DISTORTED-WAVE THEORY FOR TWO-NUCLEON TRANSFER *

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The DWBA treatment of two-nucleon transfer reactions is now entering its second decade. Many laboratories have computer codes, and these are used routinely to predict the two-nucleon transfer cross-sections that are implied by calculated wave functions.

Two-nucleon transfer codes are used in a slightly different way than one-nucleon transfer codes. Suppose one has an experimental angular distribution for, say, a (d,p) reaction in which a neutron has been stripped into a $2p_{3/2}$ orbit. The standard procedure is to calculate the DWBA angular distribution for this process, and the (hopefully) constant ratio between the experimental and calculated angular distributions gives the spectroscopic strength, which is related to the matrix element of a particle creation operator between the target and residual states. This procedure gives absolute spectroscopic strengths because the one-particle DWBA calculation gives absolute cross-sections. There is some controversy about how reliable these are, but in the most favorable cases they are believed to about $\pm 20\%$.

Now let us contrast this to the way two-nucleon transfer codes are used. Suppose we have data for the reaction $^{40}\text{Ca}(t,p)^{42}\text{Ca}$, leading to the ^{42}Ca ground state. We know the reaction has occurred by the transfer

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of a pair whose total angular momentum relative to the ^{40}Ca target is zero. However, the wave function of the transferred pair will be a superposition of zero-coupled pairs associated with any of the orbits near the Fermi surface, $(1d_{3/2}^2)_{0+}$, $(1f_{7/2}^2)_{0+}$, $(2p_{3/2}^2)_{0+}$, ..., etc. Now there is no single DWBA curve, whose ratio with the data gives a spectroscopic strength. There is no such thing as a spectroscopic strength. Thus, one generally cannot use a two-particle DWBA calculation to extract spectroscopic information direct from the data. However, if we have theoretical wave functions for the target and residual states, we can calculate spectroscopic amplitudes for the transfer of each of the contributing pairs $((1d_{3/2}^2)_{0+}, (1f_{7/2}^2)_{0+}, (2p_{3/2}^2)_{0+}, \dots$ in the above example). This spectroscopic amplitude is the matrix element of a normalized pair creation operator between the target and residual states. The predicted transformation amplitude is obtained by taking a coherent superposition of DWBA amplitudes for each of these (nlj^2) pairs, each DWBA amplitude weighted by the calculated spectroscopic amplitude. In this way, theoretical wave functions yield predictions which can be compared to two-nucleon transfer data. Since the total reaction amplitude is coherent in the partial amplitudes, we have the possibility of a strong dependence of the predicted cross-section on the relative signs and magnitudes of the spectroscopic amplitudes, and thus a sensitive test of nuclear wave functions.

Generally, the result of such a calculation is an angular distribution for each of the residual states, expressed in terms of an arbitrary unit which is common to all the levels of a given nucleus or series of nuclei, but is not easily related to an absolute unit like a mb/ster. The reason

for this is that the usual DWBA treatment involves a severe approximation for the product of the interaction and heavy projectile wave function. For the (t,p) reaction, this approximation is

$$\left[V(\vec{p} - \vec{n}_1) + V(\vec{p} - \vec{n}_2) \right] \times \Theta(\vec{p} - \vec{n}_1, \vec{p} - \vec{n}_2, \vec{n}_1 - \vec{n}_2) \\ \approx D_0 \times \delta\left(\vec{p} - \frac{\vec{n}_1 + \vec{n}_2}{2}\right) \times \Theta\left(\frac{\vec{n}_2 - \vec{n}_1}{2}, \frac{\vec{n}_1 - \vec{n}_2}{2}, \vec{n}_1 - \vec{n}_2\right). \quad (1)$$

Here $V(r)$ is the proton-neutron interaction, and Θ is the triton wave function. This approximation implies that the reaction only proceeds when the proton is at the mass-center of the transferred neutrons. It is a convenient approximation, because it enables us to do two-nucleon transfer calculations using one-nucleon transfer codes. However, it is difficult to justify this approximation in terms of the important lengths and momenta involved in the two-nucleon transfer process. And it is difficult to make a reliable estimate of the strength parameter D_0 that is implied by a realistic interaction V and a realistic triton wave function Θ . If we consider transitions to several final states, it is reasonable to hope that D_0 is fairly constant, especially if roughly the same momentum transfers are involved in the different transitions. Thus, one can hope to calculate relative cross-sections, and this has been done extensively.

But what if we were to take the trouble to evaluate the integrals exactly? Would the calculated angular distributions, and the relative cross-sections for different levels, agree with the predictions of the zero-range code? And would the absolute cross-section yielded by the DWBA and reasonable nuclear wave functions agree with the data? If not, we would have to question the adequacy of DWBA, or the adequacy of what

we thought were reasonable wave functions.

I would like to describe the results of an attempt made at the University of Minnesota to evaluate two-neutron transfer DWBA integrals exactly. The early parts of this program^{1,2} were planned and executed with the help of Ricardo Broglia and recently I have collaborated with Victor Feng. We start with the usual DWBA expression for the cross-section.

$$\frac{d\sigma}{d\Omega}(\hat{k}_f) = \frac{k_f}{k_i} \frac{\mu_i \mu_f}{(2\pi \hbar^2)^2} \cdot \frac{1}{2 \times (2J_i + 1)} \quad (2)$$

$$\times \sum_{\substack{m_p \ m_t \\ M_i \ M_f}} \left| \langle \Psi_f^{(-)}(\vec{k}_f) | V | \Psi_i^{(+)}(\vec{k}_i) \rangle \right|^2$$

If we assume that the interaction responsible for the reaction involves only the proton and the transferred neutrons, then we can, as usual, immediately integrate away the coordinates of the inactive nucleons, which leaves us with the overlap functions $\psi_M^L(\vec{n}_1, \vec{n}_2)$ that contain the nuclear structure information.

$$\sum_{\sigma_1 \sigma_2} \int d\vec{\xi}_1 \dots d\vec{\xi}_{A-1} \Phi_{M_f}^{J_f}(\vec{\xi}_1, \dots, \vec{\xi}_{A-1}, \vec{n}_1, \vec{n}_2, \sigma_1, \sigma_2) \quad (3)$$

$$\times \Phi_{M_i}^{J_i}(\vec{\xi}_1, \dots, \vec{\xi}_{A-1}) \times \left[\chi^{1/2}(\sigma_1) \chi^{1/2}(\sigma_2) \right]_0^0$$

$$= \sum_L (J_i \ L \ M_i \ M_f - M_i | J_f \ M_f) \psi_{M_f - M_i}^L(\vec{n}_1, \vec{n}_2)$$

After we do partial wave expansions for the proton and triton continuum states, we are left with the job of evaluating the following 9-dimensional integral:

$$I_{\ell_t \ell_p J_p}^{L, M} = \int d\vec{p} \, d\vec{n}_1 \, d\vec{n}_2 \, f_{\ell_p J_p}(|\vec{p} - \frac{\vec{n}_1 + \vec{n}_2}{A+2}|) \, f_{\ell_t}(|\frac{\vec{n}_1 + \vec{n}_2 + \vec{p}}{3}|) \times V(|\vec{p} - \vec{n}_1|) \quad (4a)$$

$$\times \psi_{\vec{n}_1, \vec{n}_2}^{L*} \times \bigcirc_0^0(\vec{p} - \vec{n}_1, \vec{p} - \vec{n}_2, \vec{n}_1 - \vec{n}_2) \times \left[Y_{\ell_t}(\frac{\vec{n}_1 + \vec{n}_2 + \vec{p}}{3}) \, Y_{\ell_p}(\vec{p} - \frac{\vec{n}_1 + \vec{n}_2}{A+2}) \right]_M^L$$

$$= \frac{1}{2L+1} \sum_{M=-L}^L I_{\ell_t \ell_p J_p}^{L, M} \quad (4b)$$

$$= \frac{1}{2L+1} \int d\vec{p} \, d\vec{n}_1 \, d\vec{n}_2 \, X_0^0(\vec{p}, \vec{n}_1, \vec{n}_2) \quad (4c)$$

It is an awkward integral because the geometry of the problem involves several vectors (triton relative to target, proton relative to final nucleus, captured neutrons relative to proton, and to the target). However, we can at least use the fact that it is an integral of the product of angular momentum eigenstates to reduce it to a six-dimensional integral. Since $I_{\ell_t \ell_p J_p}^{L, M}$ is independent of M , we can replace it by an average over all M (Eq. 4b) and the M -summed integrand is spherically symmetric—indicated in (4c) by $X_0^0(\vec{p}, \vec{n}_1, \vec{n}_2)$. Since X_0^0 is spherically symmetric, it will be unchanged if we subject $\vec{p}$, $\vec{n}_1$ and $\vec{n}_2$ to a simultaneous rotation.

Thus, we replace $\vec{p}, \vec{n}_1, \vec{n}_2$ by three orientation variables $\varphi, \vartheta, \alpha$, and six intrinsic variables. The orientation variables $\varphi, \vartheta, \alpha$ are such that when $\varphi = \vartheta = \alpha = 0$, the mass-center of the three-particles lies along the z-axis, with neutron 1 in the x-z plane.

$$\begin{bmatrix} \vec{p} \\ \vec{n}_1 \\ \vec{n}_2 \end{bmatrix} = R(\varphi, \vartheta, \alpha) \begin{bmatrix} \vec{p}' \\ \vec{n}_1' \\ \vec{n}_2' \end{bmatrix}$$

$$\text{with } \frac{\vec{p}' + \vec{n}_1' + \vec{n}_2'}{3} = R \hat{z}$$

$$\vec{n}_1' \cdot \hat{y} = 0$$

$$\vec{n}_1' \cdot \hat{x} \geq 0.$$

Figure 1

The intrinsic variables are the lengths of the sides of the triangle, the distance of its mass center from the origin (which is the mass-center of the target nucleus), and two angles β and γ which are needed to specify the orientation of the triangle when it is in the $\varphi = \vartheta = \alpha = 0$ position. Since $X_0^0(\vec{p}, \vec{n}_1, \vec{n}_2)$ is independent of $(\varphi, \vartheta, \alpha)$, integration over those variables gives a factor of $8\pi^2$, and we are left with a six-dimensional integral (5) which must be evaluated numerically.

$$\begin{aligned}
 & \int d\vec{p} \, d\vec{n}_1 \, d\vec{n}_2 \, X_0^o(\vec{p}, \vec{n}_1, \vec{n}_2) \\
 &= 8\pi^2 \int R^2 dR \, r_{12} dr_{12} \, r_{1p} dr_{1p} \, r_{2p} dr_{2p} \, \sin\beta \, d\beta \, d\delta \\
 & \quad \times X_0^o(\vec{p}', \vec{n}_1', \vec{n}_2') \quad (5)
 \end{aligned}$$

My first attempt^{1,2} at evaluating these six-dimensional integrals involved the use of Gaussian integration on each variable, and was very time-consuming. For $L = 0^+$ transfer, where the integrand is fairly simple and l_t and l_p must be equal, about two hours of CDC 6600 time were required for a single $(nlj)^2$ calculation. Recently Victor Feng and I have increased the speed of the calculation by using the Monte Carlo method. The $L = 0$ calculation can now be done in three minutes and an $L = 2$ calculation which involves a more complicated integrand and many more integrals (since now l_t can take on the values $l_p \pm 2$ as well as l_p), requires about 15 minutes. We believe that the Monte Carlo method shows promise of great usefulness in more complicated particle-transfer calculations. These calculations involve a realistic interaction with a hard core, that fits low-energy two nucleon scattering data, and a triton wave function obtained by a variational calculation using this same interaction.³ This triton is somewhat underbound (7.6 rather than 8.5 MeV). A stronger interaction which gave the correct binding would yield a tighter triton wave function, and a slightly smaller cross-section.

The result of these calculation is that the finite-range angular distributions and relative cross-sections agree with those given by the zero-range calculation, but the absolute cross-section is too small to

fit the data. Figure 2 shows the differential cross-section calculated for the stripping of a zero-coupled pair of $2p_{3/2}$ neutrons in the ground state transition of the reaction $^{40}\text{Ca}(t,p)^{42}\text{Ca}$. The solid line corresponds to the scale in mb/ster, and is the result of the finite range calculation. The dashed line was calculated with the zero-range code TWOPAR. The arbitrary normalization factor has been chosen so that the two curves agree in magnitude. There is remarkable agreement in the shapes of the angular distributions. Agreement was also found for the ratio of cross-section for $(2p_{3/2}^2)_{0+}$, $(1f_{7/2}^2)_{0+}$, $(1d_{3/2}^2)_{0+}$, $(2s_{1/2}^2)_{0+}$ transfer.

The calculated angular distribution is in good agreement with the data of Bjerregaard, Hansen, Nathan, Chapman, Hinds and Middleton.⁴ Note, however, that the data points in Figure 2 have been scaled down by a factor of 0.65. If a reasonable amount of configuration mixing is assumed for the ground states of ^{40}Ca and ^{42}Ca , it is predicted that the pure $(2p_{3/2}^2)$ transfer should be too strong by a factor of about 0.54. Thus, it should have been necessary in Figure 1 to scale down the curve to fit the data points, rather than vice versa. The conclusion is that the finite range calculation, with a reasonable amount of configuration mixing, underestimates the differential cross-section by a factor of $0.65 \times 0.54 = 0.35$.

Figure 3a shows the prediction of the finite range calculation for the $L = 2^+$ transition to the first excited state of ^{42}Ca , and Figure 3b shows the prediction of the zero-range code TWOPAR. Again we have good agreement between the finite and zero-range calculations, both with respect to angular distributions and relative cross-section for different

$(n_1 l_1, n_2 l_2 j_2)$ transfers. Moreover, if we compare $L = 0$ and $L = 2$ cross-sections, we again get impressive agreement between the zero- and finite-range calculations.

Barz, Hehl, Riedel and Broglia⁵ have calculated relative cross-sections for the (t,p) excitation of 2^+ and 0^+ in $^{40}\text{Ca}(t,p)^{42}\text{Ca}$. They over-predict the strength of the 2^+ transition relative to the 0^+ transition by a factor of 28/17. They used a zero-range DWBA code, but we have seen above that they would have obtained roughly the same $2^+/0^+$ ratio had they done a finite range calculation. Since the theoretical absolute cross-section for the 0^+ transition is too small by a factor of 0.35, we can conclude that the theoretical absolute cross-section for the 2^+ transition is also too small, but by a more moderate factor of 0.58.

Would the calculated cross-section be brought into closer agreement with the data if inelastic effects were included? There might be some improvement but it would be surprising if we could boost the ground state transition in this way by the required factor of three. E. Rost and D. R. Kunz⁶ are studying another two-step process which may make a significant contribution to the (t,p) cross-section, namely the sequential stripping of the two neutrons:

$$(t, p) = (t, d) + (d, p).$$

The amplitude for this process would be coherent with one-step transfer and might have a sizeable effect on the calculated cross section.

Finally, I would like to report an interesting calculation by Nick Chant⁷ at Minnesota, that is in a sense intermediate between the zero range and exact finite range calculations. Chant says, let us expand $V \oplus$ in terms of multipoles as follows

$$\begin{aligned} & \left[V(\vec{p} - \vec{n}_1) + V(\vec{p} - \vec{n}_2) \right] \ominus (\vec{p} - \vec{n}_1, \vec{p} - \vec{n}_2, \vec{n}_1 - \vec{n}_2) \\ &= \sum_k \int_k \left(\left| \vec{p} - \frac{\vec{n}_1 + \vec{n}_2}{2} \right|, \left| \vec{n}_1 - \vec{n}_2 \right| \right) \left[Y^k \left(\widehat{\vec{p} - \frac{\vec{n}_1 + \vec{n}_2}{2}} \right) Y^k \left(\widehat{\vec{n}_1 - \vec{n}_2} \right) \right]_0^0 \end{aligned}$$

The finite range calculation I have described above uses the entire sum.

The zero-range calculation replaces it by the $k = 0$ term

$$D_0 \delta \left(\vec{p} - \frac{\vec{n}_1 + \vec{n}_2}{2} \right) \ominus \left(\frac{\vec{n}_2 - \vec{n}_1}{2}, \frac{\vec{n}_1 - \vec{n}_2}{2}, \vec{n}_1 - \vec{n}_2 \right).$$

Chant also restricts himself to the $k = 0$ term, but he uses the real $k = 0$ term obtained the expansion of $V \oplus$, namely

$$\frac{1}{4\pi} * \int_0 \left(\left| \vec{p} - \frac{\vec{n}_1 + \vec{n}_2}{2} \right|, \left| \vec{n}_1 - \vec{n}_2 \right| \right).$$

This is his only approximation, and the result of his calculation is in remarkable agreement with the exact calculation (Figure 4). This comparison shows that the finite range DWBA calculation is dominated by the transfer of a neutron pair in a relative s-state. Chant's s-state approximation

leads to a large saving in computer time (a factor of about 10) and will probably be very important in the finite range calculation of more complicated transfer processes.

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FIGURE CAPTIONS

- Figure 1. The configuration of the proton and two neutrons when $\phi = \vartheta = \alpha = 0$.
- Figure 2. The solid line is the result of a finite-range calculation, using optical parameters listed in reference 2 (proton set II). The dashed line is the result of a zero-range calculation using the same optical parameters. The normalization of the zero-range curve was chosen to give a good fit to the finite-range curve. The data points of reference 4 have been scaled down by a factor of 0.65.
- Figure 3. Finite-range and zero-range calculations for some L=2 transfers. The same optical parameters were used as for Figure 2.
- Figure 4. The solid line was obtained by a Monte-Carlo finite-range calculation, and the dashed line was obtained with Chant's $k = 0$ approximation. The optical parameters were the same as those used for Figure 2, except that the spin-orbit strength for the neutron well was set equal to zero.

Figure 2

Figure 3a

Figure 3b.

Figure 4