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THICK TARGET RECOIL MEASUREMENTS

John A. Panontin

and

Nathan Sugarman

MASTER

THE UNIVERSITY OF CHICAGO

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THE UNIVERSITY OF CHICAGO
Chicago, Illinois

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John A. Panontin

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Nathan Sugarman

Enrico Fermi Institute for Nuclear Studies and
Department of Chemistry, University of Chicago,
Chicago, Illinois

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THICK TARGET RECOIL MEASUREMENTS*

John A. Panontin† and Nathan Sugarman

Enrico Fermi Institute for Nuclear Studies,
Department of Chemistry, University of Chicago, Chicago, Illinois

ABSTRACT

Corrections for scattering and edge effects affecting thick target recoil results in fission studies are presented. Measurement of the range dispersion of Sr^{89} and Mo^{99} from thermal neutron fission of U^{235} yields $18.2 \pm 0.8\%$ and $19.9 \pm 0.5\%$, respectively, for the ratio of the full-width at half maximum of the range distribution in aluminum to the mean range, assuming a Gaussian form for the distribution. Similar determinations for Cu^{67} , Sr^{89} , and Mo^{99} from 450 Mev proton fission of uranium give $27.2 \pm 6.3\%$, $25.9 \pm 1.4\%$, and $23.8 \pm 0.6\%$, respectively. The ratio of ranges in the target material, for a thick target, relative to aluminum are presented for fragments from thermal neutron fission of uranium and 450 Mev proton fission of uranium, bismuth, and tantalum. The ranges in aluminum determined by the thick target technique compare favorably with those obtained by the thin target technique.

I. INTRODUCTION

The recoil technique has been a valuable method for the study of fission processes (1-5) and other nuclear reactions (6). Range measurements determine the kinetic energy of a product nuclide. Several factors have been shown to affect the measurements made with either thin or thick targets. A correction for target thickness is required in thin target experiments (1,3) and one for scattering in thick target experiments (1,2).

This paper reports the results of experiments on the thick target method as used in fission studies. Corrections for the scattering effect based on reported data and for an edge effect found in the present studies are presented. The range dispersion of a given fragment in aluminum has been determined for Sr^{89} and Mo^{99} from thermal neutron fission of U^{235} and for Cu^{67} , Sr^{89} , and Mo^{99} from 450 Mev proton fission of U^{238} . The ratio of ranges of several fission fragments in tantalum, bismuth, and uranium relative to aluminum are also presented. Moreover, the ranges in aluminum determined by the thick target technique are compared with those obtained by the thin target technique.

II. EXPERIMENTAL PROCEDURE

The experimental procedure used is similar to that used in previous high energy recoil studies^(4,5). The target assembly is described in Fig. 1. The target foil is surrounded by a number of thin, 1/4 or 1/6-mil aluminum foils, properly aligned, the number of which is dependent upon the range to be observed. These in turn are separated from similar foils for impurity activation by high purity 1-mil aluminum and the entire assembly is wrapped in 1-mil foil.

The uranium, bismuth, and tantalum foils employed were 1-mil, 5-mil, and 1/2-mil thick, respectively. The isotopic composition of the uranium target was that of the naturally occurring material. The uranium foil surface was chemically cleaned in dilute hydrochloric acid prior to use. Chemical analysis of the bismuth foil indicated greater than 99.99% purity. Within the sensitivity of spectrographic analysis, impurities in the region of the observed fission fragments were found to be absent. The tantalum foil exhibited similar properties.

The thin aluminum foils were of a commercial grade purity. As a result, activation foils were analyzed in each experiment and appropriate corrections for impurity

activation were made. In the case of the uranium experiments, the corrections were negligible, with the exception of Cu^{67} for which corrections of the order of 1 to 2% were made, depending upon the activity of the foil. For bismuth and tantalum, the activation corrections ranged from 2 to 10%, depending upon the foil and the nuclide considered.

Foil thicknesses were determined by weighing foils cut to size and measuring areas microscopically. The uranium, bismuth, and tantalum foils were 48.4, 138.8, and 23.3 mg/cm^2 thick, respectively. The thicknesses were constant to within 1% for uranium and bismuth and to within 2% for tantalum. The 1/6 and 1/4 mil aluminum foils had thicknesses of 1.185 and 2.10 mg/cm^2 respectively, constant to within 1/2%. Target dimensions were 1.0 cm x 1.5 cm for uranium, 1.5 cm x 0.64 cm for bismuth, and 1.5 cm x 2.0 cm for tantalum. The catcher foils were 1 in. x 1 in. and overlapped the edges of the target foil to ensure the capture of all recoils leaving the target.

The thermal neutron irradiations were performed at the Argonne CP-5 reactor in a flux of 5×10^{11} neutrons/cm²/sec for a period of one week. The high energy studies used the internal circulating proton beam of the University of Chicago synchrocyclotron at a radius of 76 in., corresponding

to a nominal proton energy of 450 Mev. The period of irradiation varied from 1 hour for uranium targets to 2 hours for bismuth and tantalum. Experiments were performed with targets perpendicular, and at 10 degrees, to the beam.

After the irradiations, the target and catcher foils were separated and dissolved for radiochemical analysis. With the exception of the experiments in which tellurium and ruthenium were isolated, the uranium targets were dissolved in 6N HNO_3 in the presence of HCl. For those experiments where ruthenium or tellurium was isolated, the solution procedures of reference 2 were used, for both the target and the catcher foils. The bismuth foils were dissolved in concentrated HNO_3 . Hydrofluoric acid in the presence of concentrated nitric acid was used to dissolve the tantalum targets. The aluminum catcher foils were dissolved in a mixture of 6N HCl and HNO_3 . The target solutions with the exception of those of tantalum were diluted to 500 ml while the catcher solutions were diluted to 25 ml. Duplicate samples of the catcher foils and quadruplicate samples in the case of the uranium and bismuth target solutions agreed to within one to two percent. Due to the low activity

resulting from tantalum targets, entire catcher and target foils were analyzed.

Copper---Copper carrier (~15 mg) was added as $\text{Cu}(\text{NO}_3)_2$ to 5 ml aliquots⁽⁵⁾. Copper was precipitated as CuI , dissolved in 6M HNO_3 , and a $\text{Fe}(\text{OH})_3$ scavenge performed with NH_4OH . A AgI precipitation followed and copper was reprecipitated as the iodide and then as the hydroxide with NaOH . Following the solution of the hydroxide precipitate in 6M HNO_3 , the cycle was repeated several times. The copper was finally precipitated as CuI .

Strontium---Strontium carrier (~20 mg) was added as $\text{Sr}(\text{NO}_3)_2$ to 5 ml aliquots⁽⁵⁾. Strontium was precipitated twice as the nitrate with fuming nitric acid in the presence of sodium and fluoride hold-back carriers. A ferric hydroxide scavenge was then performed, followed by a barium chromate precipitation from a slightly acidic solution containing acetic acid-ammonium acetate buffer. The scavenge procedures were repeated and strontium carbonate precipitated.

Molybdenum---Molybdenum carrier (~15 mg) was added as

$(\text{NH}_4)_6 \text{Mo}_7\text{O}_{24}$ to 5 ml aliquots. Two procedures were used and in the case of the tantalum experiments, both procedures were employed in succession to eliminate excessive activity of Na^{24} in the aluminum catchers.

Molybdenum was precipitated with α -benzoin oxime⁽⁷⁾ in the presence of oxalic acid and the precipitate dissolved in a HClO_4 - HNO_3 acid mixture. Two $\text{Fe}(\text{OH})_3$ scavenges were performed and the cycle repeated several times. The molybdenum was precipitated for mounting as the 8-hydroxyquinolate.

Molybdenum and ferric carrier⁽⁸⁾ were extracted from a 6N HCl solution, containing tellurium as a hold-back carrier, with ether equilibrated with acid of the same molarity. NaBrO_3 was added before the extraction to ensure complete oxidation of the molybdenum to the +6 oxidation state. After back-extracting the molybdenum and iron, two $\text{Fe}(\text{OH})_3$ scavenges were performed. The molybdenum was then precipitated with 8-hydroxyquinoline.

Ruthenium---Ruthenium carrier (~30 mg) was added as RuCl_3 to 5 ml aliquots⁽²⁾. The solution was treated with

KIO_4 in alkaline solution, followed by NaOCl . After standing, $\text{Na}_2\text{S}_2\text{O}_4$ was added and the precipitate dissolved in 6N HCl containing a few drops of concentrated HNO_3 . Upon dilution, concentrated H_2SO_4 was added and the ruthenium distilled from a $\text{Na}_3\text{BiO}_3\text{-H}_2\text{SO}_4$ solution into 12 N NaOH . The ruthenium was precipitated with ethyl alcohol, dissolved in concentrated HCl , and finally precipitated as the metal with magnesium. The total ruthenium activity was counted initially. The Ru^{106} was counted separately through 40 mg/cm^2 of aluminum to absorb the weak beta rays from Ru^{103} . The Ru^{103} activity in the total was estimated by correcting for the Ru^{106} activity from its absorption characteristics.

Cadmium---Cadmium carrier (~12 mg) was added as $\text{Cd}(\text{NO}_3)_2$ to 5 ml aliquots⁽⁹⁾. CdS was precipitated in slightly acidic solution (orange to methyl red indicator) and dissolved in 6N HCl . A PdS scavenge was followed by two $\text{Fe}(\text{OH})_3$ precipitations in a buffered ammoniacal solution. The cycle was repeated several times and the cadmium was precipitated for counting as cadmium anthralate.

The total cadmium activity of $\text{Cd}^{115\text{g}}$ and $\text{Cd}^{115\text{m}}$ was counted initially and after an appropriate time, the $\text{Cd}^{115\text{m}}$, separately. The $\text{Cd}^{115\text{g}}$ activity was obtained by extrapolating the $\text{Cd}^{115\text{m}}$ activity to the time of the initial measurements and subtracting its contribution.

Tellurium---Tellurium carrier (~45 mg) in a $\text{HNO}_3\text{-HCl}$ mixture was added to 5 ml aliquots and evaporated to dryness⁽²⁾. The residue was dissolved in 4N HCl, heated and tellurium precipitated as the metal with SnCl_2 . The tellurium was dissolved in a few drops of concentrated HNO_3 and evaporated to dryness. The residue was dissolved in 2N HCl, heated, and a $\text{Fe}(\text{OH})_3$ scavenge performed. The solution was acidified to 4N HCl and tellurium precipitated with SO_2 . The ferric hydroxide and SO_2 precipitations were repeated several times. The tellurium was precipitated as the metal with SO_2 for mounting.

The final precipitates were filtered onto circular paper disks over a 2.86 cm^2 area, dried, weighed, mounted on 1/16" aluminum cards and covered with cellophane. Activity measurements were made with several end-window methane-flow

proportional counters operating at 4000 volts. Due to the large number of samples involved for each nuclide, the decay of one target sample was followed as a standard. The activities of the remaining samples were then determined relative to this standard. Errors from unlike counter characteristics were eliminated by rotating the samples among the remaining counters. This counting procedure was followed over several half-lives for each nuclide investigated. Before counting measurements were made, all counters were checked and adjusted if necessary to maintain the reading of a standard U_3O_8 source within 1/4%. Empirical self-absorption corrections were applied to the various samples which were adjusted to a constant weight for comparison. The weights of precipitates varied from experiment to experiment within the following limits: Cu^{67} , 36 ± 2 mg; Sr^{91} , 31 ± 1 mg; Mo^{99} , 41 ± 2 mg; $Ru^{103,106}$, 21 ± 2 mg; $Cd^{115g,115m}$, 40 ± 2 mg; and $Te^{129m,132}$, 40 ± 2 mg.

III. RESULTS

1. Thick Target Corrections.

A. Scattering Effect.

Ranges in aluminum for various fragments from the thermal neutron fission of 1-mil uranium foil were determined by differential experiments using approximately 2.1 mg/cm^2

aluminum for the first catcher foil. The range was determined from the relation

$$\frac{F(>t)}{F(>0)} = \left(1 - \frac{t}{R_{Al}}\right)^2 \quad (1)$$

where $F(>t)$ is the fraction of the total activity beyond a thickness t mg/cm^2 of aluminum, $F(>0)$ is the fraction of the total activity escaping from one surface of the target, and R_{Al} is the range in aluminum in mg/cm^2 . A correction for the edge effect described in (Sec. III I. B) was applied by multiplying the experimental values of $F(>0)$ by 0.9957. The results are presented in Table 1. The values of R_{Al} (column 4) for Sr^{89} and Cd^{115} were found to be 5% and 2% lower, respectively, than corresponding values obtained by the thin target technique (column 7). Range values reported by Alexander and Gazdik are used for comparison since experimental uncertainties or differences in thickness measurements of aluminum catchers between the two experiments have been minimized. Several foil thicknesses measured independently agreed to within 0.2%.

In the 450 Mev proton fission of 5-mil bismuth foil, ranges in aluminum were determined by use of different thicknesses of aluminum catchers from the relations:

Table 1

Ranges of Fission Fragments in Aluminum from Thermal Neutron Fission of Thick and Thin Uranium Targets

Nuclide	Thick Target				Thin Target ^c	
	t , mg/cm ² Al	$\frac{F(>t)^a}{F(>0)}$	R_{Al} (uncorr.) mg/cm ²	$\frac{F(>t)^b}{F(>0)}$	R_{Al} (corr ²) mg/cm ²	
Sr ⁸⁹	2.114	0.2143	3.94±0.02 ^d	0.2246	4.02±0.02 ^d	
	10.008	±0.0014		±0.0015		
		0.2166	3.95	0.2270	4.04	
	2.108	0.2188	3.96	0.2294	4.05	
		0.2162	3.94	0.2266	4.02	
	2.104	0.2152	3.92	0.2256	4.01	
		0.2168	3.94	0.2273	4.02	
		Average:	3.94±0.004 ^e		4.03±0.008 ^e	4.12±0.02
Mo ⁹⁹	2.087	0.2060	3.82±0.02 ^d	0.2152	3.89±0.02 ^d	
	±0.008	±0.0013		±0.0014		
		0.2041	3.81	0.2132	3.88	
	2.085	0.2003	3.77	0.2094	3.84	
		0.2042	3.80	0.2134	3.87	
	2.088	0.2030	3.80	0.2121	3.87	
		0.1992	3.77	0.2081	3.84	
	2.114	0.1940	3.78	0.2027	3.85	
		0.2014	3.84	0.2104	3.91	
	2.108	0.1938	3.77	0.2025	3.83	
		0.1972	3.79	0.2061	3.86	
	2.104	0.1959	3.77	0.2047	3.84	
		0.2015	3.81	0.2105	3.89	
		Average:	3.79±0.007 ^e		3.86±0.007 ^e	
Cd ¹¹⁵	2.087	0.1309	3.27±0.01 ^d	0.1361	3.31±0.01 ^d	
	±0.008	±0.0009		±0.0009		
		0.1249	3.23	0.1298	3.26	
	2.085	0.1269	3.24	0.1319	3.27	
		0.1290	3.25	0.1341	3.29	
	2.088	0.1315	3.28	0.1367	3.31	
		0.1280	3.25	0.1331	3.29	
		Average:	3.25±0.008 ^e		3.29±0.008 ^e	3.33±0.04

a. The value of $F(>0)$ has been decreased by the factor 0.9957 for the edge effect.

b. The value of $F(>0)$ has been decreased by the scattering correction factors 0.954, 0.957, and 0.962 for Sr⁸⁹, Mo⁹⁹, and Cd¹¹⁵, respectively (Fig. 2).

c. Reference 1.

d. Estimate of cumulative error.

e. Standard deviation from the mean.

$$\frac{F_{(F,B)}(>t)}{F_{(F,B)}(>0)} = \left(1 - \frac{t}{R_{Al}(F,B)}\right)^2 \quad (2)$$

$$R_{Al} = \frac{R_{Al}(F) + R_{Al}(B)}{2} \quad (3)$$

where the subscripts F and B refer to the forward and backward directions with respect to the proton beam. The ranges so determined differ by less than 1% from those obtained from expressions correcting for the velocity of the struck nucleus and for the anisotropy of the fission process. The results are presented in Table 2. The values of R_{Al} (column 7) for a given fission product were found to vary with thickness of catcher foil, t , after correction for the target edge effect.

A scattering correction was made based on reported data⁽²⁾ and data obtained in this laboratory. Differences in the scattering properties of uranium and lead were considered negligible. Ranges in uranium of fission products determined with lead catchers were considered to be unaffected by scattering. The ratio of the range in uranium for a given fission product determined with lead catchers to the range as determined with aluminum catchers was plotted vs. mass number (Fig. 2). It was assumed that the scattered recoils add only to the activity of the foils adjacent to the target since the thickness of these foils

Table 2
Ranges in Aluminum^a for Fission Products from 450 Mev Proton Fission of Bismuth

Nuclide	t mg/cm ² Al	$\frac{F(>t)^b}{F(>0)}$	R_{AlF} mg/cm ²	$\frac{F(>t)^b}{F(>0)}$	R_{AlB} mg/cm ²	R_{Al} (uncorr.) mg/cm	$\frac{F(>t)^c}{F(>0)}$	R_{AlF} mg/cm ²	$\frac{F(>t)^c}{F(>0)}$	R_{AlB} mg/cm ²	R_{Al} (corr.) mg/cm ²
Cu ⁶⁷	1.185	0.4964	4.01	0.4414	3.53	3.77	0.5242	4.29	0.4661	3.73	4.01
	$\pm 0.008^d$	± 0.0034	± 0.04	± 0.0026	± 0.03	± 0.05	± 0.0036	± 0.04	± 0.0027	± 0.03	± 0.05
	2.110	0.2457	4.16	0.1868	3.70	3.93	0.2595	4.28	0.1973	3.78	4.03
	± 0.008	± 0.0016	± 0.02	± 0.0013	± 0.02	± 0.02	± 0.0017	± 0.02	± 0.0013	± 0.02	± 0.03
	2.116	0.2420	4.17	0.1814	3.69	3.93	0.2556	4.28	0.1916	3.76	4.02
	2.118	0.2421	4.17	0.1783	3.67	3.92	0.2557	4.28	0.1883	3.74	4.01
	2.270	0.2111	4.20	0.1544	3.74	3.97	0.2229	4.30	0.1630	3.81	4.05
Mo ⁹⁹	1.185	0.3954	3.19	0.3256	2.76	2.98	0.4132	3.32	0.3402	2.84	3.08
	± 0.008	± 0.0022	± 0.02	± 0.0018	± 0.01	± 0.02	± 0.0023	± 0.02	± 0.0019	± 0.01	± 0.02
	1.185	0.3958	3.19	0.3271	2.77	2.98	0.4136	3.32	0.3418	2.85	3.09
	2.100	0.1253	3.25	0.0687	2.85	3.05	0.1309	3.29	0.0718	2.87	3.08
	± 0.008	± 0.0008	± 0.01	± 0.0005	± 0.01	± 0.01	± 0.0009	± 0.01	± 0.0005	± 0.01	± 0.01
	2.104(F)	0.1232	3.24	0.0715	2.85	3.05	0.1287	3.28	0.0747	2.87	3.08
	2.088(B)										
Cd ¹¹⁵	1.185	0.3653	3.00	0.3070	2.66	2.83	0.3797	3.09	0.3191	2.72	2.91
	± 0.008	± 0.0022	± 0.02	± 0.0019	± 0.02	± 0.03	± 0.0023	± 0.02	± 0.0019	± 0.02	± 0.03
	1.185	0.3618	2.97	0.3044	2.64	2.81	0.3761	3.06	0.3164	2.71	2.89
	2.100	0.0985	3.06	0.0525	2.72	2.89	0.1024	3.09	0.0546	2.74	2.92
	± 0.008	± 0.0007	± 0.01	± 0.0003	± 0.01	± 0.01	± 0.0007	± 0.01	± 0.0004	± 0.01	± 0.01
	2.104(F)	0.0961	3.05	0.0556	2.73	2.89	0.0999	3.08	0.0578	2.75	2.92
	2.088(B)										

- To within 1% the quoted ranges are equal to ranges determined by more specific equations which include corrections for the velocity of the struck nucleus and the anisotropy of the fission process.
- $F_F(>0)$ and $F_B(>0)$ have been decreased by the factor 0.9678 for the edge correction (Sec. III.B.).
- $F_F(>0)$ and $F_B(>0)$ have been decreased by the scattering correction factors 0.947, 0.957, and 0.962 for Cu⁶⁷, Mo⁹⁹, and Cd¹¹⁵, respectively.
- The individual errors quoted are estimates of the cumulative experimental error.

was at least 25% of the range. For a given fission fragment, $F(>0)$ was then multiplied by a scattering correction factor from Fig. 2 and a corrected range in aluminum calculated. The correction factors for scattering of Fig. 2 were used for uranium, bismuth, and tantalum targets since range-straggling parameters for all of these targets are expected to be approximately the same for a given fission product.

The effect of the scattering correction on R_{Al} of fragments from thermal neutron fission of uranium is shown in column 6 of Table 1, and is found to decrease the deviation of the differential range measurements in aluminum with thick and thin targets to 1 to 2%. The significance of this variation, if real, is discussed in Sec. III. 4. In the bismuth experiments, the scattering correction removed the apparent variation of the range as determined with different thicknesses of catcher foils (Table 2, column 12).

The scattering corrections used are based on the assumption that the uranium ranges measured with lead catchers are correct and the differences between corresponding ranges as determined with aluminum catchers can be ascribed to scattering at the interface. It is expected, however, that there should be a small scattering correction even in the case of the lead catcher experiments. The

integral recoil equations for a thick target have been determined taking into consideration isotropic scattering at the end of the range for a catcher having similar scattering properties as the target. A fractional correction given by $1/3 (R_N/R_T)^2$ where R_N is the radius of the scattering sphere and R_T is the range in the target or catcher material was calculated. For uranium, R_N/R_T is approximately 0.06, where R_N is calculated from the critical velocity at which scattering occurs and Bohr's range-energy formulation.^(6,10) The correction expected is a 0.1% decrease of the range in uranium as determined with lead catchers. A similar effect exists at the interface of two catcher foils, when isotropic scattering occurs. Values of R_N/R_T calculated from the data of Petrzhak et al.⁽¹¹⁾ for light gases is approximately 0.09 giving rise to a 0.0027 decrease in the value of $F(>t)/F(>0)$, or a decrease in $R_{Al}(\text{corr.})$ of about 0.05% for the uranium data and 0.5 to 1% in the bismuth case. The correction for ranges in uranium is insignificant and the correction for ranges in aluminum has been disregarded in view of the uncertainty in applying this scattering model to fission fragments stopping in aluminum.

B. Edge Effect

During the course of this investigation, ranges in uranium from 1-mil targets bombarded with 450 Mev protons were found to be on the average 2% lower than values obtained from 3-mil targets (Table 3 Columns 4 and 8) as determined by the equation:

$$R_U = \frac{2W [F_F(>0) + F_B(>0)]}{1 + \eta^2} \quad (4)$$

where η^2 is a small correction ($\sim 0.1\%$) resulting from the velocity of the struck nucleus, R_U is the range in uranium in mg/cm^2 , and W is the target thickness in mg/cm^2 . On the other hand, the range of Sr^{89} in aluminum for 1 mil uranium was found to be 1% higher than for 3-mil uranium. The discrepancies in range suggested an edge effect, in which fragments recoiling from the surface of the target edges increased the apparent range in the target material and decreased the range in the absorber, by increasing the activity of the foils adjacent to the target.

Recoils emanating from the edges of a target are neglected in the equations for analyzing recoil data from thick targets. A correction for these recoils is therefore required. The number of recoils escaping from a surface is proportional to its area and the beam intensity of the particles bombarding the surface.

Table 3
Ranges in Uranium Determined with 1- and 3-mil Targets
(Uncorrected for scattering)

Nuclide	1-mil				3-mil			
	W mg/cm ² U	$P_F(>0)+P_B(>0)$	R_U mg/cm ² (uncorr.)	R_U^a mg/cm ² (corr.)	W mg/cm ² U	$P_F(>0)+P_B(>0)$	R_U mg/cm ² (uncorr.)	R_U^a mg/cm ² (corr.)
Cu ⁶⁷	48.43	0.1302	12.58					
	± 0.24	± 0.0005	± 0.08					
	48.43	0.1306	12.63					
	48.02	0.1319	12.65					
		Average:	12.62	12.56		Average:	12.82 ^b	12.54 ^b
Sr ⁸⁹			± 0.02	± 0.02			± 0.06	± 0.06
					146.0	0.03859	11.26	
					± 0.7	± 0.00014	± 0.07	
					146.0	0.03885	11.33	
	48.43	0.1131	10.94	10.89		Average:	11.30	11.05
	± 0.24	± 0.0004	± 0.07	± 0.07			± 0.04	± 0.04
						Average:	11.29 ^b	11.04 ^b
Mo ⁹⁹							± 0.07	± 0.07
	48.50	0.1046	10.13					
	± 0.24	± 0.0004	± 0.06					
	48.50	0.1053	10.20					
	48.43	0.1052	10.18					
	48.43	0.1069	10.33					
		Average:	10.21	10.16				
Cd ¹¹⁵			± 0.05	± 0.05				
	48.50	0.09976	9.67					
	± 0.24	± 0.00035	± 0.06					
	48.50	0.09858	9.55					
	48.43	0.09897	9.59					
	48.43	0.09892	9.57					
		Average:	9.60	9.55		Average:	9.81 ^b	9.60 ^b
Cd ^{115m}			± 0.03	± 0.03			± 0.04	± 0.04
	48.50	0.09498	9.19					
	± 0.24	± 0.00033	± 0.06					
	48.50	0.09460	9.16					
	48.43	0.09487	9.18					
	48.43	0.09565	9.24					
		Average:	9.19	9.15		Average:	9.44 ^b	9.24 ^b
			± 0.02	± 0.02			± 0.03	± 0.03

a. The ranges quoted here are corrected only for the edge effect. No correction for scattering or for the anisotropy of the fission fragment distribution has been made. The edge effect correction factors from Table 4 are 0.9951 and 0.9783 for 1-mil and 3-mil targets, respectively.

b. The ranges are those determined by Sugarman and co-workers, University of Chicago, unpublished work.

For the case of the thermal neutron fission of uranium, the neutron intensity may be considered uniform over the target surface area. The number of recoils emanating from surfaces A, Re A (see Fig. 3), which is the number to be considered for range calculations, is related to the total recoils observed in the catcher foils, Re T, by the following expression:

$$\text{Re A} = \beta \text{ Re T} \quad (5)$$

where

$$\beta = \frac{ab}{ab + (a + b) t}$$

and a, b, t, are the target dimensions, t representing the thickness.

In the proton bombardments, the beam intensity varies radially across the target. The average intensity is αI_0 , where I_0 is the intensity at the leading edge of the target. The radial dependence of the beam intensity can be expressed by an exponential with a half-value of $T_{1/2}$ cm. This value is determined experimentally by measuring the Na^{24} activity of 1/4-inch squares cut from a 1 square inch aluminum monitor, and α is calculated from:

$$\alpha = \frac{\int_0^T e^{-0.693T/T_{1/2}} dT}{\int_0^T dT} \quad (6)$$

where T is the effective radial length over which the beam intensity is to be averaged. In this case, the corrected

number of recoils in terms of the total number is given by:

$$\text{Re } A' = \beta' \text{ Re } T \quad (7)$$

where

$$\beta' = \frac{bT}{bT + (T + \frac{b}{2\alpha}) t}$$

Expressing the recoils as a fraction of the total fragments produced, the corrected fraction, $F_{\text{corr.}}(>0)$, is then given by

$$F_{\text{corr.}}(>0) = (\beta \text{ or } \beta') F_{\text{obs.}}(>0)$$

where $F_{\text{obs}}(>0)$ is the fraction measured. Values of β and β' are given in Table 4 for the various targets employed as well as for the target position with respect to the beam. Correction of the ranges from 1-mil and 3-mil targets for this effect leads to good agreement of the values (Table 3, Columns 5 and 9).

2. Range Distributions In Aluminum

When sufficiently thin foils are employed in the differential technique, recoil activity can be observed in foils beyond the average range. Excluding such factors as non-uniformity of the foils, thermal diffusion of the fragments, Rutherford scattering of the initially charged fragments, and non-uniformity in the target surface, the effects of which were considered and found to be negligible, the observed dispersion can be attributed to scattering near

Table 4
Correction Factors for Edge Effect

Target and Bombarding Particle	t cm. (x 103)	b cm.	a or T cm.	T _{1/2} cm.		a cm.		β or β'	
				Parallel	Perp.	Parallel	Perp.	Parallel	Perp.
1-mil U + thermal neutrons	2.59	1.50	1.00					0.9957	
1-mil U + 450 Mev protons	2.59	1.50	1.00	0.334	0.318	0.422	0.407	0.9952	0.9951
3-mil U + 450 Mev protons	7.71	1.50	0.635		0.169		0.356		0.9783
5-mil El + 450 Mev protons	14.05	1.50	0.639	0.291	0.243	0.514	0.460	0.9702	0.9678
1/2-mil Ta + 450 Mev protons	1.40	2.00	1.27	0.303	0.287	0.325	0.311	0.9976	0.9976

Parallel and perpendicular refer to the target position with respect to the beam.

the end of the range from atomic and nuclear collisions and to an energy dispersion in the fission process.⁽¹²⁾

Range straggling of particles arising from nuclear and atomic collisions has been considered by Bohr⁽¹⁰⁾ and found to be expressable by a Gaussian distribution of the form.⁽¹²⁾

$$P(R)dR = \frac{1}{\sqrt{2\pi} \rho R_0} e^{-\frac{(R-R_0)^2}{2\rho^2 R_0^2}} dR \quad (8)$$

where R and R_0 refer to the range of the fragment, R_0 being the average range. The straggling parameter ρ is modified from that given for the nuclear stopping region⁽¹³⁾ and can be expressed by

$$\rho^2 = \frac{2}{3} \left(\frac{R_N}{R_0} \right)^2 \frac{M_1 M_2}{(M_1 + M_2)^2} \quad (9)$$

where R_N refers to the average range in the nuclear stopping region and M_1 and M_2 are the masses of the moving and stopping atoms respectively. Bohr's value⁽¹⁰⁾ of ρ^2 is greater by a factor of 2.

The kinetic energy distribution from fission of a given fragment has been studied by several investigators⁽¹⁴⁻¹⁷⁾. The range dispersion of a number of fragments in various materials for several fissionable nuclei have also been determined^(11,18-20). From an analysis of the range straggling of Br^{87} and I^{137} from the thermal neutron fission of U^{235} in

various light gases and the assumption of a linear range-velocity dependence, Good and Wollan calculated the ratio of the full-width at half-maximum to the mean energy for the energy distributions of the light and heavy fragments of about 8 and 5% respectively⁽¹⁸⁾. The value for Br⁸⁷ is to be compared with the value $10.8 \pm 1.2\%$, reported by Stein⁽¹⁶⁾ for mass number 97, determined by time of flight technique. Petrzhak et al.,⁽¹¹⁾ from range dispersion measurements in various gases, performed in a manner similar to that of Good and Wollan⁽¹⁸⁾, report the ratio of the full-width at half-maximum to the mean energy for the energy distributions from fission of 12.9, 12.4, 13.2, 11.6, and 9.4% for Sr⁹¹⁻⁹², Y⁹²⁻⁹³, Zr⁹⁷, Ba¹⁴⁰, and Ce¹⁴³ respectively, from the thermal neutron fission of U²³³. If the range-velocity dependence involves the velocity to a power larger than unity, then the widths of the energy distribution as calculated from the widths of the range dispersion would be smaller than the quoted values of Good and Wollan⁽¹⁸⁾ and Petrzhak et al.⁽¹¹⁾. The values of Petrzhak et al. are in good agreement with the value, $12.0 \pm 0.8\%$, reported by Cohen et al.⁽¹⁵⁾ using a high resolution spectrometer and 12.2% reported by Milton and Fraser⁽¹⁷⁾ using time of flight technique for the energy distribution of mass 97 fragments from the thermal neutron fission of U²³⁵. The values quoted have not been corrected for the dispersion resulting from prompt neutron evaporation from the initially excited

fragments except for the value of Milton and Fraser⁽¹⁷⁾.

The range distributions in aluminum of several fragments from the thermal neutron fission of U^{235} have been determined and found to be approximately Gaussian⁽²⁰⁾. In the analysis of the present data, a Gaussian distribution is assumed for the straggling of the range, with straggling transverse to the original direction of flight of the fragment assumed to be negligible. For each range, equation 1 is considered to be applicable, so that $F(>t)/F(>0)$ is given by:

$$\frac{F(>t)}{F(>0)} = \frac{\int_t^{\infty} (1 - t/R)^2 P(R) dR}{\int_0^{\infty} P(R) dR} \quad (10)$$

where $P(R)$ is defined by equation 8 and $\rho = \frac{\lambda}{2.355}$ where λ is the ratio of the full-width at half-maximum to the mean range, R_0 . $F(>t)/F(>0)$ is the ratio of the number of recoils found beyond a thickness t , to the total number of recoils escaping from one surface corrected for scattering and edge effects. Although $P(R)$ does not have the proper normalization for a set of range values from 0 to ∞ , $P(R)$ may be regarded as exact since the contribution to the normalization integral of $P(R)$ for values from 0 to $-\infty$ is negligibly small in the cases considered. If a difference in the range-velocity dependence for the target and absorber is considered, Equation 10 is altered by raising t/R to the 0.97 power (see Sec. III.4).

Values of $F(>t)/F(>0)$ for various values of λ were obtained by numerical integration of equation 10 for the observed fragments and plotted graphically as illustrated in Figures 4 and 5 for Sr^{89} and Mo^{99} for a t value of 4.19 mg/cm^2 . The two curves given in Figures 4 and 5 are for $N_{\text{U}}/N_{\text{Al}}$ ratios of 0.97 and 1.00 (see Sec. III. 4). A value of λ for a given fragment was then interpolated from the graph corresponding to the observed value of $F(>t)/F(>0)$. Dispersion widths were also calculated from the thin target data⁽¹⁾ for thermal neutron fission of U^{235} in a similar manner. Calculated values of $F(>t)/F(>0)$ for different values of λ were determined by the following expression:

$$\frac{F(>t)}{F(>0)} = \frac{\int_{t+cw}^{\infty} \left(1 - \frac{t + cw/2}{R}\right) P(R) dR + \frac{1}{2cw} \int_t^{t+cw} R \left(1 - \frac{t}{R}\right)^2 P(R) dR}{\int_0^{\infty} P(R) dR} \quad (11)$$

where cw is the target thickness equivalent mg/cm^2 Al, and $F(>t)/F(>0)$ is equal to $2F_t$ as reported by Alexander and Gazdik⁽¹⁾. The first integral in the numerator corresponds to the ordinary thin target recoil equation. The second integral is a thick target type expression for the set of ranges (t to $t + cw$). The contribution to $F(>t)/F(>0)$ from this latter integral is negligible due to the relatively small value of cw .

The values of λ corresponding to the observed values of $F(>t)/F(>0)$ for both thick and thin targets are listed in Table 5, columns 5 and 8, respectively, for various fragments from the thermal neutron fission of U^{235} . Widths for range distributions in aluminum measured directly^(12,20) are also given in column 8 for comparison. The present results are in good agreement with this data.

The value of λ calculated for a given $F(>t)/F(>0)$ may be compared with an expected value calculated from the dispersion resulting from nuclear collisions of fragments of a given energy and that resulting from the energy distribution from the fission process. If the two effects are considered to be independent and the distribution curve of each to be Gaussian, then

$$\lambda^2 = \frac{2}{3} K^2 \left(\frac{R_N}{R_0} \right)^2 \frac{M_1 M_2}{(M_1 + M_2)^2} + \lambda_{\text{Fission}}^2 \quad (12)$$

where K in the nuclear collision term is a constant (2.355) which converts p to λ . The value of λ_{Fission} was taken to be one-half of the value of the energy distribution reported by Petrzhak et al.,⁽¹¹⁾ since the range is assumed proportional to the velocity. Values for R_N/R_0 for aluminum were obtained by appropriate analysis of the dispersion measurements reported by Petrzhak et al.⁽¹¹⁾ and are about 0.09 to 0.13 in fair agreement with the ratios predicted by Bohr's expression⁽¹⁰⁾. The values of λ calculated from Equation 12

Table 5
Full Widths at Half-Maximum for Range Dispersion in Aluminum for Various Fragments
from Thermal Neutron Fission of U^{235}

Nuclide	Thick Target			Thin Target				Ref.		
	t mg/cm ² Al	R_{Al} mg/cm ²	$\frac{F(>t)}{F(>0)}$	λ (g)		R_{Al} mg/cm ²	λ (g)			
				obs.	calc ^a		obs.		calc ^a	
Sr ⁸⁹	4.19	4.03	0.00092	18.2	9.7				Present work	
			$\pm 0.00011^e$	± 0.8						
		4.12		14.5 ^b						
				± 0.8						
Sr ⁹¹						4.02	21 ⁺ 4	9.6	(1)	
Zr ⁹⁷						3.99	15 ^c	9.6	(12)	
						4.16	22 ^d		(12)	
Mo ⁹⁹	4.19	3.86	0.00039	19.9	9.6				Present work	
			$\pm 0.0004^e$	± 0.5						
		3.96		16.5 ^b						Present work
				± 0.04						
						4.16	22 ^d	9.6	(12)	
Ag ¹¹¹						3.51	24 ⁺ 3	9.4	(1)	
Cd ¹¹⁵						3.33	21 ⁺ 1	9.3	(1)	
I ¹³¹						3.37	18 ⁺ 2	10.9	(1)	
Te ¹³²						3.34	18 ^c	10.9	(12)	
						3.49	26 ^d		(12)	
Cs ¹³⁶						2.64	17.7 $\pm$ 0.5	10.8	(20)	
Cs ¹³⁷						2.91	17.0 $\pm$ 0.5	10.8	(20)	
Ba ¹⁴⁰						2.82	17.5 $\pm$ 0.5	10.7	(20)	

a. Calculated from Equation 12.

b. Dispersion width and range calculated by assuming different range-velocity dependence in Al and U.

c. Results obtained with uranium oxide target on a nickel backing.

d. Results obtained with uranium oxide target on a copper backing.

e. Estimate of cumulative experimental error.

are presented in Table 5, columns 6 and 9.

Certain factors could affect the λ calculated for $F(>t)/F(>0)$. If the actual distribution is only approximately Gaussian and has a longer tail on the high range side as suggested by several investigators^(19,20), the number of recoils calculated for this foil will be less than the number of recoils observed for a given width and would be expected to be larger than that calculated for a Gaussian distribution. Although this enhanced probability on the high range side was not observed by Petrzhak et al.⁽¹¹⁾, nevertheless, allowing for such an occurrence, the effect should be small in the present case. The non-Gaussian behavior reported^(19,20) occurs at range values larger than $1.2 R_0$. The contribution to $F(>t)/F(>0)$ for Mo^{99} and Sr^{89} in the present case for range values greater than $1.2 R_0$ is small and approximately 10% of the total for a 16% dispersion width. Increasing the probability in this region by a factor of 2, alters the calculated $F(>t)$ by 10% and lowers the corresponding calculated λ by 3% of its value.

Non-uniformity in the foil thickness could also affect the present results. Calculations showed that if one-half the recoils traversed a thickness of aluminum 5% less than the average thickness while the other half traversed a thickness 5% greater than the average, the observed fraction of recoils could be accounted for by a Gaussian of

13% full-width at half maximum. However, such a large variation in thickness is highly improbable in the present studies in which the aluminum thickness for 1" x 2" foils varied by less than 1%.

The present values for the widths obtained from thick target differential absorption measurements are in good agreement with those calculated from thin target differential measurements and those measured directly. However, they are a factor of 1.5 to 2 times the widths expected from measurements of range dispersion in gases^(11,19).

Similar experiments for range dispersion were performed for Cu^{67} , Sr^{89} , and Mo^{99} from the 450 Mev proton fission of U^{238} . The range, R , in equation 11 is replaced by R_F or R_B in this case, where F and B refer to the forward and backward direction with respect to the proton beam, respectively. The average values of R_F and R_B determined from the simple expression, equation 2, were found to be equal to $R_0 (1 \pm \eta_{||})^{1.333}$ to within 1/3%, where R_0 is the average range and $\eta_{||}$ is the ratio of the parallel component of the velocity of the struck nucleus to the velocity of the fragment as determined by more exact expressions involving R_0 , $\eta_{||}$, and $N_{\text{Al}}(1.333)$, the velocity exponent in the range-velocity expression for aluminum. Equation 2 was compared to a more exact expression describing

$F(>t)/F(>0)$ in the region of the average values of R_F and R_B and it was found that the results agreed to within 1%. The effect of a perpendicular component of the velocity of the struck nucleus has a negligible effect on $F(>t)/F(>0)$ in the present case because of its small value. The simple expression, equation 2, may therefore be regarded as a good representation of the activity distribution in the aluminum catchers.

The results for Cu^{67} , Sr^{89} , and Mo^{99} are presented in Table 6. The average value of λ is $25.6 \pm 1.4\%$ which can be compared to the average value $19.0 \pm 1.0\%$ for Sr^{89} and Mo^{99} from the thermal neutron fission of U^{235} . If λ_{Fission} is equal to $6.4\%^{(11)}$ in the thermal neutron case, λ_{Al} for straggling in aluminum is found to be $17.9 \pm 1.0\%$. The observed values of λ for fragments from the proton fission of U^{238} were corrected for straggling in aluminum and the calculated λ for fission and η_{Al} are presented in the final column of Table 6. The average value of $\lambda_{\text{Fission}} + \eta_{\text{Al}}$ is $18.3 \pm 2.0\%$.

The relatively large value of $\lambda_{\text{Fission}} + \eta_{\text{Al}}$ in the proton fission of U^{238} may be expected to result from several sources. First, there is a distribution of the nuclides undergoing fission varying in charge number and mass as a result of the nuclear cascade process and neutron evaporation competition. Second, there is also an effect arising

Table 6
Full-Widths at Half-Maximum for Distributions of R_F and R_B for Various Fragments
from the 450 Mev Proton Fission of U^{238}

Nuclide	t (mg/cm ² Al)	$\frac{F(>t)}{F(>0)}$	R_F or R_B (mg/cm ² Al)	λ (%)	$\lambda_{\text{Fission}} + \eta$ (%)
Cu^{67}	4.19	0.0211 ± 0.0021^a	4.71 ± 0.01^b	29 ± 6	22.8 ± 7.7
	4.19	0.0051 ± 0.0006	4.25 ± 0.01	25.5 ± 2.0	18.2 ± 3.0
			Average	27.2 ± 6.3	20.5 ± 8.3
Sr^{89}	4.21	0.00133 ± 0.00013	3.97 ± 0.01	24.3 ± 1.3	16.4 ± 2.2
	4.23	0.00033 ± 0.00004	3.69 ± 0.01	27.5 ± 0.5	20.9 ± 0.9
			Average	25.9 ± 1.4	18.6 ± 2.4
Mo^{99}	4.19	0.00029 ± 0.00004	3.72 ± 0.005	23.8 ± 0.6	15.7 ± 1.5

a. Errors quoted are estimates of the cumulative experimental errors.

b. Standard deviation from the mean.

from neutron evaporation from the parent fragment during its time of flight. Third, $\lambda_{\text{Fission}} + \eta_{\text{II}}$ contains a contribution from the distribution of η_{II} . Due to the relatively small average value of η_{II} (0.03), the contribution to $\lambda_{\text{Fission}} + \eta_{\text{II}}$ should be small. Fourth, a relatively large spread in $\lambda_{\text{Fission}} + \eta_{\text{II}}$ as compared to λ_{Fission} for the thermal neutron case may originate in a large spread in the kinetic energy release for a given fragment arising from the fission process, resulting from the numerous fission modes available at high excitation energies and from a large spread in the excitation energy of the fission nuclides.

3. Relative Ranges in Ta, Bi, and U for Fission Fragments.

Fission Fragment ranges in U, Bi, and Ta and corresponding ranges in aluminum are given in Table 7. The ranges have been corrected for the scattering and edge effects as discussed in Sec. 3 and include corrections for the velocity of the struck nucleus and the anisotropy of the fission fragment. The ratio of the range in the target to that in aluminum is also tabulated. These ratios can be expressed by the following equations for the fragment studied ranging from Cu^{67} to Cd^{115} for the energy region 0.75 to 1.5 Mev/A.M.U.:

$$R_{\text{U}}/R_{\text{Al}} = 2.507 + 0.002282A \quad (13)$$

$$R_{\text{Bi}}/R_{\text{Al}} = 2.477 + 0.002282A \quad (14)$$

$$R_{\text{Ta}}/R_{\text{Al}} = 2.434 + 0.002282A \quad (15)$$

Table 7

Thick Target Range Measurements for Various Fission Fragments
from U, Bi, and Ta.

Fission Fragment	Ranges (mg/cm ²) ^a		R(target)R _{Al}
	Target	Aluminum	
Thermal neutron fission of U ²³⁵			
Sr ⁸⁹	10.92 ± 0.02 ^b	4.03 ± 0.01	2.71 ± 0.01
Mo ⁹⁹	10.63 ± 0.02	3.86 ± 0.01	2.75 ± 0.01
Cd ¹¹⁵	9.12 ± 0.02	3.29 ± 0.01	2.77 ± 0.01
450 Mev Proton fission of U ²³⁸			
Cu ⁶⁷	11.88 ± 0.02	4.49 ± 0.01	2.65 ± 0.01
Sr ⁸⁹	10.43 ± 0.02	3.83 ± 0.01	2.72 ± 0.01
Mo ⁹⁹	9.78 ± 0.04	3.58 ± 0.01	2.73 ± 0.01
Cd ¹¹⁵	9.15 ± 0.02	3.32 ± 0.01	2.76 ± 0.01
Cd ^{115m}	8.67 ± 0.03	3.15 ± 0.01	2.75 ± 0.01
450 Mev proton fission of Bi ²⁰⁹			
Cu ⁶⁷	10.47 ± 0.03	4.00 ± 0.004	2.62 ± 0.01
Mo ⁹⁹	8.37 ± 0.02	3.09 ± 0.004	2.71 ± 0.01
Cd ¹¹⁵	7.89 ± 0.02	2.89 ± 0.008	2.73 ± 0.01
Cd ^{115m}	7.39 ± 0.02	2.71 ± 0.008	2.73 ± 0.01
450 Mev proton fission of Ta ¹⁸¹			
Cu ⁶⁷	8.94 ± 0.03	3.48 ± 0.01	2.57 ± 0.01
Sr ⁹¹	7.04 ± 0.02	2.68 ± 0.01	2.63 ± 0.01
Mo ⁹⁹	6.63 ± 0.02	2.48 ± 0.01	2.67 ± 0.01

a. Ranges have been corrected for scattering and edge effects.

b. Standard deviation from the mean.

where A is the mass number of the fragment. The ranges in aluminum used are those determined by the thick target-thin absorber method and are 1 to 2% less than values determined with a thin target (see Table 1). If the ratio of the range in the target to that in aluminum is normalized, for a given fragment, to that for uranium, the resulting values represent the relative ranges of fragments in Bi and Ta compared to uranium. The results are presented in Table 8. The average values for Bi and Ta are 0.989 ± 0.010 and 0.971 ± 0.010 , respectively.

From Bohr's range-velocity relation ⁽²⁾ for fission fragments stopping in matter, the ratio of ranges (mg/cm^2) of a given fragment for two different stopping materials is given approximately by:

$$R_1/R_2 = (A_1/A_2)(Z_2/Z_1)^{1/3}$$

where the subscripts 1 and 2 refer to the different materials, and A, Z, and R represent the mass number, the atomic number, and the range, respectively. For Bi and Ta with respect to uranium, the calculated values are 0.909 and 0.822, 8 and 15%, respectively, lower than the experimental values.

The range ratios in Table 7 are lower than the previously observed values for these ratios with Bi and Au as the heavier stopping material. For Sr^{91} and $\text{Ba}^{129,133\text{m}}$, from the

Table 8

Comparison of Range Ratios of U, Bi, and Ta with respect to Al

Nuclide	U	Bi	Ta
Cu ⁶⁷	2.65 (1.00) ^a	2.62 (0.989)	2.57 (0.970)
Sr ⁸⁹	2.71 (1.00)		2.63 ^b (0.970)
Mo ⁹⁹	2.74 (1.00)	2.71 (0.989)	2.67 (0.974)
Cd ^{115,115m}	2.76 (1.00)	2.73 (0.989)	

a. Values in parenthesis refer to normalization with respect to ratio for uranium.

b. Refers to value for Sr⁹¹.

450 Mev proton fission of a thick bismuth target ($\sim 40 \text{ mg/cm}^2$), Sugarman et al⁽⁴⁾ have observed range ratios of bismuth to aluminum of 2.9 ± 0.1 and 2.8 ± 0.2 , respectively. The bismuth and aluminum range measurements have not been corrected for scattering and edge effects (the latter is insignificant in this case). A 4% correction for scattering would lower the range ratio by 6%, in reasonable agreement with the present data. For Zr^{97} from the thermal neutron fission of a thin uranium target, Suzor⁽¹²⁾ has reported a value for the range ratio of Au to Al of 2.73 ± 0.15 . This value is in good agreement with that expected for Au from these studies. Employing an ionization chamber to determine the recoil activity occurring beyond a given thickness of absorber, Segre and Wiegand⁽²¹⁾, observed a value of 3.0 for the range ratio of gold to aluminum for the gross products from the neutron fission of a thin uranium target. The ranges represent extrapolated ranges and therefore contain significant errors from the extrapolation, as well as inherent experimental errors in the method, making a significant comparison with the present results difficult.

4. Comparison of Ranges in Aluminum Measured with Thin and Thick Targets.

Ranges in aluminum for various fragments from the thermal neutron fission of uranium determined by the thin target technique may be compared with experimental and

calculated values obtained with a thick uranium target in Table 9. The experimental values are from Table 7 and the calculated values were obtained by correcting ranges in uranium for scattering and edge effects from Fig. 1 and Table 4 and then employing the ratios found in Equation 13, to convert the corrected ranges in uranium to ranges in aluminum. With the exception of Ag^{111} , good agreement is found between the thick target values and the thin target values reported by Alexander and Gazdik⁽¹⁾ and the values reported by Suzor⁽¹²⁾ from measurements with thin uranium oxide targets on a nickel backing. The ranges reported by Brown and Oliver⁽²⁰⁾ and Finkle et al⁽²²⁾ appear lower than the other reported ranges by approximately constant factors in each case, suggesting a possible inherent experimental error in thickness measurements of the absorbers or in corrections for velocity losses in the target.

There appears to be a consistent difference in the aluminum ranges in Table 1 for the thick and thin target methods, the thick target ranges being somewhat lower than those observed with a thin target⁽¹⁾. If this effect is outside the experimental error, the apparent difference can be accounted for by considering different range-velocity dependencies in the target and catcher materials. If the range (R)-velocity (V) expression for a

Table 9
Ranges of Fragments from the Thermal Neutron Fission of U²³⁵

Observed Nuclide	Ranges in Uranium mg/cm ²	Ranges in Aluminum mg/cm ²		Ref.
		Thick Target	Thin Target	
Br ⁸⁷			4.05±0.03	(23)
Sr ⁸⁹	10.92±0.02 (b)	4.03±0.01 (b)	4.12±0.02	(1)
	10.97±0.02 (a)	4.05±0.02 (calc)	3.74	(22)
Sr ⁹¹	10.97±0.04 (a)	4.05±0.02 (calc)	4.02	(1)
Zr ⁹⁵	10.81±0.03 (a)	3.97±0.02 (calc)	3.64	(22)
Zr ⁹⁷	10.81±0.02 (a)	3.97±0.02 (calc)	3.99±0.15 (c)	(12)
			4.16±0.15 (d)	(12)
Mo ⁹⁹	10.63±0.02 (b)	3.86±0.01 (b)	4.16±0.15 (d)	(12)
	10.64±0.02 (a)	3.89±0.03 (calc)		
Ru ¹⁰³	10.65±0.07 (b)	3.89±0.03 (calc)	3.57	(22)
	10.76±0.04 (a)	3.93±0.02 (calc)		
Ag ¹¹¹	9.32±0.06 (a)	3.38±0.02 (calc)	3.51±0.02	(1)
Cd ¹¹⁵	9.12±0.02 (b)	3.29±0.01 (b)	3.33±0.04	(1)
	9.12±0.03 (a)	3.29±0.02 (calc)		
Te ^{129m}	9.33±0.08 (b)	3.34±0.03 (calc)	3.34	(22)
	9.39±0.02 (a)	3.36±0.01 (calc)		
I ¹³¹			3.16	(22)
			3.37±0.02	(1)
Te ¹³²	9.28±0.03 (b)	3.31±0.02 (calc)	3.34±0.15 (c)	(12)
	9.28±0.02 (a)	3.31±0.01 (calc)	3.49±0.15 (d)	(12)
Cs ¹³⁶	8.07±0.03 (a)	2.87±0.02 (calc)	2.64±0.01	(20)
Cs ¹³⁷	8.86±0.03 (a)	3.15±0.02 (calc)	2.91±0.01	(20)
I ¹³⁷			3.21±0.04	(23)
Ba ¹⁴⁰	8.44±0.03 (a)	2.99±0.01 (calc)	2.98±0.01	(1)
			2.82±0.01	(20)
			2.75	(22)
Ce ¹⁴¹	8.27±0.01 (a)	2.93±0.01 (calc)	2.69	(22)
Ce ¹⁴⁴	8.06±0.07 (a)	2.85±0.03 (calc)	2.54	(22)

a. Reference 2.

b. Present work.

c. Results obtained with uranium oxide layer on nickel backing.

d. Results obtained with uranium oxide layer on copper backing.

e. ~~Standard deviation from mean.~~

stopping material (1) is given by:

$$R_1 = k_1 V^{N_1}$$

a recalculation of the differential expression for a thick target yields

$$\frac{F(>t)}{F(>0)} = \left[1 - (t/R_{Al})^{N_U/N_{Al}} \right]^2 \quad (16)$$

The ratios of N_U/N_{Al} found for Sr^{89} and Cd^{115} to give ranges in aluminum the same as those determined with a thin target was 0.970 ± 0.003 . For N_{Al} equal to $4/3$, N_U is equal to 1.293.

5. Conclusions

Ranges in aluminum of Sr^{89} and Cd^{115} from the thermal neutron fission of U^{235} were determined by the thick target differential recoil method, and agree with ranges determined by the thin target differential method to within 1 to 2%. The agreement depends upon the application of corrections for scattering and edge effects in the thick target method and for target thickness in the thin target case. The edge effect has satisfactorily accounted for differences in uranium ranges obtained with 1-mil and 3-mil uranium targets for a number of fission fragments. Moreover, scattering and edge corrections applied to the data from the 450 Mev proton fission of bismuth eliminated an apparent dependence of the average range in aluminum on the thickness of the first aluminum catcher used in the range measurement. The 1 to 2%

remaining difference between the thick and thin target range measurements in aluminum may be attributed to a difference in the range-velocity dependence in uranium and in aluminum. However, such a difference in the range-velocity dependence between the heavier target material and aluminum was not needed to explain the bismuth range results. Consequently, the difference in the range-velocity dependence for uranium and aluminum is not definitely established.

Range dispersion widths in aluminum, based on a Gaussian distribution of ranges, have been calculated for fragments from the thermal neutron fission of U^{235} and the 450 Mev proton fission of U^{238} . The values obtained with a thick target for the thermal neutron case are in agreement with values calculated from thin target differential experiments and dispersion widths measured experimentally with thin targets. An average value of the full width at half maximum of $19.0 \pm 1.0\%$ for the range dispersion in aluminum was obtained for Sr^{89} and Mo^{99} in the thermal neutron experiments. The range dispersion widths in aluminum are a factor of 1.5 to 2 times greater than values calculated from dispersion widths determined with gases. If the range dispersion width from the fission process is 6.4% in the thermal neutron case, the range dispersion width from the straggling of fission fragments in aluminum is calculated to be $17.9 \pm 1.0\%$ in the thick target experiments. The average dispersion

width is $25.6 \pm 1.9\%$, for Cu^{67} , Sr^{89} , and Mo^{99} from the 450 Mev proton fission of U^{238} . The average dispersion width in the high energy case, corrected for straggling in aluminum, is $18.3 \pm 2.0\%$.

The large increase in the width of 18.3% as compared to that of a fragment from thermal neutron fission of 6.4% can be attributed to several sources arising from high energy fission. First, there are a variety of nuclides resulting from the nuclear cascade process and neutron evaporation competition which undergo fission as compared to the single fissioning nucleus in the thermal neutron case. Second, there is a much broader spectrum in the number of neutrons evaporated from the parent fragment during its time of flight in the high energy process. Third, there is a spread in the range arising from a distribution in the velocity imparted to the struck nucleus. (The contribution from this source, however, should be small.) Finally, an increase in the width from the fission process could result from an increase in the number of fission modes available in the high energy case as a result of the higher excitation energy deposited in the fissioning nucleus and the broad spectrum of excitation energy induced by the bombardment.

The ratio of the range in the target material to that in aluminum for U, Bi, and Ta targets for a given fission

fragment has been found to vary by 3% from uranium to tantalum. The variation in the ratio expected from the range-velocity expression given by Bohr is 15%. A general expression for converting ranges in heavy elements in this region to ranges in aluminum has been presented.

FOOTNOTES

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FIGURE CAPTIONS

- Fig. 1. Target Assembly. W: 1-mil aluminum wrapper, M: 1-mil aluminum monitor, C: 1/4 or 1/6-mil aluminum catcher foils, T: target, A: activation foils (1/4 or 1/6-mil).
- Fig. 2. Ratio of Ranges in Uranium Measured with Lead and Aluminum Catchers vs. Mass Number of Fragment. $\circ$ Data from Reference 2; $\square$ Data from N. Sugarman and K. Wielgoz (unpublished).
- Fig. 3. Diagram of target surfaces showing surfaces B and C responsible for edge effect.
- Fig. 4. $\frac{F(>t)}{F(>0)}$ vs. Range Distribution Width at Half Maximum for Sr^{89} , for $t = 4.19, \text{ mg/cm}^2 \text{ Al}$; $\oplus$ Experimental Point; — Calculated Curve.
- Fig. 5. $\frac{F(>t)}{F(>0)}$ vs. Range Distribution Width at Half Maximum for Mo^{99} ; for $t = 4.19, \text{ mg/cm}^2 \text{ Al}$; $\oplus$ Experimental Point; — Calculated Curve.

Fig. I

Fig. 2

Fig. 3

$\lambda, \%$
Fig. 4

Fig.5