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RANDOM ERROR MULTIPOLES IN SUPERCONDUCTING WINDOW-FRAME MAGNETS

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MASTER

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Abstract

The random error multipoles in the magnetic field of a window-frame magnet due to random errors in the position of the current blocks are computed. The computed random error multipoles in cosine and window-frame magnets are compared. The results of measured results for the random error multipoles in a superconducting window-frame magnet are compared with the computed results.

I. Introduction

Random errors in the positions of the current blocks of a window-frame magnet will produce random multipoles in the median plane magnetic field. An error in the position of a current block will produce an error field in the median plane which has both a vertical component ΔB_z , and a radial component ΔB_r . Each of these components can be expanded in multipoles in the median plane according to

$$\Delta B_z = B_0 \sum_{\ell=0}^{\infty} b_{\ell} r^{\ell} , \quad (1a)$$

$$\Delta B_r = B_0 \sum_{\ell=0}^{\infty} a_{\ell} r^{\ell} , \quad (1b)$$

where r is the distance from the center of the magnet and B_0 is the unperturbed dipole magnet field.

In computing the random error multipoles, one has to make some assumption as to how the current-carrying conductors can move. Figure 1 shows a window-frame magnet which has 4 current blocks. Actually, each of the 4 current blocks is made up of many conductors. In this calculation, it is assumed that each of the 4 current blocks can have random errors in their positions. This will give a larger error field than what is obtained if one assumes that all the conductors that make up each of the 4 current blocks would move in a random way.

The calculations given below show that for a window-frame magnet with a 4 in. x 4 in. aperture, whose 4 current blocks have a rms random error in their

Figure 1

position of $\epsilon = 0.002$ in., the largest error multipole generated is about 2×10^{-4} /in. The measured results for a model window-frame magnet seem to be roughly consistent with the computed error multipoles assuming a random error of 0.002 in. in each of the 4 current blocks.

A comparison of the theoretical results for the random error multipoles in window-frame and cosine magnets shows that the linear error multipoles are about equal in the two magnets but that the nonlinear error multipoles are a factor of 3 or more smaller in the window-frame magnet. Measured results on model magnets have some confusing points in them which make comparisons difficult, but it is possible that the measured results may indicate that the window-frame magnet may have smaller nonlinear multipoles.

II. Field Error Results

The error multipoles introduced by moving one of the 4 current blocks shown in Fig. 1 in either the vertical or horizontal direction was found using the GRACY magnet program.

the median plane symmetry and the error field introduced has both vertical and horizontal components in the median plane.

The rms multipoles introduced by a random rms error of $\epsilon = 0.002$ in. in the position of all 4 current blocks in either the horizontal or vertical direction is given in Table I. Table I also gives the error multipoles introduced by moving the current block in the upper right corner by the amount $\Delta x = -0.002$ in., and $\Delta y = -0.002$ in. These results are for a window-frame magnet with a 4×4 in. aperture and whose coils are 1 in. thick.

The results given in Table I may be scaled to different values of the rms error in the coil positions, ϵ , since the results are linear in ϵ , and to window-frame magnets with different square apertures since the multipoles scale like R^{ℓ} , where R is the width of the aperture.

The rms results given in the first two columns of Table I are computed by assuming each of the four coils can move randomly in the x and y directions with a rms error of 0.002 in. In an ideal window-frame magnet, the coils

1. G. Parzen and K. Jellet, Particle Accelerators 2, 169 (1971).

TABLE I. Random error multipoles in the median plane field of a window-frame magnet with a 4 in. $\times$ 4 in. aperture and 1 in. thick coils. The rms multipoles are for a random error in each of the 4 coils of $\epsilon = 0.002$ in. The multipoles are in units of 10^{-4} in. $^{-\ell}$. The $\Delta x = -0.002$ in. and $\Delta y = -0.002$ in. columns give the multipoles generated by the indicated displacement of the coil in the upper right corner. The b_{ℓ} are the multipoles in the B_z field, and a_{ℓ} are the multipoles in the B_x field.

ℓ	rms		$\Delta x = -0.002$ in.		$\Delta y = -0.002$ in.	
	b_{ℓ}	a_{ℓ}	b_{ℓ}	a_{ℓ}	b_{ℓ}	a_{ℓ}
0	0.504	2.418	0.000	0.478	0.252	-1.11
1	0.814	1.850	0.001	-0.404	-0.407	0.832
2	0.666	0.740	0.011	0.201	0.333	-0.310
3	0.340	0.213	0.000	-0.086	-0.170	0.063
4	0.134	0.102	-0.002	0.051	0.067	-0.005
5	0.048	0.038	0.000	-0.019	-0.024	-0.002
6	0.014	0.018	0.001	0.009	0.007	0.000
7	0.006	0.004	0.000	-0.002	-0.003	0.000
8	0.004	0.002	0.000	0.001	0.002	0.000

TABLE II. Error multipoles in the median plane field of a window-frame magnet with a 4 $\times$ 4 in. aperture and 1 in. thick coils. The multipoles listed are for an error in the horizontal thickness of the coil of $\Delta H = -0.002$ in. and for an error in the vertical height of the coil of $\Delta W = -0.002$ in. In each case the coil perturbed is the coil in the upper right corner. Units are 10^{-4} /in. $^{\ell}$.

ℓ	$\Delta H = -0.002$ in.		$\Delta W = -0.002$ in.	
	b_{ℓ}	a_{ℓ}	b_{ℓ}	a_{ℓ}
0	0.000	-0.067	-0.008	0.150
1	0.003	0.055	0.011	-0.138
2	0.011	-0.029	-0.005	0.078
3	-0.000	0.013	0.017	-0.038
4	-0.002	-0.006	-0.015	0.017
5	0.000	0.003	0.008	-0.007
6	-0.001	-0.001	-0.005	0.002
7	0.000	0.001	0.002	-0.001
8	0.000	0.000	-0.001	0.000

appear constrained so as not to be able to move in certain directions. However, in an actual magnet there are usually spaces between the coils and the iron so that they can move. The rms result given in Table I is thus twice the square root of the sum of squares of the multipoles generated by the displacements $\Delta x = 0.002$ in. and $\Delta y = 0.002$ in. which are also given in Table I.

Table II gives results for the multipoles generated in the median plane when the horizontal width of a coil is changed by an amount of $\Delta H = -0.002$ in., and when the vertical width of a coil is changed by an amount $\Delta W = -0.002$ in. In each case, the coil involved is the coil in the upper right-hand corner. The position of the center of the coil is unchanged. These results are given just as additional information. They were not used in computing the random error multipoles. One sees that the effect of ΔH and ΔW is considerably smaller than the effect of a corresponding Δx or Δy .

Comparison with the Cosine Magnet

It is interesting to compare the random error multipoles found for the window-frame magnet with those found for the cosine magnet. This comparison is shown in Table III for a cosine magnet^{2,3} having 4 in. circular aperture with 0.75 in. thick coils and for a window-frame magnet having a 4 x 4 in. aperture and 1 in. thick coils assuming a random error of $\epsilon = 0.002$ in. in the positions of the current blocks. One sees that the error random multipoles in each magnet are similar for the lower order multipoles $\ell = 0, 1$, but window-frame multipoles drop faster with ℓ than the cosine-magnet multipoles. Regarding the nonlinear multipoles, $\ell = 2$ and higher, the window-frame magnet is a factor of 3 or more better than the cosine magnet.

It is interesting to ask whether experimental measurements on model window-frame⁴ and cosine magnets⁵ agree with the above comparison of the

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TABLE III. A comparison of the theoretical results for the random error multipoles in a cosine magnet and a window-frame magnet for a random error of 0.002 in. in each current block. The window-frame magnet has a 4 x 4 in. aperture and 4 current blocks which are each 1 in. thick. The cosine magnet has a 4 in. aperture and 24 current blocks which are 0.75 in. thick. Units are $10^{-4}/\text{in.}^\ell$.

ℓ	Cosine Magnet rms		Window Frame rms	
	b_ℓ	a_ℓ	b_ℓ	a_ℓ
0	0.00	3.84	0.504	2.418
1	2.92	2.92	0.814	1.85
2	2.26	2.26	0.666	0.740
3	1.62	1.62	0.340	0.213
4	1.10	1.10	0.134	0.102
5	0.72	0.72	0.048	0.038
6	0.46	0.46	0.014	0.018
7	0.30	0.30	0.006	0.004
8	0.24	0.24	0.004	0.002

TABLE IV. A comparison of the measured and computed random error multipoles in a window-frame magnet with a 4 x 4 in. aperture and 1 in. thick coils. The computed result is for a random error of 0.002 in. in the 4 current blocks. Units are $10^{-4}/\text{in.}^\ell$.

ℓ	Window-Frame Magnet			
	b_ℓ		a_ℓ	
	Measured	Computed	Measured	Computed
1	2.0	0.81	0.79	1.85
3	0.84	0.34	0.20	0.21
5	0.105	0.048	0.064	0.38
7	0.044	0.006	0.011	0.004
9	0.019	0.000	0.017	0.000

cosine and window-frame magnets. The measured results so far have some confusing points in them but one might tentatively say that they indicate that the lower error multipoles in the two magnets are roughly equal, and that the window-frame magnet may have smaller nonlinear multipoles.

Comparison with Measured Results

In Table IV the computed results are compared with experimentally measured results for the 20 in. model of an 8° bending magnet.⁴ Since the computed rms result is a statistical result, one cannot expect exact agreement with a particular set of measured results. The measured results shown in Table IV appear roughly consistent with the computed results for an rms error of $\epsilon = 0.002$ in. in the current block positions.

The measured results given in Table IV are the maximum of the error multipoles measured between 5 kG and 32 kG except in the case of b_1 . The measured b_1 was very large, which was attributed to the stainless steel used in the rectangular structure and the value listed for b_1 was suggested by John Jackson. The comparison in Table IV is made for the odd b_n and odd a_n since these multipoles are likely to be due only to random errors.