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Argonne National Laboratory

TABLES OF RESONANCE INTEGRAL FUNCTIONS, $L'(Z', \xi)$

by

Edwin M. Pennington

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 $L'(Z', \xi)$

by

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ABSTRACT

Tables of the function $L'(Z', \xi)$, which occurs in the calculation of resonance integrals for absorbers without admixed moderator in the NRIA approximation, have been computed by use of the CDC-3600. The values are calculated in slab, cylindrical, and spherical geometry for $\xi = 0.02, (0.01), 0.05, (0.05), 0.50$, and ∞ , and for $j = -2, (0.5), 14$ where $Z' = 2^{-j} \times 10^4$.

I. INTRODUCTION

The function $L'(Z', \xi)$, which is defined by

$$L'(Z', \xi) = \int_0^\infty \psi(\xi, x) P_0(Z' \psi) dx, \quad (1)$$

occurs in the calculation of resonance integrals for absorbers without any admixed moderator in the NRIA approximation.⁽¹⁾ In Eq. (1), $\psi(\xi, x)$ is the Doppler-broadened line-shape function (Voigt profile), which is given by

$$\psi(\xi, x) = \frac{\xi}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp[-\xi^2(x-y)^2/4]}{1+y^2} dy; \quad (2)$$

$P_0(Z' \psi)$ is the escape probability from the absorber element for a uniform and isotropic neutron source. The parameter

$$Z' = a N \sigma_0 (\Gamma_\gamma / \Gamma),$$

where a is the radius of a cylindrical or spherical fuel element, or the thickness of a slab element, N is the absorber atom density, and σ_0 , Γ_γ , and Γ are resonance parameters having their usual definitions. Thus $Z' \psi$ is the macroscopic absorption cross section of the absorber element multiplied by the geometric quantity a .

In the NRLIA approximation for a pure absorber, the resonance integral resulting from a single resonance consists of the surface contribution

$$I(S) = (\sigma_0 \Gamma_\gamma / E_i) L'(Z', \xi).$$

Sometimes a volume contribution

$$I(V) = \xi \sigma_s,$$

is added, where $\xi \sigma_s$ is the slowing-down power of an absorber atom. However, the surface contribution alone approaches the infinite dilution limit of $\pi \sigma_0 \Gamma_\gamma / 2 E_i$ for very thin elements.

In converting the GAM-I code from the IBM-704 to the CDC-3600, it was discovered⁽²⁾ that the L' tables on the old GAM-I tape were inaccurate, especially for small values of j , where $Z' = 2^{-j} \times 10^4$. (For cylindrical geometry the tables on the GAM-I tape are the same as in Ref. 1.) It was decided to recalculate the values of L' for slab, cylindrical, and spherical geometries, and to extend the tables somewhat. A Fortran program written for the CDC-3600 was used to compute values of L' for $\xi = 0.02, (0.01), 0.05, (0.05), 0.50$, and ∞ , and for $j = -2, (0.5), 14$.

II. CALCULATIONAL METHODS

The integral in Eq. (1) was evaluated by 16-point Gaussian integration repeated over successive ranges of x from $x = 0$ to $x = 5000$, the limits of the successive ranges being determined as described in Sect. A below. The formula for 16-point Gaussian integration is

$$\int_a^b f(x) dx = \frac{(b-a)}{2} \sum_{i=1}^{16} W_i f\left[\left(\frac{b-a}{2}\right) \mu_i + \left(\frac{b+a}{2}\right)\right], \quad (3)$$

where values of W_i and μ_i are given on p. 916 of Ref. 3. Calculations of the $\psi(\xi, x)$ Voigt profile function was accomplished by use of the O'Shea-Thacher subroutine.⁽⁴⁾ The subroutines written to compute P_0 for the various geometries are described below in Sect. B. An asymptotic contribution depending on geometry and given in Sect. C was added to the integral over x from 0 to 5000 in order to yield the complete function $L'(Z', \xi)$.

A. Determination of Integration Ranges

In order to estimate the ranges of x for the Gaussian integration, we consider the case of the natural line shape and the Wigner approximation for P_0 . For this case

$$\psi(\infty, x) = 1/(1+x^2)$$

and

$$P_0^W(Z', \psi) = \frac{1}{1 + \frac{\alpha Z'}{1+x^2}};$$

here $\alpha = 2$ for slabs or cylinders and $4/3$ for spheres. Thus

$$L'^W(Z', \infty) = \int_0^\infty P_0^W(Z', \psi) \psi(\infty, x) dx = \int_0^\infty \frac{dx}{1 + \alpha Z' + x^2} = \frac{1}{(1 + \alpha Z')^{1/2}} \int_0^\infty \frac{dz}{1 + z^2}, \quad (4)$$

where

$$z = x/(1 + \alpha Z')^{1/2}.$$

A program was written for the CDC-160A computer to calculate

$$\int_{z_1}^{z_2} \frac{dz}{1 + z^2}$$

by 16-point Gaussian integration and compare results with the exact value of $\tan^{-1}z_2 - \tan^{-1}z_1$. From this program it was found that ranges in z bounded by $z = 0, 5, 10, 18, 30, 60, 120, 250, 1000$, and 5000 would yield accuracy to five or six significant figures in the integrals. The corresponding values of x are $(1 + \alpha Z')^{1/2}$ times the values of z , with the final value being taken as 5000 when a value of z is reached such that $(1 + \alpha Z')^{1/2}z$ is greater than 5000 .

It was then found, by means of another preliminary program to calculate $L'(Z', \xi)$ on the 160A computer, that halving the ranges of integration calculated as just described sometimes gave results in agreement to six figures with the original ones. For cases where ξ and j were both small (e.g., $\xi = 0.02$ and $j = -2$), there was a few tenths of a percent difference in the results. This difference arose entirely from the first integration range in x which corresponds to the range in z from 0 to 5 . It is caused by the fact that the natural line shape is much different from the Doppler-broadened line shape for small values of ξ and x , and so the integral for small values of x is not behaving as suggested by Eq. (4). A rough estimate of suitable ranges of integration for small x may be made by writing

$$\psi(\xi, x) = C/(1+x^2)$$

where $C = \psi(\xi, 0)$. On substituting this into the expression for L^1W , we get

$$C \int \frac{dx}{1 + \alpha Z^1 C + x^2}.$$

This suggests dividing the original range in x , corresponding to the range in z from 0 to 5, into M subranges, where M is the first integer greater than

$$\frac{(1 + \alpha Z^1)^{1/2}}{(1 + \alpha C Z^1)^{1/2}}.$$

For $\alpha = 2$ (slab or cylinder), values of M range from 2 for large ξ and small Z^1 , to 8 for small ξ and large Z^1 .

B. Formulae for Escape Probability

1. Slab Geometry

The escape probability in slab geometry is⁽⁵⁾

$$P_0(x) = \frac{1}{x} \left[\frac{1}{2} - E_3(x) \right]. \quad (5)$$

For values of $x < 1$, the series expansion for E_3 (see Ref. 5, p. 154) gives

$$\begin{aligned} P_0(x) = 1 - \frac{x}{2} \left(\frac{3}{2} - \gamma - \ln x \right) - \frac{x^2}{6} + \frac{x^3}{48} - \frac{x^4}{360} + \frac{x^5}{2880} - \frac{x^6}{25200} + \frac{x^7}{241,920} \\ - \frac{x^8}{2,540,160} + \frac{x^9}{29,030,400}, \end{aligned} \quad (6)$$

where $\gamma = 0.5772156649$. In the above series, terms through x^4 were included in the P_0 subroutine for $10^{-7} \leq x \leq 0.1$, terms through x^7 for $0.1 < x \leq 0.5$, and terms through x^9 for $0.5 < x \leq 1.0$.

For values of $x > 1$, the rational approximation for $E_1(x)$ given as Eq. 5.1.56 of Ref. 3 was used along with the recurrence relation for E_n (see Ref. 5, p. 154), which yields

$$P_0(x) = \frac{1}{2x} \left[1 - (1-x) e^{-x} - x^2 E_1(x) \right]. \quad (7)$$

The approximation $P_0(x) = 1/(2x)$ was used for $x > 12$.

2. Cylindrical Geometry

In cylindrical geometry, the escape probability is given by

$$\begin{aligned}
 P_0(x) &= 2x \int_x^\infty \frac{I_1(x) K_1(x)}{x^2} dx \\
 &= \frac{2}{3}x \left\{ 2 \left[x \left(K_1(x) I_1(x) + K_0(x) I_0(x) \right) - 1 \right] + \frac{K_1(x) I_1(x)}{x} \right. \\
 &\quad \left. - K_0(x) I_1(x) + K_1(x) I_0(x) \right\}. \tag{8}
 \end{aligned}$$

The following expressions were used in the P_0 subroutine for cylindrical geometry:

$$P_0(x) = 1 - \frac{4}{3}x, \quad 0 \leq x \leq 10^{-5}; \tag{9}$$

$$\begin{aligned}
 P_0(x) &= 1 - \frac{4}{3}x + \frac{1}{2}x^2 \left(\frac{5}{4} - \gamma \right) + \frac{1}{24}x^4 \left(\frac{7}{6} - \gamma \right) + \frac{1}{384}x^6 \left(\frac{11}{8} - \gamma \right) \\
 &\quad + \frac{1}{2}x^2 \left(1 + \frac{x^2}{12} + \frac{x^4}{192} \right) \ln \left(\frac{2}{x} \right), \quad 10^{-5} < x \leq 0.4; \tag{10}
 \end{aligned}$$

$$\begin{aligned}
 P_0(x) &= 1 - \left(\frac{0.29991x + 2.95455x^2 + 1.8879x^3 + 0.99821x^4}{0.22621 + 2.65415x + 4.09779x^2 + 2.37943x^3 + x^4} \right), \\
 &\quad 0.4 < x \leq 3.7; \tag{11}
 \end{aligned}$$

$$P_0(x) = \frac{1}{2x} \left(1 - \frac{3}{16x^2} - \frac{15}{128x^4} - \frac{1575}{4096x^6} \right), \quad 3.7 < x < \infty. \tag{12}$$

In the above, the expressions for small x and for large x were obtained from series and asymptotic expansions, respectively, of the Bessel functions involved. The rational approximation for $0.4 < x \leq 3.7$ was obtained from Ref. 6.

3. Spherical Geometry

The escape probability in spherical geometry is given by⁽⁵⁾

$$P_0(x) = \frac{3}{8x^3} \left[2x^2 - 1 + (1 + 2x) e^{-2x} \right]. \tag{13}$$

For $0 \leq x \leq 0.2$, the series expansion

$$P_0(x) = 1 - \frac{3}{4}x + \frac{2}{5}x^2 - \frac{1}{6}x^3 + \frac{2}{35}x^4 - \frac{1x^5}{60} \quad (14)$$

was used. For $x > 0.2$, Eq. (13) was used as it stands, with the e^{-2x} term being omitted for $x > 6$.

C. Asymptotic Contribution to L'

The asymptotic contribution to L' is given by

$$L'_{as}(Z', \xi) = \int_{x_m}^{\infty} \psi(\xi, x) P_0(Z' \psi) dx, \quad (15)$$

where x_m is 5000. In evaluating this contribution, we use the asymptotic formula⁽⁷⁾

$$\psi(\xi, x) \rightarrow \frac{1}{1+x^2} \left[1 + \frac{2}{\xi^2} \frac{3x^2-1}{(1+x^2)^2} \right] = \frac{1}{x^2} \left[1 + \left(\frac{6}{\xi^2} - 1 \right) \frac{1}{x^2} + \dots \right] \quad (16)$$

and the expansions of $P_0(Z' \psi)$ for small argument, omitting terms of higher order than $1/x^2$ in comparison with unity in the product ψP_0 . The integral in Eq. (15) is then easily evaluated analytically. The following results are obtained for the three geometries under investigation:

$$\text{Slab: } L'_{as}(Z', \xi) = \frac{1}{x_m} \left[1 - \frac{Z'}{3} \frac{\ln x_m}{x_m^2} + \left\{ \frac{2}{\xi^2} - \frac{1}{3} - \frac{Z'}{6} \left(\frac{13}{6} - \gamma - \ln Z' \right) \right\} \frac{1}{x_m^2} \right]; \quad (17)$$

$$\text{Cylinder: } L'_{as}(Z', \xi) = \frac{1}{x_m} \left[1 + \left(\frac{2}{\xi^2} - \frac{1}{3} - \frac{4Z'}{9} \right) \frac{1}{x_m^2} \right]; \quad (18)$$

$$\text{Sphere: } L'_{as}(Z', \xi) = \frac{1}{x_m} \left[1 + \left(\frac{2}{\xi^2} - \frac{1}{3} - \frac{1}{4} Z' \right) \frac{1}{x_m^2} \right]. \quad (19)$$

III. TABLES OF $L'(Z', \xi)$

Computed values of $L'(Z', \xi)$ for slabs, cylinders, and spheres are listed in Tables I, II, and III, respectively. The notation is such that $6467-2 = 0.6467 \times 10^{-2} = 0.006467$. A listing of the Fortran program for the calculation of L' along with the subroutines for calculating ψ and P_0 for the three geometries is given in the Appendix.

Table I
THE FUNCTION $L'(Z', \xi)$ FOR SLABS OF THICKNESS a ; $Z' = 2^{-j} \times 10^4$

ξ	0.02	0.03	0.04	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	∞
-2.0	6467-2	6101-2	6001-2	5964-2	5922-2	5914-2	5911-2	5910-2	5910-2	5909-2	5909-2	5909-2	5909-2	5908-2
-1.5	8017-2	7383-2	7195-2	7124-2	7049-2	7036-2	7032-2	7030-2	7028-2	7028-2	7027-2	7027-2	7027-2	7026-2
-1.0	1005-1	9011-2	8665-2	8531-2	8394-2	8372-2	8365-2	8361-2	8360-2	8358-2	8358-2	8357-2	8357-2	8355-2
-0.5	1276-1	1110-1	1050-1	1025-1	1000-1	9965-2	9952-2	9946-2	9943-2	9941-2	9940-2	9939-2	9939-2	9936-2
0	1639-1	1382-1	1283-1	1240-1	1193-1	1186-1	1184-1	1183-1	1183-1	1182-1	1182-1	1182-1	1182-1	1182-1
0.5	2127-1	1742-1	1584-1	1510-1	1424-1	1413-1	1410-1	1408-1	1407-1	1407-1	1406-1	1406-1	1406-1	1405-1
1.0	2785-1	2221-1	1978-1	1856-1	1705-1	1685-1	1679-1	1676-1	1674-1	1673-1	1673-1	1673-1	1672-1	1671-1
1.5	3673-1	2861-1	2496-1	2306-1	2048-1	2011-1	2000-1	1995-1	1993-1	1991-1	1990-1	1990-1	1989-1	1987-1
2.0	4869-1	3720-1	3185-1	2895-1	2472-1	2405-1	2385-1	2377-1	2373-1	2370-1	2368-1	2367-1	2367-1	2363-1
2.5	6477-1	4873-1	4104-1	3675-1	3003-1	2883-1	2848-1	2834-1	2826-1	2822-1	2819-1	2817-1	2816-1	2810-1
3.0	8629-1	6419-1	5333-1	4711-1	3678-1	3471-1	3408-1	3382-1	3369-1	3362-1	3357-1	3354-1	3351-1	3342-1
3.5	1150 0	8486-1	6975-1	6092-1	4546-1	4202-1	4090-1	4044-1	4021-1	4008-1	4000-1	3994-1	3990-1	3974-1
4.0	1529 0	1124 0	9164-1	7930-1	5674-1	5122-1	4930-1	4847-1	4807-1	4784-1	4769-1	4760-1	4753-1	4725-1
4.5	2024 0	1487 0	1207 0	1037 0	7149-1	6294-1	5975-1	5833-1	5760-1	5694-1	5677-1	5666-1	5666-1	5619-1
5.0	2662 0	1964 0	1590 0	1360 0	9087-1	7802-1	7291-1	7052-1	6927-1	6855-1	6812-1	6782-1	6761-1	6681-1
5.5	3463 0	2578 0	2090 0	1785 0	1163 0	9755-1	8965-1	8578-1	8368-1	8243-1	8167-1	8117-1	8082-1	7944-1
6.0	4430 0	3352 0	2733 0	2335 0	1498 0	1230 0	1111 0	1051 0	1017 0	9961-1	9828-1	9740-1	9680-1	9444-1
6.5	5540 0	4294 0	3538 0	3036 0	1933 0	1560 0	1388 0	1297 0	1243 0	1210 0	1188 0	1173 0	1163 0	1123 0
7.0	6744 0	5383 0	4508 0	3902 0	2496 0	1989 0	1746 0	1611 0	1530 0	1478 0	1444 0	1420 0	1403 0	1334 0
7.5	7977 0	6574 0	5618 0	4927 0	3208 0	2540 0	2206 0	2014 0	1896 0	1818 0	1765 0	1727 0	1700 0	1585 0
8.0	9176 0	7804 0	6820 0	6077 0	4084 0	3238 0	2793 0	2529 0	2361 0	2248 0	2169 0	2112 0	2069 0	1883 0
8.5	1029 1	9010 0	8048 0	7294 0	5114 0	4095 0	3529 0	3180 0	2950 0	2791 0	2678 0	2594 0	2531 0	2236 0
9.0	1129 1	1014 1	9241 0	8514 0	6263 0	5107 0	4423 0	3983 0	3683 0	3470 0	3314 0	3197 0	3107 0	2653 0
9.5	1215 1	1115 1	1035 1	9678 0	7473 0	6240 0	5464 0	4941 0	4571 0	4300 0	4096 0	3939 0	3817 0	3144 0
10.0	1288 1	1204 1	1134 1	1074 1	8679 0	7438 0	6611 0	6028 0	5600 0	5277 0	5027 0	4830 0	4672 0	3721 0
10.5	1349 1	1279 1	1219 1	1168 1	9826 0	8638 0	7807 0	7196 0	6733 0	6372 0	6086 0	5855 0	5666 0	4394 0
11.0	1398 1	1341 1	1292 1	1249 1	1087 1	9782 0	8988 0	8385 0	7913 0	7535 0	7229 0	6976 0	6765 0	5171 0
11.5	1437 1	1391 1	1352 1	1316 1	1179 1	1083 1	1010 1	9534 0	9078 0	8706 0	8396 0	8137 0	7916 0	6051 0
12.0	1468 1	1432 1	1400 1	1371 1	1258 1	1175 1	1111 1	1060 1	1018 1	9826 0	9531 0	9278 0	9061 0	7020 0
12.5	1493 1	1464 1	1439 1	1416 1	1323 1	1254 1	1199 1	1155 1	1117 1	1086 1	1058 1	1035 1	1015 1	8047 0
13.0	1511 1	1489 1	1470 1	1451 1	1377 1	1320 1	1275 1	1236 1	1204 1	1177 1	1153 1	1132 1	1114 1	9088 0
13.5	1526 1	1509 1	1494 1	1479 1	1421 1	1375 1	1337 1	1305 1	1278 1	1255 1	1235 1	1217 1	1201 1	1010 1
14.0	1537 1	1524 1	1512 1	1501 1	1455 1	1419 1	1388 1	1362 1	1340 1	1321 1	1304 1	1288 1	1275 1	1104 1

Table II
THE FUNCTION $L'(Z', \xi)$ FOR CYLINDERS OF RADIUS a ; $Z' = 2^{-j} \times 10^4$

ξ	0.02	0.03	0.04	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	∞
-2.0	6616-2	6278-2	6186-2	6150-2	6108-2	6101-2	6098-2	6097-2	6097-2	6096-2	6096-2	6096-2	6096-2	6095-2
-1.5	8178-2	7582-2	7410-2	7344-2	7270-2	7258-2	7254-2	7252-2	7251-2	7250-2	7250-2	7249-2	7249-2	7248-2
-1.0	1023-1	9233-2	8912-2	8789-2	8657-2	8636-2	8629-2	8626-2	8624-2	8623-2	8622-2	8622-2	8621-2	8620-2
-0.5	1295-1	1134-1	1078-1	1055-1	1031-1	1028-1	1027-1	1026-1	1026-1	1026-1	1025-1	1025-1	1025-1	1025-1
0	1660-1	1408-1	1314-1	1274-1	1230-1	1224-1	1222-1	1221-1	1220-1	1220-1	1220-1	1220-1	1219-1	1219-1
0.5	2152-1	1770-1	1618-1	1548-1	1468-1	1458-1	1454-1	1452-1	1452-1	1451-1	1451-1	1451-1	1450-1	1450-1
1.0	2814-1	2252-1	2014-1	1898-1	1757-1	1737-1	1731-1	1729-1	1727-1	1726-1	1726-1	1725-1	1725-1	1724-1
1.5	3709-1	2897-1	2536-1	2351-1	2108-1	2073-1	2063-1	2058-1	2056-1	2054-1	2053-1	2052-1	2052-1	2050-1
2.0	4916-1	3763-1	3230-1	2945-1	2541-1	2478-1	2459-1	2451-1	2447-1	2445-1	2443-1	2442-1	2441-1	2438-1
2.5	6541-1	4926-1	4157-1	3731-1	3081-1	2969-1	2936-1	2922-1	2915-1	2911-1	2908-1	2906-1	2905-1	2899-1
3.0	8717-1	6487-1	5396-1	4776-1	3765-1	3570-1	3511-1	3487-1	3474-1	3467-1	3462-1	3459-1	3457-1	3448-1
3.5	1162 0	8576-1	7054-1	6169-1	4642-1	4315-1	4211-1	4167-1	4145-1	4133-1	4125-1	4119-1	4116-1	4100-1
4.0	1546 0	1136 0	9268-1	8026-1	5780-1	5249-1	5068-1	4992-1	4953-1	4931-1	4918-1	4908-1	4902-1	4875-1
4.5	2050 0	1505 0	1221 0	1050 0	7270-1	6436-1	6133-1	6000-1	5932-1	5894-1	5870-1	5854-1	5842-1	5797-1
5.0	2701 0	1989 0	1609 0	1377 0	9227-1	7961-1	7469-1	7243-1	7126-1	7058-1	7017-1	6990-1	6970-1	6893-1
5.5	3527 0	2615 0	2118 0	1807 0	1180 0	9936-1	9166-1	8796-1	8597-1	8481-1	8408-1	8361-1	8328-1	8195-1
6.0	4542 0	3412 0	2774 0	2367 0	1519 0	1251 0	1134 0	1075 0	1043 0	1023 0	1011 0	1003 0	9968-1	9743-1
6.5	5736 0	4396 0	3604 0	3085 0	1961 0	1585 0	1414 0	1324 0	1273 0	1241 0	1220 0	1206 0	1197 0	1158 0
7.0	7062 0	5563 0	4622 0	3984 0	2533 0	2020 0	1776 0	1643 0	1564 0	1514 0	1481 0	1458 0	1442 0	1377 0
7.5	8438 0	6871 0	5818 0	5070 0	3263 0	2581 0	2243 0	2052 0	1935 0	1859 0	1807 0	1770 0	1744 0	1636 0
8.0	9774 0	8243 0	7142 0	6319 0	4172 0	3295 0	2841 0	2576 0	2408 0	2295 0	2218 0	2162 0	2121 0	1943 0
8.5	1099 1	9588 0	8513 0	7669 0	5266 0	4184 0	3596 0	3239 0	3007 0	2848 0	2735 0	2652 0	2591 0	2307 0
9.0	1205 1	1083 1	9841 0	9032 0	6516 0	5255 0	4527 0	4068 0	3759 0	3542 0	3384 0	3267 0	3177 0	2738 0
9.5	1292 1	1191 1	1105 1	1032 1	7858 0	6484 0	5635 0	5073 0	4682 0	4399 0	4189 0	4029 0	3904 0	3246 0
10.0	1362 1	1281 1	1210 1	1147 1	9206 0	7811 0	6887 0	6243 0	5777 0	5429 0	5163 0	4956 0	4791 0	3843 0
10.5	1416 1	1353 1	1296 1	1245 1	1047 1	9150 0	8215 0	7529 0	7011 0	6611 0	6297 0	6046 0	5842 0	4540 0
11.0	1458 1	1409 1	1365 1	1324 1	1160 1	1042 1	9531 0	8852 0	8319 0	7894 0	7549 0	7266 0	7032 0	5346 0
11.5	1489 1	1452 1	1419 1	1387 1	1255 1	1154 1	1076 1	1013 1	9616 0	9196 0	8846 0	8552 0	8303 0	6267 0
12.0	1512 1	1485 1	1459 1	1436 1	1332 1	1250 1	1184 1	1128 1	1082 1	1044 1	1010 1	9820 0	9575 0	7293 0
12.5	1528 1	1509 1	1490 1	1472 1	1393 1	1329 1	1274 1	1228 1	1189 1	1155 1	1125 1	1100 1	1077 1	8397 0
13.0	1540 1	1526 1	1512 1	1499 1	1440 1	1390 1	1347 1	1310 1	1278 1	1250 1	1225 1	1203 1	1183 1	9530 0
13.5	1549 1	1539 1	1529 1	1519 1	1476 1	1438 1	1405 1	1376 1	1350 1	1328 1	1307 1	1289 1	1273 1	1063 1
14.0	1555 1	1548 1	1541 1	1534 1	1502 1	1474 1	1449 1	1427 1	1407 1	1389 1	1373 1	1359 1	1346 1	1165 1

Table III
THE FUNCTION $L(\xi, \zeta)$ FOR SPHERES OF RADIUS a ; $Z' = 2^{-j} \times 10^4$

$j \backslash \xi$	0.02	0.03	0.04	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	∞
-2.0	8524-2	7885-2	7697-2	7625-2	7544-2	7530-2	7526-2	7524-2	7522-2	7522-2	7521-2	7521-2	7521-2	7520-2
-1.5	1067-1	9602-2	9262-2	9129-2	8983-2	8960-2	8953-2	8949-2	8947-2	8946-2	8945-2	8945-2	8944-2	8943-2
-1.0	1355-1	1181-1	1121-1	1097-1	1070-1	1066-1	1065-1	1065-1	1064-1	1064-1	1064-1	1064-1	1064-1	1063-1
-0.5	1738-1	1469-1	1368-1	1324-1	1277-1	1270-1	1267-1	1266-1	1266-1	1266-1	1265-1	1265-1	1265-1	1265-1
0	2256-1	1849-1	1686-1	1611-1	1524-1	1513-1	1509-1	1507-1	1506-1	1505-1	1505-1	1505-1	1505-1	1504-1
0.5	2955-1	2356-1	2102-1	1977-1	1824-1	1803-1	1797-1	1794-1	1792-1	1791-1	1790-1	1790-1	1790-1	1788-1
1.0	3899-1	3035-1	2650-1	2452-1	2190-1	2152-1	2141-1	2136-1	2133-1	2131-1	2130-1	2129-1	2129-1	2127-1
1.5	5171-1	3947-1	3380-1	3076-1	2641-1	2573-1	2553-1	2544-1	2539-1	2537-1	2535-1	2534-1	2533-1	2529-1
2.0	6882-1	5172-1	4355-1	3901-1	3205-1	3084-1	3048-1	3033-1	3025-1	3020-1	3017-1	3015-1	3014-1	3008-1
2.5	9177-1	6817-1	5660-1	5000-1	3919-1	3710-1	3646-1	3619-1	3606-1	3598-1	3593-1	3589-1	3587-1	3577-1
3.0	1233 0	9017-1	7406-1	6466-1	4838-1	4487-1	4374-1	4327-1	4303-1	4289-1	4281-1	4275-1	4270-1	4253-1
3.5	1627 0	1195 0	9736-1	8420-1	6032-1	5462-1	5267-1	5185-1	5143-1	5119-1	5104-1	5094-1	5087-1	5058-1
4.0	2156 0	1582 0	1283 0	1102 0	7595-1	6703-1	6378-1	6234-1	6161-1	6119-1	6093-1	6076-1	6063-1	6014-1
4.5	2838 0	2090 0	1692 0	1446 0	9650-1	8299-1	7773-1	7530-1	7404-1	7331-1	7286-1	7256-1	7235-1	7151-1
5.0	3701 0	2747 0	2225 0	1898 0	1236 0	1037 0	9547-1	9150-1	8936-1	8812-1	8733-1	8681-1	8645-1	8502-1
5.5	4760 0	3580 0	2912 0	2485 0	1591 0	1306 0	1182 0	1119 0	1084 0	1064 0	1050 0	1041 0	1035 0	1011 0
6.0	6003 0	4607 0	3780 0	3236 0	2055 0	1657 0	1476 0	1380 0	1325 0	1291 0	1268 0	1254 0	1243 0	1202 0
6.5	7378 0	5822 0	4841 0	4175 0	2654 0	2113 0	1855 0	1714 0	1629 0	1576 0	1540 0	1515 0	1498 0	1428 0
7.0	8798 0	7180 0	6086 0	5306 0	3417 0	2700 0	2344 0	2142 0	2017 0	1936 0	1880 0	1842 0	1813 0	1697 0
7.5	1016 1	8596 0	7459 0	6605 0	4366 0	3447 0	2969 0	2689 0	2511 0	2392 0	2309 0	2250 0	2206 0	2016 0
8.0	1138 1	9970 0	8873 0	8003 0	5505 0	4374 0	3757 0	3382 0	3137 0	2969 0	2849 0	2762 0	2696 0	2393 0
8.5	1241 1	1121 1	1022 1	9402 0	6804 0	5489 0	4728 0	4246 0	3921 0	3693 0	3526 0	3402 0	3307 0	2840 0
9.0	1325 1	1227 1	1143 1	1071 1	8193 0	6767 0	5881 0	5292 0	4882 0	4586 0	4364 0	4196 0	4064 0	3366 0
9.5	1390 1	1314 1	1246 1	1185 1	9575 0	8140 0	7180 0	6508 0	6020 0	5656 0	5377 0	5159 0	4986 0	3984 0
10.0	1439 1	1381 1	1328 1	1279 1	1086 1	9514 0	8551 0	7840 0	7301 0	6883 0	6554 0	6291 0	6077 0	4705 0
10.5	1476 1	1432 1	1392 1	1354 1	1197 1	1079 1	9899 0	9202 0	8652 0	8210 0	7851 0	7556 0	7311 0	5538 0
11.0	1502 1	1471 1	1441 1	1412 1	1289 1	1191 1	1113 1	1050 1	9979 0	9547 0	9186 0	8882 0	8623 0	6487 0
11.5	1522 1	1499 1	1477 1	1456 1	1362 1	1284 1	1220 1	1165 1	1119 1	1080 1	1047 1	1018 1	9925 0	7543 0
12.0	1536 1	1519 1	1503 1	1488 1	1418 1	1358 1	1307 1	1263 1	1224 1	1191 1	1162 1	1136 1	1113 1	8676 0
12.5	1546 1	1534 1	1523 1	1511 1	1460 1	1415 1	1376 1	1341 1	1310 1	1283 1	1259 1	1237 1	1218 1	9834 0
13.0	1553 1	1545 1	1537 1	1528 1	1491 1	1458 1	1428 1	1402 1	1378 1	1357 1	1338 1	1320 1	1305 1	1095 1
13.5	1558 1	1552 1	1546 1	1541 1	1514 1	1489 1	1468 1	1448 1	1430 1	1414 1	1399 1	1386 1	1374 1	1196 1
14.0	1562 1	1558 1	1553 1	1549 1	1530 1	1512 1	1497 1	1482 1	1469 1	1457 1	1446 1	1436 1	1426 1	1283 1

APPENDIX

Listing of Fortran Program for Calculation
of $L'(Z', \xi)$ and Associated Subroutines

```

PROGRAM LPRIME
  DIMENSION XX(16),A(16),THETA(50),XJAY(50),FINT(30),XUP(30),
  1XLOW(30),XLPR(30),ZPR(50),XLPR1(50),ASYMP(50),TOTAL(50)
  DATA(XX=-.9894009,-.9445750,-.8656312,-.7554044,
  X      -.6178762,-.4580168,-.2816036,-.9501251E=01,
  X      .9501251E=01,.2816036,.4580168,.6178762,
  X      .7554044,.8656312,.9445750,.9894009),
  X      (A=.2715246E=01,.6225352E=01,.9515851E=01,.1246290,
  X      .1495960,.1691565,.1826034,.1894506,
  X      .1894506,.1826034,.1691565,.1495960,
  X      .1246290,.9515851E=01,.6225352E=01,.2715246E=01)
  5 READ INPUT TAPE60,10,KGEOM,NTHETA,NJAY,(THETA(I),I=1,NTHETA)
  7 READ INPUT TAPE60,12,(XJAY(I),I=1,NJAY)
  10 FORMAT(3I3/(6E12,6))
  12 FORMAT(6E12,6)
  GO TO(15,16,17),KGEOM
  15 WRITE OUTPUT TAPE61,18
  GO TO25
  16 WRITE OUTPUT TAPE61,19
  GO TO25
  17 WRITE OUTPUT TAPE61,20
  18 FORMAT(14H1SLAB GEOMETRY)
  19 FORMAT(21H1CYLINDRICAL GEOMETRY)
  20 FORMAT(19H1SPHERICAL GEOMETRY)
  25 WRITE OUTPUT TAPE61,27
  27 FORMAT(82H0
  X  CONTRIBUTIONS TO L PRIME/87H      XI      J
  X  Z PRIME      INTEGRATED      ASYMPTOTIC      TOTAL)
  GO TO(29,29,30),KGEOM
  29 ALPHA=2.
  GO TO32
  30 ALPHA=1.33333333
  32 DO33I=1,NJAY
  33 ZPR(I)=10000./((2.*XJAY(I))
C  START THETA DO LOOP
  DO200IT=1,NTHETA
  XI2=THETA(IT)/2.
  XISQPI=1.7724539*XI2
  XISQ=THETA(IT)*THETA(IT)
  XMAX=5000.
  T=1./XMAX
  T2=T*T
  T3=T2*T
  ASS=T+T3*(2./XISQ-.333333333)
C  START JAY DO LOOP
  DO180IJ=1,NJAY
C  DETERMINE GAUSSIAN INTEGRATION RANGES
  XLOW(1)=0.
  XLOW(2)=5.
  XLOW(3)=10.
  XLOW(4)=18.
  XLOW(5)=30.
  XLOW(6)=60.
  XLOW(7)=120.
  XLOW(8)=250.
  XLOW(9)=1000.
  DO35I=1,8
  35 XUP(I)=XLOW(I+1)
  XUP(9)=5000.
  40 XZRAT=SQRTF(1.+ALPHA*ZPR(IJ))
  ZED=5000./XZRAT
  DO45I=1,9

```



```

      IF(XUP(I)-ZED)45,45,42
42  K=I
      XUP(K)=ZED
      GO T050
45  CONTINUE
50  IF(THETA(I) $\approx$ 1000.)54,54,52
52  M=2
      GO T056
54  XXI2=0.
      CALL W(XXI2,XI2,REW,AIW)
      C=XISQPI*REW
      XZRLOW=SQRTF(1.+ALPHA*C*ZPR(IJ))
      M=XZRAT/XZRLOW+1.
56  XM=M
      DELZ=XUP(1)/XM
      K2=K+1
      DO58 I=1,K2
      N1=M+K-1
      N2=K+1-I
      XLOW(N1)=XLOW(N2)
58  XUP(N1)=XUP(N2)
      K=K2+M
      XUP(1)=DELZ
      DO60 I=2,M
      XLOW(I)=XLOW(I-1)+DELZ
60  XUP(I)=XUP(I-1)+DELZ
65  DO67 I=1,K
      XLOW(I)=XZRAT*XLOW(I)
67  XUP(I)=XZRAT*XUP(I)
C   CALCULATE GAUSSIAN QUADRATURE CONTRIBUTIONS FROM K INTERVALS
      XLPR1(IJ)=0.
      DO100 L=1,K
      FINT(L)=0.
      C1=(XUP(L)-XLOW(L))/2.
      C2=C1+XLOW(L)
      DO80 I=1,16
      XSTAR=XX(I)*C1+C2
      XXI2=XSTAR*XI2
      IF(THETA(I) $\approx$ 1000.)70,70,69
69  PSI=1./((1.+XSTAR*XSTAR)
      GO T072
70  CALL W(XXI2,XI2,REW,AIW)
      PSI=XISQPI*REW
72  SIG=ZPR(IJ)*PSI
      GO TO(75,76,77),KGEOM
75  CALL PS LAB(SIG,POUT,PSELF)
      GO T078
76  CALL PCYL(SIG,POUT,PSELF)
      GO T078
77  CALL PSPHER(SIG,POUT,PSELF)
78  FINTEG=POUT*PSI
80  FINT(L)=FINT(L)+A(I)*FINTEG
      XLPR(L)=C1*FINT(L)
100 XLPR1(IJ)=XLPR1(IJ)+XLPR(L)
C   CALCULATE ASYMPTOTIC CONTRIBUTION
      GO TO(120,140,160),KGEOM
120 ASYMP(IJ)=ASS-ZPR(IJ)*(,2649085,166666667*LOGF(ZPR(IJ)*T2))*T3
      GO T0180
140 ASYMP(IJ)=ASS-.444444444*ZPR(IJ)*T3
      GO T0180
160 ASYMP(IJ)=ASS-.25*ZPR(IJ)*T3
180 TOTAL(IJ)=XLPR1(IJ)+ASYMP(IJ)

```

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190 FORMAT(1H 6E15.6)
200 WRITE OUTPUT TAPE61,190,(THETA(IJ),XJAY(IJ),ZPR(IJ),XLPR1(IJ),
1ASYMP(IJ),TOTAL(IJ),IJ=1,NJAY)
GO TO5
END
SUBROUTINE W(REZ,AIMZ,REW,AIMW)
C A FASTER VERSION OF W DESIGNED FOR THE CDC 3600 D. O. SHEA 8/11/64
C CONTINUED FRACTION REPRESENTATIONS OF
C THE COMPLEX PROBABILITY INTEGRAL W(Z)
C THIS ROUTINE OBTAINS 5 PLACE VALUES OF W(Z)
C FOR ARGUMENTS OF Z IN THE UPPER HALF PLANE
DIMENSIONX(12),Y(12),AA(11),BA(11)
EQUIVALENCE
1(RBK,AJP),(RNKM1,C),
2(RAK,TEMP4),(RNKM2,A),(AIBK,AK),(AINKM1,D),
3(RNK,TEMPC),(AINKM2,B),(AINK,TEMPCD),(RDKM1,AM),
4(RDKM2,G),(AIDKM,EL),(RDK,TEMPCM),(AIDKM2,H),
5(AIDK,TEML)
DATA(X=
X .1000000E 01, .4444444E-01, .5442176E=02 ,.2119093E=02,
X .1126194E=02, .6983402E=03, .4752903E=03 ,.3443545E=03,
X .2609611E=03, .2045830E=03, .1646905E=03 ,.1354280E=03),
X(Y=
X -.3333333E 00, .4761904E=01, .1298701E=01 ,.6060606E=02,
X .3508772E=02, .2288329E=02, .1610305E=02 ,.1194743E=02,
X .9216589E=03, .7326007E=03, .5963029E=03 ,.4948045E=03),
X(AA=
X .2000000E 01, .1200000E 02, .3000000E 02,
X .5600000E 02, .9000000E 02, .1320000E 03, .1820000E 03,
X .2400000E 03, .3060000E 03, .3800000E 03, .4620000E 03),
X(BA=
X .5000000E 01, .9000000E 01, .1300000E 02,
X .1700000E 02, .2100000E 02, .2500000E 02, .2900000E 02,
X .3300000E 02, .3700000E 02, .4100000E 02, .4500000E 02)
27 R2=REZ*REZ
AI2=AIMZ*AIMZ
ABREZ=ABSF(REZ)
IF(ABREZ+1.25*AIMZ=5.0)102,102,100
100 IF(ABREZ+1.1*AIMZ=6.6)117,117,116
101 IF(ABREZ+1.43333*AIMZ=4.3)119,119,118
102 IF(ABREZ+1.863636*AIMZ=4.1)111,111,104
103 IF(AIMZ=1.5)110,120,120
104 IF(AIMZ=1.4)115,115,101
105 IF(ABREZ+1.07317*AIMZ=4.4)119,119,118
106 IF(ABREZ=2.7)127,128,128
107 IF(ABREZ=3.1)106,108,108
108 IF(ABREZ=3.4)129,130,130
109 IF(R2+1.18*AI2=5.76)103,107,107
110 IF(R2+1.7227*AI2=4.41)125,126,126
111 IF(R2+1.71*AI2=2.89)113,109,109
112 IF(R2+1.69*AI2=1.69)123,124,124
113 IF(R2+2.0408*AI2=1.0)114,112,112
114 IF(R2+1.5625*AI2=.25)121,122,122
115 IF(ABREZ+1.43333*AIMZ=4.3)120,120,105
116 NMAX=1
GO TO 15
117 NMAX=2
GO TO 15
118 NMAX=3
GO TO 15
119 NMAX=4
GO TO 15

```

```

120 NMAX=6
    GO TO 15
121 NMAX=2
    GO TO 20
122 NMAX=3
    GO TO 20
123 NMAX=4
    GO TO 20
124 NMAX=5
    GO TO 20
125 NMAX=6
    GO TO 20
126 NMAX=7
    GO TO 20
127 NMAX=8
    GO TO 20
128 NMAX=9
    GO TO 20
129 NMAX=10
    GO TO 20
130 NMAX=11

```

C WA IS OBTAINED FROM ASYMPTOTIC SERIES

```

15 RV=2,*(R2-AI2)
    AIBK=4,*.REZ*AIMZ
    AIDKM1=AIBK
    AIDKM2=0,
    AINKM2=0,
    RNKM2=0,
    TEMPM=0,
    TEMEL=0,
    RDKM2=1,
    RNKM1=-1.12837917*AIMZ
    AINKM1=1.12837917*REZ
    RDKM1=RV-1,
    DO11K=1,NMAX
    RAK=AA(K)
    RBK=RV-BA(K)
    RNK=RBK*RNKM1+RAK*RNKM2-AIBK*AINKM1
    AINK=RBK*AINKM1+RAK*AINKM2-AIBK*RNKM1
    AIDK=RBK*AIDKM1+RAK*AIDKM2-AIBK*RDKM1
    RDK=RBK*RDKM1+RAK*RDKM2-AIBK*AIDKM1
    RNKM2=RNKM1
    AINKM2=AINKM1
    RDKM2=RDKM1
    AIDKM2=AIDKM1
    RNKM1=RNK
    AINKM1=AINK
    RDKM1=RDK
11 AIDKM1=AIDK
12 AMAGN=RDK*RDK+AIDK*AIDK
    REW=(RNK*RDK+AINK*AIDK)/AMAGN
    AIMW=(RDK*AINK-AIDK*RNK)/AMAGN
65 RETURN

```

C WT IS OBTAINED FROM TAYLOR SERIES

```

20 TEMP1=R2+AI2
    TEMP2=2,*.TEMP1*TEMP1
    AJ=*(R2-AI2)/TEMP2
    AK=2,*.REZ*AIMZ/TEMP2
    C=0,
    B=0,
    D=0,
    G=0,

```

```

H=0.
EL=0.
A=1.
AM=1.
EXPON=EXPF(TEMP2*AJ)
EXPC=EXPON*COSF(TEMP2*AK)
EXPS=-EXPON*SINF(TEMP2*AK)
DO13K=1,NMAX
TEMP4=X(K)
AJP=AJ+Y(K)
RNK=RBK*RNKM1+RAK*RNKM2=AIBK*AINKM1
AINK=RBK*AINKM1+RAK*AINKM2+AIBK*RNKM1
AIDK=RBK*AIDKM1+RAK*AIDKM2+AIBK*RDKM1
RDK=RBK*RDKM1+RAK*RDKM2=AIBK*AIDKM1
RNKM2=RNKM1
AINKM2=AINKM1
RDKM2=RDKM1
AIDKM2=AIDKM1
RNKM1=RNK
AINKM1=AINK
RDKM1=RDK
13 AIDKM1=AIDK
6 TEMP7=(AM*AM*EL*EL)*1.7724539
REF=(AIMZ*(C*AM*D*EL)-REZ*(AM*D=C*EL))/
1TEMP7/TEMP1
AIMF=(AIMZ*(AM*D=C*EL)+REZ*(C*AM*D*EL)
1)/TEMP7/TEMP1
REW=EXPC-REF
AIMW=EXPS-AIMF
64 RETURN
END
SUBROUTINE PSLAB(X,POUT,PSELF)
IF(X=.0000001)10,10,20
10 POUT=1.
GO TO100
20 IF(X=1.)30,30,60
30 X2=X*X
POUT=1.-.5*X*(.922784335=LOGF(X))+X2*(.166666667+X*(.0208333333
1-.00277777778*X))
IF(X=.1)100,100,40
40 X5=X2*X2*X
POUT=POUT+X5*(.000347222+X*(.000039683+.000004134*X))
IF(X=.5)100,100,50
50 X8=X5*X2*X
POUT=POUT+X8*(.000000394+.000000034*X)
GO TO100
60 POUT=.5/X
IF(X=12.)70,70,100
70 PNUM=3.95849692+X*(17.4089299+X*(13.1680640+X*(1.99938317
1+.000006395*X)))
PDEN=3.95849692+X*(21.0996531+X*(25.6329561+X*(9.57332235+X)))
POUT=POUT*(1.-EXPF(-X)*PNUM/PDEN)
100 PSELF=1.-POUT
RETURN
END
SUBROUTINE PCYL(X,POUT,PSELF)
IF(X=.00001)10,10,20
10 POUT=1.-1.33333333*X
GO TO100
20 IF(X=.4)30,30,40
30 X2=X*X
POUT=1.+X*(.1.33333333+X*(.336392168+X2*(.0245604584

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1+.00207756337*X2)))
  POUT=POUT+X2*(.5+X2*(.0416666667+.00260416667*X2))*LOGF(2,/X)
  GO TO100
40 IF(X=3.7)50,50,60
50 PNUM=X*(.29991+X*(2.95455+X*(1.8879+.99821*X)))
  PDEN=.22621+X*(2.65415+X*(4.09779+X*(2.37943*X)))
  POUT=1.-PNUM/PDEN
  GO TO100
60 T=.5/X
  T2=T*T
  POUT=T*(1.+T2*(.75+T2*(1.875+24.609375*T2)))
100 PSELF=1.-POUT
  RETURN
  END
  SUBROUTINE PSPHER(X,POUT,PSELF)
  IF(X=.2)10,10,20
10 POUT=1.+X*(.75+X*(.4+X*(.166666667*X*(.0571428571
  1+.0166666667*X))))
  GO TO100
20 T=1./X
  T2=T*T
  POUT=.75+T*(1.-.5*T2)
  IF(X=6.)30,30,100
30 POUT=POUT+.375*T*T2*(1.+2.*X)*EXP(-2.*X)
100 PSELF=1.-POUT
  RETURN
  END

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REFERENCES

1. F. T. Adler, G. W. Hinman, and L. W. Nordheim, The Quantitative Evaluation of Resonance Integrals, Proceedings of the 2nd UN Intl. Conference on the Peaceful Uses of Atomic Energy, Geneva 16, 155-171 (1958).
2. G. C. Jensen, Applied Mathematics Division, Argonne National Laboratory.
3. M. Abramowitz and I. A. Stegun, editors, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, NBS Applied Mathematics Series 55 (June 1964).
4. D. M. O'Shea and Henry C. Thacher, Jr., Computation of Resonance Line Shape Functions, Trans. Am. Nuc. Soc. 6, No. 1, p. 36-37 (June 1963).
5. K. M. Case, F. de Hoffman, and G. Placzek, Introduction to the Theory of Neutron Diffusion, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (June 1953).
6. A. Jonsson, Theseus, A One Group Collision Probability Routine for Annular Systems, AEEW-R253 (April 1963).
7. L. Dresner, Resonance Absorption in Nuclear Reactors, Pergamon Press, New York (1960).