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Upton, New York

ACCELERATOR DEPARTMENT
Informal Report

INTERSECTING STORAGE ACCELERATOR NOTES

MASTER

Electron-Proton Elastic and Inelastic Scattering at ISABELLE

Jay Orear
Cornell University

July 26, 1972

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Intersecting Storage Accelerator Notes

ELECTRON-PROTON ELASTIC AND INELASTIC SCATTERING AT ISABELLE

Jay Orear
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ABSTRACT

We will discuss both e-P elastic and inelastic scattering of a 10 GeV electron beam on a moving proton target of 200 GeV (equivalent to 4260 GeV electrons against a stationary target). With a luminosity of $10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$ we estimate that e-P elastic scattering can reach $Q^2 \approx 10 \text{ GeV}^2$ and e-P inelastic can reach $Q^2 > 3000 \text{ GeV}^2$ with rates of several events per day.

Elastic Scattering

So far $d\sigma/d\omega$ for e-P elastic scattering has been found to be S-independent for energies up to 20 GeV and momentum transfer of $Q^2 \leq 10 \text{ GeV}^2$. At ISABELLE with 10 GeV electrons against 200 GeV protons it would be possible to increase the energy range by a factor of 200 and still be able to measure $d\sigma/d\omega$ at $Q^2 = 10 \text{ GeV}^2$.

The experimental approach is to measure θ , ϕ , and F for both the scattered electron and proton. This would permit a powerful, high-resolution 4 constraint fit with which to eliminate background. Proportional wire chambers gated by e-P coincidences would be used. A BNL 48D48 (20" x 48") would be used for the scattered electrons and the dipole used to pull the electron beam away from the proton beam would be used for the scattered protons (see Fig. 1). At $Q^2 = 10 \text{ GeV}^2$ the electron scattering angle is 300 mr, the proton scattering angle is 16 mr, and the scattered electron energy is $E' = 10.2 \text{ GeV}$.

As in CRISP 71-43 we will use the subscript zero to designate quantities in the rest frame of the target proton. In this frame

$$\frac{d\sigma_o}{d\omega_o} = \frac{\alpha^2 \cos^2 \frac{\theta_o}{2}}{4E_o^2 \sin^4 \frac{\theta_o}{2}} \left[1 + \frac{2E_o}{M} \sin \frac{2\theta_o}{2} \right]^{-1} \left[\frac{1 + \mu^2 \tau}{1 + \tau} + 2\mu^2 \tau \tan^2 \frac{\theta_o}{2} \right] G^2(Q^2)$$

where $\tau = Q^2/4M^2$ assuming that $G_M/\mu = G_E = G$.

Since $\theta_o \approx 1/1500$ we have

$$\frac{d\sigma_o}{d\omega_o} \approx \frac{4\alpha^2}{Q^4} E_o^2 \left(\frac{1 + 2.23Q^2}{1 + 0.28Q^2} \right) G^2(Q^2)$$

Using $d\sigma/dQ^2 = d\sigma_o/dQ^2$, we obtain

$$\begin{aligned} \frac{d\sigma}{d\omega} &\approx \frac{E^2}{E_o^2} \frac{d\sigma_o}{d\omega_o} \quad (\text{See CRISP 71-43 for details}). \\ &= \frac{4\alpha^2 E^2}{Q^4} \left(\frac{1 + 2.23Q^2}{1 + 0.28Q^2} \right) G^2 \quad \text{where } G^2 \approx \left(1 + \frac{Q^2}{0.71} \right)^{-4} \end{aligned}$$

For $Q^2 = 10 \text{ GeV}^2$: $G^2 = 1.93 \times 10^{-5}$ and $d\sigma/d\omega = 2.5 \times 10^{-8} \text{ GeV}^{-2}$.

Converting to cm^2 gives $d\sigma/d\omega = 9.8 \times 10^{-36} \text{ cm}^2$.

The solid angle subtended by the 20" by 48" aperture is $\Delta\omega \approx 45 \text{ msr}$.

Hence the event rate is

$$N_{eP} = L \frac{d\sigma}{d\omega} \Delta\omega = (10^{32}) (9.8 \times 10^{-36}) (0.045) = 4.4 \times 10^{-5} \text{ events/sec} = 3.8 \text{ events/day}.$$

For $Q^2 \gg 1 \text{ GeV}^2$, $d\sigma/d\omega$ is dropping with Q^2 and $1/Q^{12}$; hence a doubling of Q^2 from 10 to 20 GeV^2 will decrease the event rate by almost a factor of 64. Only $\sim 10^2$ inelastic events occur per sec, so the singles rate due to hadrons in the proportional chambers will be less than this.

Inelastic Scattering

It is important to learn whether the structure function νW_2 maintains its high value of $\sim 1/3$ for large values of energy (making possible large values of $P_\perp$). So far all physical measurements show cross sections dropping many orders of magnitude in the region $P_\perp > 2 \text{ GeV}/c$. However, if νW_2 in Eq. (1) still is $1/3$ at ISABELLE it will be possible to see inelastically scattered electrons with $P_\perp \approx 30 \text{ GeV}/c$!

We will always use the subscript zero for the system where the target proton is at rest. In this system the incoming electron energy is $E_o = 4260 \text{ GeV}$. We will always use $\nu \equiv E_o - E'_o$ as the electron energy loss in this system. In invariant form it is also $\nu = \vec{P} \cdot \vec{q}/M$ where $\vec{P}$ is the proton 4-momentum and $\vec{q} = \vec{P}_e - \vec{P}_{e'}$, is the change in electron 4-momentum. $Q^2 = -\vec{q} \cdot \vec{q} = 2EE'(1 - \cos \theta)$ in all reference frames. Quantities such as E and E' without subscript are in the ISABELLE lab system. We will usually discuss $E = 10 \text{ GeV}$ electrons and $E_p = 200 \text{ GeV}$ protons. For comparison we calculate rates for $E = 15$ and $E_p = 70$ (the PEP parameters).

On the target rest frame

$$\frac{d\sigma_o}{dE'_o d\omega_o} = \frac{4\alpha^2 E_o'^2}{\nu Q^4} \left[\cos^2 \frac{\theta_o}{2} + 2 \sin^2 \frac{\theta_o}{2} \frac{Q^2 + \nu^2}{Q^2} \frac{1}{1+R} \right] \nu W_2 \quad . \quad (1)$$

In the region of interest θ_o never becomes larger than $\sim 1^\circ$; hence the quantity in the bracket is ≈ 1 . We use the SLAC results of Fig. 2 for νW_2 . It has the value 0.32 whenever $2M\nu/Q^2 > 3$. We are hoping that Fig. 2 still holds at energies 200 times those of SLAC. Then

$$\frac{d\sigma_o}{dE'_o d\omega_o} \approx \frac{4\alpha^2 E_o'^2}{\nu Q^4} (\nu W_2) \quad .$$

In the ISABELLE frame

$$\frac{d\sigma}{dP'_o d\omega'} = \frac{P'_o}{P_o} \frac{d\sigma_o}{dP'_o d\omega'_o} \quad .$$

Using $E' = P'$ gives

$$\frac{d\sigma}{dE' d\omega'} = \frac{4\alpha^2 E' (E_o - \nu)}{\nu Q^4} (\nu W_2) \quad (2)$$

On p. 6 of CRISP 71-43 we derive that

$$E' \approx E - \frac{M}{2E_p} \nu + \frac{Q^2}{4E}$$

Lines of constant E' are plotted vs ν and Q^2 in Fig. 3. In the region of $Q^2 \approx 2000 \text{ GeV}^2$ and $\nu \approx 3000 \text{ GeV}$, E' will be $\approx 50 \text{ GeV}$ at an angle of $\sim 150^\circ$. We will now show that even at such high Q^2 it would be possible to see several events/day with a luminosity of $L = 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$. We need to calculate events per bin in Q^2 and ν , so we must transform $d^2\sigma/dE'd\omega'$ to $d^2\sigma/dQ^2 d\nu d\phi$. The transformation Jacobian is $M/4EE'E_p$ as shown in CRISP 71-43. If we multiply Eq. (2) by this, we obtain

$$\frac{d\sigma}{dQ^2 d\nu d\phi} = \frac{\alpha^2 (\nu W_2) M (E_o - \nu)}{EE_p \nu Q^4}$$

For $E = 10$ and $E_p = 200$ this is

$$\frac{d\sigma}{dQ^2 d\nu d\phi} = \frac{\alpha^2 (\nu W_2) (4260 - \nu) 4.7 \times 10^{-4}}{\nu Q^4} = \frac{9.7 \times 10^{-36} (\nu W_2) (4260 - \nu)}{\nu Q^4} \frac{\text{cm}^2}{\text{GeV}^3}$$

Figure 4 shows a simple detector arrangement using gas Cerenkov and lead glass modules at $120 \pm 4^\circ$ covering essentially all 2π in azimuth. The gas Cerenkov eliminates large angle hadrons and low energy electromagnetic radiation. Since the electrons of interest are above 10 GeV, the lead glass blocks will measure the energy of each electron to a few percent. The event rate for this setup is

$$N_j = L 2\pi \int_{\text{bin}} \int_{\text{jth}} \frac{d\sigma}{dQ^2 d\nu d\phi} dQ^2 d\nu$$

$$N_j = \frac{2\pi L}{Q^2} 9.7 \times 10^{-36} \frac{\Delta Q^2}{Q^2} \int_{\text{bin}} \frac{(\nu W_2) (4260 - \nu)}{\nu} d\nu$$

Note the slow dependence on Q^2 compared to e^{-aE_1} for strong interactions. Take for example bin 2 in Fig. 4 using $L = 10^{32}/\text{sec} = 8.6 \times 10^{36}/\text{day}$:

$$N_2 \approx \frac{2\pi \times 8.6 \times 10^{36} \times 9.7 \times 10^{-36}}{700} \times 0.35 \int_{1400}^{2400} \frac{0.3(4260 - 1800)}{1800} d\nu =$$

$$= 100 \text{ events/day} .$$

We see that at $Q^2 = 700 \text{ GeV}^2$ one could obtain ~ 100 events/day per bin. Rates for other bins are shown in Fig. 5. (These calculations were done for the earlier ISABELLE energy $E_p = 216 \text{ GeV}$, but would be almost the same at 200 GeV .) Even at $Q^2 \sim 2000 \text{ GeV}^2$ and $\nu \sim 3000 \text{ GeV}$ one could obtain ~ 20 events/day. For comparison, rates were calculated for the PEP parameters of $E = 15 \text{ GeV}$ and $E_p = 70 \text{ GeV}$ at the same luminosity of 10^{32} (see Fig. 6). It is at first surprising to see that bin 2 for the PEP parameters is down by an order of magnitude compared to ISABELLE when using the same luminosity and laboratory solid angles. Even though the Q^2 and ν regions are the same, there is a kinematical advantage to the higher proton energy which permits a larger area coverage for the same apparatus. Also the factor $(E_0 - \nu)$ is larger. Another advantage of the ISABELLE $e\bar{p}$ energy of $10 \times 200 \text{ GeV}$ is that the available region in $Q^2 - \nu$ space is increased by a factor of 4 over that of PEP.

Figure Captions

- Fig. 1 Electron-proton elastic scattering set-up for $Q^2 = 10 \text{ GeV}^2$.
PC are proportional wire chambers. M_1 is the dipole used to
separate the electron and proton beams.
- Fig. 2 SLAC results for νW_2 as a function of $\omega = 2M_N/Q^2$. The heavy
line is the fit used in the rate calculations.
- Fig. 3 Lab energy E' and angle θ of scattered electron in ISABELLE
frame of reference as function of Q^2 and ν . Ten GeV electron
beam and 216 GeV proton beam was used.
- Fig. 4 Experimental setup to measure inelastic electron scattering
in region $0 < Q^2 < 1000 \text{ GeV}^2$ and $4000 > \nu > 1000 \text{ GeV}$. See
bins 1, 2, and 3 of Figs. 5 and 6. Detector is a conical
array of gas Cerenkov counters and lead glass counters at
 120° to electron beam.
- Fig. 5 Number of events/day per bin for 3 different detector angles.
- Fig. 6 Same as Fig. 5 except electron beam at 15 GeV and proton beam
at 70 GeV.

$e p$ ELASTIC
 $10 \times 200 \text{ GeV}$
 $Q^2 = 10 \text{ GeV}^2$

10 ft

Fig. 1

Fig. 2

FIG. 3

e-P INELASTIC
 $0 < Q^2 < 1000 \text{ GeV}^2$
 $1000 < \nu < 4000 \text{ GeV}$

FIG. 4

FIG. 5

Fig. 6

15 x 70 GeV e-p

$$I = 10^{32} \text{ cm}^{-2}/\text{sec}$$