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SYNCHROTRON RADIATION WITH LINE SPECTRA

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STORAGE RING FOR OBTAINING SYNCHROTRON RADIATION WITH LINE SPECTRA

ABSTRACT

It appears possible to design a very small electron storage ring in which line spectra in the optical synchrotron radiation could be obtained. These lines would be related to harmonics of the orbital frequency. This device could be used to measure the velocity of light to very high accuracy. Also the lines would be very coherent light sources for experiments on photon statistics.

The effects of betatron and synchrotron oscillations, due to gas scattering and quantum effects, on the spectrum of optical synchrotron radiation are analyzed, and the results applied to a possible design of the ring.

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The possibility of observing line spectra of optical synchrotron radiation, at the harmonics of the orbital frequency, has occurred to people previously. It is impossible to do this with any existing electron accelerators or storage rings, however it appears feasible to design a tiny electron storage ring for this purpose. If line spectra can be observed, then it should be possible to measure the velocity of light to much higher accuracy than has been done at present.

This would be done by comparing the synchrotron radiation line with a spectroscopic line whose wavelength has been measured very accurately, and adjusting the radiofrequency until they are equal. This would be a direct measurement of the line frequency and hence of the velocity of light. The synchrotron lines also would be the most coherent source of optical radiation and could be used for experiments on photon statistics.

If the electron trajectories were uniform circular motion, then the radiation spectrum would be sharp lines at harmonics of the orbital frequency. The actual motion consists of radial and vertical betatron and synchrotron oscillations about this ideal motion. The amplitude will vary with time due to the quantum nature of synchrotron radiation and due to gas scattering. The only effect of the oscillations which need be considered is the longitudinal displacement produced by these oscillations.

For purposes of analysis it may be assumed that at the point of observation the electron produces an infinitely sharp pulse of electromagnetic radiation, the time of which is determined by the longitudinal displacement of the electron.

The radiation field may then be written in the form

$$E(t) \propto 1 + 2 \sum_{n=1}^{\infty} \cos n \left[\omega_0 t + A_r \sin(\omega_r t + \phi_r) + A_s \sin(\omega_s t + \phi_s) \right]$$

The vertical betatron oscillations are in the same frequency range as the radial betatron oscillations but have a much smaller effect on the longitudinal displacement and are neglected. This may be expanded to give

$$\begin{aligned} E(t) &\propto \sum_{n=1}^{\infty} \exp(in\omega_0 t) \exp[inA_r \sin(\omega_r t + \phi_r)] \exp[inA_s \sin(\omega_s t + \phi_s)] + \text{C.C.} \\ &= \sum_{n=1}^{\infty} e^{in\omega_0 t} \left\{ J_0(nA_r) + \sum_{p=1}^{\infty} J_p(nA_r) \left[e^{ip(\omega_r t + \phi_r)} + (-1)^p e^{-ip(\omega_r t + \phi_r)} \right] \right\} \times \dots \\ &\quad \times \left\{ J_0(nA_s) + \sum_{m=1}^{\infty} J_m(nA_s) \left[e^{im(\omega_s t + \phi_s)} + (-1)^m e^{-im(\omega_s t + \phi_s)} \right] \right\} + \text{C.C.} \end{aligned}$$

Then

$$\begin{aligned} E(t) &\propto \sum_{n=1}^{\infty} \left\{ J_0(nA_r) J_0(nA_s) \cos n\omega_0 t + \dots \right. \\ &\quad \left. + \sum_{m=1}^{\infty} J_0(nA_r) J_m(nA_s) \left[\cos(n\omega_0 t + m\omega_s t + m\phi_s) + (-1)^m \cos(n\omega_0 t - m\omega_s t - m\phi_s) \right] + \dots \right\} \end{aligned}$$

$$\begin{aligned}
 & \dots + \sum_{p=1}^{\infty} J_0(nA_s) J_p(nA_r) \left[\cos(n\omega_o t + p\omega_r t + p\phi_r) + (-1)^p \cos(n\omega_o t - p\omega_r t - p\phi_r) \right] + \\
 & \dots + \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} J_m(nA_s) J_p(nA_r) \left[\cos(n\omega_o t + m\omega_s t + m\phi_s + p\omega_r t + p\phi_r) + \dots \right. \\
 & \dots + (-1)^m \cos(n\omega_o t - m\omega_s t - m\phi_s + p\omega_r t + p\phi_r) + (-1)^p \cos(n\omega_o t + m\omega_s t + m\phi_s - p\omega_r t - p\phi_r) + \dots \\
 & \left. \dots + (-1)^m (-1)^p \cos(n\omega_o t - m\omega_s t - m\phi_s - p\omega_r t - p\phi_r) \right] \Big\}
 \end{aligned}$$

This shows the not surprising result that the effect of the betatron and synchrotron oscillations is to split the spectral lines which are multiples of the orbital frequency into a number of separate lines, which are separated by combinations of multiples of the betatron and synchrotron frequencies.

If the amplitudes and phases of the betatron and synchrotron oscillations were constant, these lines would be infinitely sharp, however since the amplitude and phase are varying with time, the lines will have a finite width. The variation with time is due to gas scattering and emission of quanta of synchrotron radiation. Each of these events produces a sudden change in the amplitude and phase of the oscillation which is then damped, and results in a comparable change in the amplitude and phase of each line. Then as a

model to analyse the line shape, we assume the phase of a line is due to a large number of increments with zero rise time and exponential decay τ , of amplitude δ_0 , with equal time spacing ϵ , and random sign. The instantaneous distribution in phase is then

$$W(\phi) \propto \exp \left[-\frac{\phi^2 \epsilon}{2\delta_0^2 \tau} \right] = \exp \left[-\frac{\phi^2}{2\phi_0^2} \right]$$

The r.m.s. deviation ϕ_0 will also be comparable to the r.m.s. phase amplitude due to the betatron and synchrotron oscillations.

The signal of interest will be of the form

$$\exp i (\omega t + \delta_0 \sum_{n=-\infty}^{t/\epsilon} A_n e^{-(t-n\epsilon)/\tau})$$

A_n is of unit amplitude and random sign

ω , is the frequency of the line

The Fourier analysis for the amplitude of frequency $\omega + \omega'$ gives

$$\begin{aligned} & \int_{-\infty}^{\infty} \exp i (\omega' t + \delta_0 \sum_{n=-\infty}^{t/\epsilon} A_n e^{-(t-n\epsilon)/\tau}) \\ &= \sum_{n=-\infty}^{\infty} \int_{n\epsilon}^{(n+1)\epsilon} \exp i (\omega' t + \delta_0 \sum_{n=-\infty}^{t/\epsilon} A_n e^{-(t-n'\epsilon)/\tau}) dt \end{aligned}$$

ϵ will be very small compared with τ and $1/\omega'$, then

$$= \sum_{n=-\infty}^{\infty} \epsilon e^{i\omega' n \epsilon} \exp \left(i \delta_0 \sum_{n'=-\infty}^n A_{n'} e^{-(n-n')\epsilon/\tau} \right)$$

To obtain the power spectrum we multiply by the complex conjugate

$$p(\omega') \propto \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \epsilon^2 e^{i\omega' (n-m) \epsilon} \exp i \delta_0 \left[\sum_{n'=-\infty}^n A_{n'} e^{-(n-n')\epsilon/\tau} - \sum_{m'=-\infty}^m A_{m'} e^{-(m-m')\epsilon/\tau} \right]$$

$$p(\omega') \propto \sum_{m=-\infty}^n \epsilon^2 e^{i\omega' (n-m) \epsilon} \exp i \delta_0 \sum_{n'=m}^n A_{n'} e^{-(n-n')\epsilon/\tau} + \text{C.C.}$$

The distribution function of

$$\psi = \delta_0 \sum_{m=-\infty}^n A_{n'} e^{-(n-n')\epsilon/\tau} \text{ is given by}$$

$$W(\psi) = \exp - \left[\frac{\psi^2}{2\phi_0^2 (1 - e^{-(n-m)2\epsilon/\tau})} \right]$$

Then $x = (n-m)$

$$p(\omega') \propto \int_0^{\infty} \int_{-\infty}^{\infty} e^{i\omega' x \epsilon} \cos \psi \exp - \left[\frac{\psi^2}{2\phi_0^2 (1 - e^{-2x\epsilon/\tau})} \right] d\psi dx + \text{C.C.}$$

$$p(\omega') \propto \int_0^{\infty} e^{i\omega' x \epsilon} (1 - e^{-2x\epsilon/\tau}) \exp - \left[\frac{\phi_0^2}{2} (1 - e^{-\frac{2x\epsilon}{\tau}}) \right] dx + \text{C.C.}$$

For frequency deviations from the center of the line with $\omega' \gg 1/\tau$, the values of interest are $x \epsilon/\tau \ll 1$.

$$p(\omega') \propto \int_0^{\infty} e^{i\omega' x \epsilon} \frac{x \epsilon}{\tau} \exp - \left[\phi_0^2 \frac{x \epsilon}{\tau} \right] dx + \text{C.C.}$$

$$p(\omega') \propto \frac{1}{\left(\frac{\phi_0^2}{\tau} \right)^2 + (\omega')^2}$$

The line shape will then be Lorentzian for $\omega' \gg \frac{1}{\tau}$. This expression will be applicable if $\phi_0^2 \gg 1$.

If $\phi_0^2 \ll 1$, the integral may be approximated.

$$\int_0^{\infty} e^{i\omega' x \epsilon} (1 - e^{-2x \epsilon/\tau}) dx + \text{C.C.}$$

$$p(\omega') \propto \frac{1}{\left(\frac{2}{\tau} \right)^2 + (\omega')^2}$$

Thus if $\phi_0^2 \ll 1$, the line shape will also be Lorentzian with a width comparable to the reciprocal of the damping time.

There will also be an infinitely sharp line at the center of the lines which are harmonics of the orbital frequency due to the fact that the average phase displacement is zero. The fraction of power in the infinitely sharp line may be obtained from a Fourier analysis of the longitudinal position distribution.

$$\left[\frac{2^{\frac{1}{2}}}{\pi^{\frac{1}{2}} \phi_0} \int_0^{\infty} e^{-\phi^2/2\phi_0^2} \cos \phi d\phi \right]^2 = e^{-\phi_0^2}$$

If there are many electrons stored in the ring, the radiation in the infinitely sharp line will be coherent for all electrons. Then if the bunch length can be made smaller than the wavelength of the radiation, most of the power will be in the infinitely sharp lines. The power radiated in the infinitely sharp line in each harmonic will depend on the distribution of electrons around the circumference. If the circumference were uniformly filled, only harmonics of the radiofrequency would contain the sharp lines.

Then the effect of the synchrotron and radial betatron oscillations is to split the orbital harmonic lines into many lines which differ by combinations of multiples of the betatron and synchrotron frequencies. The width of the lines is comparable to ϕ_0^2/τ or $1/\tau$. Also there is an infinitely sharp line at the center of some of the harmonic lines.

The problem is then to design a very small storage ring which will produce some synchrotron radiation in the optical region, and in which the longitudinal motion due to synchrotron and radial betatron oscillations is comparable to the wavelength of light.

If a vertical aperture of 1 mm is assumed, longitudinal variation in magnetic field corresponding to wavelengths of 6 mm may be obtained. To obtain photons of $6000 \text{ \AA}$, $2\gamma^2 = 10^4$, $\gamma = 70$, $u = 35 \text{ mev}$.

The longitudinal motion due to radial betatron oscillations can be reduced by reducing the residual gas pressure, and by increasing the v value of the ring. The r.m.s. longitudinal

motion is given approximately by,

$$\delta_l = \theta_o \frac{R}{v^2}$$

θ_o is the r.m.s. radial scattering angle. If we assume an energy of 30 mev and an average radius of curvature of 4 in., the radiation damping time of the betatron oscillations may be estimated by scaling from 6 Gev electrons in the CEA,

$$\tau_\beta = 2 \times 10^{-3} \times (200)^3 \times \left(\frac{4}{1400}\right)^2 = .13 \text{ sec}$$

The magnet structure will have to be designed so that the radial betatron oscillations are damped by radiation. This can be done in an alternating gradient structure by making the fields higher in the radially defocussing magnets and lower in the radially focussing magnets.

The r.m.s. scattering angle for radial betatron oscillations, at a vacuum of 10^{-9} mm is given by,

$$(\theta_o^2)^{\frac{1}{2}} = \frac{1}{\sqrt{2}} \frac{21}{u_o} \left[\frac{\tau_\beta c p}{2x_o p_o} \right]^{\frac{1}{2}} = \frac{1}{\sqrt{2}} \frac{21}{30} \left[\frac{.13 \times 3 \times 10^8 \times 10^{-9}}{2 \times 300 \times 760} \right] = 1.4 \times 10^{-4}$$

The longitudinal displacement is then given by

$$\delta_l = \theta_o \frac{R}{v^2} = \frac{1.4 \times 10^{-3}}{v^2} \text{ cm}$$

Then to make the longitudinal displacement small, it is necessary to have a large v value. If it is possible to attain a v value

of 7, the phase displacement would be 2.8×10^{-5} cm. At a wavelength of $6000 \text{ \AA}$, this represents a phase of about 2.8 radians. Since the amplitude of a sideband varies as $J_n^2(\phi)$, under these conditions only two or three of the sidebands of the harmonics due to radial betatron oscillations would contain appreciable power.

The quantum effects of the synchrotron radiation cannot be changed much by the design of the ring, however the longitudinal displacement due to the synchrotron oscillations produced by the radiation can be reduced by operating very close to transition energy. From the equations for synchrotron oscillation one obtains the relation between the energy deviation and the longitudinal displacement of a synchrotron oscillation,

$$\delta_L = \frac{\delta u}{u_0} \left(\frac{2\pi R_0^2 u_0 \alpha_e}{kv} \right)^{\frac{1}{2}}$$

α_e is the effective compaction factor $\alpha_e = \frac{u_0}{\omega_0} \frac{d\omega_0}{du_0}$

By scaling from the CEA, $\frac{\delta u}{u_0} = 1.3 \times 10^{-4}$

Taking $v = 10^4$, $k = 50$, to obtain $\delta_L = 10^{-5}$ cm requires

$\alpha_e = 1.5 \times 10^{-7}$. Under these conditions the r.m.s. phase deviation is about one radian at $6000 \text{ \AA}$, then most of the power will be in the fundamental frequency, with a fraction in the nearest sidebands separated by harmonics of the synchrotron frequency. Under these conditions the synchrotron oscillation frequency will be quite low. The number of orbital revolutions in one period of the synchrotron oscillation is

given by,

$$\frac{f_o}{f_s} = \left[\frac{2\pi u_o}{\alpha_e k v} \right]^{\frac{1}{2}} = 5.1 \times 10^4$$

In order to operate very close to transition energy it is necessary to design a special magnet lattice such that the momentum compaction is very close to $(mc^2/u_o)^2 = 2.8 \times 10^{-4}$. Such a small compaction factor may be achieved by a periodically varying field around the circumference, with the length of the period being close to the betatron wavelength. This produces an orbit distortion which is reduced at higher momentum, so as to compensate for the outward movement of the orbit.

For the conditions we have assumed here it would be possible to obtain synchrotron radiation with line spectra. The harmonics of the orbital frequency would be obtained with a fraction of the power in a few sidebands due to the synchrotron and radial betatron oscillations.

The separation of the sidebands due to the radial betatron oscillations would depend on the exact betatron frequency, but would be comparable to the separation of the harmonics, which is about one part in 10^6 . These could be separated with some difficulty by optical spectroscopic methods.

The sidebands due to the synchrotron oscillations would be separated by about one part in 5×10^{10} . The width of all these lines would be about one part in 3×10^{13} .

In addition there will be an infinitely sharp line at the center of the harmonic lines. If there are many electrons stored in the ring, the radiation in these sharp lines will be omitted coherently from all the electrons. Then if the circumference is uniformly filled with electrons most of the radiation power may be in sharp lines which are harmonics of the radiofrequency. The separation between these lines would be about one part in 10^4 , which would be very easy to separate by spectroscopic methods. These very narrow lines would be useful for experiments on photon statistics in addition to measuring the velocity of light.

In measuring the velocity of light the greatest accuracy which can be obtained is about one part in 10^8 , since that is the accuracy of the wavelength measurement of the best spectral lines. Then the tolerances on the phase amplitude of the oscillations can be considerably relaxed.

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