DEC 28 1966

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BNL 10834

AADD-125

ACCELERATOR DEPARTMENT
Internal Report

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EQUIVALENT CIRCUIT AND DISPERSION RELATION
FOR THE MULTISTEM DRIFT TUBE STRUCTURE

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November 18, 1966

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

I. Introduction

S. Giordano has proposed a multistem structure for drift tube linacs which has the advantage of relative immunity from beam loading and detuning effects due to the greater mode-spacing near the TM_{010} mode.¹ In a recent paper, Giordano and Hannwacker have measured and discussed a new set of modes which are associated with the resonance in the circumferential fields of the stem systems.² These modes, called $TS(N)_{10l}$ modes, couple to the usual TM_{01l} modes and lead to the shaping of the dispersion curve about the TM_{010} resonance.

In this report, we shall consider the coupled dispersion curves for TS and TM modes in terms of an equivalent circuit for the multistem structure. It will be pointed out also that the lumped circuit constants determined from the measured frequencies have a reasonable N-dependence for the $N = 1, 2, 3$ and 4 stem structures.

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1. S. Giordano, "A New Multistem, Drift-Tube, Standing-Wave, Zero-Mode Proton Accelerating Structure," BNL Accelerator Dept. Internal Report AGSCD-7, February 25, 1966.
2. S. Giordano, "Measurements on the Multistem Drift Tube Structure for Proton Accelerators," BNL Accelerator Dept. Internal Report AGSCD-11, June 22, 1966; S. Giordano and J.P. Hannwacker, "Measurement on a Multistem Drift Tube Structure," to be published in the Proceedings of the 1966 Linac Conference, Los Alamos, October 3-7, 1966.

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II. Dispersion Relation from Equivalent Circuit

An equivalent circuit for the usual drift tube structure has been given by Hereward in order to discuss energy propagation along the linac.³ We shall modify his circuit to include the transverse stem resonances, as is shown in Fig. 1. In this circuit, the series capacitor, C , represents the gap capacity, and the series inductances, $2 \times L/2$, represent the inductance which stores the magnetic energy surrounding the drift tube and the gap. Then the resonant frequency of the TM_{010} mode is given by

$$\omega_o = \sqrt{\frac{1}{LC}} \quad (1)$$

On the other hand, the shunt capacitor, C_s , represents the capacitance between the drift tube and the outer cylinder, and the shunt inductances, $2L_s \times 1/2$, represent the inductance due to the loop consisting of the stems and the outer cylinders. L_s and C_s give the transverse stem resonance. Since the stem-loops associated with a drift tube will couple to those of the adjacent tubes, we also consider the mutual inductance, M_s , between them. It will be shown later that the resonant frequency of the $TS(N)_{100}$ mode is given by

$$\omega_s = \sqrt{\frac{1}{L_s C_s}} \times \sqrt{\frac{2L_s}{2L_s + M_s}} \quad (2)$$

Let V be the potential on the drift tube center with respect to ground (outer cylinder) and I the current from left to right through the drift tubes and gaps. We write the shunt currents from the drift tube center to ground by i , which is divided into three components, i_1 , i_2 and i_3 , corresponding to three shunt elements, $2L_s$, C_s and $2L_s$, respectively. From Floquet's theorem for the periodic structure, the voltages and all of the currents are multiplied by a phase factor $e^{j\varphi}$ when we move down the structure by one period. φ is the cell-to-cell phase shift and is given by kL_o , with the propagation constant, k ,

3. H.J. Hereward, "Some Examples of Energy Flow in the Alvarez Structure," CERN Int. Rep. MPS/DL Int. 65-1, 1965.

and the cell length, L_o . Thus, the voltage and currents are related by:

$$V_o (e^{j\varphi} - 1) = - \left(j\omega L + \frac{1}{j\omega C} \right) I_o \quad (3)$$

$$V_o = 2j\omega L_s i_1 + j\omega M_s i_3 e^{j\varphi} \quad (4)$$

$$= 2j\omega L_s i_3 + j\omega M_s i_1 e^{-j\varphi} \quad (4')$$

$$= \frac{i_2}{j\omega C_s} \quad (4'')$$

From Eqs. (4), (4') and (4''), we get

$$i = i_1 + i_2 + i_3 = j \frac{C_s}{\omega} \left[\omega^2 - \frac{4L_s}{(4L_s^2 - M_s^2) C_s} \left(1 - \frac{M_s}{2L_s} \cos \varphi \right) \right] V_o \quad (5)$$

Since the shunt current is given by $I_o (e^{-j\varphi} - 1)$, we can eliminate V_o and I_o from Eqs. (3) and (5), leading to the dispersion equation

$$B(1 - \cos \varphi) = \frac{[\omega^2 - \omega_o^2] [\omega^2 - \omega_s^2 \{ 1 + \epsilon (1 - \cos \varphi) \}]}{\omega^2 \omega_o \omega_s} \quad (6)$$

where

$$B = 2 \sqrt{\frac{C}{L}} \sqrt{\frac{L_s}{C_s}} \sqrt{1 + \frac{M_s}{2L_s}} \quad (7)$$

and

$$\epsilon = \frac{M_s}{2L_s - M_s} \quad (8)$$

The dispersion equation (6) is similar to that obtained by Allen and Kino for the resonant slot loaded waveguide structure.⁴ In addition, the

4. M.A. Allen and G.S. Kino, "On the Theory of Strongly-Coupled Cavity Chains," IRE Trans. Microwave Theory Tech., MTT-8, 362, May, 1960.

behavior of the dispersion curves shows the same types of advantages in reducing beam loading and detuning effects as discussed by Nishikawa, Giordano and Carter for the alternating periodic structures.⁵ The dispersion equation has two solutions for $\omega = 0$, i.e. $\omega = \omega_o$ and ω_s . In general, there exist two passbands which correspond to the TM_{01} and $TS(N)_{10}$ modes and are separated by the stopband between ω_o and ω_s . For the special case of $\omega_s = \omega_o$, these two bands merge into a single passband and give a finite group velocity.

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\omega_o L_o}{2} \sqrt{\frac{B}{2}} \quad (9)$$

for the zero mode operation.

For the case of $\omega_o \neq \omega_s$, the group velocity vanishes. The mode separation between TM_{010} and TM_{011} approximates to

$$\omega_1 - \omega_o \approx \frac{1}{2} \frac{\partial^2 \omega}{\partial \varphi^2} \left(\frac{\pi}{n_o} \right)^2 = \frac{\pi^2 B}{4\pi_o^2} \frac{\omega_o^2 \omega_s^2}{\omega_o^2 - \omega_s^2} \quad (10)$$

from Eq. (6), where n_o is the total number of cells. Since the separation given by (10) increases linearly as ω_s approaches ω_o , we can assure the advantages of the reduced sensitivity to beam loading, tuning errors and other perturbations for the structure which is adjusted closely enough to the same frequency.

III. Analysis of Measurements

Figure 2 shows a comparison between the calculated curves and measured frequencies. In order to determine four parameters in Eq. (6), ω_o , ω_s , B and ϵ , we use four measured frequencies. They are, in addition to ω_o and ω_s , the π -mode of the TS-band and the $\pi/2$ -mode of the TM-band for one and four stem structures, and the π -mode of the TS-band and the $\pi/3$ -mode of the TM-band for the two and three stem cases. The solid curves in Figure 2 show the calculated dispersion relations for $N = 1, 2, 3$ and 4 stem structures and the points indicated by circles, triangles, squares and crosses are the corresponding measured frequencies; those filled in or heavily written show the points used for the calculation. It

5. T. Nishikawa, S. Giordano and D. Carter, "Dispersion Relation and Frequency Characteristics of Alternating Periodic Structure for Linear Accelerators," Rev. Sci. Instr. 37, 652 (1966).

is also pointed out that, if we neglect the direct or mutual couplings between the transverse stem resonances, then the calculated curves are as shown by the dotted curve for the one stem example and no longer match the experimental data.

The variations of B and ϵ with the number of stems; N , are given in Figure 3. In this figure the frequencies of the $TS(N)_{010}$ mode, ω_s , are also shown; they are determined from a series of measurements on model cavities of various cell numbers⁶ and used for the calculations. The frequency of the TM_{010} mode, ω_o , is almost unchanged and independent of N .

Assuming that the lumped circuit constants associated with the drift tube resonance (L and C) are independent of N , we obtain from Eqs. (2), (7) and (8) the N -dependence of the lumped constants for the stem resonance, i.e. L_{SN} , M_{SN} and C_{SN} . They are shown in Figure 4 as the normalized values to the one stem case. As we can expect from the simple consideration, C_{SN} 's are less sensitive to N , while L_{SN} 's and M_{SN} 's are remarkably affected by N . Since the area of a single loop surrounded by stems and the outer cylinder is proportional to $1/N$, the mutual inductance, M_{SN} , will be proportional to $1/N^2$. This is also shown by a comparison of M_{SN}/M_{S1} with the dotted curve ($= 1/N^2$) in Figure 4. The self-inductance L_{SN} 's will be proportional to the ratio of the single loop area to the effective magnetic path length, ℓ_{mN} . In addition, since N loops are connected in shunt, they will be proportional to $(N^2 \ell_{mN})^{-1}$. Using this relation, we also plot the normalized magnetic length, ℓ_{mN}/ℓ_m , in Figure 4. The decreasing curve of ℓ_{mN} with N shows that the magnetic field becomes concentrated about the stems as the number of stems increases.

V. Acknowledgment

The author would like to thank Mr. S. Giordano for his helpful discussions and valuable information on the experimental data.

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11/18/66
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6. Private communication from S. Giordano and J.P. Hannwacker, Brookhaven National Laboratory.

Fig. 1. An equivalent circuit for the multistem drift tube structure.

Fig. 2. Comparison between measured and calculated dispersion curves (pairs of measured frequencies for the same modes of 4-stem cases were obtained by different end terminations⁶).

Fig. 3. N -dependence of B , ϵ and ω_s .

Fig. 4. N-dependence of lumped circuit constants.