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THE INTERPRETATION OF PULSED
NEUTRON EXPERIMENTS*

Noel Corngold

Brookhaven National Laboratory
Upton, L. I., N. Y.

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I. INTRODUCTION

The title of this talk is "The Interpretation of Pulsed Neutron Experiments". About nine months ago, an IAEA conference on pulsed neutron research was held in Karlsruhe. There, I delivered a paper, titled, "The Interpretation of Pulsed Neutron Phenomena" (1). My predicament today is clear; I do not wish to repeat the Karlsruhe essay, but not enough has occurred since May to warrant a completely new paper. I shall extricate myself by presenting, instead, the central ideas of the earlier paper, salted with the results of other papers presented at Karlsruhe, and with even more recent, unpublished work, of which I have become aware. Since most of the participants here, today, were not at Karlsruhe, perhaps the stratagem will succeed, and not too many will be bored by a repetition of what may be, by now, familiar material.

In the following, I shall limit myself almost entirely to experiments carried out in non-multiplying media, which are homogeneous in composition and simple in shape. (One can think of spheres of light and heavy water, and of organic liquids, and prisms of graphite, beryllium, or ice.) Much research has been performed upon these systems during the last twenty years, and almost all of it has been based upon the exponential behavior of the neutron density at times long after the cessation of the pulse. (See Fig. 1). Thus, the natural results of the experiments are decay constants, obtained as a function of the size, shape, and composition of the medium. More precisely, almost all of the experiments have dealt with the fundamental (or "dominant", or "asymptotic") decay constant, since the higher constants are difficult to extract from the data.

Recently, the use of linear accelerators of great intensity has made possible the life-history experiment - the measurement of $N(v,t)$. Data of such great detail now grant the physicist much more freedom in choosing his mathematical "point of view" in analyzing the thermalization process.

Many scientists have been disappointed in the results, to date, of the decay-constant studies. Briefly, the reason is that the experiments can be carried out with assurance only for a restricted range of sample sizes, and these data do not tell us much about properties of the Boltzmann equation or about moderator cross-sections. The high-intensity equipment is now making it possible to make measurements upon smaller, more interesting, samples. Another aspect, the determination of higher eigenvalues, is made difficult not only by intensity limitations, but by the manner in which the eigenvalues are distributed. One is led, then, to consider other points of view in analyzing the time-behavior of a neutron pulse. Time moments of the distribution may be introduced into the analysis, and one can discuss the effect of thermalization upon the dispersion of the pulse. (2) Or, one can represent the time-behavior by functions other than exponentials. (3) Both points of view have not yet been fully exploited.

II. THE EIGENVALUE PROBLEM

A. General Considerations

When we seek the set of decay constants ("time eigenvalues") for a particular system, we are led to the eigenvalue problem,

$$\lambda N(\underline{r}, \underline{v}) = (v\Sigma + \underline{v} \cdot \text{grad}) N(\underline{r}, \underline{v}) - \int d\underline{v}' v' \Sigma_s(\underline{v}', \underline{v}) N(\underline{r}, \underline{v}') \quad (1)$$

$$= AN$$

which results from the Ansatz, $N(\underline{r}, \underline{v}, t) = N(\underline{r}, \underline{v}) e^{-\lambda t}$. (The notation in Eq. 1 is conventional). The problem is made precise by specifying that $N(\underline{r}, \underline{v})$ obeys the usual transport boundary condition on the surface of the sample, and that we shall seek our eigenfunctions in a Hilbert space of functions. As we noted earlier (1), the particular choice of the function space is dictated by convenience, rather than scientific necessity. The Hilbert norm of a kernel, $K(\underline{x}, \underline{y})$ is given by

$$\int d\underline{x} \int d\underline{y} K^2(\underline{x}, \underline{y}) = ||K||^2, \quad (2)$$

and the kernel is called "square-integrable" if $||K||^2$ is finite. The kernel is bounded if

$$\int d\underline{x} (Kf)^2 = (Kf, Kf) < c (f, f) \quad (3)$$

for all f in the space (c is independent of f), and is positive-definite if $(f, Kf) > 0$ for all f . We shall also refer occasionally to the symmetrized kernel,

$$K(\underline{v}', \underline{v}) = \exp\left[\frac{v^2 - v'^2}{2kT}\right] v' \Sigma_s(\underline{v}', \underline{v}). \quad (4)$$

and its associated neutron density, $f = \exp\left(\frac{1}{2} \frac{v^2}{kT}\right) N$.

The problem posed by Eq. 1 is exceedingly complex. One

might expect from the traditional approach, which is based upon separation of variables, that an eigenvalue problem based upon functions of six variables would lead to a six-parameter family of eigenvalues. We shall have to simplify the problem before proceeding much further. However, before doing so, we should mention that very recently, important results have been obtained about the spectra of a large class of operators, A. Albertoni and Montagnini, (4), of Milan, and Bednarz, (5), of Warsaw, presented proofs, at Karlsruhe, of the assertion that when the maximum diameter of a system is sufficiently small, no discrete decay constants exist. The critical diameter is of the order of an "average" mean-free path in the sample. Thus, for even smaller samples, the distribution of neutrons in velocity and space changes continually during the evolution of the pulse. No asymptotic distribution exists. (When decay constants do exist, they are almost always limited to the range $0 \leq \lambda < \lambda^*$. This fact has been known for some time (6)).

The theorem about the critical size was proved for the ideal, monatomic gas model of the moderator. Doubtless, it is also true for simple, fluid moderators, and for solids whose scattering is totally incoherent. We shall refer to these models and their scattering kernels as "sufficiently smooth". Albertoni and Montagnini were able to prove, further, that for any system with "bounded geometry", the number of discrete eigenvalues is finite. Their proof depends upon the symmetrized kernel being positive definite, a property which follows, upon inspection, for the class of kernels mentioned above. (7)

We should add that, quite recently, Mockel (8) has proved that the behavior of λ vs. thickness in the slab (unbounded geometry!) is similar to that in bounded geometry. Thus, for

slabs sufficiently thin, no asymptotic mode exists. It is remarkable that the multi-velocity results are so different from those obtained with the one-speed model, where the fundamental mode always exists. Doubtless, the key to the distinction is the presence of neutrons of arbitrarily small speed in the former case. As the sample size is diminished, the asymptotic spectrum cools, until it becomes a distribution, singular at $v = 0$, and no longer square-integrable. Then, the asymptotic mode ceases to exist.

These recent results make it quite clear that we cannot hope to represent the evolution of a neutron pulse by a denumerably infinite sum of discrete "modes", $N(\underline{r}, \underline{v}, t) \sim \sum_{k=0}^{\infty} \phi_k(\underline{r}, \underline{v}) \exp(-\lambda_k t)$. Rather, we shall obtain a finite number of discrete modes, supplemented by some sort of "continuum contribution". A typical picture of the dependence of discrete decay constant upon sample size may be seen in Fig. 2. R denotes a characteristic dimension of the sample. When the system fills all of space, or is "semi-infinite", the discrete modes may become denumerably infinite in number, or may form a continuous infinity. One of the most interesting problems in kinetic theory today (whether neutrons or gases) is to determine the relative importance of the discrete and continuous parts of $N(\underline{r}, \underline{v}, t)$ and, in particular, their respective "physical content". To make any progress at all towards its solution, we shall have to deal with special solutions of Eq. 1. Thus, we begin with the case of the infinite medium.

B. Solutions in an Infinite Medium

1. $B^2 \neq 0$

By considering an infinite medium, we have jettisoned the transport boundary condition, and so, admitted a great number of solutions to our problem. The Ansatz following Eq. 1 is

replaced by $N(\underline{r}, \underline{v}, t) \sim \exp(i\underline{B} \cdot \underline{r} - \lambda t) N(\underline{v})$, and the equation which results contains $\underline{B}$ as a "parameter". If the scattering kernel, $v' \Sigma_s(\underline{v}', \underline{v})$ is isotropic, we have:

$$n(v) = \frac{1}{Bv} \tan^{-1} \left(\frac{Bv}{v\Sigma - \lambda} \right) \int_0^\infty dv' v' \Sigma_s(v', v) r(v'), \quad (5)$$

where $n(v)$ is proportional to $\int d\Omega N(\underline{v})$, and the isotropic kernel appears on the right-hand-side. Had we not assumed isotropy, we should obtain a set of coupled equations for $n_\ell(v)$, the angular components of $N(\underline{v})$. (9)

For our class of sufficiently smooth kernels, one can show that solutions to (5) exist for all values of B^2 in the range $0 \leq B^2 < B^{*2}$. (10) A finite number of λ 's accompany each $B^2 > 0$, depend continuously upon B^2 , and are limited to the region $0 < \lambda < \lambda^*$. Further, this continuum of λ 's is not "enough" to form a complete basis for the representation of a pulse; there are additional continuum contributions (11), needed to describe the evolution of an arbitrary, initial $N(\underline{r}, \underline{v}, 0)$ in an infinite medium.

We are accustomed, in transport theory, to build approximate solutions for finite media by combining suitable infinite-medium solutions. One chooses a sequence of bucklings, B_1^2 , B_2^2 , which correspond to harmonics of the fundamental space-distribution, whose buckling is given by the usual analysis (12) $(B_o^2 (R+Z_o)^2 \sim \pi^2)$. This approximation has always formed the basis for the interpretation of experiment. There, one considers only the long-time behavior of the pulse, when a few B_n^2 suffice. In fact, serious analysis begins only when the space-distribution achieves its final form. Then, the analysis remains complex, because of the many λ 's which may be associated with B_o^2 . To date, almost all measurements have been concerned with $\lambda_o(B_o^2)$, which appears to be well resolved. Attempts

to measure higher decay constants have been only partly successful (13). One cannot always be sure that one is seeing $\lambda_1(B_o^2)$, rather than $\lambda_o(B_1^2)$. Though the buckling approximation has worked well in practice, it is quite limited, in principle. For example, it contains the implicit assumption that the final mode is separable in its space and velocity dependence. This is surely untrue in small samples, where measurements are most interesting. In large samples, however, where the assumption is good, the eigenvalues are crowded together so closely that their measurement is difficult, and not very significant. It is remarkable that we have been able to extract so much data from the region of intermediate sizes, and it is to our good fortune that the "maximum B^2 " property of the λ 's yields the same behavior in the limit of small sizes as does the exact theory. In summary, one may say that the approximation produces a set of curves quite similar to those of Fig. 2, each labelled (n,m) (from $\lambda_n(B_m^2)$) and corresponding to a separable eigenfunction.

Thus, we are led to study Eq. (5), and λ as a function of B^2 , in some detail. We know that these quantities are limited through $0 \leq \lambda < \lambda^*$, $0 \leq B^2 < B^{*2}$, and that λ increases smoothly with B^2 , since decrease of sample size produces an effect similar to increase of capture. (See Fig. 3). We also have a formal power series expansion of $\lambda(B^2)$ about $B^2=0$ to describe part of the curve explicitly (14). When we have discussed the distribution of $\lambda_n(B^2 = 0)$, the picture will be complete. Until now, we have emphasized the transport, and geometric, aspects of the subject. The discussion of $\lambda_n(B^2 = 0)$ emphasizes the physics of the thermalization process.

$$2. \quad B^2 = 0$$

The eigenvalue problem is, in this case,

$$Jf(\underline{v}) \equiv v\Sigma_s(v)f(\underline{v}) - \int d\underline{v}' K(\underline{v}', \underline{v}) f(\underline{v}') = \lambda f(\underline{v}), \quad (6)$$

where $K(\underline{v}', \underline{v})$ is a real, symmetric, kernel, and we have omitted the effects of absorption. The operator, J , always has a non-degenerate eigenvalue $\lambda = 0$, which may be associated with the property of neutron conservation.

We shall consider the two parts of J separately. The first, $v\Sigma_s(v)$, is a positive, "multiplicative" operator, with a continuous spectrum extending from $\lambda^* = (v\Sigma_s)_{\min}$ to $(v\Sigma_s)_{\max}$. For all of the common scattering models, $v\Sigma_s$ assumes its minimum value at $v = 0$; it is a monotonic, increasing function of v when the scattering is entirely incoherent, but shows rapid variation when coherent effects are present.

In most cases, the second part of J is a "smoother" operator than the first, and the eigenvalue spectrum of J may be thought of as the spectrum of $v\Sigma_s$, perturbed by K . This will be the case when K , or one of its iterates, is square-integrable. (In the language of functional analysis, the kernel is then "compact", or "completely continuous" (15)). In the case of the monatomic gas model, one can show that $\|K^3\|^2 < \infty$ (16). No similar proof exists for modes of liquid moderators. There, as with the gas, $\|K\|^2$ diverges. But, since the divergence is "weaker" for the liquid, doubtless some iterate of the kernel has finite norm. We may safely assume that the liquid kernel is also smooth.

A proper treatment of the scattering by a poly-crystalline moderator cannot ignore the effects of coherent scattering. The

kernel is then no more smooth than the operator, $v\Sigma_s$, for it contains delta functions which cannot, in general, be subtracted. We shall have to ignore this complication at this time, and to limit ourselves to models which utilize only the incoherent scattering. These kernels have finite $\|K\|^2$, and are, in a sense, smoothest of all that we have considered.

Having established our class of "sufficiently smooth" (or compact) kernels, we may invoke a theorem due to Weyl and von Neumann (17) to conclude that J has a continuous spectrum, the spectrum of $v\Sigma_s$. The effect of K is to introduce discrete eigenvalues into the "gap", $0 \leq \lambda < \lambda^*$. One might expect that the density, ρ , of discrete λ 's will reflect the "strength" of the perturbation caused by K , and one might predict that $\rho(\text{gas}) > \rho(\text{liquid}) > \rho(\text{solid})$. Indeed, this is the case, and, as we shall show, the reasoning exhibits an elegant connection between physics and mathematics.

The discussion of the discrete λ 's is simplified by the fact that the operator, J , has eigenfunctions of the form $f(\underline{v}) = f_\ell(v)Y_\ell(\Omega)$, where Y_ℓ is a spherical harmonic of order ℓ , and $\underline{v} = v\Omega$. Thus, the λ 's may be arranged into families, characterized by $\ell = 0, 1, \dots$, each family characterized by a $(2\ell+1)$ -fold degeneracy. Preliminary calculations (18) indicate that if all of the λ 's are arranged in ascending order, the positions of the $\ell = 1, 2, \dots$ families in the sequence depends upon the mass, and effective binding of the moderator atom. For example, the lowest member of the $\ell = 1$ family follows $\ell = 0, n = 1$ in the proton gas, follows $n = 3$ for the mass-12 gas, and follows $\ell = 0, n = \text{very large}$ in the case of Parks' model for graphite. We shall, for simplicity's sake, limit ourselves to the $\ell = 0$ family.

We now re-write the equation for f_0 , and introduce an auxiliary eigenvalue, c , in the manner of Lehner and Wing (19). Thus,

$$c \Psi(v) = \int_0^\infty dv' K_\lambda(v', v) \Psi(v'), \quad (7)$$

where $\Psi(v) = \sqrt{v \Sigma_s - \lambda} f_0(v)$, and

$$K_\lambda(v', v) = \frac{v'^2}{\sqrt{v' \Sigma_s(v') - \lambda} \sqrt{v \Sigma_s(v) - \lambda}} \int d\Omega K(\underline{v}', \underline{v}). \quad (8)$$

One then regards λ as a parameter, and studies the eigenvalues $C_n(\lambda)$ as λ is varied between zero and λ^* . The values, λ_k , for which some $C_n(\lambda)$ intersects the line $C = 1$, are the eigenvalues we seek. It is not difficult to show that the number of eigenvalues lying between zero and $\lambda^{**} < \lambda^*$ is finite, for our smooth models. This follows immediately upon the demonstration that the kernels $K_\lambda(v', v)$ are compact when $\lambda < \lambda^*$. The interesting behavior occurs when we consider $\lim_{\lambda \rightarrow \lambda^*} K_\lambda$. If K_{λ^*} is compact, the number of discrete eigenvalues (belonging to $\ell=0$) will be finite. If not, one can say nothing without making a more detailed examination. (20)

The elegant connection between $\rho(\lambda)$ and the physical state of the moderator is based upon the Van Hove theory of slow neutron scattering. (21) The models we are considering are, in the terminology of the theory, incoherent, Gaussian, models. All of the information about the dynamics of the scattering atoms is contained in a single function, the "width-function", $w(t)$. This function is related to the average distance travelled by the atom in an interval of time, t , as it pursues its characteristic motion. When t is small, the width functions for all models coincide (free-particle motion); when t is large,

marked differences occur. Now, the square-integrability (compactness) of K_{λ^*} depends critically upon its behavior when v, v' are small. This is the region of quasi-elastic scattering for $K(v, v')$ and of near-complete "up-scattering" for $v \Sigma_s$. To a large extent, these aspects are controlled by the long-time behavior of $w(t)$. One finds that when the moderator is solid ($w(t)$ bounded) K_{λ^*} is compact, when the moderator is a gas ($w(t) \sim t^2$) K_{λ^*} is not compact, and when the moderator is a liquid $\|K_{\lambda^*}\|^2$ diverges "weakly". A detailed study shows that $K_{\lambda^*}(\text{liquid})$ is indeed, non-compact, and that both the gas and liquid have an infinite number of discrete eigenvalues, packed closely against the limit point, λ^* . Thus, the conjecture made earlier, that $\rho(\text{gas}) > \rho(\text{liquid}) > \rho(\text{solid})$ is borne out. In fact, the first two moderators have an infinite number of λ 's in the gap, while the solid has very few; for crystals having characteristic frequencies such that $\theta/T \gtrsim 1$, the gap is empty, except for the fundamental, $\lambda_0 = 0$.

III. THE CALCULATION OF DISCRETE EIGENVALUES

Once the range of the discrete eigenvalues is appreciated, and their general trends understood, many techniques are available for their computations. One can use multi-group methods, (22) with the understanding that eigenfunctions and eigenvalues belonging to $\lambda > \lambda^*$ refer to "packets" of singular (continuum) eigenfunctions. One can expand the density in terms of orthogonal polynomials, (23) or one can expand the scattering kernel in terms of a series of degenerate kernels. (24). There are no difficulties, in principle, in the solution of the Boltzmann equation; the various methods yield comparable accuracy for comparable effort.

The difficulties which remain, today, are connected with the choice of an adequate model for the description of the scattering. If we are to seriously consider a model for the comparison of theory and experiment over a large range of sample sizes, the model should contain the true transport mean-free path as a function of energy, correct values of the more important velocity moments of the isotropic scattering kernel, and the correct value of λ^* . These tend to guarantee the energy-dependent leakage, the approach to equilibrium (high eigenvalues), and the correct limit for the decay constants. The requirements are much more easily satisfied for incoherent scatterers than for coherent scatterers. Since we also know that experiments in the former are much easier to carry out, we conclude that we face a much more difficult situation in interpreting experiments in beryllium, beryllium oxide and graphite than we do in light water, polyethylene, or (even) heavy water.

Before turning to a discussion of particular scattering kernels, we should mention two simple kernels, of pedagogic interest, and a little more. The first is the simple, degenerate kernel, (25)

$$v' \Sigma_s(v', v) = \beta v' \Sigma_s(v') v \Sigma_s(v) M(v). \quad (9)$$

This isotropic kernel gives the correct mean free path, and the correct λ^* , when β and $\Sigma_s(v)$ are appropriately chosen. Its velocity moments are not particularly realistic; there is but a single discrete eigenvalue, and the approach to equilibrium is quite rapid. The kernel is of more than pedagogic interest in situations dominated by the leakage of neutrons. Milne's problem (26) and the decay of a pulse in very small systems, (27) are two examples.

A second kernel, which leads itself to computation, is the "secondary model" of Cadhilac. (28) One chooses the form

$$\begin{aligned} v' M(v') \sum_s (v', v) &= A(v') B(v) & v' < v \\ A(v) B(v') && v' > v, \end{aligned} \quad (10)$$

so that the scattering kernel becomes the Green's function for a differential operator of second order. Then, through choice of $A(v)$ and $B(v)$, one can cause the synthetic kernel to have a pair of correct velocity moments. Thus, Eq. (10) represents a considerable advance over Eq. (9). In recent unpublished work, Williams has discussed in detail the eigenvalue spectra produced by Eq. (10). He points out that rough, quantitative agreement with the more refined models may be obtained, through the expenditure of only a fraction of the computational effort.

If one seeks the most accurate solutions to the transport equation that are possible, one must use the kernel which duplicates - to the greatest extent - experimental data on the scattering of slow neutrons. Since it is not yet feasible to go directly from the measurement of the Scattering Law to the computing machine code for the solution of the Boltzmann equation, the connection is mediated by a "model". The model is, in some cases, physical, in others (viz., the foregoing) mathematical, and, occasionally is a combination of both. Let us consider, in turn, the most useful models which have been proposed for the common moderators.

1. Light Water: The model proposed by Nelkin (29) takes into account the translational, and optical-vibrational modes of motion of the free H_2O molecule. The effect of intermolecular forces, in the liquid state, was expressed, originally, by including a mode representative of a torsional oscillation. The

model gives quite acceptable results for the total cross-section (within, say, 5%), the transport cross-section, λ^* , and the velocity moments at energies above the $\hbar\omega$ for the torsional mode. Recent work has taken into account the anisotropic nature of the optical vibrations (30), and has replaced the single, torsional frequency by a band (31). Agreement with the total cross-section is improved, thereby, and undoubtedly, the velocity moments are also improved.

A simpler model, rather mathematical-physical, has been proposed by Egelstaff and Schofield (32). Its width-function has a particularly convenient mathematical form, and represents a particle whose motion is initially free, becoming, in time, Gaussian diffusion. Williams and Wood (33) have recently pointed out that the free-parameters in the model may be chosen so that acceptable values of the average transport mean-free path, λ^* , and the discrete eigenvalue λ_1 , are obtained. The model would appear to occupy a position between the improved dynamical model, and the simple, synthetic kernels.

2. Heavy Water: Honeck (34) has discussed the D_2O molecule in a manner analogous to Nelkin's treatment of H_2O , while Butler (35), has considered a new effect, the coherent scattering, whose inclusion improves the total cross-section below kT . The simple model of D_2O is probably less accurate below kT than is its H_2O counterpart, for it ignores coherent effects. At this moment the relation between theoretical and experimental values of λ^* is unknown, and one would guess that the D_2O model will require improvement before it can be relied upon to predict decay constants in small samples.

3. Beryllium and Graphite: The connection between neutron scattering experiments, the physical model, and the scattering kernel is quite direct in the case of the crystalline solid. The model is that of the ideal, harmonic solid, whose particles interact via central forces which extend through several nearest-neighbor distances. In the "fundamental" approach (36) phonon dispersion relations, derived from the scattering, are used to obtain inter-atomic force constants. With these, the equations of motion are solved, and $\rho(\omega)$, the distribution of vibrational modes, is obtained. In another approach (37), one gets $\rho(\omega)$ directly from the experimental data, through an extrapolation technique. Unfortunately, the extrapolation is severely complicated by coherent effects, and one obtains a $\rho(\omega)$ that is quite different from the "fundamental" $\rho(\omega)$. While it is true that the values of pulsed-neutron parameters are not sensitive to all details of $\rho(\omega)$, no systematic study of the dependence has yet been made.

The best kernels that we have, at this time, have been calculated by Parks, Young, and Koppel, using the computer code, SUMMIT (38). Inelastic scattering is calculated in the incoherent approximation for energies above the Bragg cut-off, E_B . The calculation is carried out using the phonon expansion; the "fundamental" $\rho(\omega)$ is used in the few-phonon terms, the Gaussian approximation in the remainder. The elastic scattering, which is overwhelmingly coherent, is treated exactly. As one might expect, the kernel which emerges is beyond reproach - in the interpretation of distributions in $E > E_B$. Unfortunately, pulsed neutron experiments in small, crystalline, samples are concerned with distributions in $E < E_B$, and it is in this region that the incoherent approximation fails. One

must calculate the coherent, inelastic scattering in detail to obtain a good kernel, and a reasonable value for λ^* . I understand that these important calculations are now in progress.

IV. THEORY AND EXPERIMENT

I should like to give a brief and, perhaps, over-simplified description of the status of theory vis-a-vis experiment. For an excellent, detailed analysis of the situation, one should consult the Karlsruhe paper of Beckurts (39).

1. Light and Heavy Water

In H_2O , mean free paths are small, and it is quite impractical to perform experiments upon truly small samples ($\langle \Sigma_s R \rangle \sim 1$, $\lambda_0 \sim \lambda^*$). Thus, the experimental distributions may be represented by infinite-medium solutions of the type discussed earlier, and the demands imposed upon the scattering kernel are not too stringent. The classic expression,

$$\lambda_0 = \overline{v \Sigma_a} + D_0 B^2 - C B^4 + F B^6 + O(B^8), \quad (11)$$

suits the data; the values of the coefficients, obtained in various laboratories are consistent, and the improved version of the Nelkin model predicts them within experimental error. Möller and Sjöstrand (40) have obtained a value of λ_1 ($B^2 \approx 0$), and this, too, is predicted by the model. The classic calculation of $\lambda_0(B^2)$, through the solution of the coupled equations beginning with Eq. (5), was done by Honeck (41). Perhaps the most interesting avenue that is now open, is the study of pulsed phenomena near the freezing point. It is quite possible that the diffusion cooling changes in a singular manner, as water undergoes a change in phase (42).

The situation with D_2O is not quite as satisfactory; there is reasonable, but imperfect agreement between the coefficients, D_0 and C , reported by different laboratories, and no estimates of higher eigenvalues are available. The values of D_0 and C predicted by the simple model, and calculated as in (43) are "in the range". As we turn to substances which are poorer moderators, and are characterized by interference scattering, and longer mean free paths, agreement becomes increasingly worse.

2. Beryllium and Graphite

Experimental values for the diffusion cooling in beryllium and graphite are as inconsistent today, as they were in 1962. C (graphite) ranges from 16 to $46 \times 10^5 \text{ cm}^4 \text{ sec}^{-1}$, and C (beryllium) can hardly be called known. These parameters certainly "exist"-the difficulty lies in obtaining the large- B^2 data that have great statistical significance in their determination. The time which must elapse before one can be confident of observing a fundamental mode is so great that only laboratories equipped with high-intensity sources can perform the experiments. Among these, the R.P.I. studies of the life-history of a neutron pulse in beryllium are exceptional (44). These experimenters have described the evolution of the flux distribution in time, when large and small samples were pulsed. Their observations were consistent with the prevailing theoretical picture. Equilibrium distributions were achieved after several milli-seconds in the "large" samples ($B^2 < 0.03 \text{ cm}^{-2}$, approximately). These distributions were not excessively distorted. The associated decay constants were not reported, but it is likely that $\lambda_0 \sim \lambda^*$ at 0.03 cm^{-2} (45). In still smaller samples, no fundamental distribution could be found, within the waiting time that was available. The distribution became progressively "cooler", and more distorted as it evolved, showing the effects of the irregular variation of leakage with energy (46).

A serious confrontation of experiment with theory cannot occur until we are content with the behavior of the scattering kernel for $E < E_B$. As a first step one will most likely compare the theoretical and experimental "histories", $N(v,t)$, in large samples. Later, with the use of a satisfactory scheme for incorporating exact boundary conditions, one will be able to approach the $\lambda = \lambda^*$ limit. It is not likely that the asymptotic theory (infinite medium solutions) will suffice.

Until that time, we can refer only to "computing-machine experiments", carried out with imperfect, but acceptable kernels. The most exhaustive study has been that of Ghatak and Honeck (47), who discussed both the life-history, and the eigenvalue problem by using Parks' kernel for graphite, and diffusion theory. One should also consult the studies of Wood, (48), in which a multi-phonon (Debye and Gaussian) kernel was used to represent beryllium, and the infinite medium eigenvalues ($B^2 \neq 0$) studied in some detail. Both are rich in detail, and bear out the existence of a critical buckling, B^{*2} , and a discrete spectrum empty, but for λ_0 (49).

There remains some interesting speculation about experiments carried out for $B^2 > B^{*2}$. Does the fundamental, exponential, mode disappear abruptly at B^{*2} , or does its "shadow" persevere, as a sharp peak in the distribution of continuous modes? More generally, under what conditions will the contribution from the continuous spectrum resemble a sum of discrete modes, perhaps superimposed upon a weak, and slowly-varying background? Clearly, this behavior is not as likely to occur in an incoherent scatterer, as in a coherent scatterer, where the characteristic irregularities in the cross-section are reflected as peaks and valleys in the weight function for the "continuum" contribution.

Some analytical evidence for these remarks has recently been offered (50), and the aficionado will recognize in them an expression of de Saussure's "trap" effect (51). Surely these phenomena will also appear in pulsed, fast systems, where the continuum abounds (52). But, that is another story.

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Figure Captions

Fig. 1 A typical exponential decay.

Fig. 2 Decay constants (time-eigenvalues) in samples of various sizes. R is a characteristic dimension. (n,m) refers to (λ_n, B_m^2) for the infinite medium solution best approximating the exact $\lambda(R)$.

Fig. 3 λ_n vs. B in a typical case. (Ref. 24.)

Decay of the neutron flux in a beryllium assembly
 $14\frac{3}{8} \times 14\frac{3}{8} \times 16 \text{ in.}$

FIGURE 1

FIGURE 2

FIGURE 3