MASTER

LIQUID METAL BEARINGS TECHNOLOGY FOR LARGE, HIGH-TEMPERATURE, SODIUM ROTATING MACHINERY

fifth topical report
DEVELOPMENT AND TEST OF
HIGH-TEMPERATURE FILM PRESSURE INSTRUMENTATION
FOR USE IN LIQUID SODIUM

by
I. hoogenboom
c. kissinger

prepared for

U. S. ATOMIC ENERGY COMMISSION
Division of Reactor Development and Technology
Special Technology Branch

prepared under

contract no. AT(30-1)-3930
mti project no. 48576

JUNE 1971

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Printed in the United States of America
Available from
National Technical Information Services
Springfield, Virginia 22151
Price: Printed Copy \$3.00; Microfiche \$0.65

LIQUID METAL BEARINGS TECHNOLOGY FOR LARGE, HIGH-TEMPERATURE, SODIUM ROTATING MACHINERY

fifth topical report
DEVELOPMENT AND TEST OF
HIGH-TEMPERATURE FILM PRESSURE INSTRUMENTATION
FOR USE IN LIQUID SODIUM

by
I. hoogenboom
C. kissinger

prepared for
U. S. ATOMIC ENERGY COMMISSION
Division of Reactor Development and Technology
Special Technology Branch

prepared under
contract no. AT(30-1)-3930
mti project no. 48576

JUNE 1971

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MECHANICAL TECHNOLOGY INCORPORATED

ABSTRACT

A transducer for measurement of bearing film pressures in high-temperature liquid-sodium environment was developed, built and tested. The transducer uses a hermetically sealed, high-temperature, non-contacting probe that measures the deflections of a diaphragm exposed to the sodium film pressures. The probe is a high-temperature, driven-shield, capacitance probe that had been developed earlier. It has high frequency response (to 10 kHz), uses a tri-ax lead and the commercially available Wayne-Kerr electronic amplification system. The transducers were built for operation at up to 120 psig. Extensive tests were conducted in this range in both gas and sodium environment. Two transducers were also evaluated in sodium environment in a continuous 684 hours test at a temperature range of 1050 F to 1170 F, with daily calibrations in 10 psi increments in a range of 0 to 150 psig. Post test examination showed a zero drift probably resulting from over-pressurization and consequent yielding of the 347 stainless steel diaphragm causing a change in the thickness of the pressure sensing gap. This can be prevented in future, long term 1200 hours operation by an increase in transducer size and diaphragm thickness.

FOREWORD

This is the fifth Topical Report of the program on "Liquid Metal Bearings Technology for Large, High-Temperature, Sodium Rotating Machinery." This program is currently in progress at Mechanical Technology Incorporated, under Contract AT(30-1)-3930 from the U.S. Atomic Energy Commission, Division of Reactor Development and Technology.

The program is directed at generating new technology in the areas of bearings and bearing instrumentation, that is needed for the successful development of high capacity, sodium circulation pumps and other auxiliaries of liquid metal cooled, fast breeder reactor systems. Specifically, the program comprises:

1. Extension of lubrication theory to the region of operation where bearing flow and pressure generation are governed by turbulence and convective fluid inertia forces.
2. Identification of self-acting and externally-pressurized bearing geometries that have the greatest likelihood of successful operation in high capacity sodium pumps and other auxiliaries of liquid metal cooled fast breeder reactor systems, and generation of the design data necessary for their rational design and optimization.
3. Development of accurate instrumentation for measurement of film thickness and film pressure directly in a high temperature sodium ambient.
4. Experimental evaluation of selected self-acting bearings in high temperature liquid sodium.
5. Screening, selection and evaluation of surface materials for high temperature sodium lubricated bearings.

The extension of lubrication theory, as called for in Item 1 above, was completed and described in the first topical report, NYO 3930-2. This was the basic analytical effort of the program since the sodium lubricated bearings required for high-capacity, sodium pumps will operate in the turbulent fluid inertial flow regime by virtue of their size, geometry, surface speed, lubricant properties and Reynolds number. This regime of operation had not, in the past, received comprehensive analytical treatment in part because of its complexity, but also because of the lack of practical bearing applications in this flow regime.

The analytical development was complemented by experiments conducted with a 12 inch diameter journal bearing lubricated with a non-corrosive fluid of low viscosity to provide an early check on the validity and accuracy of the analysis. Good correlation was obtained between the calculated and measured data, as described in NYO 3930-2.

The analysis and numerical solution methods were then used to generate a broad

range of bearing design data applicable to large, self-acting sodium lubricated bearings and necessary for their rational design and optimization in specific applications. This calculated performance data for self-acting bearings operating in the turbulent flow regime, was documented in the second topical report, NYO 3930-5.

In the third topical report, NYO 3930-9, the turbulent lubrication analysis was extended to cover the mode of operation of hydrostatic bearings. Calculated performance data for externally-pressurized bearings operating in turbulent flow regime was generated, based on this analysis, and presented for a wide range of operating conditions.

In the fourth topical report, the development and testing of proximity probes for operation in liquid sodium at elevated temperatures, was described. This fifth report covers the parallel development and testing of the film pressure transducers.

The concluding parts of the program, notably the development of the test facility, the self-acting bearing tests and the screening of surface materials will be described in a final topical report scheduled for issue later this year.

E. B. Arwas
Program Manager
Mechanical Technology Incorporated

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	ii
FOREWORD	iii
LIST OF FIGURES	vi
I. INTRODUCTION	1
II. CONCEPT SELECTION, DESIGN AND FABRICATION	3
III. TEST RESULTS	14
APPENDIX 1 — ELASTIC AND THERMAL PROPERTIES OF SWAGED COAXIAL AND "TRI-AX" SHEATHED CABLE	21
APPENDIX 2 — ANALYSIS OF NON-LINEARITY OF SYSTEM RESPONSE	38
APPENDIX 3 — DIAPHRAGM AND RESONANT FREQUENCY	50

LIST OF FIGURES

	<u>Page</u>
1 Bearing Pressure Probe Assembly_____	5
2 High-Temperature Capacitance Pressure Transducer, Shown with and without Swagelock Connector_____	7
3 Metallographic Section of High-Temperature Capacitance Pressure Transducers (for Use in Liquid Metals)_____	8
4 Microphotograph of Sensing End of High-Temperature Capacitance Pressure Transducer (for Use in Liquid Metals)_____	9
5 Stereomicroscope Mounted on Lathe for Precision Machining of Miniature Components for Pressure Transducers_____	12
6 High-Temperature Na Pressure Transducer Calibration_____	15
7 High-Temperature Sodium Instrumentation Test Apparatus_____	17
8 High-Temperature Sodium Instrumentation Test Apparatus_____	18
A1-1 Swaged Coaxial Cable_____	22
A1-2 Mechanical Equivalent of Swaged Coaxial Cable_____	22
A1-3 Curves for Hoop Stress vs. Temperature and Yield Stress vs. Temperature (Nonintersecting)_____	24
A1-4 Curves for Hoop Stress vs. Temperature and Yield Stress vs. Temperature with these Intersecting, Resulting in a Reduction of $\sigma_{\theta\theta}$ after Annealing to T_{max} _____	24
A1-5 Limiting Stresses in Ceramic and Metal During and After Swaging__	28
A1-6 Typical Stress Cycles in Metal-Ceramic Section During and After Swaging_____	29
A1-7 High-Temperature Capacitance Pressure Transducer for Use in Liquid Metals_____	36
A2-1 Definition of "Nonlinearity"_____	39
A2-2 Response of Wayne-Kerr DM 100, Serial #181, Connected to a High-Temperature Capacitance Pressure Transducer (Temperature 80 F) _____	41

LIST OF FIGURES (Continued)

	<u>Page</u>
A2-3 Slope of Response Curve of Wayne-Kerr DM 100, Serial #181, Connected to a High-Temperature Capacitance Pressure Transducer (Temperature 80 F)	42
A2-4 Response Curve of Wayne-Kerr TE 100 Connected to a High-Temperature Pressure Transducer	46
A2-5 Response Curve of Wayne-Kerr DM 100 Connected to a Standard .010-inch Range Probe	47
A2-6 Response Curve of Wayne-Kerr DM 100 Connected to a Standard .010-inch Range Probe	48
A2-7 Response Curve of Wayne-Kerr TE 100 Connected to a Standard .010-inch Range Probe	49

I. INTRODUCTION

The pressure transducer described in this report was developed for measurement of steady state and dynamic film pressures in high-temperature, sodium lubricated bearings. The development was necessitated by the absence of bearing film pressure transducers for this type of operation.

The transducer design concept is a hermetically sealed, high temperature capacitance probe that measures the deflection of a diaphragm exposed to the bearing film pressure. The high temperature capacitance probe element used was of a type already well developed and used in gas environments at temperatures up to 1400 F. The approach of using a hermetically sealed probe to measure the deflection of a diaphragm exposed to the pressure had been earlier employed in measurements of steam nozzle box pressures. The electronic amplification system used with the transducer is a standard, commercially available Wayne-Kerr meter.

For the purpose of the tests for which the transducer was intended, the required pressure range was 0 to 120 psi. However, similar transducers can be built for higher pressures in the range of 0 to 500 psi by suitable selection of diaphragm thickness. The target specifications that were set for the transducer were:

Environment	: Sodium
Temperature	: Up to 1200 F
Pressure Range	: 0 - 120 psig
Number of Pressure Cycles	: 5×10^8
Size Limits	: 1" dia. x 2" long
Temperature Effect on Zero Drift	: 0.01%/F
Resolution (Dynamic)	: 0.2% of F.S.
Linearity	: $\pm 5\%$ of F.S.
Frequency Response	: 0 - 200 Hz

The transducers were developed, built and extensively tested, first in a gas atmosphere over the temperature range from 80 F to 1000 F. Subsequently, two

of the transducers underwent a 684 hours continuous test in liquid sodium. During this period, the operating temperature cycled between the limits of 1050 F and 1170 F. Daily measurements of the probe calibration in 10 psi increments over a range from 0 to 150 psig, were made throughout this test. The fabrication of the transducers is described in Section II of this report and the test results are covered in Section III.

II. CONCEPT SELECTION, DESIGN AND FABRICATION

The selected concept was a hermetically sealed, high-temperature, non-contacting probe that measures the deflection of a diaphragm exposed to the sodium film pressures in the bearings. In selecting the non-contacting probe consideration was briefly given to both inductance and capacitance types.

In the parallel, proximity probe development (Report NYO 3930-11), an inductance system was selected because there, the sensed gap is filled with the electrically conductive sodium. In the pressure transducer application, however, the probe is hermetically sealed and the gap is gas filled, thus permitting the use of a capacitance probe. The selected high-temperature capacitance probe had been earlier developed and operated at temperatures up to 1400 F. It has high frequency response range and uses the commercially available Wayne-Kerr electronic amplification system. Specifically, the driven shield capacitive gaging equipment with high-temperature probe offered the following advantages:

1. Excellent long-time DC stability and freedom from lead effects (due to the driven-shield).
2. High resolution and sensitivity. Current standard probe ranges are 1.0, 10 and 100 mils with system resolution of 0.1 percent or better.
3. Good frequency response (to 10 kHz).
4. Minimal and reproducible effects of temperature since the probe contains no active elements, and is of a simple configuration which is highly favorable for mechanical stability at elevated temperatures.
5. Because of its simplicity and ruggedness, the probe can be integrated into a mechanically stable structure containing the transducing element.

A detailed assembly drawing of the developed transducer is shown in Figure 1. Photographs of the transducers and a metallographic section through the sensing end are shown in Figures 2, 3 and 4.

Fig. 1 Bearing Pressure Probe Assembly.

UNLESS OTHERWISE SPECIFIED DIMENSIONS ARE IN INCHES TOLERANCES ON: DECIMALS FRACTIONS ANGLES X .005 ± .005 ± .125 X .02 ± .02 ALL SURFACES BREAK SHARP CORNERS .005-.015 AND REMOVE BURRS	DESIGN DA JONES APR. 21/67 CHECK ES. 6-12-67 DESIGN H. JONES 5/13/67	MECHANICAL TECHNOLOGY INCORPORATED LATHAM NEW YORK	
		TITLE BRG. PRESS PROBE ASM. & CALIB. FIXTURE No BRG TEST RIG	
MATERIAL	ANALYTICAL	CODE IDENT NO.	SIZE
TREATMENT	QUALITY CONTROL	26741	D 212D194
		REVISED	BY
		10 JULY 1967	AS NOTED

Mechanical Technology Inc. Latham, N.Y.
High Temperature Capacitance Pressure Transducer For Use In Liquid Metal
Environment. Mag. Less Than 1X

Fig. 2 High-Temperature Capacitance Pressure Transducer,
Shown With and Without Swagelok Connector.

Fig. 3 Metallographic Section of High-Temperature Capacitance Pressure Transducers (for Use in Liquid Metals).

Fig. 4 Microphotograph of Sensing End of High-Temperature Capacitance Pressure Transducer (for Use in Liquid Metals).

The probe consists of a capacitance displacement sensor in a housing that is capped with a diaphragm. The displacement sensor is welded to the housing, leaving a small gap (0.4 to 0.6 mils) between the center electrode and the inside surface of the diaphragm.

The spacing between the three electrodes (diaphragm, guard, and center) is critical. The clearances are in the order of 0.001-0.002". Concentricity, flatness, and perpendicularity are accurate to within 0.0002".

The capacitance probe shown has a nominal displacement range of 0.5 mils, when used with the Wayne-Kerr DM 100 capacitance meter. Imperfections in the surfaces of the sensing electrodes must be well below 0.1 mils to prevent excessive loss of range. The smallest indicated gap with the probe inside a pressure transducer is 50-100 μ -in before contact occurs.

The probe design requires that three metal parts (electrodes) are kept in a firm position relative to each other by two layers of a high temperature dielectric. The thermal differential expansion that is inherent in such a layered structure must be accommodated without an appreciable effect on the sensing gap (center to diaphragm), and the guard shield gaps to ground (diaphragm) and center electrodes.

The methods generally used to accomplish this are not applicable because of the overall probe dimensions and the required ruggedness and frequency response. The use of ceramic cements and ceramic tubes and sleeves was considered but could not be used because one of the requirements is that the probe be stress relief annealed at 1960 F after welding. At that temperature, the differential expansion between the metal and ceramic parts is in the order of 0.3-1.0%. During the annealing cycle, no shift in relative position (particularly in the axial direction) of the electrodes could be tolerated and it appeared unlikely that this would not happen with the mismatch in thermal expansion between steel and ceramic parts. Therefore, rather than develop a workable scheme based upon the use of cements and tubular insulators, a swaging technique was used.

This method is used to fabricate high temperature coaxial cable, sheathed thermo-

couples, heating elements, etc. These swaged products are assembled from wire and tube stock and a crushable ceramic in the form of beads and tubes. Although swaging is successfully used, the approach is largely empirical. An approximate analysis of the stresses in the sheath, electrodes and compacted ceramic was therefore made and this is described in Appendix 1.

After assembly of the wire and ceramic beads into the tube, the tube is swaged in a rotary swaging (forging) machine, which reduces the diameter by 15-25% in a number of steps. Between the steps in the operation, a stress relief annealing cycle is inserted if needed.

After final annealing, a very stable material results. For the capacitance probe, a swaged section consisting of two concentric tubes and a center wire is used known as "Tri-ax".

The manufacture of the components for the displacement probe requires a high degree of control of surface finish, edge finish (between cylindrical and faced surfaces) and absolute dimensions. To obtain this control, a stereo-microscope on the lathe is used in the machining of the critical probe parts, as illustrated in Figure 5.

The assembly requires that welds are made as indicated in the section shown in Figure 1. To avoid thermal distortion, it is necessary to use electron beam welding to make the joints. The welds are made in the following sequence:

1. Diaphragm to case.
2. Guard shield to guard electrode (tube) with four tack welds.
3. Case to probe outer sheath.
4. Vent tube to case.
5. Swagelock to housing.

The third weld is the most critical, as it can result in axial displacements that directly affect the final working gap of the transducer. Fortunately, the axial displacement caused by welding is only 100-200 μ -in (gap reduction).

Fig. 5 Stereomicroscope Mounted on Lathe for Precision Machining of Miniature Components for Pressure Transducers.

After welding, the assemblies are stress relief annealed at 1950 F. The annealing operation changes the gap by 50 - 200 μ -in (increase in gap), returning it to nearly the same value as that set before making the welds between case and probe sheath.

III. TEST RESULTS

A total of nine (9) transducers were built. Four of these which were required for the bearings tests were subjected to extensive calibration and operational tests in a gas atmosphere. Two of these transducers then underwent the long term endurance test in liquid sodium.

Figure 6 shows the calibration test data in gas atmosphere over a temperature range from 80 F to 1000 F. The complete map was obtained by first applying pressure to the front face of the diaphragm and then to the back face (normally the vented side) of the diaphragm.

The zero drift is negligible at temperatures over 700 F. The total zero drift over the 80 - 700 F range is about 10% of F.S. output or $0.02/^{\circ}\text{F}$. The sensitivity change with temperature is about 25%/1000 F. Both effects can be corrected for if the operating temperature of the transducer is known.

In addition to the effects of temperature, there is the non-linearity of the response. The response curves given in Figure 6 are the combination of a calibration curve with pressure on the outside and one with pressure on the inside of the diaphragm. The design range of the transducer is 150 psi and the transducer is intended to measure external pressure. The non-linearity of the response curve in the 0 - 150 psi range (external pressure) is calculated in Appendix 2 and is $7\frac{3}{4}\%$. Due to this non-linearity, the second harmonic for a ± 80 psi sinusoidal excursion with a mean pressure of 80 psi is calculated as $5\frac{1}{2}\%$. The non-linearity is caused by excessive loading of the Wayne-Kerr DM 100 by the cable capacitance of the transducer. This capacitance cannot be reduced without a major increase in the dimensions of the swaged "Tri-ax". However, the transistorized version of the Wayne-Kerr DM 100 (Model TE 100) shows approximately five times less non-linearity in the response of the pressure transducer. Appendix 2 describes the linearity data and computations with the two Wayne-Kerr models. Because the TE 100 was not available until recently, it was evaluated with the pressure transducer only after completion of the life test in sodium. The TE 100 is recommended in future use because of its superior linearity and the reduced chance of over-

Fig. 6 High-Temperature Na Pressure Transducer Calibration

loading the electronics during long-term tests. With the TE 100 the non-linearity in the response is 1 to 2% of F.S. The frequency response of the TE 100 is DC - 2 kHz.

After completion of the tests in gas atmosphere, two probes were installed in the high-temperature sodium calibration test fixture and underwent an extended duration test in liquid sodium. The test assembly that accommodated both the pressure transducers and the proximity transducers (described in Report NYO-3930-11), is shown in Figures 7 and 8.

In this test the sodium temperature was raised to 1050 F and thereafter cycled between 1050 F and 1170 F for a period of 684 hours. The target time for this test was 1000 hours, however, it had to be ended after 684 hours when a severe leak developed in a gasketed joint of the test pot.

Throughout the duration of the test, calibration readings were made on a daily basis with one of the transducers, the data being generally recorded at a temperature of $1150\text{ F} \pm 15\text{ F}$. The calibration tests were made from 0 to 150 psi, in 10 psi increments, the pressurization being applied to the back (normally vented) face of the diaphragm. During the course of this test run, the external bridge and signal amplification system lost sensitivity. The problem was traced to a faulty capacitance detector unit, Wayne-Kerr DM 100. After repair of the detector, the pressure measurements were stable and repeatable with good sensitivity, as evidenced by the data of Table 1.

The post-test evaluation showed that a zero drift had occurred in the transducer subjected to the calibration pressures. The zero drift is a result of measurable change in the pressure sensing gap, as possible cause of which was over pressurization in the diaphragm, as discussed in Appendix 3.

Basically, the present transducers had been built for operation in a sodium bearing test rig where the long term steady state temperature would be less than 1000 F and where the maximum pressures would be 120 psi. In the 684 hours endurance test, however, the transducer was repeatedly pressurized to

Fig. 7 High-Temperature Sodium Instrumentation Test Apparatus

Fig. 8 High-Temperature Sodium Instrumentation Test Apparatus

TABLE 1
PRESSURE TRANSDUCER CALIBRATION DATA IN 1150 F SODIUM

Test Pressure (psi)	Output (Volts) at		
	<u>595 Hours</u>	<u>644 Hours</u>	<u>685 Hours</u>
0	1.053	1.056	1.053
10	1.107	1.111	1.107
20	1.159	1.163	1.161
30	1.211	1.215	1.214
40	1.261	1.264	1.264
50	1.309	1.311	1.312
60	1.355	1.358	1.361
70	1.399	1.403	1.407
80	1.442	1.445	1.452
90	1.486	1.488	1.495
100	1.526	1.530	1.537
110	1.569	1.570	1.579
120	1.612	1.606	1.618
130	1.658	1.643	1.657
140	1.683	1.677	1.693
150	(Not Recorded)	1.713	1.729

150 psi at a mean temperature of 1150 F. This results, as shown in Appendix 3, in a radial stress of 20 kpsi at the edge of the diaphragm. Since the lower portion of the tensile yield strength scatter band of 347 stainless steel (the diaphragm material) is slightly under 20,000 psi, some plastic deformation may occur. Accordingly, for future, long-term 1200 F operation and to provide for greater margin of safety at the maximum pressure required, a larger transducer and thicker diaphragm would be needed. An alternate method, when measuring bearing pressures acting on the forward face of the diaphragm is to introduce pressurized cover gas into the region at the back face of the diaphragm in order to keep the differential pressure across the diaphragm (and hence, the diaphragm stresses) very low.

APPENDIX 1ELASTIC AND THERMAL PROPERTIES OF SWAGED
COAXIAL AND "TRI-AX" SHEATHED CABLE

In the following, a qualitative analysis is made of the mechanical behavior of swaged cable with respect to its usefulness as a structural component in a displacement transducer.

Consider the section shown in Fig.A1-1, consisting of a sheath, a center conductor, and ceramic dielectric compacted by swaging the outside diameter. The state of stress of the metal and ceramic after swaging is of fundamental importance when using the residual stress in the ceramic to firmly locate the center conductor with respect to the sheath in axial direction. In the swaging operation the sheath is hammered between two dies that have a circular shape. The compressive stress generated in the work-piece is limited by the pressure needed to extrude (i.e. lengthen) the work-piece (in axial direction) while in the dies. This pressure can be expected to be in the order of 250,000 - 500,000 psi. When in the final swaging operation the ceramic is compacted completely, the work-piece becomes as hard as a solid piece of metal. Assuming that the work-piece was exposed to impact pressures of $\frac{1}{4}$ - $\frac{1}{2}$ million psi, it can be expected that after swaging the sheath is stressed in tension due to the expansion of the ceramic and conductor after removal of the high impact pressure.

Annealing the work-piece cannot remove all of the residual stresses because of the difference in expansion between metal and ceramic.

The elastic behavior of the compacted ceramic can be expected to be non-linear under low pressure (2000 psi) because of the many points of contact between individual grains, each behaving as a non-linear spring (Hertzian contact). Under high pressure, the ceramic will probably exhibit a modulus of elasticity equal to that of the individual grains. To estimate the temperature level at which the ceramic is no longer under high pressure, it is permissible to treat the ceramic material as a single cylinder of solid material.

Fig. A1-1 Swaged Coaxial Cable.

Fig. A1-2 Mechanical Equivalent of Swaged Coaxial Cable.

With this assumption, the problem reduces to that of a solid ceramic cylinder under high pressure inside a metal cylinder, i.e. a shrink fit problem as pictured in Fig. A1-2. The presence of the wire inside the ceramic is neglected as it only tends to increase the calculated temperature at which the ceramic becomes lowly stressed. The answer obtained for the case without the wire is the minimum high-temperature level at which a swaged structure can be expected to show high stability, if such a temperature level exists.

Two distinct types of behavior can be expected, depending upon material properties of the metal and ceramic:

- 1) The hoop stress in the tube decreases more rapidly with temperature than the yield stress of the tube material:

$$\sigma_{\theta}(T) < \sigma_y(T) \text{ for } 80^{\circ}\text{F} < T < T_{\max}$$

See curve for $\sigma_{\theta}(T)$ in Fig.A1-3 which is all below $\sigma_y(T)$

- 2) The hoop stress in the tube decreases less rapidly with temperature than the yield stress of the tube material:

$$\sigma_{\theta}(T) < \sigma_y(T) \quad 80^{\circ}\text{F} < T < T_y$$

$$\sigma_{\theta}(T) = \sigma_y(T) \quad T_y < T < T_{\max}$$

and thus yielding must take place.

See Fig.A1-4 curve for $\sigma_{\theta}(T)$ which follows $\sigma_y(T)$ after passing T_y . The temperature dependent material properties that interact are:

- 1) Dimensions : $L = L_0 (1 + \alpha T)$
- 2) Modulus of elasticity: $E = E_0 (1 - \beta T)$
- 3) Yield stress* : $\sigma_y = \sigma_{y0} (1 - \gamma T^2)$

The temperature coefficients (α , β , γ) all vary with temperature, but for simplicity's sake they are assumed to be constants. The results obtained will show whether this assumption was permissible.

The formulas used to obtain $\sigma_{\theta}(T)$ are as follows:

* The temperature effect on σ_y as given (γT^2) is only introduced for demonstration purposes. Actual material behavior is more complex.

Fig. A1-3 Curves for Hoop Stress vs. Temperature and Yield Stress vs. Temperature (Nonintersecting).

Fig. A1-4 Curves for Hoop Stress vs. Temperature and Yield Stress vs. Temperature with these Intersecting, Resulting in a Reduction of $\sigma_{\theta 0}$ After Annealing to $T_{\max}$.

$$r_c = r_{ci} \left\{ 1 - \frac{p}{E_c} (1 - \nu) \right\} (1 + \alpha_c T) \quad (1)$$

$$r_t = r_{ti} \left(1 + \frac{p}{E_t} \frac{D_2}{D_3 - D_2} \right) (1 + \alpha_t T) \quad (2)$$

$$p = \frac{D_3 - D_2}{D_2} \sigma_\theta \quad (3)$$

The symbols used are:

r_c = radius of solid ceramic cylinder

r_t = inner radius of tube (sheath)

α = coefficient of thermal expansion

E = modulus of elasticity

D_3 = outer diameter of tube

D_2 = inner diameter of tube

p = shrink fit pressure, or interface pressure between metal and ceramic

ν = Poisson's ratio

T = temperature (room temperature is zero)

σ_θ = hoop stress in tube

σ_y = yield stress of tube material

The subscript "c" indicates ceramic, the subscript "t" indicates material of tube, the subscript "o" indicates $T = 0$, the subscript "i" indicates ("initial"):

$T = 0$, $p = 0$. The tube is considered to be thin-walled (= uniform hoop stress) and short (= zero axial stress).

For $p > 0$, $r_c = r_t$.

Setting $r_c = r_t$, and solving (1) and (2) for p :

$$p = \frac{\frac{r_{ci}}{r_{ti}} (1 + \alpha_c T) - (1 + \alpha_t T)}{(1 - \nu) \frac{r_{ci}}{r_{ti}} \frac{(1 + \alpha_c T)}{E_c} + \frac{D_2}{D_3 - D_2} \frac{1 + \alpha_t T}{E_t}} \quad (4)$$

The numerator in (4) can be approximated and simplified:

$$p = \frac{\frac{r_{ci}}{r_{ti}} (1 + \alpha_c T) - (1 + \alpha_t T)}{\frac{1 - \nu}{E_c} + \frac{D_2}{D_3 - D_2} \frac{1}{E_t}} \quad (4a)$$

To obtain the expression for σ_θ versus T, substitute (3) in (4a):

$$\sigma_\theta = \frac{\frac{r_{ci}}{r_{ti}} (1 + \alpha_c T) - (1 + \alpha_t T)}{\frac{D_3 - D_2}{D_2} \frac{1 - \nu}{E_c} + \frac{1}{E_t}} \quad (5)$$

Substitute $\frac{r_{ci}}{r_{ti}} = \delta$, and $\frac{D_3 - D_2}{D_2} \frac{1 - \nu}{E_c} + \frac{1}{E_t} = A$

With these (5) becomes, rewritten:

$$\sigma_\theta = \frac{\delta - 1}{A} + \frac{\delta \alpha_c - \alpha_t}{A} T \quad (5a)$$

For T = 0:

$$\sigma_{\theta 0} = \frac{\delta - 1}{A} \quad (5b)$$

As δ is nearly 1, it is permissible to substitute $\delta = 1$ in the second term of (5a). With the latter substitution and with (5b) also substituted in (5a), $\sigma_\theta(T)$ becomes:

$$\sigma_\theta = \sigma_{\theta 0} - \frac{\alpha_t - \alpha_c}{A} T \quad (5c)$$

The temperature T_f for which $\sigma_\theta = 0$ is:

$$T_f = \frac{A \sigma_{\theta 0}}{\alpha_t - \alpha_c} \quad (6)$$

In the following equation (6) is used to calculate T_f for the outer sheath of the Tri-ax used in the transducer design in Fig. 1. The fact that not only ceramic but also metal is present inside the outer sheath is neglected for the moment. The numbers to be used are:

$$\alpha_t = 18 \times 10^{-6} \text{ } ^\circ\text{C}^{-1} \quad (304 \text{ SS})$$

$$\alpha_c = 14 \times 10^{-6} \text{ } ^\circ\text{C}^{-1} \quad (\text{MgO})$$

$$E_t = 28 \times 10^6 \text{ psi} \quad (304 \text{ SS})$$

$$E_c = 40 \times 10^6 \text{ psi} \quad (\text{MgO estimated})$$

$$D_2 = .200 \text{ inch}$$

$$D_3 = .290 \text{ inch}$$

$$A = \left(\frac{.290 - .200}{.200} \frac{1 - .3}{4} + \frac{1}{3} \right) 10^{-7} = .41 \times 10^{-7}$$

$$\nu = .3 \text{ (estimated)}$$

$$T_f = \frac{.41 \times 10^{-7}}{18-14} 10^6 \quad \sigma_{\theta o} = .01 \sigma_{\theta o}^{\circ C} \quad (\sigma_{\theta o} \text{ in psi}) \quad (6a)$$

For the guard shield, the dimensions are:

$$D_2 = .075, D_3 = .125, \text{ and } A = \left(\frac{.125 - .075}{.075} \frac{1 - .3}{4} + \frac{1}{3} \right) 10^{-7} = .45 \times 10^{-7}$$

And:

$$T_f = .011 \sigma_{\theta o} \quad (6b)$$

When both the guard shield and ground shield are stressless (i.e. $\sigma_{\theta}(T) = 0$), then stability obtains, that is, the (axial) gap in the pressure transducer will not change with increased temperature. If in either one $\sigma_{\theta}(T) > 0$, the temperature must still be increased further to obtain (axial, i.e. gap) stability.

The temperature at which the design in Fig. A1-1 exhibits stability experimentally is about 400°C. Substituting this temperature in (6a) or (6b) results in a $\sigma_{\theta o} \approx 40,000$ psi. To judge whether this is possible, two aspects of the manufacturing process must be investigated:

- 1) Is $\sigma_{\theta o} = 40,000$ psi possible considering the elastic-plastic behavior of the section during and after swaging.
- 2) Is $\sigma_{\theta o} = 40,000$ psi possible even though the transducers were annealed at 1950°F.

Item #1 is considered in the following and illustrated in Figs. A1-5 and A1-6.

In Fig. A1-5, a diagram is drawn of the limiting metal and ceramic stresses during a swaging cycle. In Fig. A1-6 typical cycles possible for various swaging pressures are given. The diagrams are made using the following formulas:

Fig. A1-5 Limiting Stresses in Ceramic and Metal During and After Swaging.

Fig. A1-6 Typical Stress Cycles in Metal-Ceramic Section During and After Swaging.

$$\sigma_{\theta s} = \epsilon_s E_t \quad -\sigma_y < \sigma_{\theta s} < +\sigma_y \quad (7)$$

$$\sigma_{\theta s} = \sigma_y \quad \epsilon_s > \frac{\sigma_y}{E_t} \quad (7a)$$

$$P_{cs} = \epsilon_s \frac{E_c}{1-\nu} \quad P_{cs} < P_{ce} \quad (8)$$

Eliminating ϵ_s between (7) and (8):

$$\sigma_{\theta s} = (1 - \nu) \frac{E_t}{E_c} P_{cs} \quad -\sigma_y < \sigma_{\theta s} < +\sigma_y \quad (9)$$

$$\sigma_{\theta o} = -P_{co} \frac{D_2}{D_3 - D_2} \quad (10)$$

The symbols used are:

ϵ_s = strain during swaging in ceramic and metal (assumed to be uniform throughout section)

P_{ce} = swaging pressure in ceramic at which extrusion occurs

P_{cs} = interface pressure during swaging

P_{co} = interface pressure after swaging, at R.T. (room temperature)

$\sigma_{\theta o}$ = hoop stress in tube after swaging and/or annealing as may be indicated

The assumption of uniform strain is only valid if the section has a uniform stress throughout. During most of the swaging cycle this is probably so. It is a good approximation for the purpose of the diagrams in Fig.A1-5 and A1-6. Using equation (9) and (10) the diagram in Fig.A1-6 is drawn to show the compression part of the swaging cycle and the final state of compression of the ceramic and the final tensile stress ($\sigma_{\theta o}$) in the sheath. The numbers used are:

$$\nu = .3 \quad E_t = 28 \times 10^6 \text{ psi} \quad E_c = 40 \times 10^6 \text{ psi}$$

$$D_3 = .290 \text{ in.} \quad D_2 = .200 \text{ in.}$$

With these Equations (9) and (10) become:

$$\sigma_{\theta s} = (1 - .3) \frac{28}{40} P_{cs} = .49 P_{cs} \quad -\sigma_y < \sigma_{\theta s} < \sigma_y \quad (9a)$$

$$\sigma_{\theta s} = \sigma_y \quad \epsilon_s > \frac{\sigma_y}{E_t}$$

$$\sigma_{\theta o} = - \frac{.2}{.290 - .200} P_{co} = 2.2 P_{co} \sigma_{\theta o} < \sigma_y \quad (10a)$$

Figs. A1-5 and A1-6 have the same stress scale on both axes.

Fig. A1-5 shows the limiting stresses during a cycle in a loop with 5 sections:

- Section I: Elastic compression of metal and ceramic until $\sigma_{\theta} = - \sigma_y$
- Section II: Yielding of the metal and elastic compression of the ceramic until extrusion occurs
- Section III: Decompression of the whole section, with elastic behavior of both metal and ceramic until σ_{θ} reaches $+ \sigma_y$
- Section IV: Yielding of the metal in tension and further elastic decompression of the ceramic until the pressure in the ceramic balances the stress in the metal. This occurs at the intersection of Section IV and V
- Section V: The locus of $\sigma_{\theta o}$ versus P_{co} according to Equations (10) and (10a)

Fig. A1-6 shows swaging cycles with increasingly higher pressures:

- I For low swaging pressure the whole section behaves elastically moving up and down along Section I
- II For higher swaging pressure the metal yields, and decompression follows line II terminating where II intersects the locus (V in Fig. A2-3) at $\sigma_{\theta o} = (\sigma_{\theta o})_{II}$
- III For increased swaging pressure, decompression follows line III, the metal yields and $\sigma_{\theta o}$ becomes equal to $+ \sigma_y$
- IV For maximum swaging pressure, decompression follows line IV, the metal yields more this time and again $\sigma_{\theta o} = \sigma_y$

Returning to item #1 on pg. 2/ it can be seen in Fig. A1-6 that even though large plastic deformation of the metal can occur during swaging, the resulting residual tensile stress $\sigma_{\theta o}$ can be well below the yield point. The stress number found ($\sigma_{\theta o} = 40,000$ psi) is, therefore, quite acceptable.

Item #2 is concerned with stress relief annealing of the swaged section. This process is illustrated in Figs. A1-3 and A1-4.

In the following a calculation is made of the temperature T_y at which first yielding will occur. To make this calculation requires that $\sigma_y(T)$ is available in some analytical form. A good approximation for the purpose of calculating T_y is the one introduced earlier:

$$\sigma_y(T) = \sigma_{yo} (1 - \gamma T^2) \quad (11)$$

The temperature T_y is found by equating (5c) to (11) and solving for T :

$$\sigma_{\theta o} - \frac{\alpha_t - \alpha_c}{A} T_y = \sigma_{yo} (1 - \gamma T_y^2) \quad (12)$$

$$T_y^2 - \frac{\alpha_t - \alpha_c}{A} \frac{1}{\gamma \sigma_{yo}} T_y + \frac{\sigma_{\theta o} - \sigma_{yo}}{\gamma \sigma_{yo}} = 0 \quad (12a)$$

If $\sigma_{\theta o} = \sigma_{yo}$, as could occur after the last swaging cycle then:

$$T_y^2 = \frac{\alpha_t - \alpha_c}{A} \frac{1}{\gamma \sigma_{yo}} T_y \quad (12b)$$

Equation (12b) is satisfied by $T_y = 0$, or:

$$T_{y1} = \frac{\alpha_t - \alpha_c}{A} \frac{1}{\gamma \sigma_{yo}} \quad (12c)$$

$T_y = 0$ is a trivial solution, because it is already prescribed by the assumption $\sigma_{\theta o} = \sigma_{yo}$ made above.

Actual values for γ and σ_{yo} are approximately:

$$\gamma = 10^{-6} \text{ } ^\circ\text{C}^{-2} *$$

$$\sigma_{yo} = 30 \text{ to } 60 \text{ kpsi (Annealed 304 SS)*}$$

$$\sigma_{yo} = 100 - 200 \text{ kpsi ("half hard" to "full hard" 301 SS)*}$$

* Metals Handbook, Vol. 1, p. 503, July 1967, provides yield stress vs. temperature data for 301 stainless steel. This material (301 SS) contains 17% Cr, 7% Ni and 0.15% C maximum. The lead material used in the transducers is 304 stainless steel, which is a similar alloy, except that it contains 19% Cr, 10% Ni and 0.08% C maximum.

For the tri-ax used for the outer sheath T_{y1} becomes:

$$T_{y1} = \frac{4 \times 10^6}{.44 \times 10^{-7}} \frac{1}{\sigma_{yo}} 10^6 \text{ } ^\circ\text{C} = 91 \times 10^6 \frac{1}{\sigma_{yo}} \text{ } ^\circ\text{C} \quad (12c)$$

The subscript 1 indicates T_y for $\sigma_{\theta o} = \sigma_{yo}$.

The approximation used for σ_y :

$$\sigma_y(T) = \sigma_{yo} (1 - \gamma T^2) \quad (11)$$

is limited by:

$$0 < T \leq \sqrt{\frac{1}{\gamma}}$$

Thus (12c) is only valid for:

$$\sigma_{yo} > \frac{91 \times 10^6}{\sqrt{\frac{1}{\gamma}}} \approx 90,000 \text{ psi}$$

Yielding for $\sigma_{yo} = 90,000$ psi would occur at $T_y = \sqrt{\frac{1}{\gamma}} = 1000^\circ\text{C}$

This approximate analysis indicates that it is possible that

- a) $\sigma_\theta(T) \leq \sigma_y(T)$ for $\sigma_{yo} < \sigma_y(T) < 0$ and $\sigma_{yo} < 90,000$ psi
- b) a preload can be maintained in the ceramic up to a high temperature level without yielding the metal at room temperature
- c) a high temperature annealing cycle will not remove the prestress

In the above, Equation (12b) was obtained by setting $\sigma_{\theta o} = \sigma_{yo}$. As can be seen in Fig. A1-6, $\sigma_{\theta o}$ can be well below σ_{yo} . For that case, the temperature T_y is found from:

$$T_y = \frac{\alpha_t - \alpha_c}{2A} \frac{1}{\gamma \sigma_{yo}} \pm \sqrt{\left(\frac{\alpha_t - \alpha_c}{2A} \frac{1}{\gamma \sigma_{yo}} \right)^2 - \frac{\sigma_{\theta o} - \sigma_{yo}}{\sigma_{yo}}} \quad (13)$$

The minus sign indicates yielding at a temperature $T_y < 0$, that is yielding when cooling down below room temperature. This is not a point of immediate interest here. Only in cases where the transducer would be used at very low temperatures where brittle fracture could become a problem the $T_y < 0$ becomes important.

Rewrite: (13):

$$T_y = \frac{T_{y1}}{2} \left[1 + \sqrt{1 - \frac{1}{(T_{y1})^2} \frac{\sigma_{\theta o} - \sigma_{y o}}{\gamma \sigma_{y o}}} \right] \quad (13a)$$

Approximate (13a) with:

$$T_y = T_{y1} \left[1 + \frac{1}{(T_{y1})^2} \frac{\sigma_{y o} - \sigma_{\theta o}}{4 \gamma \sigma_{y o}} \right] \quad \sigma_{\theta o} > 0 \quad (13b)$$

For example, after the last swaging operation, $\sigma_{y o}$ could be as high as 200 kpsi and $\sigma_{\theta o}$ could be 100 kpsi. For those conditions (12c) would give:

$$T_{y1} = \frac{91 \times 10^6}{2 \times 10^5} \approx 450^\circ\text{C}$$

and (13b) would give:

$$T_y = 450 \left[1 + \frac{1}{450^2} \frac{2 \times 10^5 - 10^5}{4 \times 10^{-6} \times 2 \times 10^5} \right] = 450 (1 + .6)^\circ\text{C} = 720^\circ\text{C}$$

which is 60% higher than if $\sigma_{\theta o} = \sigma_{y o}$. If after reaching 720°C heating is continued, yielding occurs until the temperature is reached at which $\sigma_y = 0$, that is for $T = \sqrt{\frac{1}{\gamma}}$. Cooling down from that temperature to room temperature will result in a stress $(\sigma_{\theta o})_{\text{ann}}$ that is calculated using equation (6) rewritten as:

$$\sigma_{\theta o} = \frac{\alpha_t - \alpha_c}{A} T_f \quad (6c)$$

Substitute (6c) $T_f = \sqrt{\frac{1}{\gamma}}$:

$$(\sigma_{\theta o})_{\text{ann}} = \frac{\alpha_t - \alpha_c}{A} \sqrt{\frac{1}{\gamma}} \quad (6d)$$

For the outer tube of the tri-ax in Fig. 1:

$$(\sigma_{\theta o})_{\text{ann}} = \frac{18-14}{.41} \frac{10^{-6}}{10^{-7}} 10^3 = 97,000 \text{ psi}$$

This result violates the fact that even though before annealing a yield stress of 200 kpsi could occur, the yield stress at room temperature after annealing is in the 30-60 kpsi range. The final state of stress of the tube after the first annealing cycle is then: $\sigma_{\theta o} = \sigma_{y o} = 30-60$ kpsi. During a second annealing cycle equation (12c) does not give a usable result (as $\sigma_{\theta o} < 90,000$ psi). The tube stress becomes zero at a temperature given by (6)

$$T_f = \frac{A\sigma_{\theta o}}{\alpha_t - \alpha_c} = \frac{.41 \times 10^{-7}}{4 \times 10^{-6}} (30 \text{ to } 60) 10^3 \approx 300 \text{ to } 600^\circ\text{C}$$

This result encompasses the observed temperature for section stability (400 C).

The magnitude of the observed temperature effect is equivalent to an axial displacement of 20 μ in between the center electrode and the diaphragm, resulting in an increase in air gap. The length of the transducer involved in the mechanical measuring circuit and affected by the thermal decompression is indicated by L_{c1} and L_{c2} in Fig. A1-7.

The effect on the gap of the compressive pressure p in the ceramic around the center electrode is approximated with:

$$\Delta L = \epsilon_z L_{c1} = -L_{c1} \frac{2\nu\Delta p}{E_t} \quad (\Delta p > 0) \quad (14)$$

in which ϵ_z = axial strain with σ_z assumed to be zero

Rewrite (14):

$$\epsilon_z = \frac{\Delta L}{L_{c1}} = - \frac{2\nu\Delta p}{E_t}$$

The actual numbers are:

$$\begin{aligned} \Delta L &= 20 \times 10^{-6} \text{ inch} & \nu &= .3 \\ L_{c1} &= \frac{1}{2} \text{ inch} & E_t &= 3 \times 10^7 \text{ psi} \end{aligned}$$

With those Δp becomes:

$$\Delta p = \frac{\Delta L}{L_{c1}} \frac{E_t}{2\nu} = \frac{20 \times 10^{-6}}{\frac{1}{2}} \times \frac{3 \times 10^7}{.6} = 2000 \text{ psi}$$

Fig. A1-7 High-Temperature Capacitance Pressure Transducer
for Use in Liquid Metals.

This Δp is an order of magnitude smaller than the pressure in the ceramic ($\sigma_{\theta 0} = 40,000$ psi) calculated earlier. It appears then that calculations predict the temperature range over which drift can be expected reasonably well. The amount of drift observed is much less than that to be expected from release of the calculated prestress $\sigma_{\theta 0}$.

The reason for this could be that the guard electrode (tube) can shield the center electrode mechanically, resulting in a radial pressure on the center wire much below the 40 kpsi calculated. When increasing the temperature, the gap between the guard electrode (face) and the diaphragm could change considerably (become larger) because relief of the (calculated) 40 kpsi radial pressure. There could be no appreciable electrical output due to this. The center electrode, which was stressed much lower than the 40 kpsi level would show a much reduced thermal displacement, resulting in the relatively minute change in output observed.

This analysis is largely qualitative and it is presented primarily in order to call attention to possible ways of behavior of the swaged material used as base stock for transducers. Further extension of the analysis are required to guide design improvements, e.g. use of different ceramics and/or metals.

APPENDIX 2
ANALYSIS OF NON-LINEARITY OF SYSTEM RESPONSE

The definition of non-linearity often used is "the deviation of the response curve from the best straight line through the data points," and it is expressed in percentage of F.S. The "best straight line" is defined as:

- a. passing through zero
- b. average of deviations from data points (or response curve) equal to zero

It is customary to approximate the data points with a smooth curve. The "best straight line" intersects the response curve in such a way that the areas enclosed by the curve and the straight line are equal. Figure A2-1 illustrates the definitions given above.

The definition of non-linearity given above is useful when considering a probe for static measurements.

If the non-linearity as defined above is given, the maximum error is known if one uses a fixed sensitivity number in converting probe output to pressure.

For dynamic measurements, the definition of non-linearity as given above is not satisfactory. The reason for this is that in dynamic measurements, non-linearity results in higher harmonics. The shape of the response curve determines the amplitude and the order of the harmonics.

In the following, the non-linearity of the transducers will be determined using the conventional (straight line) method and a method using harmonic analysis. A comparison will be made between the amounts of harmonic content found with the two

Fig. A2-1 Definition of "Nonlinearity."

methods. The calculations will extend over the full pressure range of the transducers.

Both methods use a simplified model of the response curve as follows:

$$\text{Output voltage} = E_{\text{out}} = K_1(1 + K_2p)p \text{ mv} \quad (1)$$

The slope of the response curve is:

$$\text{Slope} = \frac{\partial E_{\text{out}}}{\partial p} = K_1(1 + 2K_2p) \text{ mv/psi} \quad (2)$$

The slope is determined accurately from calibration data. The method based on harmonic analysis is applied to the data shown in Figures A2-2 and A2-3. These are the response and slope curves for two pressure transducers (#6 and #9). These transducers were identical except for the zero-pressure gap setting. Transducer #9 has a gap that is about 250 μin larger than that of #6 for the same pressure. Because of the presence of non-linearity, the two curves look different. However, comparing the curves at the same voltage levels shows that they are identical in slope.

Before discussing the cause of the non-linearity, the magnitude is determined for transducer #6. Once the non-linearity is established for a given output range of the Wayne-Kerr DM 100, it can be used directly for similar transducers with output in the same range. This is so because the non-linearity is largely due to the electrical circuit used rather than the transducing element (i.e., diaphragm).

The slope for curve #6 in Figure A2-3 is approximated with a straight line. The slope of this line $\frac{1}{70} \text{ mv/psi}^2$, is indicated by the dotted line, next to the slope curve. The slope curve for #6 intersects $p = 0$ at about 83 mv/20 psi (= 4.15 mv/psi). Using the slope and the zero crossing, equation (2) becomes:

$$\frac{\partial E_{\text{out}}}{\partial p} = - 4.15 \left(1 + \frac{1}{70} \times \frac{1}{4.15} p \right) \text{ mv/psi} \quad (2a)$$

or:

$$\frac{\partial E_{\text{out}}}{\partial p} = - 4.15 \left(1 + 3.44 \times 10^{-3} p \right) \text{ mv/psi} \quad (2b)$$

Fig. A2-2 Response of Wayne-Kerr DM 100, Serial #181, Connected to a High-Temperature Capacitance Pressure Transducer (Temperature 80 F).

Fig. A2-3 Slope of Response Curve of Wayne-Kerr DM 100, Serial #181, Connected to a High-Temperature Capacitance Pressure Transducer (Temperature 80 F).

Integration of equation (2b) and adding a constant to obtain proper voltage for $p = 0$ gives:

$$E_{out} = -4.15 (1 + 1.72 \times 10^{-3} p) p + 940 \text{ mv} \quad (1a)$$

The fit of (1a) to the data plotted in Figure A2-2 is at four points as follows:

p	eq(1a) mv	actual mv	difference mv
+100	453	454	1
+160	93	97	4
-100	1284	1284	0
-160	1422	1407	15

The fit is within 1% of F.S. over a ± 160 psi range, and better than $\frac{1}{2}\%$ of F.S. over a ± 100 psi range.

The second harmonic introduced by the second term in equation (1) is found by substituting $p = p_o (1 \sin \omega t)$ in equation (1), which is the case for a sinusoidal pressure excursion one side of zero, centered around $p = p_o$. With this, equation (1) becomes:

$$E_{out} = K_1 [p_o (1 - \sin \omega t) + K_2 p_o^2 (1 - \sin \omega t)^2] \quad (1b)$$

$$E_{out} = K_1 [p_o + K_2 p_o^2 - (1 + 2K_2 p_o) p_o \sin \omega t + K_2 p_o^2 (\sin \omega t)^2] \quad (1c)$$

Substitute $(\sin \omega t)^2 = \frac{1 - \cos 2\omega t}{2}$ in equation (1c)

$$E_{out} = K_1 [p_o + K_2 p_o^2 - (1 + 2K_2 p_o) p_o \sin \omega t + \frac{K_2 p_o^2}{2} (1 - \cos 2\omega t)] \quad (1d)$$

The second harmonic content of E_{out} relative to the first harmonic is:

$$\epsilon_2 = \frac{K_2 p_o}{2 (1 + 2K_2 p_o)} \times 100\% \quad (3)$$

$$\epsilon_2 = \frac{25}{\frac{1}{2} \frac{1}{K_2 p_o} + 1} \% \quad (3a)$$

For the response given in equation (1a), and for $p_o = 80$ psi, $1/(2K_2p_o)$ becomes $1/(2 \times 1.72 \times 10^{-3} \times 80) = 3.63$ and ϵ_2 becomes:

$$\epsilon_2 = \frac{25}{4.63} = 5.4\% \text{ for } p_o = \pm 80 \text{ psi (pressure on outside of diaphragm)}$$

$$\epsilon_2 = \frac{25}{2.63} = 9.5\% \text{ for } p_o = -80 \text{ psi (pressure on vented side)}$$

The non-linearity using the "best straight" line definition is calculated using the following equations:

$$(E_{out})_1 = K_1(p + K_2p^2) \quad (1)$$

$$(E_{out})_2 = K_3p \quad (4)$$

$$\int_0^{p_m} [(E_{out})_1 - (E_{out})_2] dp = 0. \quad (5)$$

$$\epsilon_1 = \left[\frac{(E_{out})_1 - (E_{out})_2}{(E_{out})_2} \right]_{p = p_m} \times 100\% \quad (6)$$

p_m = full scale pressure

K_3 = slope of "best straight line" = $K_1 \left(\frac{2}{3} K_2 p_m + 1 \right)$ (see below, eq.(7))

ϵ_1 = non-linearity in percentage of F.S.

Of these, equation (1) is the simplified response curve, equation (4) is the "best straight line", equation (5) implements the definition of "best straight line", and is used to calculate K_3 . Finally, equation (6) gives the maximum difference between the response curve and "best straight line."

Combining equations (1), (4), and (5) gives:

$$K_3 = K_1 \left(\frac{2}{3} K_2 p_m + 1 \right) \quad (7)$$

With this value for K_3 the non-linear error (6) becomes:

$$\epsilon_1 = \frac{1}{2 + \frac{3}{K_2 p_m}} \times 100\% \quad (8)$$

For the response given by equation (1a), ϵ_1 becomes:

$$\epsilon_1 = 7 \frac{3}{4}\% \text{ for } p_m = +160 \text{ psi (pressure on outside)}$$

$$\epsilon_1 = 11.2\% \text{ for } p_m = -160 \text{ psi (pressure on vented side)}$$

Comparison of these results shows that the percentage non-linearity as it is usually given, indicates a second harmonic content that is considerably smaller than the absolute static non-linear error (compare 5.4% to 7 3/4%).

The response curve in Figure A2-2 was obtained with the Wayne-Kerr DM-100 meter. Better linearity was obtained with the transistorized Wayne-Kerr TE 100. The response obtained is shown in Figure A2-4. The reason for the improvement is the improved cable driving circuit of the TE 100. The cable capacitance of the normal (Teflon) cable used with the DM 100 is 120 pico farads. Due to the ceramic dielectric in the swaged Tri-ax cable, its capacitance is four times greater than that of a Teflon cable. The increased load on the cable drive circuit causes a corresponding increase in non-linearity. This is illustrated by Figures A2-5 through A2-7 which show the effect of cable capacitance on the output obtained with a standard .010 inch range probe for the DM 100 and the TE 100.

As the slope of the sensitivity curve in Figure A2-4b is about five times less than that in Figure A2-3, the non-linearity and second harmonic will be decreased by the same amount, i.e. reduced to 1-2% of F.S. Thus, the TE 100 brings transducer performance well within target specifications.

Volts

(a) Output vs. Pressure

(b) Slope of Output Curve vs. Pressure

Fig. A2-4 Response Curve of Wayne-Kerr TE 100 Connected to a High-Temperature Pressure Transducer.

(Serial #6, data taken after life test in sodium)

Note: Change in slope is less than 10% over full scale pressure range (pressure on outside of diaphragm)

Fig. A2-5 Response Curve of Wayne-Kerr DM 100 Connected to a Standard .010-inch Range Probe.

Note: Curve a is obtained using the normal Teflon cable which has 128 pico farad (pf) capacitance between center and guard.

Curve b is obtained with a cable with 486 pf between center and guard and with 483 pf added between guard and ground.

Fig. A2-6 Response Curve of Wayne-Kerr DM 100 Connected to a Standard .010-inch Range Probe.

Note: Curve a is obtained using the normal Teflon cable which has 128 pico farad (pf) capacitance between center and guard.

Curve b is obtained with a cable with 486 pf between center and guard and with 483 pf added between guard and ground.

Fig. A2-7 Response Curve of Wayne-Kerr TE 100 Connected to a Standard .010-inch Range Probe.

The numbers above the curve are obtained with a normal cable (128 pf center to guard) and the numbers below the curve indicate the change caused by changing the cable to one with 486 pf between center guard and by adding 483 pf between guard and ground.

Note: The slope of the curve is 90 mv/mil $\pm$ 1 mv between any one of the data points, i.e. linearity is better than .1%.
"Check Setting" = 1.000 V.

APPENDIX 3
DIAPHRAGM STRESS AND RESONANT FREQUENCY

The stress in the diaphragm is of interest because of:

1. Long-term creep under constant pressure load
2. Yielding under momentary overload
3. Fatigue life under cyclic pressure
4. Overload capability

The stresses in a clamped diaphragm are:

at the edge

$$\sigma_r = \frac{3}{4} q \left(\frac{a}{h}\right)^2$$

$$\sigma_t = \frac{3}{4} q \nu \left(\frac{a}{h}\right)^2$$

at the center

$$\sigma_r = \sigma_t = -\frac{3}{8} q \left(\frac{a}{h}\right)^2 (1 + \nu)$$

The dimensions of the diaphragm:

a = .215 inch (radius)

h = .016 inch (thickness)

q = 125 psi (unit load)

With the dimensions and pressure given, the stresses are:

at the edge

$$\sigma_r = \frac{3}{4} \times 125 \left(\frac{.215}{.016}\right)^2 \approx 17 \text{ kpsi}$$

$$\sigma_t = .3 \times 17 \approx 5 \text{ kpsi}$$

at the center

$$\sigma_r = \sigma_t = \frac{1}{2} \times 17 \approx 8 \frac{1}{2} \text{ kpsi}$$

The diaphragm material is 347 S.S. Material properties used below were taken from the 1961 Eighth Edition of Metals Handbook, Volume I, page 503.

Item 1

The maximum operating temperature for which the design is made was 1000°F. At 1000°F, the tensile strength of annealed 346 S.S. is about 60 kpsi. It is accepted practice to estimate fatigue strength (if no experimental data is available) at 50% of the tensile strength.

The maximum stress calculated is only 30% of the tensile strength, and thus fatigue life of the diaphragm should be high ($> 10^8$ cycles).

Item 2

The .2% yield stress of annealed 347 S.S. at 1000°F is 18 - 40 kpsi. The maximum stress calculated is 17 kpsi, and it is, therefore, possible that some yielding could occur near the edge of the diaphragm. The material data shows at 1000-1200°F most yield stress data points in the 26-40 kpsi range. Therefore, yielding at 1000°F and 125 psi will be unlikely or restricted to a small amount of material. This conclusion is not altered by results obtained with the various applicable theories of failure, as the various stress combinations used, result in an equivalent stress equal to, or lower than, the maximum (principal) stress.

Item 3

The creep rate at the stress calculated at 1000°F is not directly available. However, the nearest data point given is at 1000°F and 26,000 psi, which shows a creep rate of 0.1μ strain per hour. The strain level in the diaphragm is in the order of 500μ strain maximum and thus a change in strain in the relatively small volume experiencing maximum stress should have negligible effect. This conclusion was confirmed in transducer tests made before the liquid metal pressure transducers were available, where no creep was observed at 1000°F and 17,000psi diaphragm stress over a period of 100 hours at constant load.

Item 4

During the life test in sodium, transducer #6 was tested at 1150°F, and at 150 psi load (diaphragm stress = 20.5 kpsi). During this test, a zero drift of .48 volt occurred, equivalent to about 250μ in of creep in the diaphragm. At 1200°F and 17 kp

stress, the creep rate of 347 S.S. is about 1 μ strain/hour, or about ten times greater than at 1000°F and 26 kpsi. Full scale displacement of the diaphragm is about 500 μ in. Therefore, 250 μ in of displacement due to material creep would indicate that creep occurred in large parts of the diaphragm. The creep observed in #6 pressure transducer must have been caused by repeated short-term (5 min. per cycle, less than 30 cycles) exposure to 20 kpsi at 1150°F. Such creep, though it apparently occurred is **not readily** predictable from consideration of the published material creep rate of 1 μ strain/hour at 1200 F and 17 kpsi.

It, nevertheless, seems inadvisable to expose the transducer to a load - temperature combination resulting in a bulk creep rate (at the maximum stress point) of greater than .1 μ strain/hour. For 1200°F, the stress at which .1 μ strain/hour occurs, is 10,000 psi. This stress occurs for a pressure load of 75 psi.

The resonant frequency of the diaphragm is calculated with:

$$f_1 = .47 \sqrt{\frac{E}{(1 - \nu^2) \rho}} \frac{h}{a^2}$$

E = modulus of elasticity, psi

ν = Poisson's ratio

ρ = density, lbs/in³

For a steel diaphragm, at room temperature the formula becomes:

$$f_1 = .97 \times 10^5 \times \frac{h}{a^2}$$

For the dimensions given above, the resonant frequency calculates as 33.6 KHz. This is well above the pass band of the Wayne Kerr capacitance meters, and, therefore, problems due to transducer ringing should not occur.

* see J. P. Den Hartog, Mechanical Vibrations, 2nd Edition, 1940, p.432
eq. (25)

DISTRIBUTION LIST FOR REPORTS GENERATED UNDER CONTRACT AT(30-1)-3930

U. S. Atomic Energy Commission
New York Operations Office
376 Hudson Street
New York, New York 10014

Attn: Leonard Wasser, Development Contracts Branch II
Joseph J. Dissler, Chief, Technical Branch

U. S. Atomic Energy Commission
Division of Reactor Development and Technology
Washington, D. C. 20545

Attn: Asst. Director for Plant Engineering
Asst. Director for Reactor Technology
Asst. Director for Project Management
Asst. Director for Reactor Engineering
Chief, Special Technology Branch (3 copies)
Chief, Liquid Metal Projects Branch (2 copies)
Chief, Components Branch
Chief, Instrumentation and Control Branch
Chief, Coolant Chemistry Branch
LMFBR Program Manager
FFTF Project Office

U. S. Atomic Energy Commission
Division of Technical Information Extension
P.O. Box 62
Oak Ridge, Tennessee 37830
(2) copies with form AEC-426, Block I, No. 3 will apply

U. S. Atomic Energy Commission
RDT Site Representative, AI
P.O. Box 1446
Canoga Park, California 91304

Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439

Attn: Director, LMFBR Program Office
Director, Reactor Engineering Division (3 copies)

NASA Scientific and Technology Information Facility
P.O. Box 33
College Park, Maryland 20740

Attn: Acquisition Branch

Brookhaven National Laboratory
Upton, New York 11973

Attn: Dr. O. E. Dwyer

Pacific Northwest Laboratory
P.O. Box 999
Richland, Washington 99352

Attn: FFTF Project Manager (2 copies)

Director, LMEC (2 copies)
P.O. Box 1449
Canoga Park, California 91304

Byron-Jackson Pump Division
Borg Warner Corporation
P.O. Box 2017 Terminal Annex
Los Angeles, California 90054

Westinghouse Electric Corporation
Advanced Reactors Division
P.O. Box 158
Madison, Pennsylvania 15663

Attn: H. R. Nelson

Westinghouse Electric Corporation
Atomic Equipment Division
Cheswick Avenue
Cheswick, Pennsylvania 15024

Attn: C. E. Anthony

Chief of Naval Research
Department of the Navy
Washington, D. C. 20360

Attn: Code 438

NASA Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135

Attn: Library MS 60-3 (2 copies)

Science and Technology Division
Library of Congress
Washington, D. C. 20540

Engineering Societies Library
345 East 47th Street
New York, New York 10017

Defense Documentation Center
Cameron Station
Alexandria, Virginia 22314 (2 copies)

Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78228

Attn: W. Hawkesworth