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ABSTRACT

The polarization of bremsstrahlung from electron-ion collisions is calculated for magnetically-confined electrons in the Born approximation. The polarization at high temperature is found to be of opposite sign to the low-temperature result and of smaller magnitude.

The bremsstrahlung radiation produced by electron-ion collisions is partially polarized. In a magnetically-confined anisotropic plasma there can be a net polarization of radiation emitted normal to the field direction.

The polarization, P, is defined here as

$$P = \frac{I_{\perp} - I_{\parallel}}{I_{\perp} + I_{\parallel}} ,$$

where $I_{\perp}$ and $I_{\parallel}$ are the intensities of the radiation polarized normal and parallel to the plane of observation, respectively. We define here the plane of observation to be the midplane of a magnetic mirror (in the case of a mirror-confined plasma) or else a plane normal to the magnetic axis (in the case of a toroidally-confined plasma). The plane of observation could be chosen to include the magnetic axis and this would result in a sign change from our convention.

Polarization measurements have been made on low-temperature mirror-confined plasmas[1] and on toroidally-confined plasmas.[2] These measurements have been used to infer the anisotropy of the electron distributions. In Ref. 1 agreement between the measurements and calculations based on a quantum mechanical nonrelativistic theory[3] was good. The calculations presented here were made to determine the usefulness of this technique for higher temperature plasmas. We employed the Born approximation as calculated by Gluckstern and Hull,[4] which is appropriate for the energy range for experiments at Oak Ridge on several facilities.[5,6]

This paper represents an extension of the calculations of angular distribution of bremsstrahlung in the Born approximation for high-temperature plasmas by England and Haste.[7] More exact point Coulomb cross sections have been developed by Tseng and Pratt.[8] Experiments[9] have indicated that the polarization predicted by the Born approximation may be too large and that, at least for high Z, the work of Ref. 8 is preferred.

Maxon[10] has shown that at high temperatures the total electron-electron (e-e) bremsstrahlung radiation dominates over the electron-ion bremsstrahlung for Maxwellian plasmas. However, at the present time, there are no calculations of the polarization expected from e-e bremsstrahlung.

The geometry for calculating the polarization is given in Fig. 1. The initial electron with momentum $\vec{p}_0$ is located on the x-y plane (the plane of observation). An electron-ion collision produces a photon with momentum $\vec{k}$ along x at an angle θ_0 with respect to $\vec{p}_0$. We take β as the phase angle of the electron in its gyro orbit and α as the pitch angle of the momentum relative to the x-y plane. The magnetic field is in the direction of the z-axis. The angle γ is the angle in the y-z plane between the projection of $\vec{p}_0$ onto the y-z plane and the plane of observation.

The plane p_0k containing $\vec{p}_0$ and $\vec{k}$ is used by Gluckstern and Hull[4] in defining the direction of the polarization. The differential cross section $d^2\sigma_{III}/d\Omega_0 dk$ is the cross section for the production of photons polarized normal to this plane and $d^2\sigma_{II}/d\Omega_0 dk$ is the cross section for photons polarized parallel to this plane.

The polarization of photons emitted along the z-axis is zero by symmetry arguments. Note that the magnetic field does not enter into the calculation and is only used as a reference vector.

The figure also shows schematically a Compton polarimeter. The design and calibration of such polarimeters will not be discussed here. Briefly, photons polarized in the x-y plane will tend to scatter into the detector marked $\parallel$ and photons polarized normal to the x-y plane will tend to be scattered in the x-y plane into the detector marked $\perp$. The ability of the polarimeter to analyze polarized photons is a function of the photon energy and the geometry of the polarimeter. This subject has been discussed in numerous articles.[11]

We must find the polarization as a function of the photon energy, the pitch angle distribution, and the distribution function of the trapped electrons. We have taken the monoenergetic distributions and Maxwellians as examples of distribution functions. The pitch angle distribution can be related to a specific mirror ratio, R, through the relation

$$\cos \alpha_{max} = 1/\sqrt{R} \quad ,$$

where α_{max} is the maximum angle of the distribution of p_0 vectors contained in the mirror.

We assume in the calculation that the radiation is emitted at random phases over the orbit of the electron. We first find the probability for finding the radiation polarized normal ($\pi_\perp$) and parallel ($\pi_\parallel$) to the plane of observation. The polarization is defined as the difference in probabilities for these two polarizations divided by their sum,

$$P = \frac{\pi_\perp - \pi_\parallel}{\pi_\perp + \pi_\parallel} \quad .$$

We find

$$\pi_{\perp} = \int_{E_{o\,min}}^{E_{o\,max}} dE_o \int_{\alpha_{min}}^{\alpha_{max}} \cos\alpha \, d\alpha \, f\left(kT_e, E_o, \alpha\right) \int_0^{2\pi} d\beta$$

$$\times \left(\frac{d^2\sigma_{II}}{d\Omega_o dk} \cos^2\gamma + \frac{d^2\sigma_{III}}{d\Omega_o dk} \sin^2\gamma \right)$$

and

$$\pi_{||} = \int_{E_{o\,min}}^{E_{o\,max}} dE_o \int_{\alpha_{min}}^{\alpha_{max}} \cos\alpha \, d\alpha \, f\left(kT_e, E_o, \alpha\right) \int_0^{2\pi} d\beta$$

$$\times \left(\frac{d^2\sigma_{II}}{d\Omega_o dk} \sin^2\gamma + \frac{d^2\sigma_{III}}{d\Omega_o dk} \cos^2\gamma \right) .$$

The quantity E_o is the total electron energy. We must integrate from an energy equal to the photon energy plus the rest mass energy, $E_{o\,min}$, to the maximum energy contained in the distribution, $E_{o\,max}$. We integrate over the pitch angle range contained by the field. In the case of a mirror, generally $\alpha_{min} = -\alpha_{max}$. For a beam directed along the field, we would integrate from $\pi/2$ to an angle near $\pi/2$. The quantity $f\left(kT_e, E_o, \alpha\right)$ is the distribution function of the electrons and may be a function of the electron temperature, kT_e, the electron energy, E_o, and the pitch angle distribution, α. The integration over β is through 2π radians.

The relations between the various angles are given by

$$\cos \theta_0 = \cos \alpha \cos \beta$$

$$\sin \gamma = \frac{\tan \alpha}{\sqrt{\sin^2\beta + \tan^2\alpha}}$$

$$\cos \gamma = \frac{\sin \beta}{\sqrt{\sin^2\beta + \tan^2\alpha}}$$

We can write the polarization in the following form after eliminating the angle γ:

$$P = \frac{\displaystyle\int_{E_{o\,min}}^{E_{o\,max}} dE_o \int_{\alpha_{min}}^{\alpha_{max}} \cos \alpha \, d\alpha \, f\left(kT_e, E_o, \alpha\right) \int_0^{2\pi} d\beta \left(\frac{d^2\sigma_{II}}{d\Omega_o dk} - \frac{d^2\sigma_{III}}{d\Omega_o dk}\right) \times H}{\displaystyle\int_{E_{o\,min}}^{E_{o\,max}} dE_o \int_{\alpha_{min}}^{\alpha_{max}} \cos \alpha \, d\alpha \, f\left(kT_e, E_o, \alpha\right) \int_0^{2\pi} d\beta \left(\frac{d^2\sigma_{I}}{d\Omega_o dk}\right)},$$

where

$$H = \left(\frac{\tan^2\alpha - \sin^2\beta}{\tan^2\alpha + \sin^2\beta}\right).$$

The quantity in the denominator $\frac{d^2\sigma_I}{d\Omega_o dk}$ is the sum of the differential cross sections $\frac{d^2\sigma_{II}}{d\Omega_o dk}$ plus $\frac{d^2\sigma_{III}}{d\Omega_o dk}$.

To illustrate the behavior of the polarization as a function of the electron energy for a monoenergetic distribution, the previous equation was integrated to determine P for many monoenergetic electron energies ranging from 200 keV to 4 MeV confined in a magnetic mirror field of 1.1:1 mirror ratio with the pitch angle distribution isotropic up to the loss cone. This mirror ratio corresponds to a pitch angle range of ±18° from the midplane. Figure 2 shows the polarization as a function of the electron energy for a constant photon energy. Note that the polarization is initially relatively large and positive but drops, changes sign, and tends at high electron energies to a low absolute value.

Figure 3 shows the behavior of the polarization for a Maxwellian electron distribution in the same mirror ratio where we have again assumed that the pitch angle distribution is isotropic up to the loss cone. We have plotted the polarization as a function of the normalized temperature kT_e/m_oc^2. The distribution was cut off at an electron kinetic energy of 6 MeV. Note the same general behavior as in Fig. 2; at low temperature the polarization is very large but at high temperatures it has changed sign and declines to a small absolute value.

Finally, we show the behavior of the polarization as a function of the mirror ratio (the width of the pitch angle distribution). Figure 4 displays P vs. mirror ratio for a Maxwellian with a temperature of 511 keV. The polarization in this case is always negative and is relatively large for low mirror ratios. However, for mirror ratios exceeding ~1.5, the polarization is small and very insensitive to the mirror ratio.

Other calculations have been made indicating that if the confined plasma is a bi-Maxwellian (i.e., different parallel and perpendicular temperatures) the calculated polarization can be considerably larger but that the behavior, as illustrated in Fig. 4, is roughly the same.

The behavior of the polarization as a function of the photon energy is strongly dependent upon the distribution function. Generally, for a Maxwellian plasma the polarization tends to become more positive as the photon energy is increased.

Measurements made at Oak Ridge[12] on a plasma with an electron temperature of ~1 MeV confined in a magnetic mirror with mirror ratio 2:1 have shown very small ($\leq$1%) values of polarization for photons of 100 to 400 keV. Calculations have indicated that the polarization expected under these conditions should be about −0.5%. In conclusion, it appears that polarization measurements are not useful in measuring anisotropy in mirror-confined plasmas with high temperatures and which are confined in large mirror ratio mirrors.

Polarization of bremsstrahlung radiation has been observed from runaway electrons in the ST Tokamak.[3] Calculations similar to those already shown have shown relatively good agreement between theory and experiment in the photon energy range 20 keV to 150 keV, for "free-fall"[13] distributions and one-dimensional Maxwellians. Generally, the polarization obtained is opposite in sign to that observed for a mirror-contained plasma. However, the calculations give polarization values which are very insensitive to the pitch angle distribution; a distribution of pitch angles as much as 20° from the magnetic axis gives values only a few percent different from a pitch angle distribution confined within 1° of the axis. Hence, it appears that this technique is not useful for determining the anisotropy of toroidally-contained runaway distribution.

In the low photon energy limit,[4] this work gives fair agreement with the calculations done with the nonrelativistic quantum mechanical theory.[1] However, in this low energy limit the nonrelativistic approach is preferred because the exact electron wave functions can be used.

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*Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

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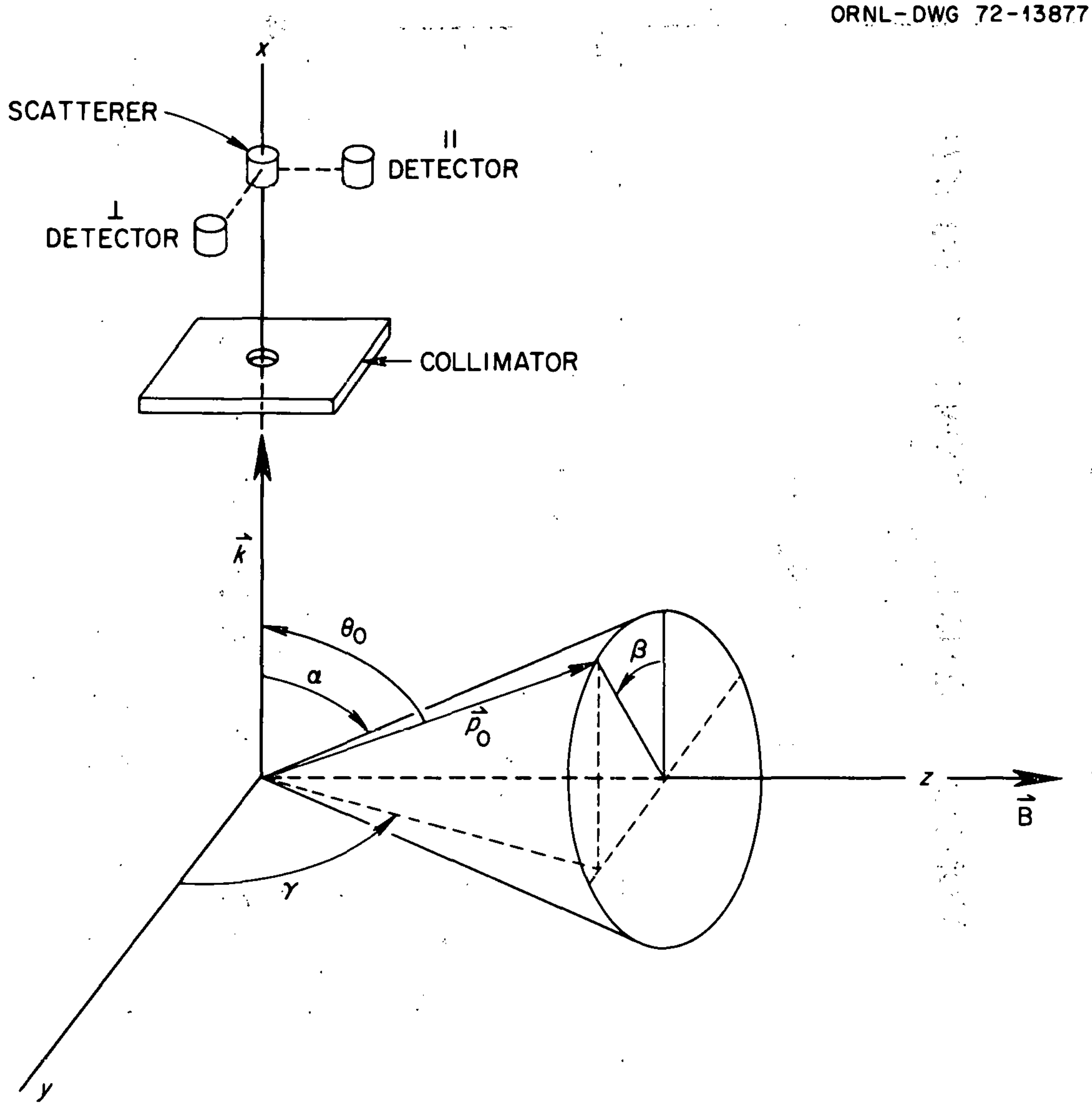

Fig. 1. The geometry used in the polarization calculation.

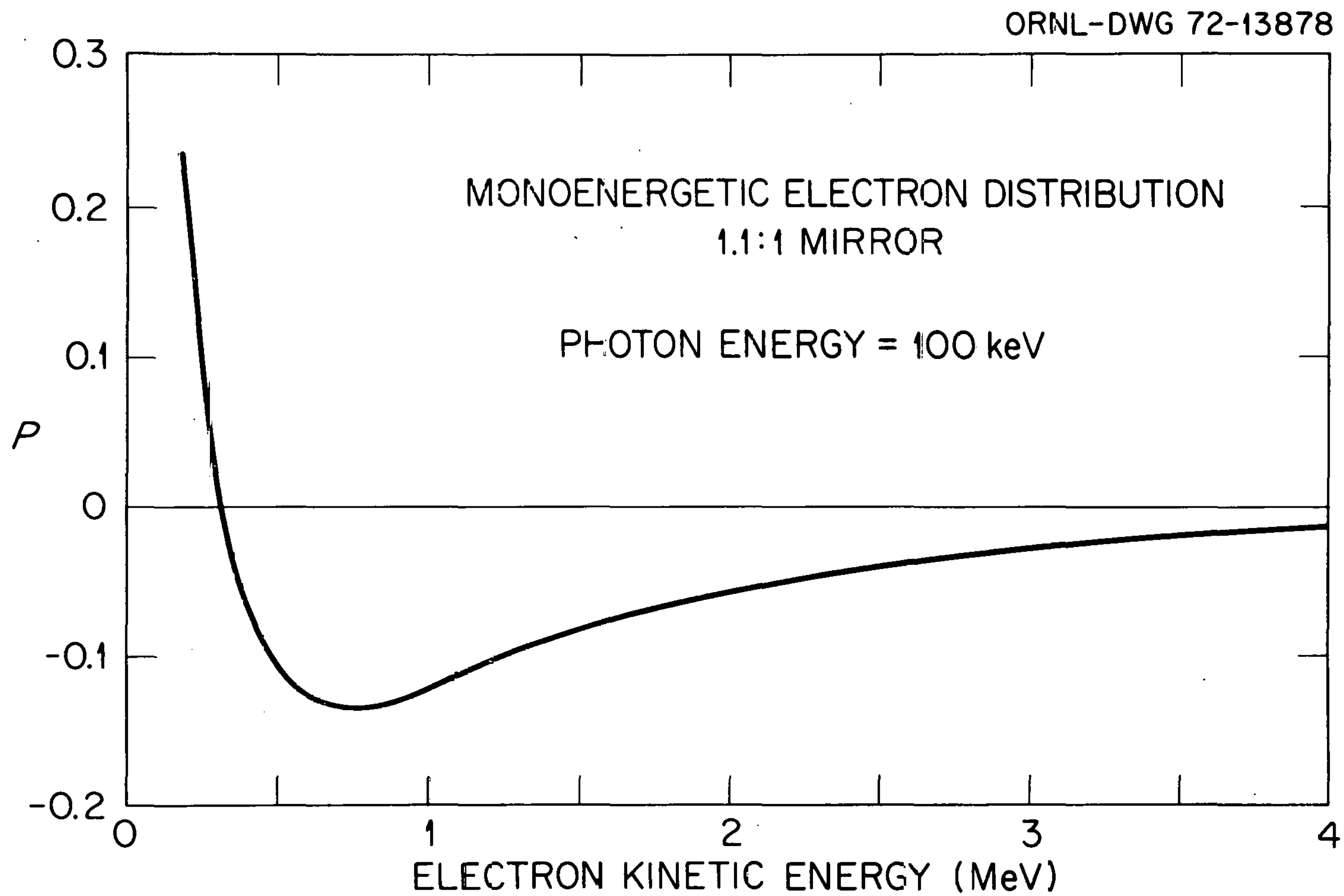

Fig. 2. Polarization of 100 keV photons vs. electron kinetic energy for a monoenergetic electron distribution contained in a mirror ratio of 1.1:1.

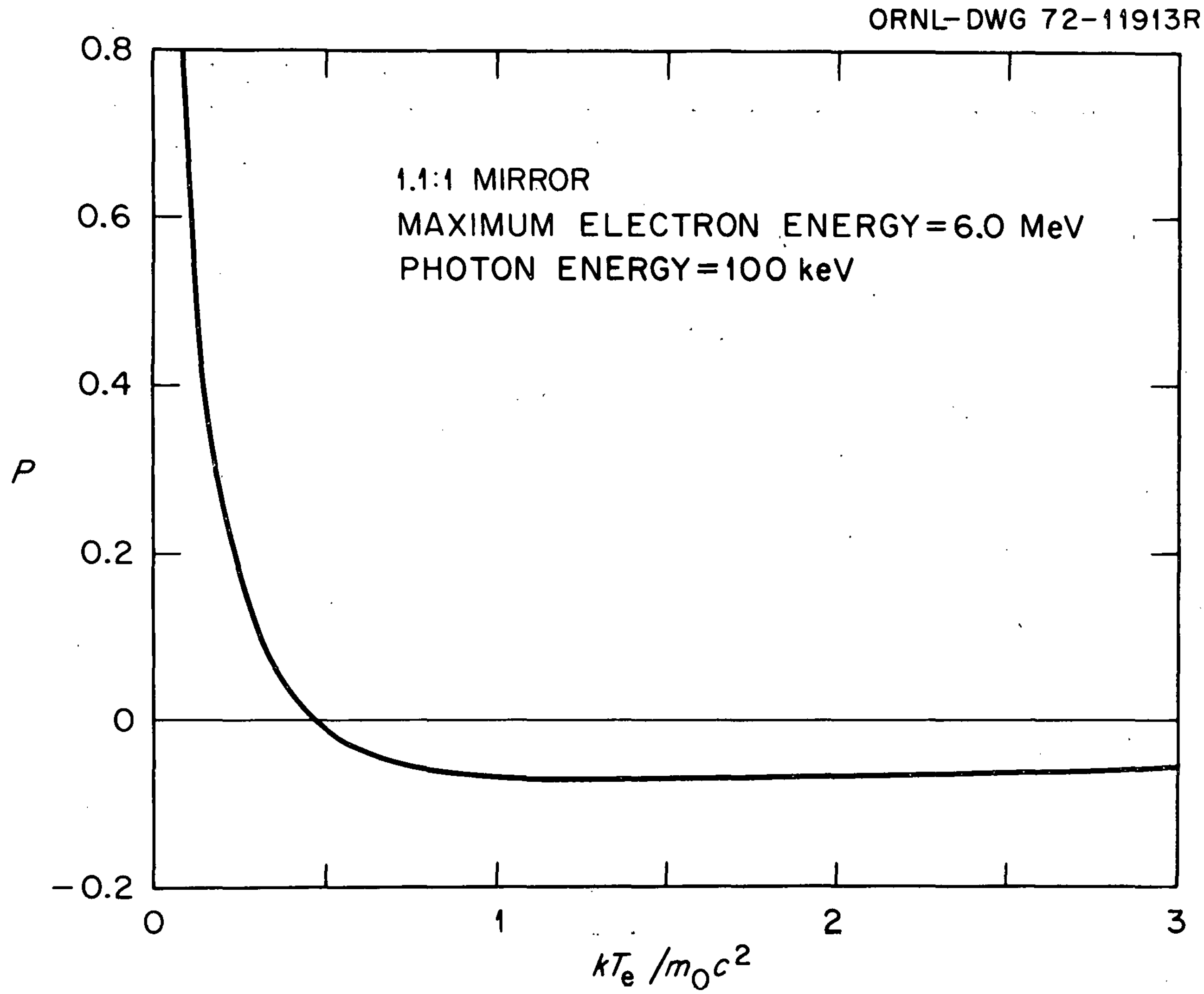

Fig. 3. Polarization of 100 keV photons vs. normalized electron temperature for a Maxwellian electron distribution contained in a mirror ratio of 1.1:1.

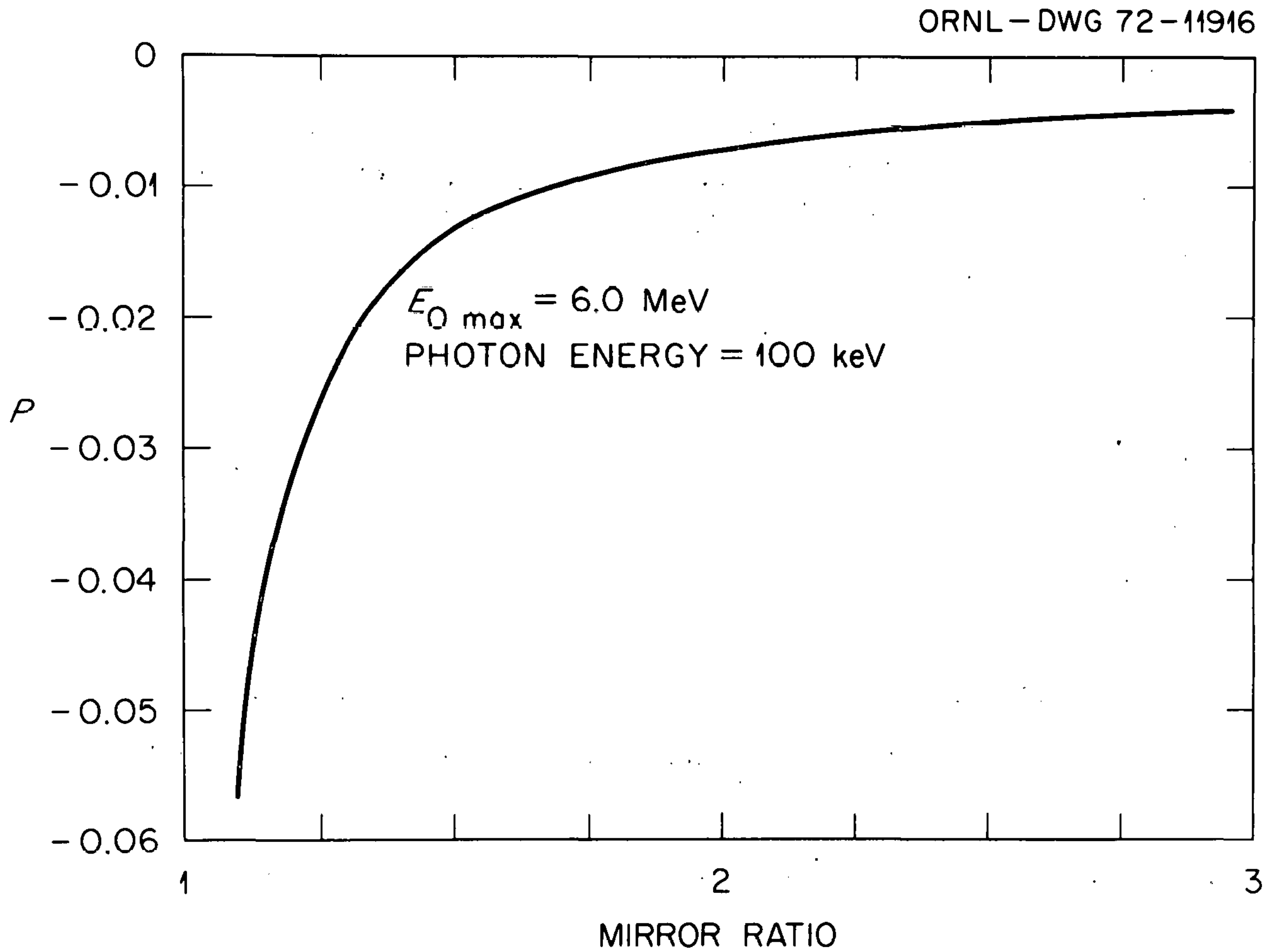

Fig. 4. Polarization of 100 keV photons vs. mirror ratio for a Maxwellian electron distribution at a temperature of 511 keV contained in a magnetic mirror.