Effects of the Polyenergetic Character of the Spectrum of ^{125}I
on the Measurement of Bone Mineral Content

by

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Abstract

The spectrum of the radionuclide ^{125}I , which is extensively used for measurements of bone mineral content (BMC), consists of five photon energies ranging from 27.2 to 35.4 keV. The effect on the measurement of BMC due to the preferential attenuation of the low energy components of this spectrum in bone mineral (hydroxyapatite) and soft tissue has been investigated. A theoretical examination of this effect was made by calculating the exponential attenuation of each component of the spectrum individually and then summing the attenuation factors weighted by the fractional contribution to the spectrum of each component. The results of similar calculations for aluminum in water were found to be in good agreement with measurements indicating the validity of the above method for predicting hardening effects. The measurements and calculations were also performed with different thicknesses of tin filtration in the beam. Values of BMC determined by the above method and by a linear estimator were found to agree to $\pm 1\%$ over the range 0.6 to 1.5 g/cm² of hydroxyapatite. Variations in soft-tissue cover changed the value of BMC by 0.5% to 1.0% per cm of unit density material. The largest effect was due to variations of tin filtration which can change the value of BMC by 0.12% to 0.16% per μm of tin. These calculations indicate that the variation between tin filters used in measurements of BMC should be limited to $\pm 2.5 \mu\text{m}$.

As a polyenergetic beam of radiation passes through matter the low energy components of the beam are usually more strongly attenuated than the high energy components. This effect is termed "hardening" since the increasing proportion of the high energy components results in more penetrating or "harder" radiation. The spectrum of ^{125}I , the radionuclide most extensively used for photon absorptiometric measurements of bone mineral content (BMC), consists of five photon energies ranging from 27.2 to 35.4 keV. The effect of hardening on BMC measurements is such that the measured bone mass does not increase linearly with the mass of the bone. That is, values of BMC measured for bones of large mass are less than the values which would be obtained by linear extrapolation from measurements of bones of smaller mass. Such an extrapolation is only valid if the photon source is monoenergetic and consequently exhibits exponential attenuation in matter.

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Although the spectrum of ^{125}I can be made more nearly monoenergetic by filtration with tin, which has a K-absorption edge at 29.2 keV, variation of the thickness of tin filtration among sources can also change the measured BMC.¹¹ Both the effect of variation of tin filtration and the effect of hardening of the ^{125}I spectrum will be discussed in this paper.

In developing the theory of the improved method of bone mineral measurement by the photon absorptiometric method, Cameron and Sorenson² assumed that the photon source is essentially monoenergetic; that is, the attenuation of its radiation is exponential. In a later article Sorenson and Cameron¹⁰ reported that an ^{125}I source filtered with 0.1 mm of tin did show exponential absorption over three decades of attenuation. However, in the same paper there was also a discussion of the observation of a slight non-linearity in the plot of the mineral content measured by the photon absorption method as a function of the mass of the measured bone section after ashing. This non-linearity was attributed partly to the finite cross-section of the photon beam. The same type of non-linearity which these investigators observed is produced by hardening of the ^{125}I spectrum as the beam passes through the measured bone and adjacent soft tissue.

Theoretical Estimate of the Hardening Effect

The attenuation of a discrete spectrum of radiation can be generally described by the equation

$$I = I_0 \sum_{i=1}^n f_i \exp \left\{ - \sum_{k=1}^m \mu_{k,i} \rho_k t_k \right\}$$

where I = the intensity of the radiation after attenuation

I_0 = the unattenuated intensity of the radiation

n = the number of components in the radiation spectrum

f_i = the fraction of photons in the beam having the energy of the i^{th} spectral component

m = the number of absorbers in the beam

$\mu_{k,i}$ = the mass attenuation coefficient of the k^{th} absorber at the energy of the i^{th} spectral component

ρ_k = the density of the k^{th} absorber

t_k = the thickness of the k^{th} absorber.

In making a determination of the BMC two measurements are required: the count rate I_0^* through a thickness t_{ST} of soft tissue only and the count rate I through the thickness t_{BM} of bone mineral plus the thickness $(t_{\text{ST}} - t_{\text{BM}})$ of soft tissue. The total thickness of soft tissue plus mineral is assumed to be constant for all points of measurement. With this assumption and the further assumption that the photon source is monoenergetic the quantity $\ln(I_0^*/I)$ can be shown to be linearly related to the mass of bone mineral in the path of the photon beam. However, considering the ^{125}I spectrum to consist of five discrete components, and defining the mass per unit area, M ,

of a substance as the product of its density, ρ , and its thickness, t , leads to a relation for $\ln(I_o^*/I)$ as a function M_{BM} and M_{ST} obtained from the previous equation, which is

$$\ln(I_o^*/I) = \ln \left\{ \frac{\sum_{i=1}^5 f_i \exp \{ -\mu_{ST,i} M_{ST} \}}{\sum_{i=1}^5 f_i \exp \{ -\mu_{ST,i} M_{ST} + [(\rho_{ST}/\rho_{BM}) \mu_{ST,i} - \mu_{BM,i}] M_{BM} \}} \right\} \quad (1)$$

where the subscripts ST and BM refer to soft tissue and bone mineral, respectively.

The effects of photon energy shifts due to inelastic scattering have been excluded in this derivation since the geometry of our apparatus permits detection of photons singly scattered through a maximum angle of 12° . Compton scattering of a 30 keV photon through this angle would yield a scattered photon decreased in energy by about 40 eV from the incident photon energy. Such an energy shift would be unresolvable in a NaI(Tl) detector.

Experimental Estimate of the Hardening Effect

To demonstrate the validity of Equation 1 for predicting the effect of hardening due to the mineral component of the system on the measured value of $\ln(I_o^*/I)$, measurements were made on an aluminum step-wedge. The wedge had eight steps ranging in thickness from 0.164 cm to 1.593 cm; it was made from 2024 aluminum alloy, which has a density of 2.77 g/cm^3 .¹ The wedge was immersed in 4 cm of water; however, the effective water depth was 4.75 cm including the lucite water container and support platform. The effect of hardening due to soft tissue surrounding the absorber was determined by measuring $\ln(I_o^*/I)$ for a 0.5 cm thick section of bovine bone covered with various thicknesses of lucite.

The third effect which was examined was the variation of $\ln(I_o^*/I)$ with the thickness of tin filtration in the ^{125}I beam. Measurements were made with 0, 1, 2, and 3 filters in the beam with each filter being $25 \mu\text{m}$ thick, having been cut from a commercially prepared sheet of tin foil with a thickness of 0.001 ± 0.0005 inches. In all of the above experiments point measurements were made to avoid the effect of the finite cross-section of the beam, an effect peculiar to scanning measurements.

The measurements were made with a NaI(Tl) crystal detector, a photo-multiplier tube, and a Baird-Atomic Model 530 Spectrometer. In order to observe the entire photopeak, the lower level of the pulse height analyzer was set at 7 keV and the window was set at 38 keV. These settings bracket the photopeak, permitting the detection of the five components of the spectrum while minimizing the contributions of equipment noise and background to the signal.

Results

As can be seen in Figures 1 and 2, Equation 1 is a valid model for predicting the hardening effects due to both bone mineral and soft tissue.

Figure 1. The effect on the measurement of bone mineral content of hardening of the ^{125}I spectrum in mineral. For these measurements the mineral was simulated by aluminum. The $\ln(I_0^*/I)$ is a measure of bone mineral content.

Figure 2. The effect on the measurement of bone mineral content of hardening of the ^{125}I spectrum in soft tissue. Soft tissue variations were simulated by varying the amount of lucite over a section of bovine bone. If there were no hardening effect, the curve would be horizontal.

Figure 1, a plot of $\ln(I_0^*/I)$ as a function of thickness of aluminum, illustrates the effect of increasing the mass of the absorber on the value of $\ln(I_0^*/I)$. The theoretical curve indicates values calculated using Equation 1 and the points are measured values using an unfiltered ^{125}I source; the error bars indicate ± 2 S.D. The effect on $\ln(I_0^*/I)$ of increasing the soft-tissue cover is shown in Figure 2, a plot of $\ln(I_0^*/I)$ as a function of the thickness of lucite covering the section of bovine bone. The points are the measured values ($\pm$ S.D.), and the curve indicates values calculated according to Equation 1.

The mass attenuation coefficients used in these calculations were obtained from interpolations of published values.⁵ Tabulations of the fractional composition of the ^{125}I spectrum have also been published.³ The values used for the calculations are shown in Table I.

The attenuation coefficients of the aluminum and the water for the K_α radiation of the ^{125}I spectrum were independently determined by the Ross balanced filters technique.^{6,8,9} This method involves measuring the transmitted intensity of the beam filtered alternately by a copper and a tin filter. The thicknesses of the two filters are chosen so that the attenuation of each filter is the same for the $K_{\beta 1}$, $K_{\beta 2}$, and γ photons in the ^{125}I beam. The difference of the transmitted intensities for the two filtrations is due to the K_α radiation alone. A plot of this difference as a function of the thickness of the absorber in the beam yielded K_α attenuation coefficients of $2.047 \pm .014$ (SD) cm^2/g for the aluminum and $0.4245 \pm .0003$ (SD) cm^2/g for the water. These mass attenuation coefficients are in good agreement with the theoretical values.

Effect of Hardening on Point Measurements

To give some approximation of the magnitude of the effect on point measurements of BMC due to hardening of the ^{125}I spectrum in both bone mineral and soft tissue, a comparison was made of the values of $\ln(I_0^*/I)$ as a function of the bone mineral mass, M_{BM} (g/cm^2), computed by Equation 1 and by a linear estimator. The linear estimator was a best-fit linear regression of a set of data calculated by Equation 1 over the range of M_{BM} from $0.45 \text{ g}/\text{cm}^2$ to $1.5 \text{ g}/\text{cm}^2$. Assuming that the total tissue thickness, which is also t_{ST} , is 7 cm and the tin filtration is $51 \mu\text{m}$, a thickness commonly used in BMC measurements, the resulting regression is:

$$\langle \ln I_0^*/I \rangle = 2.103 \times M_{\text{BM}} (\text{g}/\text{cm}^2) + 0.102 \quad (2)$$

with a standard error of estimate of 0.017. The usual practice is to derive a linear equation to obtain M_{BM} as a function of $\ln(I_0^*/I)$. Equation 2 represents the inverse of such a relation, which could be obtained if actual measurements were made of bones over a broad range of mass values with the total of the tissue plus bone thickness held constant by the addition of suitable quantities of bolus material.

Since Equation 2 has no explicit dependence on the value of t_{ST} , it will be insensitive to changes of the amount of soft-tissue cover. However, the values of $\ln(I_0^*/I)$ decrease as t_{ST} increases as is evident

TABLE I

Values used in theoretical calculations of hardening of ^{125}I spectrum

Component and Energy (keV)	Fraction of Spectrum	Mass attenuation coefficients, (cm^2/g)			
		Aluminum	Hydroxyapatite	Water	Tin
$\text{K}_{\alpha 1}$ (27.5)	.517	2.0290	2.6311	.4182	8.9739
$\text{K}_{\alpha 2}$ (27.2)	.264	2.0851	2.7061	.4259	9.2181
$\text{K}_{\beta 1}$ (31.0)	.140	1.4673	1.8821	.3430	38.4212
$\text{K}_{\beta 2}$ (31.7)	.031	1.3866	1.7744	.3344	36.0697
γ (35.4)	.049	1.0511	1.3294	.2948	26.4779

from Figure 2. Hence the ratio $\langle \ln(I^*/I) \rangle / \ln(I^*/I)$ increases with t_{ST} . This effect is shown in Figure 3, in which this ratio is plotted as a function of M_{BM} (g/cm² of hydroxyapatite) for several values of t_{ST} . For a particular value of M_{BM} the ratio increases by 0.5% to 1% per cm of soft-tissue cover. This corresponds to a decrease of $\ln(I^*/I)$ of essentially the same magnitude due to hardening in the soft tissue.

Also apparent from Figure 3 is the hardening effect due to the hydroxyapatite (HA). For $M_{BM} > 1.2$ g/cm², and for the range of t_{ST} from 3 cm to 9 cm, the ratio $\langle \ln(I^*/I) \rangle / \ln(I^*/I)$ increases at the rate of 3.3% to 5.5% per g/cm² of HA. In particular for the case $t_{ST} = 7$ cm, the calibration condition, the curve rises by 5% per g/cm² of HA. In the range of M_{BM} from 0.6 g/cm² to 1.5 g/cm² of HA, the ratio is constant to within $\pm 1\%$ for all values of t_{ST} . In the region $M_{BM} \lesssim 0.6$ g/cm² the ratio increases rapidly since $\langle \ln(I^*/I) \rangle$ approaches a finite value while $\ln(I^*/I)$ approaches zero as M_{BM} approaches zero.

Effect of Hardening on a Scan Measurement

The above comparison of a linear estimator of $\ln(I^*/I)$ and Equation 1 can be extended to yield the magnitude of the error due to hardening effects to be expected in a scan measurement. In this case values of $\ln(I^*/I)$ and $\langle \ln(I^*/I) \rangle$ were calculated at 1 mm intervals across a hypothetical upper arm to obtain values of $\sum \ln(I^*/I)$ and $\sum \langle \ln(I^*/I) \rangle$ for a cylindrical humerus. Measurements of 17 excised humeri indicated that an average outer diameter is 1.8 cm and an average inner diameter is 0.9 cm in the midshaft region. Calculations were made assuming values of $t_{ST} = 7$ cm and $t_{ST} = 8$ cm. With the assumption that cortical bone is composed of 1/3 HA and 2/3 soft tissue, by volume, values of M_{BM} were determined which ranged from 0.82 g/cm² at 1 mm from the geometrical edge of the bone, to 1.5 g/cm² at 5 mm from the center of the bone. Using Equation 2 to obtain a value of $\sum \langle \ln(I^*/I) \rangle$ and Equation 1 to obtain a value of $\sum \ln(I^*/I)$ resulted in the ratio $\sum \langle \ln(I^*/I) \rangle / \sum \ln(I^*/I) = 1.001$ for $t_{ST} = 7$ cm, the calibration condition; for $t_{ST} = 8$ cm, the ratio was 1.012. This example indicates that errors due to hardening can be minimized by calibrating the system for the same range of bone mass values over which later measurements are to be made. Also, since variation in total tissue thickness can introduce an error as large as 1% per cm deviation from the calibration tissue thickness, measurements should be made with this thickness standardized in order to maximize the precision and accuracy of the measurement.

Effects of Tin Filtration on the BMC Measurement

The calculations of the previous section were performed under the assumption that there were 51 μ m of tin filtration in the beam: a filtration in common use for BMC measurements. Tin is the element of choice because it has a K-absorption edge at 29.2 keV. Hence tin filtering makes the ¹²⁵I beam more nearly monoenergetic by selective attenuation of the $K_{\beta 1}$ (31.0 keV), the $K_{\beta 2}$ (31.7 keV), and the γ (35.0 keV) radiation. However, variations of the thickness of tin filtration in the beam can lead to considerable errors in the BMC measurements. Computations using Equation 1 indicate that the magnitude of $\ln(I^*/I)$ increases with increasing tin filtration at the rate of 0.12% to 0.16% per μ m of tin over the range of

Figure 3. Comparison of a particular linear estimator of bone mineral content and Equation 1 (see text). Values on the ordinate are ratios of $\langle \ln(I^*/I) \rangle / \ln(I_0^*/I)$ where the numerator has been calculated by the linear estimator and the denominator by Equation 1. Abscissa values represent thicknesses of hydroxyapatite.

zero to $76 \mu\text{m}$ of tin filtration and with absorber thicknesses of 0.3 to 2.1 g/cm^2 of HA. Measurements on the aluminum step-wedge over the same range of tin filtration and a similar range of absorber thickness showed an increase in $\ln(I^*/I)$ of 0.12% to 0.20% per μm of tin. This effect is displayed in Figure 4, which contains plots of the ratio of $\ln(I^*/I)$ calculated with tin filtration to the same quantity calculated without filtration as a function of the thickness of tin filtration and for several thicknesses of HA. Specifically, these measurements imply that in order to hold the variation of measured BMC due to variation of tin filtration to $\sim 1\%$ the thickness of filters used with different sources should be controlled to $\pm 2.5 \mu\text{m}$.

One method of maintaining constant filter thickness is to use the same filter on each ^{125}I source used for BMC measurements. Since such a filter would be subject to considerable handling, it should be protected, possibly by laminating it between thin sheets of lucite. Uniformity of thickness among several filters could be achieved by making transmission measurements of the filters and selecting those for which the transmitted count rates are within the required limits.

The problems of tin filter variations can be eliminated entirely by not using any filter. Although this tactic will lead to a less mono-energetic beam, the hardening effects can be minimized by calibrating the system over a wide range of BMC values. For example, a linear regression, similar to that described previously, was determined for values of $\ln(I^*/I)$ as a function of M_{BM} , but in this case no tin filtration was assumed. Computation of $\langle \ln(I^*/I) \rangle$ for the hypothetical upper arm scan produced the ratios $\langle \ln(I^*/I) \rangle / \langle \ln(I^*/I) \rangle = 0.997$ with 7 cm tissue thickness, the calibration thickness assumed, and 1.010 with 8 cm tissue thickness assumed.

Eliminating the filter will also provide a considerable increase of the photon fluence. For example, our use of $60 \mu\text{m}$ of tin filtration reduces the photon fluence by $\sim 50\%$. Removing the tin filter (and also broadening the spectrum accepted by the pulse height analyzer) will permit longer use of present sources or the purchase of lower activity, less expensive sources for future use.

Conclusion

Apparently hardening effects are sufficiently small that a linear relation between BMC and $\ln(I^*/I)$ can predict the BMC to within $\pm 1\%$. However, in order to maintain this accuracy the measuring apparatus must be calibrated over the same range of BMC values that will be encountered in practice. Significant errors in accuracy, $\sim 5\%$ per g/cm^2 of HA, can be introduced into measurements of BMC outside the range of calibration. Although the errors due to variations in soft-tissue cover are small, $\lesssim 1\%$ per cm of unit density material, some control of total tissue thickness should be instituted in order to increase the precision of the measurement. Finally, errors in accuracy and precision can be minimized by control of the amount of tin filtration used in the beam. These errors can be removed by elimination of the tin filter or at least minimized by quality control of the thickness of the filters used on the ^{125}I sources.

Figure 4. The effect of variation of tin filtration on measurement of bone mineral content. Values on the ordinate are ratios of $\ln(I_0^*/I)$ calculated with tin filtration to the same quantity calculated without tin filtration.

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Acknowledgements

I wish to thank Orlando Canto who prepared the figures for this paper.

Computer services at the Madison Academic Computing Center were made available through a grant from the Graduate School Research Committee of the University of Wisconsin.

Partial support was also provided by funds from the Biomedical Sciences Support Committee and by NASA Grant No. Y-NGR-50-002-051.