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A Study of the Structure of ^{11}B from Elastic
Scattering of Neutrons from ^{10}B

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Siegfried L. Hausladen

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A STUDY OF THE STRUCTURE OF ^{11}B FROM ELASTIC
SCATTERING OF NEUTRONS FROM ^{10}B

A Dissertation Presented to
The Faculty of the Graduate College of
Ohio University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

by
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I INTRODUCTION

Most of the experiments involving the compound nucleus, ^{11}B in highly excited states above the neutron threshold have not yielded definitive information concerning the structure of this nucleus. The availability of the Ohio University Tandem Accelerator with its excellent neutron time-of-flight facility provided an opportunity to study the energy-level structure of ^{11}B by means of the elastic scattering of neutrons from ^{10}B . This introductory chapter will first summarize what is known about ^{11}B above the neutron threshold (excitation energy $E_x = 11.456$ MeV) and give reasons why the present experiment was performed in order to give more accurate information concerning the level structure of ^{11}B . In particular such properties as spins, parities, neutron widths and α -particle widths for states in ^{11}B are deduced.

Many experiments have been done to determine the $^{10}\text{B}(n,\alpha)^7\text{Li}$ cross sections and/or the α -particle branching ratio at thermal and low energies (Fr72, Ju54, To66, Be61, Pe51, Co67, So70, Ne70, Ma68, Bo69, Da61, Gi59, Bi57). These experiments were primarily designed to measure cross sections for this reaction to be used as a standard for neutron flux measurements and gave very little information on energy levels in the compound nucleus ^{11}B . Furthermore, these experiments often gave conflicting results which have been summarized in several data compilations on this subject (Gu68, Gi69, Ir67).

The total neutron cross section as well as the low energy cross section for scattering have been measured and evaluated previously (Mo66, Bo51, As70). These experiments are quite insensitive to the parity of states in ^{11}B and have not contributed precise information

about the spins of the excited states. The ${}^7\text{Li} + \alpha$ channel in ${}^{11}\text{B}$ opens at an excitation energy, $E_x = 8.664$ MeV. The reactions ${}^7\text{Li}(\alpha, \alpha){}^7\text{Li}$ and ${}^7\text{Li}(\alpha, \alpha'){}^7\text{Li}(0.48)$ (Cu66, Bi57, Pa67) are difficult to analyze above the neutron threshold because too many partial waves contribute to these cross sections.

The present elastic scattering experiment is an extension of the work of Lane *et al.* (La67) in which differential cross section and differential polarization measurements were made for neutrons with neutron energies $0.075 \text{ MeV} < E_n < 2.24 \text{ MeV}$. The subsequent R-matrix analysis of these data (Ha70, La71, La71a, Ha72) gave a simultaneous explanation of neutron polarization, differential scattering cross section, and the ${}^{10}\text{B}(n, \alpha_0){}^7\text{Li}$ and ${}^{10}\text{B}(n, \alpha_1){}^7\text{Li}(0.48)$ reactions in terms of the resonance parameters for states of ${}^{11}\text{B}$ in a manner consistent with all other known reaction data such as ${}^7\text{Li}(\alpha, \alpha){}^7\text{Li}$, ${}^7\text{Li}(\alpha, n){}^{10}\text{B}$ and ${}^{10}\text{B}(n, n'\gamma){}^{10}\text{B}(0.717)$ as well as mirror reactions in ${}^{11}\text{C}$.

The present work consists of a measurement of elastic scattering differential cross sections taken at seven angles and 45 energies for neutrons of energy from 1.5 MeV to 4.4 MeV. This corresponds to an excitation energy range from 12.8 MeV to 15.5 MeV in ${}^{11}\text{B}$. Time-of-flight techniques were used throughout the experiment to insure that only elastically scattered neutrons were measured. The R-matrix analysis of the current data is an improvement on the previous analysis (La71) because a more complete two-level, four-channel R-matrix with non-diagonal R^0 background matrix is used. Furthermore the widths in all the channels have their explicit energy dependence included. The channels used in the present analysis are: two neutron channels corresponding to the two neutron channel channel spins, one α -particle channel to the

ground state of ${}^7\text{Li}$, and another α -particle channel to the first-excited state of ${}^7\text{Li}$. The states included in the analysis are all those that are formed with neutron orbital angular momentum $l_n < 3$. Inclusion of these states simultaneously in the two-level, four-channel formalism permits analysis of the present elastic scattering data as well as the previous data (La71) for elastic scattering and polarization below 2.25 MeV, the differential polarization data (Co73) at $E_n = 2.63$ MeV, and also the ${}^{10}\text{B}(n,\alpha){}^7\text{Li}$ data to the ground and first-excited states of ${}^7\text{Li}$. The resonance parameters which are used to fit all these data simultaneously are then used to describe the level structure of ${}^{11}\text{B}$. In addition to the indicated fit, data from other reactions such as ${}^7\text{Li}(\alpha,n){}^{10}\text{B}$ and ${}^{10}\text{B}(n,n'\gamma){}^{10}\text{B}$ (.717) are employed to help resolve any ambiguities which result from the R-matrix analysis. The open channels not considered in this analysis are:

- 1) ${}^{10}\text{B}(n,t2\alpha)$, $Q = .229$ MeV: the cross section is less than 60 mb below 3 MeV and from 3 - 4.2 MeV it rises to 105 mb (Ir67),
- 2) ${}^{10}\text{B}(n,n'){}^{10}\text{B}$ (.717): below 4.2 MeV this cross section has a maximum of 76 mb (Ir67, Da56, Da60), and
- 3) ${}^{10}\text{B}(n,p){}^{10}\text{Be}$, $Q = .228$ MeV: the cross section for fast pile neutrons is 3 mb (Eg48).

A level diagram of ${}^{11}\text{B}$ and ${}^{11}\text{C}$ is shown in Fig. 1.1. The states at 11.60 MeV, 11.79 MeV and 11.94 MeV have been previously reported (La71). The States at 12.55 MeV and 12.91 MeV have isospin $T_z = 3/2$. Their formation and decay via the channel ${}^{10}\text{B} + n$ are isospin-forbidden and these states are not included in this analysis. The emphasis of the present study is on the states at 13.05 MeV, 13.15 MeV, 14.0 MeV and 15.2 MeV.

The two states near 13.1 MeV were first seen in the ${}^{10}\text{B}(n,\alpha){}^7\text{Li}$

FIGURE 1.1 Level Diagram of ^{11}B and ^{11}C .

Final spin values from this work are included, but excitation energies have not been changed to correspond to final values given in Appendix III.

reaction as one peak (Pe51). A Breit-Wigner analysis with the assumption of only one level demands a spin greater than $9/2$. The Wigner limit is then exceeded in the α -particle channels. This anomaly was investigated for multilevel structure, again with the $^{10}\text{B}(n,\alpha)^7\text{Li}$ reaction (Da61), but none was found. The earlier analysis (La71) at this laboratory indicated that there are two overlapping states of opposite parity at this excitation energy, a p-wave state with $J^\pi = 3/2^-, 5/2^-$ or $7/2^-$ and a d-wave state with $J^\pi = 3/2^+, 5/2^+$ or $7/2^+$. A later experiment (Va72) measuring the cross section for the reactions $^7\text{Li}(\alpha, n_0)^{10}\text{B}$ and $^7\text{Li}(\alpha, n_1)^{10}\text{B}$ (.717) was done and the conclusion from only qualitative arguments was that there is one state with spin greater than $9/2$ at this excitation energy in ^{11}B . Part of the present work addresses itself to the question of whether there are one or two states near 13.1 MeV in ^{11}B . The differential cross sections measured by Lane et al. (La67) were made with energy insensitive detectors which could not discriminate against inelastically scattered neutrons. The time-of-flight techniques used in the present work allow only elastically scattered neutrons to be measured.

The broad resonance at 14.0 MeV was first seen in the total cross section (Bo51). It was later observed in the $^{10}\text{B}(n,\alpha)^7\text{Li}$ reaction (Bi57, Da61) and the $^7\text{Li}(\alpha,\alpha)^7\text{Li}$ reaction (Cu66). Nevertheless no spin assignment was made. The work of Cox et al. measuring the differential cross section and differential polarization just below the resonance at $E_n = 2.63$ MeV gave a most probable assignment of $11/2^+$ for this state. However, a $9/2^+$ assignment was not completely ruled out. The only other data in this region are those of Porter et al. (Po70) in a recent unpublished report. These data, unfortunately, were not intended

for resonance analysis. The present study and its consequent analysis was used to determine the spin and parity of this state.

The resonance at $E_x = 15.2$ MeV has been seen in the ${}^7\text{Li}(\alpha, n){}^{10}\text{B}$ reaction (Me63), the ${}^{10}\text{B}(n, n'\gamma){}^{10}\text{B}(.717)$ reaction (Da60), and the ${}^{10}\text{B}(n, \alpha){}^7\text{Li}$ reaction (Da61). The only conclusion from these investigations is that the width of this resonance is about 750 keV. The present study was made to determine the parity of and set limits on the spin of this state.

In summary, the present work is intended to clear up ambiguities in the level structure of ${}^{11}\text{B}$ above the neutron binding energy when there are conflicting results and to investigate the level structure of ${}^{11}\text{B}$ where there is no information at the present time. The R-matrix analysis will fit all the major reaction data on this nucleus. The constraints imposed by fitting all the available data lend strength to the conclusions drawn from this analysis.

II EXPERIMENTAL METHOD

Experimental Apparatus

The cross section measurements were made using pulsed beam time-of-flight (TOF) techniques. A proton beam produced by the diode source was chopped to a 40 nanosecond length at a repetition rate of 5 MHz. The beam was bunched before acceleration with a klystron buncher. The proton beam was then accelerated by the Ohio University Tandem Van de Graff accelerator and an average beam current up to 10 microamperes with a pulse width less than one nanosecond was produced at the target.

Neutrons were produced using the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction. The protons were incident on a tantalum cup which previously had had lithium metal evaporated on it to any thickness between 0.05 and 0.41 mg/cm². The tantalum cup was located on the end of the beam line on a wobble target shown in Fig. 2.1. The wobble target rotated the axis of symmetry of the tantalum cup about the proton beam at an angle of seven degrees. This prevented any local heating of the target cup which would result in lithium evaporation and eventually decreased neutron yield. It also served the purpose of averaging any irregularities which could be due to uneven evaporation of the lithium onto the target cup. The target cup was cooled by means of an air stream. This method of cooling was sufficient for beam currents up to ten microamperes. The rate of wobble of the target about the beam was continuously variable from 30 to 300 Hz and was monitored in the control room. An audible alarm system gave a warning if the target stopped for any reason.

Vacuum evaporation of lithium onto the tantalum cup was done on

FIGURE 2.1 The Wobble Target Used to Rotate the Lithium Target about the Proton Beam.

a small evaporator constructed specifically for this purpose. Uniform and reproducible targets were made with thickness ranging from 4 to 65 keV for protons with an energy of 2 MeV. Only negligible increases in target thickness were noticed after three to four days of beam on target. There was also no noticeable evidence of carbon buildup on the targets.

The scattering samples were cylinders whose exact dimensions, purities, etc. are listed in Table 2.1. Two boron samples were used for the scattering measurements, a high-purity normal boron sample and a sample enriched in ^{10}B . The boron samples were enclosed in aluminum cans of wall thickness 10 mils. Background measurements were made with an empty can identical to the boron-containing cans. Hydrogen scattering was done for the absolute calibration of the TOF spectrometer using the hydrocarbon NE 102 as a scatterer. The parameters for the NE 102 sample also appear in Table 2.1.

A floor plan of the experimental arrangement is shown in Fig. 2.2. The centers of the cylindrical samples were placed at zero degrees and 20 cm from the center of the lithium target. Lucite shadow bars shielded the detectors from the direct flux of the lithium target. The main neutron detectors were two liquid scintillators of NE 218, two inches thick and six inches in diameter each coupled to an RCA 4522 photomultiplier tube with a short lucite light pipe. Each tube was powered by an ORTEC 269 photomultiplier tube base. Each detector was surrounded with magnetic shielding and placed in the center tube of a large steel tank filled with water. A lead annulus two inches thick and 36 inches long surrounded the center tube in the vicinity of the scintillator. This served to decrease the gamma radiation background.

Table 2.1

Scattering Sample Parameters

	Enriched Sample	Normal Sample
Volume	12.87 cc	12.896 cc
Weight	12.635 g	15.226 g
Total Boron by weight	93.6%	99.99%
Weight of Boron	11.826 g	15.226 g
# ^{10}B atoms	6.35×10^{23}	1.687×10^{23}
# ^{11}B atoms	0.55×10^{23}	6.793×10^{23}
Atomic % ^{10}B	92.28	19.6
Atomic % ^{11}B	7.72	80.4
Contaminants:		
Carbon	.52%	---
Oxygen	3.30%	---
Water	1.30%	---

NE 102

Total Weight	10.891 g
H/C atomic ratio	1.105
Density	1.032 g/cc
Volume	10.553 cc
H atoms/cc	$.0523 \times 10^{24}/\text{cc}$

Dimensions:

Boron Samples: one inch by one inch cylinders

NE 102: 0.900 inch diameter by 1.003 inches high cylinder

Aluminum can thickness: 0.010 inches

FIGURE 2.2 Floor Plan of the Experimental Arrangement.

The water in the shielding tanks was used to moderate the neutron background. A top view of one of the water tanks with lead shielding, collimator, and main neutron detector is shown in Fig. 2.3.

Collimators constructed of cylindrical, foot-long sections of polyethylene were inserted into the center tube of the shielding tanks as shown in Fig. 2.3. This was done so that the detectors viewed only the scattering samples. The water tanks themselves pivoted about the axis of the scattering samples and were moved from angle to angle on air pads. Scattering measurements for the boron samples were made at seven angles: 20° , 42° , 66° , 90° , 114° , 138° , and 160° . Special shadow bars were constructed for the 20° and 160° measurements because of the particular geometrical difficulties in shielding the detectors from direct target flux at these angles. A flight path of three meters was used for the main neutron detectors throughout the experiment.

The monitor neutron detector was a NE 102 scintillator one half inch thick by one inch diameter mounted on an RCA 8575 photomultiplier tube powered by an ORTEC 265 photomultiplier tube base. This detector was placed at a flight path of five meters from the scattering sample at zero degrees relative to the proton beam. No shielding or collimation was necessary for the monitor neutron detector.

A long counter was used to monitor the yield of the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction. This was placed at a distance of two meters from the target at an angle of 90° relative to the proton beam.

Scattering Experiment Electronics

A block diagram of the neutron detection apparatus electronics is shown in Fig. 2.4. The anode pulse of each of the main neutron

FIGURE 2.3 Top View of a Main Neutron Detector in a
Shielding Tank.

POLYETHYLENE
COLLIMATOR

WATER

ANNULAR LEAD
SLEEVE

NEUTRON
DETECTOR

FIGURE 2.4 Electronics Block Diagram.

detectors was clipped with a 14 cm shorting stub so that a fast rising negative leading pulse was derived. When this pulse reached -200 mv it armed a zero-crossing discriminator which gave a fast NIM timing marker when the clipped anode pulse crossed through zero volts. This timing marker was the start pulse of an ORTEC 437 time-to-amplitude converter (TAC). The linear output of each of the main neutron detectors was stretched with an ORTEC 113 preamplifier and input into an ORTEC 460 delay line amplifier. This amplifier has the fast rise time characteristics necessary for pulse shape discrimination. The amplifier output was connected to an ORTEC 458 pulse shape analyzer. The pulse shape analyzer had a leading-edge discriminator built in and this discriminator was used to set the bias of the main neutron detectors. All pulses with an amplitude greater than the preset bias were routed through the pulse shape discrimination circuit. Only neutron pulses gave an output from the pulse shape discriminator. This output pulse was input into an AND circuit whose other inputs were from the beam current integrator (BCI) and repetition rate discriminator (RRD).

The anode pulse of the monitor neutron detector was clipped with a 12 cm shorting stub and a time marker was generated from this clipped anode pulse in the same manner as for the main neutron detectors. This time marker was the start pulse for the monitor TAC. The linear pulse was stretched with an ORTEC 113 preamplifier and amplified with a linear amplifier. The amplifier output was input into a voltage discriminator where the bias of the detector was set. Any linear pulses exceeding the preset bias gave a logic pulse which went to an AND circuit together with the BCI and RRD signals.

Before reaching the target the proton beam passed through a one

inch diameter cylindrical copper capacitative beam pickoff. The beam pickoff pulse was inverted and amplified so that a negative leading zero-crossing pulse was derived. A time marker was generated from this with a zero-crossing discriminator. This time marker was delayed 400 nsec with 80 meters of RG 213 coaxial cable and input into a LeCroy coincidence unit so that four simultaneous time markers were produced. Three of these fast timing signals were used as stop pulses for the TACs and the fourth was input into the RRD.

The RRD was a frequency-to-voltage converter (Yo73) which could be set to give an output only if the repetition rate measured at the input of the RRD was 90 to 99.9% of the true repetition rate. The selection of percentage was continuously variable over this range and a 99% criterion was selected for the experiment. If the repetition rate was in the acceptable range a logic level was output which gated the BCI on and went to the AND circuits of the detectors. In this way data would be accepted only while there was a correct relationship between collected charge and detector counts. This was particularly important in the parts of the experiment that depend on charge normalization, i.e. the efficiency measurements and the transmission measurements.

The BCI integrated the collected charge on the lithium target and put out a logic level which went to each of the AND circuits of the three detectors. Data accumulation was done for a preset charge. If the beam current dropped below a preset level data accumulation was interrupted by the BCI. This was done because large fluctuations in beam current could change the width of the TOF peaks.

The TAC output of each of the main detectors was delayed with an ORTEC 427 delay line amplifier and input into the proper ADC of the

ND 3300 analyzer. Half of the memory of the analyzer was used for each main detector. A pulse was accepted by an ADC only if the RRD, pulse shape discriminator, and BCI gave an output. The monitor detector TAC was delayed with an ORTEC delay line amplifier and input into a ND 1100 analyzer. The input to the ADC of this analyzer was gated with the monitor tube AND circuit. Pulse shape discrimination was not used for the monitor neutron detector.

Scalers measuring long counter counts, dead time of the ADCs, live time, integrated charge, and a tagword identifying the data were read into the ND 3300 analyzer. When data accumulation was completed the memory of the ND 3300 analyzer was output onto magnetic tape and data reduction was done off-line on an IBM 360/44 computer. The TOF channels of interest from the ND 1100 analyzer were typed out and punched onto computer cards for data reduction.

Experimental Procedure

The first measurements made were target thickness measurements. A typical target thickness rise curve is shown in Fig. 2.5. These measurements were made with the long counter moved to zero degrees at a distance of two meters from the lithium target. Two curves are shown taken with the same target two days apart. As can be seen from Fig. 2.5 there is no appreciable target thickening or evaporation.

The first measurements made with the TOF spectrometer were relative efficiency curves for the three neutron detectors. Each main neutron detector was placed at zero degrees at a flight path of 3.2 meters from the lithium target. The monitor detector was placed at zero degrees and had a flight path of 5.2 meters from the lithium target. A lithium

Typical Rise Curves for a Lithium Target.

FIGURE 2.5 The abscissa is given as NMR frequency which is convertible to proton energy. The ordinate is in arbitrary units. Note that target thickening was measured to be 2 keV after two days of beam. Target thickness is given by the distance from threshold to the peak in the yield.

target with target thickness < 10 keV was used to minimize dead time in the analyzer and keep the total energy spread of the neutrons at a minimum. The proton energy was slowly increased from threshold until a neutron group could be detected above the bias. Time-of-flight spectra were then taken for increasing proton energy until the two neutron groups from the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction could not be resolved. This corresponds to a neutron energy of approximately 4.5 MeV. This procedure was repeated for all three detectors. The main neutron detectors had two bias settings. A low bias corresponding to a neutron energy of 450 keV was used for scattering measurements with incident neutron energies from 1.5 to 2.2 MeV. For all scattering measurements with an incident neutron energy greater than 2.2 MeV a high bias corresponding to a neutron energy of 700 keV was used. The monitor detector had a bias setting corresponding to a neutron energy of 1.1 MeV used throughout the experiment. Data was collected for sufficient time so that the statistical error of the ground state neutron group yield was better than 1%.

Before scattering measurements were made at each energy short charge-normalized sample transmissions were measured. Charge was collected long enough so that the statistical error in the integrated yield of the peaks was of the order of 1%. The transmission measurements were made three times for each of the boron samples and for the empty can. The 5.2 meter flight path of the monitor detector was sufficient to separate the two neutron groups from the lithium target throughout the experiment. At some incident energies it was necessary to move the monitor detector to a flight path of 6.2 meters to remove gamma rays from a later beam pulse from the neutron TOF spectrum. This

was judged more convenient than changing the repetition rate to 2.5 MHz. Monitor spectra taken at the longer flight path were normalized to those from the shorter flight path by taking a spectrum at one energy at both flight paths.

The scattering measurements were done at 20° and then by angle pairs. By the term angle pairs is meant that one detector is at a forward angle and the other at a back angle. The 20° measurement was made separately so that the 20° shadow bar would not perturb the measurements at other angles. Data were taken for both the enriched sample and the normal sample and background measurements were made with the empty can. The empty can data were typically taken for 25% of the time as for the sample in measurements because there was negligible improvement in the statistical error if background measurements were taken for longer periods of time. Differential scattering measurements for absolute calibration of the time-of-flight spectrometer were made with NE 102 at an angle of 42° . Transmissions for the hydrogen samples were also taken before the differential scattering measurements.

A flight path of three meters was sufficient to separate the ground state and first excited state neutron groups from the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction (497 keV energy difference). Inelastic neutron scattering was not a factor in the experiment because the ground state of ${}^{10}\text{B}$ is 717 keV below the first excited state. At energies where the neutron flight times were such that the gamma rays from the following pulse arrived at the detector at the same time as the neutrons from the given pulse the repetition rate was changed to 2.5 MHz to completely remove the gamma rays from the elastic scattering time-of-flight peak.

III REDUCTION OF THE DATA

The method used to convert counts measured in the laboratory to cross section is to relate main detector counts per monitor count to the number of scattered neutrons per incident flux. Hydrogen scattering was used as a standard for cross section.

The placement of the monitor detector at zero degrees was a key factor in this experiment. In return for the relatively small cost of taking transmission data at every energy there was a good return. It was a sensitive indicator of sample placement and a good measure of incident flux. The differential cross section of the source reaction was not a needed quantity because there was no need to convert monitor counts at some angle relative to the proton beam to incident flux. There was also no contribution to the measured incident flux from the scattering sample. Placement of the main detectors in positions that would not shadow the monitor was also not a problem because scattering measurements were not done at zero degrees.

Hydrogen scattering was used as a standard in the absolute calibration of the time-of-flight spectrometer because cross sections were related to main detector counts per monitor count. This meant that relative, not absolute, detector efficiencies could be used. This meant an increase in statistical accuracy was possible because the relative shape of the ${}^7\text{Li}(p,n){}^7\text{Be}$ 0^0 cross section is known to about 2% accuracy while the absolute magnitude of this cross section, which would have been used if absolute efficiencies had been necessary, is known to 5 - 7% accuracy. Once the relative efficiency curves are determined the absolute normalization with hydrogen need be done at only one energy to

determine the conversion constant of counts to cross section. In practice it is best to determine the conversion constant over an energy range to be certain that it is indeed a constant and no systematic errors have been introduced into the efficiency or normalization measurements.

Efficiency Measurements

Time-of-flight spectra of the ground and first excited state neutron groups from the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction were taken. The ground state neutron peak was integrated after subtracting a constant background by extrapolating the non-peak region. Dead-time corrections were made to the data. The yields at different energies were normalized to each other using the integrated charge collected on the lithium target for each spectrum. It was absolutely necessary to have a correct relationship between integrated charge and detector counts during the efficiency measurements. The RRD again insured that this was the case.

Let Y_n be these background-subtracted, dead time corrected, charge normalized yields. The Y_n are then given by:

$$Y_n = Y_i \sigma_E(0^\circ) \epsilon_D \Omega_D t \quad 3.1$$

where Y_i = the number of incident protons on the lithium target calculated from the integrated charge,
 $\sigma_E(0^\circ)$ = the zero degree ${}^7\text{Li}(p,n){}^7\text{Be}$ cross section for the ground state neutron group at a proton energy E in b/sr,
 ϵ_D = the relative efficiency of the detector,
 Ω_D = the solid angle subtended by the detector, and
 t = the physical thickness of the lithium target in atoms/barn.

The physical target thickness was calculated from the target thickness in keV at threshold and published tables (G160). The absolute zero degree ${}^7\text{Li}(p,n){}^7\text{Be}$ cross section is known to only 5 - 7% accuracy (Bo63,Au62). However, the relative shape of the cross section is known to approximately 2%. It must be pointed out that one could solve equation 3.1 for an absolute efficiency, but this would rely on an accurate determination of the ${}^7\text{Li}(p,n){}^7\text{Be}$ cross section at zero degrees. When equation 3.1 is used to calculate a relative efficiency the absolute calibration of the time-of-flight spectrometer must be done using the scattering from some element as a standard. In this work hydrogen was used as a standard and the absolute calibration is given in a later section.

The data points of Figs. 3.1, 3.2, 3.3, and 3.4 are the experimental relative efficiencies of the main detectors for the two bias settings used for the experiment. Fig. 3.5 gives the experimental relative efficiency of the monitor detector.

For a hydrocarbon scintillator it has been shown (Ry60) that the efficiency can be expressed, for energies away from the bias, by the following expression:

$$\epsilon_D = (S/E_n)(E_n - E_b)(1 - \exp[-n\sigma(E_n)x]) \quad 3.2$$

where S = a scale factor used in the least squares fitting of equation 3.2 to the experimental data points,
 E_n = energy of the incident neutrons on the scintillator,
 E_b = the bias energy, also a parameter in the least squares fitting of equation 3.2 to the experimental data,
 n = the number of hydrogen nuclei/barn-cm in the scintillator,
 $\sigma(E_n)$ = the hydrogen total cross section for neutrons of energy E_n , and

Relative Efficiency for Main Neutron Detector 1

FIGURE 3.1 as a Function of Neutron Energy for the Low Bias,
450 keV.

Relative Efficiency for Main Neutron Detector 2
FIGURE 3.2 as a Function of Neutron Energy for the Low Bias,
450 keV.

Relative Efficiency for Main Neutron Detector 1

FIGURE 3.3 as a Function of Neutron Energy for the High Bias,
700 keV.

Relative Efficiency for Main Neutron Detector 2

FIGURE 3.4 as a Function of Neutron Energy for the High Bias,
700 keV.

Relative Efficiency for the Monitor Neutron

FIGURE 3.5 Detector as a Function of Neutron Energy for
a Bias at 1.1 MeV.

x = the detector thickness in cm.

The expression used for $\sigma(E_n)$ is due to Gammel (Ga60) for neutron energies less than 40 MeV and is given in b for E_n in MeV by:

$$\begin{aligned} \sigma(E_n) = & 3\pi\{1.206E_n + (-1.86 + 0.09415E_n + 0.0001307E_n^2)^2\}^{-1} \\ & + \pi\{1.206E_n + (0.4223 + 0.13E_n)^2\}^{-1}. \end{aligned} \quad 3.3$$

For energies away from the bias the experimental detector efficiencies were fitted with equation 3.2 by varying the parameters S and E_b . For energies close to the bias the experimental efficiencies were fitted with a polynomial of order 5 by the method of least squares. The solid lines in Figs. 3.1 - 3.5 are the fits to the experimental data. An array of relative efficiencies was determined at 1 keV increments from equation 3.2 and the 5 order polynomial fit and this array was later used to correct all the scattering data.

Flux Attenuation Correction

The finite size of the sample has two effects: attenuation of the input flux and multiple scattering. Attenuation of the input flux and the correction for this will be discussed in this section.

By input flux attenuation is meant that all the scattering sites in the sample do not experience the same incident flux. There is flux removal by the scattering sites closer to the neutron source. It has been shown (Wa54) that the average flux in the sample for the geometry of Fig. 3.6 is given by:

$$N_{avg}(E) = \frac{N_0}{\pi a^2} \int_{-a}^a \int_{-(a^2-y^2)^{1/2}}^{(a^2-y^2)^{1/2}} e^{-n\sigma_T(E)(x+(a^2+y^2)^{1/2})} dx dy \quad 3.4$$

where n = the number of nuclei/cc in the sample,

FIGURE 3.6 Geometry used to Calculate the Flux Attenuation Factor.

N_0 = the incident flux,

$\sigma_T(E)$ is the total cross section for energy E, and

a = radius of the sample.

Equation 3.4 can be rewritten, for any given energy, as:

$$N_{avg} = f N_0 \quad 3.5$$

Where f is the attenuation factor and contains everything on the right hand side of equation 3.4 except the quantity N_0 . The attenuation factor has been evaluated to be (Wa54):

$$f = (1/\sigma_T(E)a) \{I_1(2n\sigma_T(E)a) - L_1(2n\sigma_T(E)a)\} \quad 3.6$$

where I_1 is a Modified Bessel Function of the first kind and L_1 is a Modified Struve Function of the first kind. The attenuation factor f was calculated for both the boron samples and the NE 102 sample for all the incident neutron energies for the experiment. These values of f were then used to correct all the scattering data.

Absolute Normalization Using Hydrogen Scattering

A typical TOF spectrum from NE 102 is shown in Fig. 3.7. The peaks, from right to left, are due to elastic scattering of the ground state group from carbon, elastic scattering of the first excited state group from carbon, and elastic scattering of both the ground state and first excited state groups from hydrogen. The hydrogen peak is broadened in comparison to the carbon peaks due to the effects of finite sample size.

The total yield, Y_t , for a given incident energy of the hydrogen TOF peak taken at 42° was calculated from the following equation:

FIGURE 3.7 Typical TOF Spectrum for Elastic Scattering
from NE 102 at 42° .

$$Y_t = \frac{\Omega_m}{\Omega_D} \sum_i \frac{1}{\epsilon_{Di}} \left[\frac{\frac{G_i}{f_1 MG_1} + \frac{f_2 MG_2}{T_2 \epsilon_{m2}}}{\frac{MB_1}{\epsilon_{m1}} + \frac{MB_2}{\epsilon_{m2}}} - \frac{B_i}{\epsilon_{m1} + \epsilon_{m2}} \right] \quad 3.7$$

where G_i (B_i) = the number of main detector counts in TOF channel i with the sample in (out),

MG_x (MB_x) = the integrated monitor yield for TOF group x with the sample in (out),

T_x is the sample transmission of neutron group x ,

ϵ_{m_x} = monitor relative efficiency for group x ,

ϵ_{Di} = the relative efficiency of the main detector for neutrons in TOF channel i ,

Ω_D and Ω_m = solid angles of main and monitor detectors,

f_x = the attenuation factor for group x ,

and the sum over i is for the channels in the hydrogen TOF peak.

The total yield, Y_t , is composed of neutrons of both the ground state and first excited state group from the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction elastically scattered from hydrogen. The ratio of second group to first group neutrons is at most 30% (Sm72). A correction must be made to Y_t to remove the contribution to the TOF peak of the second group. If, for a given incident neutron energy, we let the total number of neutrons from group x that is scattered into a detector at an angle θ be designated as N_x , we have:

$$N_x = f_x N_{i,x} \sigma_x(\theta) n \Omega_D \quad 3.8$$

where $N_{i,x}$ is the number of incident neutrons for group x and n is the number of hydrogen atoms in the sample. The terms in equation 3.8 may be rearranged to give:

$$N_x / (f_x N_{i,x} \Omega_D) = \sigma_x(\theta) n \quad 3.9$$

where the left hand side of equation 3.9 gives the number of scattered neutrons at angle θ per average incident flux per sr. This is the same as the yield measured in the laboratory as can be seen by inspection of equation 3.7. We then have, for each group of neutrons scattered from hydrogen:

$$Y_x = n \sigma_x(\theta). \quad 3.10$$

The total measured yield given by equation 3.7 can be expressed in terms of the yields of the two neutron groups as:

$$Y_t = Y_1 + Y_2 \quad 3.11$$

and, by use of equation 3.10, the yield from the excited state group can be expressed in terms of the ground state group yield as:

$$Y_2 = (\sigma_2(\theta)/\sigma_1(\theta)) Y_1. \quad 3.12$$

Equations 3.11 and 3.12 are now solved for the yield of the ground state group in terms of the total yield of the hydrogen peak measured in the laboratory:

$$Y_1 = Y_t / (1 + (\sigma_2(\theta)/\sigma_1(\theta))). \quad 3.13$$

The hydrogen differential scattering cross section is proportional to the total cross section. Therefore the total cross section may be used in equation 3.13 and we then have:

$$Y_1 = Y_t / (1 + \sigma_2/\sigma_1) \quad 3.14$$

where σ_1 and σ_2 are calculated from equation 3.3. The yield calculated for the ground state neutron group only from equation 3.14 is then used in equation 3.10 to solve for the differential cross section per atom corrected for input flux attenuation:

$$\sigma_h^-(E_0, 42^\circ) = Y_1/n. \quad 3.15$$

A correction must now be made for the second effect of the finite sample size, multiple scattering. A Monte-Carlo computer code, MONTE-POLY, developed by A. B. Smith of Argonne National Laboratory was used for this correction. The code corrects for the effect of sample attenuation after a scattering event (output attenuation) and also for the effect of multiple collisions of neutrons before they leave the sample. The code calculates a correction factor, $Z(E_0)$, defined by the following equation:

$$Z(E_0) = N_{\text{true}}/N_{\text{obs}} \quad 3.16$$

where the measurement is done at 42° and N_{true} is the number of single scattering events and N_{obs} is the observed number of scattering events. The correction factor $Z(E_0)$ was calculated for all incident energies at which hydrogen scattering measurements were made. The differential cross sections of equation 3.15 were multiplied by the correction factor for multiple scattering and this quantity is then proportional to the true differential scattering cross section:

$$Z(E_0) \sigma_h^-(E_0, 42^\circ) = \sigma_h^-(E_0, 42^\circ). \quad 3.17$$

In terms of the yields for the ground state group we have, upon substitution of equation 3.15 into 3.17:

$$Z(E_0) Y_1(E_0, 42^\circ) / n \propto \sigma_h(E_0, 42^\circ). \quad 3.18$$

Equation 3.18 is used to calculate the constant, K, that converts yield per atom corrected for multiple scattering to cross section. The constant K is given by:

$$K = n \sigma_h(E_0, 42^\circ) / (Y_1(E_0, 42^\circ) Z(E_0)). \quad 3.19$$

This value of K should be independent of energy. The values of K for the energies that hydrogen scattering was done are shown in Fig. 3.8. These values of K are energy independent and this indicates that no systematic errors were introduced into the differential cross section measurements or the efficiency measurements. The values of K for the low bias and high bias measurements are used to convert yield per atom to differential cross sections for the measurements made with ^{10}B as will be shown in the next section.

Differential Scattering Cross Sections for $^{10}\text{B} + n$

The TOF spectra for the boron samples were reduced to yields. The yield for a given incident energy, E_0 , at an angle theta for a given sample s is given by an equation analogous to equation 3.7:

$$Y_s(E_0, \theta) = \frac{\epsilon_m \Omega_m}{f_s \Omega_D} \sum_i \frac{1}{\epsilon_{D,i}} \left(\frac{G_i(E_0, \theta) T_s}{MG(E_0, \theta)} - \frac{B_i(E_0, \theta)}{MB(E_0, \theta)} \right) \quad 3.20$$

where the sum over i is over the elastic scattering peak. The beginning and ending channels for the peak integration were chosen by a consistent inspection of background-subtracted histograms of all the TOF peaks at every energy. Yields for both the enriched and normal samples were calculated for every angle and every energy.

FIGURE 3.8 The Conversion Constant K.

The enriched sample had contaminant elements such as O, C and others which contributed to the elastic scattering yields. These contaminants composed 6.4% of the enriched sample by weight. However, only those elements whose masses are near that of boron contributed to the TOF peak. The differential cross sections for these elements do not differ very much from that of boron in the energy region investigated (Li66, Jo73). Therefore, from table 2.1 their contribution was estimated to be equal to their abundance in the sample, or 3.2%. The quantities $Y_e(E_o, \theta)$ will refer to yields corrected for these contaminants throughout the remainder of this paper. The normal boron sample was a high purity sample and the amount of contaminants was less than 1%, so no correction was made to the yields from this sample.

The normal and enriched sample yields were then converted to cross section using the conversion constant K and the isotope contributions to the yields were calculated at the same time from:

$$K Y_e(E_o, \theta) = a\sigma_u^{10}(E_o, \theta) - b\sigma_u^{11}(E_o, \theta) \quad 3.21$$

and

$$K Y_n(E_o, \theta) = c\sigma_u^{10}(E_o, \theta) - d\sigma_u^{11}(E_o, \theta) \quad 3.22$$

where $a (c) =$ the number of ^{10}B nuclei in sample $e (n)$,

$b (d) =$ the number of ^{11}B nuclei in sample $e (n)$, and

$\sigma_u^x(E_o, \theta)$ is the differential scattering cross section uncorrected for multiple scattering for isotope x at an incident neutron energy E_o and angle θ .

Solving equations 3.21 and 3.22 for $\sigma_u^{10}(E_o, \theta)$ gives:

$$\sigma_u^{10}(E_o, \theta) = \frac{K}{ad - bc} (dY_e(E_o, \theta) - bY_n(E_o, \theta)). \quad 3.23$$

The result of the calculation of equation 3.23 is then the differential cross section for a given incident energy and angle absolutely normalized and corrected for the contribution of contaminant elements and the contribution by the ^{11}B in the sample.

Multiple Scattering Corrections

A Monte-Carlo multiple scattering code, MONTE-SAMPLE, developed by A. B. Smith at Argonne National Laboratory was used to correct (Sm73) data for the effects of multiple scattering and output attenuation. The code used as input two libraries of cross sections: total cross section and differential cross section. Sample attenuation after a scattering event was calculated from the total cross section library. Absorption processes were also calculated from the difference in the total and scattering cross sections. Differential scattering events used the differential scattering cross section library. The code calculated an angle and energy dependent quantity defined as follows:

$$KI(E_o, \theta) = N_p(E_o, \theta) / N_{ob}(E_o, \theta) \quad 3.24$$

where $N_p(E_o, \theta)$ is the number of single scattering events and $N_{ob}(E_o, \theta)$ is the observed number of scattering events. The number of neutron histories to get meaningful statistical accuracy was of the order of 400,000. The maximum number of scattering events for each neutron was set equal to three. The number of triple scattering events for 400,000 neutron histories was of the order of 30 - 40, so there was no need to have the computer code follow neutrons through four scattering events.

The corrected differential cross sections were then converted to the center-of-mass system and expanded in terms of Legendre polynom-

ials. Fits to the differential cross section data were made with 4, 5, and 6 polynomials. In the 6 polynomial fits to the data the coefficient B_5 was essentially zero. In the change from a four to a five polynomial fit there was a significant change in the error function while there was no such change between the 5 and 6 polynomial fits. The final data is given in terms of a 5 polynomial fit. The coefficients B_L in the expansion of the differential cross sections and the errors on these coefficients are given in Appendix I. The differential cross sections in the center-of-mass system and the errors on these cross sections are given in Appendix II. Fig. 3.9 shows the coefficients B_L .

Errors

The errors throughout this experiment were assumed to be independent. The relative efficiency ratio for the main and monitor detector was assumed to be known to 2% and the hydrogen cross section was assumed known better than 1%. Errors due to counting statistics were of the order of 3-6% and the hydrogen normalization was accurate to 4%.

The statistical accuracy of the multiple scattering and output attenuation correction was of the order of 3% for 400,000 neutron histories. The multiple scattering correction itself was only of the order of 7% on the average. There are a few neutron energies where there are sharp resonances in $^{16}O + n$. These resonances affect only a few energy points and their effect was calculated to be less than 10%. The overall statistical accuracy of the data is approximately 8%.

FIGURE 3.9 The Coefficients B_L in the COM System.

IV DATA ANALYSIS AND THE LEVEL STRUCTURE OF ^{11}B

The R-matrix theory of nuclear reactions is used to calculate two-level, four-channel scattering matrices for all the possible states in the compound nucleus $^{11}\text{B}^*$ formed by the process $^{10}\text{B} + n$ for neutrons with orbital angular momentum less than or equal to two units. The region of excitation in $^{11}\text{B}^*$ investigated is from 11.50 - 15.46 Mev. The scattering matrices calculated from resonance parameters are used to fit the differential cross section data, polarization data, and $^{10}\text{B}(n,\alpha)^7\text{Li}$ data that is available. The level structure of ^{11}B will be deduced from the resonance parameters that fit the data.

Calculation of the Scattering Matrix Using the R-Matrix Theory

The R-matrix theory is rigorous and its application is not confined to reactions that proceed by way of the compound nucleus. However, its application to compound nucleus reactions has proven useful since the theory was presented in 1947 (W147, W149). The treatment of the problem by Lane and Thomas (La58) will be used throughout this section with occasional additions from other sources.

The assumptions used in the development of the R-matrix theory (La58) are:

- 1) Non-relativistic quantum mechanics is applicable to the problem.
- 2) The Hamiltonian equation, $H\psi = E\psi$, defined for all space must embody the three physical principles: conservation of probability flux, time reversal, and causality.
- 3) All processes in which more than two nuclei are formed are either absent or unimportant.

- 4) All processes which involve creation or destruction are absent or unimportant.
- 5) There exists a finite radial distance of separation, a_c , for any pair of nuclei c beyond which neither nucleus c experiences any polarizing field from the other.

At this point a short statement about the applicability of the third assumption to the present study is in order. The reaction $^{10}\text{B}(n, t2\alpha)$ ($Q = 0.229$ MeV) can proceed mainly by three body breakup (Da61). However, the cross section for this reaction in the energy region investigated is much smaller (Da61) than the $^{10}\text{B}(n, n)$ or $^{10}\text{B}(n, \alpha)$ cross sections. The third assumption is then satisfied because the three body breakup is unimportant, not absent.

In the development of the R-matrix theory a given channel c is defined to have the set of quantum numbers (α, s, l, J, M_J) where α refers to a pair of reaction products and their states of excitation, s is the channel spin defined by

$$\vec{s} = \vec{I} + \vec{i}, \quad 4.1$$

where I and i are the spins of the reaction particles and J is

$$\vec{J} = \vec{s} + \vec{l} \quad 4.2$$

which, together with M_J , is conserved in a nuclear reaction.

The compound state exists in an internal region v which is enclosed by a surface S . That part, S_c , of this surface which corresponds to the intersection of a given channel c with the surface S is given by a constant radial separation, a_c . The specific nuclear interactions occur almost entirely in the internal region and only reactions such as Coulomb interactions occur in the external region.

In the R-matrix theory a set of states of all nucleons is defined and cross sections can ultimately be expressed in terms of these states. The connection between the states and the cross section is very compli-

cated and so various intermediate quantities such as the scattering matrix $\tilde{U}$, the $\tilde{R}$ -matrix, and the diagonal matrices $\tilde{L}$ and $\tilde{\Omega}$ are introduced. A diagram of the general scheme is as follows:

Here we define $\sigma_{cc'}(E)$ as the cross section for the production of the pair of nuclei of type c' when two nuclei of the type c are bombarded against each other with energy E . The element $U_{c'c}(E)$ of the scattering matrix is defined as the amplitude of the outgoing waves of pair c' resulting from unit flux of bombardment with pair c . The matrix $\tilde{U}$ is expressed in terms of the matrices $\tilde{L}$, $\tilde{\Omega}$, and $\tilde{R}$. The first two are diagonal and come about from any long range nonpolarizing interactions acting between any separated nuclei. The R -matrix is nondiagonal and takes account of the effects of all interactions, both inside the compound nucleus and inside the nuclei of separated pairs. The R -matrix depends

on E , a_c and the boundary conditions b_c . However, the R-matrix is still unknown in general but it was shown by Wigner and Eisenbud (W47) that any element of the R-matrix has an energy dependence which may be expressed as:

$$R_{cc'}(E) = \sum_{\lambda} \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_{\lambda} - E} \quad 4.3$$

where λ labels the members of the complete set of states and the $\gamma_{\lambda c}$ and E_{λ} are energy dependent quantities depending on a_c and b_c . The $\gamma_{\lambda c}$ are the reduced width amplitudes and for each state λ one $\gamma_{\lambda c}$ is defined for each pair c . The E_{λ} are the energy eigenvalues of the states λ .

As an illustrative example of the channels involved in an R-matrix calculation, consider some of the nuclear reactions of ^{10}B with neutrons of energy < 4.4 MeV:

The bombardment of ^{10}B itself involves several channels c . Since the spin of ^{10}B is 3 and that of the neutron is $\frac{1}{2}$, the channel spin s is $\frac{5}{2}$ or $\frac{7}{2}$. Except at very low energies several incoming orbital angular momentum waves l from an incident plane wave can contribute to the reaction. Thus the reaction is initiated in several angular momentum channels c . When the channel separation distance falls below a_c the neutron enters the ^{10}B nucleus. This internal region of configuration

space corresponds to the compound nucleus, $^{11}\text{B}^*$. Decay of this nucleus leads to outgoing waves in all channels for which the relative energy of motion is positive. The only reactions with a significant cross section included in the present analysis are elastic scattering of neutrons and the $^{10}\text{B}(n,\alpha)^7\text{Li}$ and $^{10}\text{B}(n,\alpha_1)^7\text{Li}^*(0.48)$ reactions. There are therefore two neutron channels designated by channel spin $5/2$ and $7/2$. The ground state of ^7Li has spin $3/2$ and the α -particle has spin 0, which corresponds to a channel spin $3/2$. The first excited state of ^7Li has spin $1/2$, corresponding to a channel spin $1/2$. Both neutron channels have the same relative energy of motion. The α -particle channels do not have the same relative energy of motion because of the difference in energy of the ground state and first excited state of ^7Li .

Other additional channel characteristics are now introduced for α_1 and α_2 , the pair of reaction particles in channel c :

$$E_c \equiv E_\alpha, \text{ the energy of relative motion,} \quad 4.4$$

$$M_c \equiv M_\alpha = M_{\alpha_1} M_{\alpha_2} / (M_{\alpha_1} + M_{\alpha_2}), \text{ the reduced mass,} \quad 4.5$$

$$k_c \equiv k_\alpha = (2M_\alpha |E_\alpha|)^{1/2} / \hbar, \text{ the wave number,} \quad 4.6$$

$$v_c \equiv v_\alpha = \hbar k_\alpha / M_\alpha, \text{ the relative velocity,} \quad 4.7$$

$$\eta_c \equiv \eta_{\alpha l} = Z_{\alpha_1} Z_{\alpha_2} e^2 / \hbar v_\alpha, \text{ the Coulomb field parameter,} \quad 4.8$$

$$\omega_c \equiv \omega_{\alpha l} = \arg \Gamma(1 + l_c + i\eta_c), \text{ the Coulomb phase shift,} \quad 4.9$$

and

$$\rho_c \equiv \rho_\alpha = k_\alpha r_\alpha. \quad 4.10$$

The R-matrix specifies the form of the wave function on the surface S and the U matrix specifies the form at infinity. The connection between the two matrices (La58) is given by:

$$\tilde{U}^J = \tilde{\Omega} \tilde{W}^J \tilde{\Omega} \quad 4.11$$

where

$$\tilde{W}^J = \tilde{1} + \tilde{\beta}^{1/2} (\tilde{1} - \tilde{R} \tilde{L}^0)^{-1} \tilde{R} \tilde{\beta}^{1/2} \tilde{w} \quad 4.12$$

where $\tilde{1}$ is the unit matrix and the matrices $\tilde{\Omega}$, $\tilde{\beta}$, $\tilde{L}^0$, and $\tilde{w}$ are diagonal with respect to channels and their elements are defined by the following equations for positive energy channels:

$$\Omega_c = e^{i(\omega_c - \phi_c)}, \quad 4.13$$

$$L_c^0 = S_c + iP_c - b_c, \quad 4.14$$

$$\beta_c = P_c, \quad 4.15$$

and

$$w_c = 2i. \quad 4.16$$

The quantity ω_c is the Coulomb phase shift, the quantity $-\phi_c$ is called the hard-sphere scattering phase shift because it is the phase shift induced by an infinitely repulsive sphere of radius a_c , i.e., it corresponds to a node in the wave function at $r_c = a_c$. It is calculated from the relation:

$$\phi_c = \tan^{-1} (F_c/G_c)_{r_c=a_c} \quad 4.17$$

where, for neutrons, F_c and G_c are the regular and irregular spherical

Bessel functions and for charged particles they are the regular and irregular Coulomb wave functions. The quantity S_c is referred to as the shift factor because it appears as a factor in the level shift expansion and is defined by:

$$S_c = \rho_c (F_c F'_c + G_c G'_c) / (F_c^2 + G_c^2) \big|_{r_c=a_c} \quad 4.18$$

where the symbol (') denotes the derivative with respect to ρ_c . The quantity P_c is referred to as the penetration factor or penetrability because it appears as a factor in the level width expression. It is defined by:

$$P_c = \rho_c / (F_c^2 + G_c^2) \big|_{r_c=a_c} \quad 4.19$$

The quantities b_c are the elements of the diagonal boundary condition matrix.

The problem of calculating the scattering matrix using equations 4.11 and 4.12 can be simplified if the R-matrix is split into two matrices:

$$\tilde{R} = \tilde{R}^0 + \tilde{R}' \quad 4.20$$

where $\tilde{R}'$ is the "resonant" part of the $\tilde{R}$ matrix and has assigned to it the resonance levels considered in the problem and where $\tilde{R}^0$ is the "background" portion of the $\tilde{R}$ matrix and accounts for the background due to distant levels. If $\tilde{R}^0 = 0$ or $\tilde{R}^0$ is diagonal one can avoid the matrix inversion by calculating W_{cc} , according to the real eigenvalue expansion (La58). However, if there are any non-diagonal R_{cc}^0 , elements one loses this convenience and must do the matrix inversion anyway. The real eigenvalue expansion was used in this work because of the simplicity of

the calculation and also because the method had been used previously (La71) for analysis of similar data at lower energies. In short, one does not like to abandon a method that has been working. Matrix inversion was added to the existing program and was used only for the non-diagonal case. The program was also expanded to include any number of channels and can be easily modified to handle three levels of the same $J^{\pi R}$. The modification to three levels will be shown in this paper.

The result of the real eigenvalue expansion (La58) is:

$$\tilde{W} = \tilde{I} + \tilde{B}^{\frac{1}{2}} \{ (\tilde{I} - \tilde{R}^0 \tilde{L}^0)^{-1} \tilde{R}^0 + \sum_{\lambda \mu} (\tilde{\alpha}_\lambda \times \tilde{\alpha}_\mu) A_{\lambda \mu} \} \tilde{B}^{\frac{1}{2}} \tilde{W} \quad 4.21$$

where the components of the column vectors $\tilde{\alpha}_\lambda$ are given by:

$$\alpha_{\lambda c} = \sum_c (\tilde{I} - \tilde{R}^0 \tilde{L}^0)^{-1}_{cc} \gamma_{\lambda c}, \quad 4.22$$

where $\gamma_{\lambda c}$ is the (real) reduced width amplitude in channel c for level λ . The level matrix $\tilde{A}$ may be expressed in a level matrix equation:

$$\tilde{A} = (\tilde{e} - \tilde{E} - \tilde{\xi})^{-1} \quad 4.23$$

where $\tilde{e}$ is a real diagonal matrix with components E_λ , $\tilde{E}$ is a real diagonal matrix which is given by the product of the energy with the unit matrix and $\tilde{\xi}$ is a matrix which is a scalar with respect to channels whose elements are defined by:

$$\xi_{\lambda \mu} = (\tilde{B}_\lambda, \tilde{\gamma}_\mu) = \sum_c \beta_{\lambda c} \gamma_{\mu c} \quad 4.24$$

where $\tilde{B}_\lambda$ is a column vector whose elements are defined by:

$$\beta_{\lambda c} = \sum_c (\tilde{L}^0 (\tilde{I} - \tilde{R}^0 \tilde{L}^0)^{-1})_{cc} \gamma_{\lambda c}. \quad 4.25$$

When 1, 2, or 3 levels are assigned to the $\tilde{R}$ matrix of equation

4.20 the inversion of equation 4.23 can be carried out by the introduction of the diagonal matrix $\tilde{\epsilon}$ with components

$$\epsilon_{\lambda} = E_{\lambda} - E - \xi_{\lambda\lambda} . \quad 4.26$$

The result for one level is:

$$A = 1/\epsilon . \quad 4.27$$

For two levels the result is:

$$A_{11} = \epsilon_2/D ; A_{22} = \epsilon_1/D \quad 4.28$$

$$A_{12} = A_{21} = \xi_{12}/D$$

where

$$D = \epsilon_1\epsilon_2 - \xi_{12}^2 . \quad 4.29$$

For three levels the result is:

$$DA_{11} = \epsilon_2\epsilon_3 - \xi_{23}^2, \text{ etc.} \quad 4.30$$

$$DA_{12} = -\xi_{12}\epsilon_3 + \xi_{13}\xi_{23}, \text{ etc.},$$

where

$$D = \epsilon_1\epsilon_2\epsilon_3 - \epsilon_1\xi_{23}^2 - \epsilon_2\xi_{13}^2 + 2\xi_{12}\xi_{13}\xi_{23} . \quad 4.31$$

Differential Cross Section Expansion Coefficients B_L

It has been shown (B152) that the coefficients B_L in the expansion of the differential cross section as a sum of Legendre polynomials may be expressed in the form:

$$\begin{aligned}
B_L(\alpha's'; \alpha s) = & \frac{\pi^2(-1)^{s'-s}}{4(2l+1)(2l'+1)} \sum \bar{z}(l_1 J_1 l_2 J_2; sL) \bar{z}(l_1' J_1' l_2' J_2'; s'L) \\
& \times \operatorname{Re} \{ (\delta_{\alpha' \alpha} \delta_{s' s} \delta_{l_1' l_1} - U_{\alpha' s' l_1'}^{J_1^{l_1}}; \alpha s l_1)^* \\
& \times (\delta_{\alpha' \alpha} \delta_{s' s} \delta_{l_2' l_2} - U_{\alpha' s' l_2'}^{J_2^{l_2}}; \alpha s l_2) \}
\end{aligned} \tag{4.32}$$

where the sum is over $J_1 J_2 l_1 l_2 l_1'$ and l_2' and π is the inverse of the wave number. All sums in equation 4.32 are unrestricted and go from 0 to ∞ ; however, in practice only one of these sums is really infinite (say the sum over J_1). The other five sums are finite because of the selection rules for non-vanishing $\bar{z}$ coefficients (B152). In practical calculations the sum over J_1 is restricted by the number of states contributing to the cross section and is therefore not infinite. The $\bar{z}$ coefficients of equation 4.32 are not the z coefficients of Blatt and Biedenharn (B152) but are related to them by the following:

$$\bar{z}(l_1 J_1 l_2 J_2; sL) = i^{l_1 - l_2 - L} z(l_1 J_1 l_2 J_2; sL) \tag{4.33}$$

and are defined (La58) by the following:

$$\bar{z}(l_1 J_1 l_2 J_2; sL) = \hat{l}_1 \hat{l}_2 \hat{J}_1 \hat{J}_2 (l_1 l_2 00 | L 0) W(l_1 J_1 l_2 J_2; sL) \tag{4.34}$$

where $\hat{x} = (2x+1)^{1/2}$ and $W(l_1 J_1 l_2 J_2; sL)$ is a Racah coefficient (Ra42). The Clebsch-Gordan coefficient in equation 4.34 further reduces the terms in the sum of equation 4.32 with the following restrictions (B152):

$$l_1 + l_2' - L = \text{even and } l_1' + l_2 - L = \text{even.} \tag{4.35}$$

Once the scattering matrix is known the sum in equation 4.32 can easily be performed to fit the experimental differential scattering cross

sections.

Differential Polarization Expansion Coefficients C_L

It has been shown that the coefficients C_L in the expansion of the differential polarization as a sum of normalized associated Legendre polynomials for spin 1/2 particles is given by (Si53):

$$\begin{aligned}
 C_L = & \frac{\lambda^2}{4(2I+1)} \int \hat{\ell}_1 \hat{\ell}_2 \hat{\ell}_1' \hat{\ell}_2' \hat{s}_1' \hat{s}_2' (2J_1+1) (2J_2+1) \\
 & \times (-1)^{I' - \frac{1}{2} - s + \ell_1' + J_1 - s_1'} (\ell_1 \ell_2 00 | L0) (\ell_1' \ell_2' 00 | L0) \\
 & \times W(\frac{1}{2} s_1' \frac{1}{2} s_2'; I' 1) W(\ell_1 J_1 \ell_2 J_2; SL) \\
 & \times X(J_1 \ell_1' s_1'; J_2 \ell_2' s_2'; LLI) \\
 & \times \text{Re} \{ i (\delta_{\alpha' \alpha} \delta_{\ell_1' \ell_1} \delta_{s_1' s_1} - U_{\alpha' \ell_1' s_1'; \alpha \ell_1 s}^{J_1 \pi_1})^* \\
 & \times (\delta_{\alpha' \alpha} \delta_{\ell_2' \ell_2} \delta_{s_2' s_2} - U_{\alpha' \ell_2' s_2'; \alpha \ell_2 s}^{J_2 \pi_2}) \} \quad 4.36
 \end{aligned}$$

where the sum is over $J_1 J_2 \ell_1 \ell_2 \ell_1' \ell_2' s_1' s_2'$ s and L , and $X(\dots)$ is a Wigner 9-J coefficient (Ro57). It has been shown in the derivation of equation 4.36 (Si53) that the polarization is always normal to the scattering plane. All other selection rules follow from the requirements for the nonvanishing of the Racah and Clebsch-Gordan coefficients and are listed here for convenience:

- 1) If only channel spin 0 is effective for the final channel spin or if there is no spin-orbit coupling, the polarization is zero.
- 2) Polarization results from the interference of different final channel spins contributing to the reaction. Hence,

the polarization is zero if there is only a single nonzero element of the scattering matrix.

- 3) If there is a largest effective incident orbital wave l , final orbital wave l' or total angular momentum J , there will be a largest value of L in equation 4.36 given by the simultaneous conditions:

$$L \leq 2l; 2l'; 2J. \quad 4.37$$

In addition, L must be even if the interfering states have the same parity.

Two special cases of rule three are:

- 1) If only S waves are effective in the reaction, for either the incident or final states, the polarization vanishes.
- 2) If only levels of the compound nucleus having $J = \frac{1}{2}$ and a single parity (or $J = 0$ with any parity) are effective in the reaction, the polarization vanishes.

Again, once the scattering matrix has been calculated the sum in equation 4.36 can be easily calculated.

$^{10}\text{B}(n, \alpha_0)^7\text{Li}$ and $^{10}\text{B}(n, \alpha_1)^7\text{Li}^*(0.48)$ Cross Sections

The cross sections for the $^{10}\text{B}(n, \alpha)$ reactions are calculated (La71) from

$$\sigma(n, \alpha_x) = \frac{\pi}{k_n^2} \sum g_J |W_{n, \alpha_x}^{J^\pi}|^2 \quad 4.38$$

where g_J is the statistical factor, x represents either α_0 or α_1 and the sum is over all J^π contributing to the cross sections. This can again be easily determined once the W 's are calculated from equation 4.21.

Parametrization of the Problem

Contributions to the differential scattering cross section, differ-

ential polarization and the $^{10}\text{B}(n,\alpha)$ cross sections for neutrons incident on ^{10}B were calculated from all states in $^{11}\text{B}^*$ formed by neutrons with $l < 3$. Table 4.1 gives a summary of the spins and parities and also the orbital angular momenta in the four channels for all these states.

The parameters used to fit all the available data for these states are:

- 1) ERES_λ : the neutron resonance energy (lab system) for level λ ,
- 2) $\Gamma_{\lambda c}$: the center-of-mass partial width for level λ and channel c ,
- 3) $\text{SIGN}_{\lambda c}$: the sign ($\pm$) of the reduced width amplitude for level λ and channel c , and
- 4) R_{cc}^0 : the elements of the background matrix.

Parameters which are constant for the whole calculation are the masses of the reaction particles in the four channels and the channel radii. For the two neutron channels the radius used was 4.14 F and for the two α -particle channels a radius of 6.0 F was used. The Q values for the reaction particles in the different channels were also used as a constant input.

Boundary conditions are calculated by the following equation:

$$b_c = -S_c \big|_{E=\text{ERES}_1(\text{COM})} \quad 4.39$$

and if there is a second level it must use these boundary conditions. In the event of a bound state with respect to neutron emission the b_c are calculated at the resonance energy of the scattering state if there are two levels. If there is only one level and it is bound, equation 4.39 is used. Negative energy neutron shift functions are calculated (La58)

TABLE 4.1: Possible States in ^{11}B from $^{10}\text{B} + n$ for $\ell_n < 3$

J^π	ℓ_n	s_n	$\ell_{\alpha_0} (s_{\alpha_0} = \frac{3}{2})$	$\ell_{\alpha_1} (s_{\alpha_1} = \frac{1}{2})$
$\frac{5}{2}^+$	0	$\frac{5}{2}$	1	3
$\frac{7}{2}^+$	0	$\frac{7}{2}$	3	3
$\frac{3}{2}^-$	1	$\frac{5}{2}$	0	2
$\frac{5}{2}^-$	1	$\frac{5}{2}, \frac{7}{2}$	2	2
$\frac{7}{2}^-$	1	$\frac{5}{2}, \frac{7}{2}$	2	4
$\frac{9}{2}^-$	1	$\frac{7}{2}$	4	4
$\frac{1}{2}^+$	2	$\frac{5}{2}$	1	1
$\frac{3}{2}^+$	2	$\frac{5}{2}, \frac{7}{2}$	1	1
$\frac{5}{2}^+$	2	$\frac{5}{2}, \frac{7}{2}$	1	3
$\frac{7}{2}^+$	2	$\frac{5}{2}, \frac{7}{2}$	3	3
$\frac{9}{2}^+$	2	$\frac{5}{2}, \frac{7}{2}$	3	5
$\frac{11}{2}^+$	2	$\frac{7}{2}$	5	5

from known functions of ρ_0 . Negative energy neutron penetrabilities are 0.

The reduced width amplitudes are calculated from the following:

$$\gamma_{\lambda c} = \text{SIGN}_{\lambda c} (\Gamma_{\lambda c} / 2P_c |_{E=\text{ERES}})^{1/2}. \quad 4.40$$

In the event of a bound state the reduced width was directly input for all channels with the penetrability equal to zero.

The energy eigenvalues, E_λ , are calculated, for the case of a single level, by use of the following condition:

$$\text{Re}(\epsilon_\lambda) |_{E=\text{ERES}_\lambda(\text{com})} = 0. \quad 4.41$$

Then E_λ is calculated from equation 4.26 with the condition of equation 4.41 satisfied:

$$E_\lambda = E + \text{Re}(\xi_{\lambda\lambda}) |_{E=\text{ERES}_\lambda(\text{com})} \quad 4.42$$

For the case of two levels E_1 is calculated from equation 4.42 and a starting value for E_2 is also calculated from equation 4.42. Then the resonance condition, namely that the real part of D (equation 4.29) is a minimum, must be satisfied. In order to satisfy the resonance condition, ϵ_1 is calculated at $E = \text{ERES}_2$. Then the starting value of E_2 is used to calculate a new quantity, E_2' . This quantity E_2' is a mesh of energies at 1 keV intervals with the starting value of E_2 the middle element. For every value of E_2' there is a corresponding value of ϵ_2 and D . The value of E_2' that minimizes the real part of D is used as the energy eigenvalue for level 2. This insures that both the boundary and resonance conditions are satisfied for both levels.

Not all the states listed in table 4.1 are resonant states. Some

states contribute to the cross sections only through the hard sphere phase and the possible inclusion of an R^0 background.

Elements of the Background R^0 -Matrix

The choice of the background R^0 -matrix elements of the states used in this problem is not arbitrary. A small digression on notation for the R^0 -matrix is in order here. The elements of the R^0 -matrix will be referred to as R_{11}^0 , R_{24}^0 , etc. where the subscripts refer to the particular channels. Channels 1 and 2 are the neutron channels of channel spin $5/2$ and $7/2$ and channels 3 and 4 are the α_0 and α_1 channels. The choice of the diagonal elements of the background matrix will be discussed first.

There is a large s-wave background present throughout the energy region of interest as can be seen in the coefficient B_0 of Fig. 3.9. A value of $R_{11}^0 = 0.3$ was necessary in the $5/2+$ s-wave state to keep B_0 near the data in the fitting. The large s-wave background from both s-wave states interferes with the hard-sphere contribution of the p-wave states to give a value of B_1 which is approximately 4 times the size of the data at 3.5 MeV. The d-wave background due to hard-sphere contribution interferes with the s-wave background and this shows up in the coefficient B_2 and also interferes with the p-wave background and this also enhances the value of B_3 . It was necessary to include the following diagonal elements of the R^0 -matrices for the p-wave states:

- 1) $3/2^-$ state: $R_{11}^0 = .2$,
- 2) $5/2^-$ state: $R_{11}^0 = .2$ and $R_{22}^0 = .1$,
- 3) $7/2^-$ state: $R_{11}^0 = R_{22}^0 = .2$, and
- 4) $9/2^-$ state: $R_{22}^0 = .5$.

These choices for the diagonal background matrix elements have the effect of negating the hard-sphere contribution over the whole energy range considered. Fits to the data with these background terms are better at higher energies than at the low energies. This is to be expected because the background matrix elements were chosen so that the data and fits to the data correspond at the higher energies. It would be surprising indeed if one choice of a non-energy dependent background were to have no adverse effects over such a large energy range. The diagonal elements R_{33}^0 and R_{44}^0 were set equal to zero. These terms would be used in fits to α -particle scattering data and have no effect on the present results.

The choice of off-diagonal background matrix elements is dependent on the large non-resonant background present in the (n,α) data and only $R_{n,\alpha}^0$ off-diagonal elements can significantly contribute to this background. The form of equation 4.38 for $^{10}\text{B}(n,\alpha_x)$ reactions is such that the resonances contribute very little to the off-resonance (n,α) cross sections. This means that a sizeable background must be constructed in order to fit all the data over the whole energy range. This background can come from any of the 12 possible states. However, the $R_{n,\alpha}^0$ background elements can be selectively chosen for their interference effects with a resonance in the same state. If one wishes to construct a higher cross section at energies below the resonance one inserts a background that simulates a resonance at some higher energy. The sign of the background matrix elements are then positive as can be seen from the general form of the R-matrix of equation 4.3. When a higher cross section is necessary at energies above the resonance energy of the particular state, negative elements in the background matrix are needed

to simulate the effect of a state at an energy lower than the resonance energy of the resonant state. The large cross section between the resonances at 0.53 MeV and 1.8 MeV can be fit with negative background matrix elements in the $5/2^-$ state at 0.53 MeV and positive background matrix elements for the p-wave state at 1.8 MeV. The d-wave state at 1.8 MeV then requires negative background matrix elements to enhance the cross section at energies higher than the d-wave resonance energy. The remainder of the non-resonant (n,α) cross section comes from off-diagonal background matrix elements due to non-resonant states. The only states which were not considered for off-diagonal background contributions were the two s-wave states. This was done so that the parameters used in the previous analysis (La71) would be preserved. This assured that the fit to the thermal (n,α) cross sections would remain the same.

The off-diagonal background matrix elements R_{12}^0 and R_{34}^0 were set equal to zero because no valid reason was found to change them. The element R_{34}^0 would have an effect on calculations of the $\text{Li}(\alpha,\alpha')\text{Li}^*$ cross section and does not effect this calculation.

Data Fitting

The data that was fitted in this work is the present scattering data, the elastic scattering and polarization data of the previous analysis (La71), the polarization data at $E_n = 2.63$ MeV (Co73), and the $^{10}\text{B}(n,\alpha)^7\text{Li}$ and $^{10}\text{B}(n,\alpha_1)^7\text{Li}^*(.48)$ reaction data. The question arises, what is the true (n,α) data? There have been conflicting experiments done with this reaction and there are numerous measurements on the (n,α) cross sections below an energy of 1 MeV. However, there has been only one measurement of these cross sections over a large energy region (Da61)

but the absolute value of these cross sections does not seem to be correct (Ir67). The value of the (n,α) cross sections used in this work are those in the compilation of Irving (Ir67). However, this compilation only gives 26 energy points in the region studied in this work, so it was found necessary to take the data of Davis (Da61) and adjust this data until it reproduced the evaluated cross sections of Irving. The adjustment of the absolute magnitude of the Davis data was less than or equal to 75 mb for neutron energies up to 5 MeV. This adjusted data of Davis then gave a continuous set of data points which were judged to be correct.

The R-matrix fitting of only the present differential cross section data was judged a severe handicap in gaining valuable information about the level structure of ^{11}B in this energy region. The additional constraints imposed by the (n,α) reaction data and the polarization data as well as the low energy data of Lane lend considerable strength to the conclusions derived from this work. It was for this reason that all the known reaction and scattering data were brought into this problem.

The resonances seen in $^{10}\text{B} + n$ will be discussed individually in the next sections.

State at $E_x = 11.94$ MeV ($E_n = 0.53$ MeV)

This state will be treated briefly because the spin assignment has been made in the previous analysis (La71). Figs. 4.1 and 4.2 show this as a bump in B_1 and C_1 at 0.53 MeV. In the previous analysis all the neutron width was in the $7/2$ neutron channel. In the present analysis 0.001 MeV width was put into the $5/2$ neutron channel. The previous

analysis also had all the α -particle width in the α_0 channel. In fits to the data in the present analysis it was found necessary to put about half the α width in the α_1 channel. There is also evidence that this is the case from recent $^{10}\text{B}(n,\alpha)$ angular distribution data (Se72). The preliminary analysis of this data indicates that there is α_1 strength in the angular distributions (Se72a). Negative off-diagonal background matrix elements were necessary to keep the (n,α) cross sections from departing from the data at about 1 MeV. All the fits to the data shown in this chapter have these parameters included for this state. The R-matrix parameters describing this state are given in Appendix III. The changes made to the parameters since the previous analysis do not affect the spin assignment of this state.

State at $E_x = 13.12$ MeV ($E_{\alpha} = 1.83$ MeV)

A discussion of the history of this state is given in the first chapter. Figure 3.9 shows this resonance as a bump in B_1 at about 1.8 MeV. This resonance in B_1 can be fit only with a p-wave state. Also a p-wave state does not reproduce the rise seen in B_2 at this energy. The conclusion is that there are two closely spaced overlapping resonances here of opposite parity.

Spin values of $3/2^-$ to $9/2^-$ were used in attempted fits to the data. A spin of $3/2^-$ suffers from the fact that neither the height of the (n,α) cross sections can be reproduced at the resonance energy nor can the height of the resonance in B_1 be reproduced. For this reason a possible spin of $3/2^-$ for this state was discarded. A spin value of $5/2^-$ will fit the differential cross section data, the polarization data, but not the (n,α) data. The reason for this is that there is

another $5/2^-$ state at 0.53 MeV. It was found that for there to be constructive interference between these two levels of the same spin and parity the sign of the reduced width amplitude in the neutron channels must be opposite. Even with this enhancing the cross section between the resonances it was found that negative off-diagonal background matrix elements were necessary for these states so that a reasonable fit to the (n,α) data between the two resonances could be obtained. However, then the peak height at 1.8 MeV for the (n,α) cross sections dropped. At energies higher than the 1.8 MeV resonance the (n,α) cross section dropped severely from the data and no variation of parameters for these or the other states used in the problem could give an acceptable fit to all the data. The results of fits to the data with a $5/2^-$ state at 1.83 MeV with parameters for the other states which were considered best is shown in Figs. 4.1 and 4.2. The fit to the elastic scattering coefficients B_L shown in Fig. 4.1 is good. In Fig. 4.2 the fit to the coefficients C_L also reproduces the data well. However, the fit to the (n,α) cross sections for neutron energies $1.5 \text{ MeV} < E_n < 2.5 \text{ MeV}$ does not reproduce the data well. The conclusion from these attempts to fit the data with a $5/2^-$ state at $E_x = 13.12 \text{ MeV}$ is that the spin of this state is not $5/2$.

Since the possible $3/2^-$ and $5/2^-$ assignments for this state have been removed from further consideration, the values of $7/2^-$ and $9/2^-$ were used in subsequent fits. Both of these values will fit the (n,α) data because neither of these states will interfere with the $5/2^-$ state at $E_n = 0.53 \text{ MeV}$ for these cross sections. These (n,α) cross sections can be well reproduced by including positive off-diagonal background matrix elements in the $7/2^-$ or $9/2^-$ p-wave state at $E_n = 1.83 \text{ MeV}$ to

FIGURE 4.1 R-Matrix Fit: $5/2^-$ at 1.83 MeV

FIGURE 4.2 R-Matrix Fit: $5/2^-$ at 1.83 MeV.

FIGURE 4.3 R-Matrix Fit: $7/2^-$ at 1.83 MeV.

FIGURE 4.4 R-Matrix Fit: $7/2^-$ at 1.83 MeV.

CROSS SECTION (B)

COEFFICIENTS (B/SR)

NEUTRON LAB ENERGY (MEV)

enhance the (n,α) cross sections at energies lower than the resonance energy. Negative off-diagonal background matrix elements are used in the $5/2^-$ p-wave resonance at $E_n = 0.53$ MeV to stop the (n,α) cross sections from dropping too low between this state and the p-wave state at $E_n = 1.83$ MeV. The final background adjustment was the inclusion of negative off-diagonal background matrix elements in the d-wave state at $E_n = 1.88$ MeV. This served to keep the cross section near the data at energies higher than this resonance. Fig. 4.4 shows the fits to the (n,α) cross sections for an assignment of $7/2^-$ for this state. Figs. 4.6 and 4.8 show fits to these cross sections for a $9/2^-$ assignment for the state at $E_n = 1.83$ MeV with $5/2^+$ and $7/2^+$ states, respectively, at $E_n = 1.88$ MeV. The elastic scattering data is well reproduced when either $7/2^-$ or $9/2^-$ is used for the p-wave resonance as can be seen in Figs. 4.3, 4.5 and 4.7. However, the fit to the coefficient B_3 is much better for the $9/2^-$ case. This is due to the interference of the p-wave state with the $11/2^+$ state at $E_n = 2.82$ MeV.

The difference between the $7/2^-$ and $9/2^-$ assignment is evident in fits to the polarization data. Fig. 4.4 shows this when a $7/2^-$ state is tried at $E_n = 1.83$ MeV. The errors on the polarization data are about 1.5 times the size of the points. The discrepancy between the fit and the data in the coefficient C_1 is consistently three to four times the error on the data points for neutron energies greater than 0.75 MeV. This is due to the way that the $7/2^-$ state interferes with the s-wave backgrounds. This interference is seen in a large energy region because the s-wave background is large throughout the energy region of the calculation. Changes in the s-wave background would mean that the total scattering cross section ($4\pi B_0$) would not be fitted. Changes in the

p-wave background would cause the fit to B_1 to be too high. No improvement was seen varying the neutron widths in the two channel spins for this state. Interference of this $7/2^-$ state with the $11/2^+$ state at $E_n = 2.82$ MeV at the 2.82 MeV resonance was also not strong enough to reproduce the C_3 data in this region. However, a $9/2^-$ assignment for the p-wave state reproduces all the data well as can be seen in Figs. 4.5 through 4.8. Figs. 4.5 and 4.6 are fits with a $5/2^+$ state at $E_n = 1.88$ MeV and Figs. 4.7 and 4.8 have a $7/2^+$ state included at $E_n = 1.88$ MeV. All other parameters in these best fits to the data are those considered to describe the structure of ^{11}B best and are listed in tabular form in Appendix III. A discussion of this state is given later.

State at $E_x = 13.17$ MeV ($E_n = 1.88$ MeV)

This state is a positive parity state formed by d-wave neutrons. It shows up as a rise in the coefficient B_2 . A negative parity p-wave state will not reproduce the rise in B_2 . An s-wave state will not give this rise either. Because this state is formed by d-wave neutrons the spin of this state is $1/2^+ \sim 11/2^+$. Spin values of $3/2^+$ and $1/2^+$ could not produce enough peak height in either B_2 or the (n,α) cross sections. A spin of $11/2^+$ does not fit the (n,α) data well because of interference with the $11/2^+$ state at 2.82 MeV. Spin values of $5/2^+$, $7/2^+$ and $9/2^+$ reproduce all the data well. Fits to the data with a $5/2^+$ state at 1.88 MeV are shown in Figs. 4.5 and 4.6. Fits to the data with a $7/2^+$ state at 1.88 MeV are shown in Figs. 4.7 and 4.8. A discussion of these possible spin assignments for this state follows in the next section.

FIGURE 4.5 R-Matrix Fit: $9/2^-$ at 1.83 MeV, $5/2^+$ at 1.88 MeV,
 $11/2^+$ at 2.82 MeV, and $3/2^+$ at 4.2 MeV.

Final parameters are given in Appendix III.

FIGURE 4.6 R- Matrix Fit with Parameters of Fig. 4.5.

CROSS SECTION (B)

COEFFICIENTS (B/SR)

FIGURE 4.7 R-Matrix Fit: $9/2^-$ at 1.83 MeV, $7/2^+$ at 1.88 MeV,
 $11/2^+$ at 2.82 MeV, and $5/2^+$ at 4.2 MeV.

Parameters are given in Appendix III.

FIGURE 4.8 R- Matrix Fit with parameters of Fig. 4.7.

Discussion of States at $E_x = 13.1$ MeV

For an interpretation of the results of the R-matrix fits to these two states additional information is needed from other reactions. Inelastic neutron decay to the 1^+ first excited state from the reaction $^{10}\text{B}(n, n'\gamma)^{10}\text{B}$ (Da60) shows a peak at about 1.8 MeV as can be seen in Fig. 4.9. From this information one can conclude that this channel must be observable. These inelastic neutrons have an energy of approximately 1 MeV and it would be surprising if they have an orbital angular momentum greater than 2 units. For an orbital angular momentum of 3 the penetrability is 0.001 and the reduced width is then 34 MeV which certainly exceeds the Wigner limit of 4.0 MeV for a radius of 4.14 F. If the orbital angular momentum of these inelastic neutrons is less than three the spin of the state in ^{11}B contributing to this decay is less than $7/2^-$ if it is a negative parity state and less than $9/2^+$ if it is a positive parity state.

The $^{10}\text{B}(n, \alpha)$ reactions (Da61) show that these overlapping states in ^{11}B decay almost entirely to the $3/2^-$ ground state of ^7Li with a small width in the decay to the $1/2^-$ first excited state of ^7Li . An orbital angular momentum of less than five units for the α -particles implies that the spin is less than $11/2^+$ for the d-wave state. All the negative parity states have an α -particle orbital angular momentum less than 5. The $^7\text{Li}(\alpha, n_0)^{10}\text{B}$ and $^7\text{Li}(\alpha, n_1)^{10}\text{B}$ (.717) reaction data (Va72) are shown in Fig. 4.10. Note that the n_0/n_1 ratio is at least 3/1 which agrees with the inelastic and elastic scattering data. Note also that the n_1 peak seems displaced in energy from the n_0 peak. This displacement is toward a higher energy.

FIGURE 4.9 $^{10}\text{B}(n,n'\gamma)^{10}\text{B}(.717)$ Data (Da60).

FIGURE 4.10 Neutron Groups from ${}^7\text{Li}(\alpha, n){}^{10}\text{B}$ (Va72).

A spin assignment of $9/2^-$ for the negative parity state implies that the inelastic neutron decay originates from the d-wave state because the Wigner limit would be exceeded for the inelastic neutrons of $l = 3$ from the $9/2^-$ state. The α -particle decay to both the ground and first excited states of ${}^7\text{Li}$ is open with $l_\alpha = 4$. The spin of the positive parity d-wave state must then be $5/2^+$ or $7/2^+$ which both have $l_n' = 2$ for the inelastic neutrons. The penetrability for these neutrons is 0.04 and then the Wigner limit is not exceeded. A spin of $9/2^+$ would require $l_n' = 4$ and the reduced width would then exceed the Wigner limit. This would rule out a possible $9/2^+$ assignment for the d-wave state. A spin of $5/2^+$ would allow the α_0 channel to decay by $l_{\alpha_0} = 1$ and the α_1 channel to decay by $l_{\alpha_1} = 3$. This would mean a penetrability ratio for α_0/α_1 of 3.46/1.82 which does not explain the large α_0/α_1 ratio observed in the (n,α) cross sections. A spin of $7/2^+$ would not explain this difference either because both α -particle channels would have $l_\alpha = 3$ and the penetrability difference would be small.

A $7/2^+$ state has been seen in ${}^{11}\text{C}$ at $E_x = 12.65$ MeV in the ${}^{10}\text{B}(p,\alpha){}^7\text{Be}$ reaction (Je64). However, this state has been shown to be formed primarily by $l = 0$ protons because the α -particle angular distribution on this resonance is almost isotropic. It could, however, be formed by $l = 0$ and $l = 2$ protons and then could correspond to the d-wave resonance seen in this work. An s-wave resonance in this work will not fit the data at all. Formation of this state by a mixture of $l = 0$ and $l = 2$ neutrons is possible but is beyond the scope of this work because of programming considerations.

In conclusion, a spin assignment for the 13.12 MeV level of $9/2^-$

and $5/2^+$ or $7/2^+$ for the 13.17 MeV level gives no major discrepancies with any of the known reactions with $^{10}\text{B} + \text{n}$.

State at $E_x = 14.0$ MeV ($E_n = 2.82$ MeV)

The prominent feature of this state is the large value of B_2 . Fits to this state with s-wave and p-wave states did not reproduce the data at all. Fits were attempted with d-wave states with spin $1/2^+ - 11/2^+$. The only spin values that gave any reasonable peak height in B_2 were $9/2^+$ and $11/2^+$. A fit to the data with a spin of $9/2^+$ is shown in Figs. 4.11 and 4.12. As can be seen, the (n, α) cross sections are well reproduced but the fit to C_1 , B_2 , and B_3 is not very good. An attempt to improve this fit was made by using two levels of $9/2^+$, one at 1.88 MeV and the other at this resonance. This did not give any improvement to the fit.

Fits to the data for a spin value of $11/2^+$ are shown in Figs. 4.5 - 4.8. This was judged to give a good representation of the data. The assignment of $11/2^+$ for this state agrees with the inelastic data shown in Fig. 4.9. Inelastic neutrons would not be expected to be seen with $l_n = 4$. The penetrability is of the order of 10^{-4} and the Wigner limit would definitely be exceeded if they were observed. The α -particles have orbital angular momentum = 5 for both channels and the α -width is small. This agrees with the $^{10}\text{B}(\text{n}, \alpha)$ data and also with the $^7\text{Li}(\alpha, \text{n})^{10}\text{B}$ reaction data (Me63) which shows a small bump at this state. The d-wave state at 13.17 MeV shows a large resonance in the $^7\text{Li}(\alpha, \text{n})^{10}\text{B}$ reaction of the order of 10 times the size of the resonance of the 14.0 MeV state. The inhibition in the 14.0 MeV state in this reaction is certainly not in the $^{10}\text{B} + \text{n}$ channel as can be seen by the size of

FIGURE 4.11 R-Matrix Fit: $9/2^+$ at 2.82 MeV.

FIGURE 4.12 R-Matrix Fit: $9/2^+$ at 2.82 MeV.

B_0 at the two resonances. The α -particle penetrability ratio for the 14.0 and 13.17 MeV levels is 4.37 for a $5/2^+$ state at 13.17 MeV and 2.53 for a $7/2^+$ state at 13.17 MeV. This is a step in the right direction but does not completely explain the large peak height found in the (n,α) reaction for the 13.1 states and the small height for the 14.0 MeV state. There is, however, no conflict with any data for a spin assignment of $11/2^+$ for the 14.0 MeV level.

State at $E_x = 15.2$ MeV ($E_n = 4.2$ MeV)

This state shows up in the elastic scattering data as a bump in B_0 and B_2 . It could arise due to the interference of an s-wave state with the large d-wave background present here. However, when a fit with an s-wave for this resonance is attempted there is a large interference dip due to interference with the large s-wave background present. A fit with a negative parity p-wave state does not reproduce any of the B coefficients.

Formation by d-wave neutrons gives a qualitative representation of the data. A spin of $11/2^+$ gives too high a value of B_2 and a spin of $1/2^+$ gives too small a peak. No systematic attempt to fit this resonance was made.

This state also shows up in the inelastic neutron decay to the first excited state of ^{10}B shown in Fig. 4.9. The resonant part of this cross section is about 60 mb (Da60) out of a total of approximately 400 mb. If the total width is 700 keV this implies that the inelastic width is about 100 keV. For a state with spin greater than $7/2^+$ this means that $l_n' = 4$ and the penetrability is 0.002 which means that the reduced width is 25 MeV which exceeds the Wigner limit. This would imply that

this state has a spin smaller than $9/2^+$ which would imply that the spin of this state is $3/2^+$, $5/2^+$, or $7/2^+$.

Conclusions

The analysis performed in this work is the only comprehensive treatment of all the significant reaction data the author has seen. The constraints imposed by fitting all the available data strengthen the conclusions drawn from this analysis. The R-matrix parameters for both the resonant and the non-resonant states considered in this problem are given in Appendix III.

The results of the two-level, four-channel R-matrix analysis indicate that there is a state of spin $9/2^-$ formed by $\ell = 1$ neutrons at an excitation energy of 13.12 MeV, a state of spin $5/2^+$ or $7/2^+$ at 13.17 MeV formed by d-wave neutrons, a state of spin $11/2^+$ formed by d-wave neutrons at 14.0 MeV and a state of spin $3/2^+$, $5/2^+$ or $7/2^+$ also formed by d-wave neutrons in ^{11}B . These results are consistent with all known nuclear data on this nucleus.

In conclusion, the author wishes to point out what should be done in the future to resolve any ambiguities remaining from this work. A precision α -particle angular distribution measurement for either or both of the reactions $^{10}\text{B}(n,\alpha)^7\text{Li}$, $^{10}\text{B}(n,\alpha_1)^7\text{Li}^*(.48)$ would further clarify the results for the two overlapping resonances seen at 13.1 MeV. An angular distribution measurement of the neutron groups to the first excited state of ^{10}B for the reactions $^{10}\text{B}(n,n')^{10}\text{B}(.717)$ and $^7\text{Li}(\alpha,n)^{10}\text{B}(.717)$ would also give valuable information about this set of states. A neutron polarization measurement at a few energies near the 14.0 MeV resonance could strengthen the spin assignment of this

state with a quick check to see when the coefficient C_2 crosses zero.

The results of this work should certainly give nuclear theorists a better insight into the structure of ^{11}B and hopefully will inspire them to new calculations in this energy region.

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APPENDIX I

The following is a table of the coefficients R_L in the expansion of the differential cross section as a sum of Legendre polynomials. The coefficients are given in the center-of-mass system. The error for a particular coefficient is given below the value of the coefficient. The neutron energy is given in the laboratory system.

CENTER OF MASS B COEFFICIENTS AND EXPONENTS (M²/g)

FLAG (M ² /g)	B(0)	B(1)	B(2)	B(3)	(4)
1.555	119	43	34	1	7
	2	4	6	7	-
1.5501	127	46	36	-4	11
	3	5	6	7	-
1.540	129	54	38	10	-
	3	5	6	8	1
1.531	129	78	53	6	-
	3	5	7	8	1
1.520	123	74	56	1	17
	3	5	7	-	-
1.500	132	80	61	-11	1
	3	6	7	5	-
1.481	117	81	70	3	17
	3	5	7	5	-
1.472	125	79	102	-6	1
	3	6	7	5	-
1.455	118	75	61	12	25
	3	5	7	5	-
1.425	114	68	92	0	-
	3	5	7	5	-
1.398	115	77	60	6	-
	3	5	7	5	-
1.364	125	74	57	-10	16
	3	5	7	5	-
1.301	122	53	56	-3	-
	2	5	6	-	-
1.250	125	47	120	-25	6
	3	6	7	5	-
1.250	124	42	128	-7	10
	3	6	7	5	-
2.10	132	37	152	-5	15
	3	6	8	5	-
1.1	147	40	159	5	17
	3	7	9	5	10
1.0	149	43	196	13	20
	3	7	9	5	10
1.0	158	45	227	27	24
	3	8	10	10	11
1.070	159	54	242	31	20
	3	8	10	10	10
1.0701	161	51	245	30	21
	3	8	10	10	10
1.070	155	59	242	43	20
	3	8	10	5	10
1.0701	155	57	250	50	17
	3	8	10	5	10

COEFFICIENTS CONTINUED

$L\lambda$ (Å)	$a(0)$	$b(1)$	$c(2)$	$d(3)$	$e(4)$
2.900	152	8	245	70	10
	3	0	10		10
2.900	153	55	250	61	10
	3	7	9	3	
2.952	143	65	230	63	10
	3	7	8	6	
3.002	132	58	209	66	11
	3	0	8	5	
3.052	127	58	197	63	12
	3	5	7	7	
3.102	120	61	178	63	0
	3	6	7	7	
3.153	113	60	180	62	12
	2	6	7	7	
3.204	111	56	169	57	0
	2	5	7	7	
3.303	107	61	153	68	-1
	3	5	6		7
3.433	98	72	131	54	0
	2	5	6		
3.500	93	72	124	47	-2
	3	5	6	7	7
3.550	83	68	120	40	1
	3	5	6		
3.657	73	63	121	34	3
	2	5	6		7
3.717	69	67	110	31	0
	2	4	5	6	
3.800	55	71	110	42	13
	2	4	5		6
3.903	50	73	125	47	10
	2	5	6		
4.000	43	85	142	52	34
	2	5	7	7	7
4.100	101	92	147	60	10
	3	6	7	7	7
4.200	115	88	153	61	
	2	6	7	7	
4.300	101	82	140	61	37
	2	6	7	7	
4.500	104	101	145	64	47
	2	6	7	3	3
4.600	95	88	151	73	37
	2	5	7	7	7

APPENDIX II

The following is a table of center-of-mass differential cross sections and their errors. The incident neutron energy is given in the laboratory system.

CENTER OF MASS CROSS SECTION AND DEFLECT (MB/SR)

ANGLE	22.0	45.9	71.3	95.3	119.3	141.5	162.0
FLAS (MeV)							
1.355	193	153	121	101	97	90	110
	12	9	7	6	5	5	-
1.401	196	173	126	112	94	103	120
	12	10	7	6	5	5	-
1.440	316	175	129	105	104	100	104
	12	10	7	6	5	5	-
1.721	255	186	138	97	83	80	101
	15	10	8	6	4	5	-
1.760	253	185	123	93	72	90	120
	15	11	7	6	4	5	-
1.800	279	212	134	92	65	94	120
	16	12	8	6	4	5	7
1.851	270	189	112	71	60	86	120
	15	11	7	5	3	5	7
1.922	283	199	118	69	69	62	140
	16	12	7	4	4	5	-
1.967	272	185	104	75	69	60	130
	15	10	6	5	4	5	7
2.013	252	182	104	64	66	67	120
	15	11	7	5	4	5	7
2.081	275	173	111	73	63	65	120
	15	10	7	5	4	5	7
2.204	265	193	125	61	67	104	124
	15	11	7	5	4	5	7
2.301	248	178	112	70	72	117	122
	14	10	6	5	4	6	-
2.390	255	184	112	58	74	137	200
	15	10	7	4	4	7	11
2.450	284	178	100	52	80	158	214
	16	10	6	4	5	3	11
2.510	306	183	94	53	65	160	240
	18	10	6	4	5	5	12
2.581	270	200	95	53	96	205	270
	22	11	6	4	5	18	12
2.622	371	205	94	57	102	195	277
	21	11	6	4	6	9	12
2.640	424	220	89	55	109	210	264
	24	12	6	4	6	10	12
2.670	443	227	83	47	108	214	202
	26	12	5	3	5	11	12
2.701	452	225	80	51	114	219	207
	26	13	5	3	6	11	12
2.750	460	225	71	42	109	209	287
	26	12	4	3	6	10	14
2.801	464	223	65	43	115	211	272
	27	13	4	3	6	10	12

CENTER OF MASS CROSS SECTION AND ERROR (MB/SR)

ANGLE	22.0	45.9	71.3	95.6	119.3	141.9	162.0
ELAB (MeV)							
2.850	452 20	217 12	53 4	42 3	115 7	210 10	277 11
2.900	464 24	217 11	60 4	44 3	113 8	213 10	277 11
2.952	437 22	221 11	58 4	39 2	106 8	182 9	231 11
3.002	408 21	195 10	54 3	39 3	103 5	170 8	214 10
3.056	360 19	198 10	50 3	40 3	96 5	163 8	187 10
3.102	372 19	192 9	55 3	37 2	92 5	148 7	177 9
3.153	372 19	178 9	53 3	33 2	89 5	144 7	177 9
3.204	341 18	172 9	49 3	32 2	86 5	136 7	167 9
3.303	327 17	159 9	54 3	31 2	86 5	123 6	157 9
3.456	313 17	160 8	55 4	34 2	86 4	100 5	100 5
3.500	308 17	164 9	62 4	35 3	88 4	99 5	105 7
3.550	287 16	158 9	55 4	31 2	58 3	93 5	107 7
3.627	276 16	151 8	59 4	30 3	57 3	77 5	111 7
3.617	269 15	133 7	49 3	29 2	49 3	72 4	107 7
3.688	265 15	142 8	52 3	30 2	49 3	74 4	107 7
3.780	300 16	143 8	50 3	29 2	44 3	81 4	107 7
4.009	336 18	164 9	57 4	30 3	50 4	70 4	101 7
4.150	351 19	176 9	58 4	37 3	55 4	70 5	117 7
4.240	371 21	169 9	57 4	42 3	51 4	69 5	124 7
4.291	352 20	157 9	58 4	47 3	59 4	80 5	121 7
4.341	398 22	164 9	57 4	52 3	58 3	74 4	105 7
4.351	330 19	163 9	49 3	47 3	57 3	67 4	107 7

APPENDIX III

The parameters for the R-Matrix fits for Figs. 4.5 - 4.8
Are given in tabular form. All widths are in MeV in the center
of mass system.

PARAMETERS OF RESONANT STATES

E_x	11.60	11.79	10.60	11.94	13.12	13.17	14.0	15.2	MeV
$J^\pi(k_n)$	$5/2^+(0)$	$7/2^+(0)$	$7/2^+(0)$	$5/2^-(1)$	$9/2^-(1)$	$5/2^+, 7/2^+(2)$	$11/2^+(2)$	$3/2^+, 5/2^+, 7/2^+(2)$	
$\zeta(\alpha_0)$	1	3	3	2	4	1, 3	5	1, 1, 3	
$\zeta(\alpha_1)$	3	3	3	2	4	3	5	1, 3, 3	
$E_\lambda(\text{com})$	0.154	0.336	-1.451	0.489	1.67	1.72	2.53	3.80	
$\Gamma(n_1)$	0.004	---	---	0.001	---	0.020	---	0.250	
$\gamma^2(n_1)$	0.006	---	---	0.003	---	0.077	---	0.229	
$\Gamma(n_2)$	---	0.770	---	0.030	0.100	0.060	0.800	0.250	
$\gamma^2(n_2)$	---	0.763	0.120*	0.093	0.078	0.232	1.456	0.229	
$\Gamma(\alpha_0)$	0.296	0.001	0.030	0.080	0.275	0.200	0.045	0.100	
$\gamma^2(\alpha_0)$	0.063	0.0004	0.054	0.019	0.110	0.046	0.026	0.011	
$\Gamma(\alpha_1)$	0.0	0.113	0.070	0.090	0.050	0.150	0.010	0.100	
$\gamma^2(\alpha_1)$	0.0	0.076	0.412	0.028	0.027	0.041	0.008	0.015	
$b(n)$	0.0	0.0	0.0	-0.733	-0.437	-1.506	-1.291	-1.025	
$b(\alpha_0)$	-0.313	-1.327	-1.327	-0.579	-1.613	-0.786	-2.389	-0.110	
$b(\alpha_1)$	-1.782	-1.641	-1.641	-0.727	-1.884	-0.930	-2.689	-0.493	
R_{11}^0	0.3	0.0	0.0	0.2	0.0	0.1	0.0	0.1	
R_{22}^0	0.0	0.0	0.0	0.1	0.5	0.1	0.0	0.1	
R_{13}^0	0.0	0.0	0.0	-12	0.0	-1	0.0	0.0	
R_{23}^0	0.0	0.0	0.0	-12	0.1	-1	0.0	0.0	
R_{14}^0	0.0	0.0	0.0	-12	0.0	-1	0.0	0.0	
R^0	0.0	0.0	0.0	-12	0.1	-1	0.0	0.0	

* The reduced width amplitude for this state is negative

PARAMETERS FOR NON-RESONANT STATES

J^π	R_{11}^0	R_{22}^0	R_{13}^0	R_{23}^0	R_{14}^0	R_{24}^0
$3/2^-$	0.2	0	0	0	0	0
$7/2^-$	0.2	0.2	0	0	0	0
$+*$	0.1	0.1	0.2	0.2	0.1	0.1
$9/2^+$	0.1	0.1	0.1	0.1	0	0

* this state is a d-wave state with spin $3/2^+$ - $7/2^+$ which did not have a resonance in the particular fit shown in one of the figures in Chapter IV

All boundary conditions were chosen to be 0 for these background states.