text{for } i = 1, \dots, n \\ &j = i+1, \dots, n \end{aligned}$$

The terms of $[B^{-1}]$ replace the terms of $[C]$ to save storage space. Therefore, no more than 10,100 storage locations are needed for the terms involved in inverting a 100 x 100 symmetric matrix.

To summarize how the code handles this situation, calculate:

$$1. \quad b_{i,j} \quad \text{for } \begin{cases} i = 1, \dots, n \\ j = i, \dots, n \end{cases}$$

$$\begin{array}{l}
 2. \quad d_i = b_{i,i} - \sum_{k=1}^{i-1} [(c_{i,k})^2 d_k] \\
 \quad c_{j,i} = \left[b_{i,j} - \sum_{k=1}^{i-1} [c_{i,k} c_{j,k} d_k] \right] / d_i
 \end{array}
 \left. \vphantom{\begin{array}{l} d_i \\ c_{j,i} \end{array}} \right\} \begin{array}{l} \text{for } i = 1, \dots, n \\ j = i + 1, \dots, n \end{array}$$

$$\begin{array}{l}
 3. \quad c_{i,j}^{-1} = c_{j,i} - \sum_{k=i+1}^{j-1} (c_{j,k} c_{i,k}^{-1}) \\
 \quad d_i^{-1} = (1)/(d_i)
 \end{array}
 \left. \vphantom{\begin{array}{l} c_{i,j}^{-1} \\ d_i^{-1} \end{array}} \right\} \begin{array}{l} \text{for } i = 1, \dots, n \\ j = i + 1, \dots, n \end{array}$$

$$\begin{array}{l}
 4. \quad b_{i,i}^{-1} = d_i^{-1} + \sum_{k=i+1}^n [(c_{i,k}^{-1})^2 d_k^{-1}] \\
 \quad b_{j,i}^{-1} = c_{i,j}^{-1} d_j^{-1} + \sum_{k=j+1}^n (c_{j,k}^{-1} c_{i,k}^{-1} d_k^{-1})
 \end{array}
 \left. \vphantom{\begin{array}{l} b_{i,i}^{-1} \\ b_{j,i}^{-1} \end{array}} \right\} \begin{array}{l} \text{for } i = 1, \dots, n \\ j = i + 1, \dots, n \end{array}$$

THE CASE OF THE SINGULAR MATRIX

If a square matrix cannot be inverted it is said to be a singular matrix by the laws of matrix algebra. Occasionally in FLOPSY the matrix [B] turns out to be singular, usually through false input. The result is that the machine tries to divide by zero and the program "blows up." Thus, there arose the need for putting a check into the program to test [B] for singularity.

We shall not go through the derivation here of the laws of matrices which prove the following statement, but it can be shown that if [B] is singular and equal to the above defined product $[C_L] \times [D] \times [C_U]$, then at least one of the diagonal terms d_i will be equal to zero. Therefore, FLOPSY in calculating $[C_L]$ and $[D]$ checks to make sure that none of the terms d_i are zero. If they are, the program writes out this information and stops.

Usual Causes of Singularity

1. Not enough "external nodes"
2. Not enough "external branches"
3. Some combination of (1) and (2)
4. Too many external nodes
5. Too many external branches
6. Some combinations of (4) and (5).