APRIL 1973

MATT-979

LINEAR WAVE CONVERSION AT THE
LOWER HYBRID RESONANCE

BY

I. FIDONE

PLASMA PHYSICS
LABORATORY

MASTER

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by U. S. Atomic Energy Commission Contract AT(11-1)-3073. Reproduction, translation, publication, use, and disposal, in whole or in part, by or for the United States Government is permitted.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Linear Wave Conversion at the Lower Hybrid Resonance

I. Fidone
Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

ABSTRACT

Linear wave conversion at the lower hybrid frequency is investigated. It is shown that total conversion takes place between the components of the energy flux in the direction of the density gradient. The power of the converted plasma wave is obtained in terms of the incident flux at the plasma boundary.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

fy

Since Stix¹ pointed out the possibility of wave conversion between long wavelength cold plasma modes into short wavelength plasma waves at the hybrid resonance frequencies a number of papers have been devoted to this problem.²⁻⁵ The principal question to be answered is the relation between the energy flux of the plasma waves and the incident flux on the hybrid layers. In this note we consider the case of propagation in the plane determined by the directions of the external magnetic field and the density gradient in a plane layered plasma slab. The two directions are taken as the z and x axis of an orthogonal Cartesian system. The electric field of the waves is written as $\vec{E}(\vec{r}, t) = \vec{E}(x) \exp i\{(\omega/c)(n_z z) - \omega t\}$, where n_z is a constant since the plasma is assumed homogeneous in the z direction. For low frequency waves $\omega_i^2 \ll \omega^2 \ll \omega_{pe}^2$ in the region where $\vec{E}(x) \sim \exp i(\omega/c) \int^x q dx$ we have

$$q^2 = q_o^2 \approx - (n_z^2 - \epsilon_{xx}) - \frac{\epsilon_{xy}^2}{n_z^2 - \epsilon_{xx}}, \quad (1)$$

$$q^2 = q_x^2 \approx - \epsilon_{zz} \frac{n_z^2 - \epsilon_{xx}}{\epsilon_{xx}}, \quad (2)$$

where

$\epsilon_{xx} \approx 1 - (\omega_{pi}^2/\omega^2)$, $\epsilon_{xy} \approx -i(\omega_{pe}^2/\omega\omega_e)$, $\epsilon_{zz} = 1 - (\omega_{pe}^2/\omega^2)$, and $\omega_p^2 = (4\pi e^2 n_o/m_\alpha)$, $\omega_\alpha = (e_\alpha B_o/m_\alpha c)$. In Eqs. (1) and (2) we have ignored thermal effect and assumed $\omega_{pe}^2 \ll \omega_e^2$. The two cold plasma modes, described by q_o^2 and q_x^2 , are weakly coupled either near the slab boundaries⁶ ($\omega_{pa}^2 \rightarrow 0$) or deep in the plasma⁷ and propagate almost independently. We therefore concentrate our attention on the extraordinary mode q_x^2 which has a resonance at

$\omega_{pi}^2(x_r) \approx \omega^2$. For $n_z^2 \gg 1$, $q_x^2 = -\epsilon_{zz} n_z^2 / \epsilon_{xx}$ and it may be shown that

$$E_y(x) \ll E_x, E_z, \quad i k_z E_x = i k_x E_z = \frac{d}{dx} E_z,$$

The wave is essentially longitudinal and the problem of wave conversion can be described by the Poisson equation

$$\frac{d}{dx} Dx + i k_z \epsilon_{zz} E_z = 0. \quad (3)$$

We first use Eq. (3) to find the field of the external excited wave. Letting $D_x = \epsilon_{xx} E_x$ and $E_z = \epsilon_{xx}^{-1/2} F$ we have

$$F'' - \frac{k_z^2 \epsilon_{zz}}{\epsilon_{xx}} F \approx 0. \quad (4)$$

Eq. (4) is valid as long as ϵ_{xx} is not too small with respect to unity. Near the boundary $\epsilon_{xx} \approx 1$, $\epsilon_{zz} \approx -(x - x_c / l_c)$ where $l_c^{-1} = (1/\omega^2) (d\omega_{pe}^2/dx) x_c$ and x_c is determined by the relation $\omega_{pe}^2(x_c) = \omega^2$, Eq. (4) becomes

$$\frac{dF}{d\xi} + \xi F = 0, \quad (5)$$

where $\xi = \beta^{1/3} (\omega/c) (x - x_c)$, $\beta = (cn_z^2 / \omega l_c)$. The solution of Eq. (5) representing a backward wave in the plasma for $|\xi| \gg 1$ is

$$E_z = A \pi^{1/2} \epsilon_{xx}^{-1/2} \beta^{-1/6} \exp(i\pi/4) [B_i(-\xi) - i A_i(-\xi)], \quad (5')$$

where A is determined by $E_z(x=0) = E_z$, i.e.

$$A = \frac{\beta^{1/6} \exp(-i\pi/4)}{\pi^{1/2} [B_i(\frac{\omega}{c} x_c \beta^{1/3}) - i A_i(\frac{\omega}{c} x_c \beta^{1/3})]} E_0.$$

For $|\xi| \gg 1$ Eq. (5') becomes

$$E_z = A \epsilon_{xx}^{-1/2} q_x^{-1/2} \exp \left(-i \frac{\omega}{c} \int_{x_c}^x q_x dx \right),$$

and

$$E_x = - \frac{q_x}{n_z} E_z, \quad (6)$$

where $q_x^2 = (-\epsilon_{zz} n_z^2) / \epsilon_{xx}$. In solving Eq. (5) we have assumed that the point x_R is sufficiently far from x_c such that

$$\frac{c}{\omega} \beta^{-1/3} \ll |x - x_c| \ll |x_R - x_c|.$$

In this case the wave described by Eq. (6) propagates toward the resonance carrying the energy flux

$$\vec{S}^c = - \frac{\omega}{16\pi} \frac{\partial \epsilon}{\partial \vec{k}} |\vec{E}|^2,$$

where

$$\epsilon = \frac{k_x^2 \epsilon_{xx} + k_z^2 \epsilon_{zz}}{k^2}.$$

Then

$$S_x^c = \frac{c}{8\pi} \frac{|A|^2}{n_z^2}, \quad S_z^c = \frac{c}{8\pi} \frac{q_x}{n_z^3} |A|^2.$$

In the region where $\epsilon_{xx} \ll 1$ thermal effects must be taken into account and Eq. (3) becomes

$$\frac{d}{dx} \left(- \frac{c^2}{\omega^2} \epsilon_T E_x'' + \epsilon_{xx} E_x \right) + i k_z \epsilon_{zz} E_z = 0, \quad (7)$$

where

$$\epsilon_T = - \sum_{\alpha} \frac{3(T_{\alpha}/M_{\alpha} c^2) \omega_{p\alpha}^2 \omega^2}{(\omega^2 - \omega_{\alpha}^2)(\omega^2 - 4\omega_{\alpha}^2)} \approx - 3(T_i/m_i c^2).$$

Letting $\epsilon_{xx} = -[(x-x_R)/\ell_R] = \zeta/[(\omega/c)\ell_R]$, $b = -(\omega/c)\epsilon_{zz}(x_R)\ell_R n_z^2$,

$\rho^2 = [(c/\omega)\ell_R]/[3(T_i/M_i c^2)]$ where $\ell_R^{-1} = (1/\omega^2)(d\omega_{pi}^2/dx)_x$,

Eq. (7) becomes

$$\frac{d^4 E_x}{d\zeta^4} + \rho^2 \left(\zeta \frac{d^2 E_x}{d\zeta^2} + 2 \frac{dE_x}{d\zeta} + bE_x \right) = 0. \quad (8)$$

Eq. (8) has been discussed by Rabenstein⁸. For $|(2/3)\rho\zeta^{3/2}| \gg 1$ and $x < x_R$ the solution which includes the backward wave (6) is

$$E_x = -\pi i \left(\frac{b}{\zeta}\right)^{1/2} (\pi b^{1/2} \zeta^{1/2})^{-1/2} B \exp i(2b^{1/2} \zeta^{1/2} - \frac{3\pi}{4}) \\ - iB\pi^{1/2} \exp(-\frac{5\pi i}{4}) \rho^{1/2} \zeta^{-1/4} \exp i \frac{2}{3} \rho \zeta^{3/2}, \quad (9)$$

where B is determined by comparing the first term in Eq. (9) with Eq. (6), hence

$$B = \left(\frac{\omega \ell_R}{n_z^2 c}\right)^{1/2} \frac{A \exp i[(\omega/c) \int_{x_c}^{x_R} q dx + (\pi/4)]}{\pi^{1/2}}.$$

For the plasma wave we have

$$k_x^2 = \frac{\omega^2}{c^2} p^2 = -\frac{\omega^2}{c^2} \frac{\epsilon_{xx}}{\epsilon_T} = \frac{\omega^2 - \omega_{pi}^2}{3(T_i/\omega_i)},$$

and the directions of the group and phase velocities coincide.

The field of the plasma wave is

$$\frac{E_x^T}{E_z^T} = -iB\pi^{1/2} \exp(-\frac{5\pi i}{4}) \rho p^{-1/2} \exp - i\frac{\omega}{c} \int_{x_R}^x p dx, \quad (10) \\ \frac{E_z^T}{E_x^T} = -\frac{n_z}{p}.$$

The energy flux carried out of resonance is given by

$$\vec{S}^T = -\frac{\omega}{16\pi} \frac{\partial \epsilon}{\partial k} |\vec{E}^T|^2,$$

where

$$\epsilon = \frac{k_x^2 [(c^2/\omega^2) k_x^2 \epsilon_T + \epsilon_{xx}] + k_z^2 \epsilon_{zz}}{k^2}.$$

We have

$$\begin{aligned} S_x^T &= -\frac{c}{8\pi} \frac{|A|^2}{n_z^2} = -S_x^c, \\ S_z^T &= \frac{q_x}{p} \frac{-q_x}{n_z} S_x^T = \frac{q_x}{p} \frac{q_x}{n_z} S_x^c = \frac{q_x}{p} S_z^c. \end{aligned} \quad (11)$$

Equations (11) show that the x component of the incident flux is totally converted into plasma waves. Since $q_x/p \ll 1$ the z component of the incident flux is not converted. This results in accumulation of energy at the hybrid layer. In the long run this energy is likely to be converted into thermal energy by ordinary collisions.

For $(\omega/c)x_c \beta^{1/3} \ll 1$, $|B_i(o) - i A_i(o)|^2 \approx 0.5$, then

$$|S^T| = \frac{c}{8\pi} |E_o|^2 \frac{\beta^{1/3}}{0.5\pi n_z^2}.$$

We have found that $S_z^c/S_x^c = q_x/n_z$ hence the path of the wave-packet is given by

$$\frac{dz}{dx} = \frac{S_z^c}{S_x^c} = \frac{q_x}{n_z} \approx \frac{(m_i/m_e)^{1/2}}{\epsilon_{xx}^{1/2}}. \quad (12)$$

This equation implies that the incident wave-packet approaches the hybrid layer with an initial large angle. As x tends to x_R the direction of the wave-packet becomes parallel to the external magnetic field. However, it must be realized that Eq. (12) results from a WKB argument [Eq. (6)] which implies that the condi-

tion $(c/\omega)\beta^{-1/3} \ll |x - x_c| \ll |x_R - x_c|$ is fulfilled. In actual situations the region where Eq. (6) is valid is close to the hybrid layer and therefore Δx in Eq. (12) is small enough to make Δz small.

ACKNOWLEDGEMENT

This work was supported in part by U. S. Atomic Energy Commission Contract AT(11-1)-3073.

REFERENCES

- ¹T. H. Stix, Phys. Rev. Letters 15, 878 (1965).
- ²A. D. Piliya and V. I. Fedorov, Zh. Eksp. Teor. Fiz. 57, 1198 (1969). [Sov. Phys. JETP 30, 653 (1970)].
- ³H. H. Kuehl, Phys. Rev. 154, 124 (1967).
- ⁴Ting-Wei Tang, Phys. Fluids 13, 121 (1970).
- ⁵B. N. Moore and M. E. Oakes, Phys. Fluids 15, 144 (1972).
- ⁶V. E. Golant, Zh. Tekh. Fiz. 41, 2492 (1971). [Sov. Phys.-Tech. Phys. 16, 1980 (1972)].
- ⁷I. Fidone, Il Nuovo Cimento 5B, 219 (1971).
- ⁸A. L. Rabenstein, Arch. Ratl. Mech. Anal. 1, 418 (1958).

—LEGAL NOTICE—

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.